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Research Article

Near Optimum Multivariate Stratified Sampling Design With Random Measurement Costs

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Abstract --**Usually in sample surveys information on more than one characteristic are collected and the data obtained are analyzed to get the required estimates for the multivariate population under study. If stratified sampling design is to be applied on such a population the individual optimum allocations don't help much unless the characteristics are highly correlated. Therefore, in multivariate stratified sampling we need to work out an allocation that is optimum for all characteristics in some sense, that is, near optimum for all characteristics. Such an allocation is called a compromise allocation. Furthermore, in surveys usually the per unit measurement costs are taken as deterministic, that is, they remain constant throughout the survey. In practice the costs of measurement of different characteristics in various strata may change during the course of survey for reasons beyond the control of the sampler. Thus in some practical situations the measurement costs may become a random variable and the problem of obtaining a compromise allocation becomes a Stochastic Integer Nonlinear Programming Problem (SINLPP). The present paper addresses the problem of obtaining an integer compromise allocation for multivariate stratified sampling with random cost of measurements. A solution procedure has been developed for the formulated problem. A practical application of the procedure is also given through a numerical example to illustrate the computational details.**

Keywords- **multivariate stratified sampling; compromise allocation; random measurement costs**

I. INTRODUCTION

In the study of multivariate stratified sampling, individual optimum allocation is of no use unless the characteristics are highly correlated to each other (See Cochran[1]). With the goal of achieving a common allocation in a multivariate stratified sampling that suits all the characteristics several compromise criteria have been adopted by various authors.

The problem of determining a compromise allocation in multivariate stratified sampling was studied by many authors e.g., Neyman [2], Peter and Bucher [3] Geary [4], Dalenius [5], Ghosh [6], Yates [7], Folks and Antle [8], Kokan and Khan [9], Chatterjee [10], [11] and [12], Ahsan and Khan[13] and [14], Schittkowski [15], Chromy [16], Bethel [17] and [18], Jahan, Khan and Ahsan [19], Khan, Jahan and Ahsan[20], Rahim [21], Holmberg [22], Bosch and Wildner [23], Singh [24], Kozak [25], Diaz Garcia and Garay Tapia [26], Javed, Bakhshi and Khalid [27], Bakhshi, Khan and Ahmad [28], Khowaja, Ghufran and Ahsan [29] and others.

Assuming that the population means are of interest Chatterjee [10] proposed a compromise criteria as minimizing the sum of relative increases in the variances of the stratified sample means, when a non optimum allocation is used instead of individual optimum allocations for a fixed total cost. It was observed that in case of small strata sizes the results computed by Chatterjee's formula may became infeasible due to oversampling. Khan, Jahan and Ahsan [20], considered this problem as an Integer Nonlinear Programming Problem and proposed a method of solution using Dynamic programming technique.

The present paper deals with the problem of obtaining a compromise allocation by using Chatterjee's [10] compromise criterion when within stratum costs of measurement are random variables. A small probability of violation is attached with the cost constraint. Chance Constrained Technique is used to obtain a solution to the formulated Stochastic Integer Nonlinear Programming Problem (SINLPP).

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II. FORMULATION OF THE PROBLEM AS A SINLPP

Consider a multivariate stratified population with L strata and p characteristics. Assume that the estimation of p population means $\overline{Y}_j; j = 1,2,\dots,p,$ are of interest. In this manuscript the notations of Cochran [1] are used unless specified otherwise. The problem of finding a compromise allocation by minimizing the sum of relative increases in the variances of the stratified sample means $\bar{y}_{stj}$ of the p population means $\overline{Y}_j$; $j = 1,2,\dots,p,$ using a non optimum allocation for a fixed budget has been formulated by Chatterjee [10] as

Minimize $\frac{1}{C}\sum_{j=1}^{p}\sum_{h=1}^{L}\frac{c_h\left(n_{hj}^{*}-n_h\right)^2}{n_h}$ (1)

Subject to C = $\sum_{h=1}^{L} c_h n_h$ (2)

Where c_h denote the costs of measuring all the p characteristics in the h^{th} stratum; $h = 1,2,\dots,L.$

He obtained the continuous solution by using Lagrange Multipliers Technique as

$$n_h = \frac{C\sqrt{\sum_j n_{hj}^{*2}}}{\sum c_h\sqrt{\sum_j n_{hj}^{*2}}} \quad (3)$$

Where $n_{hj}^{*}; h = 1,2,..,L$ denote the optimum allocation for the $j^{th}; j = 1,2,\dots,p$ characteristics.

For practical implementation, one needs integer sample sizes. The continuous solution (3) may be rounded off to the nearest integer. A major problem with this approach is that the rounded off integer solution may become infeasible or non optimum. Using (3) there are also chances of encountering the problem of oversampling, that is, the sample size n_h for some h may exceed the corresponding stratum size N_h. Furthermore, for estimating the stratum variances S_h^2 we need $n_h \geq 2$ for all h.

Taking into account the above discussed points a more general formulation of the problem (1) – (2) may be given as the following Integer Nonlinear Programming Problem (INLPP).

Minimize f $(n_1, n_2, \dots, n_h) = \frac{1}{C_0}\sum_{j=1}^{p}\sum_{h=1}^{L}\frac{c_h\left(n_{hj}^{*}-n_h\right)^2}{n_h}$ (4)

Subject to $\sum_{h=1}^{L} c_h n_h \leq C_0$ (5)

$n_h \leq 2 \leq N_h$ (6)

$n_h\ integers; h = 1,2,\dots,L.$ (7)

Where the total cost of the survey C is given by

$$C = c_0 + \sum_{h=1}^{L} c_h n_h,$$

c_0 is the overhead cost and $C_0 = C - c_0$.

The cost constraint is expressed as an inequality because an equality sign may lead to an infeasible problem. In usual practice the stratum wise costs of measurement c_h are taken as deterministic, that is, they remain constant during the course of survey. However, there may be situations where c_h varies due to the random causes beyond the control of the sampler. In such cases the deterministic model (4) - (7) to obtain a compromise allocation will not be appropriate. Thus it would be more realistic to assume that c_h are random variables with a known probability distribution. We assume that c_h are independently and normally distributed random variables with means μ_{c_h} and variances $\sigma_{c_h}^2$;$h = 1, 2, \dots, L$. Under the above assumptions the INLPP (4) - (7) will become a Stochastic Integer Nonlinear Programming Problem (SINLPP).

To apply the Chance Constrained Technique the constraint in (5) is expressed as

$$P\left[\sum c_h n_h \leq C_0\right] \geq p_0 \quad (8)$$

Where $(1-p_0)$ denote a small probability of violation of the constraint. This gives the SINLPP as

Minimize f (n) = $\frac{1}{C_0}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{c_h\left(n_{hj}^{*}-n_h\right)^2}{n_h}$ (9)

Subject to $P\left[\sum c_h n_h \leq C_0\right] \geq p_0$ (10)

$n_h \leq 2 \leq N_h$ (11)

$n_h\ integers; h = 1,2, \dots , L$ (12)

Where $c_h \sim N\left(\mu_{c_h}, \sigma^2_{c_h}\right)$; $h = 1,2, \dots , L.$

III. THE DETERMINISTIC EQUIVALENT OF THE SINLPP

Assuming that the sample estimates of μ_{c_h} and $\sigma^2_{c_h}$ are available as $\bar{c}_h\ and\ \hat{\sigma}^2_h$ objective function in (4) will be a random variable with estimate of its mean and variance as

$$\frac{1}{C_0}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\bar{c}_h(n^*_{hj}-n_h)^2}{n_h} \quad (13)$$

and

$$\frac{1}{C_0^2}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\hat{\sigma}^2_{c_h}\left(n^*_{hj}-n_h\right)^4}{n_h^2} \quad (14)$$

respectively.

Using (13) and (14) the deterministic equivalent of the objective function (11) may be given as

$$\text{F (n)} = k_1\left[\frac{1}{C_0}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\bar{c}_h(n^*_{hj}-n_h)^2}{n_h}\right] + k_2\left[\sqrt{\frac{1}{C_0^2}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\hat{\sigma}^2_{c_h}\left(n^*_{hj}-n_h\right)^4}{n_h^2}}\right] \quad (15)$$

Where $k_1\ and\ k_2$ are non negative constants whose values indicate the relative importance of the mean and the standard deviation of f (n). Without loss of generality we assume that $k_1 + k_2 = 1$.

Let $\quad d = \sum_{h=1}^{L} c_h n_h$ (16)

Since c_h are normally distributed random variables, d will also be normally distributed with mean

$\bar{d} = \sum_{h=1}^{L} \bar{c}_h\, n_h$ (17)

and variance $V\,(d)\ =\ n^T V_0 n$ (18)

Where **n** = $(n_1, n_2, \dots\dots, n_L)$ and V is the variance-covariance matrix

$$\begin{bmatrix} Var(c_1) & Cov(c_1,c_2) & \cdots & Cov(c_1,c_L) \\ Cov(c_2,c_1) & Var(c_2) & \cdots & Cov(c_2,c_L) \\ \vdots & \vdots & & \vdots \\ Cov(c_L,c_1) & Cov(c_L,c_2) & & \cdots Var(c_L) \end{bmatrix} \text{ of } c_h.$$

Since $c_h; h = 1,2 \dots , L$ are independent, the covariance terms vanish.

The constraint (10) can now be expressed as

$$P[d \leq C_0] \geq\ p_0$$

or $$P\left[\frac{d-\bar{d}}{\sqrt{Var(d)}}\ \leq\ \frac{C_0-\bar{d}}{\sqrt{Var(d)}}\right] \geq\ p_0$$

or $$P\left[Z\ \leq\ \frac{C_0-\bar{d}}{\sqrt{Var(d)}}\right]\ \geq\ p_0 \quad (19)$$

where Z ~ $N(0\,,1)$.

The probability of realizing $d \leq C_0$ is given by

$$\text{P}[d \leq\ \ C_0] = \text{Ø}\left(\frac{C_0-\bar{d}}{\sqrt{Var(d)}}\right) \quad (20)$$

where $Ø(x)$ represents the cumulative distribution function of the standard normal distribution evaluated at x. If γ denotes the value of the standard normal variable at which Ø (γ)= p_0, then

$$Ø\left(\frac{C_0-\bar{d}}{\sqrt{Var(d)}}\right) \geq\ Ø(\gamma) \quad (21)$$

Inequality (21) will be satisfied if and only if

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$$\left(\frac{C_0 - \bar{d}}{\sqrt{Var(d)}}\right) \geq \gamma$$

or $$\bar{d} + \gamma\sqrt{Var(d)} - C_0 \leq 0$$

or $$\sum_{h=1}^{L} \bar{c}_h\, n_h + \gamma\sqrt{n^T V n} - C_0 \leq 0 \quad (22)$$

Inequality (22) gives the deterministic equivalent to the linear chance constraint (10) (See Rao [32]).

Using (15) and (22) the deterministic equivalent of the SINLPP (9) - (12) may be given as:

$$\text{Minimize } k_1\left[\frac{1}{C_0}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\bar{c}_h\left(n_{hj}^{*}-n_h\right)^2}{n_h}\right] + k_2\left[\sqrt{\frac{1}{C_0^2}\sum_{j=1}^{P}\sum_{h=1}^{L}\frac{\hat{\sigma}_{c_h}^2\left(n_{hj}^{*}-n_h\right)^4}{n_h^2}}\right] \quad (23)$$

$$\text{Subject to } \sum_{h=1}^{L}\bar{c}_h\, n_h + \gamma\sqrt{n^T V n} - C_0 \leq 0 \quad (24)$$

$$2 \leq n_h \leq N_h \quad (25)$$

$$n_h \ integers;\ h = 1,2,\ldots,L \quad (26)$$

When numerical values of the parameters of the INLPP (23)-(26) are available it can be solved by using an appropriate nonlinear programming technique.

IV. SOME OTHER COMPROMISE ALLOCATIONS

In this section proportional and some other well known compromise allocations are worked out, for random cost, for the sake of comparison with the proposed method.

A. Proportional allocation

In stratified sample surveys proportional allocation is the most convenient for obtaining an allocation. In this allocation the sample sizes n_h are taken proportional to the corresponding strata weights, that is,

$$n_h \propto W_h;\ h = 1,2,\ldots,L$$

$$\text{or } n_h = nW_h \quad (27)$$

where $n = \sum_{h=1}^{L} n_h$ is the total sample size.

B. Cochran's Average allocation

Cochran[1] suggested a compromise allocation by averaging the individual optimum allocations for all the characteristics. This gives

$$n_h = \frac{1}{P}\sum_{j=1}^{P} n_{hj}^{*}\ ;\ h = 1,2,\ldots,L.$$

When the costs are random variable the individual optimum allocations will be the solution to the 'p' SINLPPs

$$\left.\begin{array}{l}\text{Minimize } \sum_{h=1}^{L}\frac{W_h^2 S_{hj}^2}{n_{hj}}\\ \text{Subject to } P\left[\sum c_h n_{hj} \leq C_0\right] \geq p_j\\ \quad 2 \leq n_{hj} \leq N_h\\ \quad n_{hj}\ integers\end{array}\right\};\ j = 1,2,\ldots,p \quad (28)$$

The deterministic equivalents of the problems in (28) are given by

$$\left.\begin{array}{l}\text{Minimize } \sum_{h=1}^{L}\frac{W_h^2 S_{hj}^2}{n_{hj}}\\ \text{Subject to } \sum_{h=1}^{L}\bar{c}_h\, n_{hj} + \gamma_j\sqrt{n^T V_0 n} - C_0 \leq 0\\ \quad 2 \leq n_{hj} \leq N_h\\ \quad n_{hj}\ integers\end{array}\right\};\ j = 1,2,\ldots,P \quad (29)$$

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Minimizing Trace of variance-covariance Matrix

Sukhatme [30] obtained the compromise allocation by minimizing the trace of variance-covariance matrix of $\bar{y}_{jst}$ for deterministic cost. This problem as an SINLPP may be given as

$$\text{Minimize } \sum_{j=1}^{P} \sum_{h=1}^{L} \frac{W_h^2 S_{hj}^2}{n_h} \quad (30)$$

$$\text{Subject to } P[\sum_{h=1}^{L} c_h n_h \leq C_0] \geq p_j \quad (31)$$

$$2 \leq n_h \leq N_h \quad (32)$$

$$n_h \text{ integers} \quad (33)$$

where $c_h \sim N(\mu_{c_h}, \sigma_{c_h}^2)$.

The solution may be obtained by solving the equivalent deterministic INLPP

$$\text{Minimize } \sum_{j=1}^{P} \sum_{h=1}^{L} \frac{W_h^2 S_{hj}^2}{n_h} \quad (34)$$

$$\text{Subject to } \sum_{h=1}^{L} \bar{c}_h n_h + \gamma_j \sqrt{n^T V_0 n} - C_0 \leq 0 \quad (35)$$

$$2 \leq n_h \leq N_h \quad (36)$$

$$n_h \text{ integers} \quad (37)$$

V. AN APPLICATION OF THE PROPOSED ALLOCATION

In stratification with three strata (L=3) and two variables (p=2), the values of N_h,W_h,S_{h1},S_{h2}.$\bar{c}_h$ and $\hat{\sigma}_{c_h}^2$ are given in Table I. The total cost C of the survey is fixed as 250 units with the overhead costc_0 =50 units. Thus $C_0 = C - c_0 = 200\ units$. For the sake of simplicity $k_1 = k_2 = 0.5$ and p_j are taken as 0.9 for all $j = 1,2,\ldots,p$. Using standard normal area table we get $\gamma_j = 2.3263$ for all $j = 1,2,\ldots,p$.

TABLE I. DATA FOR THREE STRATA AND TWO CHARACTERISTICS

h	N_h	W_h	S_{h1}	S_{h2}	$\bar{c}_h$	$\hat{\sigma}_{c_h}^2$
1	18	0.3	2	1.5	3	5
2	27	0.45	4	2	4	7
3	15	0.25	20	35	5	8

On substituting the values from the data given in Table 1 the INLPP (23)-(26) takes the form

Minimize

$$0.5\left[\frac{1}{200}\left\{\frac{3(2-n_1)^2}{n_1} + \frac{4(6-n_2)^2}{n_2} + \frac{5(14-n_3)^2}{n_3} + \frac{3(1-n_1)^2}{n_1} + \frac{4(2-n_2)^2}{n_2} + \frac{5(18-n_3)^2}{n_3}\right\}\right] +$$

$$0.5\left[\sqrt{\frac{1}{40000}\left\{\begin{array}{c} 5\left(\frac{(2-n_1)^4}{n_1^2}\right) + 7\left(\frac{(6-n_2)^4}{n_2^2}\right) + 8\left(\frac{(14-n_3)^4}{n_3^2}\right) + \\ 5\left(\frac{(1-n_1)^4}{n_1^2}\right) + 7\left(\frac{(2-n_2)^4}{n_2^2}\right) + 8\left(\frac{(18-n_3)^4}{n_3^2}\right) \end{array}\right\}}\right] \quad (38)$$

Subject to

$$3n_1 + 4n_2 + 5n_3 + 2.3263\sqrt{5n_1^2 + 7n_2^2 + 8n_3^2} - 200 \leq 0 \quad (39)$$

$$\left.\begin{array}{l} 2 \leq n_1 \leq 18 \\ 2 \leq n_2 \leq 27 \\ 2 \leq n_3 \leq 15 \end{array}\right\} \quad (40)$$

$$n_1, n_2 \text{ and } n_3 \text{ are integers} \quad (41)$$

Using the optimization software LINGO[31] the solution to the INLPP (38)-(41) is obtained as:

$n_1 = 2, n_2 = 4$ and $n_3 = 15$, with a total sample size of n = 21.

The variances for the two characteristics under this allocation are $V_1 = 2.65666$ and $V_2 = 5.40795$

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A. Proportional allocation

Using the values from Table I, (27) gives the proportional allocation for n = 21 as

$n_1 = 6, n_2 = 10$ *and* $n_3 = 5$ *with* $V_1 = 5.38400$ *and* $V_2 = 15.42725$

5.2. Cochran's Average allocation

For the given data we get the INLPP (29) for j = 1 and j = 2 as follows:

For j=1

$$\text{Minimize } \frac{0.36}{n_1} + \frac{3.24}{n_2} + \frac{1.25}{n_3} \quad (42)$$

$$\text{Subject to } 3n_1 + 4n_2 + 5n_3 + 2.3263\sqrt{5n_1^2 + 7n_2^2 + 8n_3^2} - 200 \le 0 \quad (43)$$

$$\left.\begin{array}{l} 2 \le n_1 \le 18 \\ 2 \le n_2 \le 27 \\ 2 \le n_3 \le 15 \end{array}\right\} \quad (44)$$

n_1, n_2 *and* n_3 *are* integers (45)

Using LINGO the solution to the problem (42)-(45) is obtained as

$$n_1 = 5, n_2 = 12 \text{ and } n_3 = 8 \quad (46)$$

For j=2:

$$\text{Minimize } \frac{0.2025}{n_1} + \frac{0.81}{n_2} + \frac{76.5625}{n_3} \quad (47)$$

$$\text{Subject to } 3n_1 + 4n_2 + 5n_3 + 2.3263\sqrt{5n_1^2 + 7n_2^2 + 8n_3^2} - 200 \le 0 \quad (48)$$

$$\left.\begin{array}{l} 2 \le n_1 \le 18 \\ 2 \le n_2 \le 27 \\ 2 \le n_3 \le 15 \end{array}\right\} \quad (49)$$

$$n_1, n_2 \text{ and } n_3 \text{ are integers} \quad (50)$$

Using LINGO software to the problem (47)-(50) we get

$$n_1 = 2, n_2 = 4 \text{ and } n_3 = 15 \quad (51)$$

Averaging over the characters and rounding off, the Cochran's allocations are obtained as:

$n_1 = 4, n_2 = 8$ *and* $n_3 = 11$

The corresponding values of the two variances are $V_1 = 2.76772$ *and* $V_2 = 7.112102$.

C. Minimizing the Trace:

When the numerical values of $N_h, W_h, S_{h1}, S_{h2}, C, c_0, \bar{c}_h$ *and* $\hat{\sigma}^2_{c_h}$ are substituted from Table I, the INLPP (34) - (37) becomes

$$\text{Minimize } \frac{0.5625}{n_1} + \frac{4.05}{n_2} + \frac{101.5625}{n_3} \quad (52)$$

$$\text{Subject to } 3n_1 + 4n_2 + 5n_3 + 2.3263\sqrt{5n_1^2 + 7n_2^2 + 8n_3^2} - 200 \le 0 \quad (53)$$

$$\left.\begin{array}{l} 2 \le n_1 \le 18 \\ 2 \le n_2 \le 27 \\ 2 \le n_3 \le 15 \end{array}\right\} \quad (54)$$

$$n_1, n_2 \text{ and } n_3 \text{ are integers} \quad (55)$$

The corresponding compromise allocations obtained by LINGO is

$$n_1 = 2, n_2 = 4 \text{ and } n_3 = 15$$

with the value of the objective function, which is also the trace value as 8.064583

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VI. CONCLUSION

The results are summarized in Table II. The table also gives a comparative statement of the discussed allocations as compared to the proportional allocation in terms of their relative efficiencies (R. E.).

TABLE II. RELATIVE EFFICIENCIES (R. E.) OF THE COMPROMISE ALLOCATIONS AS COMPARED TO THE PROPORTIONAL ALLOCATION

S. No.	*Proportional and Compromise allocations*	n_h *h = 1, 2, 3*	V_1	V_2	*Trace*	*R.E.* T_{prop}/T_{comp}
1	Proportional	6, 10, 5	5.38400	15.42725	20.81125	1.00000
2	Cochran's	4, 8, 11	2.76772	7.112102	9.87982	2.10644
3	Minimizing Trace	2, 4 , 15	-	-	8.06458*	2.58057
4	Proposed	2, 4 , 15	2.65666	5.40795	8.06395	2.58077

*The trace value is the value of the objective function given by LINGO.

The last column of Table II shows that the relative efficiency of the proposed allocation is maximum as compared to other discussed allocations. Thus with random costs the proposed allocation gives better results.

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