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## WAVE DIGITAL MODELS OF IDEAL AND REAL TRANSFORMERS

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**Abstract.** *In this paper, the wave digital filter (WDF) theory is applied for development of the wave digital models of ideal and real transformers which can be used for modeling of more complex structures. The transformers wave digital networks are described and developed here based on scattering variables and two-port and three-port networks of parallel and series adaptors. WDF-based model of a real transformer includes parasitic resistors and inductors, which are usual in low-frequency transformer equivalent circuit.*

**Key words:** *wave digital approach, ideal transformer, real transformer, series/parallel adaptors, network synthesis*

### 1. INTRODUCTION

The basic theory of wave digital filters (WDF) was developed by Alfred Fettweis [1-3] in the early 70's. It was used for digitizing lumped electrical circuits composed of inductors, capacitors, resistors, and other elements of classical network theory. Wave digital filters offer computational efficiency, stability under finite-arithmetic conditions and facilitate interfacing with wave variables, making them a worthwhile subject of study. The wave digital structures are specially tailored with respect to hardware implementation.

In the past, wave digital filter concept has been widely used in time-discrete wave-based modeling and analysis of different physical systems. A detailed review of application of WDF structures for electromagnetic (EM) field simulation is given in [4-6]. The original intent of the author is to develop a self-written tool in MATLAB based on WDF theory for microstrip circuit simulation in digital domain. The only practical constraint is that the complex structure under study needs to have an analogous equivalent circuit representation. As stated in [7, 8], a mixed wave-digital/full-wave EM method combining the accuracy of EM modeling with the convenience of using wave digital approach can be very efficiently used to develop WDF-based network models and to get the scattering parameters of different two-port microstrip structures. The effects of discontinuity are more accurately modeled and taken into account with a full-wave EM simulation.

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Compact network models can facilitate the solution of EM field problems such as microwave structures. Recently, a systematic method for the automated extraction of lumped-element equivalent circuits for multiport microwave circuits has been presented [9]. A circuit model is extracted directly from the EM simulation or measurements. The proposed multiport-Foster synthesis method generates network topology with lumped elements and connection subnetworks based on ideal transformers.

Network-oriented modeling can also be applied to complex EM radiating structures using the segmentation technique and dividing structure into subregions. The lumped-element circuit models can be established by representing subdomains by Foster equivalent circuits, their connection by interconnection elements exhibiting only ideal transformers and the radiation modes by Cauer equivalent circuits. Some examples are shown in [10, 11]. This type of connection subnetworks based on ideal transformers can also be used for hybridization of the TLM method with other numerical methods [10, 11].

As it is mentioned earlier, the author of the paper intends to develop a software tool that can be applied to simulate microwave structures of different geometries by using their developed wave digital network models. So, in order to digitize equivalent network circuits with incorporated ideal transformers represented in [9-11] by using WDF theory, WDF-based models of an ideal transformer are developed in the first part of this paper.

In wave digital structures adaptors are used as the connection elements. A. Fettweis has introduced elementary two-port parallel and series adaptors representing parallel and series connections of two-ports [1-3, 12]). The ideal transformer is an important component in the building of complex microwave systems and its proper modeling using the methods of the WDFs is very important. In this paper, at first the  $ABCD$  and the scattering matrices parameters of the ideal transformer are introduced and interconnected, and then the results are applied to develop WDF-based models, using only two-port series or parallel wave digital adaptors and some additional multipliers. The correctness of these models is verified through a simple proof. Contrary to an ideal transformer, a real transformer has winding resistance, flux leakage, finite permeability and core losses. WDF-based model of transformer T-equivalent circuit [13, 14] which has been successfully used in steady-state studies and some low-frequency transient studies is developed here in the second part of the paper.

## 2. THE SCATTERING VARIABLE FORMALISM

The introduction of the wave variables is the key element of the wave digital approach. The correspondence between wave digital circuit and the related analog circuit is based on scattering variables, not on voltages and currents. The relevant signal quantities used in WDF representation are the so-called incident  $A$  and reflected  $B$  waves, which are defined with respect to each port of the reference network as shown in Fig. 1.

Synthesis of wave digital network of an ideal transformer can be done by starting from its  $ABCD$  parameters and their transformation into scattering parameters. For a two-port network characterized by a current and a voltage at each port (Fig. 1),  $ABCD$  matrix formulation is

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ -I_2 \end{bmatrix}, \quad (1)$$

and scattering matrix formulation is

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \mathbf{S} \cdot \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}, \quad (2)$$

where  $A_1$  and  $B_1$  are the incident and reflected wave at port 1 with port resistance  $R_1$ , and  $A_2$  and  $B_2$  are the incident and reflected wave at port 2 having port resistance  $R_2$ , [12, 15].

Classic  $ABCD$  matrix to  $S$  matrix transformation formulas for two-port case give connection [12, 15]

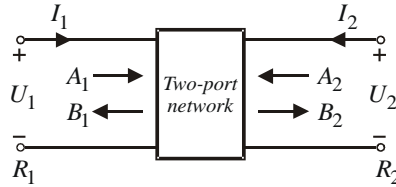
$$S_{11} = \frac{A + B \cdot G_2 - C \cdot R_1 - D \cdot R_1 \cdot G_2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (3)$$

$$S_{12} = \frac{2 \cdot R_1 \cdot G_2 \cdot (A \cdot D - B \cdot C)}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (4)$$

$$S_{21} = \frac{2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (5)$$

$$S_{22} = -\frac{A - B \cdot G_2 + C \cdot R_1 - D \cdot R_1 \cdot G_2}{A + B \cdot G_2 + C \cdot R_1 + D \cdot R_1 \cdot G_2}, \quad (6)$$

where  $G_2 = 1/R_2$ , Fig. 1.



**Fig. 1** Two-port network

### 3. TWO- AND THREE-PORT ADAPTORS

Adaptors are memoryless digital elements whose task is to perform transformations between pairs of wave variables that are referred to different port resistances [1-4, 12]. They have low sensitivity to coefficient quantization. In this section, subsequently required two- and three-port adaptors are presented.

#### 3.1. Two-port adaptors

The equations for two-port parallel adaptor are

$$B_1 = A_2 + \beta \cdot (A_1 - A_2), \quad (7)$$

$$B_2 = A_1 + \beta \cdot (A_1 - A_2), \quad (8)$$

where the adaptor coefficient  $\beta = (R_2 - R_1)/(R_2 + R_1)$  is usually written on the side corresponding to port 2. The equations for two-port series adaptor are

$$B_1 = -A_2 + \beta \cdot (A_1 + A_2), \quad (9)$$

$$B_2 = -A_1 - \beta \cdot (A_1 + A_2). \quad (10)$$

The adaptor coefficient can be defined also as  $\alpha = -\beta = (R_1 - R_2)/(R_2 + R_1)$  and written on the side corresponding to port 1. In this case, the other form of Eqs. (7)-(8) and Eqs. (9)-(10) can be written.

### 3.2. Three-port Adaptors

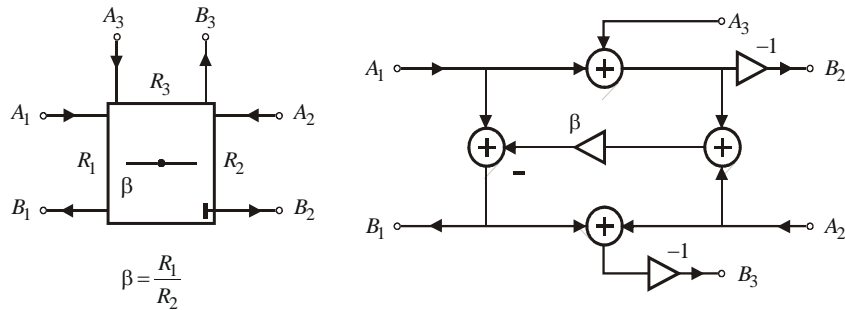
A network of three-port series adaptor with port 2 being reflection-free is depicted in Fig. 2, [1-4, 12]. The adaptor coefficient  $\beta = R_1 / R_2$ ,  $R_2 = R_1 + R_3$  is shown explicitly next to the port 1, Fig. 2a. The set of equations for the three-port series adaptor with reflection-free port 2 is

$$A_0 = A_1 + A_2 + A_3, \quad (11)$$

$$B_1 = A_1 - \beta \cdot A_0, \quad (12)$$

$$B_2 = -(A_1 + A_3), \quad (13)$$

$$B_3 = -(B_1 + A_2). \quad (14)$$



**Fig. 2** Three-port series adaptor with reflection-free port 2:  
a) symbol and b) wave digital network model

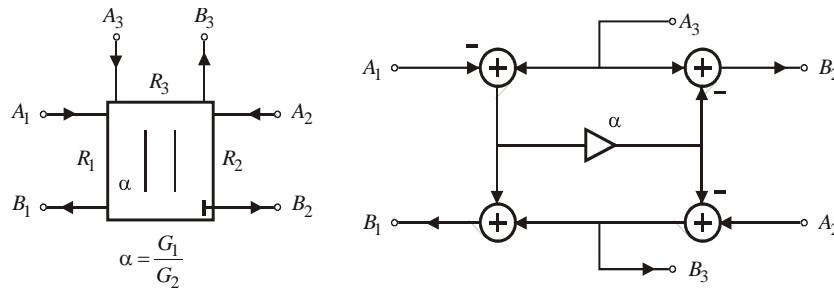
A network of three-port parallel adaptor with port 2 being reflection-free is depicted in Fig. 3, [1-4]. The adaptor coefficient  $\alpha = G_1 / G_2$ ,  $G_2 = G_1 + G_3$  is shown explicitly next to the port 1, Fig. 3a. The set of equations for the three-port series adaptor with reflection-free port 2 is

$$A_0 = \alpha \cdot A_1 + A_2 + (1 - \alpha) \cdot A_3, \quad (15)$$

$$B_1 = B_3 + A_3 - A_1, \quad (16)$$

$$B_2 = A_3 + \alpha \cdot (A_1 - A_3), \quad (17)$$

$$B_3 = A_2 + \alpha \cdot (A_1 - A_3). \quad (18)$$

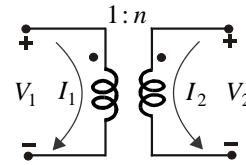


**Fig. 3** Three-port parallel adaptor with reflection-free port 2:  
a) symbol and b) wave digital network model

4. DEFINITION OF AN IDEAL TRANSFORMER AND ITS MATRICES

A transformer usually consists of two coupled windings (primary and secondary) on a magnetic iron core. An ideal transformer is an imaginary transformer which does not have any loss in it, means no core losses, copper losses and any other losses in the transformer. Efficiency of this transformer is considered as 100 %.

In general, there are different cases of an ideal transformer depending on the polarity of the voltages and current ratios. Also, the same turn ratio can be given in different ways. In general, the equation set of an ideal transformer is written according to the established reference polarities for currents (both currents entering or one entering and one leaving the dotted terminals) and voltages (both voltages positive/negative or one positive and one negative at the dotted terminals). In this section, the *ABCD* and the scattering matrices parameters for one case of an ideal transformer are introduced and interconnected first and then the results are applied to develop WDF-based models, using only two-port series or parallel wave digital adaptors with the defined adaptor coefficient  $\beta$ .



**Fig. 4** A schematic symbol of an ideal transformer

A schematic symbol of an ideal transformer with turn ration  $n$  where both currents entering (are directed into the dot-marked terminal) and both voltages are positive at dotted terminals is presented in Fig. 4. The *ABCD* parameters of the ideal transformer given in Fig. 4 can be easily obtained by considering its equation system

$$V_1/V_2 = 1/n, I_1/I_2 = -n, \tag{19}$$

and they are given in the matrix form as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/n & 0 \\ 0 & n \end{bmatrix}. \tag{20}$$

*S*-parameters of an ideal transformer can be found starting with its *ABCD* matrix given in Eq. (20) and the previously given classic formulas Eqs. (3)-(6), as follows

$$S_{11} = \frac{1-n^2 \cdot R_1 \cdot G_2}{1+n^2 \cdot R_1 \cdot G_2}, \quad S_{12} = \frac{2 \cdot n \cdot R_1 \cdot G_2}{1+n^2 \cdot R_1 \cdot G_2}, \quad (21)$$

$$S_{21} = \frac{2 \cdot n}{1+n^2 \cdot R_1 \cdot G_2}, \quad S_{22} = -\frac{1-n^2 \cdot R_1 \cdot G_2}{1+n^2 \cdot R_1 \cdot G_2}. \quad (22)$$

### 5. THE WAVE DIGITAL MODELS OF AN IDEAL TRANSFORMER

An ideal transformer is transformed to the wave digital domain by expressing its  $S$ -parameter equations in the form corresponding to the equations of the two-port parallel or series adaptor networks and by choosing value of the adaptor coefficient  $\beta$ .

In order to represent an ideal transformer with the network based on two-port parallel adaptor, adaptor coefficient is chosen to be

$$\beta = \frac{R_2 - n^2 \cdot R_1}{R_2 + n^2 \cdot R_1}. \quad (23)$$

For this case, the  $S$ -parameters of the ideal transformer given in Eqs. (21)-(22) can be written in the form

$$S = \begin{bmatrix} \beta & \frac{1}{n} \cdot (1 - \beta) \\ n \cdot (1 + \beta) & -\beta \end{bmatrix}. \quad (24)$$

For these obtained scattering parameters, according to formulation given in Eq. (2), the set of equations for ideal transformer is

$$B_1 = S_{11} \cdot A_1 + S_{12} \cdot A_2 = \beta \cdot A_1 + \frac{1}{n} \cdot (1 - \beta) \cdot A_2, \quad (25)$$

$$B_2 = S_{21} \cdot A_1 + S_{22} \cdot A_2 = n \cdot (1 + \beta) \cdot A_1 - \beta \cdot A_2. \quad (26)$$

#### 5.1. The ideal transformer model based on two-port parallel adaptor network

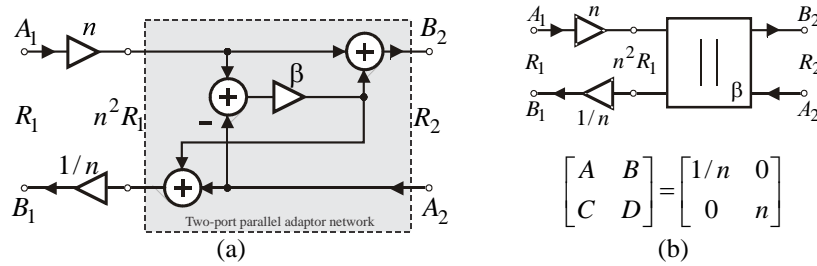
To synthesize the wave digital network of ideal transformer based on parallel adaptor network, Eqs. (25)-(26) are written in the form of Eqs. (7)-(8) as following

$$B_1 = \frac{1}{n} \cdot [A_2 + \beta \cdot (n \cdot A_1 - A_2)], \quad (27)$$

$$B_2 = n \cdot A_1 + \beta \cdot (n \cdot A_1 - A_2). \quad (28)$$

According to the last equations, wave digital network model of an ideal transformer based on two-port parallel adaptor network is depicted in Fig. 5. In addition to wave digital network of two-port parallel adaptor [1-4], this transformer model has two more multipliers. The WDF-based model can be found in [3], but description in details is not present there. In Fig. 6, the wave digital network models for different ideal transformer cases are symbolically presented. By observing the  $ABCD$  parameters and models given in Figs. 5 and 6, a generalization of multiplier values can be made. The multiplier in branch  $B_1$  always takes the value of parameter  $A$ . The multiplier in branch  $A_1$  takes the value of parameter  $D$  if parameters

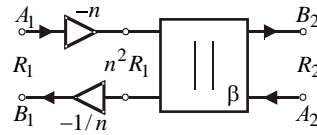
$A$  and  $D$  are both positive or negative, and value of  $-D$  in the other cases (one positive and one negative parameter).



**Fig. 5** Ideal transformer: Its realization for arbitrary choices of  $R_1, R_2$  and  $n$ , based on two-port parallel adaptor network: (a) wave digital network model and (b) its symbolic representation

$$V_1 / V_2 = -1/n, I_1 / I_2 = n$$

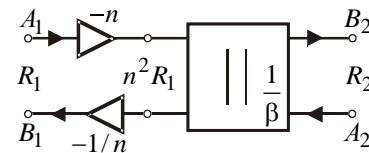
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -1/n & 0 \\ 0 & -n \end{bmatrix}$$



(a) An ideal transformer where one current entering and one leaving the dotted terminals and one voltage positive and one negative at dotted terminals

$$V_1 / V_2 = -1/n, I_1 / I_2 = -n$$

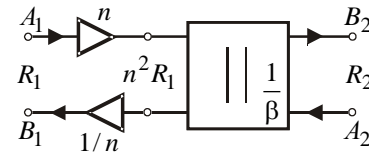
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -1/n & 0 \\ 0 & n \end{bmatrix}$$



(b) An ideal transformer where both currents entering the dotted terminals and one voltage positive and one negative at dotted terminals

$$V_1 / V_2 = 1/n, I_1 / I_2 = n$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/n & 0 \\ 0 & -n \end{bmatrix}$$



(c) An ideal transformer where one current entering and one leaving the dotted terminals and both voltages are positive at dotted terminals

**Fig. 6** Ideal transformers: Equation systems and symbolic representations of their wave digital network models for arbitrary choices of  $R_1, R_2$  and  $n$ , based on two-port parallel adaptor network with  $\beta = (R_2 - n^2 \cdot R_1) / (R_2 + n^2 \cdot R_1)$

### 5.2. The ideal transformer model based on two-port series adaptor network

In order to form the wave digital model of an ideal transformer based on series adaptor network, the previously given Eqs. (25)-(26) have to be written in the form of Eqs. (9)-(10) as shown

$$B_1 = \frac{1}{n} \cdot A_2 + \beta \cdot \left( A_1 - \frac{1}{n} \cdot A_2 \right), \quad (29)$$

$$B_2 = -n \cdot \left[ -A_1 - \beta \cdot \left( A_1 - \frac{1}{n} \cdot A_2 \right) \right]. \quad (30)$$

Starting with Eqs. (29) and (30), the wave digital network model for an ideal transformer based on two-port series adaptor network can be obtained and its representation in MATLAB Simulink toolbox can be drawn. It will contain three multipliers with constant coefficients and three adders. Its network model is modification of that one shown in Fig. 5b, multiplier  $n$  is moved from branch of  $A_1$  to branch corresponding to  $B_2$ , and multiplier  $1/n$  from  $B_1$  to branch of  $A_2$ .

### 5.3. WDF-based model possibilities

A detailed description of synthesis process of WDF-based model for an ideal transformer case given in Fig. 4 was given previously. One can follow the described procedure and develop wave digital network models for any possible case of ideal transformer and any way shown turn ratio. The choice of two-port series or parallel adaptors with defined coefficient  $\beta$  or  $\alpha$  can be made depending on the problem at hand. Solution which gives the simplest overall expressions for the particular problem is preferable.

The Simulink models of the parallel and series adaptors, as well as of different ideal transformer cases can be added in the Simulink browse library between common used blocks and those blocks can be used further in generating wave digital network models of complex structures.

### 5.4. Proof of the ideal transformer feature

Let's consider first an ideal transformer shown in Fig. 4 closed by a resistor  $R_2 = R_p$ . Its input impedance is

$$Z_{in} = \frac{V_1}{I_1} \Big|_{R_p} = \frac{A \cdot R_p + B}{C \cdot R_p + D} = \frac{R_p}{n^2}.$$

If  $R_1$  and  $R_2$  are port resistances, the reflection coefficient is

$$\Gamma(z) = \frac{B_1}{A_1} = \frac{Z_{in} - R_1}{Z_{in} + R_1} = \frac{R_p - n^2 \cdot R_1}{R_p + n^2 \cdot R_1}. \quad (31)$$

The wave digital network of an ideal transformer based on two-port parallel adaptor shown in Fig. 5 will be used in order to prove the main transformer feature. At the far end (port 2), it is terminated by a wave digital element corresponding to resistor (multiplier  $\gamma$ ). By substituting relation



$$A_2 = \gamma \cdot B_2 \quad (32)$$

where  $\gamma = \frac{R_p - R_2}{R_p + R_2}$ , in the set of Eqs. (25)-(26), the reflection coefficient is found to be

$$\Gamma(z) = \frac{B_1}{A_1} = \frac{\beta^2(1-\gamma) + \gamma(1+\beta)}{\beta + \gamma}. \quad (33)$$

In case when resistance of the resistor at the far end is chosen to be equal to its port resistance  $R_p = R_2$ , the coefficient  $\gamma$  takes zero value, and finally the reflection coefficient is

$$\Gamma(z) = \beta = \frac{R_2 - n^2 \cdot R_1}{R_2 + n^2 \cdot R_1}. \quad (34)$$

It is evident that the reflection coefficients given by Eqs. (31) and (34) are the same. In this way, it is shown that this wave digital network operates exactly like ideal transformer, it modifies load impedance.

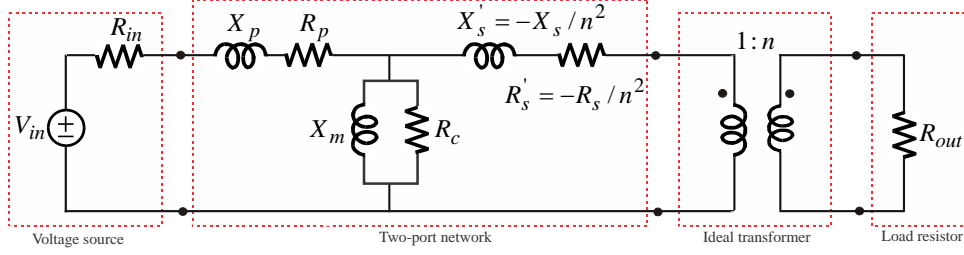
## 6. DEFINITION OF A REAL TRANSFORMER

Although transformers are used for different purposes, the fundamental theory and concepts of all transformers are same.

A real transformer is non-ideal or practical iron-core transformer. Contrary to an ideal transformer, a practical transformer has winding resistance, flux leakage, finite permeability and core losses. All these things have to be considered to derive equivalent circuit of a real transformer. A real transformer's behaviour may be represented by an equivalent circuit model which contains an ideal transformer. The remaining elements are those elements that contribute to the non-ideal characteristics of the device. The linear model of a transformer, with core losses and load losses taken into account, is observed here. The ideal transformer can be shifted to either side of the circuit, and the observed equivalent circuit with secondary parameters referred to primary side is shown in Fig. 7. This model is known as transformer T-equivalent circuit (T-model) [13, 14] and has been successfully used for many years in steady-state studies and some low-frequency transient studies.

In the equivalent circuit,  $R_p$  and  $R_s$  are primary and secondary winding resistances (series resistances including conductor losses of each ending),  $X_p = \omega L_p$  and  $X_s = \omega L_s$  are primary and secondary leakage reactances. The parallel branch (so-called magnetizing branch of the model) represents the magnetic core of the transformer, where  $R_c$  is core losses and  $X_m = \omega L_m$  is magnetizing reactance. All parameters values in the transformer model shown in Fig. 7 can be determined by carrying out experiments.

The equivalent model with included capacitances between windings and between each winding and ground, and self capacitance of each winding, can be found in [13]. The wave digital model of such an equivalent circuit with included parasitic capacitances will be derived in the author's future work.



**Fig. 7** Equivalent circuit of a practical transformer referred to the primary side

### 7. THE WAVE DIGITAL MODEL OF A REAL TRANSFORMER

The analysis presented for an ideal transformer is helpful when digitizing the real transformer. The WDF-based model of real transformer is developed following the rules of interconnection of wave elements. To form WDF-based model from the transformer T-equivalent circuit from Fig. 7, it is advisable to divide it into several parts representing a real voltage source, three-port series/parallel adaptors with series/parallel element branches, an ideal transformer and load resistance. Block connection in digital form, i.e. symbolic representation of wave digital model of real transformer is shown in Fig. 8. In that way, the circuit elements are separated from their connection network. Then, series and parallel element branches with resistor and inductor are represented by corresponding models. Wave digital one-ports corresponding to the classical one-ports such as resistor, inductor and real voltage source can be found in [1-4, 12]. The one-ports are transformed into the wave digital domain by expressing their underlying physical law in terms of wave variables, applying the Bilinear transform, and choosing the port resistances.

In the literature [14], one can also find transformer  $\pi$ -model. It can also be transformed to wave digital domain by following the interconnection rules and by using wave digital one-port models marked in Fig. 8.

The wave digital network model parameters (port resistances and adaptor coefficients) are defined as

$$R_1 = R_p, R_2 = L_p, R_{11} = R_1 + R_2, \beta_2 = \frac{R_1}{R_{11}}, \quad (35)$$

$$R_3 = R_c, R_4 = L_m, G_{22} = G_3 + G_4, \alpha_2 = \frac{G_3}{G_{22}}, \quad (36)$$

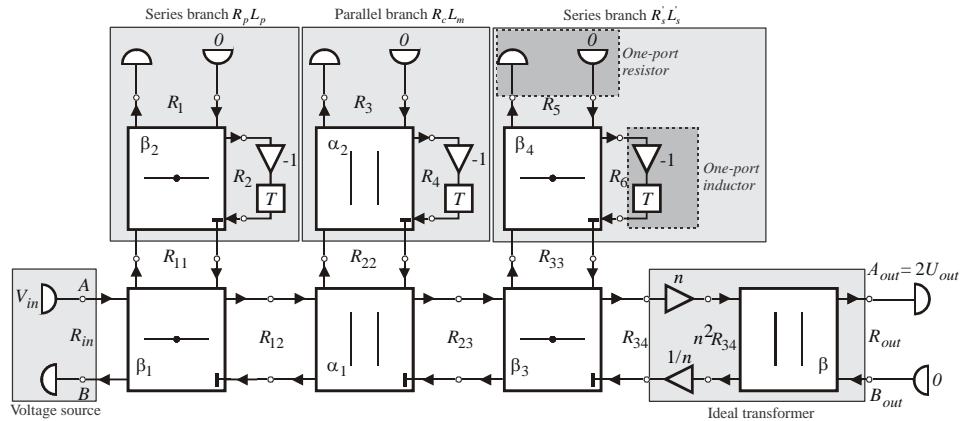
$$R_5 = R'_s, R_6 = L'_s, R_{33} = R_5 + R_6, \beta_4 = \frac{R_5}{R_{33}}, \quad (37)$$

$$R_{12} = R_{in} + R_{11}, \beta_1 = \frac{R_{in}}{R_{12}}, \quad (38)$$

$$G_{23} = G_{12} + G_{22}, \alpha_1 = \frac{G_{12}}{G_{23}}, \quad (39)$$

$$R_{34} = R_{23} + R_{33}, \beta_3 = \frac{R_{23}}{R_{34}}, \text{ and } \beta = \frac{R_{out} - n^2 R_{34}}{R_{out} + n^2 R_{34}}. \quad (40)$$

Response in the formed network model can be easily found by use of block-diagram network drawn in Simulink toolbox and some basic MATLAB functions allowing for accurate and fast modeling and analyzing of circuits.



**Fig. 8** Real transformer: Symbolic representation of WDF-based network model

## 8. CONCLUSION

The application of wave digital approach in modeling and analysis of microstrip structures at high operating frequency is considered in [7, 8]. The two-port microstrip structures under investigation are represented by their equivalent circuits. Generation of the equivalent circuit models is also presented in those papers. Further in case of multi-port microwave circuits, one of the techniques to construct an equivalent-circuit model is based on numerical data from either full-wave analysis or measurement [9] in wide frequency range. For this purpose, system identification is performed on the impedance/admittance function. The generated equivalent circuit consists of connection network and lumped elements. The connection network does not store energy and can be represented by canonical form which only makes use of ideal transformers [16]. The element values and turns ratio of transformers are determined automatically [9]. Equivalent circuit contains lossy elements, i.e. losses in the microwave structure are included through resistances and conductances.

The main goal here is to digitize these devices (ideal and real transformers) based on their known equivalent circuits. Because of that, one possible equivalent circuit of real transformer and its corresponding wave digital network model are shown. A schematic representation of the major low-frequency parasitic elements in a generalized transformer is shown in this paper. In most textbooks and literature, this model is used to describe the transformer behaviour. In general, this equivalent circuit is more accurate than that one of an ideal transformer, but less accurate than the exact one. Also, it is possible to form wave digital network model of real transformer with included all high-frequency parasitic effects. Such an equivalent circuit is capable of representing a practical design with considerable accuracy, but actual calculations would be very difficult in some cases. The wave digital model of such an equivalent circuit will be derived in the author's future work.

In general, this paper shows how wave digital networks representing digital models of ideal and real transformers are synthesized based on scattering variables. Wave digital model of the ideal transformer is developed based on two-port parallel/series adaptor networks. The WDF-based network model of the ideal transformer is modified to include the effects of winding resistance, flux leakage, finite permeability and core losses. In that way, real transformer is digitizing and its WDF-based model is formed.

The author's intention is to develop a software tool embedded in MATLAB for simulation of a wide variety of microwave structures based on their equivalent circuits and wave digital approach. A transformer is a very common magnetic structure which can be found in many applications. In the complex EM structures, such as multiport microwave or radiating structures, connection subnetworks in their equivalent circuits can be represented based on ideal transformer [6, 9-11]. Developed wave digital models of ideal and real transformers can be added in the Simulink browse library between common used blocks and those blocks can be used further in generating wave digital network models of the mentioned complex structures containing transformers as interconnecting elements. Now, an increase in a number of types of structures that can be efficiently modeled by using wave digital approach is obtained.

Compact wave digital network models, embedded into MATLAB, can provide considerable lower computational effort and run time in comparison with full-wave EM analysis. Due to the complexity of modern microwave structures and systems, an EM full-wave analysis of these structures/systems is often prohibitive due to limitations in processing and memory capabilities.

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