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STRUCTURAL THEOREMS FOR (m,n)-QUASI-IDEAL SEMIGROUPS *

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Abstract. The definitions of (m, n)-ideal and (m, n)-quasi-ideal on a semigroup are given in [5]. They have been studied in many papers, recently in papers [7], [8] and [9]. In this paper we introduce the notion of an (m, n)-quasi-ideal semigroup and consider some general properties of this class of semigroups. Also, we introduce the notion of an (m, n)-duo quasi-ideal semigroup and give its structural description. In section 3 we describe a class of (m, n)-quasi-ideal semigroups by matrix representation.

Keywords: (m, n)-quasi-ideal, periodic semigroup, rectangular band, power breaking partial semigroup

1. Introduction and some general properties for (m, n)-quasi ideal semigroups

Let S be a semigroup, $a \in S$, then by $< a >$ we denote *monogenic subsemiroup* on S generated by a. Either $< a >$ is isomorphic to the natural numbers N under addition or there exist positive integers r, m such that $< a >= \{a, a^2, \dots, a^r, \dots, a^{r+m-1}\}$ with $K_a = \{a^r, a^{r+1}, \dots, a^{r+m-1}\}$ a cyclic group of order m. The integers r and m are called *index* and *period* of $< a >$, respectively. A semigroup S is *periodic* if all its monogenic subsemigroups are finite. By $| < a > |$ we denote the cardinal of $< a >$.

By $E(S)$ we denote a set of all idempotents on S.

For nondefined notions we refer to [2] and [3].

A subsemigroup A of a semigroup S is an (m, n)-ideal on S if $A^m SA^n \subseteq A$, where $m, n \in N \cup \{0\}$ $(A^0SA^n = SA^n,\ A^m SA^0 = A^m S)$, [5].

A semigroup S is an (m, n)-ideal semigroup if each subsemigroup of S is an (m, n)-ideal on S, [11, 13].

Definition 1.1. A subsemigroup Q of a semigroup S is an (m, n)-quasi-ideal on S if $Q^m S \cap SQ^n \subseteq Q$, where $m, n \in \mathbb{N} \cup \{0\}$ $(Q^0 S \cap SQ^n = SQ^n,\ Q^m S \cap SQ^0 = Q^m S)$.

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Remark 1.1. Let Q be an (m,n)-quasi-ideal on S, then

$$Q^mSQ^n = Q^mSQ^n \cap Q^mSQ^n \subseteq Q^mS \cap SQ^n \subseteq Q$$

and so Q is an (m,n) ideal on S.

Definition 1.2. A semigroup S is an (m,n)-quasi-ideal semigroup if each subsemigroup of S is an (m,n)-quasi-ideal on S.

For $m = 1,\ n = 0$ ($m = 0,\ n = 1$) we have right (left) ideal semigroups considered in [4] and [5]. For $m = n = 1$ we have quasi-ideal semigroups.

It is clear that if S is an (m,n)-quasi-ideal semigroup, then each subsemigroup of S is an (m,n)-quasi-ideal semigroup as well.

Remark 1.2. By Remark 1 we conclude that a class of (m,n)-quasi-ideal semigroups is a subclass of the class of (m,n)-ideal semigroups.

Remark 1.3. Let S be an (m,n)-quasi-ideal semigroup, $a \in S$, then

$$a^mS \cap Sa^n \subseteq < a >^m S \cap S < a >^n \subseteq < a > .$$

The following proposition is proved in [11].

Proposition 1.1. *Let S be an (m,n)-ideal semigroup. Then the following statements are true:*

(i) S is periodic, $K_a = \{e\}$, e is zero in $< a >$ for every $a \in S$ and $| < a > | \leqslant 2m + 2n + 1$;

(ii) The set $E(S)$ is a rectangular band and an ideal of S;

(iii) S is a disjoint union of the maximal unipotent subsemigroups $S_e = \{x \in S \mid (\exists p \in N)\ x^p = e,\ e \in E(S)\}$ and e is zero in S_e.

Remark 1.4. By Remark 2 the above proposition is valid for (m,n)-quasi-ideal semigroups.

Theorem 1.1. *Let S be an (m,n)-quasi-ideal semigroup, then for every $a \in S$ is $| < a > | \leqslant 2\max\{m,n\} + 1$.*

Proof. By Proposition 1 $< a >= \{a, a^2, a^3, \ldots, a^p = e\}$ and $| < a > | = p$. Suppose that $p > 2m + 2k + 1 = 2n + 2s + 1$ where $k, s \in \mathbb{N} \cup \{0\}$ are minimal such that $m + k = n + s$. Let $B = \{a^2, a^4, \ldots, a^p\}$, then B is a subsemigroup and (m,n)-quasi-ideal of $< a >$. So

$$B^m a \cap aB^n \subseteq B^m < a > \cap < a > B^n \subseteq B$$

and

$$\{a^{2m+1}, \ldots, a^{2m+2k+1}, \ldots, a^p\} \cap \{a^{2n+1}, \ldots, a^{2n+2s+1}, \ldots, a^p\} \subseteq B.$$

Now $a^{2m+2k+1} = a^{2m+2s+1} \in B$ where $k, s \in \mathbb{N} \cup \{0\}$ are minimal such that $m + k = n + s$ and $a^{2m+2k+1} \neq a^p$ which is a contradiction.

Hence, $p \leqslant 2m + 2k + 1 = 2m + 2s + 1$ or $p \leqslant \max\{m,n\} + 1$. □

2. A structural theorems for (m, n)-quasi-ideal semigroup

Let P be a partial semigroup, then $Q \subseteq P$ is a partial subsemigroup of P if for $x, y \in Q$ and $xy \in P$ it follows that $xy \in Q$. Then if $Q^m P \cap PQ^n \subseteq Q$ we say that Q is a partial (m, n)-quasi-ideal of P.

A partial semigroup P is a partial (m, n)-quasi-ideal semigroup if each partial subsemigroup of P is an (m, n)-quasi-ideal.

A partial semigroup P is called a power breaking partial semigroup if for every $a \in P$ there exists some $k \in \mathbb{N}$ such that $a^k \notin P$.

Example 2.1. Let semigroup $S = \{i, j, k, m, n, p, q, e, f, g, h\}$ be a given by the following table:

	i	j	k	m	n	p	q	e	f	g	h
i	j	k	m	n	p	q	e	e	g	g	e
j	k	m	n	p	q	e	e	e	g	g	e
k	m	n	p	q	e	e	e	e	g	g	e
m	n	p	q	e	e	e	e	e	g	g	e
n	p	q	e	e	e	e	e	e	g	g	e
p	q	e	e	e	e	e	e	e	g	g	e
q	e	e	e	e	e	e	e	e	g	g	e
e	e	e	e	e	e	e	e	e	g	g	e
f	h	h	h	h	h	h	h	h	f	f	h
g	e	e	e	e	e	e	e	e	g	g	e
h	h	h	h	h	h	h	h	h	f	f	h .

Then S is a $(4, 1)$-quasi-ideal semigroup, $E(S) = \{e, f, g, h\}$ is a rectangular band and an ideal of S and set $P = \{i, j, k, m, n, p, q\}$ is a power breaking partial $(4, 1)$-quasi-ideal semigroup.

Theorem 2.1. *Let S be an (m, n)-quasi-ideal semigroup, then $S = P \cup E(S)$ where P is a partial power breaking (m, n)-quasi-ideal semigroup and $E(S)$ is a rectangular band and an ideal of S, i.e. S is a retractive extension of the rectangular band $E(S)$ by the partial power breaking (m, n)-quasi-ideal semigroup P.*

Proof. Let S be an (m, n)-quasi-ideal semigroup, then by Remark 1.2 and Proposition 1.1 we have that S is a periodic semigroup, $E(S)$ is a rectangular band and an ideal of S and $P = S \setminus E(S)$ is a power breaking partial semigroup.

Let Q be a subsemigroup of P. We denote by $R =< Q >$ a subsemigroup of S generated by Q. Then R is an (m, n)-quasi-ideal of S, and so

$$Q^m P \cap PQ^n \subseteq R^m S \cap SR^n \subseteq R,$$

whence

$$Q^m P \cap PQ^n \subseteq R \setminus E(S) = Q.$$

Hence, Q is a partial (m,n)-quasi-ideal of P and so P is an (m,n)-quasi-ideal semigroup.

By Proposition 1 the mapping $\varphi : S \to E(S)$ defined by $\varphi(a) = e_a$, where e_a is a zero in $< a >$, is a retraction. □

Next, we give the following construction.

Construction 1. Let P be a power breaking partial (m,n)-quasi-ideal semigroup, E be a rectangular band, $P \cap E \neq \emptyset$ and φ be a partial homomorphism from P into E. Let us put $\varphi(e) = e$ for each $e \in E$. Then the mapping $\varphi : S = P \cup E \longrightarrow E$ is a surjective.

Let us define an operation $\cdot$ on S by

$$x \cdot y = \begin{cases} xy, & \text{if } x, y \in P \text{ and } xy \text{ is defined in } P, \\ \varphi(x)\varphi(y) & \text{otherwise.} \end{cases}$$

Then $(S, \cdot)$ is a semigroup which will be denoted by $[P, E, \varphi]$.

Theorem 2.2. *The semigroup $S = [P, E, \varphi]$ given by the above construction is an (m,n)-quasi-ideal semigroup.*

Proof. Let Q be a subsemigroup of S and $Q^* = Q \setminus E$. Let $s \in Q^m S \cap S Q^n$. Then it is $s = pd = tq$ where $p = x_1 x_2 \ldots x_m \in Q^m$, $q = y_1 y_2 \ldots y_n \in Q^n$, $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n \in Q$ and $d, t \in S$. It is enough to consider the following cases:

1. $p, q \in Q^*$, $d, t \in P$

1.1. $s = pd = tq \in P$, then $s \in (Q^*)^m P \cap P(Q^*)^n \subseteq Q^*$ since P is an (m,n)-quasi-ideal partial semigroup.

1.2. $s = pd = tq \in E$, then

$$\begin{aligned} s &= s^2 = pdtq = \varphi(p)\varphi(d)\varphi(t)\varphi(q) = \varphi(p)\varphi(q) \\ &= \varphi(p^{\max\{m,n\}+1})\varphi(q) = \varphi(p^{\max\{m,n\}+1}q) = p^{\max\{m,n\}+1}q \in Q. \end{aligned}$$

Other cases can be considered analogously. Hence, Q is an (m,n)-quasi-ideal of S and so S is an (m,n)-quasi-ideal semigroup. □

Definition 2.1. A subsemigroup Q of a semigroup S is an (m,n)-duo quasi-ideal of S if

$$Q^m S \cap S Q^n \subseteq Q \quad \text{and} \quad Q^n S \cap S Q^m \subseteq Q$$

where $m, n \in \mathbb{N} \cup \{0\}$. A semigroup S is an (m,n)-duo quasi-ideal semigroup if each subsemigroup of S is an (m,n)-duo quasi-ideal of S.

It is clear that a class of (m,n)-duo quasi-ideal semigroups is a subclass of a class of (m,n)-quasi-ideal semigroups. Putting $n = 1$, we have $(m,1)$-duo quasi-ideal semigroups.

If P is a partial semigroup then the notions of (m,n)-duo quasi-ideal and (m,n)-duo quasi-ideal partial semigroup are defined analogously by the above.

The semigroup S given in Example 1 is an $(4,1)$-duo quasi-ideal semigroup and $P = S \setminus E(S)$ is an $(4,1)$-duo quasi-ideal partial semigroup.

Let in Construction 1 P be an $(m,1)$-duo quasi-ideal power breaking partial semigroup. Then we prove the following

Theorem 2.3. *A semigroup S is an $(m,1)$-duo quasi-ideal semigroup if and only if $S \cong [P,E,\varphi]$.*

Proof. Let S be an $(m,1)$-duo quasi-ideal semigroup. The proof that $S = P \cup E(S)$ where P is a partial power breaking $(m,1)$-duo quasi-ideal semigroup and $E(S)$ is a rectangular band and an ideal of S is analogous with the proof in Theorem 1.

Let $\varphi : S = P \cup E(S) \longrightarrow E(S)$ be a mapping defined by $\varphi(x) = e_x$ such that e_x is the idempotent in $< x >$. Since a class of $(m,1)$-duo quasi-ideal semigroups is a subclass of a class of $(m,1)$-duo ideal semigroup, we have that mapping φ is epimorphism [9].

The operation on S is defined in the following way:

$$xy = \begin{cases} xy, \quad \text{as in } P \text{ if } x, y \in P \text{ and } xy \text{ is defined in } P, \\ z = \varphi(z) = \varphi(xy) = \varphi(x)\varphi(y) \quad \text{otherwise.} \end{cases}$$

It follows that the operation on S is defined in the same way as Construction 1, and so $S \cong [S,P,\varphi]$.

Conversely, suppose that $S \cong [P,E,\varphi] = T$ where P is an $(m,1)$-duo quasi-ideal power breaking partial semigroup. The proof that T is an $(m,1)$-duo quasi-ideal semigroup is analogous with the proof of Theorem 2. Consequently, S is an $(m,1)$-duo quasi-ideal semigroup. □

3. The matrix representation for (m,n)-quasi ideal semigroups

In this section we describe the class of (m,n)-quasi-ideal semigroups by matrix representation.

Construction 2. Let $E = I \times Y$ be a rectangular band and let Q be a partial (m,n)-qusi-ideal power breaking semigroup such that $E \cap Q = \emptyset$. Let $\xi : p \longrightarrow \xi_p$ be a mapping from Q into the semigroup $T(I)$ of all mappings from I into itself and $\eta : p \to \eta_p$ be a mapping from Q into $T(J)$.

For all $p,q \in Q$ let:

(i) $pq \in Q \implies \xi_{pq} = \xi_p\xi_q, \ \eta_{pq} = \eta_p\eta_q,$

(ii) $pq \notin Q \implies \xi_p\xi_q = \text{cons.},\ \eta_p\eta_q = \text{cons.},$

Let us define a multiplication on $S = E \cup Q$ with:

(a) $(i, j)(k, l) = (i, l)$,

(b) $p(i, j) = (i\xi_p, j)$,

(c) $(i, j)p = (ij\eta_p)$,

(d) $pq = r \in Q \implies pq = r \in S$,

(e) $pq \notin Q \implies pq = (i\xi_q\xi_p, j\eta_p\eta_q)$.

Then S with this multiplication is a semigroup, [1], [6] and [12, 14].

A subsemigroup B of S is of the form $B = E_B \cup Q_B$, where $E_B = I_B \times J_B$, ($I_B \subseteq I, J_B \subseteq J$) is a rectangular band and Q_B is a partial subsemigroup of Q.

If for $p, q \in Q_B, p \in Q_B^m, q \in Q_B^n$ the following condition holds:

(iii) $\xi_p : I \longrightarrow I_B,\ \eta_q : J \longrightarrow J_B$

then a semigroup which is constructed in this way will be denoted by $M(I, J, Q, \xi, \eta)$.

Theorem 3.1. *Semigroup S is an (m, n)-quasi-ideal semigroup if and only if S is isomorphic to some $M(I, J, Q, \xi, \eta)$.*

Proof. Let S be an (m, n)-quasi-ideal semigroup, then S is an (m, n)-ideal semigroup. By Theorem, [1], we have that S is isomorphic to a semigroup $M(I, J, Q, \xi, \eta)$.

Conversely, let $S = M(I, J, Q, \xi, \eta)$ and let B be a subsemigroup of S. Then $B = E_B \cup Q_B$, where $E_B = I_B \times J_B$, $(\mathrm{I_B} \subseteq \mathrm{I},\ \mathrm{J_B} \subseteq \mathrm{J})$ is a rectangular band and Q is a partial subsemigroup of Q.

We will prove that B is an (m, n)-quasi-ideal of S.

Since E_B is an ideal in B, we have

$$B^2 = (E_B \cup Q_B)(E_B \cup Q_B) = E_B^2 \cup E_BQ_B \cup Q_BE_B \cup Q_B^2 = E_B \cup Q_B^2,$$

and so, by induction, we conclude that $B^p = (E_B \cup Q_B)^p = E_B \cup Q_B^p$ for every $p \in \mathbb{N}$. Now

$$\begin{aligned} &B^mS \cap SB^n = (E_B \cup Q_B)^mS \cap S(E_B \cup Q_B)^n \\ &= (E_B \cup Q_B^m)S \cap S(E_B \cup Q_B^n) \\ &= (E_BS \cap SE_B) \cup (E_BS \cap SQ_B^n) \cup (Q_B^m \cap SE_B) \cup (Q_B^m \cap SQ_B^n). \end{aligned}$$

Now we will consider the set

$$E_BS \cap SE_B = \{ea = be | e, f \in E_B, a, b \in S\}.$$

Since $E = I \times J$ is a rectangular band and ideal on S we have

$$ea = e^2 a = eea = ebf = e(eb)f = ef \in E_B.$$

Hence

$$E_B S \cap SE_B \subseteq E_B. \tag{3.1}$$

We will consider the set

$$E_B S \cap SQ_B^n = \{ea = bq \mid e \in E_B, a, b \in S, q \in Q_B^n\}.$$

Then $ea = bq \in E$. Let this be $e = (i, j) \in E_B = I_B \times J_B$. We have the following cases:

(2.1) If $a, b \in Q, q \in Q_B^n \subseteq Q_B$ then $(i, j)a = bq$ and

$$(i, j)a = (i, j)^2 a = (i, j)bq = (i, j\eta_b)q = (i, j\eta_b\eta_q),$$

so by (iii) it follows that

$$(i, j\eta_a) = (i, j\eta_b\eta_q) \in E_B \subseteq B.$$

(2.2) If $a, b \in Q, q \in E_B$, then it is case (1).

(2.3) Let $a \in E, b \in Q, q \in Q_B$, then $a = (k, l)$ and $(i, l) = (i, j)(k, l) = bq$. Now

$$(i, l)^2 = (i, l)bq = (i, l\eta_b\eta_q) \in E_B.$$

(2.4) Let $a \in Q, b \in E, q \in Q_B$, then $b = (k, l)$ and $(i, j)a = (k, l)q$. Now

$$(i, j\eta_a) = (i, j\eta_a)^2 = (i, j\eta_a)(k, l\eta_q) = (i, l\eta_q) \in E_B.$$

(2.5) Let $a, b \in E,\ q \in Q_B$, then $a = (k, l),\ b = (k', l'), (i, j)(k, l) = (k', l')q$ and

$$(i, l) = (k', l'\eta_q) \quad \Longrightarrow \quad (i, l'\eta_q) \in E_B.$$

(2.6) If $a \in E,\ b \in Q,\ q \in E_B$, then it is case (1).

(2.7) If $a \in Q,\ b \in E,\ q \in E_B$, then it is case (1).

(2.8) If $a, b \in E,\ q \in E_B$, then it is case (1).

By the above it follows that

$$E_B S \cap SQ_B^n \subseteq E_Q \subseteq B. \tag{3.2}$$

Similarly we have that

$$Q_B^m S \cap SE_B \subseteq B. \tag{3.3}$$

Finally, we will consider the set

$$Q_B^m S \cap SQ_B^n = \{pa = bq \mid p \in Q_B^m,\ q \in Q_B^n,\ a, b \in S\}$$

Then we have the following cases:

(4.1) If $p, q \in Q_B,\ a, b \in Q$ then

$$pa = bq \in Q_B^m Q \cap QQ_B^n \subseteq Q_B \subseteq B.$$

(4.2) If $p, q \in Q_B,\ a, b \in Q$ and $pa = bq \in E$, then by (e) it follows

$$pa = (i\xi_a\xi_p, j\eta_p\eta_a) = bq = (i\xi_q\xi_b, j\eta_b\eta_q)$$

and so by (iii) we have

$$pa = bq = pabq = (i\xi_a\xi_p, j\eta_b\eta_q) \in E_B \subseteq B.$$

(4.3) If $p \in E_B,\ a, b \in Q,\ b \in Q_B$ then it is case (2).

(4.4) From $p, q \in Q_B,\ a \in E,\ b \in Q$ follows $bq \in E$ and, for $a = (i, j)$ and (e), it follows that $pa = (i\xi_p, j),\ bq = (i\xi_q\xi_b, j\eta_b\eta_q)$. So

$$pa = bq = pabq = (i\xi_p, j\eta_b\eta_q) \in E_B \subseteq B.$$

(4.5) If $p, q \in Q_B,\ a \in Q,\ b = (i, j) \in E$ then $pa = (i\xi_a\xi_p, j\eta_p\eta_a) = bq = (i, j\eta_q)$ and so $(i\xi_a\xi_p, j\eta_q) \in E_B$.

(4.6) If $p = (i, j) \in E_B,\ a = (k, l) \in E,\ b \in Q,\ q \in Q_B$ then it is case (2).

(4.7) If $p = (i, j) \in E_B,\ a \in Q,\ b \in E,\ q \in Q_B$ then it is case (2).

(4.8) If $p = (i, j) \in E_B,\ a, b \in Q,\ q = (k, l) \in E_B$ then it is case (1).

(4.9) If $p \in Q_B,\ a = (k, l) \in E,\ b \in Q,\ q \in E_B$ then it is case (3).

(4.10) If $p \in Q_B,\ a =\in Q,\ b \in E,\ q \in E_B$ then it is case (3).

(4.11) If $p \in Q_B,\ a = (i, j) \in E,\ b = (k, l) \in E,\ q \in E_B$ then $pa = p(i, j) = (i\xi_p, j) = bq = (k, l)q = (k, l\eta_q)$ and so

$$pa = bq = (i\xi_p, l\eta_q) \in E_B \subseteq B.$$

(4.12) If $p \in E_B,\ a, b \in E,\ q \in Q_B$ then it is case (2).

(4.13) If $p, q \in E_B,\ a \in E,\ b \in Q$ then it is case (1).

(4.14) If $p, q \in E_B,\ a \in Q,\ b \in E$ then it is case (1).

(4.15) If $p \in Q_B,\ a, b \in E,\ q \in E_B$ then it is case (3).

(4.16) If $p, q \in E_B,\ a, b \in E$ then it is case (1).

There are no other cases. Hence

$$Q_B^m S \cap SQ_B^n \subseteq B. \tag{3.4}$$

By (1), (2), (3) and (4) we have that $B^m S \cap SB^n \subseteq B$, B is an (m, n)-ideal of S and S is an (m, n)-quasi-ideal semigroup. □

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