Generalized classical thermodynamic analysis of a Stirling engine

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Abstract

This paper provides a theoretical investigation on thermodynamic analysis of a Stirling engine. An isothermal model for an imperfect regeneration Stirling engine with dead volumes of hot space, cold space and regenerator that the regenerator effective temperature is an arithmetic mean of the heater and cooler temperature is developed. The effects of the regenerator effectiveness and dead volumes are studied. Results from this study indicate that the engine net work is affected by only the dead volumes while the heat input and engine efficiency are affected by both the regenerator effectiveness and dead volumes. The engine net work decreases with increasing dead volumes. The heat input increases with increasing dead volumes and decreasing regenerator effectiveness. The engine efficiency decreases with increasing dead volumes and decreasing regenerator effectiveness.

Keyword: Stirling engine, hot-air engine, regenerative heat engine.

1. Introduction

The Stirling engine is a simple type of external-combustion engine that uses a compressible fluid as the working fluid. The Stirling engine can theoretically be a very efficient engine to upgrade from heat to mechanical work with the Carnot efficiency. The thermal limit of the operation of Stirling engine depends on the material used for construction. In most instances the engines operate with a heater and cooler temperature of 923 and 338 K [1]. Engine efficiency range from about 30 to 40% resulting from a typical temperature range of 923-1073 K, and normal operating speed range from 2000 to 4000 rpm [2].

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Fig. 1 shows a simplified schematic diagram of an imperfect-regeneration Stirling engine. The p-v diagram for the Stirling cycle with imperfect regeneration is shown in Fig. 2. For an ideal regeneration, the total heat rejected during process 4-1 is absorbed by a perfect regenerator and released to the working fluid during process 2-3. However, in the ideal regeneration, the infinite heat-transfer area or the infinite regeneration time is needed.

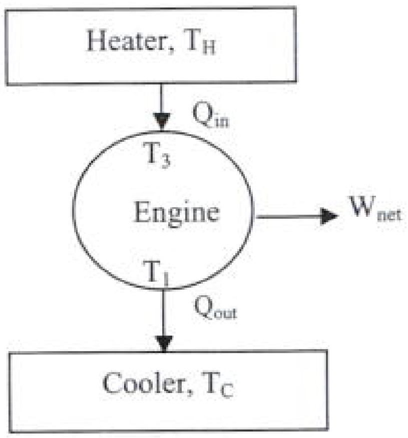

Fig. 1 Schematic diagram

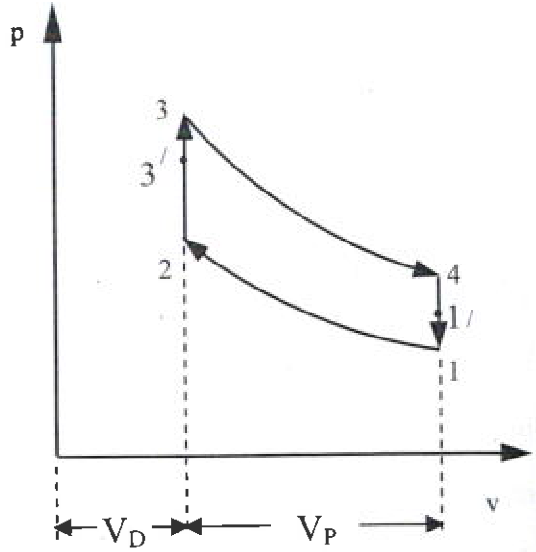

Fig. 2 p-v diagram

For an imperfect regenerator, the working fluid temperature at the regenerator outlet and inlet will be T_3' and T_1', respectively. An external heat input and output is required to increase T_3' to T_3 and decrease T_1' to T_1 [1]. Although the regenerator effectiveness of 95%, 98-99%, and 99.09% are reported [3-7], the engine developers who do not have in hand the efficient-regenerator technology should be taken into account the regenerator effectiveness, then the analysis with imperfect regeneration should be made.

Dead volume is defined as the total void volume in a Stirling engine. In general, the dead volume is referred to the volume of working fluid contained in the total dead space in engine, including regenerator and transfer ports. It is evidenced that real Stirling engine must have some unavoidable dead volume. In normal Stirling engine design practice, the total dead volume is approximately 58% of the total volume [8].

Although many researchers have analyzed the Stirling engines; there still remains a room for further development. The imperfect regeneration Stirling engine including dead volumes is one that received comparatively little attention in literature and should be study in detail. Many works on common Stirling engines, low temperature differential Stirling engines and solar-powered Stirling engines including technology and optimization have been investigated in the authors' former works [9-12]. Investigation on Stirling engine analysis showed that almost literatures are treated with ideal regeneration and zero dead volume.

The Stirling engine including dead volume can be analyzed by the Schmidt technique [13]: However, the ideal regeneration is assumed in Schmidt analysis [1, 14]. For the Stirling engines with large dead volumes, the correct working fluid temperature in regenerator is important [8]. The effective temperature of the working fluid contained in the regenerator dead space can be calculated by separate the dead volume into two halves, the hotter half is at T_3' and the cooler half is at T_1'. The regenerator effective temperature in this case can be calculated from [8, 14]:

$$\frac{1}{T_R} = \frac{1}{2T_3'} + \frac{1}{2T_1'} \tag{1}$$

The regenerator effective temperature in the second way is the arithmetic mean of working fluid at the outlet and inlet of regenerator [1, 8, 14]:

$$T_R = \frac{T_3' + T_1'}{2} \tag{2}$$

The third regenerator effective temperature is the log mean value of temperature of working fluid at the outlet and inlet of regenerator [8, 14]:

$$T_R = \frac{T_3' - T_1'}{\ln \frac{T_3'}{T_1'}} \tag{3}$$

Martini [8] claimed that the log-mean regenerator effective temperature is the most realistic and he used this regenerator effective temperature in his analysis. But the log-mean regenerator effective temperature will be infinite at 50% regenerator effectiveness, since T_3' equals T_1', therefore the log-mean regenerator effective temperature should not be used in the analysis of the Stirling engine with variable regenerator effectiveness. However, the definition of the regenerator effectiveness used by Martini [8] is different from the definition commonly used by the others [1, 14]. According to [14], the arithmetic-mean regenerator effective temperature should be a good approximation.

The objective of this article is to formulate an isothermal model for an imperfect regeneration Stirling engine with dead volumes, base on the classical thermodynamics. The model formulated is in a general form that put in to account both the regenerator effectiveness and the dead volume. The results obtained in this article will provide a generalized analytical method to evaluate the Stirling engine performance.

2. Isothermal model of an imperfect regeneration Stirling engine with dead volumes

2.1 Dead volumes

Assume that the hot space, regenerator and cold space dead volume, in m^3, is respectively V_{SH}, V_{SR} and V_{SC}, therefore, the total dead volume is:

$$V_S = V_{SH}+V_{SR}+V_{SC} = (k_{SH}+k_{SR}+k_{SC})\,V_S \qquad (4)$$

where $k_{SH} = V_{SH}/V_S$ is hot-space dead volume ratio, $k_{SR} = V_{SR}/V_S$ is regenerator dead volume ratio and $k_{SC} = V_{SC}/V_S$ is cold-space dead volume ratio.

Let the total dead volume to total volume ratio is represented by $k_{ST} = V_S/V_I$. Then the total dead volume can be expressed in term of total volume as:

$$V_S = k_{ST}\,V_I = k_{ST}\,(V_S+V_D+V_P) \qquad (5)$$

where V_D and V_P is displacer and power-piston swept volume in m^3, respectively. The ratio $k_{CR} = V_D/V_P$ is called the compression ratio. The dead volume is more convenience to express in term of the total swept volume and power-piston swept volume as:

$$V_S = k_{SDP}\,(V_D+V_P) = (k_{CR}+1)\,k_{SDP}V_P \qquad (6)$$

The dead volume to the total volume ratio and the dead volume to the total swept volume ratio are related by:

$$k_{ST} = \frac{k_{SDP}}{1+k_{SDP}} \quad \text{or} \quad k_{SDP} = \frac{k_{ST}}{1-k_{ST}} \qquad (7)$$

2.2 Imperfect regenerator

The regenerator effectiveness, e, of an imperfect regenerator is defined as [1, 14]:

$$e = \frac{T_3^{/} - T_1}{T_3 - T_1} \tag{8}$$

The value of e = 1, for 100% effectiveness or ideal regeneration and e = 0, for 0% effectiveness or no regeneration. The working fluid temperature at regenerator outlet can be express in term of the regenerator effectiveness as:

$$T_3^{\prime} = T_1 + e\,(T_3 - T_1) \tag{9}$$

For a regenerator that having equal effectiveness in heating and cooling, $Q_{23}^{\prime} = Q_{41}^{\prime}$, the working fluid temperature at regenerator inlet is:

$$T_1^{\prime} = T_3 + e\,(T_1 - T_3) = T_3 - e\,(T_3 - T_1) \tag{10}$$

Substitute Eqs. (9) and (10) into Eq. (2) gives:

$$T_R = \frac{T_3 + T_1}{2} \tag{11}$$

It can be seen that by using the arithmetic mean the regenerator effective temperature does not depend on the regenerator effectiveness.

2.3 Common pressure

Assume that the hot-space and cold-space volumes are respectively V_H and V_C and that the working fluid temperatures in the hot space, regenerator, and cold space are respectively T_3, T_R and T_1. The common pressure in an engine with dead volumes V_{SH}, V_{SR} and V_{SC} is [8]:

$$p = \frac{mR}{\frac{V_H}{T_3} + \frac{V_{SH}}{T_3} + \frac{V_{SR}}{T_R} + \frac{V_{SC}}{T_1} + \frac{V_C}{T_1}} = \frac{mR}{\frac{V_H}{T_3} + K + \frac{V_C}{T_1}} \tag{12}$$

where $$K = \frac{V_{SH}}{T_3} + \frac{V_{SR}}{T_R} + \frac{V_{SC}}{T_1} \tag{13}$$

m is the total working fluid mass contained in the engine in kg, R is the gas constant in J/kg K. Substitute Eqs. (4), (6) and (11) into Eq. (13) gives:

$$K = \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{0.5(T_3 + T_1)} + \frac{k_{SC}}{T_1}\right)(k_{CR}+1)\,k_{SDP}\,V_P \tag{14}$$

It is clear that, for a given compression ratio, power piston swept volume, hot-side and cold-side working fluid temperature, the factor K in general is a function of the dead volumes.

2.3 Working fluid mass contained in engine

In isothermal compression process 1-2, the power piston compresses the working fluid from state 1, p_1 and $V_{C1} = V_D+V_P$ to state 2, p_2 and $V_{C2} = V_D$. At state 1, total working fluid contained only in cold space and dead spaces, $V_H = 0$, mass of working fluid at state 1 then can be calculated from Eq. (12):

$$m = \frac{p_1}{R}(\frac{V_{C1}}{T_1} + K) = \frac{p_1}{RT_1}(V_{C1} + KT_1) \qquad (15)$$

Substitute Eq. (14) and $V_{C1} = V_D+V_P = (k_{CR}+1)\,V_P$ into Eq. (15) gives:

$$m = \frac{(k_{CR}+1)p_1 V_P}{RT_1}\left[1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1\right] \qquad (16)$$

It should be noted that the working fluid mass depends on the dead volumes. In the case of zero dead volume Eq. (16) will become the equation of state that can be found in thermodynamics textbooks.

2.4 Isothermal compression process

In compression process, the cold-side working fluid is compressed from $V_{C1} = V_D+V_P = (k_{CR}+1)V_P$ to $V_{C2} = V_D = k_{CR}\,V_P$. The hot-space working fluid swept volume $V_H = 0$ throughout this process. Then the heat rejected during the isothermal compression process 1-2 is:

$$Q_{1\text{-}2} = W_{1\text{-}2} = \int_{V_{C1}}^{V_{C2}} p\,dV_C = mRT_1\int_{V_{C1}}^{V_{C2}}\frac{d\,V_C}{V_C+KT_1} = m\,R\,T_1\ln\frac{V_{C2}+KT_1}{V_{C1}+KT_1}$$

$$= m\,R\,T_1\ln\frac{\dfrac{k_{CR}}{(k_{CR}+1)}+\left(\dfrac{k_{SH}}{T_3}+\dfrac{k_{SR}}{T_R}+\dfrac{k_{SC}}{T_1}\right)k_{SDP}T_1}{1+\left(\dfrac{k_{SH}}{T_3}+\dfrac{k_{SR}}{T_R}+\dfrac{k_{SC}}{T_1}\right)k_{SDP}T_1} \qquad (17)$$

It should be noted that the compression work depends only on the dead volumes.

2.5 Isochoric heating process

In principle, the heat added during the isochoric heating process 2-3 is:

$$Q_{2\text{-}3} = m\,C_V\,(T_3 - T_2) = m\,C_V\,(T_3 - T_1) \tag{18}$$

where C_V is the specific heat at constant volume in J/kg K, and is assumed to be constant. Without regeneration, this amount of heat is added by an external source and for ideal regeneration this amount of heat is released from an ideal regenerator.

The regeneration heat released from an imperfect regenerator during this process is:

$$Q_{2\text{-}3}^{\prime} = m\,C_V\,(T_3^{\prime} - T_2) = e\,m\,C_V\,(T_3 - T_1) \tag{19}$$

Heat added from an external source during process $3^{\prime}$-3 is:

$$Q_{3^{\prime}\text{-}3} = m\,C_V\,(T_3 - T_3^{\prime}) = (1\text{-}e)\,m\,C_V\,(T_3 - T_1) \tag{20}$$

It can be seen that, since the working fluid mass depends on dead volumes, the heat input to this process depends on both the regenerator effectiveness and dead volumes.

2.6 Isothermal expansion process

In expansion process, the hot-side working fluid volume changes from $V_{H3} = V_D = k_{CR}\,V_P$ to $V_{H4} = V_D + V_P = (k_{CR}+1)\,V_P$. The cold-space working fluid swept volume $V_C = 0$ throughout this process. The heat added to the cycle during the isothermal expansion process 3-4 is:

$$Q_{3\text{-}4} = W_{3\text{-}4} = m\int_{V_{H3}}^{V_{H4}} p\,dV_H = mRT_3\int_{V_{H3}}^{V_{H4}} \frac{d\,V_H}{V_H + KT_3} = m\,R\,T_3\,\ln\frac{V_{H4} + KT_3}{V_{H3} + KT_3}$$

$$= m\,R\,T_3\,\ln\frac{1 + \left(\dfrac{k_{SH}}{T_3} + \dfrac{k_{SR}}{T_R} + \dfrac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\dfrac{k_{CR}}{(k_{CR}+1)} + \left(\dfrac{k_{SH}}{T_3} + \dfrac{k_{SR}}{T_R} + \dfrac{k_{SC}}{T_1}\right)k_{SDP}T_3} \tag{21}$$

It is evidenced that, similarly to the compression work, the expansion work is only depends on the dead volumes.

2.7 Isochoric cooling process

The heat rejected during the isochoric cooling process 4-1 is:

$$Q_{4-1} = m\,C_V\,(T_1 - T_4) = -m\,C_V\,(T_3 - T_1) \qquad (22)$$

Without regeneration, this amount of heat is rejected to an external sink and for ideal regeneration; an ideal regenerator absorbs this amount of heat.

For an imperfect regeneration, the heat absorbed by a regenerator is:

$$Q_{4-1}' = m\,C_V\,(T_1'-T_4) \qquad = -\,e\,m\,C_V\,(T_3 - T_1) \qquad (23)$$

The heat rejected to an external sink, an adequate cooler, during process $1'$-1:

$$Q_{1'-1}' = m\,C_V\,(T_1-T_1') \qquad = -\,(1\text{-}e)\,m\,C_V\,(T_3 - T_1) \qquad (24)$$

It can be seen that, except the minus sign, Eqs. (23) and (24) is respectively the same as Eqs. (19) and (20). Therefore, similarly to the isochoric heating process, the heat transfer in cooling process depends on both the regenerator effectiveness and dead volumes.

2.8 Total heat added

For an imperfect regeneration, the total heat added from an external source to the cycle is:

$$Q_{in} = Q_{3'-3}' + Q_{3-4} \qquad (25)$$

Substitute Eqs. (20) and (21) into Eq. (25) gives:

$$Q_{in} = m\,C_V\,\left[(1\text{-}e)\,(T_3-T_1) + (k\text{-}1)\,T_3\,\ln\frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3}\right] \qquad (26)$$

where k is the specific heat ratio. Therefore, the heat input to the engine is depends on both the regenerator effectiveness and dead volumes. The heat input increases with increasing dead volumes and decreasing regenerator effectiveness.

Without regeneration, the total heat added from an external source is:

$$Q_{in} = Q_{2-3} + Q_{3-4} \qquad (27)$$

However, for an ideal regeneration, the total heat added from an external source will be:

$$Q_{in} = Q_{3\text{-}4} \tag{28}$$

2.9 Total heat rejected

For an imperfect regeneration, the total heat rejected from the cycle to an external sink is:

$$Q_{out} = Q^{/}_{1\text{-}1} + Q_{1\text{-}2} \tag{29}$$

Substitute Eqs. (17) and (24) into Eq. (29) gives:

$$Q_{out} = -m C_V[(1\text{-}e)(T_3\text{-}T_1)+(k\text{-}1)T_1 \ln \frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}] \tag{30}$$

The heat rejected from the engine is also depends on both the dead volumes and regenerator effectiveness.

Without regeneration, the total heat rejected to an external sink is:

$$Q_{out} = Q_{4\text{-}1} + Q_{1\text{-}2} \tag{31}$$

For an ideal regeneration, the total heat rejected to an external sink can be only:

$$Q_{out} = Q_{1\text{-}2} \tag{32}$$

It is evidenced that the amounts of heat added to the cycle and rejected from the cycle are depend on the regeneration heating and cooling.

2.10 Net work

The surplus energy of two isothermal processes 1-2 and 3-4 is converted into a useful mechanical work. The net work for an imperfect regeneration engine with dead volumes can be determined from:

$$W_{net} = \Sigma Q = Q_{in} - Q_{out} = Q^{/}_{3\text{-}3} + Q_{3\text{-}4} + Q^{/}_{1\text{-}1} + Q_{1\text{-}2} = Q_{3\text{-}4} + Q_{1\text{-}2} \tag{33}$$

Substitute Eqs. (17) and (21) into Eq. (33) gives:

$$W_{net} = m\,R\,[T_3 \ln \frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3} - T_1 \ln \frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}] \tag{34}$$

It is evidenced that the cycle net work is only depends on the dead volumes. The engine net work decreases with increasing dead volumes.

In a case of zero dead volume, the cycle net work is:

$$W_{net} = m\,R\,(T_3\text{-}T_1) \ln \frac{(k_{CR}+1)V_P}{k_{CR}V_P} = m\,R\,(T_3\text{-}T_1) \ln \frac{V_1}{V_2} \tag{35}$$

Eq. (35) can be found in many thermodynamics textbooks. It should be noted that a case of zero dead volume the cycle net work does not depend on the regenerator effectiveness.

2.11 Mean-effective pressure

The engine net work can be determined from the cycle mean-effective pressure, p_m, and total volume changed, $V_{H4}\text{-}V_{H3} = V_{C1}\text{-}V_{C2} = V_1\text{-}V_2 = V_P$:

$$W_{net} = p_m V_P \tag{36}$$

Equate Eq. (36) to Eq. (34), then:

$$p_m = \frac{mR}{V_P}\,[T_3 \ln \frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_3} - T_1 \ln \frac{1+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}{\frac{k_{CR}}{(k_{CR}+1)}+\left(\frac{k_{SH}}{T_3}+\frac{k_{SR}}{T_R}+\frac{k_{SC}}{T_1}\right)k_{SDP}T_1}] \tag{37}$$

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It should be noted that, same as the engine net work, the mean-effective pressure is only depends on the dead volumes.

For zero dead volume, by using the perfect gas law and noted that $T_1 = T_2$ and $V_D = V_2 = V_3$:

$$p_m = (p_3 - p_2)\frac{V_3}{V_1 - V_2}\ln\frac{V_1}{V_2} = \frac{(p_3 - p_2)}{(\frac{V_1}{V_2} - 1)}\ln\frac{V_1}{V_2} \quad (38)$$

Again, Eq. (38) can be found in many thermodynamics textbooks. It can be seen that the greater pressure change in the constant volume heating process 2-3, the larger the volume ratio, V_1/V_2, and the larger in volume after compression, V_2, the value of cycle net work will be more. Since the mean-effective pressure is the net work divided by the power-piston swept volume, therefore, its characteristics should be the same as the net work.

2.12 Thermal efficiency

The Stirling engine thermal efficiency can be determined from:

$$E_S = W_{net}/Q_{in} \quad (39)$$

Substitute Eqs. (26) and (34) into Eq. (39) gives:

$$E_S = \frac{T_3 \ln \dfrac{1 + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\frac{k_{CR}}{k_{CR}+1} + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_3} - T_1 \ln \dfrac{1 + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_1}{\frac{k_{CR}}{k_{CR}+1} + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_1}}{T_3 \ln \dfrac{1 + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_3}{\frac{k_{CR}}{k_{CR}+1} + \left(\frac{k_{SH}}{T_3} + \frac{k_{SR}}{T_R} + \frac{k_{SC}}{T_1}\right)k_{SDP}T_3} + (T_3 - T_1)\dfrac{(1-e)}{(k-1)}} \quad (40)$$

It can be seen that the Stirling engine efficiency also depends on both the regenerator effectiveness and dead volumes. The engine efficiency decreases with increasing dead volumes and decreasing regenerator effectiveness.

In a case of zero dead volume, Eq. (40) will be reducing to:

$$E_S = \frac{(T_3 - T_1)}{T_3 + (T_3 - T_1)\dfrac{(1-e)}{(k-1)\ln\dfrac{V_1}{V_2}}} \tag{41}$$

Even the different notation is used in Eq. (41) the results are the same as the former works approached by the classical thermodynamics [8, 14, 15] and the finite-time thermodynamics [16]. Without regeneration, e = 0, the worst case of Stirling cycle efficiency:

$$E_S = \frac{(T_3 - T_1)}{T_3 + \dfrac{(T_3 - T_1)}{(k-1)\ln\dfrac{V_1}{V_2}}} \tag{42}$$

For an ideal regeneration, e = 1, the best case of Stirling cycle efficiency:

$$E_S = 1 - T_1/T_3 \tag{43}$$

That is the endo-reversible Carnot-like engine efficiency [17].

It is evidenced that, in theory, the Stirling engine can be a very efficient device for converting heat into mechanical work with high efficiency requiring high-temperature difference. The efficiency of the Stirling engine with imperfect regeneration and zero dead volume equals that of the endo-reversible Carnot-like engine efficiency. The endo-reversible Carnot-like engine efficiency is lower than the complete reversible Carnot engine efficiency; however it produces useful power output [12].

3. Conclusions

The thermodynamic analysis for an imperfect regeneration Stirling engine with dead volumes is presented in this article. This study shows that an imperfect regeneration Stirling engine with dead volumes can be analyzed by using the basic classical thermodynamics. The analysis presented should provide a more generalized and more realistic analytical method for Stirling engines performance evaluation and improvement.

Results from this study indicate that for an imperfect regeneration Stirling engine with dead volumes and an arithmetic-mean regenerator effective temperature, the working fluid mass, net work and mean-effective pressure are affected only by the dead volumes and the heat added, heat rejected and thermal efficiency are affected by both the dead volumes and the regenerator effectiveness. From this study, we can conclude that:

1) For a Stirling engine with a given dead volume, an inefficient regenerator will not affect the engine net work (see Eq. (34)). But an engine with an inefficient regenerator need more heat input and better cooling than an efficient one (see Eqs. (26) and (30)).

2) The dead volume will decrease both the engine net work and the thermal efficiency (see Eqs. (34) and (37)) and will increase both the external heat input and output (see Eqs. (26) and (30)). However, the real engine must have some unavoidable dead volume.

3) Some small figure of the engine net work can be produced even the engine has a large dead volume (see Eq. (34)).

4) To attain high efficiency, a good regenerator is needed. However, the Stirling engine has some small efficiency without a regenerator (see Eq. (42)).

Nomenclature

C_V = specific heat at constant volume, J/kg K

e = regenerator effectiveness

E_S = Stirling engine thermal efficiency

K = a factor defined by Eq. (13)

k = specific heat ratio

$k_{CR} = V_D/V_P$ is compression ratio

$k_{SH} = V_{SH}/V_S$ is hot-space dead volume ratio

$k_{SR} = V_{SR}/V_S$ is regenerator dead volume ratio

$k_{SC} = V_{SC}/V_S$ is cold-space dead volume ratio

$k_{SDP} = V_S/(V_D+V_P)$ is total dead volume to total volume ratio

$k_{ST} = V_S/V_1$ is total dead volume to total volume ratio

m = total working fluid mass contained in the engine, kg

p = absolute pressure, N/m^2

p_m = mean-effective pressure, N/m^2

Q_{in} = total heat added from an external source to the cycle, J

Q_{out} = total heat rejected from the cycle to an external sink, J

R = gas constant, J/kg K

T_3 = working fluid temperature in the hot space, K

T_3' = working fluid temperature at regenerator outlet, K

T_1 = working fluid temperature in the cold space, K

T_1' = working fluid temperature at regenerator inlet, K

T_R = effective working fluid temperature in regenerator dead space, K

T_C = cooler temperature, K

T_H = heater temperature, K

V_{SH} = hot-space dead volume, m^3

V_{SR} = regenerator dead volume, m^3

V_{SC} = cold-space dead volume, m^3

V_S = total dead volume, m^3

V_D = displacer swept volume, m^3

V_P = power-piston swept volume, m^3

W_{net} = engine net work, J

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