Deriving Tree Crown Distributions from Diameter at Breast Height

Ogana F.N.[1*], Dau J.H.[2]

[1]*Department of Social and Environmental Forestry, University of Ibadan, Nigeria*
[2]*Department of Forest Production and Products, Federal University of Agriculture, Makurdi, Nigeria*

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Abstract

The distribution of crown diameter is important for assessing crown social class, monitoring forest health and wildlife management. However, the direct measurement of crown diameter is relative difficult, and as such, it is often predicted from diameter at breast based on a simple relationship. Therefore, in this study, the crown diameter distribution of *Parkia biglobosa* was derived from Dbh using Weibull and Log-Logistic functions. A total of 284 trees were measured from *Parkia biglobosa* plantation in Makurdi, Nigeria. Four methods were used for Weibull distribution including maximum likelihood (MLE), moments, percentiles and cumulative distribution function regression (CDFreg). MLE and CDFreg were used for Log-Logistic function. Transformation technique was used to transform the Dbh to crown distribution based on a simple allometric relationship between the variables. Kolmogorov-Smirnov (D_n), Cramer-von Mises statistic (W^2) and Reynolds error index (EI) were used to assess the derived crown diameter distribution. The result showed that the underlying diameter distribution followed Weibull and Log-Logistic distributions. The fitted allometric equation was of the form: $Cd = 2.521 + 0.099Dbh$. MLE and CDFreg were the best methods for Weibull and Log-Logistic functions, respectively. The D_n, W^2 and EI were 0.071, 0.0265 and 0.3434, respectively for MLE; and 0.0931, 0.0367 and 0.4171, respectively for CDFreg. In all methods, the observed and derived crown distributions were not significant at 20% (D_α=0.339). Thus, given the diameter distribution, the tree crown distribution of *Parkia biglobosa* can be derived. This would be useful for determination of the crown social class.

Keywords: Crown diameter, crown distribution, Log-Logistis, Parkia biglobosa, Weibull

1. Introduction

Tree size distribution remains one of the most researched areas in forestry. It forms the basis for analysing the structure, volume production, value and even growth of forest stand (Gorgoso-Varela and Rojo-Alboreca, 2014). One easily measured variable frequently used in size distribution is diameter at breast height i.e., 1.3 m above the ground (hereafter referred to Dbh). It correlates with other tree variables that are difficult to measure, e.g. height, volume, biomass, crown diameter etc. (West, 2015).

Several distribution functions have been applied at different times to describe the structure of forest stand using Dbh as impute variable. Some of these functions include beta, gamma, Johnson's S_B, Logit-Logistic, lognormal and Weibull distributions. The Weibull distribution has been consistently used in forestry because of its simplicity, relative flexibility, ease of parameter estimation and computation of proportion of trees in diameter class (Burkhart and Tomé, 2012). Beside Dbh, there are different alternatives used for size distribution, such as height, basal area, tree volume, crown diameter and crown area (Mehtatalo, 2013). If for example, the tree size specifies tree height, then the underlying distribution is called height distribution.

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*Correspondence: fn.ogana@ui.edu.ng
Tel: +23 47037679328
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It could be basal area distribution, volume distribution, crown area or crown diameter distribution. Mirzaei et al. (2016) modelled the frequency distributions of crown area, diameter and height of *Quercus persica* stands in Iran. Chuckwu et al. (2018) used the Weibull distribution to describe the crown diameter distribution in the natural stand of Shasha Forest Reserve, Nigeria.

Crown diameter is the "average of the widest axis of the crown and its perpendicular axis" (Zarnoch et al., 2004). Its distribution is relevant to both forest and wildlife managers. The larger the tree crown diameter, the more foliage for photosynthesis; in consequence, a more potential for carbon fixation (Zarnoch et al., 2004). Also, larger tree crowns are important for bird nest and perching (Deng et al., 2003). Tree crown is a good indicator of forest health because it shows the first symptom of deterioration when either natural or anthropogenic factors impact a forest (Adesoye and Ezenwenyi, 2016; Kershaw et al., 2017). In addition, crown diameter is a key variable for computing crown surface area, volume (van Laar and Akca, 2007 p 96), canopy cover (Gill et al., 2000) and crown profile analysis (Marshall et al., 2003). To the wildlife manager, a forest with larger crowns could affect the abundance and distribution of fauna species, especially the herbivores. Because low light penetration to the forest floor means less grasses and shrubs for the herbivores (Chukwu et al., 2018).

The measurement of tree crown diameter remains a major challenge to foresters, especially in forest stand with contiguous canopy and irregular edges (Gering, 1995; van Laar and Akça, 2007). In view of this, crown diameter is obtained as photo-derived variable from remote sensing (Gering, 1995). Furthermore, foresters have also relied on the ability to predict crown diameter from Dbh/stump diameter through a simple linear relationship (e.g. Sonmez, 2009; Adesoye and Ezenwenyi, 2014; Raptis et al., 2018 etc.). Adesoye and Ezenwenyi (2014) developed a crown diameter-Dbh prediction model for *Tectona grandis* Linn. In Omo forest reserve using a simple linear equation. Recently, Chukwu et al. (2017) used a simple linear relationship to predict crown diameter from stump diameter of *Parkia biglobosa* Benth. Crown diameter-Dbh relationship can be used to determine tree spacing, stand density, basal area per ha, thinning regimes etc. (Hemery et al., 2005).

Mehtatalo (2013) introduced the idea of transformation in forestry. It involves changing from one tree size distribution to another. One major application of transformation is that it helps in the formulation of diverse distributions based on allometric relationships of trees (Mehtatalo, 2013). Transformation could be from diameter to crown diameter, diameter to height, diameter to basal area, or diameter to volume; in so far there is a functional relationship between the variables. This is an important tool that can be used to predict size distribution for variables that are difficult to measure. Therefore, the main objective of this study was to derive tree crown distribution from diameter at breast height using transformation.

2. Methodology

2.1 Data

The data were obtained from the *Parkia biglobosa* plantation in Makurdi, Nigeria. Its lies between Latitudes $7^o21^/$ and $8^o0^/$ N and Longitudes $8^o21^/$ and $9^o0^/$ E of Benue State, Nigeria and occupies an area of 7,978 km^2 (Chukwu et al., 2017). A total of 284 trees were measured on nine temporary sample plots (TSPs) of 1ha size. Diameter at breast height (Dbh), height and crown diameter (Cd) were measured to an accuracy of 0.1 cm, 0.1 m and 0.3 m, respectively. Crown diameters were measured as the linear distance between edges of the tree crown in a north-south and east-west direction. The average value of north-south and east-west measures was taken as the crown diameter. The descriptive statistics of the inventoried data is presented in Table 1.

Table 1: Descriptive statistic of data set.

Tree Variables	Mean	Max	Min	Standard deviation
Dbh (cm)	40.45	95.43	14.60	15.00
Height (m)	6.85	17.80	2.11	2.07
Crown diameter (m)	6.64	15.10	2.80	2.10
BA (m^2)	0.15	0.72	0.02	0.11
CPA (m^2)	38.13	179.08	6.16	25.17
Volume (m^3)	0.011	0.360	$4.6\text{x}10^{-5}$	0.038

CPA=crown projection area.

2.2 Transformation of tree Dbh to crown diameter distribution

Let *d* and *Cd* be diameter at breast height (Dbh) and crown diameter, respectively. And the *Dbh* data is assumed to follow the 2-parmeter Weibull and Log-Logistic distributions. Their cumulative distribution functions (cdf) are expressed as:

Weibull cdf,

$$F_D(d) = 1 - e\left[-\left(\frac{d}{\beta}\right)^{\alpha}\right] \tag{1}$$

Log-Logistic cdf,

$$F_D(d) = \left[1 + \left(\frac{\beta}{d}\right)^{\alpha}\right]^{-1} \tag{2}$$

The crown diameter-Dbh relationship is given as linear

$$Cd(d) = a + bd \tag{3}$$

Making *d* in equation (3) the subjection of the equation gives

$$g^{-1}(Cd) = \frac{Cd - a}{b} \tag{4}$$

Substituting *d* in equation 1 with equation 4 gives the distribution of tree crown diameter

$$F(Cd) = 1 - e\left[-\left(\frac{\frac{Cd-a}{b}}{\beta}\right)^{\alpha}\right] \tag{5}$$

Similarly, substituting equation (4) into equation 2 yielded the transformed Log-Logistic tree crown diameter expressed as:

$$F(Cd) = \left[1 + \left(\frac{\beta}{\left(\frac{Cd-a}{b}\right)}\right)^{\alpha}\right]^{-1} \tag{6}$$

where;

α and β=the shape and scale parameters of the Weibull and Log-Logistic distributions;

a and *b*=the regression parameters.

The fit of the transformed Dbh to crown distribution were compared with the observed crown distribution to ascertained the adequacy of the method.

2.3 Fitting methods

Four methods were considered for estimating the parameters of the Weibull function including maximum likelihood, moment, percentiles (40 and 82) (Bailey and Dell, 1973) and CDF regression (Cao, 2004). Maximum likelihood and CDF regression methods were used for the Log-Logistic distributions. Distributions were fitted in R (R Core Team, 2017). Kolmogorov-Smirnov (D_n), Cramer-von Mises statistic (W^2) and Reynolds error index (EI) were used to assess the goodness of fit of the derived crown diameter distribution. The smaller the fit indices are, the better the distribution. Also, the fits from the different methods were tested with the Kolmogorov-Smirnov test at 20% significant level ($D\alpha$). The null hypothesis (H_0) is that the observed and estimated/derived distributions are not significant. Thus, H_0 is rejected when $D_n > D\alpha$.

$$D_n = Supx|F(x_i) - F_0(x_i)| \tag{7}$$

$$D_\alpha = \sqrt{\frac{-ln\left(\frac{1}{2}\alpha\right)}{2n}} \tag{8}$$

$$W^2 = \sum_{i=1}^{n} \left\{\hat{F}(x_i) - \frac{(i-0.5)}{n}\right\}^2 + \frac{1}{12n} \tag{9}$$

$$EI = \sum_{i=1}^{n} \left|\hat{F}(x_i) - \frac{(i-0.5)}{n}\right| \tag{10}$$

where;

sup$x$=Supremum value for x
$F(x_i)$=Cumulative frequency distribution observed for the sample x_i (i =1, 2, …, n)
$F_0(x_i)$=Probability of the theoretical cumulative frequency

3. Results and Discussion

3.1 Fitted diameter distributions

The suitability of Weibull and Log-Logistic distributions in describing the diameter distributions of *Parkia biglobosa* was first assessed. This was done to ascertain the claim that the underlying diameter distribution followed the Weibull and Log-Logistic distributions. The results showed that Log-Logistic fitted with MLE and CDFreg had the smallest D_n, W^2 and EI values (Table 2). However, no significant difference was observed between predicted and observed diameter distribution for all methods at 20% level based on $D\alpha$ test (0.118). Thus, all methods can be regarded as equally good and that the underlying diameter distribution followed the Weibull and Log-Logistic distributions.

Cao (2004) commented on the advantages and disadvantages of some of these methods, especially the methods for fitting Weibull distributions. The author reported that CDF-reg was the most preferred method for the Weibull distributions. The MLE has be found to produce consistent result and provides a means of estimating the standard error of the parameter estimate compared to other methods, but it computation procedure can be laborious (Nord-Larson and Cao, 2006; Mehtatalo, 2013). The method of moment and percentiles have also been applied to estimate the parameters of the Weibull distributions with some level success including Zhang et al. (2003), Cao (2004), Gorgoso et al. (2007), Carretero and Alvarez (2013) etc.

The graph of the observed and Weibull distributions fitted with MLE, moments, percentiles and CDF-reg is presented in Figure 1. While that of Log-Logistic fitted with MLE and CDF-reg is shown in Figure 2. The performances of the estimation methods were relatively the same for the two distributions. Both Weibull and Log-Logistic distributions approximate the observed tree diameter distribution of *Parkia biglobosa*.

Table 2: Fitted Weibull and Log-Logistic to the Diameter Distributions of *Parkia boglobosa*.

Distributions	Methods	Fit indices		
		D_n	W^2	EI
Weibull	CDF reg	0.0675	0.0590	1.8349
	Percentiles	0.0839	0.0654	1.8400
	Moments	0.0754	0.0721	1.9121
	MLE	0.0736	0.0750	1.9219
Log-Logistic	MLE	0.0358	0.0186	0.9943

	CDF reg	0.0441	0.0190	1.0443

Dα=0.118

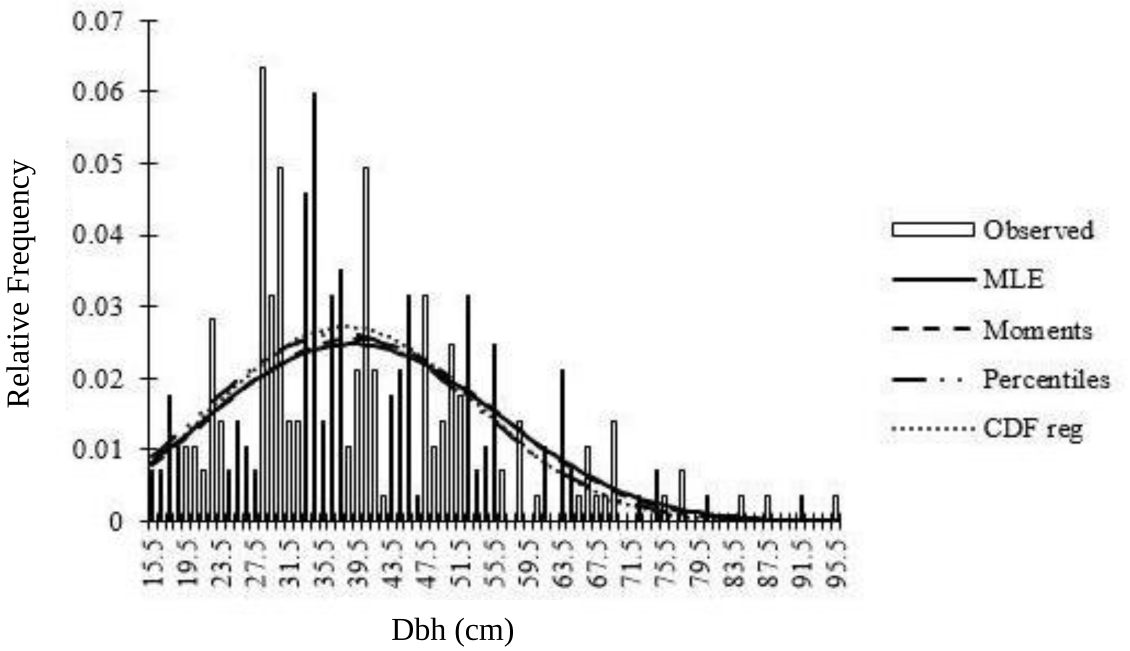

Figure 1: Observe and fitted Weibull distribution using different methods to the Dbh data.

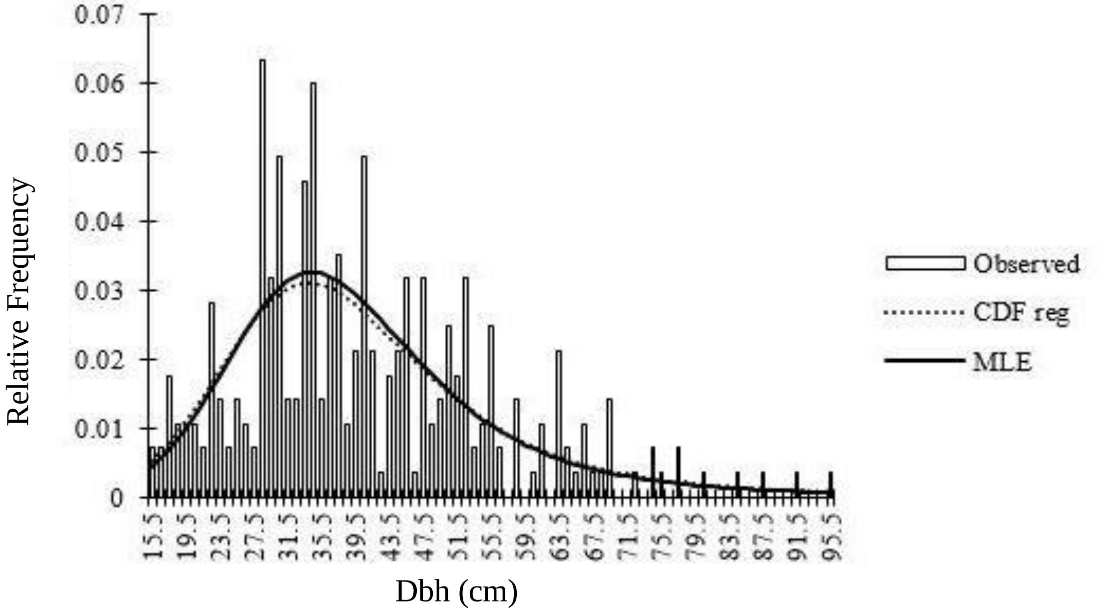

Figure 2: Observe and fitted Log-Logistic distribution using different methods to the Dbh data.

3.2 Fitted derived crown distributions

The fitted Weibull and Log-Logistic distributions and the established crown diameter-Dbh relationship were used to derived the tree crown distribution vis-à-vis the different estimation methods. The fitted simple linear crown diameter-Dbh relationship was of the form:$Cd = 2.521 + 0.099Dbh$; with adjusted coefficient of determination (R^2_{adj}) and root mean square error (RMSE) of 0.609 and 1.182, respectively. The results of the transformed tree diameter to crown distributions are presented in Table 3. The Weibull distribution fitted with MLE and moments were more suitable than percentiles and CDF reg methods. The Kolmogorov-Smirnov (D_n), Cramer-von Mises (W^2) and Error index (EI) were 0.071, 0.0265 and 0.3434, respectively for MLE and 0.0781, 0.0305 and 0.3793, respectively for moments. In the case of Log-Logistic, the CDF reg was better than MLE. Its D_n, W^2 and EI were 0.0931, 0.0367 and 0.4171, respectively. Generally, the value of EI ranged between 0 (perfect match) and 2 (mismatched) (Mehtatalo, 2017). The EI value of the methods considered for Weibull and Log-Logistic distributions were within 0 > EI < 1 (i.e., < 0.5). This shows that the derived crown distribution matched the observed crown distribution of *Parkia biglobosa*. The Kolmogorov-Smirnov test also shows no significant difference between the derived crown distribution and observed distributions at 20% level (0.339) for all methods.

Table 3: Fitted Weibull and Log-Logistic to the Derived Crown Distributions.

Distributions	Methods	Parameters				Fit indices		
		a	b	α	β	D_n	W^2	EI
Weibull	MLE	2.521	0.099	2.839	45.256	0.0710	0.0265	0.3434
	Moments	2.521	0.099	2.932	45.178	0.0781	0.0305	0.3793
	Percentile	2.521	0.099	2.886	43.385	0.1083	0.0357	0.4008
	CDF reg	2.521	0.099	3.029	43.628	0.1131	0.0400	0.4462
Log-Logistic	CDF reg	2.521	0.099	4.445	37.803	0.0931	0.0367	0.4171
	MLE	2.521	0.099	4.706	37.804	0.1055	0.0442	0.4687

$D\alpha$=0.339

The graphs of the derived tree crown distributions resulting from different estimation methods are shown in Figure 3 and 4. The graphs showed the relative cumulative distribution of trees per 1 m crown diameter class. The shape of the derived tree crown distribution (dash line) followed the observed distribution (thick line). This shows the effectiveness of transformation technique (Cd-Dbh transformation). Thus, given an assumed diameter distribution, the tree crown distribution of *Parkia biglobosa* can be derived. However, Mehtatalo (2013) asserted that no randomness is assumed with the transformation technique. This implies that tree crown diameter depends on diameter at breast height (Dbh) deterministically according to the simple linear Cd-Dbh relationship with the given values of the parameters. There is a positive relationship between tree crown diameter and Dbh. The Cd-Dbh relationship of *Parkia biglobosa* had R^2_{adj} =0.609. Gering (1995) stated that “the ability to predict crown diameter from Dbh provides an efficient method of obtaining an estimate of crown diameter”. Similarly, Bechtold (2003) reported high positive relationship for these variables for 87 tree species. Foli et al. (2003) also reported R^2=0.606 for *Entandrophragma angolense* in Ghana.

The distribution of tree crown is germane to both forest and wildlife managers. Tree crown diameter is one of the variables used for wildlife suitability index model (Sousa, 1987; Gering, 1995). Also crown diameter is a veritable tool for monitoring forest health and a determinant of wildlife habitat value (Kershaw et al., 2017). It forms the basis for determining tree crown structure and in consequence, crown social class. Information on crown diameter is used together with other variables to derive other crown indices including crown thickness index, linear crown index, crown spread ratio, etc. (van Laar and Akça, 2007).

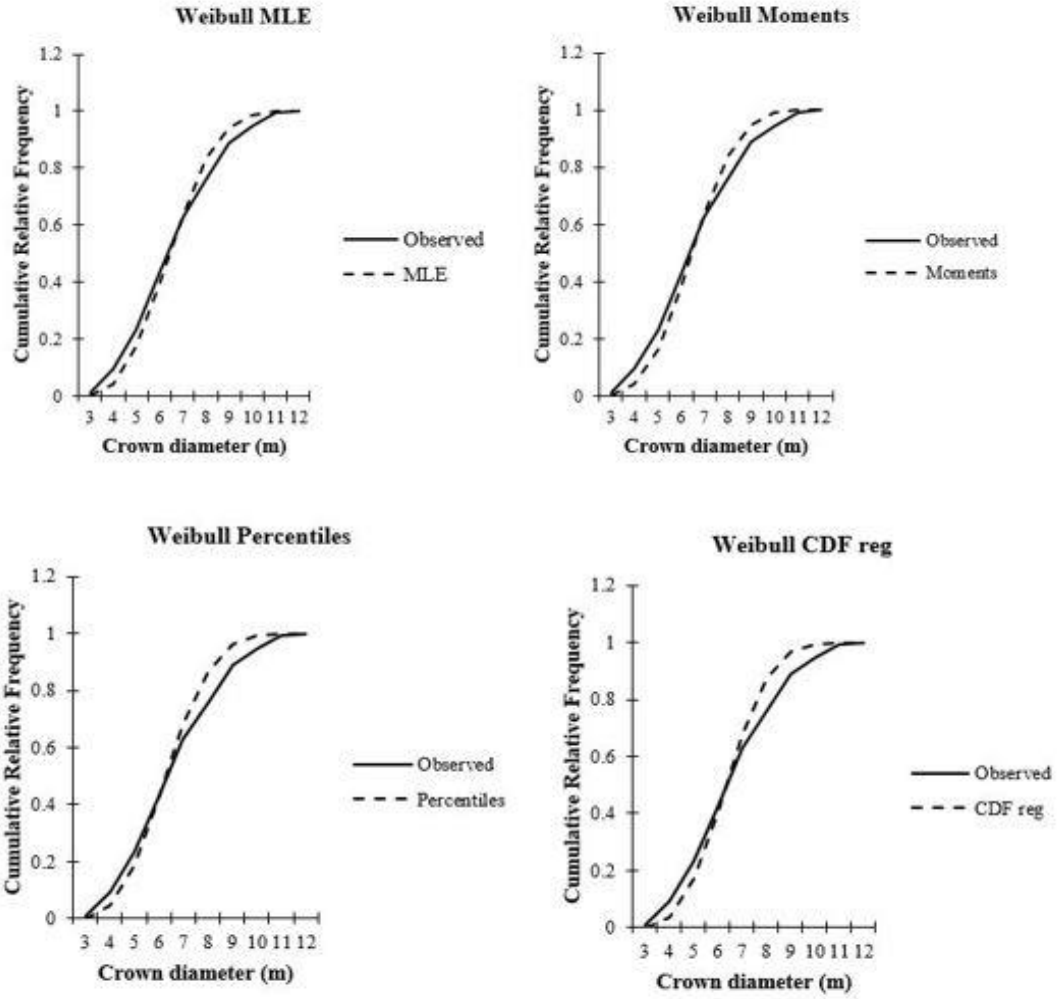

Figure 3: Observed and fitted Weibull of the derived crown diameter distribution from Dbh.

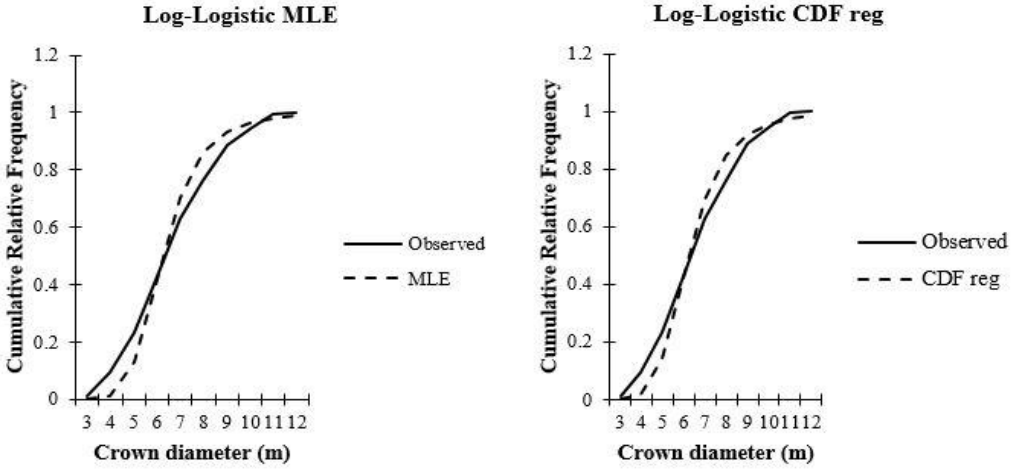

Figure 4: Observed and fitted Log-Logistic of the derived crown diameter distribution from Dbh.

4. Conclusion

This study has formulated a new crown distribution function based on the allometric relationship between tree crown diameter and diameter at breast height of *Parkia biglobosa.* Weibull and Log-Logistic distribution functions were used as the underlying distributions. The derived tree crown distribution was comparable to the actual crown distribution of the specie. Thus, given the diameter distribution, the tree crown distribution of *Parkia biglobosa* can be derived. This information can be used to assess the crown social class of the specie. The method of transformation could be extended to diameter to basal area, or diameter to volume etc., in so far there is a functional relationship between the variables.

References

Adesoye, P.O. and Ezenwenyi, J.U., 2014. Crown diameter prediction models for *Tectona grandis* Linn. Omo forest reserve. *Journal of Forestry Research and Management,* 11:72-87.

Bailey, R.L. and Dell, T.R., 1973. Quantifying diameter distributions with the Weibull function. *Forest Science*, 19:97-104.

Bechtold, W., 2003. Crown-diameter prediction models for 87 species of stand-grown trees in the eastern United States. *Southern Journal of Applied Forestry,* 27:269-278.

Cao, Q.V., 2004. Predicting parameters of a Weibull function for modelling diameter distribution. *Forest Science*, 45:506-511.

Carretero, A.C. and Alvarez, E.T., 2013. Modelling diameter distributions of *Quercus suber* L. stands in “Los Alcornocales” Natural Park (Cadiz-Malaga, Spain) by using the two parameter Weibull functions. *Forest Systems* 22:15-24.

Chukwu, O., Chenge, I.B. and Ezenwenyi, J.U., 2018. Tree crown distribution for Wildlife Management of Shasha forest reserve, Nigeria. *Nigeria Journal of Wildlife Management*, 1:86-91.

Chukwu, O., Dau, J.H. and Ezenwenyi, J.U., 2017. Crown-stump diameter model for *Parkia biglobosa* Benth species in Makurdi, Benue State, Nigeria. *Journal of Tropical Forestry and Environment*, 7:43-53.

Deng, W.H., Wei, G. and Mei, Z.G., 2003. Nest and roost habitat characterisitcs of the grey-faced buzzard in northeastern China. *Journal of Raptor Research*, 37:228-235.

Foli, E.G., Alder, D., Miller, H.G. and Swaine, M.D., 2003. Modelling growing space requirements form some tropical forest tree species. *Forest Ecology and Management*, 173:79-88

Gering, L.R., 1995. The relationship of diameter at breast height and crown diameter for species groups in Hardin County, Tennessee. *Southern Journal of Applied Forestry*, 19:177-184.

Gorgoso, J.J., González, J.Á., Rojo, A. and Grandas-Arias, J.A., 2007. Modelling diameter distributions of *Betula alba* L. stands in northwest Spain with the two-parameter Weibull function. *Forest Systems*, 16:113-123.

Hemery, G.E., Savill, P.S. and Pryor, S.N., 2005. Applications of the crown-stem diameter relationship for different species of broadleaved trees. *Forest Ecology and Management*, 215:285-294

Kershaw, J.A., Ducey, M.J., Beers, T.W. and Hush, B.M., 2017. *Forest mensuration*. 5th Ed. Chichester, UK; Hoboken, NJ; John Wiley & Sons, 632.

Mehtatalo, L., 2017. Lmfor: Functions for Forest Biometrics. R packages version 1.2.

Mehtätalo, L., 2013. Forest biometrics with examples in R. University of Eastern Finland. School of Computing, 406.

Mirzaei, M., Aziz, J., Mahdavi, A. and Rad, A.M., 2016. Modelling frequency distributions of tree diameter, height and crown area by six probability functions for open forests of *Quercus persica* in Iran. *Journal of Forest Research,* 27:1-6.

Nord-Larson, T. and Cao, Q.V., 2006. A diameter distribution model for even-aged beech in Denmark. *Forest Ecology and Management*, 231:218-225.

R Core Team, 2017. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria.

Raptis, D., Kazana, V., Kazaklis, A. and Stamatious, C., 2018. A crown width-diameter model for natural even-aged black pine forest management. *Forests*, 9:610-629.

Sonmez, T., 2009. Diameter at breast height-crown diameter prediction models for *Picea orientalis*. *African Journal of Agriculture Research*, 4:215-219.

Van Laar, A., Akça, A., 2007. Forest Mensuration. Dordrecht, Netherlands, Springer, 389.

West, P.W., 2015. *Tree and Forest Measurement, 3rd Edition*. Springer Cham Heidelberg New York Dordrecht London, 218.

Zarnoch, S.J., Bechtold, W.A. and Stoke, K.W., 2004. Using crown condition variables as indicators of forest health. *Canadian Journal of Forest Research*, 34:1057-1070.

Zhang. L., Packard, K.C. and Liu, C.H., 2003. A comparison of estimation methods for fitting Weibull and Johnson’s S_B distributions to mixed spruce-fir stands in northeastern North America. *Canadian Journal of Forest Research*, 33:1340-1347.