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Capital Asset Pricing Model: An application to the estimation of undivided real estate shares*

The estimate of undivided and indivisible real estate shares, in relation to different building types represents, in practice application, a recurring underestimated by the professional appraisers.

This problem requires finding a logical solution that can to rationalize the process the process of estimation of undivided shares, given that these shares are more difficult to sell and, consequently, there is a decrease of their market value.

It follows that in this estimative case the problem can be lead, both in theoretical and practical terms, to assessment of specific real estate investment's risk and how to convert the measure of risk into an expected return that can compensate it.

In this framework, the proposed contribution aims to implement the *Capital Asset Pricing Model* with *Penalized Spline Semiparametric Method* in order to obtain an estimation algorithm that allows to rationalize the approach to the problem of estimation of undivided real estate shares using data easy to find.

1. Introduction

In professional practice an increasingly recurring question concerns the estimation of undivided and indivisible shares of real estate property, with reference to different building types (residential, office buildings, commercial, etc.).

This problem, often overlooked by professional appraisers, is seen frequently in judicial real estate execution where the liquidation of the undivided share of goods subject to compulsory sale procedure (with the remaining shares belonging to other unrelated third parties to legal prosecution), is governed by the Articles 599 et seq. the Code of Civil Procedure with three different possible alternatives, namely¹:

- sale of single undivided share;
- separation in nature;
- division.

* Organization and coordination of work are due to V. Del Giudice, the writing of memory and the application of the model are attributed to P. De Paola.

¹ In particular, in Italy the Code of Civil Procedure states that “[...] the judge of enforcement proceeding, on request by the distrainer creditor or co-owners and heard all interested parties, provides, when possible, the separation of the share in kind due to the debtor. If the separation is not possible, judge can order the sale of single undivided share or provide for the division to proceed under the regulation of Civil Code [...]” (C.P.C.; Chapter V: The expropriation of property undivided; Art. 600).

Of the three alternatives provided by the law, there is no doubt that the sale of undivided share is the only option that poses fewer problems of coordination with the ordinary enforcement proceedings, having, thus, merely a partial modification of the subjective structure of the communion in the absence of its dissolution.

At present, partly justify by economic crisis to national level, corresponding an increasing number of property enforcement proceedings arising mostly from unpaid mortgages or recurring estimative circumstances (firstly the case of productive property leased to third companies) that demand the estimation of individual building ownership shares, imposes the need to seek a solution with logic and transparency is able to rationalize and solve the problem for the estimation of undivided shares, taking also into account that for such shares, being clearly the greatest difficulties in selling, there is a reduction of its market value².

At this point it is necessary premise that each trader expresses a different propensity for financial risk, understood as a willingness to accept possible changes in the value of its investment in a more or less sensitive over time. It's just so obvious besides that each specific mode of investment has a different level of risk, understood as the probability of change in expected return.

Can therefore assume that the purchase of an undivided share of property has a mainly speculative purposes, given that the buyer invests its financial resources in an operation without immediate utility understood, for example, such as residential use of the same buyer.

The purchase of an undivided share of property may be compared to an alternative form of financial investment, albeit at high risk for the possible existence of legal constraints and burdens of various types (marital agreements, court applications, other limitations and burden), faults and defects inherent in the judicial process and the uncertainty of the time period needed to monetize the expected return. It follows then that the problem in question can logically be focused on how to assess the specific risk of speculative investment, and how to convert the risk measure into an expected return that can compensate it.

Determination of expected return demand, preliminarily, a careful risk analysis in reference to following aspects:

- the objective probability that occurred, following the liquidation of single undivided ownership share, an event of dissolution of the communion, in the short or medium term, for economic good;
- determination of expected value for each possible outcome of the legal dissolution of the communion;
- variability so the outcome of judicial events may differ for uncertainties inherent the judicial proceedings.

From the foregoing it can logically be concluded that the expected return on the investment in question is expressed as the sum of two rates, an *risk-free interest rate* and an extra return to compensate for the risk of not marginal entrepreneur.

² Del Giudice V., (2010). *Estimo e Valutazione Economica dei Progetti*. Loffredo Editore. Napoli.

In general, the *risk-free interest rate* represents the payment of a risk-free financial activity, thereby assuming that in financial markets there is always a title with guaranteed performance and known (in formal terms the “bond yield” is a random variable with constant expected value and worthless variance³). In practice, application of these securities are identified by the government bonds in the short to medium term which, for example, the German Bund, the U.S. Treasury Bonds or the Italian B.O.T. and B.T.P.

Another problem concerns the measure of market risk due to property investment on the purchase of a undivided share (*undiversifiable risk*⁴), with the intention to investigate what investors require as a risk premium over the risk-free rate.

In the literature the risk premium is in most cases determined by recourse to the *Capital Asset Pricing Model* (C.A.P.M.), which represents a static, linear and monofactorial model that brings down the risk premium of an investment by comparing its expected return with that of the entire target market measured, precisely, by a single risk factor called *beta*.

In the remainder of this paper will therefore implemented the *Capital Asset Pricing Model* by *Penalized Spline Semiparametric Method* (P.S.S.M.) for the determination of beta risk factor, in order to obtain an estimation algorithm which allows to rationalize the approach to the problem the estimation of undivided shares of ownership and is, at the same time, market-driven, which means that they reflect current subjective perceptions of market using data readily available.

2. Literature Review

The classical theory of equilibrium prices for years has addressed issues related to knowledge of the expected return of securities, the appraisal of capital's cost in investment decisions and company valuations, the management of a portfolio of financial assets including the evaluation of its performance, all developing the concept of balance of the capital market and trying to find the appropriate measure of expected return on an investment respect to risk.

The relationship between risk and return of a security has been identified by the *Capital Asset Pricing Model* (C.A.P.M.), first proposed by Sharpe (1964) and later developed independently by Lintner (1965) and Mossin (1966).

These contributions have been joined in the time by numerous empirical investigations aimed at verifying the consistency of the risk-return relationship ex-

³ An investment can be defined as risk-free when effective returns are always equal to the expected return and, therefore, the risk can be defined in terms of variance in actual returns compared to an expected return.

⁴ Adding a risk premium to the risk-free interest rate is necessary to distinguish between diversifiable risk, which can be eliminated by portfolio diversification by investors in various financial assets, and non-diversifiable risk (called market risk also), that is implicit risks of investment (having regard to the specific financial activities) and can not be eliminated through diversification.

placit by the C.A.P.M., although the empirical evidence in favor of the C.A.P.M. is contradictory or at least only partly statistically significant, as evidenced by an extensive international literature [Lintner (1972), Douglas (1968), Black, Jensen and Scholes (1972), Blume and Friend (1970), (1973), Fama and McBeth, (1973), Litzenberger and Ramaswamy, (1979), Cristini, (1978), Caprio (1989); Murgia (1989); Caparelli and Viviani (1990)].

This is determined, primarily, by specific factors used in empirical tests methodologies of the C.A.P.M., where the empirical tests are conducted, in general, explaining a cross-sectional relation between average returns and historical *beta* of analyzed securities. In this regard, it notes that the goal of obtaining better estimates of the *beta* coefficient has helped to identify the critical issues inherent its estimation, thus correcting the errors in the regression phase [Miller-Scholes (1972), Roll (1977); Murgia (1989)] or deriving the *beta* coefficient in function of company fundamentals that determine it (*capital beta*) [Beaver, Kettler and Scholes (1970)].

On the other hand, a second critical issue concerns the assumptions underlying the original formulation of the C.A.P.M., which is not supported by empirical evidence because there are certain inefficiencies in the market that would not be consistent with the restrictive assumptions of the model [Brennan (1970); Litner, (1971), Black (1972), Mayers, (1972), Lindenberg (1979); Mayshar, (1981)].

Some experts invoke, therefore, that the *beta* may not be sufficient to evaluate the variations of returns as it may not be the only component of risk that investor asks like remuneration.

So, assuming that the return on investment can not be totally explained by its *beta*, subsisting risk factors in addition to market risk, alternative approaches have been developed multifactorial extensions of the C.A.P.M. Among these are mentioning Merton's Multi-Beta C.A.P.M. (1973), the Consumption C.A.P.M. of Breeden (1979) and finally the Arbitrage Pricing Model (A.P.T) of Ross (1976) that, among the risk factors related to returns, in addition to considering market risk including macroeconomic and financial variables. Numerous other studies emphasize the risk-return relationship is not only a function of the *beta*, but also in function of other variables such as the microeconomic *proxy* variables, for example, the *price/earnings ratios* (P/E), the *book-to-market value* (B/M), the *cash flow per security/price* (CF/P), etc.

It is results, in point, supporting the validity or less of multifactorial pricing models, studies conducted by Basu, (1977), Blume (1980), Jaffe, Keim and Westerfield, (1989), Aggarwal, Rao and Hiraki, (1990), Chan, Hamao and Lakonishok, (1991), Aggarwal, Rao and Hiraki, (1992), Fama and French, (1992), Fama and French, (1993), Davis (1994), Lakonishok, Shleifer and Vishny, (1994), Kothari, Shanken and Sloan (1995), Fama and French, (1996); Barontini, (1997), Daniel and Titman, (1997), Davis, Fama and French, (2000), and Yook McCabe (2001), Campbell and Voulteenaaho (2004). In summary, the main trend that emerged from these studies is summarized in the fact that the *beta*, alone, is almost never significant in explaining variations in yields, but gains significance when it is aggregated to proxies variables. This suggests, therefore, that the yield can be expressed in function of three prizes for the

risk: market risk premium, risk premium linked to the size and the risk premium related to *book to market ratio*. This model, characterized by three factors, was introduced, in particular, by Fama and French (1993) for to explain the variations of time-series returns in a very large number of practical cases.

Ultimately, the most obvious aspect that emerges from the studies and analysis conducted at the international level is certainly the not uniqueness besides the fragmentation of theoretical explanations to support the empirically observed relationship between performance and determinants of risk [Fidanza, (2003)]. These reports could in fact be simply a result of a not efficient capital market or non-rational behavior of investors, limiting, therefore, the formulation of a truly alternative to the C.A.P.M. model, the application of which remains in any case characterized by a remarkable ease of interpretation.

3. Capital Asset Pricing Model

The research of the C.A.P.M. is the *Mean-Variance Criterion* of Markowitz (1952) for the selection of efficient portfolios of securities (also known as modern portfolio theory)⁵.

More specifically, deriving from the theory of Markowitz and assumed an expected return desired, the C.A.P.M. provides that the risk premium in a market equilibrium need not be proportionate to all risk borne by the investor but the only risk that can not be eliminated, operationally, with the diversification of the portfolio securities.

The residual risk derived by the strategy of diversification (*systematic risk*) is therefore a risk that affects all financial assets, including the return of the market portfolio (which, being composed of a combination of various financial assets is therefore representative also the entire financial market).

In the C.A.P.M., the link between the extra return of the market portfolio ($R_M - r$), and extra return of *i-th* stock ($R_i - r$), is represented by a linear relationship⁶:

$$R_i - r = \alpha_i + \beta_i (R_M - r) + \varepsilon_i \quad (1.3)$$

where R_M is the return on market portfolio, R_i is the *i-th* security yield, r is the return on risk-free investments, α_i e β_i are coefficients, and ε_i is the residue of

⁵ The Markowitz's study is based, in particular, on the analysis of the process that generates demand and supply of financial assets depending by risk/return ratio; the fundamental principle of his theory predicts that, in order to constitute a portfolio of securities effectively, it needs to find a combination of securities for maximize the overall return and minimize, at the same time, the risk, offsetting asynchronous trends of individual stocks. It follows then that the risk can potentially be eliminated or greatly reduced with an operation of portfolio diversification (specific risk).

⁶ For a complete discussion of C.A.P.M. see the contribution of Cochrane J., *Asset Pricing* H. (2001), Princeton University Press, NJ.

the i -th security, moreover, we have that $E[\varepsilon_i] = 0$ with ε_i stochastically independent by R_M and ε_j (for each $j \neq i$).

In equation (1.3) can therefore distinguish the systematic part of extra yield of the i -th security, defined by the term $[\alpha_i + \beta_i (R_M - r)]$, from the non-systematic part, which coincides with the ε_i parameter⁷.

In the event of balance of the financial market result that $\alpha_i = 0$ for each i -th security; on the other hand, the β_i parameter may properly assume, therefore, as a measure of market risk or systemic risk (or rather determine the extent of the linear relationship that exists between the return of market portfolio and the i -th security yield)⁸.

From equation (1.3) comes, moreover, that the expected return of i -th stock (μ_i) and the return of the market portfolio (μ_M) are bound by the following formula:

$$\mu_i = r + \beta_i (\mu_M - r) \quad (2.3)$$

The formulation (2.3) also defines the *Security Market Line* (S.M.L.), and identifies, more precisely, the relationship between the expected return of a security and its systematic risk, measured by the β_i coefficient. Therefore, according to the *Securities Market Line*, the risk premium of a security (Φ_i) is related to the risk premium on the market portfolio in reason to the only systematic risk measured by β_i factor:

$$\Phi_i = \beta_i (\mu_M - r) \quad (3.3)$$

According to the assumptions of the model we assume that the S.M.L. includes all financial activities present in the market in balance condition, and also for each activity listed in the financial market in equilibrium is valid the relation (2.3).

For the calculation of security's β_i coefficient, or for the determination of the risk premium corresponds to a specific financial activity, we proceed with the relation (1.3) considering, in addition, the hypothesis of normal distribution of returns. According to this assumption and procedure of least squares, we obtain:

$$\hat{\beta}_i = \frac{Cov(R_i, R_M)}{Var(R_M)} \quad (4.3)$$

Therefore, the S.M.L. can be rewritten as:

⁷ Assuming that the distribution of returns is multivariate normal type we have that the stochastic independence between ε_i and R_M is equivalent to the condition: $Cov[\varepsilon_i, R_M] = 0$.

⁸ The beta coefficient measures the responsiveness of yield compared to the movements of the market in which the security is traded, and therefore, its non-diversifiable risk: in other words, the value of the deviation of expected returns than the benchmark.

$$\varphi_i = Cov \left[R_i, (\mu_M - r) \frac{R_M}{Var(R_M)} \right]$$

$$\varphi_i = \lambda \rho_{i,M} \sigma_i \tag{5.3}$$

being used in (5.3) the relation $Cov [R_i, R_M] = \rho_{i,M} \sigma_i \sigma_M$ where different terms have the following meanings: $\sigma_i = [Var (R_i)]^{1/2}$; $\rho_{i,M}$ is the coefficient of linear correlation between R_i and R_M ; and, finally, $\lambda = [(\mu_M - r) / \sigma_M]$ represents the market price for systematic risk unit, which is a characteristic quantity of financial market in balance condition and is an independent factor of security considered. In its original coverage the resolution of the C.A.P.M. consists of two phases⁹. First there is the determination of the systematic risk factor through the application of a *time-series market model regression*:

$$R_{jt} = \alpha_j + \beta_j \cdot R_{mt} + \varepsilon_{it} \tag{6.3}$$

where: R_{jt} is the *j-th* security yield at time *t*; R_{mt} is the return of market portfolio at time *t*; α_j is the intercept; β_j is the systematic risk factor of *j-th* security; ε_{it} is the residual term.

In the second phase, the systematic risk factor (β_j) obtained by the relation (6.3) is then used in a *cross-sectional regression model* to determine the average yield of the security in question:

$$R^*_j = a + b \cdot \beta_j + \eta_i \tag{7.3}$$

where: R^*_j is the average yield of *j-th* security respect to time period of observation; β_j is the systematic risk factor for *j-th* security derived from the first phase of resolving the CAPM; the terms *a* and *b* represent, respectively, the intercept (*risk-free rate*) and the slope (*market risk premium*) of the interpolated line of regression; and finally, η_i indicates the residual variable.

4. Penalized Spline Semiparametric Method¹⁰

The Penalized Splines Semiparametric Method (P.S.S.M.) can be briefly defined by the following general formulation:

⁹ Cummins J.D. and Lee A.C. Alternative models for estimatine the cost of equità capital for property/casualty insurers. *Review of Quantitative Finance and Accounting*. Kluwer Academic Publishers. 10 (1998). pp. 235-267.

¹⁰ For an exhaustive and analytical review of the literature on semiparametric models, please refer to followings contributions: Del Giudice V., (2010); Del Giudice V. and De Paola P., (2011); Ruppert D., Wand M.P. and Carroll R.J., (2003).

$$y = X\beta + Zu + \varepsilon \tag{1.4}$$

where:

$$y = (y_1, \dots, y_N)^T$$

$$X = [1 \ x_i]_{1 \leq i \leq N}$$

Z contains $T \leq N$ truncated power basis function of degree p :

$$Z = \begin{bmatrix} (x_1 - \kappa_1)_+^p & \dots & (x_1 - \kappa_k)_+^p \\ \dots & \dots & \dots \\ (x_n - \kappa_1)_+^p & \dots & (x_n - \kappa_k)_+^p \end{bmatrix}$$

or in reduced form:

$$Z = \left[(x_i - \kappa_k)_+^p \right]_{\substack{1 \leq k \leq K \\ 1 \leq i \leq n}}$$

$u = (u_1, \dots, u_k)^T$ is the vector of random effects, with:

$$E(u) = 0, \quad Cov(u) = \sigma_u^2 I, \quad Cov(\varepsilon) = \sigma_\varepsilon^2 I,$$

considering u_k coefficients of κ_k knots as the random effects independent by ε .

For non linear components of the model are used *penalized spline* functions qualified by the following general expression:

$$f(x) = \alpha_0 + \alpha_1 x + \dots + \alpha_p x^p + \sum_{k=1}^K \alpha_{pk} (x - \kappa_k)_+^p \tag{2.4}$$

Also, the base of the generic function (2.4) is represented by the following terms:

$$1, x, \dots, x^p, (x - \kappa_1)_+^p, \dots, (x - \kappa_k)_+^p$$

where the generic function $(x - \kappa_k)_+^p$ has $(p - 1)$ continuous derivatives.

For $p > 0$ the expression to determine the fitted values is as follows:

$$\hat{y} = X(X^T X + \lambda^{2p} D)^{-1} X^T y \tag{3.4}$$

where:

$$X = \begin{bmatrix} 1 & x_1 & \dots & x_1^p & (x_1 - \kappa_1)_+^p & \dots & (x_1 - \kappa_k)_+^p \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_p & \dots & x_n^p & (x_n - \kappa_1)_+^p & \dots & (x_1 - \kappa_k)_+^p \end{bmatrix}$$

$$D = \text{diag} (0_{p+1}, 1_k) \tag{4.4}$$

More simply, the relationship (3.4) becomes:

$$\hat{y} = S_\lambda \cdot y \tag{5.4}$$

Assuming and denominated as *smoother matrix* as follows:

$$S_\lambda = X(X^T X + \lambda^{2p} D)^{-1} X^T \tag{6.4}$$

In relations (3.4) and (6.4) the λ parameter is defined as *smoothing parameter*, whose selection, for a *spline* function of degree p , is done through the application of *Restricted Maximum Likelihood* (Re.M.L.).

Obtained the λ parameter as above, the estimates of β coefficients and the predictions of u variables can be determined as follows:

$$\hat{y} = X\hat{\beta} + Z\hat{u} \tag{7.4}$$

5. Model's Calibration

As specified in the paragraphs above, the C.A.P.M. is expressed, in hypothesis of financial market's balance, through an expected relationship between the performance of considered security and the performance of the market portfolio:

$$R_i - r = \beta_i (R_M - r) + \varepsilon_i \tag{1.5}$$

In the framework clear-cut, an approach to the problem of estimation of undivided ownership shares may, ultimately, attributable to the estimate of the premium for high-risk financial investments (*equity premium*) through the use of historical yields, considering, in particular, the difference between the mean performance of closed and listed real estate trusts and low-risk bonds (usually gov-

ernment bonds) measured on appropriate period of time¹¹. This difference, given the β parameter, permits to determine the expected risk premium that can to be extend into the future.

Estimated the extra returns associated to the risk of speculative investment as above, in order to determination of expected return can logically take appropriate real estate profitability indices instead of risk-free interest rate or, alternatively, can reference to the yearly average increase of property prices for the specific market segment in which the good is included and whose ownership is undivided.

In analytical terms the problem can therefore be resolved as follows¹²:

$$R_A = \Delta_{QI} + \beta_{spline} \cdot (R_{HR} - R_{TS}) \quad (2.5)$$

$$R_A = \Delta_{QI} + \left[\sum_{b=1}^H f_b(\bar{x}_b) / H \right] \cdot (R_{HR} - R_{TS}) \quad (3.5)$$

Where the β_{spline} coefficient is estimated by *Penalized Spline Semiparametric Method*:

$$\beta_{spline} = \left[\sum_{b=1}^H f_b(\bar{x}_b) / H \right] \quad (4.5)$$

with:

$$f_b(x_b) = \alpha_0 + \alpha_1 x_b + \dots + \alpha_p x_b^p + \sum_{k=1}^K \alpha_{pk} (x_b - \kappa_k)^p_+$$

Consequently, determined the expected average yield (R_A), we have:

$$CG_{(n)} = R_A \cdot [(q^n - 1) / i_L] \quad (5.5)$$

$$V_{m(n)} = [V_0 \cdot (1 + CG_{(n)})] \quad (6.5)$$

$$V_0 = [V_{m(n)} / (1 + CG_{(n)})] \quad (7.5)$$

To result, ultimately, that the single undivided share of considered property is depreciated, if subjected to forced judicial sale, for example, in measure of:

¹¹ Del Giudice V., (2010). *Estimo e Valutazione Economica dei Progetti*. Op. cit.

¹² Considering negligible the residual term (ϵ).

$$D = 1 - [1 / (1 + CG_{(n)})] \quad (8.5)$$

$$C_D = [1 / (1 + CG_{(n)})] \quad (9.5)$$

Therefore the most probable market value of undivided ownership share is equal to:

$$V_Q = V_0 \cdot Q = V_{m(n)} \cdot C_D \cdot Q \quad (10.5)$$

Having defined:

- R_A Expected average annual return of the speculative investment;
- Δ_{QI} Average annual increase of the property prices for the specific market segment to which the asset belongs and which the ownership is undivided, based on historical data relating to a reasonable period of time;
- β_{spline} *beta* coefficient obtained using the P.S.S.M.;
- R_{HR} Average annual return for high-risk financial investment (closed and listed real estate trusts);
- R_{TS} Average annual return for government bonds;
- $CG_{(n)}$ *Capital gain*, that is the difference between the current possible purchase price and expected future sale price for the real estate property whose ownership is undivided;
- n Expected time horizon needed to monetize the expected return¹³;
- q Binomial interest $(1+i_L)$;
- i_L Current legal interest rate;
- $V_{m(n)}$ Market value at current prices for entire property at the time n ;
- V_0 Present value for entire property in a condition of undivided ownership;
- D Depreciation percentage of single undivided share;
- C_D Coefficient of depreciation for the single undivided share;
- V_Q Present value for the single undivided share of property;
- Q Undivided share of property which we are estimating.

6. Data description and empirical analysis

A. Data Description

The model described in the previous section was implemented on the basis of data referring to a time period that extends from the second half of 2002 to the

¹³ The time horizon to considered to be commensurate in function both the time required for to conclude the judicial procedure, both of time ordinarily be expected for terminating the legal action for the division of the good whose ownership is undivided (usually not less than 2 or 3 years).

second half of 2010, in order to assess with congruous measure the dynamics of *beta* and housing and financial markets.

Therefore, we have been developed and standardized for analysis to be conducted the following variables (see Table 1 and Figure 1):

- *REIndexNA*: price index for Naples's residential real estate market derived from data by Tecnocasa, Stock Realty of Naples (by the Chamber of Commerce of Naples) and F.I.A.P. (Italian Federation of Professional Estate Agents)¹⁴;
- *REIM*: BNP Paribas REIM index (*BNP Paribas Italian Real Estate Funds*), which measures the overall performance of all funds listed closed-end real estate investment on the Milan stock, calculated using the formula of total return; it comes to indices such value weighted open all share, receptive of new securities listing, that consider in order to calculation of yield the temporal variation of the price level, income distribution and capital repayments¹⁵;
- *DTN*: DTN index BNP Paribas REIM (*BNP Paribas REIM Real Estate Funds Discount to NAV*), which measures at level of entire Italian real estate funds sector the trend of the difference between the share price on the stock exchange and the NAV (*Net Asset Value*); they are "unfrozen" index because the values are modified in subsequent periods, considering the NAV actually calculated at the last assessment¹⁶;
- *FTSEMIB*: MIB stock index of the Italian Stock Exchange formulated on the basis of a basket comprising the securities of the 40 Italian and foreign largest companies listed on markets managed by the Italian Stock Exchange, representing approximately 80% of the Italian stock market capitalization¹⁷;
- *R (TS)*: average annual return of the main Italian government bonds such as B.O.T., B.T.P., C.C.T., C.T.Z., Rendistato¹⁸.

The use in analysis of six-month data results from essentially practical reasons, wanting to highlight in this contribution, in operational terms, the potential offered by the proposed model. It is also taking into account that in the real estate markets and closed-end real estate trusts the traders often based their choices and strategies of investment on time horizons rather broad, so as not to require such frequent changes in portfolio composition. For these reasons, the choice of using semi-annual returns in the estimation of the beta coefficient is logical with the housing and financial market dynamics.

The sample data are analytically presented in Table 1, while the same data in Figure 1, appropriately normalized, are shown in graphic design.

Figure 1 shows how the FTSE MIB index, BNP Paribas REIM index and BNP Paribas DTN index, have almost similar and overlapping temporal trends, dem-

¹⁴ Del Giudice V. and d'Amato M., (2008). *Principi metodologici per la costruzione di indici dei prezzi nel mercato immobiliare*. Maggioli Editore.

¹⁵ <http://www.realestate.pnbparibas.it>

¹⁶ <http://www.realestate.pnbparibas.it>

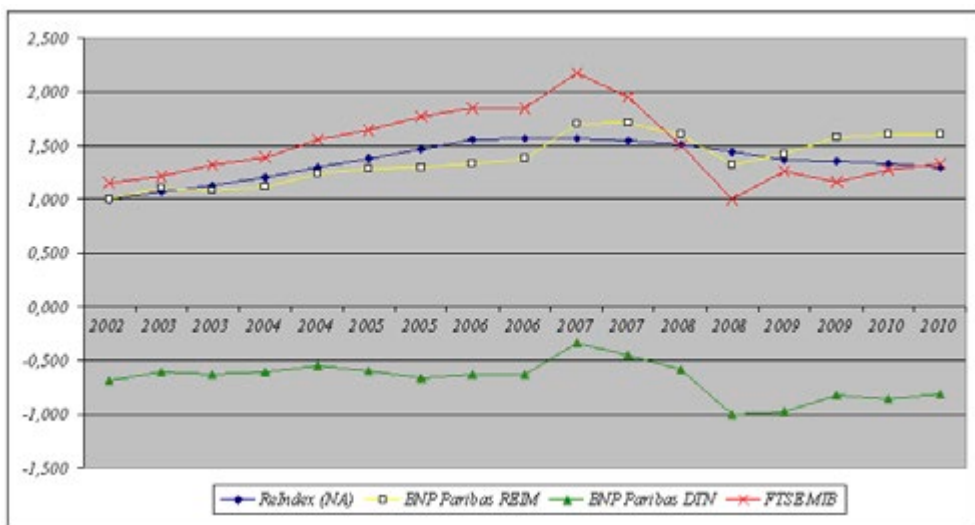
¹⁷ <http://www.borsaitaliana.it>

¹⁸ <http://www.bancaditalia.it>

Table 1. Database (our elaboration on data derived from different sources).

Year (Semester)	REIndexNA	BNP Paribas REIM	BNP Paribas DTN	FTSE MIB	R(TS)
2002 (II)	100,000	100,000	-27,520%	17485	3,901%
2003 (I)	107,278	111,280	-24,040%	18360	2,931%
2003 (II)	113,524	107,800	-25,090%	19922	
2004 (I)	119,768	111,320	-24,040%	21113	2,866%
2004 (II)	129,828	124,200	-21,580%	23534	
2005 (I)	137,877	128,260	-23,560%	24736	2,695%
2005 (II)	146,426	129,900	-26,270%	26778	
2006 (I)	155,504	132,960	-25,260%	27923	3,857%
2006 (II)	157,370	138,320	-24,980%	27886	
2007 (I)	157,370	170,130	-13,140%	32886	4,218%
2007 (II)	154,852	171,300	-17,850%	29402	
2008 (I)	151,136	160,750	-22,950%	22721	4,190%
2008 (II)	144,486	131,820	-39,700%	15096	
2009 (I)	137,262	141,510	-38,600%	19063	1,993%
2009 (II)	135,752	157,750	-32,820%	17652	
2010 (I)	133,170	160,700	-33,820%	19312	2,014%
2010 (II)	129,710	160,370	-32,160%	20173	

Figure 1. Temporal trend of explicative variables used in the P.S.S.M.



onstrating the link between the same indices; also, with such clear evidence it is possible to notice as the price index of residential real estate market on the city of Naples gives rise to a line of influence can interpolate correctly the trend of the remaining indices.

B. Empirical Analysis

The analysis of the real estate prices index carried out using a semiparametric additive model provides the adoption of statistical tools (tests of significance, measures of residues, etc.) able to select both the data set sampled, both to verify the reliability and predictive validity of the results of the estimation by the match of the representative function of the index to the observed data.

The explicative variables were appropriately normalized to their minimum value in order to avoid even the appearance of multicollinearity phenomena between the same variables, in the presence of which requires use of appropriate testing procedures to define correctly the function of the price index.

The algebraic structure of the model, specified on the basis of estimative evidence of empirical-argumentative nature, is obtained by implementing the following semiparametric additive model¹⁹:

$$REIndexNA = f_1(Semester) + f_2(REIM) + f_3(DTN) + f_4(FTSE.MIB) + \varepsilon_i$$

In the absence of multicollinearity phenomena in reason of the low correlation between the explanatory variables, the main indices of model verification are shown for completeness in the prospectus and in the graph below (see Table 3 and Figure 2).

Under the estimative profile, the amounts of mean percentage error (0.24%) and average standard error (0.32%) appear congruent, this is because the values of the forecasts obtained using the estimated model show a trend conforms to the observed data, the residual analysis also not shows abnormality (see Figure 2).

From the statistical point of view, the index of determination is significant, amounting to 0.99 (correct index equal to 0.99), as well as the *F* test is significant at a confidence level of 95%.

In turn, the estimate of the non-linear components of the model results statistically significant, not finding anomalies in the values of the degrees of freedom (*df*) and the smoothing parameters (*spar*) (see Table 3).

The amount that each variable assumes in the model is achieved through the development and examination of the estimated functions; for brevity of discussion, however, the trend by each variable of the model is only shown in the fol-

¹⁹ The software which has been developed the elaboration of semiparametric model is represented by the R program, free online available (<http://www.R-project.org>).

lowing graphic, being primary goal of the contribution the implementation of the C.A.P.M. (See Figure 2).

In correspondence of average values given by the explicative variables is possible to obtain the contribution to the variable explained by each function estimated in the semiparametric model and, therefore, it is possible to determine, by the formula (4.5), the systematic risk factor (β_{spline}) that can be used in the relation (2.5), equal, namely, at 1,142.

Can therefore be determined at this point, the expected average return on speculative investment (R_A) using the relationship (2.5):

$$R_A = [4,47\% + 1,142 \cdot (7,22\% - 3,18\%)] = 9,08\%$$

Determined the R_A term, from relationship (5.5) we obtain the expected profit margin for the purchase of undivided share of ownership²⁰:

$$CG_{(n)} = 9,08\% \cdot \{(1,015)^3 - 1\} / 0,015\} = 27,64\%$$

In consequence, using the relation (8.5) the depreciation, in percentage terms, the single share respect to the arithmetic fraction of undivided ownership is equal to:

$$D = 1 - [1 / (1 + 27,64\%)] = 21,65\% .$$

Table 2. Statistical description of the explicative variables used in P.S.S.M.

Explicative Variables	Std. Dev.	Median	Mean	Min	Max
REIndexNA	0,177	1,373	1,360	1,000	1,574
Semester	2,525	4,000	4,000	0,000	8,000
REIM	0,227	1,330	1,376	1,000	1,713
DTN	0,174	-0,632	-0,672	-1,000	-0,331
FTSEMib	0,329	1,399	1,496	1,000	2,178

7. Concluding remarks

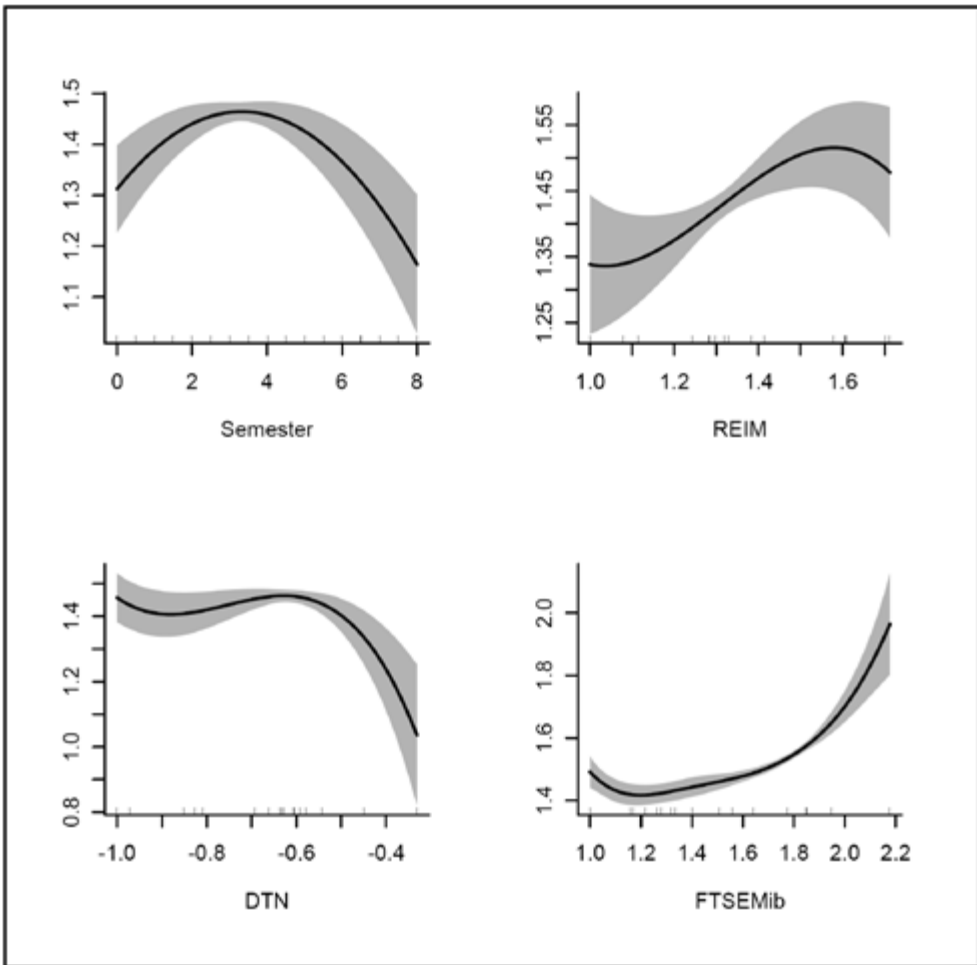
In the present contribution the *Capital Asset Pricing Model* is implemented through the *Penalized Spline Semiparametric Method* for the determination of the systematic risk factor in the estimate of undivided and indivisible real estate shares.

²⁰ In relation (5.5) the time horizon needed to monetize the expected return is estimated, with carefulness, in 3 years; on the other hand, we have that the legal interest rate is currently equal to 1,50%.

Table 3. Verification results of Penalized Spline Semiparametric Method (estimates of the degrees of freedom, smoothing parameters and knots number).

Penalized Spline Functions	df	spar	knots
f (Semester)	2,000	187,800	3,000
f (REIM)	3,000	6,483	3,000
f (DTN)	3,000	1,786	3,000
f (FTSEMib)	4,083	0,142	3,000

Figure 2. Effects of non-linear components on residential real estate prices index (REIndexNA) with representation of the confidence interval at 95%.



The applied importance of the proposed model is confirmed by the positive

results achieved with the analysis carried out, consistent with the dynamics of the housing and financial markets.

For trader, is undoubted in fact the importance of operational tools useful for rapid adjustment of the composition of their investment portfolios and thus enabling them to reactively modify specific positions using temporary trend also relatively small extent.

In this study we have been processed six-months data relating to residential property market in the city of Naples with only aim of showing the ease of interpretation and the potentiality offered by the proposed model, applied, in the case, to real estate properties with ordinary characteristics in an area sufficiently wide.

Further opportunity of research concerning the possibility to calibrate the proposed model respect to the specific nature of the undivided property and its market. This can be achieved using appropriate econometric analysis able to adequately express the real estate revaluation rate, even on a monthly basis, with specific reference to the undivided good.

Looking ahead, other lines of research and application developments in the estimative field relating the proposed model, can will concern the concept of "efficiency" of real estate market and will depend largely from the results of research relating to procedures for the detection and processing of property information and financial data, through more appropriate analysis tools can also guide the choice of functional form of the model more right to the examined trading phenomenon.

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