XLI Incontro di Studio del Ce.S.E.T.: 697-720

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A fuzzy measure of the ability of a real estate capital to increase in value. The real estate decision problem for Ortigia

Key words: *Real estate decision problem, rough set, fuzzy measure*

The valorization processes of real estate capital are strongly influenced by the ability of the different buildings to capture and incorporate some liquidity flows, this capacity, is closely related to the dynamics of the market, of its segments and the single building.
The study proposes in this regard, a new methodology, warranting a continuity process with some traditional economic and evaluation approach, using some new tools, such as the developed rough set by the Operations Research for other issues, but, in this case, that have an instrumental role for the identification of the capacity to increase in value of a real estate capital and to propose a fuzzy measure of this capacity. The study case is that of the market of Ortigia, the historic center of Syracuse.

1. Introduction

The market analysis, in the economic and in the evaluation sphere, represents the preliminary stage to express the value judgments and the judgments of the economic affordability. In particular in the real estate market, the dynamics of the values, do not follow a uniform and consistent pattern, but follow a structured pattern in relation to the ability of the different segments to capture and incorporate liquidity flows. The ability of some real estate capital to capture and to trap some liquidity flows is closely related to the market dynamics and the its segments, and then is related to internal and external market mechanisms, of its the single segment and of a real estate capital. Surely, the state condition of a building and its belonging to a segment or a market is the condition base from which it can be measured the value and its ability to increase in value, thus from which can be measured the inducted plus and the minus increase in value by the evolution of the dynamic segment. Then, to check the susceptibility of a real estate to increase or to decrease in values, it is necessary to know its specific market, its position within the market in the physical-spatial sense and its level of quality in technological-architectural sense and then it is necessary to know the dynamics of the optional capitals. Furthermore, it is also important to note that, in the economic-territorial analysis, the real estate market is instrumental in producing a variety of useful data to locate addresses of economic policy, to plan and to implement the processes of planning, therefore, this analysis can be considered as a strategic factor for a macro-scale planning processes, which relate to the real estate market, the different types and different market segments and for a micro scale that relate to the single building. In particular, given the great importance of a measure of the ability to raise the value of real estate capital such as support for the identification of the public and private value judgments and the public and private judgments of the economic affordability, it is considered important to focus on a methodological-operative proposal from enabling the identification of this capacity, correlating it to all elements of the market and all the economic and territorial induced effects on it. The problem of defining and characterizing the mechanisms that regulate the increase or decrease in value of real estate capital is a prerequisite in order to support the choice or the decision to undertake a public and private planning process or an investment property rather than another. The problem to identify the capacity to raise the value of the real estate, can be solved by the implementation of a decision problem, by where, the public or private entity may be aided to choice the best investment. Then, the solution of the problem is the solution of the real estate decision problem. In this regard, the study addresses a real estate decision problem that allows you to define a model representation of the real estate market, aimed to identify the capacity to increase in value of the belonging real estate capital to a specific market. In particular, by the real estate decision problem will be possible correlate the characteristics of the condition with the level of the market value of the property in question. The target market of this study is that the historical center of Ortigia, the city of Syracuse.

2. Methods and tools

2.1 The market model

The real estate market, using systems theory, can be represented as a system open to trade, but with a closure which is determined by an internal code that generate the same system and that is to capable to assist it in the control of the relationship system that the market interlaces with all the other systems.

According to the systemic-relational approach, a model representation of the real estate market is possible by the identification of all the elements that characterize the system of external relations of the market, ie the communicative-relational mechanisms of the system market with all other systems, and of all the elements that characterize the system of internal relationships between the various components of the system-market. Thus, in a first moment, a fundamental point for the market definition is to identify the systems with which it interacts and then the mechanisms of adjustment of systemic intermediaries.

The most significant interaction systems of the market may be the following:

- The urban system;
- The mobility and transport system;
- The economic and productive system;
- The historical and architectural system;

- The cultural system;
- The environmental system;
- The political-administrative system;
- The urban-regulatory system;
- The regulatory and environmental system;
- The energy-regulatory system;
- The tax system;
- The banking system;
- The macro-economic system;
- The micro-economic system.

The regulatory mechanisms of relationship systemic represent the laws of the communications system that govern the synapses of the network systems. The synapses are the place where happening the transmutations of liquidity from one system to another or from one capital to another and still they are the context in which they can enable process to increase in value and in which they can enable process to decrease in value, of the system and of the interacting capitals. So it possible understand, that the laws that govern the synapses are the laws that govern and regulate the transmutations of liquidity and they are able to trigger processes of enhancement of some systems to the detriment of others.

2.2 The real estate decision problem

If the laws that govern the synapses are those that govern and regulate the transmutations of liquidity and they are able to trigger processes of enhancement of some systems to the detriment of others, o if the dynamics of the real estate market value born as the result of systemic interaction, then, for an effective modeling and efficient of these trends that are shifty, it is possible to consider a new road that is not usual in literature, ie, the way that uses to define a decision problem to represent and to summarize the decision-making processes for the different systems involved. So, if the dynamics of the market, ie, what underlies the decision in the market, is defined as a process of the interaction of the decisions for of the systems that interact with the market (Trovato 2010), then, to model the decision, that then determines the choice in the market, will need define a decision problem to support to real estate decision. Therefore, the sought decision problem, must be able to support the choice of the public and private operator to identify the real estate capital that will have a greater susceptibility to increase in value over time.

2.3 The rough set and the DRSA algorithm

The rough sets theory introduced by Pawlak (1982, 1991) has often proved to be an excellent mathematical tool for the analysis of a vague description of objects (called actions in decision problems).

The adjective vague, referring to the quality of information, means inconsistency or ambiguity which follows from information granulation. The rough sets philosophy is based on the assumption that with every object of the universe there is associated a certain amount of information (data, knowledge), expressed by means of some attributes used for object description. Objects having the same description are indiscernible (similar) with respect to the available information. The indiscernibility relation thus generated constitutes a mathematical basis of the rough sets theory; it induces a partition of the universe into blocks of indiscernible objects, called elementary sets, that can be used to build knowledge about a real or abstract world. The use of the indiscernibility relation results in information granulation. Any subset X of the universe may be expressed in terms of these blocks either precisely (as a union of elementary sets) or approximately only. In the latter case, the subset X may be characterized by two ordinary sets, called lower and upper approximations. A rough set is dened by means of these two approximations, which coincide in the case of an ordinary set. The lower approximation of X is composed of all the elementary sets included in X (whose elements, therefore, certainly belong to X), while the upper approximation of X consists of all the elementary sets which have a non-empty intersection with X (whose elements, therefore, may belong to X). Obviously, the difference between the upper and lower approximation constitutes the boundary region of the rough set, whose elements cannot be characterized with certainty as belonging or not to X, using the available information. The information about objects from the boundary region is, therefore, inconsistent or ambiguous. The cardinality of the boundary region states, moreover, to what extent it is possible to express X in exact terms, on the basis of the available information. For this reason, this cardinality may be used as a measure of vagueness of the information about X.

Some important characteristics of the rough set approach make of this a particularly interesting tool in a number of problems and concrete applications. With respect to the input information, it is possible to deal with both quantitative and qualitative data, and inconsistencies need not to be removed prior to the analysis. With reference to the output information, it is possible to acquire a posteriori information regarding the relevance of particular attributes and their subsets to the quality of approximation considered in the problem at hand, without any additional inter-attribute preference information. Moreover, the final result in the form of *"if..., then..."* decision rules, using the most relevant attributes, is easy to interpret.

Several attempts have already been made to use the rough sets theory to decision support (Pawlak, Slowinski, 1994). The original rough set approach is not able, however, to deal with preference-ordered attribute domains and decision classes. Solving this problem was crucial for application of the rough set approach to multicriteria decision analysis (MCDA). For this reason, Greco (Greco 1999) have proposed an extension of the rough sets theory that is able to deal with inconsistencies typical to exemplary decisions in MCDA problems. This innovation is mainly based on substitution of the indiscernibility relation by a dominance relation in the rough approximation of decision classes. An important consequence of this fact is a possibility of inferring from exemplary decisions the preference model in terms of decision rules being logical statements of the type *"if..., then..."* The separation of certain and doubtful knowledge about the DM's preferences is done by distinction of different kinds of decision rules, depending whether they are induced from lower approximations of decision classes or from the boundaries of these classes composed of inconsistent examples that do not observe the dominance principle. Because of the little available space, it is necessary to refer in the literature as regards the DRSA, Dominance Approach Rough Set (Greco 2007a, 2007b), and it is possible propose only some basic elements to understand the operation algorithm of the rough set

2.4 The data table and indiscernibility relation

For algorithmic reasons, the information regarding the objects is supplied in the form of a data table, whose separate rows refer to distinct objects (actions), and whose columns refer to different attributes considered. Each cell of this table indicates an evaluation (quantitative or qualitative) of the object placed in that row by means of the attribute in the corresponding column.

Formally, a data table is the 4-tuple $S=\{U,Q,V,f\}$, where U is a finite set of objects (universe), $Q=\{q_1,\ldots,q_m\}$ is a finite set of attributes, V_q is the domain of the attribute q, $V=\bigcup_{q\in Q} V_q$ and $f:U\times Q\rightarrow V$ is a total function such that $f(x,q)\in V$ for each, $q\in Q, x\in U$, called information function. Therefore, each object x of U is described by a vector (string) $Des_Q\{f(x,q_1),\ldots f(x,q_m)\}$, called description of x in terms of the evaluations of the attributes from Q; it represents the available information about x. To every (non-empty) subset of attributes P is associated an indiscernibility relation on U, denoted by I_P:

$$I_P=\{(x,y)\in U\times U : f(x,q)=f(y,q)\forall q\in P\} \tag{1}$$

If $(x,y)\in I_P$, it is said that the objects x and y are P-indiscernible. Clearly, the indiscernibility relation thus defined is an equivalence relation (reflexive, symmetric and transitive). The family of all the equivalence classes of the relation I_P is denoted by U/I_P and the equivalence class containing an element $x\in U$ by $I_P(x)$. The equivalence classes of the relation I_P are called P-elementary sets. If $P=Q$, the Q-elementary sets are called atoms.

2.5 The approximations

Let S be a data table, X a non-empty subset of U and $\emptyset\neq P\subset Q$.The P-lower approximation and the P-upper approximation of X in S are defined, respectively, by:

$$\underline{P}(X) = \{x \in U : I_P(x) \subseteq X\}, \tag{2}$$

$$\overline{P}(X) = \{x \in U : I_P(x) \cap X \neq \emptyset\} \tag{3}$$

The elements of $\underline{P}(X)$. are all and only those objects $x \in U$ which belong to the equivalence classes generated by the indiscernibility relation I_P, contained in X; the elements of $\overline{P}(X)$ are all and only those objects $x \in U$ which belong to the equivalence classes generated by the indiscernibility relation I_P, containing at least one object x belonging to X. In other words, $\underline{P}(X)$. is the largest union of the P-elementary sets included in X, while $\overline{P}(X)$ is the smallest union of the P-elementary sets containing X.

The P-boundary of X in S, denoted by

$$Bn_P(X) \text{is } BN_P(X) = \overline{P}(X) - \underline{P}(X) \tag{4}$$

The following relation holds:

$$\underline{P}(X) \subseteq X \subseteq \overline{P}(X) \tag{5}$$

Therefore, if an object x belongs to $\underline{P}(X)$, it is certainly also an element of X, while if x belongs to $\overline{P}(X)$, it may belong to the set X. $Bn_P(X)$ constitutes the "doubtful region" of X: nothing can be said with certainty about the belonging of its elements to the set X.

The following relation, called complementarity property, is satisfied:

$$\underline{P}(X) = U - \overline{P}(U - X) \tag{6}$$

If the P-boundary of X is empty, $Bn_P(X) = \emptyset$ then the set X is an ordinary (exact) set with respect to P, that is, it may be expressed as the union of a certain number of P-elementary sets; otherwise, if $Bn_P(X) \neq \emptyset$, the set X is an approximate (rough) set with respect to P and may be characterized by means of the approximations $\underline{P}(X)$ and $\overline{P}(X)$. The family of all the sets $X \subseteq U$ having the same P-lower and P-upper approximations is called a rough set. The following ratio defines an accuracy of the approximation of X, $X \neq \emptyset$, by means of the attributes from P:

$$\alpha_P(X) = \frac{|\underline{P}(X)|}{|\overline{P}(X)|} \tag{7}$$

where $|Y|$ indicates the cardinality of a finite set Y. Obviously, $0 \leq \alpha_P(X) \leq 1$; if $\alpha_P(X) = 1$, X is an ordinary (exact) set with respect to P; if $\alpha_P(X) < 1$, X is a rough (vague) set with respect to P.

Another ratio defines a quality of the approximation of X by means of the attributes from P:

$$\gamma_P(X) = \frac{|\underline{P}(X)|}{|X|} \tag{8}$$

is called quality of the approximation of classification Y by set of attributes P, or in short, quality of classification. It expresses the ratio of all P-correctly classified objects to all objects in the system.

The main preoccupation of the rough sets theory is approximation of subsets or partitions of U, representing a knowledge about U, with other sets or partitions built up using available information about U.

From the viewpoint of a particular object $x \in U$, it may be interesting, however, to use the available information to assess the degree of its membership to a subset X of U. The subset X can be identified with a concept of knowledge to be approximated. Using the rough set approach one can calculate the membership function $\mu_X^P(X)$ (rough membership function) as

$$\mu_X^P(x) = \frac{|X \cap I_P(x)|}{|I_P(x)|} \tag{9}$$

The value of $\mu_X^P(X)$ may be interpreted analogously to conditional probability and may be understood as the degree of certainty (credibility) to which x belongs to X. Observe that the value of the membership function is calculated from the available data, and not subjectively assumed, as it is the case of membership functions of fuzzy sets.

Between the rough membership function and the approximations of X the following relationships hold:

$$\underline{P}(X) = \left\{x \in X : \mu_X^P(x) = 1\right\} \tag{10}$$

$$\overline{P}(X) = \left\{x \in X : \mu_X^P(x) > 0\right\} \tag{11}$$

$$BN_P(X) = \left\{x \in X : 0 <_X^P (x) < 1\right\} \tag{12}$$

In the rough sets theory there is, therefore, a close link between vagueness (granularity) connected with rough approximation of sets and uncertainty connected with rough membership of objects to sets.

2.6 The dependence and reduction of attributes

A very important concept for concrete applications is that of dependence of attributes. Intuitively, a set of attributes $T \subseteq Q$ totally depends on a set of attributes $P \subseteq Q$ (notation $P \rightarrow T$) if all the values of the attributes from T are uniquely determined by the values of the attributes from P, that is, if a functional dependence exists between evaluations by the attributes from P and by the attributes from T. In other words, the partition generated by the attributes from P is at least as "fine" as that generated by the attributes from T, so that it is sufficient to use the attributes from P to build the partition U / T.Formally, T totally depends on P if ..

Therefore, T is totally (partially) dependent on P if all (some) elements of the universe U may be univocally assigned to classes of the partition U / T, using only the attributes from P.

Another issue of great practical importance is that of "superfluous"data in a data table. Superfluous data can be eliminated, in fact, without deteriorating the information contained in the original table.

Let $P \subseteq Q$ and $p \in P$. It is said that attribute p is superfluous in P if $I_P = I_{P-\{p\}}$; otherwise, p is indispensable in P. The set P is independent (orthogonal) if all its attributes are indispensable. The subset P' of P is a reduct of P (denotation $\mathrm{Re}d(P)$) if $P'$is independent and $I_{P'} = I_P$.

A reduct of P may also be defined with respect to an approximation of a partition Y of U. It is then called Y-reduct of P (denotation $\mathrm{Re}d_Y(P)$.) and specifies a minimal subset P' of P which keeps the quality of classification unchanged, i.e. $\gamma_{P'}(Y) = \gamma_P(Y)$. In other words, the attributes that do not belong to Y-reduct of P are superfluous with respect to the classification Y of objects from U. More than one Y-reduct (or reduct) of P may exist in a data table. The set containing all the indispensable attributes of P is known as the Y -core. Formally

$$Core_Y(P) = \cap \mathrm{Re}d_Y(P) \tag{13}$$

Obviously, since the Y-core is the intersection of all the Y-reducts of P, it is included in every Y-reduct of P. It is the most important subset of attributes of Q, because none of its elements can be removed without deteriorating the quality of classification.

2.7 The decision table and the decision rules

If in a data table the attributes of set Q are divided into condition attributes (set $C \neq \emptyset$) and decision attributes (set $D \neq \emptyset$), $C \cup D = Q$ and $C \cap D = Q$, such a table is called a decision table. The decision attributes induce a partition of U deduced from the indiscernibility relation I_D in a way that is independent of the condition attributes. D-elementary sets are called decision classes. There is a tendency to reduce the set C while keeping all important relationships between C and D, in order to make decisions on the basis of a smaller amount of information. When the set of condition attributes is replaced by one of its reducts, the quality of approximation of the classification induced by the decision attributes is not deteriorating. Since the tendency is to underline the functional dependencies between condition and decision attributes, a decision table may also be seen as a set of decision rules. These are logical statements (implications) of the type "*if..., then...*", where the antecedent (condition part) specifies values assumed by one or more condition attributes (description of C-elementary sets) and the consequence (decision part) specifies an assignment to one or more decision classes (description of D-elementary sets). Therefore, the syntax of a rule is the following:

if $f(x,q_1)$ is equal to r_{q_1} and $f(x,q_2)$ is equal to r_{q_2} and. . . $f(x,q_P)$ is equal to r_{q_P}, then x belongs to Y_{j1} or

Y_{j2} or . . . Y_{jK}, where $(q_1,q_2,...q_P)\subseteq C$; . $(r_{q_1},r_{q_2},...r_{q_P})\in V_{q_1}\times V_{q_2}\times ...V_{q_P}$ and $Y_{j1},Y_{j2}...,Y_{jK}$ are some decision classes of the considered classification (D-elementary sets). If the consequence is univocal, i.e. k=1, then the rule is exact, otherwise it is approximate or ambiguous. An object $x\in U$ supports decision rule r if its description is matching both the condition part and the decision part of the rule. We also say that decision rule r covers object x if it matches at least the condition part of the rule. Each decision rule is characterized by its strength, defined as the number of objects supporting the rule. In the case of approximate rules, the strength is calculated for each possible decision class separately. Let us observe that exact rules are supported only by objects from the lower approximation of the corresponding decision class. Approximate rules are supported, in turn, only by objects from the boundaries of the corresponding decision classes.

2.8 The fuzzy measures and rough sets

From a formal point of view, the quality of classification satisfies the properties of set functions called fuzzy measures. As observed by Grabisch (1997), fuzzy measures constitute a useful tool for modeling the importance of coalitions. In Greco (1999), fuzzy measures have been used to assess a relative value of information supplied by each attribute and to analyze the interactions among attributes, basing on the quality of classification calculated from the rough set approach. Let us explain this point in more detail. Let $N=\{1,2...n\}$ be a finite set, whose elements could be players in a game, criteria in a multicriteria decision problem, attributes in a data table, etc., and let $P(N)$. denote the power set of N, i.e. the set of all subsets of N.

A fuzzy measure on N is a set function $\mu : P(N)\rightarrow[0,1]$ satisfying the following axioms:

1. $\mu(\emptyset)=0, \mu(N)=1$ (14)

2. $A\subseteq B$ implies $\mu(A)\leq\mu(B)$ for all $A,B\in P(N)$ (15)

In the following, the first axiom is relaxed by considering the condition $\mu(N) \leq 1$ instead of $\mu(N) = 1$.

Within game theory, the function $\mu(A)$ is called characteristic function and represents the payooff obtained by the coalition $A \subseteq N$ in a cooperative game (Shapley 1953; Banzhaf 1965); in a multicriteria decision problem, $\mu(A)$. can be interpreted as the conjoint importance of the criteria from $A \subseteq N$ (Grabisch 1997).

Some indices have been introduced in game theory as specific solutions of cooperative games. The most important are the Shapley value and the Banzhaf value. The Shapley and Banzhaf values have also been proposed to represent the average importance of particular criteria within multicriteria decision analysis, when for the conjoint importance of criteria fuzzy measures are used (Murofushi, 1992).

In addition to the indices concerning particular criteria, other indices have been proposed to measure the interaction between pairs of criteria. Interaction indices have been suggested by Murofushi and Soneda (1993) and Roubens (1996) with respect to Shapley value and Banzhaf value, respectively.

In addition to the interaction indices, another concept useful for the interpretation of the fuzzy measures is the Möbius representation of μ, i.e. the set function $m : P(N) \rightarrow R$ defined by

$$m(A) = \sum_{B \subseteq A} (-1)^{|A-B|} \mu(B) \tag{16}$$

for any. $A \subseteq N$ within Dempster-Shafer theory of evidence, $m(A)$ is interpreted as basic probability assignment.

The Möbius representation $m(A)$ of μ can be interpreted as the conjoint contribution of the subset of attributes $A \subseteq C$ to the quality of classification.

All of these indices are useful to study the informational dependence among the considered attributes and to choose the best reduct.

2.9 The model to support the valuation of the real estate capital

For the definition of a model to support the valuation of real estate capital is necessary to recall some basic elements. Therefore we set out below some reference instruments to promote the understanding of the model for the evaluation of real estate capital.

2.10 The theory capital

For the definition of a model to support the valuation of real estate capital it is necessary to recall the Rizzo's theory capital (Rizzo 2006), by which the value of a capital asset can be expressed by the following formula:

$$V = K + aK \tag{17}$$

Where:
K represents the "theoretical" value or balance of a capital;
K represents the surplus value or capital gain on the assumption that $a>0$, and represents the minusvalore or capital loss on the assumption that $a<0$; in the latter case must $0<|a|<1$ occur because the loss of capital in absolute value can not be greater than *K*.

Thus, the positive or negative value of a depends of real and monetary factors.

From the capitalization formula $V = \frac{R_n}{r}$:

$r = \frac{R_n}{K(1+a)}$, in the event occurs or is anticipated earnings of capital-value;

$r = \frac{R_n}{K(1-|a|)}$, in the event occurs or is anticipated losses of capital-value.

If $ra = C$ we have: $r' = r + C$; $C = r' - r$. C is due to the hicksian concept of growing.

In fact: $C = r' - \frac{1}{P} = r' - \frac{1}{1/r} = r' - r$ when $r' > r$.

If $-|a|r = D$ we have $r' = r + D$; $D = r' - r$. D is due to the hicksian concept of diminishing.

In fact: $D = r' - \frac{1}{P} = r' - \frac{1}{1/r} = r' - r$ in cui $r' < r$.

However, this approach than the hicksian (Hicks 1959), differs in that in addition to considering the change in the profit rate on an investment of capital in relation to the time variable considers the combined effect of the monetary and the temporal component, in fact it considers the temporal and monetary dimension of the investment. Then, starting from this approach to value theory in which each investment capital has a its temporal and monetary shape and to its temporal and monetary dimension, for the modeling of the mechanisms of regulation of real estate capital will need to understand the factors that influence and change the values of the profit deemed normal rate. But, the study does not propose a dynamic analysis, rather it will focus on the factors that influence the values of the capitalization rate in a market, causing increases or decreases in the value of real estate capital. The problem is that you're studying, concerns the identification of factors that influence *a*. The factors that influence *a*, are instrumental to identify the dynamics that determine in the market, values of a <0 or> 0.

2.11 The analytical estimation by the Carlo Forte's procedure

In literature among, the economic and estimation proposed techniques for the estimation of real estate value, surely the analytical estimate by the C. Forte's procedure (Forte 1968) is among the best known.

Through the C. Forte's process is possible to estimate a real estate value of a building in a specific market by using the capitalization technique of income.

To determine the market value of a building by the Forte's procedure is necessary to make two distinct market research:

An investigation about the building for sale, to be performed on property as far as possible similar to that being evaluated, in an area with similar characteristics to that where the property is situated. Considering, however, a sample of sufficient amplitude to find a reasonable number of cases;

An investigation about on leased properties, to be made on properties as similar as possible to that being evaluated, in an area with similar characteristics to that where the property is situated. Considering, however, a sample of sufficient amplitude to find a reasonable number of cases.

The market value of the object of evaluation will be given by following formula:

$$Vm = Rn/r \tag{18}$$

Where: Vm = the market value of the property being assessed; Rn = the net income of the property being assessed; r = the capitalization rate of the property being assessed.

The net income is determined from the gross income which you will deduct the master's expenses[1], by using the following formula:

$$\mathrm{Rn} = \mathrm{Rl} - \mathrm{Spp} \tag{19}$$

The capitalization rate shall calculate from the average rate of the market by using the following the formula:

$$r = rmed + \sum_i A_i - \sum_i D_i \tag{20}$$

Where: r = the capitalization rate of the property being assessed; $rmed$ = average rate;

[1] The master's expenses are the following:
- The costs for the services;
- The insurance costs;
- The expenses for the depreciation;
- The cost of the administration;
- The expenses for the maintenance;
- The costs due to non-collectability and the costs due to vacancies;
- The tax expense.

$\sum_i A_i$ = the summation of the factors that increase the rate e $\sum_i D_i$ the summation of the factors that decrease the rate. The factors which increase and which decrease the rate of capitalization are influenced by the following elements:

- the capital risks, then, the economic risks of the income and the economic risks of the technical capital;
- the capital productivity;
- the capital scarcity;
- the capital regeneration;
- the duration of the investment;
- the monetary effects.

The factors which give growth and decrease the capitalization rate represent a synthesis of all the elements that can change the value of *a*. Obviously, they were recalled only a few elements of the *C*. Forte's procedure, the rest can be found in literature.

2.12 The real estate decision problem

An important point of the problem addressed is to define the mechanisms that regulate and govern the dynamics of the real estate market, in order to understand how it enhances the real estate capital in the market. For the definition of a model to support the valuation of real estate capital is necessary to define a decision problem, in fact in this case it is necessary to define one specific real estate problem decision to support.

Therefore, the structure of a model to support the valuation of real estate capital can be summarized in Figure 2.

2.13 The condition system of the real estate market

At this point of the study must be identified with the earlier reference to systemic-relational approach, to generate a modeling the real estate market, all the elements that characterize the system of the external relations of the market and all the elements that characterize the system of the internal relations between the different components of the system-market. For the resolution of real estate decision problem using the algorithm of DRSA (Greco 2007), that is part of the family of rough sets, it is necessary to identify the characteristics of the condition of the universe, ie of the real estate market. The identification of the condition characteristics of the real estate market can be made starting from the identification of the elements that characterize the system (Trovato 2010) of external relations and internal market. In particular in the following table has been identified the family of elements of conditions supporting the modeling of the real estate market.

Figure 1. The real estate decision problem.

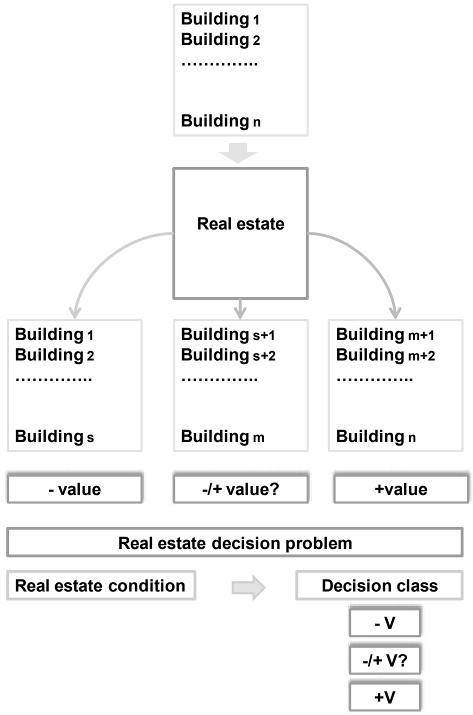

Figure 2. The evaluation real estate capital model.

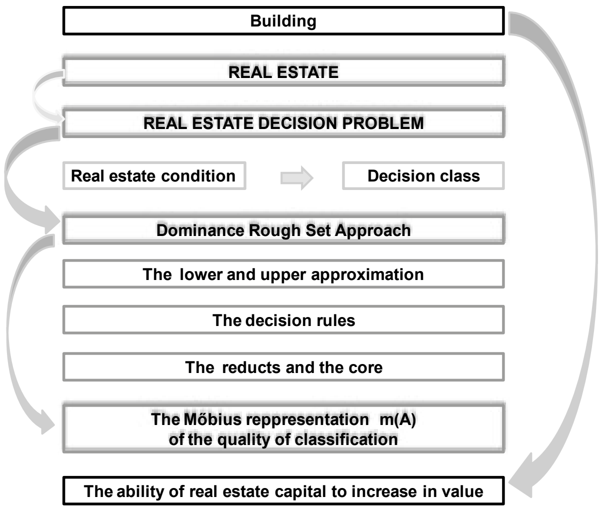

Table 1. Condition characteristics of the real estate market.

Condition characteristics of the real estate market	
1	Positional extrinsic features
2	Environmental characteristics
3	Geomorphological characteristics
4	Inherent positional features
5	Technology features
6	Productive features
7	Features of the lease
8	Characteristics of use
9	Internal management features
10	Social characteristics
11	Architectural features
12	Functional characteristics
13	Historical-cultural features
14	Characteristics of the city planning legislation
15	Characteristics of the mobility and the transport
16	Transactional features
17	Financial characteristics
18	Typological characteristics of the market
19	Characteristics of demand
20	Characteristics of offer
21	General macroeconomic features
22	External management features

The family of the condition characteristics of the real estate market allows you to identify the condition criteria at the base of real estate decision problem. In particular for each area of characterization of the real estate market has been identified a family of condition criteria. Each condition criterion was associated with a judgement and for each judegement a number that gives a concise evaluation of the considered criteria and helps to define the information table to support to the decision problem in the question. Below, the condition criteria and the possible assessments in the form of judgments and numbers associated with them.

Figure 3. The condition criteria a) for the real estate decision model.

CONDITION CHARACTERISTICS		REF.	CONDITION CRITERIA
1	POSITIONAL EXTRINSIC FEATURES	A1	Centrality
		A2	Spatial quality of settlement
		A3	Quality of equipments
		A4	Quality of services
		A5	Mix of the functions
		A6	Mix socio-professional
		A7	Urban maintenance
		A8	External accessibility of the public transport
		A9	External accessibility of the pirvate transport
		A10	Internal accessibility by public transport
		A11	Internal accessibility by private vehicles
2	ENVIROMENTAL CHARACTERISTICS	A12	Air pollution
		A13	Noise pollution
		A14	The efficiency level of the system of urban solid waste disposal
3	GEOMORPHOLOGICAL CHARACTERISTICS	A15	Seismic vulnerability
		A16	Hydrogeological vulnerability
4	INHERENT POSITIONAL FEATURES	A17	Panoramic views
		A18	Which looks out across
		A19	Brightness
		A20	Exposure
		A21	Internal security
5	TECHNOLOGY FEATURE	A22	Level of the supplied equipment
		A23	Level of finish
		A24	State of the conservation building
		A25	Level of technological and constructive characteristics
		A26	State of the conservation for the units
6	PRODUCTIVE FEATURES	A27	Ability to produce income
		A28	Level of generated income
7	FEATURES OF THE LEASE	A29	Type of contract for subject
		A30	Coverage of the contract
		A31	Duration of the contract
8	CHARACTERISTICS OF USE	A32	Level of fitness of use
		A33	Level of flexibility
9	INTERNAL MANAGEMENT FEATURES	A34	Efficiency level of the managing by administrator
		A35	Imposed constraints by the condominium lease
		A36	Condominial amount
		A37	Level of cover of the condominial amount
10	SOCIAL CHARACTERISTICS	A38	Characterization of the resident population
		A39	Condominiums with pets
		A40	Level of contentiousness of the condominiums
		A41	Cultural level
		A42	Level of perception of condominial space as a common property
11	ARCHITECTURAL FEATURES	A43	Typology
		A44	Planimetric and distributivity quality in relation to the typology
		A45	Altimetrical quality in relation to the typology
		A46	Quality of the direct accessories (terrace, balconies, etc.)
		A47	Quality of the indirect accessories (garage or parking space)
12	FUNCTIONAL CHARACTERISTICS	A48	Level of the disengagement
		A49	Level of the ventilation
		A50	Illumination level
		A51	Level of supplied of the direct accessories
		A52	Level of supplied of the indirect accessories
		A53	Level of access to the direct accessories (terrace or balcony)
		A54	Level of access to the indirect accessories (garage or parking space)
13	HISTORICAL-CULTURAL FEATURES	A55	Construction period
		A56	Perceived symbolic level
		A57	Level of conservation of the original characters
14	CHARACTERISTICS OF THE CITY PLANNING LEGISLATION	A58	Compatibility of the urban destination of the adjacent buildings
		A59	Forecastof some infrastructure construction
		A60	Forecast of some construction equipment
		A61	Forecast of some environmental improvements that determinate a private cost
		A62	Forecast of some environmental deteriorations
		A63	Forecast of some likely expropriation
		A64	Forecast of some urban policies that limit yes / no the expansions
		A65	Level of acceptance / denial of the requests
		A66	Presence or absence of a equalization scheme
		A67	Level of incentives in the equalization system in terms of money / sup.

Figure 4. The condition criteria b) for the real estate decision model.

CONDITION CHARACTERISTICS		REF.	CONDITION CRITERIA
15	CHARACTERISTICS OF THE MOBILITY AND OF THE TRANSPORT	A69	Level of access to parking areas in the physical sense
		A70	Level of access to parking areas in the economic sense (ticket price)
		A71	Contract for the use of parking areas (exist or not exist)
		A72	Type of contract for use of parking
		A73	Level of access to urban transport in the physical sense (the location of stops)
		A74	Level of access to urban transport in the economic sense (ticket price)
		A75	Quality of service of urban public transport (synthesis of safety, of cleanliness, of timing and of reliability)
16	TRANSACTIONAL FEATURES	A76	Type of transaction
17	FINANCIAL CHARACTERISTICS	A77	Level of access to mortgage
		A78	Type of mortgage
		A79	Duration of mortgage
		A80	Level of access to credit for firms in the construction industry
18	TYPOLOGICAL CHARACTERISTICS OF THE MARKET	A81	Market type
		A82	Rate of the growth market
19	CHARACTERISTICS OF DEMAND	A83	Characterization of demand in relation to the typology building
		A84	Characterization of the demand in relation to the localization
		A85	Characterization of the demand in relation to the technological level
		A86	Characterization of the demand in relation to the level of the accessories
		A87	The WTP of the consumer
20	CHARACTERISTICS OF OFFER	A88	Characterization of the offer in relation to the typology building
		A89	Characterization of the offer in relation to the localization
		A90	Characterization of the offer in relation to the technological level
		A91	Characterization of 'offer in relation to the level of the accessories
		A92	Cost of production
		A93	Level of reliability of the undertaking
21	GENERAL MACROECONOMIC FEATURES	A94	Employment growth rate
		A95	Level of the household income
		A96	Level of the consumers's perception about the general economic situation
		A97	Level of the undertaking's perception about the general economic situation
22	EXTERNAL MANAGEMENT FEATURES	A98	Leadership skills of the public administration
		A99	Efficiency of public administration

3. Application. The real estate of Ortigia

3.1 The context

The market under study is to Ortigia, the historic center of Syracuse. Ortigia is the place where they are synthesized some of the historical, architectural, cultural and environmental elements of the heritage of Syracuse's city, which belongs to the WHL. In recent years the real estate of Ortigia has picked up local and external interests , attracted here by the great potential development that this center coudl offer, but that had not yet been be fully exploit till now. Also, in recent years, the number of university seats that have been located in Ortigia, have helped to revitalize this urban context, which for years had been considered marginal. Then the real estate capital of Ortigia, in recent years, has become an object of interest of investors for the its susceptibility as capital complex.

3.2 The application of the model

For a definition of the sample of examples that is instrumental for the application of the algorithm DRSA, you must locate the sample of real estates, that within the market have recorded the highest real estate value and those have recorded the lower value real estate. Thus, by identifying the range of oscillation of the capitalization rate you can find the law that places the real estate sample in a class of decision. Defined the values that characterize the decision class of real estate decision problem, for each real estate of the sample will be detected the qualitative and quantitative assessments for the criteria of condition, and then, will be possible to associate the assessments on the criteria of conditions with the found decision class.

In this regard have been carried out a series of general market surveys, to characterize the judgments on the condition criteria and in particular on real estate that represent the sample for the example characterizing for the real estate decision problem for Ortigia.

3.2.1 The survey sample

In particular, there were two type of real estate samples, a sample relates to real estate for rent and the other relates to real estate for sale.

Table 2. The rents of the real estate sample.

The real estate market of di Ortigia: the summary sheet of the rents								
Id.	Street/Square	number	n. room	square meters	year of construction	monthly rent	gross annual income	gross annual income/room
1	Via Castello Maniace	5	2	70	1700	€ 420,00	€ 5.040,00	€ 2.550,00
2	Santa Teresa	3	2	45	1800	€ 700,00	€ 8.400,00	€ 4.200,00
3	Lungomare	1	2	40	1800	€ 350,00	€ 4.200,00	€ 2.100,00
4	Vicolo Mastrarua	13	1	50	1800	€ 500,00	€ 6.000,00	€ 6.000,00
5	Vicolo Mastrarua	13	2	75	1800	€ 700,00	€ 8.400,00	€ 4.200,00
6	Vittorio Veneto	207	2	60	1800	€ 700,00	€ 8.400,00	€ 4.200,00
7	Corso Matteotti	45	4	75	1970	€ 960,00	€ 11.520,00	€ 2.900,00
8	Corso Matteotti	45	5	200	1970	€ 2.450,00	€ 29.400,00	€ 5.900,00
9	Piazza San Giuseppe	4	3	60	1900	€ 500,00	€ 6.000,00	€ 2.000,00
10	Ronco Bentivegna	19	5	200	1883	€ 1.440,00	€ 17.280,00	€ 3.500,00
11	Giudecca	13	5	150	1900	€ 1.000,00	€ 12.000,00	€ 2.400,00
12	Vergini	18	3	80	end '800	€ 730,00	€ 8.760,00	€ 2.950,00
13	Gargallo	26	6	85	end '800	€ 1.100,00	€ 13.200,00	€ 2.200,00
14	Trieste	13	6	200	end '800	€ 1.350,00	€ 16.200,00	€ 2.700,00
						Values average	€ 11.000,00	€ 3.400,00

$$R_n med = S_i Rn_i/Sroom = €2.042 \quad (21)$$

It may notice that belong to the family of real estate that are likely to decrease in value the following buildings: 2, 5, 6, 11.

$$\Delta V(-) r=2\text{-}4\% \quad (22)$$

Table 3. Real estate values of the sample.

The real estate market of Ortigia: the summary sheet of the market values									
Id.	Street/Square	number	n. room	square meters	year of construction	market value	market value/room	market value/s. m.
1	Ruggero VII	18	2	65	1800	€ 200.000,00	€ 100.000,00	€ 3.000,00
2	Castello Maniace	21	2	50	1800	€ 125.000,00	€ 62.000,00	€ 2.500,00
3	Lungomare Alfeo	57	1	50	1800	€ 250.000,00	€ 250.000,00	€ 5.000,00
4	Castello Maniace	74	2	90	1900	€ 260.000,00	€ 130.000,00	€ 2.500,00
5	Lungomare	44	2	80	1800	€ 145.000,00	€ 72.000,00	€ 1.500,00
6	Vicolo III alla Mastrarua	15	2	90	1800	€ 105.000,00	€ 52.000,00	€ 1.000,00
7	Corso Matteotti	45	5	200	1970	€ 700.000,00	€ 140.000,00	€ 3.500,00
8	Cesare Battisti	16	3	90	1900	€ 700.000,00	€ 233.000,00	€ 7.500,00
9	Porta Marina	2	5,5	120	1900	€ 700.000,00	€ 127.000,00	€ 5.500,00
10	Ronco Bentivegna	19	5	200	1883	€ 1.000.000,00	€ 200.000,00	€ 5.000,00
11	Maestranze	97	4	190	1800	€ 270.000,00	€ 67.000,00	€ 1.000,00
						Values average	€ 130.000,00	€ 3.450,00

Table 4. Calculation of net income.

Id.	gross annual income	Master's expenses (% of the gross annual income)								net income
		expenses for the maintenance	costs for the services	costs due to non-collectability and the costs due to vacancies	insurance costs	expenses for the depreciation	cost of the administration	tax expense	total	
1	€ 5.040,00	3%	4%	0%	1%	0%	3%	25%	36%	€ 3.225,60
2	€ 8.400,00	3%	3%	0%	1%	0%	3%	25%	35%	€ 5.460,00
3	€ 4.200,00	4%	2%	2%	1%	0%	2%	25%	36%	€ 2.688,00
4	€ 6.000,00	3%	4%	0%	1%	0%	3%	25%	36%	€ 3.840,00
5	€ 8.400,00	3%	3%	0%	1%	0%	3%	25%	35%	€ 5.460,00
6	€ 8.400,00	4%	2%	3%	1%	0%	2%	25%	37%	€ 5.334,00
7	€ 11.520,00	3%	3%	5%	1%	0%	3%	25%	40%	€ 6.912,00
8	€ 29.400,00	3%	3%	0%	1%	0%	3%	25%	35%	€ 19.110,00
9	€ 6.000,00	3%	3%	0%	1%	0%	3%	25%	35%	€ 3.900,00
10	€ 17.280,00	4%	3%	0%	1%	0%	3%	25%	36%	€ 11.059,20
11	€ 12.000,00	2%	4%	3%	1%	0%	2%	25%	37%	€ 7.560,00
12	€ 8.760,00	3%	3%	0%	1%	0%	3%	25%	35%	€ 5.737,80
13	€ 13.200,00	4%	3%	2%	1%	0%	2%	25%	37%	€ 8.316,00
14	€ 16.200,00	4%	4%	5%	1%	0%	3%	25%	42%	€ 9.396,00
								Values average		€ 6.999,90

Table 5. Distribution of the capitalization rates, the maximum rate, the minimum rate and the centroid distribution of the rates.

id.	capitalization rate	
1	r1	2%
2	r2	3%
3	r3	1%
4	r4	2%
5	r5	4%
6	r6	4%
7	r7	1%
8	r8	1%
9	r9	2%
10	r10	1%
11	r11	3%
	rmin	1%
	rmax	4%
	r centroide	2%

Instead belong to the family of real estate that are likely to increase in value the following buildings: 3, 7, 8, 10.

Δ V(+) r=1-2% (23)

And still belong to the family of real estate that aren't likely to increase or decrease in value, the following buildings:2, 4, 9.

Δ V(-,+) r=2% (24)

Table 6. The distribution of capitalization rates and the decision class.

id.	capitalization rate		decision class
1	r1	2%	ΔV(-,+)?
2	r2	3%	ΔV(-)
3	r3	1%	ΔV(+)
4	r4	2%	ΔV(-,+)?
5	r5	4%	ΔV(-)
6	r6	4%	ΔV(-)
7	r7	1%	ΔV(+)
8	r8	1%	ΔV(+)
9	r9	2%	ΔV(-,+)?
10	r10	1%	ΔV(+)
11	r11	3%	ΔV(-)

As can be seen from the tables of the criteria of condition, the previous views, have been identified 99 condition criteria for the characterization of the market, that they may be write in more compact form in the following way: $A_1 .... A_{99}$. So the information table in the real estate decision problem may be write in the following way:

Table 7.The information table.

		CONDITION CRITERIA							DECISION
EXAMPLE	Id. Ex.	A1	A2	A3	A4	A5	…………	A99	D
Ruggero VII	1	4	4	4	3	3		2	ΔV(-,+)?
Castello Maniace	2	4	4	4	3	3		2	ΔV(-)
Lungomare Alfeo	3	4	4	4	3	3		2	ΔV(+)
Castello Maniace	4	4	4	4	3	3		2	ΔV(-,+)?
Lungomare	5	4	4	4	3	3		2	ΔV(-)
Vicolo III alla Mastrarua	6	4	4	4	3	3		2	ΔV(-)
Corso Matteotti	7	4	4	4	3	3		2	ΔV(+)
Cesare Battisti	8	4	4	4	3	3		2	ΔV(+)
Porta Marina	9	4	4	4	3	3		2	ΔV(-,+)?
Ronco Bentivegna	10	4	4	4	3	3		2	ΔV(+)
Della Maestranze	11	4	4	4	3	3		2	ΔV(-)

But, once identified for the sample being analyzed the values of the condition criteria, it was possible to see that many of the assessments were the same, this is due to the persistence of some characteristics of the sample of the real estate, such as location, typology, size, relationship context with other urban systems, etc. Then, it was necessary to define a table of information in which they were present only the most important provided condition criteria for the being analyzed sample.

Obviously the larger structure is a general support to the modeling of a real estate market, which from time to time as the case may be reformulated.

So it was necessary to pass from the information table $A_1........A_{99} \rightarrow D$ to the information table with the following reductions:

$$A_6, A_7, ...A_{9,} A_{10}, A_{11}, ..., A_{17}, A_{18}, A_{19}, A_{20}, A_{21}, A_{22}, A_{23}, A_{24}, A_{25}, A_{26}, A_{27}, A_{28}, A_{29}, A_{30}, A_{31}...,$$
$$A_{33}, A_{34}, A_{35}, A_{36}, A_{37}, A_{38}; ...A_{42}, A_{43}, A_{44}, A_{45}, A_{46}, A_{47}, A_{48}, A_{49}, A_{50}, A_{51}; A_{52}; A_{53}, A_{54}, A_{55}, A_{56}, A_{57},$$
$$...., A_{69}, ...A_{71}, A_{72}, A_{73}, A_{74}, ..., A_{76}, A_{77}, ..., A_{79}, ..., A_{81}, A_{82}, A_{83}, ..., A_{85}, A_{86}, A_{87}, ...A_{90}, A_{91}, A_{92}, ... \rightarrow D \quad (25)$$

Once you define the table of information it may be implement the DRSA algorithm for the determination of the minimum set of decision rules, for the determination of reduced and the core and at the end were obtained the following results.

Table 8. The decision rules.

	Decision rules
1	***If*** the Panoramic views is excellent, ***then*** the real estate has capacitance to grow in value
2	***If*** the level of supplied of the indirect accessories is good, ***then*** the real estate has capacitance to grow in value
3	***If*** the contract for use of parking has for the residents and those working in the area, ***then*** the real estate has capacitance to grow in value
4	***If*** the brightness is good, ***then*** the real estate maintains its value
5	***If*** the level of the disengagement is good, ***then*** the real estate maintains its value
6	***If*** the rate of the growth market is constant, ***then*** the real estate maintains its value
7	***If*** the building has one exposure at west, ***then*** the real estate maintains its value
8	***If*** the level of finish is è mediocre, ***then*** the real estate loses value
9	***If*** the level of the disengagement is poor, ***then*** the real estate loses value

The dominance-based rough approximations of upward and downward unions of decision classes can serve to induce a generalized description of sorting decisions in terms of "*if . . . , then . . .* " decision rules. For a given upward or downward union of classes, $Cl_t^{\geq}$ or $Cl_s^{\leq}$, the decision rules induced under a hypothesis that objects belonging to $\underline{P}(Cl_t^{\geq})$ or $\underline{P}(Cl_s^{\leq})$ are positive examples (that is objects that have to be matched by the induced decision rules), and all the others are negative (that is objects that have to be not matched by the induced decision rules), suggest a certain assignment to "class Cl_t or better", or to "class Cl_s or worse", respectively. On the other hand, the decision rules induced under a hypothesis that objects belonging to $\overline{P}(Cl_t^{\geq})$ or $\overline{P}(Cl_s^{\leq})$ are positive examples, and all the others are negative, suggest a possible assignment to "class Cl_t or better", or to "class Cl_s or worse", respectively. Finally, the decision rules induced under a hypothesis that objects belonging to theintersection $\overline{P}(Cl_t^{\geq}) \cap \overline{P}(Cl_s^{\leq})$ are positive examples, and all the others are negative, suggest an assignment to some classes between Cl_s and Cl_t (s < t).

Note that in the above rules, each condition profile defines a dominance cone in $|P|$-dimensional condition space $R^{|P|}$ where $P = \{i_1, \ldots, i_P\}$ is the set of criteria considered in the rule, and each decision defines a dominance cone in one-dimensional decision space {1, . . .,m}.

About the reducts, in this case have been generate the following reducts sets:

Table 9. The reducts.

Reducts	
166.944 reducts	Cardinality 4 -9

The core include the indispensable criteria, but in this case the core of are empty, because the $Core_Y(P) = \cap \mathrm{Red}_Y(P) = O$.

This means that the granules of knowledge for the sample of the real estate have empty intersection, that is not possible to obtain a minimum and essential set of criteria for representing the object of the universe, ie the mechanisms that increase in value the real estate in the market.

Table 10. The core of approximation.

Core
Empty

This is due to the chosen sample, which returns a jagged knowledge discovery, then it will necessary using a new sample a better implementation of the model.

Or it can highlight that the mechanisms that determine the value of the real estate within the market, does not allow to determine one law for the market. In addition, since the core is empty is not possible to calculate the sought quality of the approximation, that is instrumental in determining the fuzzy measure through the Mobius representation $m(A)$ of μ. So it can not propose a exemplify of the fuzzy measure of the ability to increase in value of a real estate capital in a market.

4. Conclusion

The identified proposed operational methodology in this study represents a synthesis of some traditional valuation methodologies that are proposed in the literature estimation with new valuation methodologies, developed in areas such as Operations Research, and therefore in one different area from that of real estate valuation, but that the offer the opportunity to create an evaluation structure as a result of a transdisciplinary approach that can mitigate some gaps or weaknesses of some traditional estimation techniques. The obtained results are not entirely satisfactory, because the analyzed sample has showed a variety of imperfections, which did not allow to extrapolate a number value for the fuzzy measure of the capacity of the real estate to increase in value within the real estate market of Ortigia, but the structure of the condition criteria is very helpful for the analysis of any real estate market and the procedure with the mechanism of generation of decision rules, the core, the reducts, is an effective proposed methodology-operational for the analysis of the real estate market. In addition, it necessary to emphasize that the procedure is able to jointly develop quantitative and qualitative variables and is a neutral procedure, in that it develops as a "glass box". In fact in this case, it is not the subject that carries out the assessment that interprets the laws by which the real estate can to increase or to decrease in value in the market, but is the market that defines its laws from its structure and from its characterization, in fact in this case the algorithm has be use to return the growth mechanisms or the decrease mechanisms in the value in the market. It is understood that the proposed study, will be improvements in order to obtain a support for an exhaustive evaluation of the real estate market for public and private subject, and therefore as support for the development of a database of an observatory building.

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