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SPACE TESTS OF THE STRONG EQUIVALENCE PRINCIPLE: BEPICOLOMBO AND THE SUN-EARTH LAGRANGIAN POINTS OPPORTUNITY

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The validity of General Relativity, after 100 years, is supported by solid experimental evidence. However, there is a lot of interest in pushing the limits of precision by other experiments. Here we focus our attention on the equivalence principle, in particular the strong form. The results of ground experiments and lunar laser ranging have provided the best upper limit on the Nordtvedt parameter η that models deviations from the strong equivalence principle. Its uncertainty is currently $\sigma[\eta] = 4.4 \times 10^{-4}$. In the first part of this paper we will describe the experiment, to measure η , that will be done by the future mission BepiColombo. The expected precision on η is $\approx 10^{-5}$. In the second part we will consider the ranging between the Earth and a spacecraft orbiting near the Sun-Earth Lagrangian points to get an independent measurement of η . In this case, we forecast a constraint similar to that achieved by lunar laser ranging.

Keywords: Relativity; Radioscience; Mercury.

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1. Introduction

The equivalence principle (EP)¹ states the equivalence between inertial and gravitational mass. This fact is a mere coincidence in classical physics, but it has some important consequences, for example:

- the free fall of *any* object in the same gravity field depends only on their initial status and not on their composition or structure;

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- it is impossible to detect the difference between a uniform static gravitational field and a uniform acceleration: free-fall and inertial motion are physically equivalent.

As a consequence, the EP allows the geometrical description of spacetime, which is at the basis of General Relativity (GR).

The weak form of the EP (WEP) is limited to strong and electroweak interactions. It can be verified by measuring the free fall of test masses with different chemical compositions. Tests are performed on ground with, for instance, torsion balances² or in space with low Earth orbits (e.g. with the MICROSCOPE mission³). The strong form (SEP) extends the validity of the weak principle to self-gravitating bodies. The EP violation for the body i can be parametrized as follows^{4,5}

$$m_i^G = m_i^I(1 + \delta_i + \eta \Omega_i), \quad (1)$$

where m_i^I (m_i^G) is the inertial (gravitational) mass, and

$$\Omega_i = \frac{E_g}{m_i^I c^2} = -\frac{G}{2m_i^I c^2} \iint \frac{dm_i'^G dm_i''^G}{|\mathbf{r}' - \mathbf{r}''|}, \quad (2)$$

where c is the speed of light, and E_g is the self-gravity energy, which is obtained by double-integrating over the mass of the body. The WEP involves only the case $\Omega_i = 0$ and corresponds to $\delta_i = 0$, while the SEP is valid, for each Ω_i , when both δ_i and η are equal to zero.

With experiments on ground, the typical Ω_i can be so small (see Table 1) that only the WEP can effectively be tested. The only means by which the SEP can be constrained is evidently in space involving celestial bodies.

Table 1. Self-gravity coefficients Ω_i for some celestial bodies and a reference test mass.

Sun	-3.52×10^{-6}
Jupiter	-1.21×10^{-8}
Earth	-4.64×10^{-10}
Moon	-1.88×10^{-11}
test mass (1 kg, size 5 cm)	$\approx -8.90 \times 10^{-27}$

Thanks to retroreflectors placed on the facing side of the Moon it is possible to measure the Earth-Moon distance and detect a possible SEP violation signal. This experiment was proposed by Nordtvedt.⁶ In this case, a violation of the SEP will introduce a signal in the Earth-Moon range, its amplitude being proportional to $\Omega_{\text{Earth}} - \Omega_{\text{Moon}}$.

Over the last 46 years, the Lunar Laser Ranging (LLR) project has carried out a long sequence of range measurements, and the precision on the Earth-Moon relative differential accelerations is currently⁷ $\sigma[\delta a/a_{\text{sun}}] = 1.3 \times 10^{-13}$, but this result includes possible violations of both SEP and WEP. Since ground experiments can test only the weak form of the EP, the parameter η can be measured only by

using the results of both experiments, ground and LLR. No disproofs of the SEP have still been found and the error associated to η is currently⁷ $\sigma[\eta] = 4.4 \times 10^{-4}$. The BepiColombo mission is expected to improve this result by about an order of magnitude⁸ – a prediction is given in the first part of this paper. Instead, an alternative ranging experiment towards the Sun-Earth Lagrangian points – recently proposed in Ref. 9 – could easily reach the LLR’s performance in a very short time span, which is investigated in the second part of the paper.

Therefore we will describe two experiments for the estimation of η . The first one in Section 2 is the well-known Relativity experiment of the BepiColombo mission, while in Section 3 we will study the same measurement performed on range data between the Earth and a spacecraft (SC) orbiting around a Sun-Earth Lagrangian point.

2. MORE with BepiColombo

BepiColombo (BC) is a joint ESA/JAXA mission to Mercury with challenging objectives regarding geophysics, geodesy, and fundamental physics.¹⁰ Currently, the launch is scheduled for the end of 2018, with a nominal duration of one year plus a possible one-year extension.

The Mercury Orbiter Radioscience Experiment (MORE) is one of the on-board experiments that focus on gravimetry, rotation and Relativity.^{4, 11, 12} The goal is the measurement of key parameters by means of orbit determination techniques using the Earth-MPO^a radio link observables, i.e. range and range rate. The parameters for gravitation and rotation experiments are the Mercury gravity field coefficients, Love numbers, obliquity and libration. Instead the Relativity experiment consists in the measurement of the Parametrized Post-Newtonian (PPN) parameters, which account for possible small deviations from GR – η is one of them.

All parameters will be estimated by a global nonlinear least-squares fitting of all the *observed* signals (range, range-rate, accelerometer readings, etc.) along with the *computed* signals that are calculated by using mathematical models as accurate as possible. The main characteristics of the Radioscience experiment are summarized in Table 2. For further details see Refs 4, 13. The observed data of gravity and rotation experiments are primarily range-rate signals, which are poorly correlated with those of the Relativity experiment, i.e. Earth-MPO range only, because the frequency domains are very different. Since we are interested in the Relativity experiment, we can neglect the motion of the MPO around Mercury (the orbital period is approximately 2 hrs) and consider only the Mercury-Earth range.

2.1. Analytical model and sources of uncertainties

We aim at calculating the expected root-mean-square (RMS) error of η after the whole duration of the BC mission. Since the data are obviously not available, we

^aMercury Planetary Orbiter.

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Table 2. Summary of the main characteristics of the radioscience experiments on-board Bepi-Colombo.

	gravimetry	rotation	Relativity
parameters	- gravity field coeffs (up to the 25th deg.) - k_2	- longitude libration - obliquity	- $\gamma, \beta, \alpha_1, \alpha_2, \eta$ - $\mu_0, \dot{\mu}_0/\mu_0, J_{2\odot}$ - initial cond. of Earth and Mercury
observables	range-rate	range-rate	range
precision	3.0×10^{-4} cm/s @ 1000 s	3.0×10^{-4} cm/s @ 1000 s	30 cm @ 300 s
freq. domain	$\gtrsim 1.2 \times 10^{-4}$ Hz (MPO mean motion)	$\gtrsim 1.2 \times 10^{-4}$ Hz (MPO mean motion)	$\approx 10^{-7}$ Hz (planetary mean motions)

need to simulate them. To this end, we are going present a simplified heliocentric analytical model that yields the perturbations on the Earth-Mercury range due to η and all the parameters that are expected to correlate with. This is a typical Fisher/covariance analysis: the RMS of the parameters will be given by the square root of the diagonal elements of the covariance matrix.

We adopt the notation of Ref. 14: we define $\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i$ and $r_{ij} = \|\mathbf{r}_{ij}\|$, where \mathbf{r}_i is the coordinate of the i th-body in an inertial reference frame. Planets are numbered from 1 (Mercury) to 8 (Neptune), while 0 refers to the Sun. We also define the gravitational parameters for all bodies in the same way: $\mu_i = Gm_i^G$. The equations of motion for the i th-planet i , in the case $\eta \neq 0$, are^{4,8,9,13,15,16}

$$\ddot{\mathbf{r}}_{0i} = -\frac{\mu^*}{r_{0i}^3} \mathbf{r}_{0i} + \sum_{j \neq i \neq 0} \mu_j \left[(1 + \eta \Omega_i) \frac{\mathbf{r}_{ij}}{r_{ij}^3} - (1 + \eta \Omega_0) \frac{\mathbf{r}_{0j}}{r_{0j}^3} \right], \quad (3)$$

where the summation includes all solar system bodies (planets, dwarves planets, asteroids, etc.), and $\mu^* = \mu_0 + \mu_i + \eta(\mu_i \Omega_0 + \mu_0 \Omega_i)$. We can write a similar equation for body k and afterwards calculate the range $\rho_{ik} = \|\mathbf{r}_{0i} - \mathbf{r}_{0k}\|$ where i and k are Earth and Mercury. Since $\Omega_i \ll \Omega_0$ for all i , the leading term is the last one, which is proportional to Ω_0 . It is an apparent term, essentially a perturbation on the acceleration of the Sun with respect to the Solar System Barycenter (SSB). Note that there is a non-zero signal even if $\Omega_i = 0$, which means that the experiment can be done also if the body i is a drag-free test mass, e.g. a SC with an onboard accelerometer (see Section 3). It is worth mentioning that the signals due to other PPN parameters, such as $\beta, \gamma, \alpha_1, \alpha_2$, along with the effect due to ζ (the rate of change of μ_0), $J_{2\odot}$ (gravitational “flattening” of the Sun) and the initial conditions of Earth and Mercury (see Ref. 8 for details), must all be calculated and included in the global fit. Also from Eq. (3), a high correlation among planetary perturbations (proportional to $\mu_j s$) and SEP violation is evident.

In order to avoid systematic effects, the $\mu_j s$ must be added to the set of parameters to be estimated, and their errors must be taken into account in terms of prior constraints in the global covariance analysis. Current uncertainties of planetary $\mu_j s$ range from 2.8×10^{-4} (Mars) to $10.5 \text{ km}^3/\text{s}^2$ (Neptune).¹⁷ Regarding asteroids,

their relative errors can be very large (50% or more).

To summarize, we will calculate the signatures on the Earth-Mercury range due to all the following effects:

- (1) initial conditions of Earth and Mercury;
- (2) SEP violation – free parameter: η ;
- (3) planets/dwarf planets/asteroids – free parameters: μ_j ;
- (4) secular variation of the Sun's gravitational parameter μ_0 – free parameters: $\delta\mu_0$ (bias of the measured μ_0 from the true value at the starting epoch), and its rate of change in time $\zeta = \dot{\mu}_0/\mu_0$,
- (5) PPN – free parameter: $\bar{\beta} = \beta - 1$,
- (6) Sun's quadrupole coefficient: free parameter $J_{2\odot}$, whereas higher order terms are negligible.

The PPN parameter γ , which is related to the curvature produced by unit rest mass, has not been considered here for simplicity. However, this is not reductive since the best estimate of γ ($\sigma[\gamma] = 2.0 \times 10^{-6}$) is expected to be given right after the dedicated superior conjunction experiment (SCE) during the cruise phase of BC. The value of the Nordtvedt parameter can be derived from the Nordtvedt quation

$$\eta = 4\beta - \gamma - 3, \quad (4)$$

which will be used as a prior. We also neglect the preferred frame parameters α_1 and α_2 since they are poorly correlated with the other parameters of the Relativity experiment, in particular η . For more details compare the results of experiments A, B, C and D in Ref. 4. Finally, we assume that the unperturbed orbits of planets and asteroids are circular with radius R_{0i} , and co-planar. We define \mathbf{q} as the vector of all N_p parameters, $q_m \delta \mathbf{r}_{i,m}$ is the displacement from the circular reference orbit $\mathbf{R}_i = R_{0i} \mathbf{u}_r^i$ for the i th-body due to the (linearized) force $q_m \delta \mathbf{f}_{i,m}$ relative to the (small) parameter q_m .

The procedure is as follows:

- (1) write the heliocentric position of the i th-body as

$$\mathbf{r}_i = \mathbf{R}_i + \sum_{n=1}^{N_p} q_n \delta \mathbf{r}_{i,n}; \quad (5)$$

- (2) for each q_m , decompose $\delta \mathbf{r}_{i,m}$ and the perturbative force $\delta \mathbf{f}_{i,m}$ into radial, along-track and out-of-plane components

$$\begin{aligned} \delta \mathbf{r}_{i,m} &= x_i \mathbf{u}_r^i + y_i \mathbf{u}_t^i + z_i \mathbf{u}_w^i, \\ \delta \mathbf{f}_{i,m} &= R_m^i \mathbf{u}_r^i + T_m^i \mathbf{u}_t^i + W_m^i \mathbf{u}_w^i; \end{aligned} \quad (6)$$

- (3) solve the Hill's equations for $i = 1$ and $i = 3$

$$\begin{aligned} \ddot{x}_i - 2n_i \dot{y}_i - 3n_i^2 x_i &= R_m^i, \\ \ddot{y}_i + 2n_i \dot{x}_i &= T_m^i, \\ \ddot{z}_i + n_i^2 z_i &= W_m^i, \end{aligned} \quad (7)$$

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where n_i is the mean motion of the i th-body;

(4) finally calculate the Earth-Mercury range as

$$\rho_{13}(t, \mathbf{q}) = \|\mathbf{r}_{13}\| \approx R_{13} + \sum_n q_n \frac{\delta \mathbf{r}_{13,n} \cdot \mathbf{R}_{13}}{R_{13}} \quad (8)$$

where $\delta \mathbf{r}_{13,n} = \delta \mathbf{r}_{3,n} - \delta \mathbf{r}_{1,n}$ and the factor $1/R_{13}$ can be rewritten in Legendre polynomials P_n

$$\frac{1}{R_{13}} = \frac{1}{R_{03}} \sum_{l=0}^{\infty} \left(\frac{R_{01}}{R_{03}} \right)^l P_l(\cos \Phi_{13}), \quad (9)$$

where $\Phi_{ij} = (n_j - n_i)t + \varphi_j - \varphi_i$.

Due to visibility windows, range and range-rate data contain several gaps. A gap occurs approximately every day and lasts about 9.3 h. A low-frequency sampling ($f_s = 10^{-4}$ Hz) is therefore sufficient for our purposes since the involved signals have frequencies of the same order of planetary mean motions. We can then calculate the range at epochs t_i and obtain the looked-after $N_p \times N_p$ Fisher matrix, or *normal matrix*. Including all prior information, it is given by

$$F_{jk} = \sum_{i=1}^N \frac{1}{\sigma_i^2} \frac{\partial \rho_{13}(t_i, \mathbf{q}_0)}{\partial q_j} \frac{\partial \rho_{13}(t_i, \mathbf{q}_0)}{\partial q_k} + \frac{1}{2} \frac{\partial^2 P(\mathbf{q})}{\partial q_j \partial q_k}, \quad (10)$$

where N is the number of range measurements; σ_i is the RMS error on each data point^b; $P(\mathbf{q})$ is a function that contains all prior information (the Nordtvedt equation Eq. (4) and the uncertainties on all the μ_m s) and is given by

$$P(\mathbf{q}) = \frac{(\eta - 4\bar{\beta})^2}{\sigma_N^2} + \sum_m \frac{(\mu_m - \mu_m^P)^2}{\sigma_{\mu_m}^2}; \quad (11)$$

μ_m^P are the measured values of μ_m and σ_{μ_m} are the corresponding errors; the summation over m is extended to all *GMS*; $\sigma_N = 2.0 \times 10^{-6}$ is the expected RMS error of γ after the expected performance of the SCE. The inverse of F_{jk} yields the covariance matrix, whose diagonal elements give us the expected RMS errors, and correlations, of all the parameters.

2.2. Results

As well as standard parameters, we include the μ_j s of all the planets and the 343 more massive asteroids (the total number of parameters was 362). Since some of the μ_j s are expected to be improved by GAIA¹⁹ and JUICE, we calculate the global covariance by using the expected RMS errors of μ_j at the epoch of the mission. The RMS error of all parameters, including the initial conditions of Mercury and Earth, are reported in Table 3. Regarding the SEP violation, we found $\sigma[\eta] = 3.13 \times 10^{-5}$.

^bFor the Ka-band we adopted $\sigma_i = 15\sqrt{300f_s}$ cm = 2.6 cm.¹⁸

If we were to compare this result with the “idealistic case” where the μ_j s have all zero errors,^{4,20,21} we would find that the uncertainties degrade the precision of most of the PPN parameters by about an order of magnitude. However, since the current RMS error of η , from LLR measurements, is $\sigma[\eta] = 4.4 \times 10^{-4}$, we can conclude that the BC Relativity experiment will improve the current constraint on η by a factor of 10 at least, having included uncertainties on the planetary masses.

Table 3. Expected formal errors for the Relativity experiment on-board Bepi-Colombo.

parameter	units	RMS error
β	-	7.81×10^{-6}
η	-	3.13×10^{-5}
μ_0	[cm ³ s ⁻²]	5.50×10^{13}
$J_{2\odot}$	-	8.03×10^{-10}
$\zeta = \dot{\mu}_0/\mu_0$	[yr ⁻¹]	1.78×10^{-14}
X_1	[cm]	2.49×10^3
Y_1	[cm]	1.18×10^4
Z_1	[cm]	5.15
\dot{X}_1	[cm s ⁻¹]	2.36×10^{-3}
\dot{Y}_1	[cm s ⁻¹]	1.68×10^{-3}
\dot{Z}_1	[cm s ⁻¹]	4.72×10^{-6}
\dot{X}_3	[cm s ⁻¹]	1.77×10^{-3}
\dot{Y}_3	[cm s ⁻¹]	9.41×10^{-5}

3. An opportunity with the Lagrangian points

When testing for a SEP violation, the advantage of the ranging between two planets over that between Earth and Moon is twofold: a longer baseline (≈ 1 vs $\approx 3 \times 10^{-3}$ AU) and $\delta a/a_{\text{sun}} \propto \Omega_0$ instead of $\Omega_{\text{earth}} - \Omega_{\text{moon}}$. This in turn implies a much bigger ranging signal amplitude (about three orders of magnitudes better than the Nordtvedt effect^{15,22}). In fact, even if the time span and the precision of the data will be worse, a bigger self-energy and a stronger signal will certainly allow better measurements of η . For example, consider the BC experiment: the expected measurement precision on the SEP is $\sigma[\delta a/a_{\text{sun}}] \approx 10^{-11}$, which will be roughly two orders of magnitude worse than WEP measurements achieved by LLR and torsion balances experiments.² However, since the signal is $\propto \Omega_0$, the parameter η will be constrained with an accuracy of 10^{-5} – 10^{-6} (see Section 2 and also Ref. 4), which is of course better than LLR. This is also the case of the Lagrangian points ranging, with the only difference that a smaller baseline will give us an RMS error that will be similar in magnitude to LLR.

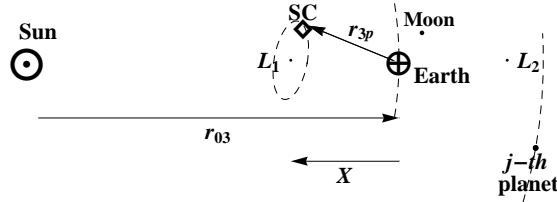


Fig. 1. Spacecraft ranging towards L_1 or L_2 as a means by which to test the SEP (not in scale). We calculate the SEP signature as a perturbation on the Earth's orbit around the Sun (\mathbf{r}_{03}) as well as on the SC ranging (\mathbf{r}_{3p}). We also include perturbations from other planets.

3.1. Detailed calculations

In the Earth's reference frame, the positions of the collinear Lagrangian points are the solutions of the following equation

$$-\frac{\mu_0}{|R-X|^3}(R-X) + \mu_3 \left(\frac{X}{|X|^3} - \frac{1}{R^2} \right) + n_3^2(R-X) = 0, \quad (12)$$

where R is the Earth-Sun distance, and n_3 is the mean motion of the Earth. Eq. (12) has three solutions: $X_{1,2} \approx \pm 0.01$ AU that correspond to L_1 and L_2 , and $X_3 \approx 2$ AU that corresponds to L_3 . We will consider only the case of L_1 and L_2 as these are the spots where many missions fly to. Consider a SC, hereafter identified with the index p , near L_1 (or L_2). Its mass and self-gravity energy are negligible with respect to those of the Sun and all planets. The SC's equation of motion relative to the Sun can be obtained by Eq. (3) after this substitution: $(\Omega_3, \mu_3, \mathbf{r}_{03}, \mathbf{r}_{3j}) \rightarrow (0, 0, \mathbf{r}_{0p}, \mathbf{r}_{pj})$. We subtract the SC's equation of motion from Eq. (3) to finally derive the relative motion, \mathbf{r}_{3p} , between the SC and Earth, which is given by

$$\ddot{\mathbf{r}}_{3p} = -\mu_0 \left(\frac{\mathbf{r}_{0p}}{r_{0p}^3} - \frac{\mathbf{r}_{03}}{r_{03}^3} \right) - \mu_3 \frac{\mathbf{r}_{3p}}{r_{3p}^3} + \sum_{j \neq 0,3} \mu_j \left(\frac{\mathbf{r}_{pj}}{r_{pj}^3} - \frac{\mathbf{r}_{3j}}{r_{3j}^3} \right) + \eta \Omega_3 \sum_{j \neq 3} \mu_j \frac{\mathbf{r}_{j3}}{r_{j3}^3}, \quad (13)$$

where $\mathbf{r}_{0p} = \mathbf{r}_{03} + \mathbf{r}_{3p}$. It is worth noting that we are in fact solving the equation of motion for the observed SC ranging, \mathbf{r}_{3p} . As it was done for the Earth-Mercury range in the previous section, we decompose $\mathbf{r}_{3p} = \{\delta x, \delta y\}$ in radial and along-track components (but now only δx can be measured). For simplicity we assume that the SC is very near to the Lagrangian point, such that the gravity field can be linearized in this case, and all trajectories are Lissajous orbits. Details of the calculation can be found in Ref. 9. In Fig. 2 we plot δx (normalised to $\eta = 1$) for the two scenarios of a SC orbiting around either L_1 or L_2 .

In order to compute our prediction for a measurement of the SEP around the Lagrangian point, we assume we have N equally-spaced observations of the SC's range distance, over a total observation of $T = 5$ yr, sampling interval $\delta t = 1$ h^c. We

^cHereafter we assume an hour integration time for all range measurements.

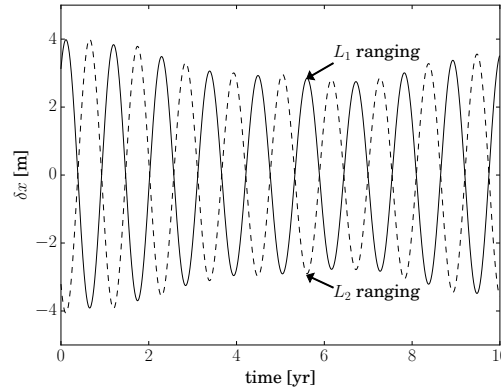


Fig. 2. Range perturbations (normalised to $\eta = 1$) for a SC orbiting either L_1 or L_2 .

can then calculate the Fisher matrix from Eq. (10). The free parameters considered in our analysis are: η , the initial position and velocity of the Earth and the initial position and velocity of the SC. We distinguish between two possible scenarios. In the *realistic scenario* (A) we use a nominal range error typical for two-way ranging in the X-band, $\sigma_i = 0.1$ m^d. Additionally, we assume the following prior uncertainties on the orbital initial conditions:

- (1) 2 m and 3×10^{-5} m/s for the Earth's heliocentric radial position and velocity, from a great abundance of radio tracking data;²⁴
- (2) 145 m for the Earth's heliocentric along-track position as this is less well constrained;²⁴
- (3) no assumed prior on both the Earth's heliocentric along-track velocity as this is very weakly constrained by current data, and the parameters of the SC's orbit relative to Earth.

In the *optimistic scenario* (B) we use the range error typical of the Ka-band, $\sigma_i = 0.04$ m, as well as a factor 10 improvement in the knowledge of the Earth's initial position and velocity, 0.2 m and 3×10^{-6} m/s, which is likely to be achieved in the near future.

3.2. Results

Neglecting errors in planetary masses and ephemerides, we forecast $\sigma[\eta] = 6.4(2.0) \times 10^{-4}$ (5 yr integration time) via Earth- L_1 ranging in a realistic (optimistic) scenario depending on current (future) range capabilities and knowledge of the Earth's

^dAs obtained from a degradation of a conservative factor 2.5 of the Ka-band range error $\sigma_i = 0.15\sqrt{300/\delta t} \approx 0.04$ m,^{20,21,23} owing to the lower frequencies typical of the X-band.

ephemerides. A combined measurement, $L_1 + L_2$, gives instead an improved constraint of $4.8(1.7) \times 10^{-4}$, which would be comparable with those already achieved by LLR. It is worth noting that the performances could be much improved if data were integrated over time and over the number of satellites flying around either of the two Lagrangian points. We point out that some systematics (gravitational perturbations of other planets or figure effects) are much more in control compared to other experiments. This SC ranging would be a new and complementary probe to constrain the strong equivalence principle in space.

3.3. Conclusions

In this work we described two experiments devoted to testing the SEP in space. In both cases we performed a global covariance analysis based on simulated data.

The first test is the BC Relativity experiment: we calculated the effect of the uncertainties on the masses of the Solar System's bodies on the estimation of PPN parameters. We forecast a degradation for the RMSs of all parameters, including η for the strong equivalence principle, of about an order of magnitude with respect to the nominal case where uncertainties are not taken into account. Nonetheless this result, in terms of η , represents an improvement of a factor 10 over the current precision achieved by LLR.

In the second part of the paper we calculated the signal due to SEP violation on the ranging between a ground station and a SC orbiting near an Earth-Sun collinear Lagrangian point. With a covariance analysis based on a 5 years mission, we forecast an RMS error for η that would be around the same level of current measurements by LLR and ground experiments. We conclude that this recently proposed experiment would serve as a direct test of the SEP that is both independent from other experiments, and at least comparable in terms of performances achieved in a relatively short time span.

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