LEGAL QUANTA: A MATHEMATICAL ROMANCE OF MANY DIMENSIONS†

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TABLE OF CONTENTS

I. LEGAL NUMERACY: A MANIFESTO

Truth routinely manifests itself by mathematical means. Many (perhaps most) things can be measured and expressed numerically.

> Of all the miracles available for inspection, none is more striking than the fact that the real world may be understood in terms of the real numbers, time and space and flesh and blood and dense primitive throbbings sustained somehow and brought to life by a network of secret mathematical nerves. . . .[1]

Just as law uses words to breathe life into "the enterprise of subjecting human conduct to the governance of rules,"[2] "[n]ature talks to us in the language of mathematics."[3] This symposium, *Legal Quanta*, demonstrates several distinct applications of mathematics to law and the use of quantitative techniques to model, describe, and predict legal phenomena.

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† *Cf.* EDWIN A. ABBOTT, FLATLAND: A ROMANCE OF MANY DIMENSIONS (1884).

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1. DAVID BERLINSKI, A TOUR OF THE CALCULUS, at xiii (1995).
2. LON L. FULLER, THE MORALITY OF LAW 122 (rev. ed. 1969).
3. Peter Hilton, *The Mathematical Component of a Good Education*, *in* MISCELLANEA MATHEMATICA 145, 149 (Peter Hilton, Friedrich Hirzebruch & Reinhold Remmert eds., 1991); *accord* Peter Hilton, *Foreword: Mathematics in Our Culture*, *in* JAN GULLBERG, MATHEMATICS: FROM THE BIRTH OF NUMBERS, at xvii-xxii (1997).

Admittedly, "[i]t is an open secret that lawyers," including those who teach rather than practice for a living, (stereo)typically "don't like math."[4] In a legal culture whose leaders shamelessly confess their ignorance of the "fine details of molecular biology,"[5] lawyers and their teachers run a dire risk of falling behind "the extraordinary rate of scientific and other technological advances that figure increasingly in litigation" and, for that matter, in daily life.[6] American law labors under an "extraordinary condition . . . which makes it possible for [someone] without any knowledge of even the rudiments of chemistry to pass upon" scientifically or technologically sophisticated questions.[7] The legal academy should aspire to a level of "numeracy," one that is "less about numbers per se and more about statistical inference or how to interpret and understand scientific . . . studies."[8]

Innumeracy is flatly unacceptable.[9] Indeed, a mastery of basic mathematical concepts should serve as a prerequisite to attain membership in the legal academy. As a group that not only digests but also delivers postmodern criticism,[10] legal scholars can surely grasp mathematics, which after all is merely another branch of philosophy.[11] Notwithstanding their profession's reputation for

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4. Lisa Milot, *Illuminating Innumeracy*, 63 CASE W. L. REV. 769, 769 (2013).

5. Ass'n for Molecular Pathology v. Myriad Genetics, Inc., 133 S. Ct. 2107, 2120 (2013) (Scalia, J., concurring in part and concurring in the judgment) ("I join the judgment of the Court, and all of its opinion except Part I-A and some portions of the rest of the opinion going into fine details of molecular biology. I am unable to affirm those details on my own knowledge or even my own belief.").

6. Jackson v. Pollion, 733 F.3d 786, 788 (7th Cir. 2013).

7. Parke-Davis & Co. v. H.K. Mulford Co., 189 F. 95, 115 (S.D.N.Y. 1911).

8. Edward K. Cheng, *Fighting Legal Innumeracy*, 17 GREEN BAG 2D 271, 272 (2014).

9. *See generally* JOHN ALLEN PAULOS, INNUMERACY: MATHEMATICAL ILLITERACY AND ITS CONSEQUENCES (2d ed. 2001).

10. *Cf.* STANLEY FISH, DOING WHAT COMES NATURALLY: CHANGE, RHETORIC, AND THE PRACTICE OF THEORY IN LITERARY AND LEGAL STUDIES (1989).

11. *See* U.S. PATENT & TRADEMARK OFFICE, GENERAL REQUIREMENTS BULLETIN FOR ADMISSION TO THE EXAMINATION FOR REGISTRATION TO PRACTICE IN CASES BEFORE THE UNITED STATES PATENT AND TRADEMARK OFFICE 7 (2015), http://www.uspto.gov/sites/default/files/OED_GRB.pdf [https://perma.cc/XR66-Z8N5] (describing mathematics as a philosophical discipline and therefore insufficient by itself to satisfy the technical training requirement for eligibility to take the Patent and Trademark Office examination); *see also* 37 C.F.R. § 11.7(a)(2)(ii) (2010) (requiring practitioners before the USPTO to "[p]ossess[] the

quantitative ineptitude, mathematical thinking should come naturally to legal academics. As social scientists who have nurtured "something like a third culture" between science and literature in order to improve the circumstances under which real "human beings are living,"[12] legal academics enjoy a special opportunity to unite the literary culture's "canon of works and expressive techniques" with the scientific culture's "guiding principles of quantitative thought and strict logic."[13] At their best, legal scholars as social scientists bridge all of contemporary civilization's intellectual subcultures.[14]

II. LEGAL QUANTA: A SYMPOSIUM

On October 29, 2015, the *Michigan State Law Review* convened a day-long symposium called *Legal Quanta*. Although this symposium by no means exhausted all applications of mathematics to law, its seven papers illustrate four distinct variations on this theme: empirical legal analysis, machine learning, legal networks, and analytical modeling.

The first of those themes, the application of empirical methods to discrete legal problems, is perhaps the most familiar and deeply rooted form of mathematically informed legal scholarship. More than a century after Oliver Wendell Holmes declared that "the man of the future is the man of statistics and the master of economics,"[15] and three decades after Richard Posner celebrated the decline of law as an autonomous discipline,[16] the primacy of empiricism in the contemporary legal academy lies beyond dispute. Indeed, the last half-century has witnessed the rise of the Ph.D. as the leading credential for law school professors.[17]

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legal, scientific, and technical qualifications necessary . . . to render . . . valuable service" to patent and trademark applicants).

12. C.P. SNOW, THE TWO CULTURES: A SECOND LOOK 70 (2d ed. 1965).

13. Frank Wilczek, *The Third Culture*, 424 NATURE 997, 997 (2003).

14. *See* Jim Chen, *The Midas Touch*, 7 MINN. J.L. SCI. & TECH., at i, ii (2005); *cf. Cultural Divides, Forty Years On*, 398 NATURE 91, 91 (1999) (recognizing how C.P. Snow's depiction of two cultures "still resonates" in a world "where cultural antipathies are very much alive and kicking").

15. Oliver Wendell Holmes, *The Path of the Law*, 10 HARV. L. REV. 457, 470 (1897), *reprinted in* 110 HARV. L. REV. 990, 1001 (1997).

16. *See generally* Richard A. Posner, *The Decline of Law as an Autonomous Discipline: 1962-1987*, 100 HARV. L. REV. 761 (1987).

17. *See generally* Justin McCrary, Joy Milligan & James Cleith Phillips, *The Ph.D. Rises in American Law Schools, 1960-2011: What Does It Mean for Legal Education?*, 65 J. LEGAL EDUC. 543 (2016).

Two of this symposium's articles advance the spirit of the landmark *Supreme Court Compendium*, which has sought since 1994 to "rectify" the historical "absence of reliable data" enabling quantitative analysis of decisionmaking at the highest court of the United States.[18] Ryan Black and Ryan Owens contest the conventional wisdom that attorneys with experience in the Office of the Solicitor General are likelier to win cases before the Supreme Court.[19] Far from the high court, J.J. Prescott, Norman Bishara, and Evan Starr have conducted a nationwide survey to gauge the experiences and understanding of employment noncompetition agreements within the American labor force.[20]

Empirical analysis leads each article to a surprising finding of no effect. Professors Black and Owens conclude that the "Supreme Court is no more likely to rule in favor" of alumni of the Office of the Solicitor General than in favor of other attorneys.[21] For their part, Professors Prescott, Bishara, and Starr find "little credible evidence of any relationship between the strength of enforcement" in a state and the use of noncompetition agreements by employers in that state.[22]

Two further papers document different aspects of the impact of *machine learning* on law. Certain mathematical algorithms are more readily expressed through computer programming languages than by conventional notation.[23] Seth Chandler examines the Actuarial Value Calculator,[24] the spreadsheet that the Obama administration has used to implement the "extraordinarily complex" Affordable Care Act of 2010.[25] A spreadsheet is "a directed graph of computations" in which quantitative values and computational "programming at each node in

18. LEE EPSTEIN ET AL., THE SUPREME COURT COMPENDIUM: DATA, DECISIONS AND DEVELOPMENTS, at xxi (5th ed. 2012).

19. Ryan C. Black & Ryan J. Owens, *The Success of Former Solicitors General in Private Practice: Costly and Unnecessary?*, 2016 MICH. ST. L. REV. 325, 327.

20. J.J. Prescott, Norman D. Bishara & Evan Starr, *Understanding Noncompetition Agreements: The 2014 Noncompete Survey Project*, 2016 MICH. ST. L. REV. 369.

21. Black & Owens, *supra* note 19, at 327.

22. Prescott, Bishara & Starr, *supra* note 20, at 377.

23. *See* STEPHEN WOLFRAM, A NEW KIND OF SCIENCE 368 (2002).

24. Seth J. Chandler, *Regulation by Calculator: Experience Under the Affordable Care Act*, 2016 MICH. ST. L. REV. 465, 467-68.

25. *See* Patient Protection and Affordable Care Act, Pub. L. No. 111-148 (2010); Health Care Education and Reconciliation Act, Pub. L. No. 111-152 (2010).

the graph" are depicted in a two- or three-dimensional array.[26] Professor Chandler concludes that the Actuarial Value Calculator falls short of the ideal that would inhere in a properly constructed, easily implemented, and procedurally transparent approach to "regulation by calculator."

In his paper on prototypical perceptual semantics, L. Thorne McCarthy offers a more theoretical response to the limitations of machine learning.[27] Nearly two decades ago, Professor McCarty recognized that "[l]egal reasoning is a form of *theory construction*."[28] At that time, machine learning had not attained sufficient sophistication to handle complex legal reasoning.[29] Professor McCarty now argues that the answers to these "great mysteries of cognitive science" and "legal theory" lie in "manifold learning."[30] Presenting this "very technical" subject "in a more intuitive and informal way,"[31] Professor McCarty argues that "prototypical perceptual semantics," or the coding of legal prototypes according to a probabilistic model and a geometric model, enables the development of a logical language for law.[32] The recognition that such a language for representing legal knowledge can be *learned*, even if it remains momentarily unrealized, fulfills the esthetic if not the practical goals of information theory.[33]

The third distinct topic represented in this symposium is that of legal networks. In one of two papers addressing this subject, Ryan Whalen presents an overview of "The Promises and Challenges of Legal Network Analysis."[34] Legal networks typically take the form of "nodes or vertices joined by links or edges," with modifications "such as weighted links" enabling "those structures [to] contain

26. Chandler, *supra* note 24, at 467 n.8.

27. L. Thorne McCarty, *How to Ground a Language for Legal Discourse in a Prototypical Perceptual Semantics*, 2016 MICH. ST. L. REV. 511.

28. L. Thorne McCarty, *Some Arguments About Legal Arguments*, *in* PROCEEDINGS OF THE SIXTH INTERNATIONAL CONFERENCE ON ARTIFICIAL INTELLIGENCE AND LAW 215, 221 (1997).

29. *Id.*

30. *Id.*

31. McCarty, *supra* note 27, at 512.

32. *See id.* at 514; L. Thorne McCarty, *How to Ground a Language for Legal Discourse in a Prototypical Perceptual Semantics*, *in* FIFTEENTH INTERNATIONAL CONFERENCE ON ARTIFICIAL INTELLIGENCE AND LAW 89, 89 (2015).

33. *See generally* ABRAHAM MOLES, INFORMATION THEORY AND ESTHETIC PERCEPTION (Joel E. Cohen trans., 1968); FRIEDER NAKE, ÄSTHETIK ALS INFORMATIONSVERARBEITUNG [Esthetics as Information Processing] (1974).

34. Ryan Whalen, *Legal Networks: The Promises and Challenges of Legal Network Analysis*, 2016 MICH. ST. L. REV. 539.

different types of information."[35] Mr. Whalen urges legal scholars "to retain more detailed and nuanced data, to analyze those data in more sophisticated ways, and to embrace the explanatory power of network analyses."[36]

In their contribution to this symposium, Anne Lippert and Justin Wedeking use network theory to measure the growth of judicial expertise as a response to growing complexity within law.[37] It is widely accepted that complexity within American legal system is growing.[38] Ms. Lippert and Professor Wedeking have published the first effort to measure the expertise of Supreme Court Justices according to "the content of their work product—their written opinions."[39] Their methodology "generat[es] a new, dynamic measure that incorporates network indicators derived from texts of Supreme Court opinions."[40]

My own contribution to this symposium applies a more traditional set of mathematical tools to a longstanding set of related problems in the law of finance.[41] The conventional capital asset pricing model (CAPM) remains the preferred approach in many fields of risk management. Behavioral economics has placed the CAPM's neoclassical assumptions under severe pressure. To alleviate that tension, I specify a generalized higher-moment capital asset pricing model that expresses the emotional impact of odd and even moments of statistical distributions. A four-moment version of this generalized model explains the common preference for lottery-like returns from actuarially unfavorable gambles.

III. FROM ROMANCE TO REALISM IN MATHEMATICAL ANALYSIS OF LAW

The themes and papers comprising this symposium portend a future of staggering amounts of data and fierce competition among

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35. *Id.* at 540.

36. *Id.*

37. Anne Lippert & Justin Wedeking, *Is Judicial Expertise Dynamic? Judicial Expertise, Complex Networks, and Legal Policy*, 2016 MICH. ST. L. REV. 567.

38. *See, e.g.*, RICHARD A. POSNER, SIMPLE LAW (2013); Michael J. Bommarito II & Daniel Martin Katz, *A Mathematical Approach to the Study of the United States Code*, 389 PHYSICA A 4195 (2010).

39. Lippert & Wedeking, *supra* note 37, at 571.

40. *Id.*

41. James Ming Chen, *Momentary Lapses of Reason: The Psychophysics of Law and Behavior*, 2016 MICH. ST. L. REV. 607.

mathematical models for analyzing that information. We may anticipate a parade of imperfectly articulated hypotheses and efforts at verification or falsification that will likely lead neither to elegant closed-form solutions nor to pathological functions. In our world, fundamental forces defy analog definition, invite quantification, and still leave ample room to pursue theories of everything.[42] Even with the most thoughtfully elaborated empirical tests, knowledge about law and its underlying logic remains incomplete. "Every year, if not every day, we have to wager our salvation upon some prophecy based upon imperfect knowledge."[43]

The mathematical analysis of law reflects a dialectic of romance, frustration, and eventual reconciliation of the internal logic of this scientific enterprise with reality. All of science follows a familiar progression. "Normal science does not aim at novelties of fact or theory and, when successful, finds none."[44] But when "fundamental novelties of fact and theory" arise, "[d]iscovery commences with the awareness of anomaly, *i.e.*, with the recognition that nature has somehow violated the paradigm-induced expectations that govern normal science."[45] Once an "awareness of anomaly ha[s] lasted so long and penetrated so deep" as to plunge a scientific discipline into "a state of growing crisis," a succeeding "period of pronounced professional insecurity" over "the persistent failure of the puzzles of normal science" prompts a fruitful search for new rules.[46]

The metaphysical arc of legal quanta exhibits the seductive symmetry of mathematics as a source of beauty and sensory delight. Uniquely among human endeavors, mathematics boasts "a beauty cold and austere, . . . without any appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight, the exaltation, the sense of being more than Man, which is the touchstone of the highest

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42. *See, e.g.*, GEORGE GREENSTEIN & ARTHUR ZAJONC, THE QUANTUM CHALLENGE: MODERN RESEARCH ON THE FOUNDATIONS OF QUANTUM MECHANICS 215 (2d ed. 2006) (describing the inevitably probabilistic nature of quantum mechanics).

43. Abrams v. United States, 250 U.S. 616, 630 (1919) (Holmes, J., dissenting).

44. *See* THOMAS S. KUHN, THE STRUCTURE OF SCIENTIFIC REVOLUTIONS 52 (2d ed. enlarged, 1970).

45. *Id.* at 52-53.

46. *See id.* at 66-67.

excellence, is to be found in mathematics as surely as poetry."[47] As the poet Edna St. Vincent Millay expressed the sentiment: "Euclid alone has looked on Beauty bare."[48] What Justice Potter Stewart said of obscenity (that he knew it when he saw it)[49] finds a parallel in Paul Erdős's definition of mathematical beauty: "Why are numbers beautiful? It's like asking why Beethoven's Ninth Symphony is beautiful. If you don't see why, someone can't tell you."[50]

The most beautiful mathematical results, in law or elsewhere, are those that exhibit "a very high degree of unexpectedness, combined with inevitability and economy."[51] Deep mathematical beauty subsists in connections that appear at first to be unrelated,[52] but upon further examination reveals "[m]etaphoric combination[s]" that "leap[] beyond systematic placement" and "explore[] connections that before were unsuspected."[53] The most "useful and fertile combinations" of ideas are almost certainly those "which present themselves to the mind in a sort of sudden illumination, after an unconscious working somewhat prolonged, . . . which seem the result of a first impression."[54]

The real world, however, often inconveniently fails to align itself with mathematically beautiful models. In the face of anomalous results, even the most rigorous, comprehensively elaborated approach to legal quanta "can no longer understand [itself] because the theories . . . of [a] former age no longer work and the theories of

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47. BERTRAND RUSSELL, *The Study of Mathematics*, *in* MYSTICISM AND LOGIC, AND OTHER ESSAYS 58, 60 (1988); *accord* Jim Chen, *Truth and Beauty: A Legal Translation*, 41 U. TOLEDO L. REV. 261, 265 (2010).

48. EDNA ST. VINCENT MILLAY, *Euclid Alone Has Looked on Beauty Bare*, *in* SELECTED POEMS 52 (J.D. McClatchy ed., 2003).

49. *See* Jacobellis v. Ohio, 378 U.S. 184, 197 (1964) (Stewart, J., concurring) ("[P]erhaps I could never succeed in intelligibly [defining obscenity]. But I know it when I see it . . . .").

50. PAUL HOFFMAN, THE MAN WHO LOVED ONLY NUMBERS: THE STORY OF PAUL ERDŐS AND THE SEARCH FOR MATHEMATICAL TRUTH 42 (1998) (quoting Erdős); *accord* KEITH DEVLIN, THE MATH GENE: HOW MATHEMATICAL THINKING EVOLVED AND WHY NUMBERS ARE LIKE GOSSIP 140 (2000).

51. G.H. HARDY, A MATHEMATICIAN'S APOLOGY 29 (1940).

52. *See* GIAN-CARLO ROTA, THE PHENOMENOLOGY OF MATHEMATICAL BEAUTY 173 (1997).

53. JEROME S. BRUNER, *The Conditions of Creativity*, *in* ON KNOWING: ESSAYS FOR THE LEFT HAND 17, 20 (1963).

54. HENRI POINCARÉ, *Mathematical Creation*, *in* THE FOUNDATIONS OF SCIENCE: SCIENCE AND HYPOTHESIS, THE VALUE OF SCIENCE, SCIENCE AND METHOD 383, 391 (George Bruce Halstead trans., 1913).

the new age are not yet known."[55] That challenge leaves exactly one path forward: to "start afresh as if [we] were newly come into a new world."[56]

Financial economics, which supplies the quantitative underpinnings of my contribution to this symposium, has undergone precisely this sort of scientific crisis. Much of the edifice of contemporary mathematical finance—from the capital asset pricing model to the Black-Scholes model of option pricing,[57] Merton's distance-to-default model of credit risk,[58] the original RiskMetrics specification of value-at-risk,[59] and the Gaussian copula[60]—is built on the Gaussian "normal" distribution.[61]

These elegant models—absent elaborate modifications that ruin their spare, symmetrical form—are treacherously wrong in their reporting of the true nature of risk. Many of the predictive flaws in contemporary finance arise from reliance on the mathematically elegant but practically unrealistic construction of "beautifully Platonic models on a Gaussian base."[62] Gaussian mathematics suggests that financial returns are smooth, symmetrical, and predictable. In reality, returns are skewed[63] and exhibit heavier than

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55. WALKER PERCY, *The Delta Factor*, *in* THE MESSAGE IN THE BOTTLE: HOW QUEER MAN IS, HOW QUEER LANGUAGE IS, AND WHAT ONE HAS TO DO WITH THE OTHER 3, 7 (1986).

56. *Id.* at 7.

57. *See* Fischer Black & Myron S. Scholes, *The Pricing of Options and Corporate Liabilities*, 81 J. POL. ECON. 637 (1973); Robert C. Merton, *The Theory of Rational Option Pricing*, 4 BELL J. ECON. 141 (1973).

58. *See* Robert C. Merton, *On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*, 29 J. FIN. 449 (1974).

59. *See* JORGE MINA & JERRY YI XIAO, RETURN TO RISKMETRICS: THE EVOLUTION OF A STANDARD (2001); Jeremy Berkowitz & James O'Brien, *How Accurate Are Value-at-Risk Models at Commercial Banks?*, 57 J. FIN. 1093 (2002).

60. *See* ROGER B. NELSEN, AN INTRODUCTION TO COPULAS (1999); David X. Liu, *On Default Correlation: A Copula Function Approach*, 9 J. FIXED INCOME 43 (2000).

61. *See generally* BENOIT B. MANDELBROT & RICHARD L. HUDSON, THE (MIS)BEHAVIOR OF MARKETS: A FRACTAL VIEW OF RISK, RUIN, AND REWARD (2004).

62. NASSIM NICHOLAS TALEB, THE BLACK SWAN: THE IMPACT OF THE HIGHLY IMPROBABLE 279 (2007).

63. *See, e.g.*, JOHN Y. CAMPBELL, ANDREW W. LO & A. CRAIG MACKINLAY, THE ECONOMETRICS OF FINANCIAL MARKETS 17, 81, 172, 498 (1997); Felipe M. Aparicio & Javier Estrada, *Empirical Distributions of Stock Returns: European Securities Markets, 1990-95*, 7 EUR. J. FIN. 1 (2001); Geert Bekaert et al., *Distributional Characteristics of Emerging Market Returns and Asset Allocation*, 24 J. PORTFOLIO MGMT. 102 (1998); Pornchai Chunhachinda et al., *Portfolio Selection and Skewness: Evidence from International Stock Markets*, 21 J. BANKING & FIN.

normal tails.[64] The attraction in law and finance to formal elegance reflects a love affair with the Gaussian mathematics that has traditionally dominated the culture of the natural and social sciences.[65] Grasping the uncomfortable truth that Gaussian models of risk and return belong to "a system of childish illusions" forces our infatuation with the seductive symmetry of traditional risk modeling to pass "like first love . . . into memory."[66]

This conflict is often portrayed as an irreconcilable struggle between the romance of beauty and the realism of truth. Hermann Weyl admonished physics (and presumably all other pursuits informed by mathematics) that any necessary choice between truth and beauty should favor beauty.[67] Practical versus philosophical "conflict" over "the purpose of scientific inquiry" is "an ancient [struggle] in science."[68] Although law ordinary seeks "knowledge . . . for purely practical reasons, to predict and control some part of nature for society's benefit," the knowledge unveiled through mathematical analysis "may serve more abstract ends for the contemplative soul" and "[u]ncover[] new relationships" that prove "aesthetically satisfying" insofar as they "bring[] order to a chaotic world."[69]

This tension is ultimately illusory. Mathematics itself delivers an elegant denouement. Mathematical analysis of law is ordinarily associated with—indeed, equated with—the application of established empirical techniques to ever-growing bodies of legal

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143 (1997); Amado Peiró, *Skewness in Financial Returns*, 23 J. BANKING & FIN. 847 (1999).

64. *See, e.g.*, J. Brian Gray & Dan W. French, *Empirical Comparisons of Distributional Models for Stock Index Returns*, 17 J. BUS. FIN. & ACCOUNTING 451 (1990); Stanley J. Kon, *Models of Stock Returns—A Comparison*, 39 J. FIN. 147 (1984); Harry M. Markowitz & Nilufer Usmen, *The Likelihood of Various Stock Market Return Distributions, Part 1: Principles of Inference*, 13 J. RISK & UNCERTAINTY 207 (1996); Harry M. Markowitz & Nilufer Usmen, *The Likelihood of Various Stock Market Return Distributions, Part 2: Empirical Results*, 13 J. RISK & UNCERTAINTY 221 (1996); Terence C. Mills, *Modelling Skewness and Kurtosis in the London Stock Exchange FT-SE Index Return Distributions*, 44 STATISTICIAN 323 (1995). *See generally* TERENCE C. MILLS, THE ECONOMETRIC MODELLING OF FINANCIAL TIME SERIES (2d ed. 1999).

65. *See* TALEB, *supra* note 62, at 279.

66. BERLINSKI, *supra* note 1, at 239.

67. *Obituaries*, 177 NATURE 457, 458 (1956) (quoting Weyl: "My work always tried to unite the true with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."); *accord* EDWARD O. WILSON, BIOPHILIA 61 (1984).

68. SHARON E. KINGSLAND, MODELING NATURE: EPISODES IN THE HISTORY OF POPULATION ECOLOGY 4-5 (1985).

69. *Id.*

data. This symposium, however, demonstrates that the enterprise of legal quanta enjoys a far broader scope and entertains vastly deeper ambitions. In stark "contrast with soulless calculation," "[g]enuine mathematics . . . constitutes one of the finest expressions of the human spirit."[70] Admittedly, the "great areas of mathematics"—including "combinatorics, probability theory, statistics," and other fields of greatest interest to law—"have undoubtedly arisen from our experience of the world around us."[71] The law applies these mathematical tools "in order to systematize that experience, to give it order and coherence, and thereby to enable us to predict and perhaps control future events."[72] But it is the mathematical reasoning beneath the application of quantitative tools to law, rather than raw real-world data that propels scientific "progress . . . in response to what might be called the mathematician's apprehension of the natural dynamic of mathematics itself."[73]

In other words, mathematics *as doing* delivers the answers that we most pressingly seek—not simply according to the data describing the legal world as we find it, but also according to the own internal logic of mathematics. "[T]here is nothing in the world of mathematics that corresponds to an audience in a concert hall, where the passive listen to the active. Happily, mathematicians are all *doers*, not spectators."[74] Through its quest for the "universal interest[s]" of the law, legal quanta may yet "catch an echo of the infinite, a glimpse of its unfathomable process, a hint of the universal law."[75]

70. Hilton, *The Mathematical Component of a Good Education*, *supra* note 3, at 151; *accord* Hilton, *Mathematics in Our Culture*, *supra* note 3, at xxi.

71. *Id.*

72. *Id.*

73. *Id.*

74. GEORGE M. PHILLIPS, MATHEMATICS IS NOT A SPECTATOR SPORT, at vii (2005).

75. Holmes, *supra* note 15, at 478, *reprinted in* 110 HARV. L. REV. at 1009.

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