FACTA UNIVERSITATIS
Series: **Automatic Control and Robotics** Vol. 15, N° 1, 2016, pp. 33 - 42

ANALYTICAL APPROACH IN ESTIMATING ERROR PERFORMANCE OF PARTIALLY COHERENT PSK RECEIVER OVER KAPPA-MU FADING

UDC (621.395.38:519.724)

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Abstract. *In this paper, a novel analytical expression for the symbol error probability (SEP) of M-ary Phase-Shift Keying (MPSK) signal transmission is derived, by using the Fourier series method (FSM). The signal is transmitted over κ-μ fading channel. The hardware imperfections in PSK demodulator are taken into consideration, and are presented through the phase noise described by Tikhonov distribution. Based on the derived SEP expression, numerical results are presented and discussed. It is illustrated that the existence of phase noise leads to the irreducible SEP floor which degrades the system performance to a large extent.*

Key words: *κ-μ distribution, M-ary Phase-Shift Keying (MPSK), phase noise, symbol error probability (SEP), Tikhonov distribution.*

1. INTRODUCTION

The phenomenon of fading, which represents the variation of instantaneous value of the received signal, is one of the main reasons for degradation of the system performance. In order to properly describe the different propagation environments imposed by fading phenomena, many statistical models have been proposed in technical literature [1]. The κ-μ distribution has recently been proposed and adopted as a convenient model for describing linear, line-of-sight (LOS) environments with several clusters [2]−[5]. It is also proved to be a general model, since many other fading distributions can be obtained for different distribution parameters. For example, when κ=0 κ-μ distribution is reduced to

Received December 31, 2015
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the Nakagami-m distribution, for μ=1 it approximates Rice distribution, and for κ=0 and μ=1, it is reduced to the Rayleigh distribution.

In order to provide high spectral efficiency, the M-ary phase-shift keying (MPSK) modulation is commonly used in practical communication systems. The main flaw of the systems based on PSK modulation is the imperfect extraction of the carrier reference phase at the receiver during coherent detection [1], [6]. Although the main cause of the performance degradation is the existence of additive white Gaussian noise (AWGN), as well as the unfavorable fading channel conditions, the hardware imperfections of PSK modulator and demodulator are also of great importance. The PSK modulator imperfections are usually weaker than the PSK demodulator imperfections. The main reason for the PSK demodulator imperfections is the imperfect reference signal recovery when coherent detection is performed, which is presented through the existence of phase noise [7]−[9].

Considering MPSK signal transmission influenced by κ-μ fading channel, the novel symbol error probability (SEP) expression is derived. The hardware imperfections in PSK demodulator represented through the phase noise are taken into account. The Tikhonov probability density function (PDF) is used for describing the impact of the phase noise. The SEP expression is derived by using the Fourier series method (FSM) presented in [7]−[9]. Furthermore, the SEP expression for the system under investigation, when the phase noise is small and neglected, is also derived. On the basis of the derived expressions, numerical results are presented.

The rest of the paper is organized as follows. Section 2 describes the system and channel model. The average SEP analysis is presented in Section 3. Numerical results with discussions are offered in Section 4. Section 5 gives some concluding remarks.

2. System and Channel Model

The analysis of the system under investigation considers the PSK signal transmission over the κ-μ fading channel. Since κ-μ distribution is general and can be reduced to simpler cases, such as Nakagami-m, Rayleigh, and Rice fading distributions, the following analysis is convenient for describing PSK signal transmission influenced by different fading conditions. Beside the impact of κ-μ fading, the PSK system performance analysis takes into account hardware imperfections represented through phase noise.

It is assumed that x denotes the fading envelope of the desired signal, which experiences κ-μ distribution, so the PDF of x is given by [2]

$$f_x(x) = \frac{2\mu(\kappa+1)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\kappa\mu)\Omega^{\frac{\mu+1}{2}}} x^{\mu} \exp\left(-\frac{\mu(\kappa+1)}{\Omega}x^2\right) I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)}{\Omega}}x\right), \quad (1)$$

where κ and μ are Rice factor and fading severity factor of the signal envelope respectively, Ω is the local mean power of x, and $I_v(\cdot)$ denotes the modified Bessel function of the first kind and order v [10, eq. (8.406)].

After applying the standard technique of transforming random variables, the PDF of the instantaneous signal-to-noise-ratio (SNR) is derived by using $f_\gamma(\gamma) = \left. \frac{f_x(x)}{|\partial\gamma/\partial x|} \right|_{x=\sqrt{\gamma}}$ as

$$f_\gamma(\gamma) = \frac{\mu(\kappa+1)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}} \exp(\kappa\mu)\bar{\gamma}^{\frac{\mu+1}{2}}} \gamma^{\frac{\mu-1}{2}} \exp\left(-\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\right) I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)\gamma}{\bar{\gamma}}}\right), \tag{2}$$

where γ denotes the instantaneous SNR per symbol of κ-μ distribution, and $\bar{\gamma}$ represents the average SNR per symbol. Since the considered system employs M-ary PSK (M is modulation order), note that $\gamma_b = \gamma/\log_2 M$ and $\bar{\gamma}_b = \bar{\gamma}/\log_2 M$, where γ_b and $\bar{\gamma}_b$ are the instantaneous and the average SNR per bit, respectively.

The hardware imperfections are represented through the phase noise φ, which is modelled by the Tikhonov PDF given by [7]−[9]

$$f_\varphi(\varphi) = \frac{\exp(\cos\varphi/\sigma_\varphi^2)}{2\pi I_0(1/\sigma_\varphi^2)}, \quad |\varphi| \le \pi, \tag{3}$$

where σ_φ^2 is the variance of the phase error. Since the following analysis is performed by using FSM method, the Tikhonov PDF is presented in the Fourier series form as [7]−[9]

$$f_\varphi(\varphi) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} c_n \cos(n\varphi), \quad |\varphi| \le \pi, \tag{4}$$

where the Fourier coefficient is defined as [7]−[9]

$$c_n = \frac{I_n(1/\sigma_\varphi^2)}{\pi I_0(1/\sigma_\varphi^2)}. \tag{5}$$

3. The SEP Analysis

In order to determine the analytical expression for the average SEP of the system under consideration, the FSM is applied, based on the PDF of the phase ψ of the composite received signal given in the Fourier series form as [7]−[9]

$$f_\psi(\psi) = \frac{1}{2\pi} + \sum_{n=1}^{\infty} b_n \cos(n\psi), \quad |\psi| \le \pi, \tag{6}$$

where the Fourier coefficient b_n is dependent on the fading channel conditions. For the considered system influenced by κ-μ fading, the PDF of the phase ψ can be obtained as

$$f_\psi(\psi) = \int_0^\infty f_\psi(\psi/\gamma) f_\gamma(\gamma)\, \mathrm{d}\gamma, \tag{7}$$

where $f_\gamma(\gamma)$ represents the PDF of the instantaneous SNR given by eq. (2), and $f_\psi(\psi/\gamma)$ is a Fourier series form of the conditional PDF of the received signal phase due to AWGN, defined as

$$f_\psi(\psi/\gamma)=\frac{1}{2\pi}+\sum_{n=1}^{\infty}a_n(\gamma)\cos(n\psi),\quad |\psi|\le\pi. \tag{8}$$

The Fourier coefficient for AWGN $a_n(\gamma)$ is presented as [9]

$$a_n(\gamma)=\frac{1}{n!\pi}\Gamma\left(\frac{n}{2}+1\right)\gamma^{\frac{n}{2}}\exp(-\gamma)\,{}_1F_1\left(\frac{n}{2}+1;n+1;\gamma\right), \tag{9}$$

where $\Gamma(\cdot)$ is Gamma function defined as [10, eq. (8.310.1)] and ${}_1F_1(\cdot;\cdot;\cdot)$ is the confluent hypergeometric function defined as [10, eq. (9.21)].

After substituting eqs. (2), (8) and (9) into (7), the PDF of the phase ψ is obtained as

$$\begin{aligned}f_\psi(\psi)=&\frac{1}{2\pi}+\frac{\mu(\kappa+1)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\kappa\mu)\bar{\gamma}^{\frac{\mu+1}{2}}}\sum_{n=1}^{\infty}\frac{1}{n!\pi}\Gamma\left(\frac{n}{2}+1\right)\cos(n\psi)\\&\times\int_0^{\infty}\gamma^{\frac{n}{2}+\frac{\mu-1}{2}}\exp\left(-\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\right)\exp(-\gamma)\,{}_1F_1\left(\frac{n}{2}+1;n+1;\gamma\right)I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)\gamma}{\bar{\gamma}}}\right)\mathrm{d}\gamma.\end{aligned} \tag{10}$$

In order to solve the previous integral, after utilization [11, eq. (03.02.06.0002.01)], the modified Bessel function of the first kind in eq. (10) is transformed as

$$I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(\kappa+1)\gamma}{\bar{\gamma}}}\right)==\sum_{d=0}^{\infty}\frac{1}{d!\Gamma(d+\mu)}\left(\mu\sqrt{\frac{\kappa(\kappa+1)\gamma}{\bar{\gamma}}}\right)^{2d+\mu-1}, \tag{11}$$

so the PDF of the phase ψ in eq.(10) is now given in the form

$$\begin{aligned}f_\psi(\psi)=&\frac{1}{2\pi}+\sum_{d=0}^{\infty}\sum_{n=1}^{\infty}\frac{\mu^{2d+\mu}(\kappa+1)^{d+\mu}\kappa^{d}}{\pi n!d!\Gamma(d+\mu)\exp(\kappa\mu)\bar{\gamma}^{d+\mu}}\Gamma\left(\frac{n}{2}+1\right)\cos(n\psi)\\&\times\int_0^{\infty}\gamma^{d+\frac{n}{2}+\mu-1}\exp\left(-\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\right)\exp(-\gamma)\,{}_1F_1\left(\frac{n}{2}+1;n+1;\gamma\right)\mathrm{d}\gamma.\end{aligned} \tag{12}$$

The exponential function in eq. (12) is represented in terms of Meijer's *G* function [10, eq. (9.301)] by using [11, eq. (01.03.26.0004.01)] as

$$\exp\left(-\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\right)=G_{0,1}^{1,0}\left(\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\,\middle|\,\begin{matrix}-\\0\end{matrix}\right), \tag{13}$$

while the product of the exponential function and the confluent hypergeometric function with the same argument is transformed in terms of Meijer's *G* function by [11, eq. (07.20.26.0015.01)] as

$$\exp(-\gamma)\,{}_1F_1\left(\frac{n}{2}+1;n+1;\gamma\right)=\frac{\Gamma(n+1)}{\Gamma\left(\frac{n}{2}\right)}G_{1,2}^{1,1}\left(\gamma\left|\begin{matrix}1-\frac{n}{2}\\0,\quad -n\end{matrix}\right.\right). \tag{14}$$

After replacing eqs. (13) and (14) into eq. (12), and applying [11, eq. (06.05.16.0002.01)] as $\Gamma\left(\frac{n}{2}+1\right)=\frac{n}{2}\Gamma\left(\frac{n}{2}\right)$, the PDF of the phase ψ is obtained as

$$\begin{aligned}f_\psi(\psi)=\frac{1}{2\pi}+\sum_{d=0}^{\infty}\sum_{n=1}^{\infty}\frac{n\Gamma(n+1)\mu^{2d+\mu}(\kappa+1)^{d+\mu}\kappa^d}{2\pi n!d!\Gamma(d+\mu)\exp(\kappa\mu)\bar{\gamma}^{d+\mu}}\cos(n\psi)\\ \times\int_0^\infty\gamma^{d+\frac{n}{2}+\mu-1}G_{0,1}^{1,0}\left(\frac{\mu(\kappa+1)}{\bar{\gamma}}\gamma\left|\begin{matrix}-\\0\end{matrix}\right.\right)G_{1,2}^{1,1}\left(\gamma\left|\begin{matrix}1-\frac{n}{2}\\0,\quad -n\end{matrix}\right.\right)\mathrm{d}\gamma.\end{aligned} \tag{15}$$

The integral in eq. (15) is solved by using [11, eq. (07.34.21.0011.01)], so the final form of the PDF of the phase ψ is derived as

$$\begin{aligned}f_\psi(\psi)=\frac{1}{2\pi}+\sum_{n=1}^{\infty}\sum_{d=0}^{\infty}\frac{n\Gamma(n+1)e^{-k\mu}\mu^{d-\frac{n}{2}}k^d(k+1)^{-\frac{n}{2}}}{2\Gamma(d+\mu)d!n!\pi\bar{\gamma}^{-\frac{n}{2}}}\\ \times G_{2,2}^{1,2}\left(\frac{\bar{\gamma}}{\mu(k+1)}\left|\begin{matrix}1-\frac{n}{2},\quad 1-d-\frac{n}{2}-\mu\\0,\quad -n\end{matrix}\right.\right)\cos(n\psi).\end{aligned} \tag{16}$$

Representing the previous PDF in the Fourier series form as in eq. (6), the Fourier coefficient b_n is derived as

$$b_n=\sum_{d=0}^{\infty}\frac{n\Gamma(n+1)e^{-k\mu}\mu^{d-\frac{n}{2}}k^d(k+1)^{-\frac{n}{2}}}{2\Gamma(d+\mu)d!n!\pi\bar{\gamma}^{-\frac{n}{2}}}G_{2,2}^{1,2}\left(\frac{\bar{\gamma}}{\mu(k+1)}\left|\begin{matrix}1-\frac{n}{2},\quad 1-d-\frac{n}{2}-\mu\\0,\quad -n\end{matrix}\right.\right). \tag{16}$$

Since a perfect local reference phase is not realizable in most practical scenarios, the impact of the phase noise is considered. The conditional SEP of MPSK is found as [8]

$$P_M(\varphi)=1-\int_{\varphi-\pi/M}^{\varphi+\pi/M}f_\psi(\psi)\,\mathrm{d}\psi=1-\frac{1}{M}-\sum_{n=1}^{\infty}\frac{2b_n}{n}\sin\left(\frac{n\pi}{M}\right)\cos(n\varphi). \tag{17}$$

After averaging the previous conditional SEP over φ, the final form of the average SEP of MPSK signal transmission, in the presence of Tikhonov distributed phase error over κ-μ fading channel can be found as

$$P_M=\int_{-\pi}^{\pi}P_M(\varphi)f_\varphi(\varphi)\,\mathrm{d}\varphi=1-\frac{1}{M}-\sum_{n=1}^{\infty}\frac{2\pi b_n c_n}{n}\sin\left(\frac{n\pi}{M}\right). \tag{18}$$

When the phase noise is very small and can be neglected, the SEP of MPSK system can be easily obtained as [8]

$$P_M(\varphi=0)=1-\int_{-\pi/M}^{\pi/M} f_\psi(\psi)\,\mathrm{d}\psi = 1-\frac{1}{M}-\sum_{n=1}^{\infty}\frac{2b_n}{n}\sin\left(\frac{n\pi}{M}\right). \tag{19}$$

4. Numerical Results

In this section, numerical results obtained by the derived expression are presented. The average SEP dependences on modulation order M, phase noise standard deviation σ_φ and κ-μ distribution parameters are observed.

Fig. 1 shows QPSK SEP dependence on average SNR per bit for different values of the phase error standard deviation. The system performance degradation is noticed when the value of σ_φ is greater, meaning that the impact of the phase noise is stronger. Furthermore, the unrecoverable error rate floor exists, which is an important limiting factor that should be taken into consideration during the PSK system design. The error rate floor is evident at lower values of SNR per bit when the parameter σ_φ has greater values.

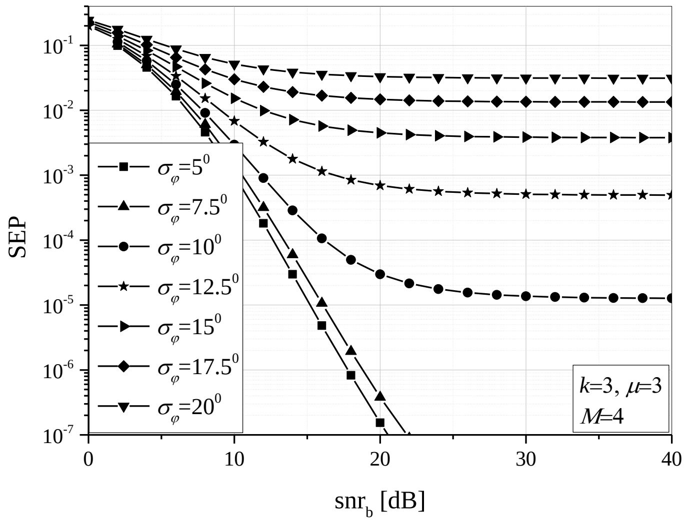

Fig. 1 QPSK SEP dependence on average SNR per bit for different values of σ_φ

The SEP dependence on average SNR per bit for different values of the phase error standard deviation, considering BPSK, QPSK and 8PSK modulation, is presented in Fig. 2. It can be noticed that the influence of the phase noise is more expressed when the modulation order is higher. Observing the results obtained for BPSK signal transmission, it can be concluded that the SEP values are almost equal when the phase error standard deviation is $\sigma_\varphi=5^0$ and $\sigma_\varphi=15^0$, in the range of low values of SNR_b. Furthermore, the existence of error rate floor is noticed, appearing firstly when the phase noise is stronger and/or with greater modulation order.

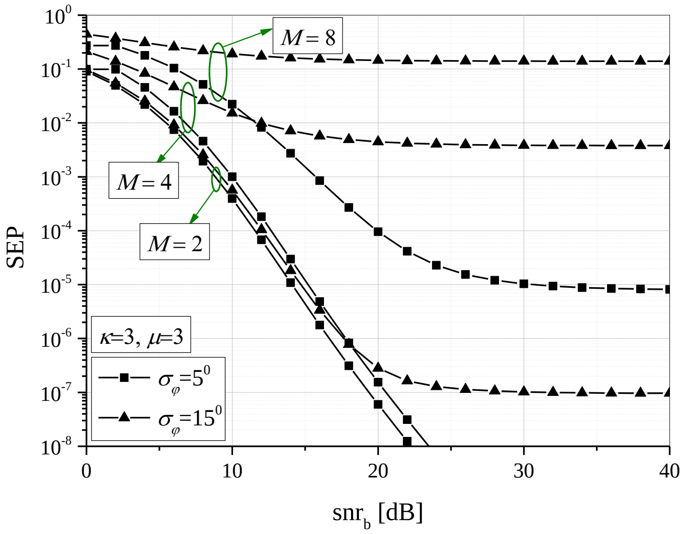

Fig. 2 MPSK SEP dependence on average SNR per bit for different values of σ_φ parameters and various modulation orders

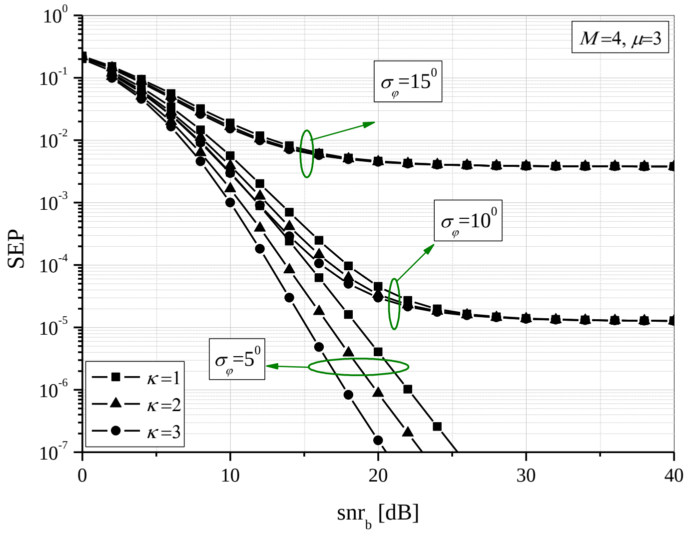

Fig. 3 QPSK SEP dependence on average SNR per bit for different values of σ_φ and parameter κ

Fig. 3 presents QPSK SEP dependence on average SNR per bit for different values of the κ-μ distribution parameter κ. The influence of hardware imperfections is determined by the phase noise standard deviation, which takes values σ_φ=5^0, 10^0 and 15^0. When the parameter κ takes greater values, the feding strength is weaker and system has better performance. In addition, it can be observed that the impact of feding strength, determined by the parameter κ, is more dominant when the phase noise standard deviation is lower and hardware imperfections are smaller. With greater values of σ_φ, the phase noise is stronger, and the parameter κ has a minimal impact on the average SEP performance. In addition, there exists the error floor, which is not dependent on the value of the parameter κ.

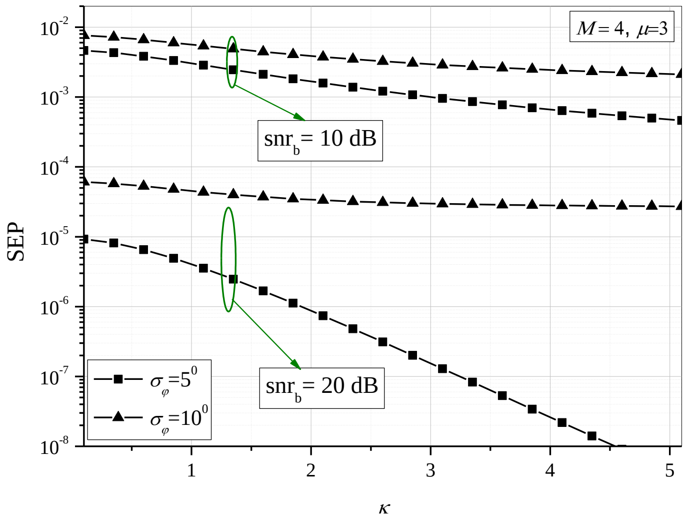

Fig. 4 QPSK SEP dependence on the fading parameter κ for different values of average SNR per bit and the parameter σ_φ

Fig. 4 shows QPSK SEP dependence on the fading parameter κ for different values of average SNR per bit, when the phase noise standard deviation is $\sigma_\varphi=5^0$ and $\sigma_\varphi=10^0$. As it has already been concluded, greater values of the parameter κ correspond to the weaker fading and better conditions for transmission. Also, when the average SNR per bit is greater, system has better performance. Furthermore, it can be observed that the influence of the phase noise is more dominant when the average SNR is greater. The SEP curve for $\sigma_\varphi=10^0$ and SNR_b= 20 dB has almost a constant value (there is a slight improvement with increasing the parameter κ). When the phase noise is very strong, the value of the parameter κ has a minor impact on the system performance determination. This is in agreement with the conclusions from Fig. 3.

The MPSK SEP dependence on the phase error standard deviation for different values of the average SNR per bit is observed in Fig. 5, assuming BPSK, QPSK and 8PSK signal transmission. Since there are no changes in SEP values when $\sigma_\varphi \rightarrow 0°$, the phase noise is weak and can be neglected. This SEP value is equal to the SEP value of the system without phase noise. The increasing of σ_φ results in the SEP performance worsening, so the hardware imperfections in PSK demodulator are an important factor during the determination of the system performance. The greater values of the fading parameters κ and μ correspond to the weaker fading conditions, which reflects in better system performance.

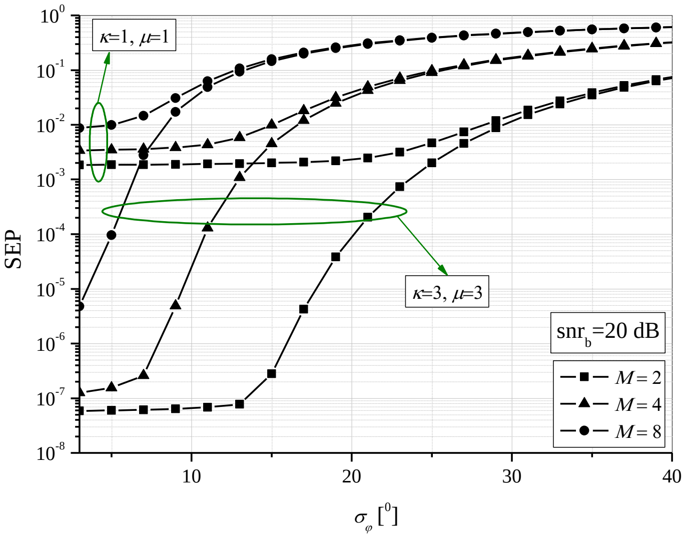

Fig. 5 MPSK SEP dependence on σ_φ for different values of the fading parameters and various modulation orders

5. Conclusion

In this paper, we derive a novel analytical SEP expressions of MPSK signal detection, which is influenced by κ-μ fading channel. The hardware imperfections in PSK demodulator are considered, with the phase noise modeled by Tikhonov distribution. The presented system performance analysis is based on FSM. The derived SEP expression is reduced to the case when the phase noise is small and neglected. On the basis of the derived results, numerical results are obtained and discussed. The influence of phase noise standard deviation, fading parameters and PSK modulation order is observed. From the presented results, it is concluded that the existence of hardware imperfections in PSK demodulator can cause important system performance deterioration, especially when a higher PSK order is considered. Furthermore, the unrecoverable error rate floor appears as a consequence of the phase noise existence, which can seriously limit the SEP performance of the observed system.

Acknowledgement: *This paper was supported by Ministry of Education, Science and Technology Development of the Republic of Serbia under Grants TR-32028 and III-44006.*

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