LEAPS, METES, AND BOUNDS: INNOVATION LAW AND ITS LOGISTICS

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> If you came this way,
> Taking any route, starting from anywhere,
> At any time or at any season,
> It would always be the same . . . .
> T.S. ELIOT, *Little Gidding*, *in* FOUR QUARTETS (1943)[1]

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INTRODUCTION

This symposium addresses legal policies designed to promote innovation. Economic analysis of technological innovation, diffusion, and decline often proceeds according to sigmoid (S-shaped) models, either directly or as a component in more elaborate mathematical representations of the creative process. The three topics addressed in this symposium—Aereo's failed attempt to retransmit television broadcasts, agricultural biotechnology, and network neutrality—invite analysis according to one variant or another of the logistic function, the simplest of sigmoid functions. Mindful that mathematics is a philosophical discipline as well as a scientific tool,[2] this Article will extract lessons for the law of innovation from a qualitative application of sigmoid modeling. Innovation and legal policies designed to foster it follow the leaps, metes, and bounds of sigmoid functions.

Part I of this Article introduces the logistic function as the simplest analytical expression of a sigmoid function. Its parameters provide very clear interpretations grounded in physical principles. Part II evaluates the Aereo controversy and agricultural biotechnology as instances of logistic substitution between competing products. The deployment of plant-incorporated protectants and herbicide-resistant crops arguably follows the Hubbert curve, a related function that describes the peak production and eventual exhaustion of depletable resources. Part III proposes

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Torrance. Christian Diego Alcocer Argüello provided very capable research assistance. Special thanks to Heather Elaine Worland Chen.

[1] T.S. ELIOT, *Little Gidding*, *in* FOUR QUARTETS 49, 50 (Harcourt Brace Jovanovich 1971) (1943).

[2] *See* U.S. PATENT & TRADEMARK OFFICE, GENERAL REQUIREMENTS BULLETIN FOR ADMISSION TO THE EXAMINATION FOR REGISTRATION TO PRACTICE IN PATENT CASES BEFORE THE UNITED STATES PATENT AND TRADEMARK OFFICE 7 (2014), *available at* http://www.uspto.gov/sites/default/files/ip/boards/oed/exam/OED_GRB.pdf (describing mathematics as a philosophical discipline and therefore insufficient by itself to satisfy the technical training requirement for eligibility to take the Patent and Trademark Office examination); *see also* 37 C.F.R. § 11.7(a)(2)(ii) (2014) (requiring practitioners before the USPTO to "[p]ossess[] the legal, scientific, and technical qualifications necessary . . . to render [patent and trademark] applicants valuable service"); *cf.* SHARON E. KINGSLAND, MODELING NATURE: EPISODES IN THE HISTORY OF POPULATION ECOLOGY 4-5 (1985) ("On the one hand, knowledge may be sought for purely practical reasons, to predict and control some part of nature for society's benefit. On the other hand, knowledge may serve more abstract ends for the contemplative soul. Uncovering new relationships is aesthetically satisfying in that it brings order to a chaotic world.").

multiple ways of understanding network neutrality as a problem of multilayered innovation. The presence of two different types of nonlinear growth, in network operating costs and in expressive diversity, suggests that the law should prescribe independent rather than bundled solutions to these conceptually distinct legal concerns.

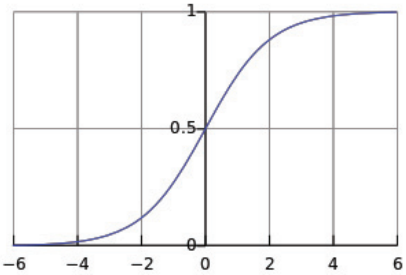
I. SIGMOID MODELING AND THE LOGISTIC FUNCTION

A. Sigmoids Across the Sciences

Across the physical, biological, and social sciences, bounded growth processes are modeled according to sigmoid functions. A sigmoid function is a bounded differentiable real function that is supported across the entire domain of real numbers and whose first derivative is either consistently positive or consistently negative.[3] The logistic function and the error function represent two of the most familiar sigmoid functions:[4]

Logistic function

$$f(t) = \frac{1}{1+e^{-t}}$$



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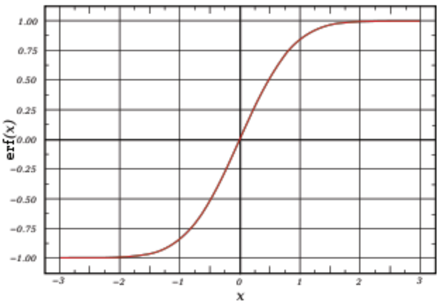
[3] *See generally* Jun Han & Claudio Moraga, *The Influence of the Sigmoid Function Parameters on the Speed of Backpropagation Learning*, *in* FROM NATURAL TO ARTIFICIAL NEURAL COMPUTATION 195 (José Mira & Francisco Sandoval eds., 1995).

[4] The following images come from *Sigmoid Function*, WIKIPEDIA, http://en.wikipedia.org/wiki/Sigmoid_function (last modified Feb. 18, 2015, 5:54 a.m.).

Error function

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$



A sigmoid model portrays a system's potential for accelerated growth at the outset, while simultaneously accounting for negative feedback mechanisms that prevent indefinite, unsustainable growth beyond the system's carrying capacity. Despite the complexity of many of these processes, many instances of technological growth, substitution, and decline can be elegantly described by a simple mathematical model. The simplest sigmoid model follows the logistic function. The logistic function and the ordinary differential equation for which it supplies an analytical solution play a crucial role in the mathematics of nonlinear, concave growth curves.

Sigmoid modeling in the physical, biological, and social sciences has a celebrated history spanning two centuries.[5] In the tradition of Thomas Malthus's work on overpopulation and food shortages[6] and Benjamin Gompertz's demographic "law of human mortality,"[7] nineteenth-century Belgian scientist Pierre-François Verhulst developed a population model based on the intrinsic rate of reproduction (r) and an ecosystem's carrying capacity (K).[8]

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[5] *See* Dmitry Kucharavy & Roland De Guio, *Logistic Substitution Model and Technological Forecasting*, 9 PROCEDIA ENGINEERING 402, 403-04 (2011).

[6] *See generally* THOMAS MALTHUS, AN ESSAY ON THE PRINCIPLE OF POPULATION (1798), *available at* http://www.esp.org/books/malthus/population/malthus.pdf.

[7] *See generally* Benjamin Gompertz, *On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies*, 115 PHIL. TRANSACTIONS ROYAL SOC'Y LONDON 513 (1825).

[8] *See generally* P.-F. Verhulst, *Notice sur la Loi que la Population suit dans son Accroissement*, 10 CORRESPONDANCE MATHÉMATIQUE ET PHYSIQUE DE L'OBSERVATOIRE DE BRUXELLES 113 (1838) (Fr.); P.-F. Verhulst, *Recherches Mathématiques sur la Loi d'Accroissement de la Population*, 18 NOUVEAUX MÉMOIRES DE L'ACADÉMIE ROYALE DES SCIENCES ET BELLES-LETTRES DE BRUXELLES 1 (1845) (Fr.); P.-F. Verhulst, *Deuxième Mémoire sur la Loi*

Verhulst's work lay dormant until it was rediscovered in the early twentieth century.[9] T. Brailsford Robertson used the logistic function to model growth in an individual organism[10] and in microbial populations.[11] With evangelistic zeal, Raymond Pearl and Lowell Reed based an entire theory of human populations on the logistic function.[12] With even greater impact, Alfred J. Lotka and Vito Volterra independently extended Verhulst's model to interspecific competition between predators and prey.[13] The celebrated Lotka–Volterra equations dominated ecology for half a century.[14] In 1950 Theodosius Dobzhansky suggested that climate, especially in the contrast between tropical and temperate regions,

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d'Accroissement de la Population, 20 MÉMOIRES DE L'ACADÉMIE ROYALE DES SCIENCES, DES LETTRES ET DES BEAUX-ARTS DE BELGIQUE 1 (1847) (Fr.).

[9] *See* KINGSLAND, *supra* note 2, at 66 ("Verhulst died in 1849, . . . and his population work remained largely unnoticed."). *See generally* Anthony F.J. van Raan, *Sleeping Beauties in Science*, 59 SCIENTOMETRICS 461 (2004).

[10] *See* T. Brailsford Robertson, *On the Normal Rate of Growth of an Individual, and Its Biochemical Significance*, 25 ARCHIV FÜR ENTWICKLUNGSMECHANIK DER ORGANISMEN 581, 584 (1908) (Ger.); T. Brailsford Robertson, *Further Remarks on the Normal Rate of Growth of an Individual, and Its Biochemical Significance*, 26 ARCHIV FÜR ENTWICKLUNGSMECHANIK DER ORGANISMEN 108, 109 (1908) (Ger.); T. Brailsford Robertson, *On the Nature of the Autocatalyst of Growth*, 37 ARCHIV FÜR ENTWICKLUNGSMECHANIK DER ORGANISMEN 497, 498 (1913) (Ger.).

[11] *See* T. BRAILSFORD ROBERTSON, THE CHEMICAL BASIS OF GROWTH AND SENESCENCE (1923).

[12] *See* Raymond Pearl & Lowell J. Reed, *On the Rate of Growth of the Population of the United States Since 1790 and Its Mathematical Representation*, 6 PROC. NAT'L ACAD. SCI. 275, 280-82 (1920); Raymond Pearl, *The Growth of Populations*, 2 Q. REV. BIOLOGY 532, 537 (1927); RAYMOND PEARL, INTRODUCTION TO MEDICAL BIOMETRY AND STATISTICS 243 (1923); P.J. Lloyd, *American, German and British Antecedents to Pearl and Reed's Logistic Curve*, 21 POPULATION STUD. 99, 99 (1967).

[13] *See* ALFRED J. LOTKA, ELEMENTS OF PHYSICAL BIOLOGY 123 (1925); Vito Volterra, *Variazioni e Fluttuazioni del Numero d'Individui in Specie Animali Conviventi*, *in* 2 MEMORIE RENDICONTE DELL'ACADEMIA NAZIONALE DEI LINCEI 1, 31 (1926).

[14] *See generally* ROBERT M. MAY, STABILITY AND COMPLEXITY IN MODEL ECOSYSTEMS (1973); M.L. Rosenzweig & R.H. MacArthur, *Graphical Representation and Stability Conditions of Predator–Prey Interactions*, 97 AM. NATURALIST 209 (1963); James A. Yorke & William N. Anderson, Jr., *Predator–Prey Patterns*, 70 PROC. NAT'L ACAD. SCI. 2069 (1973). *See also* Theodore Modis, *Strengths and Weaknesses of S-Curves*, 74 TECHNOLOGICAL FORECASTING & SOC. CHANGE 866, 872 (2007) ("S-curves sit in the heart of the Volterra–Lotka equations, which describe the predator–prey relations and other forms of competition.").

could limit the ecological carrying capacity.[15] Robert H. MacArthur and E.O. Wilson incorporated the contributions of Verhulst, Lotka, and Volterra into their theory of island biogeography.[16]

Logistic modeling spread from ecology to the social sciences. From the 1950s onward, economists applied logistic modeling to technological competition and substitution.[17] By the 1970s, numerous studies on advertising and marketing applied logistic models.[18] Energy and transportation infrastructure has proved an irresistible subject for logistic analysis.[19] At sufficiently high levels of abstraction, even sweeping economic transformations such as the Industrial Revolution may be evaluated as the logistic displacement of agricultural labor by mechanical substitutes.[20]

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[15] *See* Theodosius Dobzhansky, *Evolution in the Tropics*, 38 AM. SCIENTIST 209, 219-21 (1950).

[16] *See* ROBERT H. MACARTHUR & EDWARD O. WILSON, THE THEORY OF ISLAND BIOGEOGRAPHY 84, 94-95 (1967).

[17] *See* EDWIN MANSFIELD, INNOVATION, TECHNOLOGY AND THE ECONOMY 744, 755 (1995); Robert U. Ayres, *What Have We Learned?*, 62 TECHNOLOGICAL FORECASTING & SOC. CHANGE 9, 9-10 (1999) (tracing the use of logistic models in technological forecasting throughout the 1960s).

[18] *See* AMBAR G. RAO, QUANTITATIVE THEORIES IN ADVERTISING 16-17 (1970); Russell L. Ackoff & James R. Emshoff, *Advertising Research at Anheuser-Busch, Inc. (1963-68)*, 16 SLOAN MGMT. REV. 1, 2-4 (1975); James R. Freeland & Charles B. Weinberg, *S-Shaped Response Functions: Implications for Decision Models*, 31 J. OPERATIONAL RES. SOC'Y 1001, 1002 (1980); Johny K. Johansson, *A Generalized Logistic Function with an Application to the Effect of Advertising*, 68 J. AM. STAT. ASS'N 824, 824 (1973); Gary L. Lilien & Ambar G. Rao, *A Model for Allocating Retail Outlet Building Resources Across Market Areas*, 24 OPERATIONS RES. 1, 2-5 (1976); John D.C. Little, *Aggregate Advertising Models: The State of the Art*, 27 OPERATIONS RES. 629, 637-40 (1979); C.B. Weinberg, *Response Curves for a Leaflet Distribution—Further Analysis of the DeFleur Data*, 22 OPERATIONAL RES. Q. 177, 177 (1971).

[19] *See generally* ARNULF GRÜBLER, THE RISE AND FALL OF INFRASTRUCTURES: DYNAMICS OF EVOLUTION AND TECHNOLOGICAL CHANGE IN TRANSPORT (1990); Jesse H. Ausubel & Cesare Marchetti, Elektron*: Electrical Systems in Retrospect and Prospect*, 125 DÆDALUS 139 (1996).

[20] *See* Robert Herman & Elliott W. Montroll, *A Manner of Characterizing the Development of Countries*, 69 PROC. NAT'L ACAD. SCI. 3019, 3019 (1972); Joel Mokyr, *The Industrial Revolution and the Economic History of Technology: Lessons from the British Experience, 1760–1850*, 41 Q. REV. ECON. & FIN. 295, 298 (2001); *cf.* G.N. von Tunzelmann, *Innovation and Industrialization: A Long-Term Comparison*, 56 TECHNOLOGICAL FORECASTING & SOC. CHANGE 1 (1997) (arguing that economic and social differences in the three industrial revolutions of the eighteenth, nineteenth, and twentieth centuries impose severe limitations on the reliability of long-term forecasts of technological and social change).

B. The Logistic Law of Jurisdynamics

"[T]ime is the longest distance between two places."[21] And nothing as complex and contested as time should be expected to yield its secrets to mathematically crude methods. Economic time series routinely demand empirical tools befitting the full warp and woof of human experience.[22] As a form of legal signal processing that is at once sophisticated and tractable, sigmoid modeling offers some hope of "find[ing] in motion what was lost in space."[23]

"Anything that begins and ends an existence will fit a logistic."[24] Many phenomena in physics, biology, and the social sciences—from the "population of a species, height of a plant, [or] power of an engine"[25] to language acquisition[26] and linguistic change[27]—exhibit an initial spurt of exponential growth. But "natural systems cannot sustain exponential growth indefinitely."[28] Such systems routinely reflect "negative feedback mechanisms or signals from the environment."[29] The interaction between initial exponential growth and negative feedback often generates "a single growth process" following "a single sigmoidal curve."[30]

The "three-parameter S-shaped logistic growth model" provides the analytical foundation for projecting technological growth (and decline) according to one or more sigmoid functions.[31] S-shaped functions can and should be defined as ordinary differential

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[21] TENNESSEE WILLIAMS, THE GLASS MENAGERIE 114 (New Directions Books 1970) (1945); *accord* Jim Chen, *Liberating* Red Lion *from the Glass Menagerie of Free Speech Jurisprudence*, 1 J. ON TELECOMM. & HIGH TECH. L. 293, 307 (2002).

[22] *See generally* MICHAEL P. CLEMENTS & DAVID F. HENDRY, FORECASTING ECONOMIC TIME SERIES (1998).

[23] WILLIAMS, *supra* note 21, at 115.

[24] Modis, *supra* note 14, at 866; *accord* Jesse H. Ausubel & Cesare Marchetti, *The Evolution of Transport*, 7 INDUS. PHYSICIST 20 (2001).

[25] Perrin S. Meyer, Jason W. Yung & Jesse H. Ausubel, *A Primer on Logistic Growth and Substitution: The Mathematics of the Loglet Lab Software*, 61 TECHNOLOGICAL FORECASTING & SOC. CHANGE 247, 247 (1999).

[26] *See* Cesare Marchetti, *Society as a Learning System: Discovery, Invention, and Innovation Cycles Revisited*, 18 TECHNOLOGICAL FORECASTING & SOC. CHANGE 267, 267-68 (1980).

[27] *See* PROBABILISTIC LINGUISTICS § 5.3 (Rens Bod, Jennifer Hay & Stefanie Jannedy eds., 2003).

[28] Meyer, Yung & Ausubel, *supra* note 25, at 247.

[29] *Id.*

[30] *Id.*

[31] *Id.* at 248 (emphasis omitted).

equations. "Physicists first used [ordinary differential equations] to model the trajectories of moving objects."[32] Ordinary differential equations, "[w]hen applied to populations or technologies," comparably "describe continuous 'trajectories' of growth or decline through time."[33] At their most ambitious, projections of physical and social trajectories give rise to "'laws of social dynamics' based on Newton's laws of mechanics."[34]

Consider the following ordinary differential equation:[35]

$$\frac{d\,P(t)}{dt} = \alpha \cdot P(t)$$

where $P(t)$ represents population as a function of time. The analytical solution to this differential equation reveals a model of exponential growth:

$$P(t) = \beta e^{\alpha t}$$

where α is a growth rate constant and $\beta = P(0)$ is the baseline population at $t = 0$.

Although this model might depict the initial stages of seemingly exponential growth, "no bounded system can sustain exponential growth indefinitely unless the parameters or boundaries of the system are changed."[36] Even though a "simple exponential growth model can provide an adequate approximation" for growth during an "initial period," considerations such as predation and "intraspecific competition for environmental resources such as food and habitat" render "unrealistic" any model that assumes "unrestricted growth."[37] To achieve a more realistic model, we must

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[32] *Id.*

[33] *Id.*

[34] *Id. See generally* Elliott W. Montroll, *Social Dynamics and the Quantifying of Social Forces*, 75 PROC. NAT'L ACAD. SCI. 4633 (1978). Expressing linear regression as a first-order ordinary differential equation exposes the limitations of this popular quantitative technique:

$$\frac{dN}{dt} = \beta$$

The analytical solution to a first-order ordinary differential equation specified as a constant is the family of linear functions following the form, $N(t) = \beta + C$.

[35] Meyer, Yung & Ausubel, *supra* note 25, at 249.

[36] *Id.*

[37] A. Tsoularis, *Analysis of Logistic Growth Models*, 2 RES. LETTERS INFO. & MATHEMATICAL SCI. 23, 23 (2001).

modify the basic exponential equation "with a limit or a carrying capacity."[38] The result is a sigmoid curve that resembles the logistic function or the error function.[39]

The simplest and perhaps most widely used sigmoid modification of exponential growth is the logistic function. Analytical expressions of logistic growth are transformations of the basic logistic function:

$$f(t) = \frac{1}{1+e^{-t}}$$

To model logistic rather than exponential growth, we revisit our original formulation of exponential growth as an ordinary differential equation. A logistic growth model adopts the *P(t)* and *α* terms of the exponential growth function "but adds a 'negative feedback' term $\left(1 - \frac{P(t)}{\kappa}\right)$ that slows the growth rate of a population as the limit *κ* is approached":[40]

$$\frac{d\,P(t)}{dt} = \alpha \cdot P(t) \cdot \left(1 - \frac{P(t)}{\kappa}\right)$$

The negative feedback term exerts greater limits on the differential equation as *P(t)* increases:

$$1 - \frac{P(t)}{\kappa} \approx 1, \;\; P(t) << \kappa$$
$$1 - \frac{P(t)}{\kappa} \rightarrow 0, \;\; P(t) \rightarrow \kappa$$

To restate the foregoing analysis in English: "[T]he growth rate begins exponentially but then decreases to zero as the population *P(t)* approaches the limit *κ*, producing an S-shaped (sigmoidal) growth trajectory."[41]

The analytical solution to the foregoing differential equation is a logistic function:

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[38] Meyer, Yung & Ausubel, *supra* note 25, at 249.

[39] *See* KINGSLAND, *supra* note 2, at 64-76.

[40] Meyer, Yung & Ausubel, *supra* note 25, at 249.

[41] *Id.*

$$P(t) = \frac{\kappa}{1 + e^{-\alpha(t-\beta)}}$$

where parameters α, β, and κ are all necessary for the expression of the equation.[42] This ordinary differential equation and its analytical solution provide "a parsimonious model [whose] three parameters have clear, physical interpretations."[43] Economic applications of the logistic function are therefore consilient with models applying that function to physical or biological phenomena.[44]

Growth rate parameter α, which describes the "width or steepness" of the logistic function, is often replaced by the so-called "characteristic duration," or Δt. The characteristic duration Δt specifies the amount of time needed for the function to progress from $P(t_1) = 0.1\kappa$ to $P(t_2) = 0.9\kappa$. Δt is a straightforward transformation of α:[45]

$$\Delta t = \frac{\ln(81)}{\alpha}$$

The reciprocal relationship between characteristic duration Δt and growth rate α is precisely what we would expect of time and rate parameters in any mathematical model.

Location parameter β indicates the point in time when the function reaches its midpoint. Formally: $P(\beta) = \kappa/2$. Consequently, $\beta = t_m$, or the midpoint of the logistic function. The standard logistic model "is symmetric around the midpoint t_m."[46] To overcome this limitation, numerous alternative sigmoid models relax the symmetric assumption embedded in the standard logistic function.[47]

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[42] *Id*. at 250.

[43] *Id*. at 248.

[44] *See generally* EDWARD O. WILSON, CONSILIENCE: THE UNITY OF KNOWLEDGE (1998).

[45] Meyer, Yung & Ausubel, *supra* note 25, at 250. Numerically, the natural logarithm of 81 is approximately 4.3944.

[46] *Id*. at 250.

[47] *See, e.g*., Gompertz, *supra* note 7. *See generally* ROBERT B. BANKS, GROWTH AND DIFFUSION PHENOMENA: MATHEMATICAL FRAMEWORKS AND APPLICATIONS 149-62 (1994) (describing the Gompertz and Weibull distributions); M. Nawaz Sharif & M. Nazrul Islam, *The Weibull Distribution as a General Model for Forecasting Technological Change*, 18 TECHNOLOGICAL FORECASTING & SOC. CHANGE 247, 247-48 (1980) (describing the Weibull distribution).

κ indicates "the asymptotic limit that the growth curve approaches," whether that limit is defined as a "market niche or [as the] carrying capacity" of an ecological system.[48] In a traditional application of the logistic growth model to "the multiplication of bacteria consuming sugar and minerals in a closed petri dish," κ describes the system's "carrying capacity," which "is limited by available space" in the dish.[49] "[S]tagnation" sets in "[a]s the bacteria exhaust the nutritious area of the dish" and "befoul their environment."[50] The resulting reduction in their rate of growth "produc[es] the S-shaped logistic growth trajectory":[51]

$$N(t) = \frac{\kappa}{1 + e^{-\frac{\ln(81)}{\Delta t}(t - t_m)}}$$

C. Modeling (and Visualizing) Diffusion as Cumulative Adoption

Why growth processes, including the diffusion of innovation, take sigmoid form warrants a brief but dramatic and persuasive

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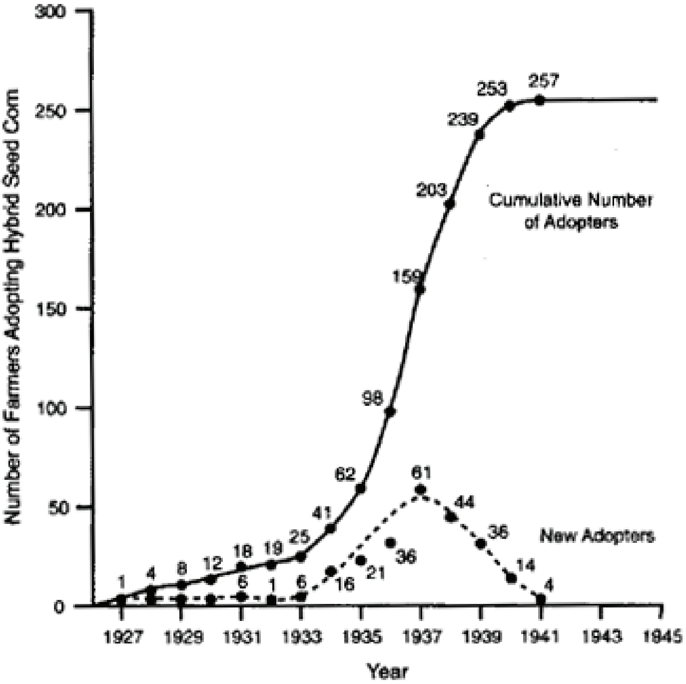
[48] Meyer, Yung & Ausubel, *supra* note 25, at 250.

[49] *Id.*; *see* JACQUES MONOD, RECHERCHES SUR LA CROISSANCE DES CULTURES BACTÉRIENNES 123-27 (2d ed. 1942). For applications of logistic growth models to other organisms, see Tor Carlson, *Über Geschwindigkeit und Größe der Hefevermehrung in Würze*, 57 BIOCHEM. ZEITUNG 313 (1913) (yeast); CHARLES J. KREBS, ECOLOGY: THE EXPERIMENTAL ANALYSIS OF DISTRIBUTION AND ABUNDANCE 368-69 (4th ed. 1994) (anchovies); Byron J.T. Morgan, *Stochastic Models of Grouping Changes*, 8 ADVANCES APPLIED PROBABILITY 30, 31-34 (1976) (elephants); Cesare Marchetti, Perrin S. Meyer & Jesse H. Ausubel, *Human Population Dynamics Revisited with the Logistic Model: How Much Can Be Modeled and Predicted?*, 52 TECHNOLOGICAL FORECASTING & SOC. CHANGE 1 (1996) (humans); Lilian Shiao-Yen Wu & Daniel B. Botkin, *Of Elephants and Men: A Discrete, Stochastic Model for Long-Lived Species with Complex Life Histories*, 116 AM. NATURALIST 831 (1980) (elephants and humans).

[50] Meyer, Yung & Ausubel, *supra* note 25, at 250. Bacteria are hardly the only species that befouls its environment. Perhaps the most vivid application in the legal literature of the κ parameter as the designation of a system's carrying capacity is Douglas A. Kysar's distinction between "'spaceman'" and "'cowboy'" approaches to ecological economics. Douglas A. Kysar, *Sustainability, Distribution, and the Macroeconomic Analysis of Law*, 43 B.C. L. REV. 1, 9-11 (2001) (quoting and discussing Kenneth E. Boulding, *The Economics of the Coming Spaceship Earth*, *in* ENVIRONMENTAL QUALITY IN A GROWING ECONOMY 3 (Henry Jarrett ed., 1966), *reprinted in* VALUING THE EARTH: ECONOMICS, ECOLOGY, ETHICS 297 (Herman E. Daly & Kenneth N. Townsend eds., 1993); Kenneth E. Boulding, *Spaceship Earth Revisited*, *in* VALUING THE EARTH: ECONOMICS, ECOLOGY, ETHICS, *supra*, at 311)).

[51] Meyer, Yung & Ausubel, *supra* note 25, at 250.

graphic demonstration. In *Diffusion of Innovations*, Everett Rogers recognized that "the rate of adoption for an innovation can be represented by either a bell-shaped (frequency) curve or an S-shaped (cumulative) curve."[52] "These are just two different ways to display the same data."[53] Consider the following illustration from *Diffusion of Innovations*,[54] drawn from a study of the uptake of hybrid corn by Iowa farmers:[55]



The sigmoid representation of cumulative adoption held its shape even when a "chi square goodness-of-fit test" showed that "the rate of adoption deviated significantly from a cumulative normal curve . . . in the years 1935 and 1936."[56] "Nevertheless, the overall rate of adoption over time generally approached a normal S-curve,"

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[52] EVERETT M. ROGERS, DIFFUSION OF INNOVATIONS 272 (5th ed. 2003).

[53] *Id*.

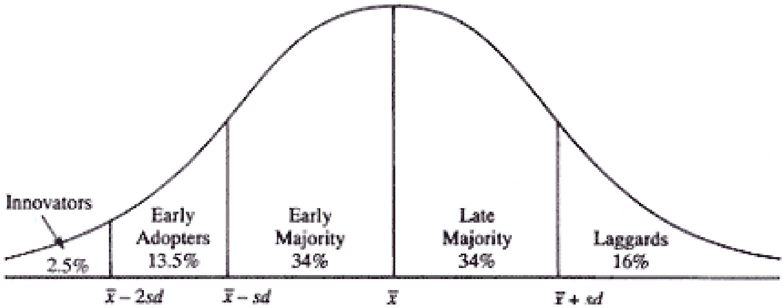
[54] *Id*. at 273 (fig.7-1).

[55] *See* Bryce Ryan & Neal C. Gross, *The Diffusion of Hybrid Seed Corn in Two Iowa Communities*, 8 RURAL SOC. 15, 16-17 (1943).

[56] ROGERS, *supra* note 52, at 274.

as episodic departures from the trend "tend[ed] to cancel one another out over the total diffusion process."[57]

Diffusion of Innovations categorized adopters according to the time at which they took on a new invention:[58]



Using four divisions within the normal Gaussian distribution ($\mu - 2\sigma$, $\mu - \sigma$, μ, and $\mu + \sigma$), Rogers separated adopters into five categories: innovators, early adopters, early majority, late majority, and laggards.[59] As his depiction of the hybrid corn study demonstrated, it is the progression through time of the bell-shaped distribution of adopters that generates the sigmoid cumulative distribution.

Frank Bass developed a more sophisticated variant of Rogers's model of diffusion.[60] For each product, Bass identified not one but two drivers of adoption: *innovation* attributable to external influence by mass media and *imitation* attributable to interpersonal communications.[61] For our purposes, the practical difference between the models is the greater flexibility of Bass's approach. Whereas "the Rogers classification . . . assumes the percentage of adopters for the five categories is invariant across innovations, the Bass classification is innovation specific," and the "percentage of adopters in each . . . categor[y] varies across innovations":[62]

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[57] *Id.* at 274-75.

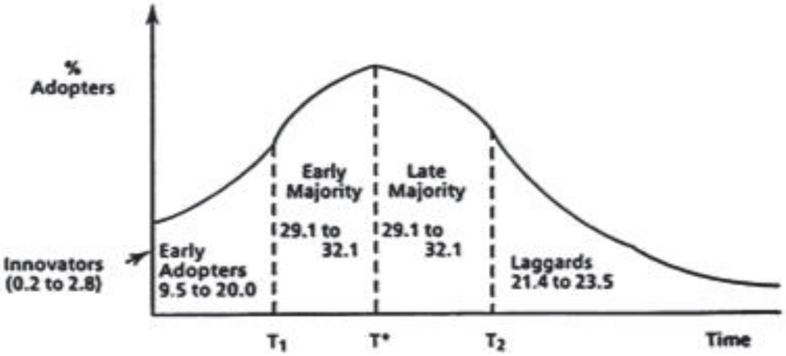
[58] *Id.* at 281 (fig.7-3).

[59] *See id.* at 280-81.

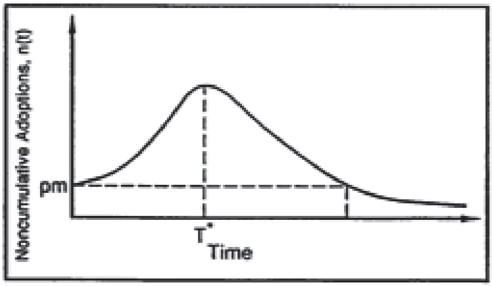
[60] *See* Frank M. Bass, *A New Product Growth for Model Consumer Durables*, 15 MGMT. SCI. 215, 215 (1969).

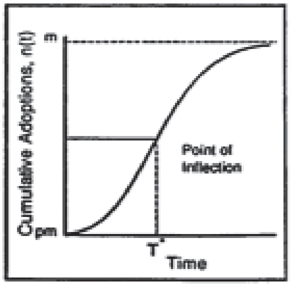
[61] *See* Vijay Mahajan, Eitan Muller & Frank M. Bass, *New Product Diffusion Models in Marketing: A Review and Directions for Research*, 54 J. MARKETING 1, 2 (1990).

[62] Vijay Mahajan, Eitan Muller & Yoram Wind, *New-Product Diffusion Models: From Theory to Practice*, *in* NEW-PRODUCT DIFFUSION MODELS 3, 4 (Vijay Mahajan, Eitan Muller & Yoram Wind eds., 2000).



Frequency curves specifying distinct time series by which adopters take up each new innovation generate cumulative sigmoid functions that likewise differ by innovation, with a point of inflection that does not necessarily occur at precisely the fiftieth percentile of the distribution of adopters:[63]





The logistic distribution used in this Article and favored in much of the biological and economic literature is not materially different from the simpler, purely parametric normal distribution that Everett Rogers used to model the adoption of new technology.[64]

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[63] The images in text illustrating the Bass model of diffusion appear in *id*. at 5, 6; Vijay Mahajan, Eitan Muller & Frank M. Bass, *Diffusion of New Products: Empirical Generalizations and Managerial Uses*, 14 MARKETING SCI. G79, G81, G83 (1995).

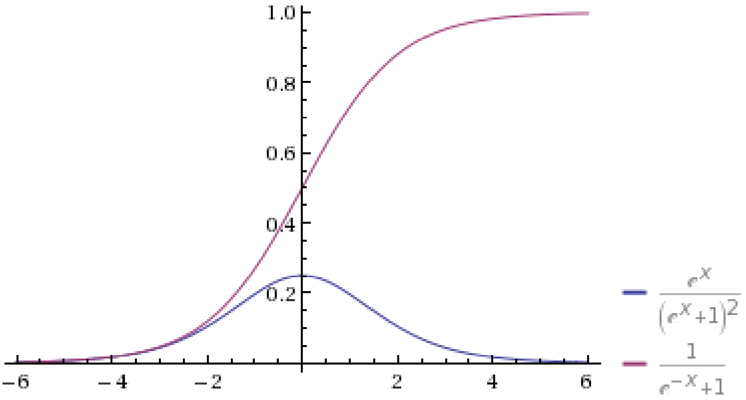
[64] For purely qualitative uses such as this Article, the logistic and Gaussian distributions are essentially equivalent. At a more technical level, these two symmetrical distributions differ in important ways. Whereas the logistic distribution can be specified in closed form with elementary functions, the Gaussian distribution requires use of a special function, the error function. The logistic distribution is slightly more leptokurtic, which makes it a convenient choice (among

Even Frank Bass's more elaborate model of diffusion relies on parametric statistical boundaries within the temporal distribution adopters.[65] If we adopt the simplest available values for the logistic function's three parameters—growth rate $\alpha = 1$, midpoint (or location parameter) $\beta = 0$, and carrying capacity $\kappa = 1$—then the simplest variant of the logistic function specifies the cumulative distribution, and its first derivative describes the distribution of adopters:

$$f(t) = \frac{1}{1+e^{-t}}$$

$$f'(t) = \frac{e^{t}}{\left(1+e^{t}\right)^{2}}$$

Or, in graphic form:[66]



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others) for testing samples expected to have fatter tails and for improving robustness relative to the Gaussian distribution. *Compare* N. BALAKRISHNAN & V.B. NEVZOVOV, A PRIMER ON STATISTICAL DISTRIBUTIONS 197-207 (2003) (logistic distribution) *with id*. at 209-34 (Gaussian distribution).

[65] *See* Vijay Mahajan, Eitan Muller & Rajendra K. Srivastava, *Determination of Adopter Categories by Using Innovation Diffusion Models*, 27 J. MARKETING RES. 37, 41-43 (1990).

[66] I generated the following graphic using Wolfram Alpha using the single command line, *plot e^x/(e^x+1)^2 and 1/(1+e^(-x)) for x=-6 to 6*. *See* WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+e%5Ex%2F%28e%5Ex%2B1%29%5E2+and+1%2F%281%2Be%5E%28-x%29%29+for+x%3D-6+to+6 (last visited Apr. 13, 2015).

Logistic analysis enhances our understanding of technological innovation even where, as here, the model is enlisted for purely qualitative use. I have concededly made no effort to gather economic data, let alone to fit them on a formal, fully specified model.[67] But logistic analysis across a wide range of fields has given rise to useful empirical generalizations, or "pattern[s] or regularit[ies] that repeat[] over different circumstances and that can be described simply by mathematical, graphic, or symbolic methods."[68] As a result, even strictly qualitative interpretation of the logistic model's physically cogent parameters can provide "rare insights and intuitive understanding" of technological evolution and the law's proper response to innovation.[69]

II. LOGISTIC MODELS OF TECHNOLOGICAL SUBSTITUTION AND SUCCESSION

A. The Logistic Substitution Model

Perhaps the most common manifestation of logistic analysis in economics and the social sciences, especially in the evaluation of technological diffusion, is the logistic substitution model. This model, pioneered by Fisher and Pry[70] and by Nakicenovic and Marchetti,[71] extends biological work in which Verhulst, Pearl, Lotka, Volterra, and others evaluated organisms and populations according to logistic models. Indeed, the logistic substitution model provides an

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[67] *Cf*. Dmitry Kucharavy & Roland De Guio, *Application of S-Shaped Curves*, 9 PROCEDIA ENGINEERING 559, 564 (2011) (acknowledging that the majority of uses of sigmoid models involve "pure[ly] qualitative analysis of arbitrary parameters" and that some studies "do not consider any parameters on the vertical scale at all").

[68] Frank M. Bass, *The Future of Research in Marketing: Marketing Science*, 30 J. MARKETING RES. 1, 2 (1993); *see also* Mahajan, Muller & Bass, *supra* note 63, at G79 (observing that the description underlying an empirical generalization "may be approximate rather than exact, and the pattern need not always hold").

[69] Modis, *supra* note 14, at 869.

[70] *See generally* J.C. Fisher & R.H. Pry, *A Simple Substitution Model of Technological Change*, 3 TECHNOLOGICAL FORECASTING & SOC. CHANGE 75 (1971).

[71] *See generally* N. NAKICENOVIC, INT'L INST. FOR APPLIED SYS. ANALYSIS, SOFTWARE PACKAGE FOR THE LOGISTIC SUBSTITUTION MODEL (1979); C. MARCHETTI & N. NAKICENOVIC, INT'L INST. FOR APPLIED SYS. ANALYSIS, THE DYNAMICS OF ENERGY SYSTEMS AND THE LOGISTIC SUBSTITUTION MODEL (1979).

elegant way of illustrating the dynamics that govern competition between individual products or even entire lines of technology.[72]

The logistic substitution model rests upon the following assumptions: First, "[n]ew technologies enter the market and grow at logistic rates." Second, "[o]nly one technology saturates the market at any given time." Third, a saturated technology "follows a non-logistic path that connects the period of growth to its subsequent period of decline." Fourth, "[d]eclining technologies fade away steadily at logistic rates."[73] Specifically, substitution, once begun, "will proceed to completion," and the fractional rate at which new technology replaces "old is proportional to the . . . amount of the old" technology that remains.[74] "The speed with which a substitution takes place is not a simple measure of the pace of techn[ological] advance . . . ."[75] Instead, logistic substitution reflects the imbalances in manufacturing, marketing, and distribution that fuel the eventual displacement of an incumbent technology.[76]

The first and fourth assumptions suggest that growth and decline can both be modeled in logistic terms, simply by substituting a negative for a positive characteristic duration (the quantity represented by Δt). The second and third assumptions treat saturation as a phenomenon influenced or even dictated by the emergence and growth of new technologies. The model's qualitative implications are straightforward: "If a new technology is introduced, its growth must come at the cost (primarily) of the leading technology, causing it to saturate and decline."[77] Mutual rivalry between the technologies is also implicit in the logistic substitution model's reduction of the standard logistic function's three parameters to two. In the logistic substitution model, κ, the carrying capacity, is normalized as 1, or 100% market share.[78] Any gains in market share by the new technology necessarily come at the expense of existing technology.[79]

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[72] *See* Meyer, Yung & Ausubel, *supra* note 25, at 263.

[73] *Id*. at 262; *see also* Fisher & Pry, *supra* note 70, at 75.

[74] *See* Fisher & Pry, *supra* note 70, at 75.

[75] *Id*. at 88.

[76] *See id*.

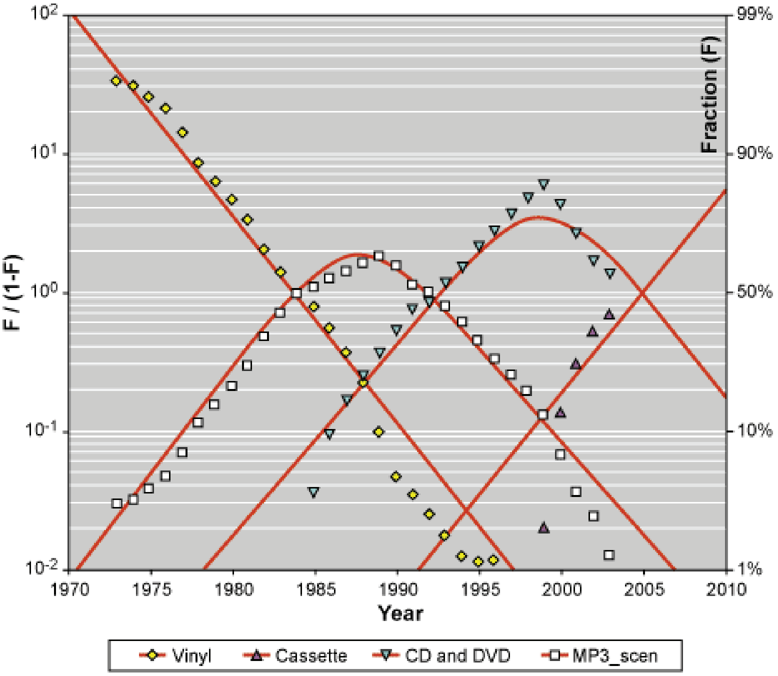
[77] *See* Meyer, Yung & Ausubel, *supra* note 25, at 265.

[78] *See id*.

[79] *See id*.

B. Multi-Product Interactions Across Multiple "Broadcasting" Contexts

The logistic substitution model has described and forecast technological interactions as diverse as recorded music media[80] and natural versus synthetic fibers in clothing.[81] Meyer, Yung, and Ausubel accurately anticipated that compact disks (CDs) would give way to some other medium for recorded music, but mistakenly predicted (as of 1999) that the replacement technology would be digital versatile disks (DVDs).[82] Their logistic substitution model proved correct in forecasting the magnitude and timing of CDs' retreat, but not in identifying the precise technology (MP3 recordings) that would take their place:[83]



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[80] *See id*. at 263-66.

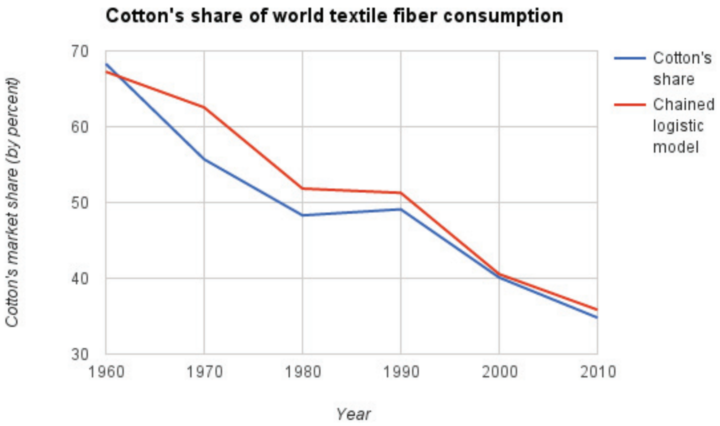
[81] *See* Fisher & Pry, *supra* note 70, at 77-79.

[82] *See* Meyer, Yung & Ausubel, *supra* note 25, at 266.

[83] *See* Kucharavy & De Guio, *supra* note 5, at 409-11 (fig.5) (recording media sales with MP3 data to 2003). The data in Kucharavy and De Guio's figure are plotted semilogarithmically. The so-called Fisher-Pry transform renders logistic data so that it appears linear. *See* Fisher & Pry, *supra* note 70, at 77.

In short, even where the logistic substitution model successfully "predict[s] logistic growth and decline, it is a challenging task [to] nam[e] . . . a new technology []in the long-term."[84]

For its part, Fisher and Pry's assessment of the clothing fiber market in 1973 appears to have captured only the first among multiple stages of partial logistic substitution of synthetic fibers for cotton. Very crude market data by decade from 1960 through 2010 suggest that cotton has lost market share to synthetic fibers in not one but two consecutive cycles of logistic decline:[85]



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[84] Kucharavy & De Guio, *supra* note 5, at 411.

[85] Brian Kantz, *Cotton Versus Synthetics*, COTTON GROWER (Mar. 31, 2010), http://www.cottongrower.com/uncategorized/cotton-versus-synthetics. The chained logistic model of cotton's loss of market share is based on my own calculations. For more detailed evaluation of Fisher and Pry's forecasts regarding cotton and synthetic fibers, see Banks, *supra* note 47, at 35-38.

World textile fiber consumption, 1960-2010
Source: International Cotton Advisory Committee

Year	Kilograms per capita		Total consumption in millions of metric tons		**Cotton's share (in %)**	**Chained logistic model**
	Cotton	Non-cotton	Cotton	Non-cotton		
1960	3.43	1.59	10.36	4.8	**68.3**	**67.3**
1970	3.28	2.61	12.11	9.64	**55.7**	**62.5**
1980	3.23	3.45	14.3	15.29	**48.3**	**51.8**
1990	3.54	3.67	18.6	19.28	**49.1**	**51.3**
2000	3.3	4.92	19.98	29.81	**40.1**	**40.6**
2010	3.29	6.15	22.45	42	**34.8**	**35.8**

$$\hat{f}(t) = m - \frac{\kappa}{1+e^{-\frac{\ln(81)}{\Delta t}(t-\beta)}}$$

$$f(1960) - f(2010) = 33.5\% \text{ (total change in cotton's market share)}$$

$$\frac{f(1960) - f(2010)}{2} = 16.75\% \text{ (half of the total market share change)}$$

$$\frac{f(1960) + f(2010)}{2} = 51.55\% \text{ (midpoint of cotton's market share)}$$

$$\kappa = \frac{f(1960) - f(2010)}{2} = 16.75\%$$

$$\Delta t = 12.5;\ \alpha = \frac{\ln(81)}{12.5} \approx 0.3516 \text{ (characteristic duration; growth rate)}$$

for first logistic decline:

$$m = 51.55\% + \frac{19\kappa}{20} = 67.4625\%$$

$$\beta = 1972.5$$

for second logistic decline:

$$m = 51.55\% + \frac{\kappa}{20} = 52.3875\%$$
$$\beta = 1997.5$$

This chained logistic model is very rudimentary. It draws upon six data points to implement, on a purely parametric basis, two concatenated logistic functions. Its *R*-squared statistic, relative to actual market share data for cotton from 1960 through 2010, is 0.9480.

Notwithstanding the subtleties of logistic forecasting for music and fiber, these markets provide appealing analogies to Aereo's abortive attack on incumbent multichannel video programming distributors (MVPD)[86] and the diffusion of plant-incorporated protectants and herbicide-resistant seeds in agriculture.[87] Aereo's programming platform and Monsanto's genetically modified seeds represent standard instances of competitive substitution. In each instance, innovators threatened to displace incumbents from saturated markets. In the biological idiom of logistic analysis, Aereo and Monsanto are predators. Their rivals, from incumbent

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[86] *See* Am. Broad. Cos. v. Aereo, Inc., 134 S. Ct. 2498, 2503-04 (2014); Bruce E. Boyden, Aereo *and the Problem of Machine Volition*, 2015 MICH. ST. L. REV. 485; Annemarie Bridy, *Aereo: From Working Around Copyright to Thinking Inside the (Cable) Box*, 2015 MICH. ST. L. REV. 465; Jennifer Carter-Johnson, *Defining Limites to the Application of the Statutory Experimental Use Exception Within the Agricultural Biotechnology Industry*, 2015 MICH. ST. L. REV. 509. The Communications Act defines an MVPD as "a person such as, but not limited to, a cable operator, a multichannel multipoint distribution service, a direct broadcast satellite service, or a television receive-only satellite program distributor, who makes available for purchase, by subscribers or customers, multiple channels of video programming." 47 U.S.C. § 522(13) (2012). "MVPDs include, but are not limited to, cable systems, direct broadcast satellite ('DBS') systems, and other entities that sell multiple channels of video programming to consumers." Annual Assessment of the Status of Competition in the Mkt. for the Delivery of Video Programming, 26 FCC Rcd. 14,091, 14,092 n.5 (2011).

[87] *See* Bowman v. Monsanto Co., 133 S. Ct. 1761, 1764 (2013); Yaniv Heled, *Regulatory Competitive Shelters as Incentives for Innovation in Agrobiotech*, 2015 MICH. ST. L. REV. 553; Daryl Lim, *Living with Monsanto*, 2015 MICH. ST. L. REV. 559; J. Janewa Osei-Tutu, *Agricultural Biotechnology: Drawing on International Law to Promote Progress*, 2015 MICH. ST. L. REV. 531.

broadcasters to suppliers of nonmodified soybean and cotton seeds, are prey.[88]

The prey–predator relationship between old and new technology represents merely one of four, six, or nine distinct ways in which products can compete with each other.[89] Logically, interaction between two products that may have positive, zero, or negative effects on either the incumbent product or the entrant can fall into nine categories, as illustrated in the following 3×3 matrix:

Vertical scale: effect of new product on the existing product	Horizontal scale: effect of the existing product on the new product		
	+ Positive	0 Zero	– Negative
+ Positive	*Complementary products*	*Facilitating products*	*Predator–prey product relationship*
0 Zero	*Auxiliary products*	*Independent products*	*New product failure*
– Negative	***Prey–predator product relationship***	*Technological product substitution*	*Product substitutes-in-use*

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[88] The possible application of the Hatch–Waxman Act's exemption for experimental use to agricultural biotechnology would treat follow-on innovators exploiting the experimental use exception as parasites. *See* 35 U.S.C. § 271(e)(1) (2012). A useful contrast lies in the Federal Insecticide, Fungicide, and Rodenticide Act, which withholds safety data submitted by the proponent of a pesticide from would-be follow-on inventors for ten years after approval of the pesticide. *See* 7 U.S.C. § 136a(c)(1)(F)(i) (2012); Heled, *supra* note 87 (manuscript at 2).

[89] *See* Barry L. Bayus, Namwoon Kim & Allan D. Shocker, *Growth Models for Multiproduct Interactions: Current Status and New Directions*, *in* New-Product Diffusion Models, *supra* note 62, at 141, 153-55. The first matrix in the text is derived directly from this source. *See id*. at 155. The second matrix is merely a simplification of the grid provided by Bayus, Kim, and Shocker. On the mathematics of interactions between two products or two biological populations, see generally Felix Albrecht et al., *The Dynamics of Two Interacting Populations*, 46 J. Mathematical Analysis & Applications 658 (1974).

We can simplify further by emphasizing only the corners of the table and ignoring multi-product interactions having zero effect on either an incumbent or an entrant:

Vertical scale: effect of new product on the existing product	Horizontal scale: effect of the existing product on the new product	
	+ Positive	– Negative
+ Positive	*Complementary products*	*Predator–prey relationship*
– Negative	***Prey–predator relationship***	*Product substitutes-in-use*

A distinct classification of competitive relationships emphasizes combinations (rather than permutations) of "coupling parameters" between two competitors, *A* and *B*, based on each competitor's potential impact on the other's growth rate:[90]

Mode	Definition	Coupling parameter	
		A	B
Pure competition	Each species suffers from the other's existence.	–	–
Predator–prey	One species serves as food for the other.	+	–
Mutualism	Symbiosis: A win-win situation.	+	+
Commensualism	A parasitic relationship in which one species benefits, but the other remains unaffected.	+	0
Amensualism	One species suffers from the existence of the other, which remains impervious to the loss.	–	0
Neutralism	No interaction between species.	0	0

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[90] *See* Theodore Modis, *A Scientific Approach to Managing Competition*, 9 INDUS. PHYSICIST 22, 22 (2003).

As is evident from any of these classifications of multi-product interactions—either the full 3×3 or the condensed 2×2 matrix, or the six combinations of positive, negative, and neutral coupling parameters—the controversies involving Aereo and agricultural biotechnology implicate only the most dramatic of multi-product interactions, the prey–predator relationship in which the introduction of a new product benefits the entrant at the expense of the incumbent. In the industries that straddle both senses of the word *broadcasting*, which evolved during the twentieth century from a term describing an agricultural technique into a term designating a mass communications medium,[91] innovation has simultaneously delivered gains to new technology and ruin to the old.

But no more than the law can bear. "To the economic victor belong only those spoils that may be [lawfully] obtained."[92] Before *Aereo*, conventional broadcast television had already endured decades of pitched legal battles against cable television.[93] Broadcast television has survived the emergence of cable and direct broadcast satellite, due in no small part to the federal government's repeated efforts, from *Southwestern Cable*[94] through the *Turner Broadcasting* decisions,[95] to shelter conventional broadcasters from unrestrained competition by multichannel distributors.[96] A Supreme Court that had given such solicitude to must-carry and retransmission consent regimes unsurprisingly rejected Aereo's interpretation of the Copyright Act. Despite the federal government's efforts to preserve the "free" broadcast model against competition by the MVPD market, broadcast television has undoubtedly entered the downward

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[91] *Etymology Online* traces the use of "broadcast," as an adjective describing "the spreading of seed," to 1767. ONLINE ETYMOLOGY DICTIONARY, http://www.etymonline.com/index.php?term=broadcast (last visited Apr. 13, 2015). The word's "[f]igurative use is recorded from 1785." *Id*. "Modern media use began with radio (1922, adjective and noun)." *Id*. The use of *broadcast* "[a]s a verb, [is] recorded from 1813 in an agricultural sense, 1829 in a figurative sense, 1921 in reference to radio." *Id*.

[92] *Cf*. Rutan v. Republican Party of Ill., 497 U.S. 62, 64 (1990) ("To the victor belong only those spoils that may be constitutionally obtained.").

[93] *See, e.g*., DANIEL L. BRENNER & MONROE E. PRICE, CABLE TELEVISION AND OTHER NONBROADCAST VIDEO: LAW AND POLICY § 1.02 (1986); Jim Chen, *The Last Picture Show (On the Twilight of Federal Mass Communications Regulation)*, 80 MINN. L. REV. 1415, 1459-72 (1996).

[94] United States v. Sw. Cable Co., 392 U.S. 157 (1968).

[95] Turner Broad. Sys., Inc. v. FCC, 512 U.S. 622 (1994); Turner Broad. Sys., Inc. v. FCC, 520 U.S. 180 (1997).

[96] *See Sw. Cable Co*., 392 U.S. at 159-60, 181; *Turner Broad. Sys*., 512 U.S. at 667-68; *Turner Broad. Sys*., 520 U.S. at 224-25.

sloping phase of the logistic substitution model.[97] The extent of that medium's decline and its viability in certain niches (such as network news) remain sources of controversy.[98]

C. Peak Glyphosate

The market for broad-spectrum herbicides (and genetically modified crops that resist them) has likewise witnessed multiple cycles of technological rise, decline, and displacement. "Atrazine yesterday, glyphosate today, glufosinate tomorrow."[99] Agricultural biotechnology does differ from video-programming delivery platforms in a crucial way. Genetic engineering of widely cultivated crop plants eventually reaches a biological limit on the deployment of technology, as target organisms (whether insect pests[100] or weeds[101]) develop resistance under severe selective pressure. More species in an increasing number of locations will evolve their own defenses against any agricultural technology. The proliferation of resistance across biological taxa and geographic space imposes a

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[97] *See* Jim Chen, *From* Red Lion *to Red List: The Dominance and Decline of the Broadcast Medium*, 60 ADMIN. L. REV. 793 (2008).

[98] *Compare* Thomas E. Patterson, *Young People Flee from the News, Whatever the Source*, 38 TELEVISION Q. 32, 33 (2008) (identifying television news as the lone "marginally brighter" element of a bleak diagnosis of younger Americans' disengagement from mass media), *with* Lynn Vavreck, *Why Network News Still Matters*, N.Y. TIMES (Feb. 18, 2015), http://www.nytimes.com/2015/02/19/upshot/why-network-news-still-matters.html (acknowledging the decline in network news programs' share of young viewers despite extolling the viability of such broadcasts).

[99] James Ming Chen, *An Agricultural Law Jeremiad: The Harvest Is Past, the Summer Is Ended, and Seed Is Not Saved*, 2014 WIS. L. REV. 235, 263.

[100] *See, e.g.*, EPA Proposal to Improve Corn Rootworm Resistance Management, 80 Fed. Reg. 4564 (Jan. 28, 2015); NOW OR NEVER: SERIOUS NEW PLANS TO SAVE A NATURAL PEST CONTROL 1 (Margaret Mellon & Jane Rissler eds., 1998) (describing the emergence of entomological resistance to *Bacillus thuringiensis* after the incorporation of *Bt* into genetically modified crop plants); Carlos A. Blanco et al., *An Empirical Test of the F_2 Screen for Detection of* Bacillus thuringiensis-*Resistance Alleles in Tobacco Budworm (Lepidoptera: Noctuidae)*, 101 J. ECON. ENTOMOLOGY 1406, 1406 (2008); Jennifer L. Price, Jeffrey Hyde & Dennis D. Calvin, *Insect Resistance Management for Bt Corn: An Assessment of Community Refuge Schemes*, 9 AGBIOFORUM 129, 129 (2006).

[101] *See, e.g.*, Grace A. Hite et al., *Differential Response of a Virginia Common Lambsquarters (*Chenopodium Album*) Collection to Glyphosate*, 56 WEED SCI. 203, 203 (2008); Micheal D.K. Owen & Ian A. Zelaya, *Herbicide-Resistant Crops and Weed Resistance to Herbicides*, 61 PEST MGMT. SCI. 301, 301 (2005).

ceiling on the usefulness of any plant-incorporated protectant or herbicide-resistance trait that is embedded within crop seeds.

Moreover, technologies differ within agriculture itself. Intervention in plant genetics does not necessarily start a biological countdown to commercial extinction. Hybrid corn, whose uptake during the 1930s inspired one of the most influential lines of research into the innovative process and the diffusion of inventions,[102] represented the vanguard of an agronomic technique that produced "the predominant form of cultivar in many crops."[103] But other forms of agricultural technology have a distinct half-life, as it were. Forms of agricultural biotechnology that exert evolutionary pressure on predators, parasites, or competitors should be regarded as depletable rather than renewable resources. Chief among these technologies are plant-incorporated protectants and herbicide-resistant crop varieties. To the extent that medical technologies such as antibiotics face similar evolutionary limits on their effectiveness,[104] apart from economic pressure from rival innovations, those forms of biotechnology should also be evaluated with a similar sensitivity to declines in market share. The mathematical modeling of the diffusion of biological technologies should account not only for the usual problems of logistic substitution, but also for such technologies' vulnerability to evolutionary pressure.

The leading mathematical model of the rise and fall of depletable resources is the Hubbert curve. Geologist M. King

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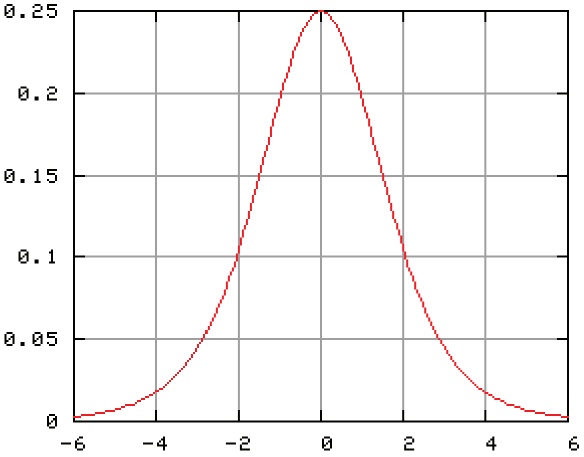
[102] *See* Ryan & Gross, *supra* note 55, at 15; Zvi Griliches, *Hybrid Corn: An Exploration in the Economics of Technological Change*, 25 ECONOMETRICA 501, 501-02 (1957); Zvi Griliches, *Research Costs and Social Returns: Hybrid Corn and Related Innovations*, 66 J. POL. ECON. 419, 419 (1958); Zvi Griliches, *Hybrid Corn and the Economics of Innovation*, 132 SCIENCE 275, 275 (1960); Zvi Griliches, *Hybrid Corn Revisited: A Reply*, 48 ECONOMETRICA 1463, 1464 (1980); *cf.* Zvi Griliches, *Research Expenditures, Education, and the Aggregate Agricultural Production Function*, 54 AM. ECON. REV. 961, 961 (1964); *see also* ROGERS, *supra* note 52, at 54-60 (reviewing the rise and fall of the rural sociology tradition within diffusion literature).

[103] Arnel R. Hallauer, *Breeding Hybrids*, *in* ENCYCLOPEDIA OF PLANT AND CROP SCIENCE 186, 186 (Robert M. Goodman ed., 2004).

[104] *See, e.g.*, Cesar A. Arias & Barbara E. Murray, *Antibiotic-Resistant Bugs in the 21st Century—A Clinical Super-Challenge*, 360 NEW ENG. J. MED. 439, 439 (2009); Peter M. Hawkey & Annie M. Jones, *The Changing Epidemiology of Resistance*, 64 J. ANTIMICROBIAL CHEMOTHERAPY i3, i3 (Supp. 2009); Ernest J. Soulsby, *Resistance to Antimicrobials in Humans and Animals*, 331 BRIT. MED. J. 1219, 1219 (2005); *cf., e.g.*, Frank M. Aarestrup et al., *Changes in the Use of Antimicrobials and the Effects on Productivity of Swine Farms in Denmark*, 71 AM. J. VETERINARY RES. 726, 726 (2010).

Hubbert predicted in 1956 that peak production of petroleum would signal its eventual exhaustion:[105]



The Hubbert curve is simply the probability distribution function of the logistic distribution. That function is the first derivative of the basic logistic function (which in turn serves as the cumulative distribution function of the logistic distribution).[106] The formal specification of the Hubbert curve reveals its relationship to the logistic function:

$$h(t) = \frac{e^t}{\left(1+e^t\right)^2} = \frac{d}{\left(1+e^{-t}\right)dt}$$

$$\text{i.e., } h(t) = f'(t), \text{ where } f(t) = \frac{1}{1+e^{-t}}$$

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[105] *See* M. KING HUBBERT, NUCLEAR ENERGY AND THE FOSSIL FUELS 22 (1956), *available at* http://www.hubbertpeak.com/hubbert/1956/1956.pdf; Adam R. Brandt, *Testing Hubbert*, 35 ENERGY POL'Y 3074, 3074-75 (2007). The image in text is derived from *Hubbert Curve*, WIKIPEDIA, http://en.wikipedia.org/wiki/Hubbert_curve (last modified Nov. 25, 2014, 3:35 p.m.).

[106] *See Hubbert Curve*, *supra* note 106; *Logistic Distribution*, WIKIPEDIA, http://en.wikipedia.org/wiki/Logistic_distribution (last modified Mar. 4, 2015, 9:35 p.m.). *See generally* Jean Laherrère & Paul Deheuvels, *Distributions de Type "Fractal Parabolique" dans la Nature*, 322 COMPTES RENDUS DE L'ACADÉMIE DES SCIENCES 535 (1996) (reviewing the mathematical relationships between the logistic function, the Hubbert curve, the normal distribution, and parabolic fractals).

Recognizing the Hubbert peak, or the maximum value of the Hubbert curve, holds the key to predicting when a depletable resource, or a theoretically renewable resource harvested so aggressively as to be depletable, will be exhausted. The Hubbert curve reaches its peak where its first derivative equals zero. One formal specification defines the year of peak production according to the parameters of the Hubbert curve. Cumulative production, *Q(t)*, may be defined as a logistic function:

$$Q(t) = \frac{Q_{\max}}{1 + ae^{-bt}}$$

where $Q_{\max}$ defines the total available amount of a depletable resource (such as petroleum) and *a* and *b* are empirically determined constants.[107] The year of peak production, $t_{\max}$, is predicted according to this formula:[108]

$$t_{\max} = \frac{\ln a}{b}$$

Analysis along these lines predicts that global supplies of phosphorus, a critical ingredient in fertilizer, will peak in 2030 and will be exhausted within 50 to 100 years of the present.[109] Inasmuch as phosphorus is one of three macronutrients in plant fertilizers (along with nitrogen and potassium),[110] the Supreme Court case that anticipates "peak phosphorus" is *Funk Bros. Seed Co. v. Kalo Inoculant Co.*,[111] just as *Bowman v. Monsanto Co.*[112] presages "peak glyphosate." The Malthusian specter of global famine may yet return

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[107] *See* Alfred J. Cavallo, *Hubbert's Petroleum Production Model: An Evaluation and Implications for World Oil Production Forecasts*, 13 NAT. RESOURCES RES. 211, 212 (2004).

[108] *See id.*

[109] *See* Stuart White & Dana Cordell, *Peak Phosphorus: The Sequel to Peak Oil*, SUSTAINABLE PHOSPHORUS FUTURES, http://phosphorusfutures.net/peak-phosphorus.html (last updated Apr. 25, 2010, 10:11 p.m.); *see also* Dana Cordell, Jan-Olof Drangert & Stuart White, *The Story of Phosphorus: Global Food Security and Food for Thought*, 19 GLOBAL ENVTL. CHANGE 292, 292 (2009); Tina-Simone S. Neset & Dana Cordell, *Global Phosphorus Scarcity: Identifying Synergies for a Sustainable Future*, 92 J. SCI. FOOD & AGRIC. 2, 3 (2012).

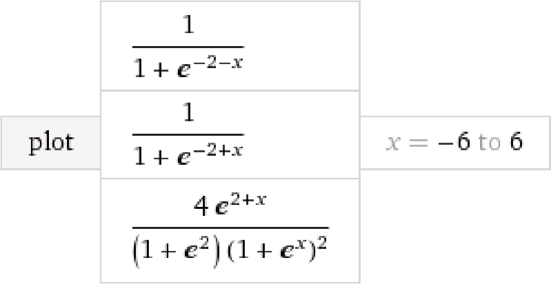
[110] *See* Heinrich Dittmar et al., *Fertilizers, 2. Types*, *in* ULLMAN'S ENCYCLOPEDIA OF INDUSTRIAL CHEMISTRY 200 (7th ed. 2011).

[111] 333 U.S. 127 (1948).

[112] 133 S. Ct. 1761 (2013).

in the guise of agricultural asymptotes imposed by absolute limits on exhaustible resources and terrestrial carrying capacity.

It may be possible to model technological successions in agriculture according to either the Hubbert curve or the standard logistic substitution model. The Hubbert curve suggests that peak deployment of a plant-incorporated protectant or an herbicide-resistant crop variety is a function of evolved resistance in target organisms. The logistic substitution model, by contrast, implies that the logistic decline in market share is a function of economic competition between consecutive generations of agricultural technology. In practice, the economic standing of agricultural technologies reflects not only the impact of innovation and adoption, but also any environmental constraints on the effectiveness of those technologies. If logistic growth and decline occur in close succession, then market flow predicted by the logistic substitution model will closely resemble a forecast conducted according to a Hubbert curve:[113]



plot

$$\frac{1}{1+e^{-2-x}}$$

$$\frac{1}{1+e^{-2+x}}$$

$$\frac{4\,e^{2+x}}{\left(1+e^{2}\right)\left(1+e^{x}\right)^{2}}$$

$x = -6$ to 6

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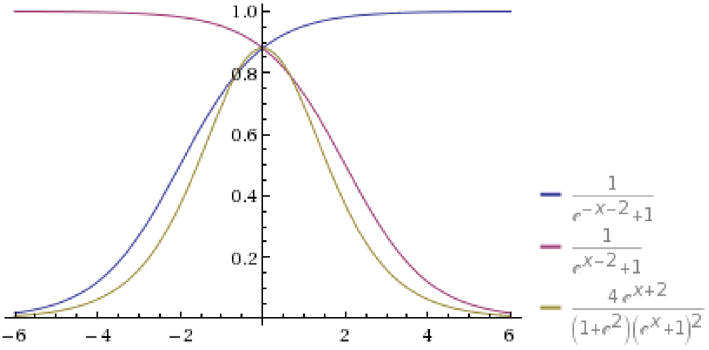
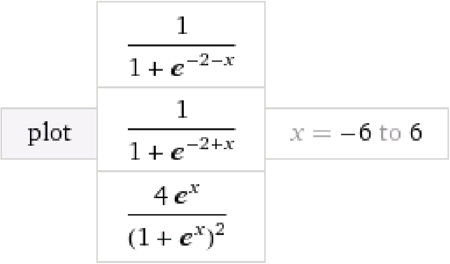
[113] I generated the following graphic through Wolfram Alpha with the single command line, _plot 1/(1+e^(-x-2)) and 1/(1+e^(x-2)) and 4*e^2/(e^2+1)*e^x/(1+e^x)^2 for x=-6 to 6_. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+1%2F%281%2Be%5E%28-x-2%29%29+and+1%2F%281%2Be%5E%28x-2%29%29+and+4*e%5E2%2F%28e%5E2%2B1%29*e%5Ex%2F%281%2Be%5Ex%29%5E2+for+x%3D-6+to+6 (last visited Apr. 13, 2015). The blue curve indicates the first half or growth phase of the logistic growth model. The red curve indicates the second half or decline phase of that model. The gold curve indicates a Hubbert curve. The logistic growth model here is indicated by the simplest available parameters: growth rate $\alpha = \pm 1$; total market share or carrying capacity $\kappa = 1$. The location parameter β, which indicates the midpoint of the growth and decline phase, is set to ±2. For its part, the Hubbert curve, which ordinarily reaches its maximum value at $h(0) = 1/4$, has been magnified by a factor of $4e^2/(e^2+1)$, which is approximately 3.5232, so that the Hubbert peak intersects with both of the logistic curves.



Scaling the Hubbert curve so that it peaks at $h(0) = 1$ improves the fit between the two models even more:[114]



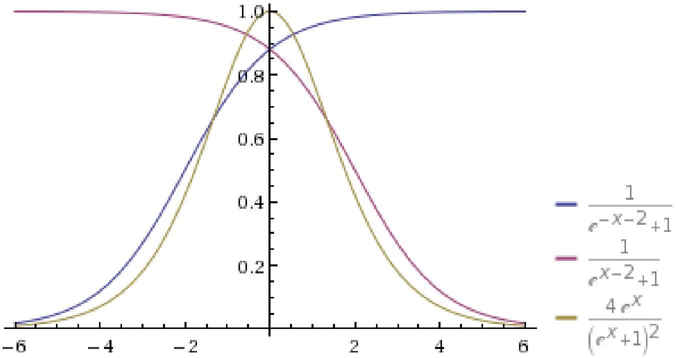
plot

$$\frac{1}{1+e^{-2-x}}$$

$$\frac{1}{1+e^{-2+x}}$$

$$\frac{4e^{x}}{(1+e^{x})^2}$$

$x = -6$ to 6



The exact scaling of these two models, however, is not central to the argument. The point, rather, is that the two models and the

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[114] I generated the following graphic with Wolfram Alpha using the single command line, *plot 1/(1+e^(-x-2)) and 1/(1+e^(x-2)) and 4*e^x/(1+e^x)^2 for x=-6 to 6*. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+1%2F%281%2Be%5E%28-x-2%29%29+and+1%2F%281%2Be%5E%28x-2%29%29+and+4*e%5Ex%2F%281%2Be%5Ex%29%5E2+for+x%3D-6+to+6 (last visited Apr. 13, 2015).

legal narratives they represent are hard to distinguish from one another. The close resemblance between the Hubbert curve and rapid logistic substitution suggests that makers of innovation policy (and, for that matter, environmental regulators) may have trouble discerning whether it is economic competition or evolutionary pressure that has put an agricultural or biomedical technology into eclipse.

D. Inflecting Innovation Policy

Whether technological succession proceeds according to the logistic substitution model or the Hubbert curve, the first step in legally meaningful evaluation of these models consists of determining the point in time on which the technological model pivots. The Hubbert peak marks that moment in the Hubbert curve. In the logistic substitution model, the parameter specified as β or t_m, either of which designates the inflection point of the logistic function, identifies the key moment. In the standard model of logistic growth, β not only indicates the point at which growth reaches half of a system's carrying capacity—specifically, $P(\beta) = \kappa/2$—but also the moment at which the initial spurt of seemingly exponential growth begins to be overtaken by negative feedback that eventually imposes an asymptotic limit on logistic growth. In more sophisticated extensions of sigmoid modeling, logistic-style growth may not reach its midpoint at the exact center of the time series; that benchmark may be reached either before or after "halftime."[115] In that event, the real goal is to find inflection point β, even where $\beta \neq t_m$. The real objective is to ascertain the moment at which initial growth begins to give way to negative feedback and the eventual triumph of resource constraints can be foreseen.

The close mathematical relationship between the logistic substitution model and the Hubbert curve enables us to use the identical mathematical technique to identify the pivotal moment in either model. The Hubbert peak occurs where the first derivative of the Hubbert curve equals zero and the Hubbert curve itself reaches its maximum. Inflection point β of a standard logistic growth function is determined according to the *second* derivative. When the second derivative of a logistic growth function reaches zero, acceleration of the growth rate yields to deceleration. Because the Hubbert curve is the probability density function of a logistic

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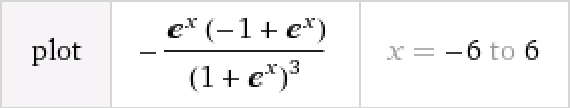
[115] *See* Islam & Sharif, *supra* note 47, at 247-48.

distribution and the first derivative of a logistic function, the second derivative of a logistic growth function is equivalent to the first derivative of the Hubbert curve:

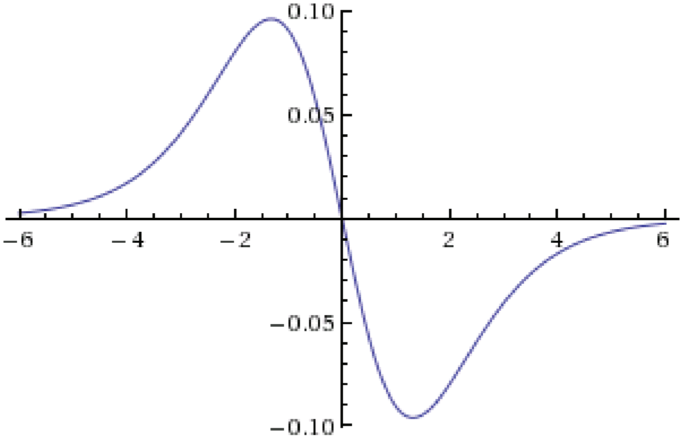
$$h(t) = f'(t),\ \text{where}\ f(t) = \frac{1}{1+e^{-t}}$$

$$\therefore h'(t) = f''(t) = \frac{e^{t}(e^{t}-1)}{(e^{t}+1)^{3}}$$

Or, in graphic terms:[116]



plot	$-\dfrac{e^{x}(-1+e^{x})}{(1+e^{x})^{3}}$	$x = -6$ to 6



The plot of *f''(t)* makes it clear that *f''(0)* = 0 and that *f''(t)* is an odd function and is rotationally symmetrical about the origin:

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[116] I plotted the following image through Wolfram Alpha with the single command line, *second derivative of 1/(1+e^(-x)) for -6<x<6*. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=second+derivative+of+1%2F%281%2Be%5E%28-x%29%29+for+-6%3Cx%3C6 (last visited Apr. 13, 2015). In one step, Wolfram Alpha computed the second derivative of the basic logistic function and plotted it for the range $-6 < x < 6$.

$$f''(-t) = -f''(t)$$

$$t < 0, f''(t) > 0$$

$$t = 0, f''(t) = 0$$

$$f > 0, f''(t) < 0$$

Knowledge of the location parameter, or inflection point *β*, of a logistic growth curve or of the Hubbert peak immediately enables the computation of the growth rate and characteristic duration of either of these functions. Recall that many specifications of the logistic growth function in economic evaluations of technology replace growth rate *α* with characteristic duration *Δt*, where *Δt* indicates the time needed for a logistic model to grow from one-tenth to nine-tenths of its maximum value, or from 0.1κ to 0.9κ. In that instance, $\Delta t = \ln(81)/\alpha$. Since the logistic substitution model normalizes carrying capacity *κ* at 1 as an expression of the total market share available to competing technologies, the only parameters necessary to the specification of that model are inflection point *β* and characteristic duration *Δt*.

In turn, knowledge of characteristic duration *Δt* provides legally significant insight into the commercial lifespan of a technology. Within this symposium, Yaniv Heled has taken pains to demonstrate that patents are not the only legal tool for conferring economic incentives to innovate.[117] For various forms of agricultural biotechnology, regulatory competitive shelters augment or replace patents. However the law elects to incubate innovation, whether by patent or by regulatory competitive shelter, a key question is the appropriate duration.[118] If the term of legal protection is too short, prospective inventors may not realize enough of an incentive to develop new technology. If the term of protection is too long, inventors will suppress supplies and gouge consumers, and delays in the transition of patented or otherwise protected technology into the

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[117] *See* Heled, *supra* note 87 (manuscript at 1); *see also* Yaniv Heled, *Introducing Regulatory Competitive Shelters, the New Patents*, 76 OHIO ST. L.J. (forthcoming 2015), *available at* http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2598129.

[118] *See, e.g.*, GRAHAM DUTFIELD, INTELLECTUAL PROPERTY RIGHTS AND THE LIFE SCIENCES INDUSTRIES: PAST, PRESENT, AND FUTURE 218-20 (2d ed. 2009); Rebecca S. Eisenberg, *Why the Gene Patenting Controversy Persists*, 77 ACAD. MED. 1381, 1381 (2002); Michael A. Heller & Rebecca S. Eisenberg, *Can Patents Deter Innovation? The Anticommons in Biomedical Research*, 280 SCIENCE 698, 698-99 (1998).

public domain will retard future innovation.[119] Dan Burk and Mark Lemley have hinted that patent law should be technology-specific.[120] Logistic modeling of technological transitions may shed light on questions of timing in the law of innovation, especially the expected economic lifespan of any single invention.

III. LOGISTIC ANALYSIS BEYOND TWO-PRODUCT SUBSTITUTION

The qualitative and quantitative application of logistic analysis to legal subjects is nearly boundless. "In the real world there are many wiggles, speedups, and setbacks, new S-curves growing out of old, separate curves for different sectors and regions of a national economy . . . ."[121] And even though "[m]ost innovations [do] have an S-shaped rate of adoption . . . the slope of the 'S' [varies] from innovation to innovation."[122] For those "ideas [that] diffuse relatively rapidly . . . the S-curve is quite steep. Other innovations have a slower rate of adoption, and the S-curve is more gradual, with a slope that is relatively lazy."[123] In all instances, the sigmoid function of diffusion "is innovation-specific and system-specific, describing the diffusion of a particular new idea" within a "specific system" or market.[124] "[O]nly [instances] of successful innovation, in which an

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[119] *See* Lim, *supra* note 87 (manuscript at 19-26) (evaluating the legal and economic effects of the expiration of the first wave of Monsanto's biotechnology patents, especially on its Roundup Ready herbicide-resistant seeds). Of particular concern is the prospect that patentees may try to use licensing restrictions to reclaim rights otherwise extinguished by exhaustion of their patents. *Compare* Fed. Trade Comm'n v. Actavis, Inc., 133 S. Ct. 2223, 2231 (2013) (applying antitrust scrutiny to reverse payments between patent-holding drug developers and their generic competitors even if such payments fell within the scope of the drug patents), *with* Lim, *supra* note 87 (manuscript at 67-72) (outlining the application of *Actavis* to potential anticompetitive behavior by Monsanto after the expiration of patents on the first generation of its Roundup Ready technologies).

[120] *See* Dan L. Burk & Mark A. Lemley, *Inherency*, 47 WM. & MARY L. REV. 371, 372 (2005); Dan L. Burk & Mark A. Lemley, *Is Patent Law Technology-Specific?*, 17 BERKELEY TECH. L.J. 1155, 1156 (2002).

[121] CHARLES P. KINDLEBERGER, WORLD ECONOMIC PRIMACY: 1500 TO 1990, at 16 (1996); *accord* Meyer, Yung & Ausubel, *supra* note 25, at 247.

[122] ROGERS, *supra* note 52, at 23.

[123] *Id.*

[124] *Id.* at 275; *see also id.* at 11 ("[*D*]*iffusion* [i]s the process by which (1) an *innovation* (2) is *communicated* through certain *channels* (3) *over time* (4) among the members of a *social system*.").

innovation spreads to almost all of the potential adopters in a social system," generate an S-curve.[125]

The limitations of the simple logistic function (especially its rotational symmetry about the midpoint t_m, where $P(\beta) = \kappa/2$) are easily overcome; the cumulative distribution function of the generalized Weibull distribution can generate a sigmoid model that is either symmetrical or asymmetrical, and one whose inflection point can reflect greater growth in the earlier or later phases of development.[126] The fractal nature of logistic functions enables models of this sort to describe exponential growth,[127] sinusoid cycling in the tradition of the Lotka–Volterra equations,[128] or even stochastic chaos.[129] The generalization of the logistic function aspires to treat sigmoid "growth, chaos, self-organization, [and] complex adaptive systems . . . as special cases" of logistic analysis.[130]

A particularly ambitious variation on this theme, "loglet" analysis, decomposes logistic functions by analogy to wavelet theory as an extension of Fourier analysis.[131] ("Loglet" is a portmanteau word that combines "logistic" with "wavelet."[132]) Distinct aspects of a single industry, such as natural gas production and consumption in

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[125] *Id.* at 275; *see also id.* (acknowledging that unsuccessful innovations that are "adopted by only a few people . . . [are] ultimately . . . rejected, so that [their] rate[s] of adoption level[] off and, through discontinuance," plummet).

[126] *See* Sharif & Islam, *supra* note 47, at 247-48. *See generally* Waloddi Weibull, *A Statistical Distribution Function of Wide Applicability*, 18 J. APPLIED MECHANICS 293 (1951).

[127] *See* Perrin Meyer, *Bi-Logistic Growth*, 47 TECHNOLOGICAL FORECASTING & SOC. CHANGE 89, 90 (1994).

[128] *See* Modis, *supra* note 14, at 872; Carl W.I. Pistorius & James M. Utterback, *The Death Knells of Mature Technologies*, 50 TECHNOLOGICAL FORECASTING & SOC. CHANGE 215, 226-31 (1995) (identifying conditions under which predator–prey relationships in ecology and economics result in cyclical oscillations); C.W.I. Pistorius & J.M. Utterback, *Multi-Mode Interaction Among Technologies*, 26 RES. POL'Y 67, 68 (1997) (same).

[129] *See* Theodore Modis & Alain DeBecker, *Chaoslike States Can Be Expected Before and After Logistic Growth*, 41 TECHNOLOGICAL FORECASTING & SOC. CHANGE 111, 111-12 (1992).

[130] *See* Theodore Modis, *Genetic Re-Engineering of Corporations*, 56 TECHNOLOGICAL FORECASTING & SOC. CHANGE 107, 107 (1997). Fourier analysis provides a mathematical method for calculating the frequencies and amplitudes of a signal comprising a sound or an image. Traditional Fourier methods are less successful in reconstructing highly noisy signals. Wavelet analysis focuses on the local aspects of a signal and provides algorithms for filtering signals from noise. *See generally* James S. Walker, *Fourier Analysis and Wavelet Analysis*, 44 NOTICES AM. MATH. SOC'Y 658 (1997).

[131] *See* Meyer, Yung & Ausubel, *supra* note 25, at 248.

[132] *Id.*

Brazil, lend themselves to exponential, Hubbert, von Bertalanffy, and simple logistic models.[133]

A. Classifying Network Neutrality as a Problem of Innovation Policy

The expansion of logistic analysis to more ambitious goals demands a broader legal and economic canvas.[134] Product substitution situations such as those presented by *Aereo* and *Monsanto* are among the simpler legal issues that sigmoid modeling and regression can inform. The prey–predator relationship in those scenarios represents one of four, six, or nine types of two-product interaction within the economics of innovation. This symposium's extended discussion of network neutrality[135] provides some hints on the possible expansion of logistic analysis in law beyond basic models of multi-product interaction.

At an absolute minimum, the economic impact of net neutrality may be evaluated according to the assumption that innovation results from the endogenous diffusion of successive generations of a single product.[136] Perhaps broadband infrastructure should be evaluated

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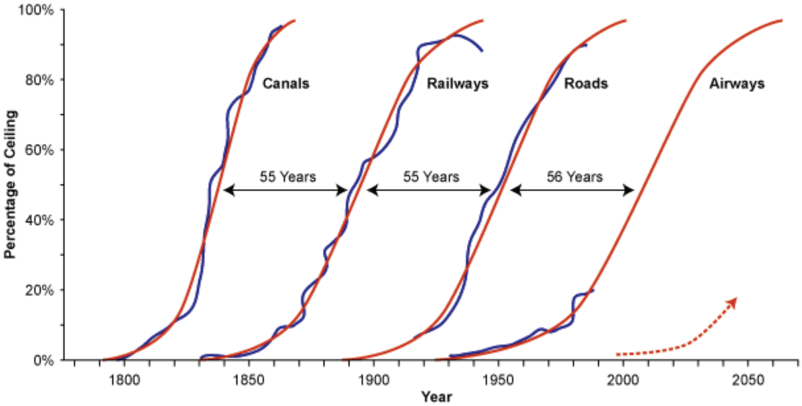
[133] *See* Antonio Carlos Gracias, Sérgio Ricardo Lourenço & Marat Rafikov, *Estimation of Natural Gas Production, Import and Consumption in Brazil Based on Three Mathematical Models*, 3 NAT. RESOURCES 42, 43 (2012). On the von Bertalanffy growth model, see generally Ludwig von Bertalanffy, *A Quantitative Theory of Organic Growth (Inquiries on Growth Laws. II)*, 10 HUM. BIOLOGY 181 (1938).

[134] *Cf.* PIERRE SIMON, MARQUIS DE LAPLACE, A PHILOSOPHICAL ESSAY ON PROBABILITIES 17 (Frederick Wilson Truscott & Frederick Lincoln Emery trans., 1902) ("[T]he more extraordinary the event, the greater the need of its being supported by strong proofs.").

[135] *See* John Blevins, *The Risks and Rewards of Network Neutrality Under § 706*, 2015 MICH. ST. L. REV. 723; Adam Candeub, *Is There Anything New to Say About Network Neutrality?*, 2015 MICH. ST. L. REV. 455; Rob Frieden, *What's New in the Network Neutrality Debate*, 2015 MICH. ST. L. REV. 739; Justin (Gus) Hurwitz, *Net Neutrality: Something Old; Something New*, 2015 MICH. ST. L. REV. 665.

[136] *See* Bayus, Kim & Shocker, *supra* note 89, at 144, 146-48 (distinguishing between product diffusion models that emphasize potential interactions among multiple products and models that evaluate the creation and diffusion of successive generations of a single product). If a broadband network is imagined as being successive generations of a single product, each defined by unique features and improvements, then the cooperative approach to innovation acquires greater relevance to the network neutrality debate. *See* Jorge L. Contreras, *Patent Pledges: Between the Public Domain and Market Exclusivity*, 2015 MICH. ST. L. REV. 787. Nothing prevents a broadband network operator, after all, from eschewing paid prioritization of traffic, throttling, or any other form of

according to a multi-stage logistic substitution model, much as physical transportation infrastructure in the United States has followed successive generations of logistic diffusion and substitution, from canals to railroads to highways to airports:[137]



On the other hand, one might argue that any technological progression of this sort has not yet had a significant impact on providers or consumers of broadband service in the United States.[138] Cable emerged early as the premier fixed broadband technology in America and has never yielded that dominant position.[139] From the perspective of residential customers who are effectively locked into a single geographic market, high-speed broadband options are limited; few American consumers have more than a single provider from which to choose.[140] At download speeds of 25 megabits per second

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discriminatory treatment based on the origin of content downloaded over its facilities.

[137] *See* Kucharavy & De Guio, *supra* note 5, at 413 (fig.7).

[138] *Cf*. NICK RUSSO ET AL., THE COST OF CONNECTIVITY 2014, at 12-17 (2014), *available at* http://static.newamerica.org/attachments/229-the-cost-of-connectivity-2014/OTI_The_Cost_of_Connectivity_2014.pdf (comparing broadband costs in selected cities in the United States, Europe, and Asia).

[139] *See* FCC, CONNECTING AMERICA: THE NATIONAL BROADBAND PLAN 42 (2010), *available at* http://download.broadband.gov/plan/national-broadband-plan-chapter-4-broadband-competition-and-innovation-policy.pdf (predicting that consumers "in areas that include 75% of the population . . . will likely have only one service provider"—namely, cable companies—"that can offer very high peak download speeds").

[140] *See* DAVID N. BEEDE, U.S. DEP'T OF COMMERCE, COMPETITION AMONG U.S. BROADBAND SERVICE PROVIDERS (2014), *available at* http://www.esa.doc.gov/sites/default/files/competition-among-us-broadband-

(Mbps) or greater, which the FCC has proposed to set as the definition of high-speed broadband,[141] few Americans enjoy a choice among providers.[142] Cable broadband has yet to face the sort of competition and resulting erosion of market share that would make it plausible to contemplate logistic substitution of an aging technology whose operator is clinging to a declining position. Eventually, perhaps, municipal broadband networks or even fiber optic networks installed by wealthy private rivals will confront cable operators with the prospect of plummeting profits, stranded investments, and chaotic exit. But that day has not yet arrived.

Charting the progression of high-speed broadband as a story of technological succession would involve extensive data gathering beyond this cursory glance. Moreover, I have spoken so far almost entirely of fixed broadband infrastructure. Rapidly improving mobile devices have expanded the range of tasks that smartphones and tablets may divert from the desktop—and has concomitantly made mobile Internet access a more viable competitor to cable and other fixed broadband platforms. In addition to tracking the rise and ebb of different platforms' market shares, empirical evaluation of broadband as a possible instance of logistic succession should establish trends in the growth of download and upload speeds, not only in absolute terms, but also in terms of Mbps per dollar spent on monthly subscription fees. In a testy exchange over broadband policy, Christopher Yoo and Susan Crawford embraced one shared technological assumption: cable operators were capable of offering 160 Mbps download speeds *in 2006*.[143] Yet dissenting

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service-providers.pdf; Anne Neville, *Faster Broadband, Reaching More*, NTIA (July 17, 2014), http://www.ntia.doc.gov/print/blog/2014/faster-broadband-reaching-more.

[141] *See* FCC, 2015 BROADBAND PROGRESS REPORT AND NOTICE OF INQUIRY ON IMMEDIATE ACTION TO ACCELERATE DEPLOYMENT 29 (2015) [hereinafter 2015 BROADBAND PROGRESS REPORT], *available at* http://transition.fcc.gov/Daily_Releases/Daily_Business/2015/db0224/FCC-15-10A1.pdf.

[142] *See* BEEDE, *supra* note 140, at 5 (observing that "only 37 percent of persons had a choice of two or more fixed ISPs" providing service at "25 Mbps or greater speeds," and that "only 9 percent had a choice of three or more").

[143] *Compare* Christopher S. Yoo, *Technological Determinism and Its Discontents*, 127 HARV. L. REV. 914, 919 (2014) (reviewing SUSAN CRAWFORD, CAPTIVE AUDIENCE: THE TELECOM INDUSTRY AND MONOPOLY POWER IN THE NEW GILDED AGE (2013)), *with* Susan Crawford, *Was That a Book Review?*, 127 HARV. L. REV. F. 137, 140 (2014).

Commissioners could be heard in 2015, decrying the FCC's proposed threshold of 25 Mbps as aggressive and oppressive.[144]

B. A Multi-Layered Approach to Logistic Analysis of Network Neutrality

As much as network neutrality has eluded legal classification,[145] the neutrality concept also defies easy categorization within innovation policy. It seems naive to force network neutrality, an epochal policy choice decades in the making, into the simple logistic substitution model. At a minimum, cable broadband operators are simultaneously competing on a horizontal basis against potential providers of alternative channels of high-speed Internet access and on a vertical basis against content providers.[146] An understanding of the multiple levels of competition at stake begins with an evaluation of the layered nature of broadband networks.

Every communications medium consists of at least *three* layers: a physical layer consisting of network infrastructure, a logical layer consisting of software and standards for connection, and a content layer.[147] Opponents of network neutrality obligations emphasize the capital-intensiveness of network construction, maintenance, and expansion. The specter of "torrent[s] of bandwidth-intensive downstream traffic, such as Internet Protocol Television and other Over the Top applications," haunts broadband system operators.[148] These arguments ring of threats to the physical and logical layers.

By contrast, the competing perspective typically emphasizes end-to-end design in information science, which propels all "intelligence" to the edges of the network (where creators load

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[144] *See* 2015 BROADBAND PROGRESS REPORT, *supra* note 141, at 111 (Pai, Comm'r, dissenting); *id*. at 114 (O'Rielly, Comm'r, dissenting).

[145] *See* Verizon v. FCC, 740 F.3d 623, 628 (D.C. Cir. 2014); Comcast Corp. v. FCC, 600 F.3d 642, 645 (D.C. Cir. 2010). During the editing of this symposium, the FCC adopted regulations classifying broadband Internet service as a telecommunications service and imposing network neutrality rules under Title II of the Communications Act. *See* Protecting and Promoting the Open Internet, 2015 WL 1120110 (2015).

[146] *See* Hurwitz, *supra* note 135 (manuscript at 4-5).

[147] *See* Yochai Benkler, *From Consumers to Users: Shifting the Deeper Structures of Regulation Toward Sustainable Commons and User Access*, 52 FED. COMM. L.J. 561, 562 (2000); *cf., e.g.*, Kevin Werbach, *A Layered Model for Internet Policy*, 1 J. ON TELECOMM. & HIGH TECH. L. 37, 59 (2002) (interjecting a fourth layer, applications, between logic and content).

[148] Frieden, *supra* note 135 (manuscript at 11-12) (footnotes and abbreviations omitted).

content and where consumers access that information) in order to keep the physical and logical architecture of the network as simple and general as possible.[149] The end-to-end principle's aspiration of a network with intelligent edges connected by dumb pipe[150] seeks to "maximize[] innovation" through network "architecture that maximizes the *opportunity* for innovation."[151] In this symposium, Andrew Torrance and Eric von Hippel's paean to "innovation wetlands" similarly privileges user-initiated innovation over legal regimes that would enable the inventors and guardians of other creative platforms to retain greater control over downstream innovation.[152]

As a first step toward resolving this debate on the basis of evidence rather than rhetoric, we might conduct logistic analysis of growth rates within the physical and content layers of the Internet. A project that grand exceeds the scope of this Article on a symposium spanning three or four discrete subjects within the law of innovation. According to various formulations of Moore's law, processing speed in computing doubles every 18, 24, or 36 months.[153] Moore's law is therefore a classic instance of exponential growth.

But Moore's *second* law, also known as Rock's law (in honor of investor Arthur Rock), holds that the capital cost of inventing and testing each new generation of semiconductors also rises exponentially.[154] Logistic analysis posits that there is no such thing as indefinite exponential growth.[155] As Gordon Moore acknowledged in 2005, on the fortieth anniversary of his original 1965 magazine article describing the periodic doubling of transistor density on integrated circuits, "'It can't continue forever. The nature of exponentials is that you push them out and eventually disaster

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[149] *See* J.H. Saltzer, D.P. Reed & D.D. Clark, *End-To-End Arguments in System Design*, *in* INNOVATIONS IN INTERNETWORKING 195, 196 (Craig Partridge ed., 1988).

[150] *See* David S. Isenberg, *The Dawn of the "Stupid Network,"* 2 NETWORKER 24, 26 (1998) (describing an end-to-end network as a "stupid network").

[151] Mark A. Lemley & Lawrence Lessig, *The End of End-To-End: Preserving the Architecture of the Internet in the Broadband Era*, 48 UCLA L. REV. 925, 938 (2001).

[152] *See* Andrew W. Torrance & Eric von Hippel, *Protecting the Right to Innovate: Our "Innovation Wetlands,"* 2015 MICH. ST. L. REV. 793.

[153] *See* Gordon E. Moore, *Cramming More Components onto Integrated Circuits*, 38 ELECTRONICS 4, 5 (1965).

[154] *Rock's Law*, WIKIPEDIA, http://en.wikipedia.org/wiki/Rock%27s_law (last visited Aug. 27, 2015).

[155] *See supra* text accompanying notes 28-30, 41, and 48-51.

happens.'"[156] Serious estimates of the ultimate limits on Moore's law have ranged from twenty or forty years[157] to 600 years.[158]

Growth within the content layer poses an even more intriguing problem for logistic analysis and cognate forms of nonlinear modeling. Absolute levels of diversity in online content may escape any physical constraint. Perhaps we can find a solution to problems of instantaneous queuing in modern information technology in the deepest of historical studies.[159] Marine paleontology supplies a remote but relevant analogy. Although total biomass on earth is assuredly bounded by the planet's carrying capacity, biodiversity as measured by the number of distinct species is not necessarily constrained. Logistic models of biodiversity over geologic time assume that limits on ecospace provide negative feedback and impose some ceiling on total levels of diversity.[160] Exponential models assume no such limit and allow the number of species to grow subject only to the ability of biological taxa to occupy new ecospace.[161]

But a third school within paleontology asserts that "the entire Phanerozoic history of marine biodiversity at genus level" is not only unconstrained by putative physical limits on ecospace, but also best described by a *hyperbolic* growth model whose underlying first-order ordinary differential equation is:[162]

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[156] Manek Dubash, *Moore's Law Is Dead, Says Gordon Moore*, TECHWORLD (Apr. 13, 2010), http://www.techworld.com/news/operating-systems/moores-law-is-dead-says-gordon-moore-3576581.

[157] *See* Michio Kaku, *Parallel Universes, the Matrix, and Superintelligence*, KURZWIEL ACCELERATING INTELLIGENCE (June 26, 2003), http://www.kurzweilai.net/parallel-universes-the-matrix-and-superintelligence; Suhas Kumar, *Fundamental Limits to Moore's Law* (June 9, 2012), http://large.stanford.edu/courses/2012/ph250/kumar1/.

[158] *See* LAWRENCE M. KRAUSS & GLENN D. STARKMAN, UNIVERSAL LIMITS ON COMPUTATION (2004), *available at* http://arxiv.org/pdf/astro-ph/0404510.pdf.

[159] *Cf*. New York v. United States, 505 U.S. 144, 149 (1992) ("These cases implicate one of our Nation's newest problems of public policy and perhaps our oldest question of constitutional law."). *See generally* DAVID CHRISTIAN, MAPS OF TIME: AN INTRODUCTION TO BIG HISTORY (2011).

[160] *See, e.g.*, J. John Sepkoski, Jr., *Phylogenetic and Ecologic Patterns in the Phanerozoic History of Marine Biodiversity*, *in* SYSTEMATICS, ECOLOGY, AND THE BIODIVERSITY CRISIS 77, 89 (Niles Eldredge ed., 1992).

[161] *See, e.g.*, Michael J. Benton, *The History of Life: Large Databases in Palaeontology*, *in* NUMERICAL PALAEOBIOLOGY: COMPUTER-BASED MODELLING AND ANALYSIS OF FOSSILS AND THEIR DISTRIBUTIONS 249, 267 (David A.T. Harper ed., 1999).

[162] Alexander V. Markov & Andrey V. Korotayev, *Phanerozoic Marine Biodiversity Follows a Hyperbolic Trend*, 16 PALAEOWORLD 311, 312 (2007).

$$\frac{dN}{dt} = kN^2$$

Analytical solutions to this differential equation take the general form, $N(t) = \frac{C}{t_0 - t}$.[163] Unlike logistic growth, which obeys a *horizontal* asymptote, hyperbolic growth observes a *vertical* asymptote at the mathematical singularity where $t = t_0$.

Paleontology has therefore proposed three mathematically distinct answers to the riddle of diversity. Marine biodiversity at the appropriate taxonomic level (species or genus) may follow logistic, exponential, or hyperbolic growth over the course of geologic history. Although all three of these growth models are convex functions (at least in their initial stages), they behave in dramatically different ways as input grows:[164]

- Logistic growth is constrained: Even as time goes to infinity, logistic growth obeys a finite limit.
- Exponential growth grows to infinity as time goes to infinity, but remains finite as long as time remains finite.
- Hyperbolic growth has a singularity in finite time: It grows to infinity within a finite time.[165]

Or, in graphic form with stylized logistic (blue), exponential (red), and hyperbolic (gold) curves, each scaled so that the function equals 1 at $t = 0$:[166]

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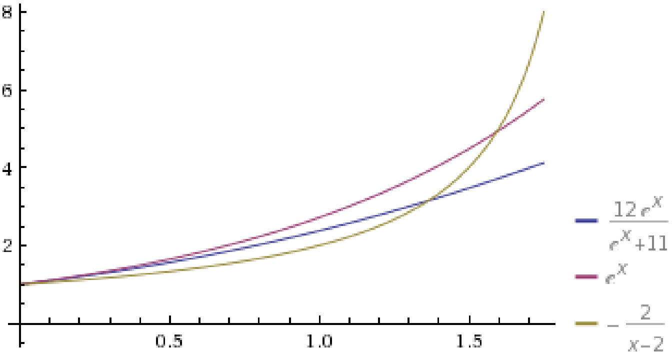
[163] *See id*. at 313; A.V. Markov & A.V. Korotayev, *Hyperbolic Growth of Marine and Continental Biodiversity Through the Phanerozoic and Community Evolution*, 69 ZHURNAL OBSHCHEI BIOLOGII 175 (2008).

[164] *See* ANDREY KOROTAYEV, ARTEMY MALKOV & DARIA KHALTOURINA, INTRODUCTION TO SOCIAL MACRODYNAMICS: COMPACT MACROMODELS OF THE WORLD SYSTEM GROWTH 7-8, 19-20 (2006).

[165] *Hyperbolic Growth*, WIKIPEDIA, http://en.wikipedia.org/w/index.php?title=Hyperbolic_growth (last modified Oct. 3, 2014, 12:08 a.m.).

[166] I plotted the following image using Wolfram Alpha using the single command line, *plot 12/(1+e^(ln(11)-x)) and e^x and 2/(2-x) for x=0 to 1.75*. WOLFRAMALPHA, http://www.wolframalpha.com/input/?i=plot+12%2F%281%2Be%5E%28ln%2811%29-x%29%29+and+e%5Ex+and+2%2F%282-x%29+for+x%3D0+to+1.75 (last visited Apr. 13, 2015).



If expressive diversity online resembles biological diversity in the sense that new forms, artistic or biological, continue to evolve and occupy new niches without regard to physical constraints on ecospace, then online content may be growing at a hyperbolic pace that outstrips even the exponential rates of Moore's law. Markets, after all, are metaphysical as well as physical spaces,[167] and online expression demands none of the physical variety.

Any decoupling of growth rates in the physical and the content layers of the Internet supports a corresponding decoupling of legal remedies. Whereas growth in content diversity is an unmitigated good, growth in network traffic is not. Congestion and the costs of its management, to say nothing of the capital required to replace the physical infrastructure of existing networks, are the prime movers of incumbent operators' strident opposition to net neutrality regulation. Usage-based pricing of broadband service can address network operators' concerns with traffic, stability, and utilization[168] without resort to paid prioritization, the throttling of otherwise lawful content based on its origin, or tiered levels of service—all of which are anathema to net neutrality. But if the costs of network management observe some constraint short of the growth rate of diversity in online content, even if the costs of network operation are rising exponentially within foreseeable time horizons, the case for tying network control to content—as a matter of rhetoric as well as economics—becomes much weaker.

Three counterarguments remain available to opponents of network neutrality. First, and most fundamentally, biological

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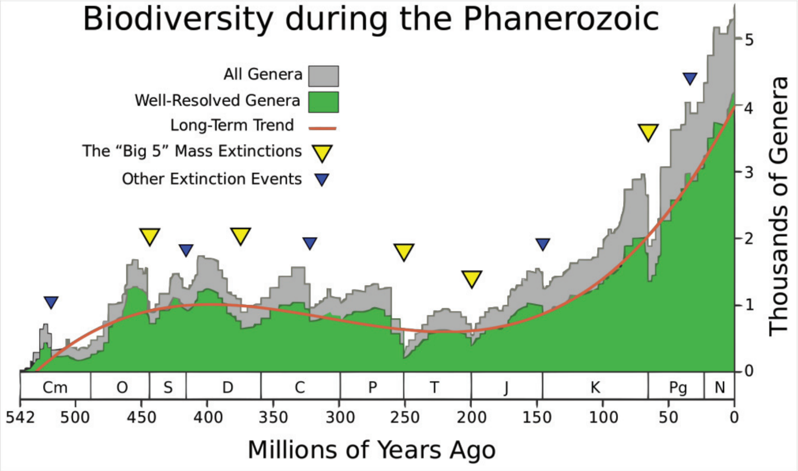
[167] *See* H. Hotelling, *Stability in Competition*, 39 ECON. J. 41, 44-45 (1929).

[168] *See* Daniel A. Lyons, *Internet Policy's Next Frontier: Usage-Based Broadband Pricing*, 66 FED. COMM. L.J. 1, 35-36 (2013).

diversity within ecospace may simply be an inapt basis for modeling, let alone measuring, expressive diversity. Second, as a descriptive matter, it is not at all clear that even biological diversity has grown at hyperbolic rates over the 542 million-year span of the Phanerozoic Eon. In addition to logistic, exponential, and hyperbolic models of species diversity over the course of natural history, a *fourth* method of evaluating biodiversity argues that the fossil record reveals double periodicity, with marine genera cresting and ebbing according to overlapping 62 million-year and 140 million-year cycles.[169] Given the immense number of segments into which the Phanerozoic may be divided, decomposing the entire Eon into discrete components supports "any possible pattern" of exponential, logistic, periodic, or stochastic growth in biodiversity.[170] Perhaps it is best just to let the paleontological data speak for itself:[171]



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[169] *See* Robert A. Rohde & Richard A. Muller, *Cycles in Fossil Diversity*, 434 NATURE 208, 209 (2005); *see also* Marc Davis, Piet Hut & Richard A. Muller, *Extinction of Species by Periodic Comet Showers*, 308 NATURE 715, 715 (1984); David M. Raup & J. John Sepkoski, Jr., *Periodicity of Extinctions in the Geologic Past*, 81 PROC. NAT'L ACAD. SCI. 801, 805 (1984); David M. Raup & J. John Sepkoski, Jr., *Periodic Extinction of Families and Genera*, 231 SCIENCE 833, 836 (1986). *But see* Antoni Hoffman, *Patterns of Family Extinction Depend on Definition and Geological Timescale*, 315 NATURE 659, 660 (1985); Richard A. Kerr, *Periodic Extinctions and Impacts Challenged*, 227 SCIENCE 1451, 1451 (1985).

[170] Markov & Korotayev, *supra* note 162, at 311-12.

[171] The following graphic is displayed at *Phanerozoic*, WIKIPEDIA, http://en.wikipedia.org/wiki/Phanerozoic (last modified Mar. 21, 2015, 8:09 a.m.) (drawing from data reported in Rohde & Muller, *supra* note 169).

Third and finally, wholly apart from the clarity of the paleontological record and its suitability as a guide to the measurement of expressive diversity, there is a colorable argument that network traffic has also grown at a hyperbolic rate. "[W]hen the average arrival rate approaches the server capacity," "classical queuing theory" predicts "hyperbolic growth" in users' average waiting time.[172] In the formal notation of queuing theory:[173]

$$\hat{U} = \frac{\lambda}{\mu}, \text{ where } \lambda = \text{arrival rate and } \mu = \text{service rate}$$

$$\lambda \to \mu,\ \hat{U} \to 1$$

The nightmarish worst-case scenario presented by unchecked growth of Internet traffic is what queuing theory would describe as loss of network stability: impossibly long queues for service, triggering intolerable delays and outright losses of service.[174] Network operation and maintenance under the burden of rules demanding neutrality among different sources of traffic, so the argument proceeds, must labor under a legal prescription "for stealthy low-rate denial-of-service (DoS) attacks inducing arbitrary long queues in . . . target network[s], which in turn cause high delays and loss."[175]

We might demand more evidence before decrying the "financial ruin" that net neutrality would inflict "upon the simplest

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[172] DANIEL S. BERGER, MARTIN KARSTEN & JENS SCHMITT, ON THE RELEVANCE OF ADVERSARIAL QUEUEING THEORY IN PRACTICE 11 (2014), *available at* https://disco.informatik.uni-kl.de/discofiles/publicationsfiles/BKS14.pdf; *see also Hyperbolic Growth*, *supra* note 165.

[173] *See* MOSHE ZUKERMAN, INTRODUCTION TO QUEUEING THEORY AND STOCHASTIC TELETRAFFIC MODELS § 3.2 (2015).

[174] *See, e.g.*, ROBERT B. COOPER, INTRODUCTION TO QUEUEING THEORY (3d ed. 1990); Robert B. Cooper, *Queueing Theory*, *in* ENCYCLOPEDIA OF COMPUTER SCIENCE 1496 (Anthony Ralston, Edwin D. Reilly & David Hemmendinger eds., 4th ed. 2003). *Compare* Allan Borodin et al., *Adversarial Queuing Theory*, 48 J. ACM 13, 14 (2001) ("[F]or independent and time-invariant input distributions (say, for example, Poisson arrivals), FIFO [first in, first out] scheduling is stable for *any* class-independent service time distribution . . . as long as the necessary load conditions (i.e., total expected arrival rate at any server is less than the expected service rate) are satisfied."), *with* Maury Bramson, *A Stable Queueing Network with Unstable Fluid Model*, 9 ANNALS APPLIED PROBABILITY 818, 818 (1999) (identifying "a family of queueing networks that are stable, but whose fluid models are unstable, that is, there exists an unstable solution of the fluid model equations").

[175] *See* BERGER, KARSTEN & SCHMITT, *supra* note 172, at 1.

[Internet service provider] who finds his [network] conscripted to national [informational] use."[176] The burden of persuasion, in my judgment, remains with opponents of net neutrality. Despite this debate's superficial emphasis on network management and incentives to maintain, expand, and build digital networks, the real innovative stakes reside in the content layer. The ultimate question is whether independent suppliers or the networks themselves will excel in meeting demand for novel, engaging content. In his contribution to this symposium, Thomas Jeitschko emphasizes economic distinctions between patents and other forms of property and urges makers of innovation policy not to equate intellectual property with rights in land or other tangible property.[177] In a symposium dominated by considerations of economic rivalry and resource-based constraints, diversity in the Internet's content layer stands out as the lone element of innovation that potentially heeds no carrying capacity.

IV. LAW'S ECOLOGY

Qualitative evaluation of the debate over network neutrality, as informed by the quantitative insights of logistic analysis, leaves this controversy in the deep ideological trenches dug by partisans in American debates over innovation policy. Network operators preach Joseph Schumpeter's gospel of creative destruction through monopoly, or at least through the quest for supracompetitive returns.[178] Creators of content enjoying neither control of the network

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[176] *See* Babbitt v. Sweet Home Chapter of Cmtys. for a Great Or., 515 U.S. 687, 714 (1995) (Scalia, J., dissenting). I shall make clear what my allusion to Justice Scalia's *Sweet Home* dissent merely implies: Much of the opposition to net neutrality regulation arises from the supposition that this form of regulation would effect an unconstitutional taking of network operators' private property. *See* Daniel A. Lyons, *Virtual Takings: The Coming Fifth Amendment Challenge to Net Neutrality Regulation*, 86 NOTRE DAME L. REV. 65 (2011).

[177] *See* Thomas D. Jeitschko, *Beyond Intellectual Property: Economic Concerns in Patents Policy and Practice*, 2015 MICH. ST. L. REV. (forthcoming) (manuscript at 2) (on file with the Michigan State Law Review).

[178] *See* JOSEPH A. SCHUMPETER, CAPITALISM, SOCIALISM AND DEMOCRACY 81-86 (2003); *see also* Robert P. Merges, *Commercial Success and Patent Standards: Economic Perspectives on Innovation*, 76 CALIF. L. REV. 803, 843 (1988) (describing the pursuit of "temporary monopoly profits" facilitated by "technological innovation" as the primary "spur[for] the tremendous growth of the Western economies").

nor an affiliation with any network operator invoke Kenneth Arrow's competing vision of innovation through robust competition.[179]

The unrelenting battle between Schumpeterian and Arrovian accounts of innovation,[180] so pivotal to debates over the optimal scope of intellectual property rights,[181] reprises debates over *r*/*K* selection theory in ecology and evolutionary biology. *r*-selection in biological species (analogous to Arrovian competition) favors low-cost reproduction of numerous offspring, while *K*-selection (analogous to Schumpeterian competition) favors the expenditure of enormous energy in the production of a low number of high-quality offspring.[182] The *r*- and *K*-selection strategies derive their names from the ecological literature's preferred rendering of the differential form of the logistic function:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

r and *K* are the preferred names in ecology and evolutionary biology for the growth rate and carrying capacity parameters we have designated as α and κ.[183] "[T]he end of all our exploring / Will be to arrive where we started / And know the place for the first time."[184]

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[179] *See* Kenneth J. Arrow, *Economic Welfare and the Allocation of Resources for Invention*, *in* THE RATE AND DIRECTION OF INVENTIVE ACTIVITY: ECONOMIC AND SOCIAL FACTORS 609 (Princeton Univ. Press 1962), *reprinted in* 5 COLLECTED PAPERS OF KENNETH J. ARROW: PRODUCTION AND CAPITAL 104 (1985).

[180] *See, e.g.*, Jonathan B. Baker, *Beyond Schumpeter vs. Arrow: How Antitrust Fosters Innovation*, 74 ANTITRUST L.J. 575, 575 (2007); Richard Gilbert, *Looking for Mr. Schumpeter: Where Are We in the Competition–Innovation Debate?*, *in* 6 INNOVATION POLICY AND THE ECONOMY 159, 164-65 (Adam B. Jaffe, Josh Lerner & Scott Stern eds., 2006).

[181] *See, e.g.*, Wendy J. Gordon, *Of Harms and Benefits: Torts, Restitution, and Intellectual Property*, 21 J. LEGAL STUD. 449, 475-76 (1992); Mark A. Lemley, *The Economics of Improvement in Intellectual Property Law*, 75 TEX. L. REV. 989, 1050-51 (1997); Robert P. Merges & Richard R. Nelson, *On the Complex Economics of Patent Scope*, 90 COLUM. L. REV. 839, 875 (1990).

[182] *See, e.g.*, Jennifer H. Fewell & Susan M. Bertram, *Evidence for Genetic Variation in Worker Task Performance by African and European Honey Bees*, 52 BEHAV. ECOLOGY & SOCIOBIOLOGY 318, 324 (2002); Eric C. Keen, *Tradeoffs in Bacteriophage Life Histories*, 4 BACTERIOPHAGE 1, 5 (2014).

[183] *See, e.g.*, Rosenzweig & MacArthur, *supra* note 14, at 217. *Compare* KINGSLAND, *supra* note 2, at 74-75 (extolling "the differential form" of "the logistic curve" as "easier to interpret and to analyze" than its differentiated, analytical version), *with id.* at 85-86 (describing Alfred Lotka's rendering of the differential form of the logistic function according to *r*, the function's rate of increase). *See*

This Article's abbreviated and strictly qualitative application of sigmoid models presents merely a few special cases of generalized logistic functions.[185] It represents a modest contribution to the "larger effort to move legal science toward a first law of jurisdynamics."[186] Law, like ecology, is a "search for patterns of repetition."[187] The enterprise is fraught with some danger: Just as the invocation of the word *law* in biology "sets off emotional resonances," as though the announcement of a scientific law "suggest[s] that we have discovered a truth of some importance,"[188] the reverse process of intellectual osmosis—the infusion of concepts from the physical and biological sciences into law—sets its own philosophical trap. Whenever "ideas taken from physics have been transferred to a biological context," including the life science called law, proponents of these ideas risk "reject[ing] . . . history in favor of" populations and markets "in equilibrium" as "a harmonious, unifying concept."[189] We can save through motion what we might otherwise lose to stasis:

> The detail of the pattern is movement, ...
> Desire itself is movement
> Not in itself desirable;
> Love is itself unmoving,
> Only the cause and end of movement,
> Timeless, and undesiring
> Except in the aspect of time
> Caught in the form of limitation
> Between un-being and being.[190]

The consilient application of logistic analysis to cycles of growth, competition, decay, and renewal supports a general system theory of physics, biology, and human society.[191] To better

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generally J.M. Jeschke, W. Gabriel & H. Kokko, *r-Strategist/K-Strategists*, *in* ENCYCLOPEDIA OF ECOLOGY 3113 (Sven Erik Jorgensen & Brian Faith eds., 2008).

[184] ELIOT, *supra* note 1, at 59.

[185] *See* Banks, *supra* note 47; Tsoularis, *supra* note 37.

[186] Daniel Martin Katz et al., *Reproduction of Hierarchy? A Social Network Analysis of the American Law Professoriate*, 61 J. LEGAL EDUC. 76, 101 (2011).

[187] KINGSLAND, *supra* note 2, at 8 (describing Robert H. MacArthur's approach to ecology); *see* G. Evelyn Hutchinson, *The Concept of Pattern in Ecology*, 105 PROC. ACAD. NAT. SCI. PHILA. 1 (1953); Robert H. MacArthur, *Patterns of Species Diversity*, 40 BIOLOGICAL REVS. 510 (1965).

[188] KINGSLAND, *supra* note 2, at 68.

[189] *Id*. at 8.

[190] ELIOT, *Burnt Norton*, *in* FOUR QUARTETS, *supra* note 1, at 13, 19-20.

[191] *See generally* LUDWIG VON BERTALANFFY, GENERAL SYSTEM THEORY: FOUNDATIONS, DEVELOPMENT, APPLICATIONS (1968).

"organiz[e] our understanding of economic, ecological, and institutional systems," such a theory would exploit "notions of hierarchies across scales to represent structures that sustain experiments, test results, and allow adaptive evolution."[192] At its most ambitious, logistic analysis in law extends *r*/*K* selection theory beyond its venerable but increasingly brittle origins in biology[193] into a more comprehensive life-history paradigm[194] or even an all-encompassing theory of "panarchy,"[195] on earth as in the heavens.[196]

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[192] C.S. Holling, Lance H. Gunderson & Donald Ludwig, *In Quest of a Theory of Adaptive Change*, *in* PANARCHY: UNDERSTANDING TRANSFORMATIONS IN HUMAN AND NATURAL SYSTEMS 3, 5 (Lance H. Gunderson & C.S. Holling eds., 2002) [hereinafter PANARCHY].

[193] *See* ROGER ARDITI & LEV R. GINZBURG, HOW SPECIES INTERACT: ALTERING THE STANDARD VIEW ON TROPHIC ECOLOGY (2012); Roger Arditi & Lev R. Ginzburg, *Coupling in Predator–Prey Dynamics: Ratio-Dependence*, 139 J. THEORETICAL BIOLOGY 311 (1989). Full elaboration of Arditi and Ginzburg's ratio-dependent view of interspecific competition lies well beyond the scope of this Article. This much is worth noting here: If the Lotka–Volterra model of interspecific competition has come under paradigm-shifting attack, then a similarly dynamic reworking of the logistic substitution model and other economic applications of logistic analysis is assuredly in order. *See generally* THOMAS S. KUHN, THE STRUCTURE OF SCIENTIFIC REVOLUTIONS (1962). I shall leave that task for another time, but not another scholar. *Cf.* Jim Chen, *A Vision Softly Creeping: Congressional Acquiescence and the Dormant Commerce Clause*, 88 MINN. L. REV. 1764, 1795 (2004).

[194] *See, e.g.*, STEPHEN C. STEARNS, THE EVOLUTION OF LIFE HISTORIES (1992); David Reznick, Michael J. Bryant & Farrah Bashey, *r- and K-Selection Revisited: The Role of Population Regulation in Life-History Evolution*, 83 ECOLOGY 1509 (2002); Stephen C. Stearns, *The Evolution of Life History Traits: A Critique of the Theory and a Review of the Data*, 8 ANN. REV. ECOLOGY & SYSTEMATICS 145 (1977).

[195] *See* PANARCHY, *supra* note 192.

[196] *See* LEV GINZBURG & MARK COLYVAN, ECOLOGICAL ORBITS: HOW PLANETS MOVE AND POPULATIONS GROW 3-10 (2004).