

FACTA UNIVERSITATIS

Series: **Electronics and Energetics** Vol. 27, N° 4, December 2014, pp. 601 - 611

DOI: 10.2298/FUEE1404601S

## FUZZY MODEL REFERENCE ADAPTIVE CONTROL OF VELOCITY SERVO SYSTEM

**Momir R. Stanković<sup>1</sup>, Milica B. Naumović<sup>2</sup>, Stojadin M. Manojlović<sup>1</sup>,  
Srđan T. Mitrović<sup>1</sup>**

<sup>1</sup>Military Academy, University of Defence in Belgrade, Serbia

<sup>2</sup>Department of Automatic Control, Faculty of Electronic Engineering, University of Niš,  
Serbia

**Abstract.** *The Implementation of fuzzy model reference adaptive control of a velocity servo system is analysed in this paper. Designing the model reference adaptive control (MRAC) and the problem of choosing adaptation gain is considered. Tuning the adaptation gain by fuzzy logic subsystem and a simple synthesis procedure of fuzzy MRAC are proposed. Several simulation runs show the advantages of fuzzy MRAC approach. Experimental validation on laboratory speed servo is realized by the acquisition system. The results confirm benefits of the proposed controller in comparison with the standard MRAC.*

**Key words:** MRAC, fuzzy MRAC, adaptation gain.

### 1. INTRODUCTION

The major conventional controllers design concepts are model based. However, process modelling is a complex procedure, which at best, provides only an approximate model of the real process, followed with some level of model uncertainty. The controllers with constant parameters in most cases are unable to cope with parameters perturbations, unmodelled dynamics and external disturbances. In order to provide acceptable system behaviour in the presence of internal and external disturbances, the appropriate adaptation of controller parameters is necessary [1].

Adaptive control contains a proper adjustment mechanism of controller parameters in accordance with working conditions and the current state of the system. Recall that the adaptive systems are divided in two classes: self-tuning systems and model reference adaptive systems based on parameters adaptation technique [2], [3].

In self-tuning control systems some of the recursive methods for on-line process identification are used and controller parameters are adjusted in real time based on the estimated values and predefined algorithm [4].

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Received May 14, 2014; received in revised form October 13, 2014

**Corresponding author:** Momir R. Stanković

Military Academy, University of Defence in Belgrade, Generala Pavla Jurišića Šturma 33, 11000 Belgrade, Serbia  
(e-mail: momir\_stankovic@yahoo.com)

In model reference adaptive control (MRAC) the system performance is given by the reference model. Tuning of controller parameters is based on the error, defined as the difference between reference model and real process responses [2], [5]. The main weakness of the standard MRAC, with MIT rule, is non-existent clearly defined rules for the adaptation gain selection. In most cases, however, it is chosen based on the large number of simulations and trial and error methods [2].

The use of fractional order parameter adjustment rule instead of the gradient approach with MIT rule and the employment of fractional order reference model is proposed in [6]. The different modifications of the MRAC, with a variable structure design [7], based on performing repetitive tasks [8] and with a time-varying reference model [9] have successfully been applied for plants with unmodeled dynamics, external disturbances and unknown parameters and for a system with control effort bounded.

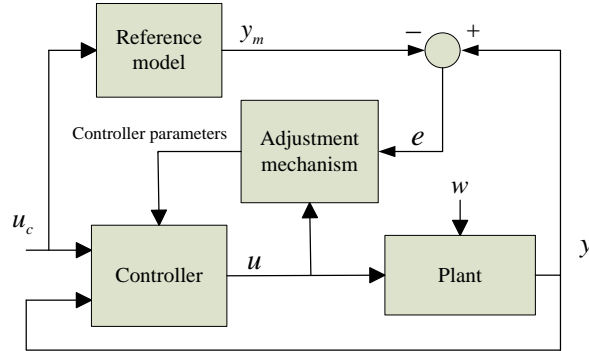
Since Ichikawa presented the novel design of model reference adaptive fuzzy control [10], many authors have made progress in the application of fuzzy theory in MRAC [11], [12]. The fuzzy set theory allows the use of experience in system control design. The great contribution of fuzzy logic is the possibility of modelling unstructured heuristic assertions, which are expressed linguistically [13]. Fuzzy adaptive concept becomes closer to the designer and it allows the use of expert knowledge and experience in designing control systems. As a result, the performance/complexity ratio is better for fuzzy adaptive controllers [14], [15].

Fuzzy MRAC is suitable for application in industrial control systems, where the influence of internal and external disturbances is high. In [16] simple design procedure of fuzzy MRAC using error signal as fuzzy subsystem input was presented and this controller has shown better results than conventional ones.

In this paper the fuzzy MRAC of the speed servo system is proposed. In speed servo systems the main influence on system performances has varying load torque. Based on the estimation of load torque and its first derivative the adaptation gain is adjusted by fuzzy logic subsystem (FLsS). Through MATLAB/Simulink® simulation models, the proposed and standard MRAC of speed servo system with different load disturbance profiles are compared. The proposed controller is implemented in laboratory DC velocity servo system, and experimental validation of simulation results is obtained.

## 2. A REVISIT TO THE MODEL REFERENCE ADAPTIVE CONTROL (MRAC)

A block diagram of the model reference adaptive control is shown in Fig. 1. The desired behaviour of the system is expressed by reference model. Parameters of controller are adjusted based on error  $e = y - y_m$ , which is the difference between plant output  $y$  and reference model output  $y_m$ . The main sources of error  $e$  are difference of reference and plant dynamics and external disturbances, denoted with  $w$  in Fig. 1. The system has two feedback loops: an ordinary one composed of the plant and controller and a feedback loop for controller parameters adjustment [2].



**Fig. 1** Block diagram of model reference adaptive control

Adjusting the controller parameters in the direction of the negative gradient of  $e$  is realized by the well-known MIT rule:

$$\frac{d\theta}{dt} = -\gamma e \frac{de}{d\theta}, \quad (1)$$

where  $\theta$  is the parameter of controller, the partial derivative  $de/d\theta$  is called the sensitivity derivative of the system and  $\gamma$  presents the adaptation gain [2].

Consider a first order system described by the model [2]:

$$\frac{dy}{dt} = -ay + bu, \quad (2)$$

where  $u$  is the control variable.

Let the reference model be given as:

$$\frac{dy_m}{dt} = -a_m y_m + b_m u_c, \quad (3)$$

where  $u_c$  is the command signal.

The perfect model reference following can be achieved by controller:

$$u = \theta_1 u_c - \theta_2 y, \quad (4)$$

with parameters:

$$\theta_1 = \theta_1^0 = \frac{b_m}{b} \text{ and } \theta_2 = \theta_2^0 = \frac{a_m - a}{b}. \quad (5)$$

The sensitivity derivatives directly follow from partial derivations of  $e$  with respect to the controller parameters  $\theta_1$  and  $\theta_2$  [2]:

$$\frac{de}{d\theta_1} = \frac{b}{p+a+b\theta_2} u_c \quad (6)$$

$$\frac{de}{d\theta_2} = \frac{b^2\theta_1}{(p+a+b\theta_2)^2} u_c = -\frac{b}{p+a+b\theta_2} y \quad (7)$$

where  $p = d/dt$  is the differential operator. Equations (6) and (7) cannot be used directly because  $a$  and  $b$  represent parameters of the system, which are uncertain or unknown. In order to exclude parameters  $a$  and  $b$  the following approximations are required:

$$p+a+b\theta_2 \approx p+a_m \quad (8)$$

Based on (8) and MIT rule (1) the following equations for updating the controller parameters are obtained:

$$\frac{d\theta_1}{dt} = -\gamma \left( \frac{1}{p+a_m} u_c \right) e, \quad (9)$$

$$\frac{d\theta_2}{dt} = \gamma \left( \frac{1}{p+a_m} y \right) e. \quad (10)$$

It can be noted that parameter  $b$  is absorbed in adaptation gain  $\gamma$  [2].

From (9) and (10) it can be seen that MRAC has only one parameter, the adaptation gain  $\gamma$ , which has to be chosen *a priori* and its selection influences system performances significantly [15].

By substitution of (2), (3) and (4) in (9) and (10),  $y$  and  $e$  are excluded, and the following equations are obtained:

$$\frac{d\theta_1}{dt} = -\gamma y_m \left( \frac{G_m(p)u_c}{1+G_m(p)\theta_2} \right) \theta_1 + \gamma y_m^2, \quad (11)$$

$$\frac{d\theta_2}{dt} = -\gamma G_{ref}(p) \left( \frac{G_m(p)\theta_1 u_c}{1+G_m(p)\theta_2 u_c} \right)^2 - \gamma y_m G_{ref}(p) \frac{G_m(p)\theta_1 u_c}{1+G_m(p)\theta_2 u_c}, \quad (12)$$

where  $G_m(p)$  and  $G_{ref}(p)$  are equivalent to transfer functions of the plant and reference model, respectively. The influence of adaptation gain  $\gamma$  on the convergence rates of  $\theta_1$  and  $\theta_2$  to  $\theta_1^0$  and  $\theta_2^0$  cannot be analytically derived from (11) and (12). If  $\gamma$  is constant, the convergence rates depend only on uncertainty of plant transfer function. It is known that system performances differ for different values of  $\gamma$  [2] and therefore it is assumed that varying  $\gamma$ , as a function of external disturbances influencing the system, can significantly increase the convergence rates.

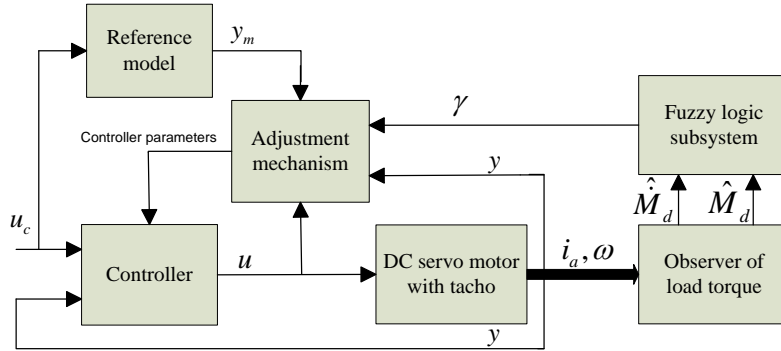
### 3. FUZZY MRAC OF VELOCITY SERVO SYSTEM

#### 3.1. Concept of fuzzy MRAC

It is known that external disturbances significantly influence the convergence rates of controller parameters. The main external disturbance for speed servo system is the varying load torque on the motor shaft and it can be shown that convergence rates depend on disturbance and its dynamics. For example, the constant load torque can be effectively compensated with a small value of  $\gamma$ , while the compensation of small load torque of high frequency dynamics requires a significantly larger value  $\gamma$ . Depending on load torque and its dynamics some different rules can be formed based on experience, but the exact mathematical solution cannot be easily found. This is one of the required prerequisites for fuzzy logic subsystem design: the system can be described through set of rules based on experience, while its mathematical model is too complicated or does not exist at all [18]. This fact was the main motivation to include a special fuzzy logic subsystem (FLsS) in the control loop.

#### 3.2. Application of fuzzy MRAC

The block diagram of a velocity servo system with a fuzzy model reference adaptive control is shown in Fig. 2.



**Fig. 2** Block diagram of fuzzy model reference adaptive control

The performance of the system is given by the first order reference model:

$$G_{ref}(s) = \frac{Y_m(s)}{U_c(s)} = \frac{10}{0.01s+1}. \quad (13)$$

The perfect model reference following the controller (4) is designed with parameters adjustment rules (9) and (10) and fuzzy logic based tuning of gain  $\gamma$ . DC servo motor transfer function with armature voltage  $u$  as input and tachometer voltage  $u_{tg}$  as output is given with:

$$G_m(s) = \frac{U_{tg}(s)}{U(s)} = \frac{k_m k_{tg}}{T_m s + 1}, \quad (14)$$

where  $k_m = k_{em} / (R_a F_e + k_{em} k_{me})$  and  $T_m = J_e R_a / (R_a F_e + k_{em} k_{me})$  are DC motor static gain and time constant, respectively [19]. The values of tacho constant  $k_{tg}$  and DC motor electrical and mechanical parameters are previously identified [20] and shown in Table 1.

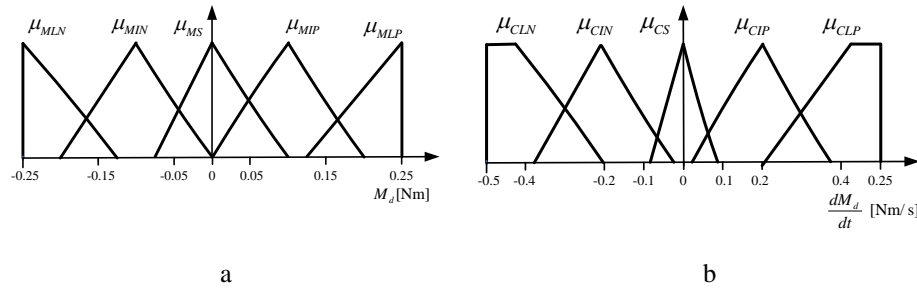
**Table 1** Parameters of DC motor and tacho

Parameter		Value
Armature resistance	$R_a$	8.91 $\Omega$
Armature inductivity	$L_a$	4.5 mH
Moment of inertia	$J_e$	2.93e-5 kg m <sup>2</sup>
Coefficient of viscous friction	$F_e$	11.7e-5 kg m <sup>2</sup> /rad/s
Electromechanical constant	$k_{em}$	0.103 Nm/A
Mechanical-electrical constant	$k_{me}$	0.103 V/rad/s
Tacho constant	$k_{tg}$	0.0191 V/rad/s

The observer for load torque and its first derivative estimation is designed based on DC motor moment equation:

$$\begin{aligned} \hat{M}_d(t) &= k_{em} i_a(t) - J_e \frac{d\omega(t)}{dt} - F_e \omega(t) \\ \hat{\dot{M}}_d(t) &= \frac{d\hat{M}_d(t)}{dt} \end{aligned} \quad (15)$$

where  $i_a(t)$  and  $\omega(t)$  are measured armature current and shaft angular velocity, respectively. The estimated values  $\hat{M}_d$  and  $\hat{\dot{M}}_d$  are inputs to fuzzy subsystem for  $\gamma$  tuning, and the corresponding membership functions are shown in Fig. 3. The linguistic variable  $\hat{M}$  (load torque) is described by five membership functions: *small* (MS), *intermediate positive* (MIP), *large positive* (MLP), *intermediate negative* (MIN) and *large negative* (MLN). Linguistic variable  $\hat{CM}$  (the first derivation of load torque) is defined by membership functions: *small* (CS), *intermediate positive* (CIP), *large positive* (CLP), *intermediate negative* (CIN) and *large negative* (CLN).



**Fig. 3** Membership functions of: a) linguistic variable  $\hat{M}$  and b) linguistic variable  $\hat{CM}$

The developed FLsS is of the Takagi–Sugeno type, with two inputs:  $\hat{M}_d$  and  $\dot{\hat{M}}_d$ , and one output, adaptation gain  $\gamma$ . For T and S norm, the minimum and maximum method was selected, respectively [21].

The fuzzy rules base for suggested FLsS for adaptation gain selection is shown in Table 2. The set of rules is comprised of 25 rules, and the rules are defined experimentally, based on repeated simulations with different values of  $\gamma$ . FLsS output  $\gamma$  is nonnegative scalar, and can assume values from  $\gamma_{\min} = 0.005$  to  $\gamma_{\max} = 0.25$ .

**Table 2** Fuzzy rules base of FLS

	<i>MLN</i>	<i>MIN</i>	<i>MS</i>	<i>MIP</i>	<i>MLP</i>
<i>CLN</i>	$0.6\gamma_{\max}$	$0.8\gamma_{\max}$	$0.9\gamma_{\max}$	$0.8\gamma_{\max}$	$0.4\gamma_{\max}$
<i>CIN</i>	$10\gamma_{\min}$	$0.6\gamma_{\max}$	$0.4\gamma_{\max}$	$0.6\gamma_{\max}$	$2\gamma_{\min}$
<i>CS</i>	$2\gamma_{\min}$	$10\gamma_{\min}$	$0.4\gamma_{\max}$	$10\gamma_{\min}$	$\gamma_{\min}$
<i>CIP</i>	$10\gamma_{\min}$	$0.8\gamma_{\max}$	$0.4\gamma_{\max}$	$0.6\gamma_{\max}$	$2\gamma_{\min}$
<i>CLP</i>	$0.6\gamma_{\max}$	$\gamma_{\max}$	$\gamma_{\max}$	$\gamma_{\max}$	$0.4\gamma_{\max}$

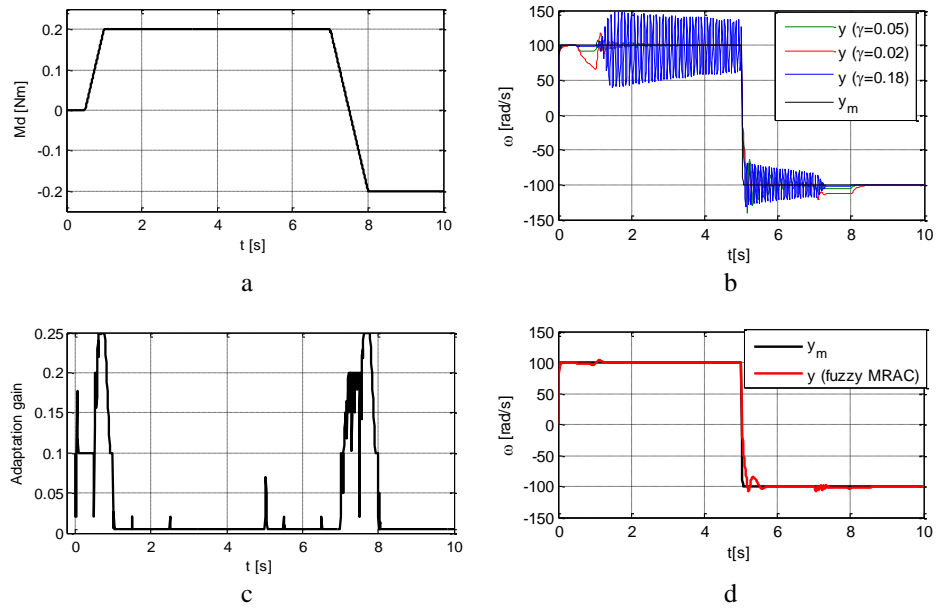
### 3.3. Simulation results

Based on MATLAB/Simulink® simulation models, the MRAC and proposed fuzzy MRAC of the velocity servo system with parameters given in Table 1 are compared. Performances of tracking of square reference with magnitude of  $\pm 100$  rad/s and period of 10s are analyzed.

In the first case the trapezoidal load, shown in Fig. 4a, is applied on a motor shaft. Responses of the speed servo system with MRAC and fuzzy MRAC are presented in Fig. 4b and Fig. 4d. It can be seen that during the transient of the load disturbance, when it has a constant rate of change, MRAC with greater  $\gamma$  provides smaller reference tracking error, but when load disturbance becomes of constant value, the response is more oscillatory. The fuzzy MRAC has better reference tracking performances during both periods in load disturbance profile, due to fuzzy adjustment of  $\gamma$  in which the information of load disturbance derivative is included. The change of  $\gamma$  is shown in Fig. 4c.

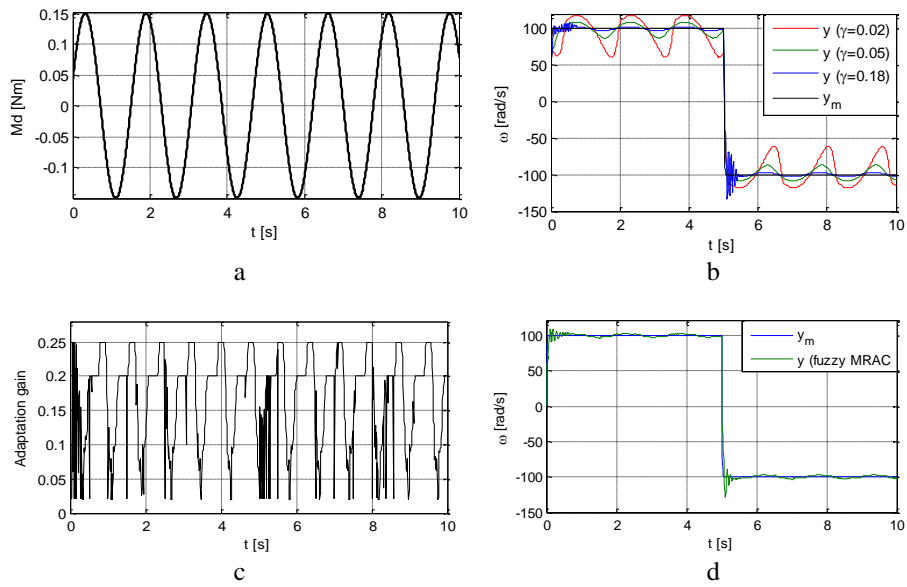
The tracking performances in the presence of the sinusoidal load disturbance with angular frequency of 4 [rad/s], presented in Fig. 5a, are also analysed. The responses of MRAC and proposed controller are shown in Fig. 5b and Fig. 5d, respectively. It can be noted that MRAC with greater  $\gamma$  almost completely eliminates the influence of load disturbance of steady state, but the transient is more oscillatory. The proposed controller enables acceptable overshoot and steady state reference tracking performances. The adaptation gain for this case is shown in Fig 5c.

The integral of absolute error  $e = y - y_m$  for all cases with trapezoidal load and sinusoidal load disturbance is summarized in Table 3, and the results confirmed the advantages of the proposed fuzzy MRAC controller.



**Fig. 4** Simulation results for tracking reference model:

a) load torque, b) MRAC for different  $\gamma$ , c) Change  $\gamma$  of Fuzzy MRAC, d) Fuzzy MRAC



**Fig. 5** Simulation results for tracking reference model:

a) load torque, b) MRAC for different  $\gamma$ , c) Change  $\gamma$  of Fuzzy MRAC, d) Fuzzy MRAC



Table 3 Integral absolute error

	MRAC			Fuzzy MRAC
	$\gamma=0.02$	$\gamma=0.05$	$\gamma=0.18$	
Trapezoidal load	43.4	37.5	149.2	15.1
Sinusoidal load	182.6	70	27.5	24.5

#### 4. EXPERIMENTAL VALIDATION

The experimental validation of simulation results is realized with a laboratory velocity servo system. In Fig. 6 the experimental setup is shown. A DC servo motor with outputs for angular rate and armature current signals is used. The motor is equipped with a magnetic brake for variable load torque generating. The communication between the personal computer and the DC servo motor is provided with the acquisition card DT 9812. The control signals from the acquisition card before applying to the armature of the motor are amplified by the power amplifier.



Fig. 6 Experimental setup

MRAC and fuzzy MRAC are designed in MATLAB/Simulink® environment. The Simulink model of the proposed fuzzy MRAC is shown in Fig. 7.

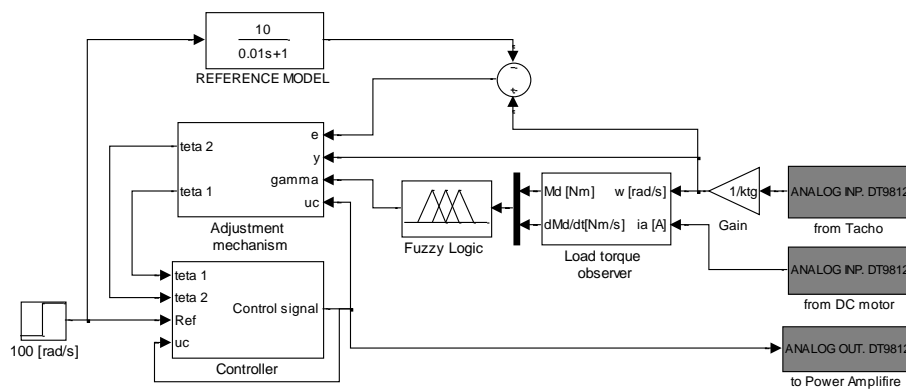
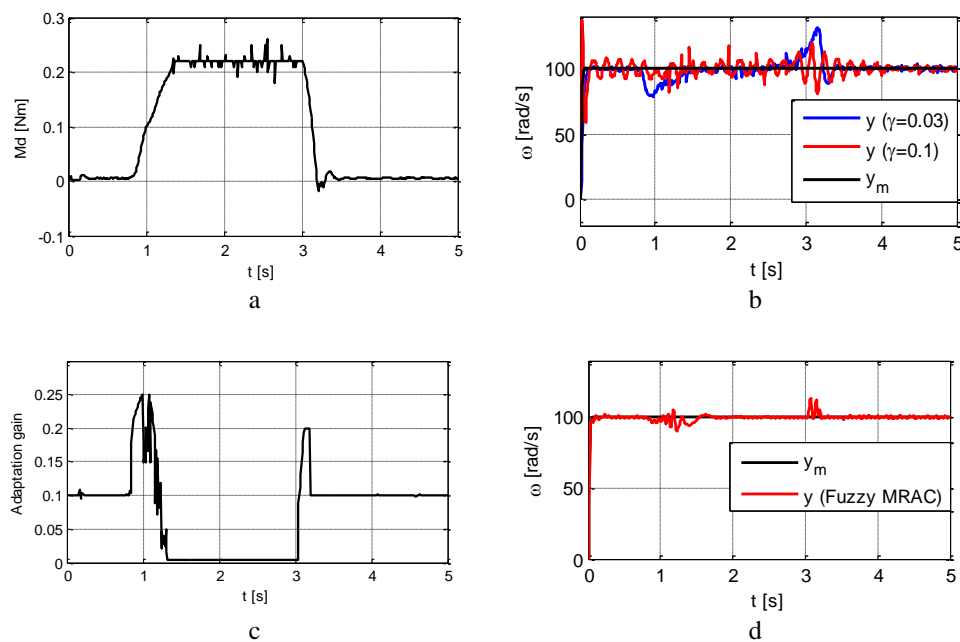


Fig. 7 Simulink model of fuzzy MRAC

The step reference with magnitude of 100 rad/s is software generated. The signals of tacho and DC motor armature current from the acquisition card are introduced in Simulink environment by analog input blocks. The control signal from the controller is passed to acquisition card by analog output block.

Varying load torque, generated by magnetic brake, is estimated with the observer and is shown in Fig. 8a. Angular rate of motor shaft is acquired and graphically presented in Fig 8b and 8d. From figures it can be seen that the experimental results are very similar to the simulation results. Speed servo system performances are much better with the proposed fuzzy MRAC then with conventional MRAC. In Fig. 8c the changing of the adaptation gain  $\gamma$  is shown.



**Fig. 8** Experimental results for tracking reference model:

a) load torque, b) MRAC for different  $\gamma$ , c) Change  $\gamma$  of Fuzzy MRAC, d) Fuzzy MRAC

## 5. CONCLUSION

The synthesis procedure of fuzzy logic model reference adaptive control (MRAC) is realized in this paper. Fuzzy MRAC is suitable for use in industrial control applications under all disturbance conditions. The implementation of the proposed control algorithm is analysed on the laboratory velocity servo system where the varying load torque has the main influence on system performances. The influence of varying load disturbance is compensated by changing the adaptation gain parameter by using a relatively simple T-S fuzzy logic subsystem. Some simulation results show the advantages of the fuzzy MRAC concept. The experimental validation confirms the simulation results.

**Acknowledgement:** *The paper is a part of the research supported by the Ministry of Education, Science and Technology Development within the project III44004 (2011-2014).*

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