Unsteady Flow through Porous Media Past on Moving Vertical Plate with Variable Temperature in the Presence of Inclined Magnetic Field

GAURAV KUMAR SHARMA
Dept. of Maths
BBDNIIT, Lucknow
UP India.

LAV KUSH KUMAR
Dept. of Maths
BBDNIIT, Lucknow
UP India.

NAVNEET KUMAR SINGH
Dept. of Maths
BBDNIIT, Lucknow
UP India.

Abstract- **In this paper, we have investigated the motion of unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field. The fluid considered is viscous, electrically conducting, incompressible, absorbing-emitting radiation in a non-scattering medium. The Laplace transform technique has been used to find the solutions for the velocity profile. The velocity profile has been studied for different parameters like Hartmann number, Prandtl number, thermal Grashof number and time. The effects of variable parameters to the velocity profiles are discussed graphically and the numerical values obtained for skin-friction has been tabulated.**

Keywords: **Unsteady Flow, Heat Transfer, Porous Media, Inclined Magnetic Field.**

I. INTRODUCTION

The study of flow of an electrically conducting fluid has important applications in many branches of engineering and applied sciences such as MHD accelerators and power generation systems, plasma studies, cooling of nuclear reactors, geothermal energy extraction, electromagnetic propulsion and the boundary layer control in the field of aerodynamics. Unsteady hydro-magnetic flow in a rotating channel in the presence of inclined magnetic field was studied by Seth and Ghosh[1]. Inclined magnetic field and chemical reaction effects on flow over a semi infinite vertical porous plate through porous medium was studied by Sugunamma et al[7]. Sandeep and Sugunamma[8] has analyzed effect of inclined magnetic field on unsteady free convective flow of dissipative fluid past a vertical plate and further [6] Studied effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium. Sandeep and Sugunamma [11] has also analyzed radiation and inclined magnetic field effects on unsteady hydromagnetic free convection flow past an impulsively moving vertical plate in a porous. Kim[2] has analyzed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction. Seddeek[3] has presented the effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation. Unsteady flow through a highly porous medium in the presence of radiation was analyzed by Raptis and Perdikis[4]. Heat transfer to MHD oscillatory flow in a channel filled with porous medium was presented by Makinde and Mhone [5]. Alia et al[9] has worked on heat and mass transfer with free convection MHD flow past a vertical plate embedded in a porous medium. MHD free convective heat and mass transfer of fluid flow past a moving variable surface in porous media was studied by Singh and Benu[10]. Effect of an inclined magnetic field on steady Poiseuille flow between two parallel porous plates was studied by Kuiryand and Bahadur[12]. In this paper, we have analyzed the motion of unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field.

II. MATHEMATICAL ANALYSIS

We considered an unsteady viscous incompressible electrically conducting fluid past vertical plate and the vertical plate impulsively started moving with velocity u_0 . The plate is electrically non-conducting. A uniform inclined magnetic field B_0 is assumed to be applied on the plate with angle α. Initially the fluid and plate are at the same temperature T_∞. At time t>0, temperature of the plate is raised to T_w. The flow modal is as under:

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2 u \,\mathrm{Sin}^2\alpha}{\rho} - \frac{\upsilon}{K} u, \qquad (1)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2}. \qquad (2)$$

With the boundary conditions:

$$\left.\begin{array}{l} t \le 0 : u = 0,\ T = T_\infty \quad \forall \text{ y} \\ t > 0 : u = u_0, \\ T = T_\infty + \left(T_w - T_\infty\right)\dfrac{u_0{}^2 t}{\upsilon} \text{ at y} = 0, \\ u \to 0,\ T \to T_\infty \text{ as y} \to \infty. \end{array}\right\} \quad (3)$$

Here u is the velocity of the fluid in, g – acceleration due to gravity, β - volumetric coefficient of thermal expansion, t- time, T_∞ - the temperature of the fluid near the plate, T_w - temperature of the plate T - the temperature of the fluid far away from the plate, k - the thermal conductivity, υ is the kinematic viscosity, ρ - the fluid density, σ - electrical conductivity, μ - the magnetic permeability, K - the permeability of the medium and C_P is the specific heat at constant pressure.

To write the equation (1) and (2) in dimensionless form, we introduce the following non - dimensional quantities:

$$\left.\begin{array}{l} \bar{u} = \dfrac{u}{u_0}, \bar{w} = \dfrac{w}{u_0}, \bar{y} = \dfrac{y u_0}{\upsilon}, \\ \Pr = \dfrac{\mu C_P}{k}, \bar{t} = \dfrac{t u_0{}^2}{\upsilon}, \\ \bar{K} = \dfrac{K u_0}{\upsilon^2}, Gr = \dfrac{g\beta^* \upsilon (T_w - T_\infty)}{u_0{}^3}, \\ \theta = \dfrac{T - T_\infty}{T_w - T_\infty}, Ha^2 = \dfrac{\sigma B_0{}^2 l^2}{\mu} \end{array}\right\} \quad (4)$$

Here the symbols used are:

$\bar{u}$ -dimensionless velocity, Pr - Prandtl number, Ha -Hartmann number, M - magnet field parameter, $\bar{y}$ - dimensionless coordinate axis normal to the plate, θ -dimensionless temperature, Gr – thermal Grashof number.

The dimensionless form of Equation (1) and (2) are as follows:

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gr\,\theta - M\bar{u} - \frac{1}{k}\bar{u}, \quad (5)$$

$$\frac{\partial \theta}{\partial \bar{t}} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \bar{y}^2}. \quad (6)$$

Here $M = Ha^2 \sin^2 \alpha$.

The corresponding boundary conditions are:

$$\left.\begin{array}{l} \bar{t} \le 0,\ \bar{u} = 0,\ \theta = 0 \quad \forall \text{ y}, \\ \bar{t} > 0,\ \bar{u} = 1,\ \theta = t \quad \text{at } \bar{\text{y}} = 0, \\ \bar{u} \to 0, \theta \to 0, \quad \text{as } \bar{\text{y}} \to \infty. \end{array}\right\} \quad (7)$$

Dropping the bars and combining the Equations (5) and (6), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta - Mu - \frac{1}{k}u, \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{pr}\frac{\partial^2 \theta}{\partial y^2}, \quad (9)$$

with corresponding boundary conditions

$$\left.\begin{array}{l} t \le 0, \quad u = 0,\ \theta = 0, \quad \forall\ y, \\ t > 0, \quad u = 1, \quad \theta = t, \ \ at\ y = 0, \\ u \to 0,\ \theta \to 0, \qquad as\ y \to \infty. \end{array}\right\} \quad (10)$$

On solving above equations analytically, we have following solutions:

$$\begin{array}{l} u = \dfrac{1}{2}e^{-\sqrt{A}y}(1 + A_1 + e^{2\sqrt{A}y}A_5) \\ + \dfrac{1}{4A^2}G_r(2e^{-\sqrt{A}y}(1 + e^{2\sqrt{A}y}A_1 \\ - e^{2\sqrt{A}y}A_2)(1 - At) + y\sqrt{A}e^{-\sqrt{A}y} \\ (1 - e^{2\sqrt{A}y} + A_1 + e^{2\sqrt{A}y}A_2) \\ + 2e^{\frac{At}{P_r - 1} - y\sqrt{\frac{AP_r}{P_r - 1}}}(-1 - e^{2y\sqrt{\frac{AP_r}{P_r - 1}}} + A_3 \\ + e^{2y\sqrt{\frac{AP_r}{P_r - 1}}}A_4)(1 - P_r) + 2e^{-\sqrt{A}y} \\ (-1 - e^{2\sqrt{A}y} - A_1 + e^{2\sqrt{A}y}A_2)P_r) \end{array}$$

$$\begin{array}{l} \theta = t\{(1 + \dfrac{z^2 P_r}{2t})erfc[\dfrac{\sqrt{P_r}}{2\sqrt{t}}] \\ - \dfrac{z\sqrt{P_r}}{\sqrt{\pi\sqrt{t}}}e^{-\frac{y^2}{4t}}P_r\}, \end{array}$$

III. SKIN FRICTION

The dimensionless skin friction at the plate y=0 is obtained as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= -\frac{1}{2}\sqrt{A}(1 + erf[\sqrt{At}] + erfc[\sqrt{At}])$$
$$+\frac{1}{2}(-\frac{2e^{At}}{\sqrt{\pi t}} + 2\sqrt{A}erfc[\sqrt{At}])$$
$$+\frac{1}{4A^2}G_r(-4\sqrt{A} - 4e^{\frac{At}{P_r-1}} + 4A^{\frac{3}{2}}t$$
$$+2\sqrt{A}erf[\sqrt{At}] + 2(2\sqrt{A} - \frac{2e^{At}}{\sqrt{\pi t}}$$
$$-2\sqrt{A}erf[\sqrt{At}]) + 2At(-2\sqrt{A} + \frac{2e^{At}}{\sqrt{\pi t}}$$
$$+2\sqrt{A}erf[\sqrt{At}]) + 4\sqrt{A}P_r$$
$$+4e^{\frac{At}{P_r-1}}P_r + 2(-2\sqrt{A} + \frac{2e^{At}}{\sqrt{\pi t}} + 2\sqrt{A}erf[\sqrt{At}])P_r)$$

IV. DISCUSSION AND RESULTS

The velocity profile for different parameters like angle of inclination of magnetic field α, Hartmann number Ha, thermal Grashof number Gr, permeability parameter K, prandtl number Pr and time t is shown in figures 1 to 6. On increasing the angle of inclination of magnetic field $(\alpha = 15^\circ, 30^\circ, 60^\circ)$ the decreasing velocity are shown in figure1. If the Hartmann number (Ha=2, 4, 6) is increased then the velocity are decreased observed from figure2. Thermal Grashof number (Gr=2, 5, 10) is increased, the velocity are decreased observed from figure3. On increasing the permeability parameter (K = 0.1, 0.4, 2.0), velocity are increased observed from figure4. If the Prandtl number (Pr = 0.71, 7.0) is increased then the velocity are increased shown by figure5. Velocity are increased with time (t= 0.1, 0.15, 0.2) observed from figure6.

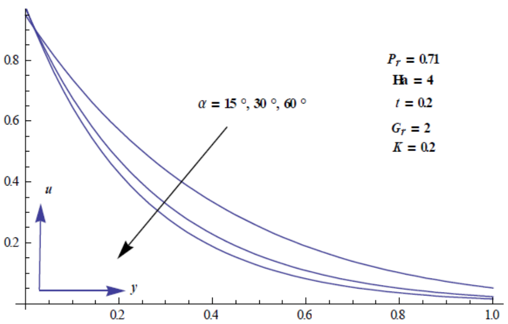

Figure1: u for different values of α

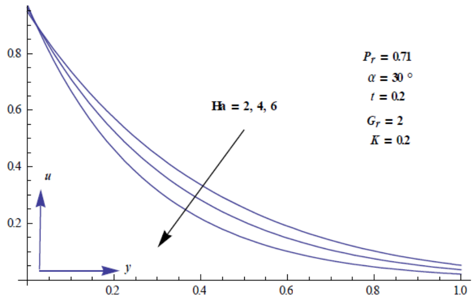

Figure2: u for different values of Ha

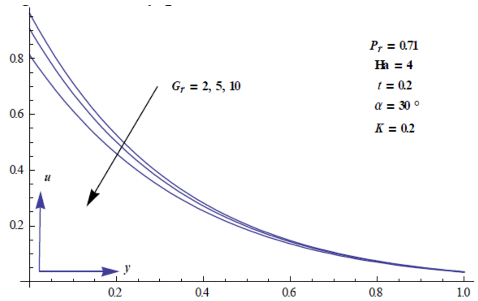

Figure3: u for different values of Gr

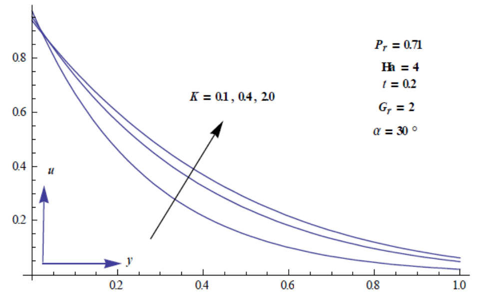

Figure4: u for different values of K

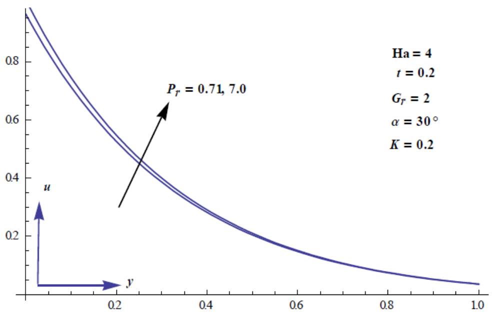

Figure5: u for different values of Pr

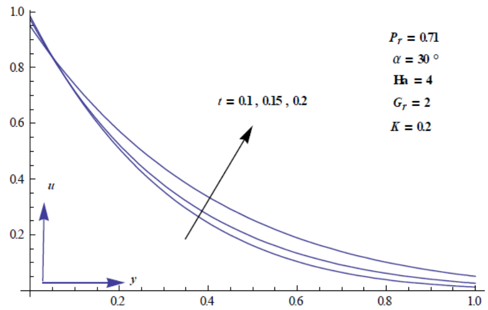

Figure6: u for different values of t

Table of Skin friction

α	Ha	Gr	Pr	K	t	τ
15°	4	2	0.71	0.2	0.2	-2.35885
30°	4	2	0.71	0.2	0.2	-2.88715
30°	6	2	0.71	0.2	0.2	-3.63635
30°	4	4	0.71	0.2	0.2	-2.17258
30°	4	2	7.00	0.2	0.2	-2.21576
30°	4	2	0.71	0.5	0.2	-2.15716
30°	4	2	0.71	0.2	0.1	-3.03013

V. CONCLUSION

Some conclusions of the study are as under:

1. Velocity increases with the increase in permeability parameter, Prandtl number Pr and time t.
2. Velocity decreases with the increase in angle of inclination of magnetic field α, Hartmann number Ha, thermal Grashof number Gr.
3. Skin friction increases with the increase angle of inclination of magnetic field α, thermal Grashof number Gr, Prandtl number Pr, permeability parameter and time t.
4. Skin friction decreases with the increase Hartmann number Ha.

VI. APENDIX

$$A_1 = erf[\frac{2\sqrt{At}-y}{2\sqrt{t}}],$$

$$A_2 = erf[\frac{2\sqrt{At}+y}{2\sqrt{t}}],$$

$$A = M + \frac{1}{K},$$

$$M = Ha^2 Sin^2\alpha,$$

$$A_3 = erf[\frac{y-2t\sqrt{\frac{AP_r}{P_r-1}}}{2\sqrt{t}}],$$

$$A_4 = erf[\frac{y+2t\sqrt{\frac{AP_r}{P_r-1}}}{2\sqrt{t}}],$$

$$A_5 = erfc[\frac{2\sqrt{At}+y}{2\sqrt{t}}],$$

VII. REFERENCES

[1]. Seth, GS., and Ghosh, SK., 1986. Unsteady hydro-magnetic flow in a rotating channel in the presence of inclined magnetic field, International *journal of engineering science, Vol.24 (7), pp.* 1183-1193.

[2]. Kim, YJ., 2000. Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, *Int. J. Eng. Sci.,* Vol.38, pp.833–845.

[3]. Seddeek, M.A., 2000. The effect of variable voscosity on hydromagnetic flow and heat transfer past a contnuously moving porous boundary with radiation, *Int. Commun Heat Mass, Vol.27(7), pp.1037-1046.*

[4]. Raptis A. and Perdikis C., 2004. Unsteady flow through a highly porous medium in the presence of radiation, *Transport Porous Media, Vol.57,pp.171-179,*

[5]. Makinde, O. D. and Mhone P. Y., 2005. Heat transfer to mhd oscillatory flow in a channel filled with porous medium, *Rom Journal of Physics, Vol.*50(10), *pp.931-938.*

[6]. N.Sandeep and V. Sugunamma., 2013. Effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by a porous medium, *International Journal of Applied Mathematics and Modeling, Vol.1(1), pp.16-33.*

[7]. Sugunamma V. et al., 2013. Inclined magneticfield and chemical reaction effects on flow over a semi infinite vertical porous plate through porous medium, *Communications in Applied Sciences, Volume 1(1), pp. 1-24*

[8]. N, Sandeep and Sugunamma V., 2013. Effect of inclined magnetic field on unsteady free convective flow of dissipative fluid past a vertical plate, *Journal Open Journal of Advanced Engineering Techniques OJAET, Vol.1(1), pp.6-23.*

[9]. Ali, F. et al., 2013. Heat and mass transfer with free convection MHD flow past a vertical plate embedded in a porous medium, Mathematical Problems in Engineering. Article ID 346281, pp1-13

[10]. Ashok Kumar Singh and Benu Madhab, 2013. MHD Free Convective Heat and Mass transfer of fluid flow past a moving variable surface in porous media, International Journal of Engineering Trends and Technology (IJETT) – Vol.4 (4), pp.1151-1157

[11]. N. Sandeep and V. Sugunamma, 2014. Radiation and inclined magnetic field effects on unsteady hydro-magnetic free convection flow past an impulsively moving vertical plate in a porous medium, *Journal of Applied Fluid Mechanics, Vol. 7(2), pp.275-286.*

[12]. Kuiryand D.R. and Bahadur, S., 2014. Effect of an inclined magnetic field on steady Poiseuille flow between two parallel porous plates, *IOSR Journal of Mathematics, Vol. 10(5), pp.90-96*