Fuzzy g- Super Continuous Mappings

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Abstract—**In this paper we study and introduced the concepts of generalized super continuous mappings and explore some of its characterization in fuzzy topological spaces**

Keywords-**Fuzzy Topology fuzzy super closed set, fuzzy super closure, fuzzy super interior ,fuzzy super open set fuzzy super continuous mapping fuzzy g super continuous mappings, fuzzy $T_{1/2}$ -spaces, fuzzy g-super open set fuzzy g-super closed set , fuzzy GO- super compactness, fuzzy GO-super connectedness.**

I. PRELIMINARIES

Let X be a non-empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y)=β for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi –coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$. A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1[5]:- Let (X,τ) fuzzy topological space and A⊆X then

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior sint(A) $=\{x\in X:cl(U)\leq A\neq\phi\}$

Definition 1.2[5]: -A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) Fuzzy super closed if $scl(A) \leq A$.

(b) Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space (X,ℑ), then $A \cup B$ is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space (X,ℑ) may not be fuzzy super closed.

Definition 1.3[1,5,6,7]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:

(a) fuzzy semi super open if there exists a super open set O such that $O\leq A\leq cl(O)$.

(b) fuzzy semi super closed if its complement 1-A is fuzzy semi super open.

Remark 1.4[1,5,7]:- Every fuzzy super open (resp. fuzzy super closed) set is fuzzy semi super open (resp. fuzzy semi super closed) but the converse may not be true .

Definition 1.4[5]:- The intersection of all fuzzy super closed sets which contains A is called the semi super closure of a fuzzy set A of a fuzzy topological space (X,τ) . It is denoted by scl(A).

Definition 1.5[3,8,9,10, 11]:- A fuzzy set A of a fuzzy topological space (X,τ) is called:

1. fuzzy g- super closed if $cl(A) \leq G$ whenever $A \leq G$ and G is super open.
2. fuzzy g- super open if its complement 1-A is fuzzy g- super closed.
3. fuzzy sg- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy semi super open.
4. fuzzy sg- super open if if its complement 1-A is sg- super closed.
5. fuzzy gs- super closed if $scl(A) \leq O$ whenever $A \leq O$ and O is fuzzy super open.
6. fuzzy gs- super open if if its complement 1-A is gs- super closed.

Remark 1.5[10,11]:- Every fuzzy super closed (resp. fuzzy super open) set is fuzzy g- super closed (resp. fuzzy g- super open) and every fuzzy g-super closed (resp. fuzzy g-super open) set is fuzzy gs-super closed (resp. gs –super open) but the converses may not be true.

Remark 1.6[10,11]:- Every fuzzy semi super closed (resp. fuzzy semi super open) set is fuzzy sg-super closed (resp. fuzzy sg-super open) and every fuzzy sg-super closed (resp. fuzzy sg-super open) set is fuzzy gs-super closed (resp. gs - super open) but the converses may not be true.

Definition 1.6.[3,8,9,10, 11] A fuzzy set A of (X,τ) is called:

(1) Fuzzy semi super open (briefly, Fs- super open) if $A \leq cl(int(A))$ and a fuzzy semi super closed (briefly, Fs- super closed) if $int(cl(A)) \leq A$.

(2) Fuzzy pre super open (briefly, Fp- super open) if $A \leq int(cl(A))$ and a fuzzy pre super closed (briefly, Fp- super closed) if $cl(int(A)) \leq A$.

(3) Fuzzy α super open (briefly, Fα- super open) if $A \leq IntCl(Int(A))$ and a fuzzy α- super closed (Briefly, Fα- super closed) if $cl(int(cl(A))) \leq A$.

(4) Fuzzy semi-pre super open (briefly, Fsp- super open) if $A \leq cl(int(cl(A))$ and a fuzzy semi-pre super closed (briefly, Fsp- super closed) if $int(cl(int(A))) \leq A$]. By FSPO(X,τ), we denote the family of all fuzzy semi-pre super open sets of fts X .

The semi closure (resp α- super closure , semi-pre super closure of a fuzzy set A of (X, τ) is the intersection of all Fs- super closed (resp. Fα- super closed, Fsp- super closed) sets that contain A and is denoted by scl(A) (resp. α cl(A) and spcl(A)).

Definition 1.7. [3,8,9,10, 11]:- A fuzzy set A of (X, τ) is called:

(1) Fuzzy generalized super closed (briefly, Fg- super closed) if $cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy super open set in X;

(2) Generalized fuzzy semi super closed (briefly, gFs- super closed) if $scl(A) \leq H$, whenever $A \leq H$ and H is Fs- super open set in X.

(3) Fuzzy generalized semi super closed (briefly, Fgs- super closed) if $scl(A) \leq H$, whenever $A \leq H$ and H is fuzzy super open set in X;

(4) Fuzzy α generalized super closed (briefly, Fαg- super closed) if $\alpha\ cl(A) \leq H$, whenever $A \leq H$ and H is fuzzy super open set in X;

(5) Fuzzy generalized α- super closed (briefly, Fg_- super closed) if $\alpha\ cl(A) \leq H$, whenever $A \leq H$ and H is Fα- super open set in X;

(6) Fuzzy generalized semi-pre super closed (briefly, Fgsp- super closed) if $spcl(A) \leq H$, whenever $A \leq H$ and H is fuzzy super open set in X.

Definition 1.8. [3,8,9,10, 11]:- A fuzzy point $x_p \in A$ is said to be quasi-coincident with the fuzzy set A denoted by $x_p qA$ iff $p + A(x) > 1$. A fuzzy set A is quasi-coincident with a fuzzy set B denoted by $A_q B$ iff there exists $x \in X$ such that $A(x) + B(x) > 1$. If A and B are not quasi-coincident then we write $A_q B$. Note that $A \leq B$, $Aq(1-B)$.

Definition 1.9. [3,8,9,10, 11]:- A fuzzy topological space (X, τ) is said to be fuzzy semi super connected (briefly, Fs- super connected) iff the only fuzzy sets which are both Fs- super open and Fs- super closed sets are 0 and 1.

II. FUZZY G-SUPER CONTINUOUS MAPPINGS

Definition 2.1: A mapping $f: (X,\tau) \rightarrow (Y,\sigma)$ is said to be fuzzy g-super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy g-super closed in X.

Theorem 2.1:A mapping $f: (X,\tau) \rightarrow (Y, \sigma)$ is fuzzy g-super continuous if and only if the inverse image of every fuzzy super open set of Y is fuzzy g- super open in .

Proof: It is obvious because $f^{-1}(1 - U)\ 1 - f^{-1}(U)$ for every fuzzy set U of Y. Remark 2.1: Every fuzzy super continuous mapping is fuzzy g-super continuous, but the converse may not be true. For, Example 2.1 : Let X = {a,b}, Y={x,y} and the fuzzy sets $U \subset X$, $V \subset Y$ defined as follows U(a) = 0.5 U(b) = 0.7, V(x) = 0.3 V(y) = 0.2, Let $\tau = \{0,U,1\}$ and $\sigma = \{0,V,1\}$ be topologies on X and Y respectively. Then the mapping $f: (X,\tau) \rightarrow (Y, \sigma)$ defined by f(a) = x and f(b) =y is fuzzy. g-super continuous but not fuzzy super continuous.

Theorem 2.2: If $f: (X,\tau) \rightarrow (Y, \sigma)$ is fuzzy g-super continuous then for each fuzzy point x_β of X and each fuzzy super open set $f(x_\beta) \in V$ there exists a fuzzy g- super open set U such that $x_\beta \in U$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy super open set such that $f(x_\beta) \in V$ put $U = f^{-1}(V)$ then by hypothesis U is a fuzzy g- super open set of X such that $x_\beta \in U$ and $f(U) = (F^{-1}(V)) \leq V$.

Theorem 2.3: If $f: (X,\tau) \rightarrow (Y,\sigma)$ is fuzzy g-super continuous, then for each fuzzy point $x_\beta \in X$ and each fuzzy super open set V of Y such that $f(x_\beta)q\ V$, there exists a fuzzy g- super open set U of X such that $x_\beta qU$ and $f(U) \leq V$.

Proof: Let x_β be a fuzzy point of X and V be a fuzzy super open set such that $f(x_\beta)qV$. Put $U = f^{-1}(V)$.

Then by hypothesis U is a fuzzy g- super open set of X such that x_β qU and $f(U) = f(f^{-1}(V)) \leq V$.

Definition 2.2:Let (X,τ) be a fuzzy topological space. The generalized super closure of a fuzzy set A of X denoted by gcl(A) is defined as follows: gcl(A) = inf {B: B ≥ A, B is fuzzy g- super closed set of (X,τ))}

Remark 2.2: It is clear that, A ≤ gcl(A) ≤clA) for any fuzzy set A of X.

Theorem 2.4: If f: (X,τ) →(Y,σ) is fuzzy g-super continuous, then f(gcl(A)) ≤cl(f(A)) for every fuzzy set A of X.

Proof: Let A be a fuzzy set of X. Then cl(f(A)) is a fuzzy super closed set of Y. Since f is fuzzy g-super continuous f^{-1}(clf(A)) is fuzzy g- super closed in X. Clearly A $\leq f^{-1}$(cl(f(A))). Therefore gcl(A) ≤ gcl(f^{-1}(cl(f(A)))) = f^{-1} (cl(f(A))). Hence f(gcl(A))≤ cl(f(A)).

Remark 2.2: The converse of theorem 3.4 may not be true. For

Example 2.2: Let X={a,b,c}, Y={x,y,z} and the fuzzy set U and V are defined as U(a) = 1, U(b) = 0, U(c) = 0 ,V(x) = 1, V(y) = 0, V(z) = 1 ,Let τ = {0,U,1} and σ = {0,V,I} be fuzzy topologies on X and Y respectively and f: (X,τ) →(Y,σ) be a mapping defined by f(a)= y, f(b) = x, f(c) = z. Then f(gcl(A)) ≤ cl(f(A)) holds for every fuzzy set A of X, but I is not fuzzy g-super continuous.**Definition 2.2:** A fuzzy topological space (X,-r) is said to be fuzzy T112 if every fuzzy g- super closed set in X is fuzzy closed in X.

Theorem 2.5: A mapping f from a fuzzy $T_{1/2}$ space (X,τ) to a fuzzy topological space (Y,σ) is fuzzy super continuous if and only if it is fuzzy g-super continuous.

Proof: Obvious.

Remark 2.4: The composition of two fuzzy g-super continuous mappings may not be fuzzy g-super continuous. For,

Example 2.3 : Let X = {a, b}, Y = {x, y} and Z = {p,q} and the fuzzy sets U ⊂ X,V ⊂ Y and W ⊂ Z are defined as follows U(a) = 0.5,. U(b) = 0.7,V V(x) = 0.3, V(y) = 0.2,,W(p) = 0.6, W(q) = 0.4,Let τ = {0,U,l}, σ = {0,V,I} and η = {0,W,l} be fuzzy topologies on X, Y and Z respectively. Let the mapping f: (X,τ) →(Y,σ) be defined by f(a)=x, f(b)=y and the mapping g : (Y,σ) — (Z,η) be defined by g(x)=p and g(y)=q. Then f and g are fuzzy g-super continuous but gof is not fuzzy g-super continuous. However,

Theorem 2.6: If f: (X,τ) →(Y,σ)is fuzzy g-super continuous and g:(Y,σ) →(Z,η) is fuzzy super continuous. Then gof: (X,τ)→(Z,η) is fuzzy g-super continuous.

Proof: If A is fuzzy closed in Z, then g^{-1}(A) is fuzzy closed in Y because g is fuzzy super continuous. Therefore $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$ is fuzzy g-closed in X. Hence gof is fuzzy g-super continuous.

Theorem 2.7: If f: (X,τ) →(Y,σ) and g : (Y,σ) — (Z,η) are two fuzzy g-super continuous mappings and (Y,σ) is fuzzy $T_{1/2}$ then gof: (X,τ)→ (Z,η) is fuzzy g-super continuous.

Proof: Obvious.

Definition 2.3: A collection {A_i: i ∈ Λ} of fuzzy g-super open sets in a fuzzy topological space (X,τ) is called a fuzzy g- super open cover of a fuzzy set B of X if B ≤ ∪ {A_i: i ∈ Λ}

Definition 2.4: A fuzzy topological space (X,τ) is said to be fuzzy GO- super compact if every fuzzy g- super open cover of X has a finite sub cover.

Definition 2.5: A fuzzy set B of a fuzzy topological space (X,τ) is said to be fuzzy GO- super compact relative to X, if for every collection {A_i: i ∈ Λ}of fuzzy g- super open subsets of X such that B ≤ ∪{A_i: i ∈ Λ} there exists a finite subset Λ_0 of A such that B ≤∪ {A_i: i ∈ Λ_0}

Definition 2.6: A crisp subset B of a fuzzy topological space (X,τ) is said to be fuzzy GO- super compact if B is fuzzy GO- super compact as a fuzzy subspace of X.

Theorem 2.8: A fuzzy g-closed crisp subset of fuzzy GO- super compact space is fuzzy GO- super compact relative to X.

Proof: Let A be a fuzzy g- super closed crisp set of a fuzzy GO- super compact space (X,τ). Then 1-A is fuzzy g-open in X. Let M be a cover of A by fuzzy g- super open sets in X. Then {M, 1-A} is a fuzzy g-super open cover of X. Since X is fuzzy GO- super compact, it has a finite sub cover say {G_1, G_2 ,…..G_n}, if this sub cover contains 1-A, we discard it. Otherwise leave the sub cover as it is, thus we have obtained a finite fuzzy g- super open sub cover of A. Therefore A is fuzzy GO- super compact relative to X.

Theorem 2.9: A fuzzy g-super continuous image of a fuzzy GO- super compact space is fuzzy super compact.

Proof: Let f: (X,τ) →(Y, σ) be a fuzzy g-super continuous map from a fuzzy GO- super compact space (X,τ) onto a fuzzy topological space (Y, σ). Let {A_i : i∈Λ} be a fuzzy g- super open cover of Y then {$f^{-1}(A_i)$: i∈Λ} is a fuzz1y g- super open cover of X.. Since X is fuzzy GO- super compact it has finite fuzzy sub cover say { f^{-1} (A_1), f^{-1}(A_2),….. f^{-1}(A_n)}. Since f is onto {A_1 ,A_2….A_n} is an fuzzy open cover of Y and so (Y, σ) is fuzzy super compact.

Definition 2.7: A fuzzy topological space X is fuzzy GO- super connected if there is no proper fuzzy set of X which is both fuzzy g- super open and fuzzy g-super closed.

Remark 2.5: Every fuzzy GO-super connected space is fuzzy super connected but the converse may not be true. For, -

Example 2.5:Let X = {a,b} and U be defined as U(a) = 0.5, U(b) = 0.7,Let $\tau = \{0,U,1)$ be a topology on X, then (X,τ) is fuzzy super connected but not fuzzy GO-super connected.

Theorem 2.10: A fuzzy $T_{1/2}$ space (X,τ) is a fuzzy super connected if and only if it is fuzzy GO-super connected.

Proof: Obvious.

Theorem 2.11: If $f : (X,\tau)\rightarrow (Y,\sigma)$ is a fuzzy g-super continuous surjection and X is fuzzy GO-connected then Y is fuzzy connected.

Proof: Suppose Y is not fuzzy connected. Then there exists a proper fuzzy set G of Y which both fuzzy super open and fuzzy super closed. Therefore $f^{-1}(G)$ is a proper fuzzy set of X, which is both fuzzy g-super open and fuzzy g-super closed, because f is fuzzy g-super continuous surjection. Hence X is not fuzzy GO-super connected, which is a contradiction.

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