# Integrated Supply Chain Optimization Model Using Mixed Integer Linear Programming

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This article presents an integrated approach to optimize the different functions in a supply chain on strategic tactical and operational levels. The integrated supply chain model has been formulated as a cost minimization problem in the form of MILP (Mixed Integer Linear Programming). The costs of production, transport, distribution and environmental protection were adopted as optimization criteria. Timing, volume, capacity and mode of transport were also taken into account. The model was implemented in the LINGO package. The implementation model and the numerical tests are presented and discussed. The numerical experiments were carried out using sample data to show the possibilities of practical decision support and optimization of the supply chain.

**Keywords:** discrete optimization, supply chain management, multimodal transportation, decision support.

## 1. INTRODUCTION

The issue of the supply chain is the area of science and practice that has been strongly developing since the ‘80s of the last century. Numerous definitions describe the term, and a supply chain reference model has been designed [1,2]. The supply chain is commonly seen as a collection of various types of companies (raw materials, production, trade, logistics, etc.) working together to improve the flow of products, information and finance. As the words in the term indicate, the supply chain is a combination of its individual links in the process of supplying products (material and services) to the market.

Huang et al. [3] studied the shared information of supply chain production. They considered and proposed four classification criteria: supply chain structure, decision level, modeling approach and shared information.

**Supply chain structure:** It defines the way in which various organizations within the supply chain are arranged and related to each other. The supply chain structure falls into four main types [4]: Convergent: each node in the chain has at least one successor and several predecessors. Divergent: each node has at least one predecessor and several successors. Conjoined: which is a combination of each convergent chain and one divergent chain. Network: which cannot be classified as convergent, divergent or conjoined, and is more complex than the three previous types.

**Decision level:** Three decision levels may be distinguished in terms of the decision to be made: strategic, tactical and operational, with their corresponding period, i.e., long-term, mid-term and short-term.

**Supply chain analytical modeling approach:** This approach consists in the type of representation, in this case, mathematical relationships, and the aspects to be considered in the supply chain. Most literature describes and discusses the linear programming-based modeling approach, mixed integer linear programming models in particular [5, 6, 7, 8, 9].

**Shared information:** This consists in the information shared between each network node determined by the model, which enables production, distribution and transport planning dependent on the purpose. The shared information process is vital for effective supply chain production, distribution and transport planning. In terms of centralized planning, the information flows from each node of the network where the decisions are made. Shared information includes the following groups of parameters: resources, inventory, production, transport, demand, etc. Minimization of total costs is the main purpose of the models presented in the literature [9, 10, 11, 12, 13], while maximization of revenues or sales is considered to a smaller scale [7, 14].

This paper deals with a mathematical model for supply chain costs optimization in the form of MILP (Mixed Integer Linear Programming Problem) [15] from the perspective of logistic provider. In this model, shared information process includes such parameters as resources, inventory, production, transport, demand etc. In previous works, the authors studied models and algorithms for combinatorial optimization of cost in a supply chain. This paper focuses on the multimodal transport in the supply chain and its implementation aspects. It should be emphasized that the presented model can be the basis for the decision support in the supply chain management. Optimization results of this model relate to two types of decision. These are short-term decisions, about how to supply at minimum cost (operational level), and long-term decisions on the capacity of individual distributors or production capacity of individual producers (tactical and strategic level). The article also presents various models of outsourced logistics management. The rest of the paper is organized as follows: Section II describes the decision-making problems that occur in SCM (Supply Chain Management) . Section III analyses the state of the art in this domain. Section IV gives the problem statement and provides an optimization model for the considered supply chain with multimodal transport. The implementation aspects of the optimization model are explained briefly in Section V. Computational examples and tests of the implemented model are presented in Section VI. The discussion on possible extensions of the proposed approach and conclusions is included in Section VII.

## 2. SUPPLY CHAIN MANAGEMENT

Supply chain management (SCM) is the systematic analysis and educated decision-making within the different business functions of an organization resulting in smooth and cost-effective flows of resources – materials, information, and finance.

The supply chain is the network that sources raw material from suppliers, transforms it into finished products at the manufacturing facilities, and distributes the finished products to final customers through distribution centers.

The aim of supply chain management (SCM) is to increase sales, reduce costs and take full advantage of business assets by improving interaction and communication between all the actors forming the supply chain. Supply chain management is a decision process that not only integrates all of its participants but also helps to coordinate the basic flows: products/services, information and funds. Changes in the global economy and the increasing globalization lead to the widespread use of IT tools, which enables continuous, real-time communication between the supply chain links. One of the objectives is to optimize logistics and entrust it to specialized companies.

Decisions are made across the supply chain on three levels: strategic, tactical and operational. Strategic decisions are long term decisions where the time horizon may be anything from one year to several years i.e. it involves multiple planning horizons. These decisions may be made on an organizational level or the supply chain level with the aim for global optimization. Tactical decisions are taken over a shorter period of time, maybe a few months. These are more localized decisions taken to keep the organization on the track set at the strategic level. Operational decisions are similar to day-to-day decisions for planning a few days' worth of operations. These take into consideration the most profitable way to carry out daily activities for satisfying immediate requirements. Some authors also suggest a set of best organization structures for efficient supply chain management [16, 17].

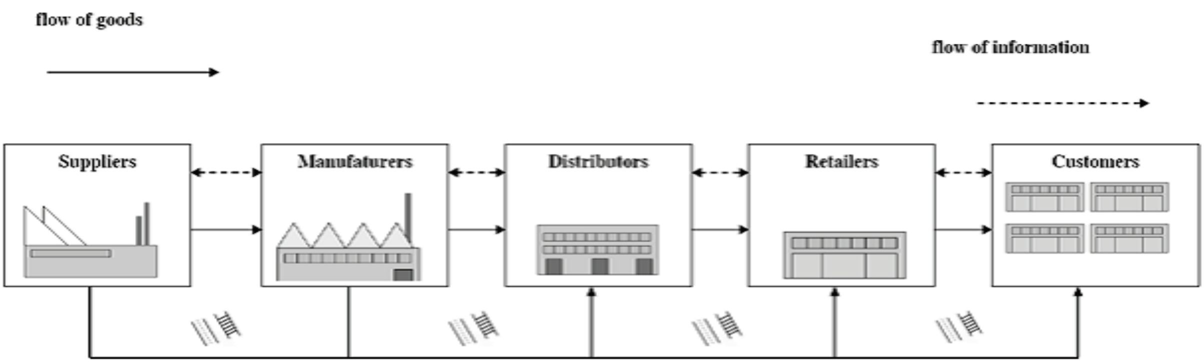


Fig. 1. The chart of the supply chain with multimodal transport

## 3. STATE OF ART AND MOTIVATION

Simultaneous considering the supply chain production, distribution processes in distribution centers and transport-planning problems greatly advances the efficiency of all processes. The literature in the field is vast, so an extensive review of existing research on the topic is extremely helpful in modeling and research. Comprehensive surveys on these problems and their generalizations were published, for example in [3].

In our approach, we are considering a case of the supply chain where (Fig.1):

- the shared information process in the supply chain consists of resources (capacity, versatility, costs), inventory (capacity, versatility, costs, time), production (capacity, versatility, costs), product (volume), transport (cost, mode, time), demand, etc. (Fig.2, Fig.3);
- the transport is multimodal. (several modes of transport, a limited number of means of transport for each mode);
- the environmental aspects of use of transport modes;
- different products are combined in one batch of transport;
- the cost of supplies is presented in the form of a function (in this approach linear function of fixed and variable costs);
- different decision levels are considered simultaneously;
- depending on the time between the execution of orders and the transport and distribution (Fig.4).

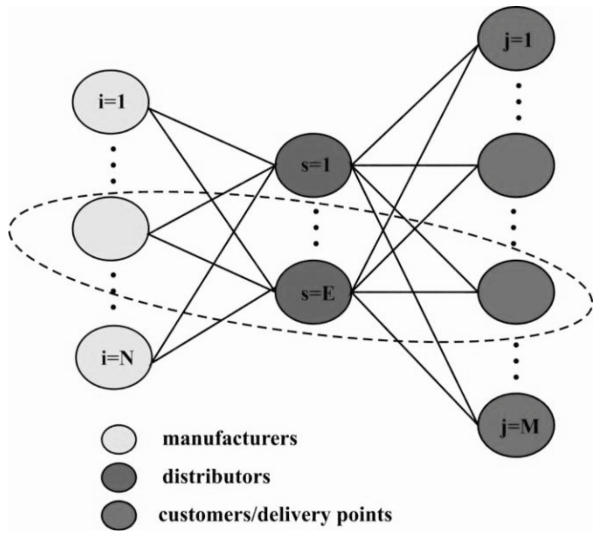


Fig. 2. The part of the supply chain network with marked indices of individual participants (elements). Dashed line marks one of the possible routes of delivery.

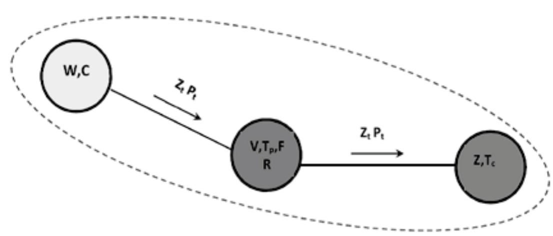


Fig. 3. The selected path of the supply chain along with the parameters that describe the individual elements and its dependencies (shared information) - Tab.1.

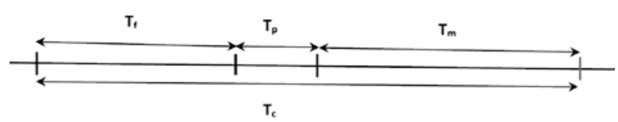

Fig. 4. Depending on the time parameters between the execution of orders and the transport and distribution (Table 1)

Decision levels in supply chains are mainly classified by the extent or effect of the decision to be made in terms of time. For instance, at the strategic level, the decisions made in relation to selecting production, storage and distribution locations, etc should be identified. At the tactical level however, the aspects such as production and distribution planning, assigning production and transport capacities, inventories and managing safety inventories are identified. At the operational level, replenishment and delivery operations are classified [3]. Most of the reviewed works focus on the tactical decision level [6, 7, 8, 10, 11, 12, 18]. Only few works deal with the problems taken together for the different decision levels [5, 13].

Therefore, the motivation behind this work is to suggest an approach to multilevel supply chain cost optimization with multimodal transport from the perspective of a logistics provider, and to propose an optimization model in the form of integer programming problem [15] which facilitates its solution using specialized software available on the market (LINGO, CPLEX). Many aspects of the proposed model implementation are featured here, including additional decision variables introduced at the level of implementation, optimization model in LINGO language, etc.

The aim of this paper is to design and implement the model that can become the basis for making optimal decisions at different levels of supply chain management. The proposed solution will also enable a comprehensive examination of the impact on cost and performance of various parameters of the shared information.

## 4. PROBLEM STATEMENT

### A. BACKGROUND

The key step in many decision-making and design processes is the optimization phase, which itself contains several stages. The purpose of the optimization process in our approach is to help determine realistic and practical outcomes of management decision-making and design processes in the supply chain. There are two basic ways to optimize the problem, either the qualitative approach or the quantitative approach. Using only a qualitative approach, the problem optimization, when making a decision, relies on personal judgment or experience acquired in dealings with similar problems in the past. In a few cases this approach may be adequate; however, there are many situations where a quantitative approach to the problem provides a better-structured and logical path through the decision-making process.

We propose the quantitative approach for the cost optimization supply chain network model (Fig. 2).

### A. PROBLEM FORMULATION

The mathematical optimization model was formulated as a mixed-integer program [15] with the minimization of costs (1) under constraints (2) .. (23).

A mixed-integer program is the minimization or maximization of a linear function subject to linear constraints. If all the variables can be rational (the set D is empty), this is a linear programming problem, which can be solved in polynomial time. In practice linear programs can be solved efficiently for reasonable-sized problems, or even for big problems with special structure. However when some or all of the variables must be integer, corresponding to pure integer and mixed integer programming respectively, the problem becomes NP-complete (formally intractable). Indices, parameters and decision variables in the model together with their descriptions are provided in Table 1. The proposed optimization model is a cost model that takes into account three other types of parameters, i.e., the spatial parameters (area/volume occupied by the product, distributor capacity and capacity of transport unit), time (duration of delivery and service by distributor, etc.) and transport mode. The position of each parameter against the subsequent links of the supply chain is shown in Figure 3.

Table 1. Summary indices, parameters and decision variables of the mathematical optimization model

| Symbol | Description |
|---|---|
| *Indices* | |
| k | product type (k=1..O) |
| j | delivery point/customer/city (j=1..M) |
| i | manufacturer/factory (i=1..N) |
| s | distributor /distribution center (s=1..E) |
| d | mode of transport (d=1..L) |
| N | number of manufacturers/factories |
| M | number of delivery points/customers |
| E | number of distributors |
| O | number of product types |
| L | number of mode of transport |
| *Input parameters* | |
| $F_s$ | the fixed cost of distributor/distribution center *s* (s=1..E) |
| $P_k$ | the area/volume occupied by product *k* (k=1..O) |
| $V_s$ | distributor *s* maximum capacity/volume (s=1..E) |
| $W_{i,k}$ | production capacity at factory *i* for product *k* (i=1..N) (k=1..O) |

| | |
|---|---|
| $C_{i,k}$ | the cost of product *k* at factory *i* (i=1..N) (k=1..O) |
| $R_{s,k}$ | if distributor *s* (s=1..E) can deliver product *k* (k=1..O) then $R_{sk}$=1, otherwise $R_{sk}$=0 |
| $Tp_{s,k}$ | the time needed for distributor *s* (s=1..E) to prepare the shipment of product *k* (k=1..O) |
| $Tc_{j,k}$ | the cut-off time of delivery to the delivery point/customer *j* (j=1..M) of product *k* (k=1..O) |
| $Z_{j,k}$ | customer demand/order *j* (j=1..M) for product *k* (k=1..O) |
| $Zt_d$ | the number of transport units using mode of transport *d* (d=1..L) |
| $Pt_d$ | the capacity of transport unit using mode of transport *d* (d=1..L) |
| $Tf_{i,s,d}$ | the time of delivery from manufacturer *i* to distributor *s* using mode of transport *d* (i=1..N) (s=1..E) (d=1..L) |
| $K1_{i,s,k,d}$ | the variable cost of delivery of product *k* from manufacturer *i* to distributor *s* using mode of transport *d* (d=1..L) (i=1..N) (s=1..E) (k=1..O) |
| $R1_{i,s,d}$ | if manufacturer *i* can deliver to distributor *s* using mode of transport *d* then $R1_{i,s,d}$=1, otherwise $R1_{i,s,d}$=0 (d=1..L) (s=1..E) (i=1..N) |
| $A_{i,s,d}$ | the fixed cost of delivery from manufacturer *i* to distributor *s* using mode of transport *d* (d=1..L) (i=1..N) (s=1..E) |
| $Tm_{s,j,d}$ | the time of delivery from distributor *s* to customer *j* using mode of transport *d* (d=1..L) (s=1..E) (j=1..M) |
| $K2_{s,j,k,d}$ | the variable cost of delivery of product*k*from distributor *s*tocustomer *j*usingmode of transport *d*(d=1..L)(s=1..E) (k=1..O) (j=1..M) |
| $R2_{s,j,d}$ | if distributor *s* can deliver to customer *j* using mode of transport *d* then $R2_{s,j,d}$=1, otherwise $R2_{s,j,d}$=0 (d=1..L) (s=1..E) (j=1..M) |
| $G_{s,j,d}$ | the fixed cost of delivery from distributor *s* to customer *j* using mode of transport *d* (s=1..E) (j=1..M) (k=1..O) |
| $Od_d$ | the environmental cost of using mode of transport *d* (d=1..L) |
| CW | Arbitrarily large constant |
| *Decision variables* | |
| $X_{i,s,k,d}$ | delivery quantity of product k from manufacturer i to distributor s using mode of transport d |
| $Xa_{i,s,d}$ | if delivery is from manufacturer *i* to distributor *s* using mode of transport *d* then $Xa_{i,s,d}$=1, otherwise $Xa_{i,s,d}$=0 |
| $Xb_{i,s,d}$ | the number of courses from manufacturer i to distributor s using mode of transport d |
| $Y_{s,j,k,d}$ | delivery quantity of product *k* from distributor *s* to customer *j* using mode of transport *d* |
| $Ya_{s,j,d}$ | if delivery is from distributor *s* to customer *j* using mode of transport *d* then $Ya_{s,j,d}$ =1, otherwise $Ya_{s,j,d}$ =0 |
| $Yb_{s,j,d}$ | the number of courses from distributor *s* to customer *j* using mode of transport *d* |
| $Tc_s$ | if distributor *s* participates in deliveries, then $Tc_s$=1, otherwise $Tc_s$=0 |
| *Auxiliary parameters-calculated* | |
| $Koa_{s,j,d}$ | the total cost of delivery from distributor *s* to customer *j* using mode of transport *d* (d=1..L) (s=1..E) (j=1..M) |
| $Kog_{s,j,d}$ | the total cost of delivery from distributor *s* to customer *j* using mode of transport *d* (d=1..L) (s=1..E) (j=1.M) (k=1..O) |

## B. OPTIMIZATION CRITERIA

The objective function (1) defines the aggregate costs of the entire chain and consists of five elements. The first is the fixed costs associated with the operation of the distributor involved in the delivery (e.g. distribution center, warehouse, etc.). The second part sets out the environmental costs of using various means of transport. They are dependent on the number of runs of the means of transport, and on the environmental levy, which may depend on the use of fossil fuels and carbon-dioxide emissions.

The third component determines the cost of supply from the manufacturer to the distributor. Another component is responsible for the costs of supply from the distributor to the end user (the store, the individual client, etc.). The last component of the objective function determines the cost of manufacturing the product by the given manufacturer.

$$\sum_{s=1}^{E} F_s * Tc_s + \sum_{d=1}^{L} Od_d \Big(\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M} Yb_{j,s,d}\Big) + \\ + \sum_{i=1}^{N}\sum_{s=1}^{E}\sum_{d=1}^{L} Koa_{i,s,d} + \sum_{s=1}^{E}\sum_{j=1}^{M}\sum_{d=1}^{L} Kog_{s,j,d} + \sum_{i=1}^{N}\sum_{k=1}^{O}(C_{ik} * \sum_{s=1}^{E}\sum_{d=1}^{L} X \quad (1)$$

Simplifying the objective function is possible. To do this, just reset the factors such as $F_s$, $Od_d$, $C_{i,k}$, etc. Nevertheless, such a complex objective function represents an integrated approach.

## C. CONSTRAINTS

The model was developed subject to constraints (2) .. (23). Constraint (2) specifies that all deliveries of product *k* produced by the manufacturer *i* and delivered to all distributors *s* using mode of transport *d* do not exceed the manufacturer's production capacity.

$$\sum_{s=1}^{E}\sum_{d=1}^{L}(X_{i,s,k,d}) \le W_{i,k} \text{ for } i = 1..N, k = 1..O \quad (2)$$

Constraint (3) covers all customer j demands for product $k(Z_{j,k})$ through the implementation of supply by distributors *s* (the values of decision variables $Y_{i,s,k,d}$). The constraint was designed to take into account the specificities of the distributors resulting from environmental or technological constraints (i.e., whether the distributor *s* can deliver the product *k* or not ).

$$\sum_{s=1}^{E}\sum_{d=1}^{L}(Y_{s,j,k,d} \cdot R_{s,k}) \ge Z_{j,k} \text{ for } j = 1..M, k = 1..O \quad (3)$$

The balance of each distributor s corresponds to constraint (4).

$$\sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d} = \sum_{j=1}^{M}\sum_{d=1}^{L} Y_{s,j,k,d} \text{ for } s = 1..E, k = 1..O \quad (4)$$

The delivery, dependent on technical capabilities – in the model represented by distributor's volume/capacity- is defined by constraint (5).

$$\sum_{k=1}^{O}(P_k \cdot \sum_{i=1}^{N}\sum_{d=1}^{L} X_{i,s,k,d}) \le Tc_s \cdot V_s \text{ for } s = 1..E \quad (5)$$

Constraint (6) ensures the fulfillment of the terms of delivery time.

$$Xa_{i,s,d} \cdot Tf_{i,s,a} + Xa_{i,s,d} \cdot Tp_{s,k} + Ya_{s,j,d} \cdot Tm_{s,j,d} \le Tc_{j,k} \quad \text{for } i = 1..N,\ s = 1..E,\ j = 1..M,\ k = 1..O, d = 1..L \tag{6}$$

Constraints (7a), (7b), (8) guarantee deliveries with available transport taken into account.

$$R1_{i,s,d} \cdot Xb_{i,s,d} \cdot Pt_d \ge X_{i,s,k,d} \cdot P_k \quad \text{for } i = 1..N, s = 1..E, k = 1..O, d = 1..L \tag{7a}$$

$$R2_{s,j,d} \cdot Yb_{s,j,d} \cdot Pt_d \ge Y_{s,j,k,d} \cdot P_k \quad \text{for } s = 1..E, j = 1..M, k = 1..O, d = 1..L \tag{7b}$$

$$\sum_{i=1}^{N}\sum_{s=1}^{E} Xb_{i,s,d} + \sum_{j=1}^{M}\sum_{s=1}^{E} Yb_{j,s,d} \le Zt_d \quad \text{for } d = 1..L \tag{8}$$

Constraints (9), (10), (11) set values of decision variables based on binary variables $Tc_s$, $Xa_{i,s,d}$, $Ya_{s,j,d}$ respectively.

$$\sum_{i=1}^{N}\sum_{d=1}^{L} Xb_{i,s,d} \le CW \cdot Tc_s \quad \text{for } s = 1..E \tag{9}$$

$$Xb_{i,s,d} \le CW \cdot Xa_{i,s,d} \quad \text{for } i = 1..N, s = 1..E, d = 1..L \tag{10}$$

$$Yb_{s,j,d} \le CW \cdot Ya_{s,j,d} \quad \text{for } s = 1..E, j = 1..M, d = 1..L \tag{11}$$

Dependencies (12) and (13) represent the relationship by which total costs are calculated. In general, these may be any linear functions. In this model represent the fixed costs associated with the course and the variables which depend on the size of the transported goods. If your country or region environmental costs are not fixed but dependent upon the traveled route, then part of the parameter *A* and *G* is an environmental charge.

$$Koa_{i,s,d} = A_{i,s,d} \cdot Xb_{i,s,d} + \sum_{k=1}^{O} K1_{i,s,k,d} \cdot X_{i,s,k,d} \quad \text{for } i = 1..N, s = 1..E, d = 1..L \tag{12}$$

$$Kog_{s,j,d} = G_{s,j,d} \cdot Yb_{j,s,d} + \sum_{k=1}^{O} K2_{s,j,k,d} \cdot Y_{s,j,k,d} \quad \text{for } s = 1..E, j = 1..M, d = 1..L \tag{13}$$

The remaining constraints (14)..(23) arise from the nature of the model.

$$X_{i,s,k,d} \ge 0 \text{ for } i = 1..N, s = 1..E, k = 1..0,\ d = 1..L \tag{14}$$

$$Xb_{i,s,d} \ge 0 \text{ for } i = 1..N, s = 1..E, d = 1..L, \tag{15}$$

$$Yb_{s,j,d} \ge 0 \text{ for } s = 1..E, j = 2..M,\ d = 1..L, \tag{16}$$

$$X_{i,s,k,d} \in C \ \text{ for } i = 1..N, s = 1..E,\ k = 1..0, d = 1..L, \tag{17}$$

$$Xb_{i,s,d} \in C \ \text{ for } i = 1..N, s = 1..E, d = 1..L \tag{18}$$

$$Y_{s,j,k,d} \in C \text{ for } s = 1..E, j = 1..M,\ k = 1..0, d = 1..L \tag{19}$$

$$Yb_{s,j,d} \in C \ \text{ for } s = 1..E, j = 1..M,\ d = 1..L, \tag{20}$$

$$Xa_{s,j,d} \in \{0,1\} \ \text{ for } i = 1..N, s = 1..E,\ d = 1..L, \tag{21}$$

$$Ya_{s,j,d} \in \{0,1\} \ \text{ for } s = 1..E, j = 1..M,\ d = 1..L, \tag{22}$$

$$Tc_s \in \{0,1\} \ \text{ for } s = 1..E \tag{23}$$

## 5. METHOD DEVELOPED

The model was implemented in "LINGO" by LINDO Systems [19]. "LINGO" Optimization Modeling Software is a powerful tool for building and solving mathematical optimization models. "LINGO" package provides the language to build optimization models and the editor program including all the necessary features and built-in "solvers" in a single integrated environment. "LINGO" is designed to model and solve linear, nonlinear, quadratic, integer and stochastic optimization problems. Model implementation is possible in two basic ways. The first way is to enter the model into the "LINGO" editor in the explicit form, that is a full function of the objective with all the constraints, parameters, etc. Although it is an intuitive approach consistent with the standard form of linear programming [15], it is not very useful in practice. This is due to the size of models implemented in practice. For the example presented in chapter Computational examples, the number of decision variables and constraints was 559 (361 integers) and 809, respectively. The other way is to use the "LINGO" language of mathematical modeling, an integral part of the "LINGO" package.

A general description of the “LINGO” solvers, tools and language has been shown in [19].

## 6. COMPUTATIONAL EXAMPLES

The cost optimization model (1)..(23) was implemented in the “LINGO” environment. Fig.5 shows the implicit model. Optimization was performed for six examples: P1,P2,P3,P4,P5 and P6.

All the cases relate to the supply chain with two manufacturers *(i=1..2)*, three distributors *(s=1..3)*, four customers *(j=1..4)*, three mode of transport *(d=1..3)* and five types of products *(k=1..5)*. The examples differ in capacity available to the distributors *($V_s$)*, number of transport units using mode of transport *d($Zt_d$)* and the environmental cost of using mode of transport *d ($Od_d$)*. The numeric data for all the model parameters from Table I are presented in Appendix A and [21]. In the examples, distributors capacities are: $V_1=V_2=V_3=$ 1500 (P1); $V_1=V_2=V_3=$ 1200 (P2),. $V_1=V_2=V_3=$ 950 (P3,P4,P5,P6), Parameters $Zt_d$ are: $Zt_1=14$ $Zt_2=Zt_3=$ 10 (P1,P2,P3), $Zt_1=10$ $Zt_2=8$ $Zt_3=$ 6 (P4), $Zt_1=12$ $Zt_2=10$ $Zt_3=$ 8 (P5), $Zt_1=8$ $Zt_2=6$ $Zt_3=$ 4 (P6). Other details are the same for all six examples.

Optimization follows the implementation of the model in the “LINGO” mathematical modeling language (Fig. 5).

```
Model:
Sets:
 products          /1..@file(size.ldt)/:p;
 factories         /1..@file(size.ldt)/;
 customers         /1..@file(size.ldt)/;
 distributors
   /1..@file(size.ldt)/:f,v,vx,T;
 mode       /1..@file(size.ldt)/:pt,zt,od,dx;
 orders            (customers,products):z,tc;
 production        (factories,products):c,w,wx;
 locations         (distributors,products):r,tp;
 route_1
   (factories,distributors,mode):a,r1,tf,Xb,Xa,k
o_1;
 route_2
   (distributors,customers,mode):g,r2,tm,Yb,Ya,k
o_2;
 delivery_1
   (factories,distributors,products,mode):X,k1;
 delivery_2
   (distributors,customers,products,mode): Y,k2;
 delivery_3  (factories,distributors,products);
 delivery_4  (distributors,customers,products);
EndSets
Data:
 p=@file(data.ldt);f=@file(data.ldt); r1
=@file(data.ldt);
v=@file(data.ldt);pt =@file(data.ldt); tp
=@file(data.ldt);
zt =@file(data.ldt);z=@file(data.ldt);
a=@file(data.ldt);
tc =@file(data.ldt);c=@file(data.ldt);
w=@file(data.ldt);
r=@file(data.ldt);
...............
EndData
! Objective function;
Min= @sum(distributors(s):f(s)*T(s))+
 @sum(delivery_1(i,s,k,d):ko_1(i,s,d))+
 @sum(delivery_2(s,j,k,d):ko_2(s,j,d))+

@sum(production(i,k):c(i,k)*(@sum(distributors(s
):
@sum(mode(d):X(i,s,k,d)))))+
 @sum(mode(d):od(d)*(@sum(factories(i):
@sum(distributors(s):Xb(i,s,d)))+@sum(distributo
rs(s):
@sum(customers(j):Yb(s,j,d)))));
@for(route_1(i,s,d):
ko_1(i,s,d)=a(i,s,d)*Xb(i,s,d)+
@sum(products(k):k1(i,s,k,d)*X(i,s,k,d)));
@for(route_2(s,j,d):
 ko_2(s,j,d)=g(s,j,d)*Yb(s,j,d)+
@sum(products(k):k2(s,j,k,d)*Y(s,j,k,d)));
! Constraint(1);
@for(production(i,k):@sum(distributors(s):
 @sum(mode(d): X(i,s,k,d))) <=w(i,k) ;
! calculation of the auxiliary variable Wx;
@sum(distributors
(s):@sum(mode(d):X(i,s,k,d)))=wx(i,k);); 
! Constraint (2);
@for(orders(j,k):@sum(distributors
 (s):@sum(mode(d):r(s,k)*Y(s,j,k,d)))>=z(j,k));
................
! binary Ts;
 @for(distributors (s):bin(T(s)));
End
```

Fig. 5. Part of the file scm.lng (the supply chain cost optimization model in LINGO).

Optimization results for all decision variables are shown in Appendix B (Tab. 4) for P4, P5, P6. The optimization process involves finding the global solution for the specific data Appendix A (Tab. 3), which in this case means the lowest cost of satisfying customer needs through the supply chain and amounts to $Fc^{opt}$=65345 for P1, $Fc^{opt}$=65450 for P2, $Fc^{opt}$=65565 for P3, $Fc^{opt}$=65875 for P4,. $Fc^{opt}$=65565 for P5 and $Fc^{opt}$=66555 for P6. Transportation networks diagrams showing the number of hauls (no number means one) corresponding to the optimal solutions for P1, P2, P3, P4, P5, P6 are shown sequentially in Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11.

At the same time, the specific values of decision variables that minimize the cost are determined (Tab.4 only for P4, P5, P6). These values represent, among other things, the volume of supplies from the manufacturer to the distributor of selected products using mode of transport *($X_{i,s,k,d}$)* and the supply of products from specific distributors to selected customers/recipients*($Y_{s,j,k,d}$)*.

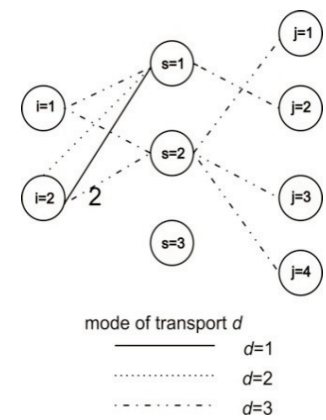


Fig. 6 Transport network of multimodal optimal solution ($Fc^{opt}$=65345) for P1

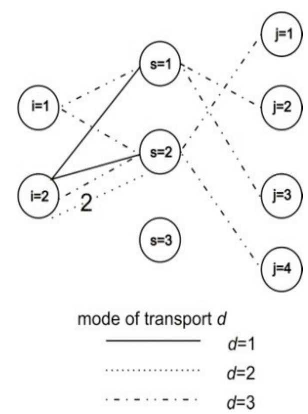


Fig. 7. Transport network of multimodal optimal solution ($Fc^{opt}$= 65450) for P2

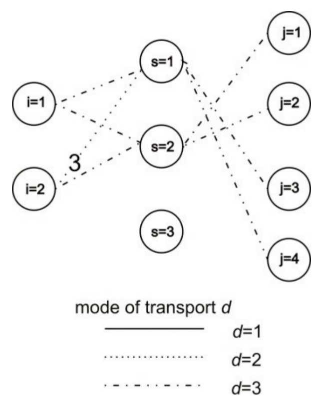


Fig. 8. Transport network of multimodal optimal solution ($Fc^{opt}$=65565) for P3

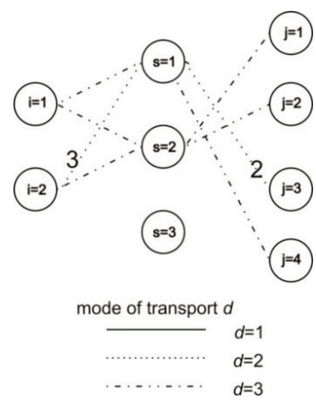


Fig. 9. Transport network of multimodal optimal solution ($Fc^{opt}$=65875) for P4

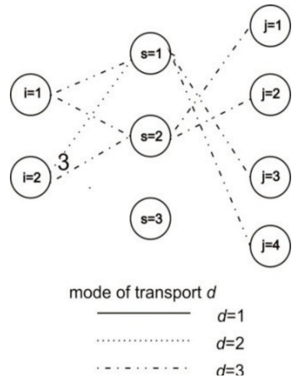


Fig. 10. Transport network of multimodal optimal solution ($Fc^{opt}$=65565) for P5

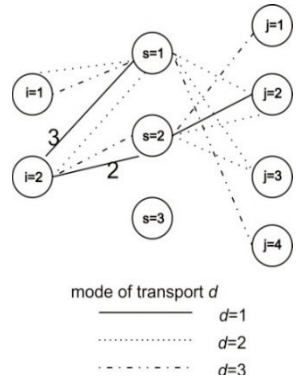


Fig. 11. Transport network of multimodal optimal solution ($Fc^{opt}$=66555) for P6

Based on these variables, one can make a decision at the current operating level.

The values of decision variables $Yb_{s,j,d}$, $Xb_{i,s,d}$ determine the number of runs using transport mode. Based on these variables one can make a decision from the tactical level, which includes the mode of transport and the need for different means of transport.

Another way to use the implemented model is to determine the effect of the change in the model parameters on the cost. One can analyze in detail the sensitivity of solutions depending on the parameters *Ko*, *A, G, C, T, V, Zt* etc. The article focused on the effect of parameter *V ,Zt, Od.* Numerous analyses of that kind can be conducted. For these studies and especially long-term decision support, the optimization model was extended at the implementation stage. Auxiliary variables were introduced at implementation stage $Vx_s$ (the value corresponds to the distributor's uptake capacity), $Wx_{ik}$ (production capacity utilization rates for manufacturer *i* of product *k*) and $Dx_d$ (the cumulative number of courses given mode *d*). The analysis of the decision variables values $Vx_s$ ,$Wx_{ik}$ and $Dx_d$ Appendix B (Tab. 5) has an impact on strategic decision making level of production capacity or distributor location, capacity, the number of transport units etc.

To estimate the influence of parameters (*V ,Zt, Od, COd*) on the optimal solution, additional experiments were carried out (S1..S7, Q1..Q6). Other parameters are taken as an example P1. The effect of selected parameters on the solution is presented on charts (Fig.12, Fig.13, Fig.14) and Tab. 6, Tab. 7 in Appendix B.

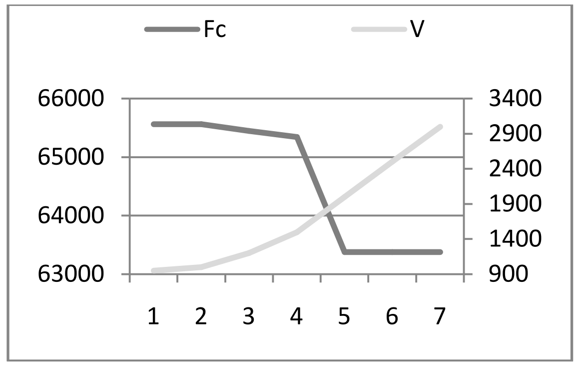


Fig. 12. The impact of parameter *V* for the solution (Tab.VI Appendix B)

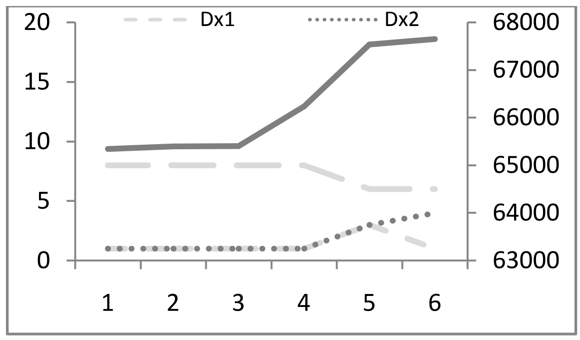


Fig. 13. The impact of parameter *Od* for the solution $Fc^{opt}$ and the cumulative number of courses (*Dx*) using mode of transport *d*

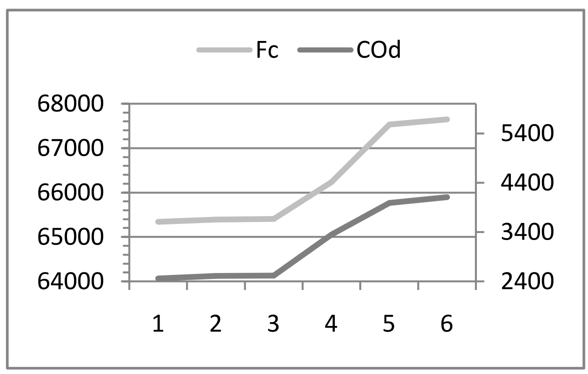


Fig. 14. The impact of parameter *COd* for the solution $Fc^{opt}$ (*COd*-the total cost of environmental charges for the means of transport used)

## 7. CONCLUSION

The paper presents the model of optimizing supply chain costs. Creating the model in the form of a MILP problem undoubtedly facilitates its solution using mathematical programming tools available in "LINGO" package [19] or "CPLEX" [20] and others. Of course, the model should be implemented in one, selected environment package. Implementation of the model in the "LINGO" package and the computational experiments were presented.

After the implementation of the language from the mathematical modeling package "LINGO", a number of computational experiments were conducted. Six of them in the form of examples P1 .. P6 were described in detail in the article. Based on these and other experimental results, analysis and previous experience, the authors can state that the proposed model and its implementation ensure a very large range of applications. First, they allow finding the distribution flows (decision variables) for the modeled supply chain, which minimize the global cost satisfying the customer demands. Second, they offer numerous possibilities for decision support in supply chain management through the solutions sensitivity analysis, determination of the range and quality of the impact of various parameters on the cost and even on the structure of the supply chain. The analysis presented in the article, only in terms of the capacity available to distributors, the number of transport units and environmental costs, fully confirms this statement.

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# APPENDIX A

## Data for computational examples P1, P2, P3, P4, P5, P6

Table 2. The set of parts of data tables for examples P1, P2, P3, P4, P5, P6

| s | $F_s$ | P1 - $V_s$ | P2 - $V_s$ | P3, P4, P5, P6 - $V_s$ |
|---|---|---|---|---|
| 1 | 1 200 | 1 500 | 1 200 | 950 |
| 2 | 1 500 | 1 500 | 1 200 | 950 |
| 3 | 1 000 | 1 500 | 1 200 | 950 |

| d | $Pt_d$ | $Ot_d$ | P1, P2, P3 -$Zd_d$ | P4- $Zd_d$ | P5- $Zd_d$ | P6- $Zd_d$ |
|---|---|---|---|---|---|---|
| 1 | 60 | 10 | 14 | 10 | 12 | 8 |
| 2 | 180 | 50 | 10 | 8 | 10 | 6 |
| 3 | 600 | 300 | 10 | 6 | 8 | 4 |

| j | k | $Z_{jk}$ | $Tc_{jk}$ | j | k | $Z_{jk}$ | $Tc_{jk}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 10 | 10 | 2 | 1 | 10 | 10 |
| 1 | 2 | 10 | 10 | 2 | 2 | 20 | 10 |
| 1 | 3 | 15 | 10 | 2 | 3 | 0 | 10 |
| 1 | 4 | 20 | 10 | 2 | 4 | 20 | 10 |
| 1 | 5 | 15 | 20 | 2 | 5 | 0 | 20 |
| 3 | 1 | 10 | 10 | 4 | 1 | 10 | 10 |
| 3 | 2 | 10 | 10 | 4 | 2 | 30 | 10 |
| 3 | 3 | 10 | 10 | 4 | 3 | 10 | 10 |
| 3 | 4 | 10 | 10 | 4 | 4 | 20 | 10 |
| 3 | 5 | 15 | 20 | 4 | 5 | 20 | 20 |

| i | k | $C_{ik}$ | $W_{ik}$ |
|---|---|---|---|
| 1 | 1 | 100 | 100 |
| 1 | 2 | 200 | 100 |
| 1 | 3 | 200 | 100 |
| 1 | 4 | 300 | 100 |
| 1 | 5 | 300 | 100 |
| 2 | 1 | 150 | 100 |
| 2 | 2 | 210 | 100 |
| 2 | 3 | 150 | 100 |
| 2 | 4 | 250 | 100 |
| 2 | 5 | 350 | 100 |

| k | $P_k$ |
|---|---|
| 1 | 5 |
| 2 | 5 |
| 3 | 10 |
| 4 | 10 |
| 5 | 5 |

| s | k | $R_{sk}$ | $Tp_{sk}$ | s | k | $R_{sk}$ | $Tp_{sk}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 |
| 1 | 2 | 1 | 2 | 2 | 2 | 1 | 1 |
| 1 | 3 | 1 | 2 | 2 | 3 | 1 | 1 |
| 1 | 4 | 1 | 2 | 2 | 4 | 1 | 1 |
| 1 | 5 | 1 | 2 | 2 | 5 | 1 | 1 |
| 3 | 1 | 1 | 3 | 3 | 3 | 1 | 3 |
| 3 | 2 | 1 | 3 | 3 | 4 | 1 | 3 |
| 3 | 5 | 1 | 3 | | | | |

# APPENDIX B

## Detailed results of optimization for computational examples P1, P2, P3, P4, P5, P6

Table 3. The set of parts of tables with results for examples P4, P5,P6

***Example P4 $Fc^{opt}$ = 65875***

| i | s | k | d | $X_{iskd}$ | i | s | d | $Xb_{isk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 20.00 | 1 | 1 | 3 | 1.00 |
| 1 | 1 | 2 | 3 | 50.00 | | | | |
| 1 | 1 | 5 | 3 | 20.00 | | | | |
| 1 | 2 | 1 | 3 | 20.00 | 1 | 2 | 3 | 1.00 |
| 1 | 2 | 2 | 3 | 20.00 | | | | |
| 1 | 2 | 5 | 3 | 30.00 | | | | |
| 2 | 1 | 3 | 2 | 10.00 | 2 | 1 | 2 | 3.00 |
| 2 | 1 | 4 | 2 | 40.00 | | | | |
| 2 | 2 | 3 | 3 | 25.00 | 2 | 2 | 3 | 1.00 |
| 2 | 2 | 4 | 3 | 30.00 | | | | |

| s | j | k | d | $Y_{sjkd}$ | s | j | d | $Yb_{sjk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 2 | 10.00 | 1 | 3 | 2 | 2 |
| 1 | 3 | 2 | 2 | 20.00 | | | | |
| 1 | 3 | 4 | 2 | 20.00 | | | | |
| 1 | 4 | 1 | 3 | 10.00 | 1 | 4 | 3 | 1 |
| 1 | 4 | 2 | 3 | 30.00 | | | | |
| 1 | 4 | 3 | 3 | 10.00 | | | | |
| 1 | 4 | 4 | 3 | 20.00 | | | | |
| 1 | 4 | 5 | 3 | 20.00 | | | | |
| 2 | 1 | 1 | 3 | 10.00 | 2 | 1 | 3 | 1 |
| 2 | 1 | 2 | 3 | 10.00 | | | | |
| 2 | 1 | 3 | 3 | 15.00 | | | | |
| 2 | 1 | 4 | 3 | 20.00 | | | | |
| 2 | 1 | 5 | 3 | 15.00 | | | | |
| 2 | 2 | 1 | 3 | 10.00 | 2 | 2 | 3 | 1 |
| 2 | 2 | 2 | 3 | 10.00 | | | | |
| 2 | 2 | 3 | 3 | 10.00 | | | | |
| 2 | 2 | 4 | 3 | 10.00 | | | | |
| 2 | 2 | 5 | 3 | 15.00 | | | | |

***Example P5 $Fc^{opt}$ = 65565***

| i | s | k | d | $X_{iskd}$ | i | s | d | $Xb_{isk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 20.00 | 1 | 1 | 3 | 1 |
| 1 | 1 | 2 | 3 | 50.00 | | | | |
| 1 | 1 | 5 | 3 | 20.00 | | | | |
| 1 | 2 | 1 | 3 | 20.00 | 1 | 2 | 3 | 1 |
| 1 | 2 | 2 | 3 | 20.00 | | | | |
| 1 | 2 | 5 | 3 | 30.00 | | | | |
| 2 | 1 | 3 | 2 | 10.00 | 2 | 1 | 2 | 3 |
| 2 | 1 | 4 | 2 | 40.00 | | | | |
| 2 | 2 | 3 | 3 | 25.00 | 2 | 2 | 3 | 1 |
| 2 | 2 | 4 | 3 | 30.00 | | | | |

| s | j | k | d | $Y_{sjkd}$ | s | j | D | $Yb_{sjk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 3 | 10.00 | 1 | 3 | 3 | 1 |
| 1 | 3 | 2 | 3 | 20.00 | | | | |
| 1 | 3 | 4 | 3 | 20.00 | | | | |
| 1 | 4 | 1 | 3 | 10.00 | 1 | 4 | 3 | 1 |
| 1 | 4 | 2 | 3 | 30.00 | | | | |
| 1 | 4 | 3 | 3 | 10.00 | | | | |
| 1 | 4 | 4 | 3 | 20.00 | | | | |
| 1 | 4 | 5 | 3 | 20.00 | | | | |
| 2 | 1 | 1 | 3 | 10.00 | 2 | 1 | 3 | 1 |
| 2 | 1 | 2 | 3 | 10.00 | | | | |
| 2 | 1 | 3 | 3 | 15.00 | | | | |
| 2 | 1 | 4 | 3 | 20.00 | | | | |
| 2 | 1 | 5 | 3 | 15.00 | | | | |
| 2 | 2 | 1 | 3 | 10.00 | 2 | 2 | 3 | 1 |
| 2 | 2 | 2 | 3 | 10.00 | | | | |
| 2 | 2 | 3 | 3 | 10.00 | | | | |
| 2 | 2 | 4 | 3 | 10.00 | | | | |
| 2 | 2 | 5 | 3 | 15.00 | | | | |

*Example P6 $Fc^{opt}$ = 66 555*

| i | s | k | d | $X_{iskd}$ | i | s | d | $Xb_{isk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 25.00 | 1 | 1 | 3 | 1 |
| 1 | 1 | 2 | 3 | 60.00 | | | | |
| 1 | 1 | 5 | 3 | 35.00 | | | | |
| 1 | 2 | 1 | 2 | 15.00 | 1 | 2 | 2 | 1 |
| 1 | 2 | 2 | 2 | 6.00 | | | | |
| 1 | 2 | 5 | 2 | 15.00 | | | | |
| 2 | 1 | 3 | 1 | 10.00 | 2 | 1 | 1 | 3 |
| 2 | 1 | 4 | 1 | 7.00 | | | | |
| 2 | 1 | 4 | 2 | 18.00 | 2 | 1 | 2 | 1 |
| 2 | 2 | 2 | 3 | 4.00 | 2 | 2 | 3 | 1 |
| 2 | 2 | 3 | 3 | 25.00 | | | | |
| 2 | 2 | 4 | 3 | 33.00 | | | | |
| 2 | 2 | 4 | 1 | 12.00 | 2 | 2 | 1 | 2 |

| s | j | k | d | $Y_{sjkd}$ | S | j | d | $Yb_{sjk}$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 2 | 5.00 | 1 | 2 | 2 | 1 |
| 1 | 2 | 2 | 2 | 10.00 | | | | |
| 1 | 2 | 4 | 2 | 3.00 | | | | |
| 1 | 2 | 5 | 2 | 15.00 | | | | |
| 1 | 3 | 1 | 2 | 10.00 | 1 | 3 | 2 | 1 |
| 1 | 3 | 2 | 2 | 20.00 | | | | |
| 1 | 3 | 4 | 2 | 2.00 | | | | |
| 1 | 4 | 1 | 3 | 10.00 | 1 | 4 | 3 | 1 |
| 1 | 4 | 2 | 3 | 30.00 | | | | |
| 1 | 4 | 3 | 3 | 10.00 | | | | |
| 1 | 4 | 4 | 3 | 20.00 | | | | |
| 1 | 4 | 5 | 3 | 20.00 | | | | |
| 2 | 1 | 1 | 3 | 10.00 | 2 | 1 | 3 | 1 |
| 2 | 1 | 2 | 3 | 10.00 | | | | |
| 2 | 1 | 3 | 3 | 15.00 | | | | |
| 2 | 1 | 4 | 3 | 20.00 | | | | |
| 2 | 1 | 5 | 3 | 15.00 | | | | |
| 2 | 2 | 1 | 1 | 1.00 | 2 | 2 | 1 | 1 |
| 2 | 2 | 4 | 1 | 1.00 | | | | |
| 2 | 2 | 3 | 2 | 10.00 | 2 | 2 | 2 | 1 |
| 2 | 2 | 1 | 2 | 4.00 | | | | |
| 2 | 2 | 4 | 2 | 6.00 | | | | |
| 2 | 3 | 4 | 2 | 18.00 | 2 | 3 | 2 | 1 |

Table 4. The set of parts of tables with results for examples P1, P2, P3, P4, P5, P6 –decision variables $Vx_s$, $Dx_d$

| P1 | | P2 | | P3 | | P4 | | P5 | | P6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| s | $Vx_s$ | s | $Vx_s$ | s | $Vx_s$ | s | $Vx_s$ | s | $Vx_s$ | s | $Vx_s$ |
| 1 | 375 | 1 | 725 | 1 | 950 | 1 | 950 | 1 | 950 | 1 | 950 |
| 2 | 1475 | 2 | 1125 | 2 | 900 | 2 | 900 | 2 | 900 | 2 | 900 |
| 3 | 0 | 3 | 0 | 3 | 0 | 3 | 0 | 3 | 0 | 3 | 0 |

| P1 | | P2 | | P3 | | P4 | | P5 | | P6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| s | $dx_s$ | s | $dx_s$ | s | $dx_s$ | s | $dx_s$ | s | $dx_s$ | s | $dx_s$ |
| 1 | 1 | 1 | 2 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 6 |
| 2 | 1 | 2 | 2 | 2 | 3 | 2 | 5 | 2 | 3 | 2 | 6 |
| 3 | 8 | 3 | 7 | 3 | 7 | 3 | 6 | 3 | 7 | 3 | 4 |

Table 5. The tables with results for examples from S1 to S7 (parameter *V*)

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Fc | 65565 | 65565 | 65450 | 65345 | 63380 | 63380 | 63380 |
| V | 950 | 1000 | 1200 | 1500 | 2000 | 2500 | 3000 |

Table 6.The tables with results for examples from Q1 to Q6 (parameters *Od,COd*)

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Fc | 65345 | 65395 | 65405 | 66235 | 67535 | 67650 |
| Dx1 | 1 | 1 | 1 | 1 | 3 | 1 |

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