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CONTROL OF FLOW AND HEAT TRANSFER USING SUCTION, MAGNETIC AND ELECTRIC FIELD

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Abstract. *Flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient or constant flow rate has been considered in the paper. Effects of magnetic field, suction/injection and load factor have been studied in order to control the flow rate, shear stress and heat transfer on the plates. Applied magnetic field is perpendicular to the plates, the channel plates are electrically insulated and through the plates perpendicular to the surface the fluid of the same physical characteristics as the fluid in the basic flow is injected or ejected. An exact solution of governing equation has been obtained in a closed form. The influences of each of the governing parameters on flow rate, shear stress and heat transfer are discussed with the aid of graphs.*

Key words: *MHD, suction-injection, temperature, velocity, porous plates*

1. INTRODUCTION

Since the last century, many researchers are interested in magnetohydrodynamics MHD, due to its important applications in different scientific and technology fields. The application of outer magnetic and electric field appears in power plants, flow measurement, nuclear fusion reactor, cooling blankets, accelerators, MHD pumps etc.

The exact solutions of steady MHD flows are available only for some simple geometries subject to simple boundary conditions [1,2]. The geometry of the section has been taken as a circle, rectangle, ellipse, sector, etc. Hartmann [3] and Hartmann and Lazarus [4] wrote the first experimental and theoretical work on MHD channel flow. After this work there were several other investigators and works which carried out model studies on MHD channel flow [5,6]. Shercliff [7] was the first one who noticed that for

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high Hartmann numbers (*Ha*) the velocity distribution consists of a uniform core with a boundary layer near the walls. After solving the problem of rectangular duct, this result enabled him to solve the problem for a circular pipe in an approximate manner for large *Ha* assuming walls of zero conductivity and, subsequently, walls with small conductivity. Gold [8] has obtained an analytical solution for the MHD flow in a circular tube with zero wall conductivity while Gardner and Lykoudis [9] experimentally acquired some results for circular tube without heat transfer.

All the above studies investigated the problem of flow in electrically conducting fluid under the influence of a magnetic field, and heat transfer was not considered. The phenomenon of heat transfer plays an important role in many industrial processes, in the production and transformation of energy, then the protection of elements in high-temperature regions, in the design of propulsion systems and cryogenic technology. For these reasons, the problems of heat transfer in the MHD flow of electrically conducting fluid channels and control of heat transfer using suction, magnetic and electric field are becoming very popular and important for the study.

The problem of heat transfer for steady MHD flow in a rectangular channel with wall temperature discontinuity, was numerically discussed by Singh and Lal [10]. Petrykowski and Walker [11] considered the MHD flows of liquid metals in rectangular channels, where the lower and upper walls were considered non-conductive, while the side walls were ideally conductive. They presented solutions related to fluid layers that are parallel to the magnetic field. During the last years Singh with a group of co-authors [12,13] considered the problems of the effect of magnetic fields on the flat plate thermometer problem and unsteady MHD free convection in a channel with one moving wall and their symmetric or asymmetric heating.

Following the interest for control of flow and heat transfer [14-16] several interesting features have been considered in the paper: effects of magnetic field and suction/injection on the flow rate, velocity profile, temperature and skin friction at the walls.

2. Mathematical Analysis

As mentioned in the introduction, flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient has been considered in the paper. Fully developed flow takes place between parallel plates that are at a distance h, as shown in Figure 1. Electrically conductive fluid flows through the channel due to the constant pressure gradient where the velocity has one component only depending on the coordinate x. Channel and the fluid are exposed to the externally applied magnetic field. The walls are non-conductive, while their temperature is maintained at constant values T_{w1} and T_{w2}.

The fluid velocity $\mathbf{v}$, magnetic field $\mathbf{B}$ and electric field $\mathbf{E}$ are:

$$\mathbf{v}=(u(y),-v_0,0)\,;\,\mathbf{B}=(B_x(y),B_0,0)\,;\,\mathbf{E}=(0,0,E_z)\,. \tag{1}$$

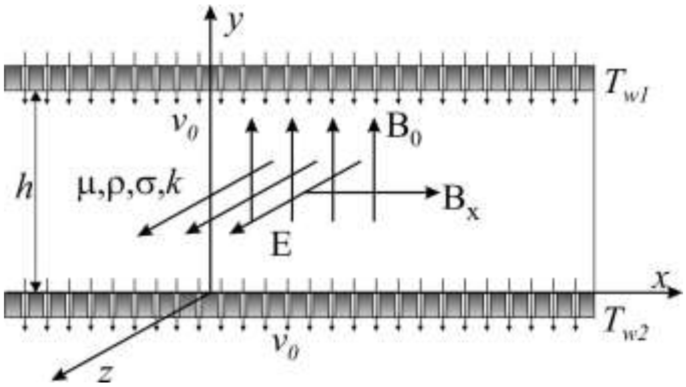

Fig. 1 Physical model and coordinate system

Described laminar MHD flow and heat transfer is mathematically presented with following equations:

$$\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}=-\frac{1}{\rho}\nabla p+\frac{\mu}{\rho}\nabla^2\mathbf{v}+\frac{1}{\rho}\mathbf{j}\times\mathbf{B}\ , \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t}-\nabla\times(\mathbf{v}\times\mathbf{B})-\frac{1}{\sigma\mu_e}\nabla^2\mathbf{B}=0\ \ , \tag{3}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+\mathbf{v}\nabla T\right)=k\nabla^2 T+\mu\Phi+\frac{\mathbf{j}^2}{\sigma}\ , \tag{4}$$

where:

$$\Phi=2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)^2-\frac{2}{3}(\nabla\mathbf{v})^2. \tag{5}$$

All the symbols used in previous general equations, are well known for the theory of MHD flows. The last term of equation (4) is the magnetic body force and $\mathbf{j}$ is the current density vector defined by:

$$\mathbf{j}=\sigma(\mathbf{E}+\mathbf{v}\times\mathbf{B})\,. \tag{6}$$

Using the velocity, magnetic and electric field distribution as stated above in the equation (1), the equations (2) to (4) are as follows:

$$\frac{1}{\rho}P+\frac{\mu}{\rho}\frac{d^2u}{dy^2}+v_0\frac{du}{dy}-\frac{\sigma}{\rho}B_0(E_z+uB_0+v_0B_x)=0\ , \tag{7}$$

$$-\frac{\partial p}{\partial y}+\sigma v_0 B_x^2+\sigma B_0 u B_x+\sigma E_z B_x=0\,, \tag{8}$$

$$\frac{d^2B_x}{dy^2} + \sigma\mu_e v_0 \frac{dB_x}{dy} + \sigma\mu_e B_0 \frac{du}{dy} = 0\,, \tag{9}$$

$$\rho c_p v_0 \frac{\partial T}{\partial x} + k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma(E_z + uB_0 + v_0 B_x)^2 = 0\,, \tag{10}$$

where: $P = -\partial p/\partial x$ is pressure gradient, μ - dynamic viscosity, B_0 -intensity of applied magnetic field, B_x -induced magnetic field, ρ -fluid density, v_0 -suction velocity, σ -electrical conductivity of fluid, E_z -applied electric field, μ_e -magnetic permeability, k -thermal conductivity and c_p -specific heat capacity.

Boundary conditions are represented with the following equations:

$$u(0) = 0\,;\ u(h) = 0\,;\ B_x(0) = 0\,;\ B_x(h) = 0\,,\ T_1(h) = T_{w1}\,;\ T_2(-h) = T_{w2}\,. \tag{11}$$

It is evident from the equation (11), that the flow and thermal boundary conditions have been unchanged by the addition of electromagnetic fields. By the no slip conditions, the fluid velocities are equal to the plate's velocities and boundary conditions on temperature are isothermal conditions. Since we have assumed that the plate is electrically non-conducting, no electric current flows in the plates. Therefore, the magnetic field at the plate should be curl free, that is $\nabla \times \mathbf{B} = 0$. Thus, we have $-dB_x/dy = 0$ or $B_x = const$. This constant must be taken to be zero, since applied magnetic field is normal to the plates.

Introducing the dimensionless quantities as:

$$y^* = \frac{y}{h}\,;\ U_0 = \frac{h^2 P}{\mu}\,;\ u^* = \frac{u}{U_0}\,;\ b = \frac{B_x}{B_0}\,;\ \beta = \frac{v_0}{U_0}\,,$$

$$Re = \frac{U_0 \rho h}{\mu}\,;\ p^* = \frac{p}{p_0}\,;\ K = \frac{E_z}{U_0 B_0}\,;\ Ha = B_0 h\sqrt{\frac{\sigma}{\mu}}\,,$$

$$G = \frac{Ph^2}{\mu U_0}\,;\ Rm = U_0 h\sigma\mu_e\,;\ Eu = \frac{p_0}{\rho U_0^2}\,;\ Pr = \frac{c_p \mu}{k}\,,$$

$$\Theta = \frac{T - T_{w2}}{T_{w1} - T_{w2}}\,;\ Ec = \frac{U_0^2}{c_p(T_{w1} - T_{w2})}\,. \tag{12}$$

Equations (7-10) get the following form:

$$\frac{d^2u^*}{dy^{*2}} + \beta Re\frac{du^*}{dy^*} - Ha^2 u^* - \beta Ha^2 b = KHa^2 - G\,, \tag{13}$$

$$\frac{\partial p^*}{\partial y^*} = \frac{Ha^2}{ReEu}(\beta b^2 + bu^* + Kb)\,, \tag{14}$$

$$\frac{d^2b}{dy^{*2}} + \beta Rm\frac{db}{dy^*} + \beta Rm\frac{du^*}{dy^*} = 0\,, \tag{15}$$

$$\frac{d^2\Theta}{dy^{*2}} + \beta PrRe\frac{d\Theta}{dy^*} + PrEc\left(\frac{du^*}{dy^*}\right)^2 + Ha^2PrEc(K + u^* + \beta b)^2 = 0 . \tag{16}$$

Now the boundary dimensionless conditions for previous equations are:

$$u^*(0) = 0,\ u^*(1) = 0 ;\ b(0) = 0,\ \ b(1) = 0 ,\ \Theta(0) = 0,\ \ \Theta(1) = 1 . \tag{17}$$

The solution of transformed equations (13) to (16), with boundary conditions, has the following form:

$$u^*(y^*) = \frac{C_1}{r_1^2}\exp(r_1y^*) + \frac{C_2}{r_2^2}\exp(r_2y^*) + C_3y^* + C_4 , \tag{18}$$

$$b(y^*) = B_1\exp(r_1y^*) + B_2\exp(r_2y^*) - B_3y^* + B_4 , \tag{19}$$

$$\begin{aligned}\Theta\left(y^*\right) &= Z_1\exp(2r_1y^*) + Z_2\exp(2r_2y^*) + Z_3\exp((r_1 + r_2)y^*) + \\ &+ Z_4\exp(r_1y^*) + Z_5\exp(r_2y^*) + Z_6y^* - \frac{S_1}{q}\exp(-qy^*) + S_2 ,\end{aligned} \tag{20}$$

where:

$$r_1 = m_1 + m_2 ;\ r_2 = m_1 - m_2 ,$$

$$m_1 = -\frac{1}{2}\beta(Re + Rm) ;\ m_2 = \frac{1}{2}\sqrt{\beta(Re - Rm)^2 + 4Ha^2} ,$$

$$C_1 = R\frac{D_4 - D_2}{D_2D_3 - D_1D_4} ;\ C_2 = R\frac{D_1 - D_3}{D_2D_3 - D_1D_4} ,$$

$$C_3 = \frac{C_1}{r_1^2}\left[1 - \exp\left(r_1\right)\right] + \frac{C_2}{r_2^2}\left[1 - \exp\left(r_2\right)\right] ;\ C_4 = -\left(\frac{C_1}{r_1^2} + \frac{C_2}{r_2^2}\right) ,$$

$$D_1 = \left(A_1 + \frac{Ha^2 - \beta Re}{r_1^2}\right)\exp(r_1) + \frac{\beta Re}{r_1^2} ;\ \ D_2 = \left(A_2 + \frac{Ha^2 - \beta Re}{r_2^2}\right)\exp(r_2) + \frac{\beta Re}{r_2^2} ,$$

$$D_3 = A_1 + \frac{Ha^2 - \beta Re}{r_1^2} + \frac{\beta Re}{r_1^2}\exp(r_1) ;\ \ D_4 = A_2 + \frac{Ha^2 - \beta Re}{r_2^2} + \frac{\beta Re}{r_2^2}\exp(r_2) ,$$

$$A_1 = 1 + \beta\frac{Re}{r_1} - \frac{Ha^2}{r_1^2} ;\ A_2 = 1 + \beta\frac{Re}{r_2} - \frac{Ha^2}{r_2^2} ,$$

$$R = G - Ha^2K ,$$

$$B_1 = \frac{C_1A_1}{\beta Ha^2} ;\ B_2 = \frac{C_2A_2}{\beta Ha^2} ;\ B_3 = \frac{C_3}{\beta} ;\ B_4 = \frac{1}{\beta Ha^2}(\beta ReC_3 - Ha^2C_4 + R) ,$$

$$S_1 = q\frac{\chi_1 - \chi_2}{1-\exp(-q)};\; S_2 = \frac{\chi_1 \exp(-q) - \chi_2}{1-\exp(-q)},$$

$$q = \beta PrRe,$$

$$\chi_1 = Z_1 + Z_2 + Z_3 + Z_4 + Z_5,$$

$$\chi_2 = Z_1\exp(2r_1) + Z_2\exp(2r_2) + Z_3\exp(r_1+r_2) + Z_4\exp(r_1) + Z_5\exp(r_2) + Z_6 - 1,$$

$$Z_1 = \frac{\Im_1}{2r_1(2r_1+q)};\; Z_2 = \frac{\Im_2}{2r_2(2r_2+q)},\; Z_3 = \frac{\Im_3}{(r_1+r_2)(r_1+r_2+q)},$$

$$Z_4 = \frac{\Im_4}{r_1(r_1+q)};\; Z_5 = \frac{\Im_5}{r_2(r_2+q)};\; Z_6 = \frac{\Im_6}{q},$$

$$\Im_1 = -PrRe\left[\frac{C_1^2}{r_1^2}\left(1+\frac{Ha^2}{r_1^2}\right) + \beta Ha^2 B_1\left(\beta B_1 + 2\frac{C_1}{r_1^2}\right)\right],$$

$$\Im_2 = -PrRe\left[\frac{C_2^2}{r_2^2}\left(1+\frac{Ha^2}{r_2^2}\right) + \beta Ha^2 B_2\left(\beta B_2 + 2\frac{C_2}{r_2^2}\right)\right],$$

$$\Im_3 = -2PrRe\left[\frac{C_1C_2}{r_1r_2}\left(1+\frac{Ha^2}{r_1r_2}\right) + \beta Ha^2\left(\frac{C_1B_2}{r_1^2} + \frac{C_2B_1}{r_2^2} + \beta B_1B_2\right)\right],$$

$$\Im_4 = -2PrEc\left[\frac{C_1}{r_1}\left(C_3 + \frac{Ha^2C_4}{r_1}\right) + \frac{C_1}{r_1^2}Q_1 + B_1Q_2\right],$$

$$\Im_5 = -2PrEc\left[\frac{C_2}{r_2}\left(C_3 + \frac{Ha^2C_4}{r_2}\right) + \frac{C_2}{r_2^2}Q_1 + B_2Q_2\right],$$

$$\Im_6 = -PrEc(C_3^2 + Ha^2C_4^2 + 2C_4Q_1 + Q_3),$$

$$Q_1 = Ha^2(\beta B_3 + K);\; Q_2 = \beta Ha^2(\beta B_3 + K + C_4);\; Q_3 = Ha^2(\beta B_3 + K)^2. \tag{21}$$

With the aid of the expressions for velocity and temperature following important characteristics of the flow and heat transfer are derived in the forms:

The flow rate:

$$Q = \int_0^1 u^*(y^*)dy^* = \int_0^1\left(\frac{C_1}{r_1^2}\exp(r_1y^*) + \frac{C_2}{r_2^2}\exp(r_2y^*) + C_3y^* + C_4\right)dy^*,$$

$$Q = \frac{C_1}{r_1^3}(\exp(r_1)-1) + \frac{C_2}{r_2^3}(\exp(r_2)-1) + \frac{C_3}{2} + C_4. \tag{22}$$

The shear stress:

$$\tau = \mu \frac{du^*}{dy^*} = \mu\left(\frac{C_1}{r_1}\exp(r_1 y^*) + \frac{C_2}{r_2}\exp(r_2 y^*) + C_3\right), \tag{23}$$

$$\tau_{lp} = \mu \frac{du^*}{dy^*}\bigg|_{y^*=0} = \mu\left(\frac{C_1}{r_1} + \frac{C_2}{r_2} + C_3\right), \tag{24}$$

$$\tau_{up} = \mu \frac{du^*}{dy^*}\bigg|_{y^*=1} = \mu\left(\frac{C_1}{r_1}\exp(r_1) + \frac{C_2}{r_2}\exp(r_2) + C_3\right). \tag{25}$$

Dimensionless heat transfer coefficient-Nusselt number on the plates:

$$Nu_{lp} = \frac{d\Theta}{dy^*}\bigg|_{y^*=0} = 2r_1 Z_1 + 2r_2 Z_2 + (r_1 + r_2)Z_3 + r_1 Z_4 + r_2 Z_5 + Z_6 + S_1, \tag{26}$$

$$Nu_{up} = \frac{d\Theta_2}{dy^*}\bigg|_{y^*=1} = 2r_1 Z_1 \exp(2r_1) + 2r_2 Z_2 \exp(2r_2) + (r_1 + r_2)Z_3 \exp(r_1 + r_2) + \\ + r_1 Z_4 \exp(r_1) + r_2 Z_5 \exp(r_2) + Z_6 + S_1 \exp(-q). \tag{27}$$

3. RESULTS AND DISCUSSION

After definition of the mathematical model for flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient, part of results obtained with numerical integration are presented graphically in this chapter.

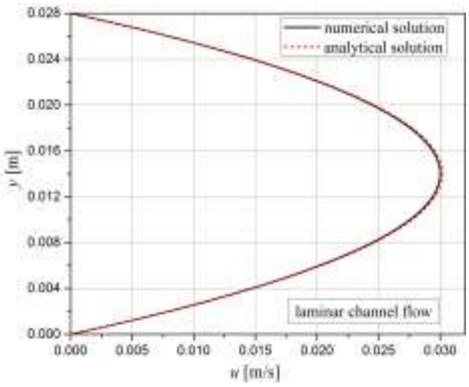

Fig. 2 Laminar flow velocity distribution

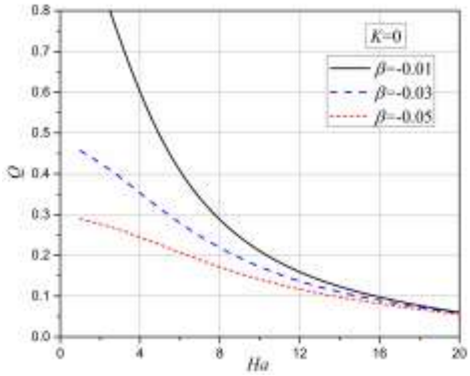

Fig. 3 Influence of Hartmann number on flow rate *Q*

The obtained solutions for velocity distribution were compared with numerical simulations results in *ANSYS CFX* software. Results comparison is presented in fig. 2, where the dashed line presents the numerical results from *ANSYS CFX* software for the influence of magnetic field on the flow of electrically conducting fluid [17]. Fig. 3 to 5 presents variations of flow *Q* depending on Hartmann number, load factor *K* and suction velocity.

It can be noted in Fig. 3 that decreasing of Hartmann number, cause the increasing of flow rate. The same influence has the suction velocity v_0 (Fig. 5), but the influence is more expressive. In the case when load factor is greater than zero, decrease of parameter β, ratio of suction velocity to mean velocity, increases the reverse flow rate.

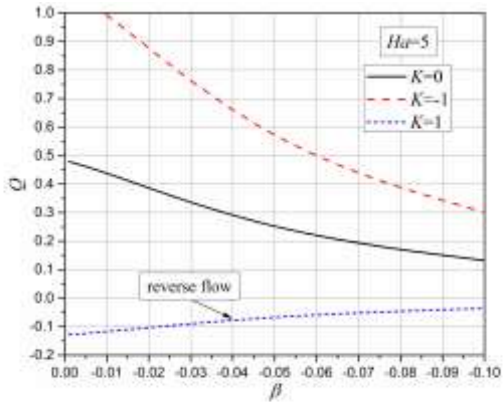

Fig. 4 Influence of suction on flow rate *Q*

The value of load factor defines the system regime: pumping, generator or braking. Negative values of load factor increase the flow rate as pump, for the values between 0 and 1 system act as generator, while for values greater than one flow rate is increased in the opposite direction.

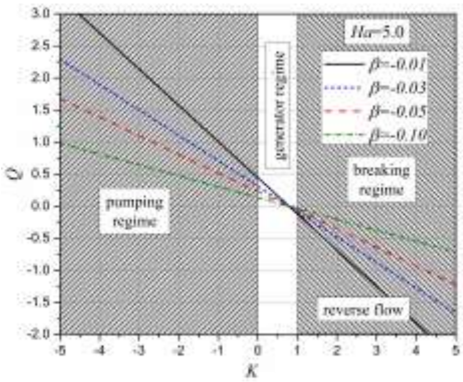

Fig. 5 Influence of load factor K on flow rate Q

Next four figures (Figs. 6 to 9) shows influence of Hartmann number, load factor and suction velocity on shear stress. For the influence of Hartmann number, two cases are analysed for constant pressure drop and constant flow rate.

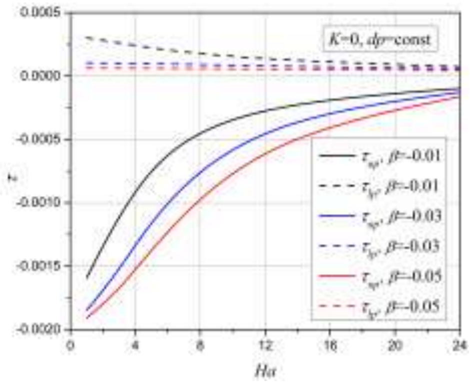

Fig. 6 Influence of Hartmann number on shear stress (constant pressure drop)

The Hartmann number (*Ha*) affects the shear stress in such a way that increasing the *Ha* causes the lower shear stress on the porous plates for the case of constant pressure drop (Fig. 6). This conclusion is valid for the case of suction (lower plate) and for the injection (upper plate). In the case of constant flow In the case of upper plate, influence of Hartmann number is function of suction velocity.

From Fig. 8 it can be noted that the suction ($_{lp}$-lower plate) decreases the shear stress for all the values of load factor, while the injection ($_{up}$-upper plate) has quite the opposite effect on the shear stress. For the case of load factor (Fig. 9) the shear stress is decreased in one small area between zero and one, and for all other values of load factor the shear stress is increased. Increase of the suction velocity reduces the shear stress on lower plate, and injection has an opposite effect on the upper plate.

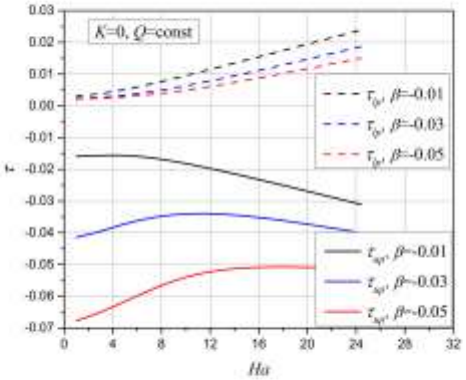

Fig. 7 Influence of Hartmann number on shear stress (constant flow rate)

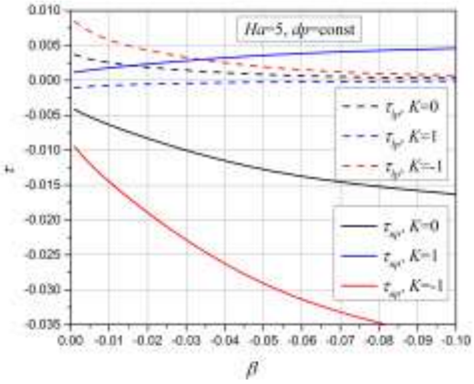

Fig. 8 Influence of suction velocity on shear stress (constant pressure drop)

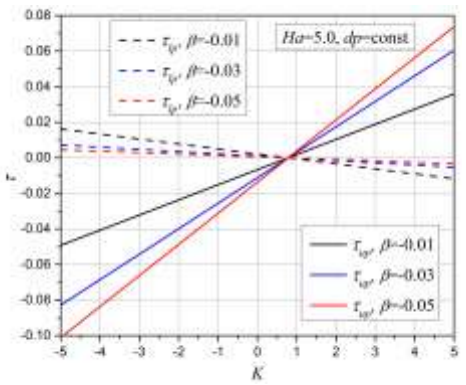

Fig. 9 Influence of load factor on shear stress (constant pressure drop)

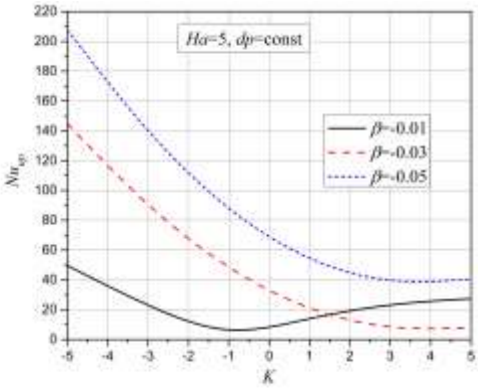

Fig. 10 Influence of load factor on heat transfer on upper plate

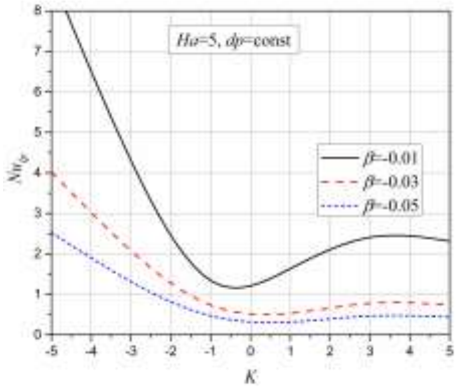

Fig. 11 Influence of load factor on heat transfer on lower plate

Figures 10 and 11 show the behavior of the Nusselt number on the lower and upper plate for different values of load factor and suction parameter.

An external electric field as an additional pressure gradient increases significantly the heat transfer (negative values of load factor). At both plates, the heat transfer decreases with the increase of load factor, in the case when suction parameters have higher values. For the low values of suction parameter with increase of load factor heat transfer slightly increases at both plates.

Figures 12 and 13 show the behavior of the Nusselt number on the lower and upper plate for different values of magnetic field intensity (Hartmann number) and suction parameter. Both cases are considered a constant pressure drop and a constant flow rate. At the upper plate Nusselt number decreases with the increase of Hartmann number. Only for the case of very low suction parameter and constant pressure drop an increase of Hartmann number causes an increase of Nusselt number. At the lower plate an increase of magnetic field intensity, significantly increases the heat transfer in the case of constant flow rate.

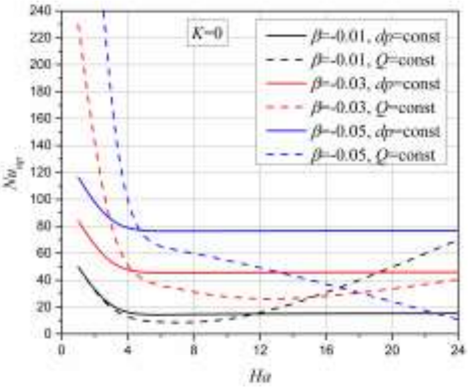

Fig. 12 Influence of Hartmann number on heat transfer on upper plate

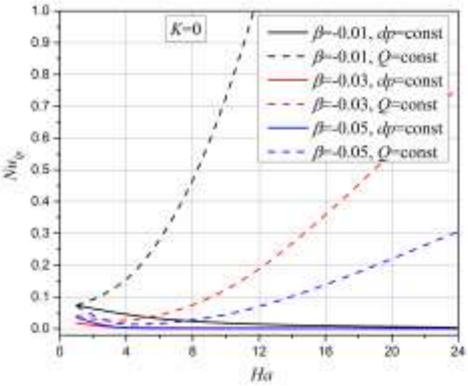

Fig. 13 Influence of Hartmann number on heat transfer on the lower plate

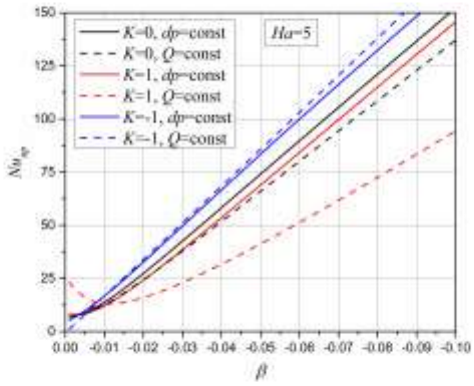

Fig. 14 Influence of suction parameter on the upper plate heat transfer

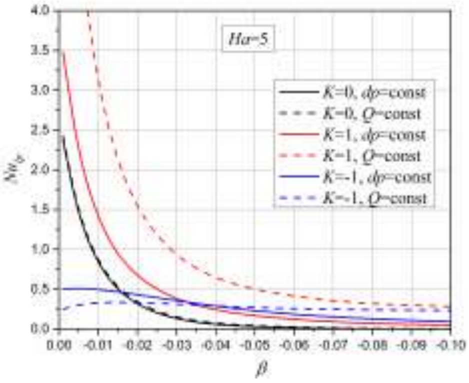

Fig. 15 Influence of suction parameter on lower plate heat transfer

An increase of the suction parameter decreases the Nusselt number for both considered cases at the lower plate for all values of the load factor. Convective heat transfer is more intense at the upper plate. At the upper plate Nusselt number changes rapidly with the increase of suction parameter. This conclusion is valid for all values of load factor and both considered cases.

4. CONCLUSION

In this paper, a flow of a viscous incompressible electrically conducting fluid between two infinite horizontal parallel porous plates under a constant pressure gradient or constant flow rate has been considered. An exact solution of governing equation has been obtained in a closed form. The influences of each of the governing parameters on velocity, temperature, flow rate and shear stress are discussed with the aid of graphs. The obtained results show that the control of flow and heat transfer for the observed case can be realized by changing the Hartmann number, the loading factor and the suction velocity.

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