The role of collapsed matter in the decay of black holes

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ABSTRACT

We try to shed some light on the role of matter in the final stages of black hole evaporation from the fundamental frameworks of classicalization and the black-to-white hole bouncing scenario. Despite being based on very different grounds, these two approaches attempt at going beyond the background field method and treat black holes as fully quantum systems rather than considering quantum field theory on the corresponding classical manifolds. They also lead to the common prediction that the semiclassical description of black hole evaporation should break down and the system be disrupted by internal quantum pressure, but they both arrive at this conclusion neglecting the matter that formed the black hole. We instead estimate this pressure from the bootstrapped description of black holes, which allows us to express the total Arnowitt–Deser–Misner mass in terms of the baryonic mass still present inside the black hole. We conclude that, although these two scenarios provide qualitatively similar predictions for the final stages, the corpuscular model does not seem to suggest any sizeable deviation from the semiclassical time scale at which the disruption should occur, unlike the black-to-white hole bouncing scenario. This, in turn, makes the phenomenology of corpuscular black holes more subtle from an astrophysical perspective.

1. Introduction

The possible existence of black holes is one of the most striking predictions of General Relativity and understanding their true nature has become one of the hottest topics of current theoretical research after the detection of gravitational waves and the reconstruction of their "shadow". In particular, the famous result that black holes could evaporate [1] raised a number of paradoxes and made it apparent that a consistent quantum description of strong gravitational fields might require going beyond the (up to then) very successful application of quantum field theory on classical curved backgrounds. A different approach is certainly necessary in order to understand the late stages of the evaporation (see, *e.g.* [2] and references therein).

A quantum bounce is a typical scenario which emerges when one tries to include quantum mechanical effects in the description of self-gravitating systems (see *e.g.* [3,4] and references therein). Thus, it is not hard to believe that this sort of effects should turn out to be massively relevant in the very late stages of the life of a black hole. On similar considerations are based the models of the Planck star [5] and of the black-to-white hole quantum transitions [6]. More specifically, the fundamental idea is that classical General Relativity becomes unreliable when the system enters regimes of very high curvature, such as regions close to black hole singularities. In a Planck star, the classical metric manifold inside regions of large curvature is therefore replaced by a fully quantum space-time as it would emerge in Loop Quantum Gravity. One naively expects that quantum mechanical effects will dominate at a scale $\ell \sim \ell_{\rm P}$, where $\ell_{\rm P}$ denotes the Planck length. However, if one considers the typical bouncing scenarios within the mini-superspace approach to quantum cosmology [7], the general conclusion is that the bounce should occur when a certain model-dependent *critical density* $\rho_{\rm c}$ is reached. By applying this result to the gravitational collapse, one expects that the core of the system, where the matter is supposed to end after crossing the event horizon, stops contracting at a comparable critical density $\rho_{\rm c}$. For example, in Loop Quantum Cosmology [8], one finds $\rho_{\rm c} \simeq \rho_{\rm P} \simeq m_{\rm P}/\ell_{\rm P}^3$, where $m_{\rm P}$ is the Planck mass. Hence, in spherical symmetry, the size of the inner quantum mechanical region, representing the core of a Planck star, is naively given by $\ell \sim (M/m_{\rm P})^{1/3}\ell_{\rm P}$, with M denoting the Arnowitt–Deser–Misner (ADM) mass [9] of the

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system. Notice that ℓ is several orders of magnitude larger than ℓ_P for an astrophysical black hole, which, therefore, offers a window for detecting quantum gravitational effects at scales far from the Planck regime. If we buy this argument, the semiclassical picture of black hole evolution driven by the Hawking effect should proceed smoothly up to the scale ℓ, at which point the horizon hits the boundary of the quantum gravity region and the quantum pressure tears the core apart, thus freeing up what remains of its energy content.

The natural extension of this very simple argument consists in describing the final stage of black hole evaporation as a tunnelling process from a black hole geometry to a white hole geometry [6]. More precisely, in this model the two classical space-time patches are glued together through an intermediate region, dominated by quantum gravitational effects, resembling a sort of Oppenheimer-Snyder collapse [10] with an extra bounce into a white hole [6,11] (see also [12] and references therein for similar results in canonical Einstein gravity). One of the most interesting results following from this effective description of the tunnelling process is that the quantum bounce might actually be realized after a time

$$\tau_b \sim \ell_P \frac{M^2}{m_P^2} , \tag{1}$$

which is way shorter than the standard Hawking evaporation time

$$\tau_H \sim \ell_P \frac{M^3}{m_P^3} . \tag{2}$$

In the semiclassical picture, black hole should evaporate semiclassically up to the Page time [2]. If we trust the Hawking formula for a time $\tau \sim \tau_b$, its total mass at the time of the bounce is given by

$$M_b \simeq M \sqrt[3]{1 - (m_P/M)} \simeq M , \tag{3}$$

which means that the entire black hole mass "bounces out" for $M \gg m_P$.

Several phenomenological studies concerning these loopy bouncing black holes have been carried out in the last few years (see e.g. [13,14] and references therein), with a natural focus on the final stages of evolution of primordial black holes. In these studies it was found that the mass of primordial black holes "exploding" today, after a time $\tau \sim \tau_b$ from formation, should range from 10^{11} kg to 10^{20} kg [5,13]. The aim of this letter is to discuss the bouncing scenario of black-to-white hole transitions within the framework of the classicalization scheme [15,16] and of the corpuscular picture of gravity [18,20] by taking explicitly into account the role of matter collapsed inside the black hole [21].

2. Corpuscular gravity

The goal of classicalization [15] is to tackle the problem of the UV completion of effective field theories without forcing in any new (hard) degree of freedom, as it would happen in the Wilsonian approach. This idea lies at the very foundation of the corpuscular model of black holes, also known as black hole's quantum N-portrait [16]. According to this view, a black hole can be described as a leaky bound state of a large number N_G of soft virtual gravitons. In particular, while the collective gravitational coupling

$$g = N_G \alpha \simeq 1 \tag{4}$$

and the system is *globally* in the strong coupling regime, the effective coupling among the constituents

$$\alpha \simeq \frac{1}{N_G} \tag{5}$$

remains extremely small, which suppresses the contribution of loop corrections *locally*. It is also worth recalling, for the sake of argument, that in this framework one finds the fundamental scaling relations

$$M \simeq \sqrt{N_G}\, m_P , \qquad \varepsilon \simeq \frac{m_P}{\sqrt{N_G}} , \tag{6}$$

where ε denotes the typical energy of each graviton in the bound state, provided the typical length scale of all the constituents is $\lambda \simeq R_H$ (see, e.g. [17] and references therein). It is then easy to conclude that the classical description of gravity can emerge naturally from this picture as a direct consequence of the large value of the occupation number N_G, ultimately leading to an effective classical behaviour.

On the other hand, semiclassical effects like the Hawking process and the generalized second law of black hole thermodynamics arise inherently from the softness of the constituents and their combinatorics. Indeed, in this scenario the Hawking radiation is understood as the leakage of the bound state due to $2 \rightarrow 2$ scattering processes among the gravitons in the system. The total depletion rate caused by the scattering of each graviton with the other $N_G - 1$ constituents is roughly given by [16,18]

$$\Gamma \simeq \alpha^2 N_G (N_G - 1) \frac{\varepsilon}{\hbar} + \mathcal{O}\left(\frac{1}{\ell_P N_G^{3/2}}\right)$$
$$\simeq \frac{1}{\sqrt{N_G}\, \ell_P} + \mathcal{O}\left(\frac{1}{\ell_P N_G^{3/2}}\right) , \tag{7}$$

where ε is, indeed, the typical energy involved in this process. On recalling the scaling laws (6) and that $\dot{N}_G \simeq -\Gamma$, this yields

$$\varepsilon \dot{N}_G \sim \dot{M} \sim -\frac{m_P}{\ell_P} \left(\frac{m_P}{M}\right)^2 , \tag{8}$$

and we conclude that, in the corpuscular picture, the (apparent) thermal behaviour of the Hawking radiation follows from the softness of the gravitons leaving the system and from their combinatorics. It is then important to recall that one also expects a flux of comparable energy for ordinary matter from the scattering of baryons by gravitons [18]. Consequently, the description of Hawking evaporation as quantum depletion allows for a natural resolution of the information loss problem since the features of the collapsed matter are stored within the system, for a very long time, before they start to gradually leak out. In other words, the unitarity of the time evolution should be restored through the emission of quantum hair [19], which is subjected to a suppression of order $1/M^2$, contrary to the standard semiclassical picture in which these fields are exponentially faint.

From this quantum field theoretic description of black holes and the Hawking radiation we also get a taste of when such a model should break down. A careful inspection of Eq. (7) tells us that the quantum gravitational corrections to the semiclassical picture of gravity should become relevant when the condition $N_G \gg 1$ ceases to be valid. In other words, when the number of gravitons in the black hole has become small enough that the collective gravitational interaction (4) can no more overcome the increasing quantum pressure, the system will not remain confined within the Schwarzschild radius

$$R_H = 2\, \ell_P \frac{M}{m_P} \simeq \sqrt{N_G}\, \ell_P . \tag{9}$$

The bound state should break off at that point, freeing up all the remaining matter and gravitons. For a "purely gravitational" black

hole made only of gravitons, we expect that this occurs for N_G of order one, when $R_H \sim \ell_P$. However, if one realistically requires the presence of regular baryonic matter up to the latest stages of the evaporation, the typical values of N_G at which the semiclassical behaviour breaks down could be much bigger.

We have seen that corpuscular black holes share some similarities with the typical dynamics expected for Planck stars. One can then investigate the phenomenological implications for primordial black holes, considering the scenario of black-to-white hole transitions, if we add the classicalization scheme to the mix. For example, the smallest possible mass of a primordial black hole exploding today should be $M \simeq 10^{11}$ kg according to Ref. [14]. From Eq. (6) one immediately finds $N_G \simeq 10^{38} \gg 1$ at the time of the bounce. According to the black-to-white hole picture, the semiclassical behaviour should therefore break down at scales $\ell \sim 10^{19} \ell_P$, about a tenth of the proton's size.

The huge discrepancy between $N_G \sim 1$ and $N_G \sim 10^{38}$ at the time of the departure from the semiclassical evolution estimated in the two scenarios leads us to the fundamental questions: What is the shortest scale of gravitational confinement for ordinary matter? In order to answer this question, we need an estimate of the quantum pressure that prevents the baryons from reaching the infinitely dense classical singularity [22,23].

3. Bootstrapped black holes

The bootstrapped description [22] of stars and black holes [21] was recently suggested as an effective realisation of the classicalization scheme for the gravitational interaction. This approach is simply constructed by introducing the leading order non-linearities predicted by General Relativity in Newtonian gravity. The effective Lagrangian for the gravitational potential is found to be [21]

$$
L[V] = L_N[V] - 4\pi \int_0^\infty r^2 \, dr \left[J_V \, V + J_\rho \, (\rho + p) \right]
$$

$$
= -4\pi \int_0^\infty r^2 \, dr \left[\frac{m_P \, (V')^2}{8\pi \, \ell_P} \, (1 - 4V) \right.
$$

$$
\left. + (\rho + p) \, V \, (1 - 2V) \right], \tag{10}
$$

where

$$
L_N[V] = -4\pi \int_0^\infty r^2 \, dr \left[\frac{m_P \, (V')^2}{8\pi \, \ell_P} + \rho \, V \right] \tag{11}
$$

is the Newtonian part,

$$
J_V = -\frac{m_P \, (V')^2}{2\pi \, \ell_P} \tag{12}
$$

is the gravitational current and $J_\rho = -2V^2$ the higher order correction to the matter part. The corresponding field equation

$$
\triangle V = 4\pi \frac{\ell_P}{m_P} \, (\rho + p) + \frac{2 \, (V')^2}{1 - 4V} \tag{13}
$$

and the conservation equation

$$
p' = -V' \, (\rho + p) \ , \tag{14}
$$

allow for finding explicit (classical) solutions generated by arbitrarily compact matter sources. In fact, it was recently shown that

the baryonic pressure can, in principle, counterbalance the gravitational pull for any radius R of the matter source in this framework [21]. In other words, there is no Buchdahl limit and, for $R \lesssim R_H$, one also finds that the total proper mass of N_B baryons $M_0 = N_B \, \mu$ must relate with the ADM mass according to[1]

$$
\frac{\ell_P \, M_0}{R \, m_P} \sim \left(\frac{\ell_P \, M}{R \, m_P} \right)^\alpha \ , \tag{15}
$$

where $\alpha = 2/3$ for a homogeneous matter distribution. For the sake of generality, we will just assume $0 < \alpha < 1$. First of all, we need

$$
R \lesssim R_H \simeq 2 \, \ell_P \, \frac{M}{m_P} \ , \tag{16}
$$

in order for the system to be a black hole. For the initial configuration, we can assume $R \ll R_H$ and the baryons are in a highly relativistic regime, so that the initial size of the baryon source

$$
R_{in} \sim \lambda_B \sim \ell_P \, \frac{N_B \, m_P}{M_{in} - M_0} \sim \ell_P \, \frac{N_B \, m_P}{M_{in}} \ , \tag{17}
$$

in which we used $M_{in} \gg M_0$ for $R \ll R_H$, as follows from Eq. (15). Due to the Hawking evaporation of gravitons, the ADM mass M decreases. Since the (initial) Hawking temperature is very low for astrophysical black holes, we can safely assume no baryon is emitted and M_0 remains constant. From Eq. (15) we then infer that

$$
\frac{\Delta R}{R} \simeq -\frac{\alpha}{1 - \alpha} \, \frac{\Delta M}{M} > 0 \ , \tag{18}
$$

and the size of the baryon source inside the black hole increases while the hole evaporates semiclassically.

Let us then consider a final configuration in which the size of the source $R_{fin} \sim R_H$, or, again from Eq. (15),

$$
M_{fin} \simeq M_0 \ . \tag{19}
$$

In this case, the baryons are no more highly relativistic and

$$
R_{fin} \sim \lambda_B \sim \ell_P \, \frac{m_P}{\mu} \ , \tag{20}
$$

where μ is the proper mass of one of the N_B baryons. Clearly, after this point, the size of the baryon source could exceed the gravitational radius and the object would not be a black hole any more. From Eqs. (15) and (17), we also have

$$
\frac{M_0}{m_P} \sim \left(\frac{R_{in}}{\ell_P} \right)^{1-\alpha} \left(\frac{M_{in}}{m_P} \right)^\alpha
$$

$$
\sim N_B^{1-\alpha} \left(\frac{M_{in}}{m_P} \right)^{2\alpha - 1} , \tag{21}
$$

or

$$
\frac{M_{in}}{m_P} \sim \left(\frac{M_0}{\mu} \right)^{\frac{\alpha}{2\alpha-1}} \left(\frac{\mu}{m_P} \right)^{\frac{1}{2\alpha-1}}
$$

$$
\sim N_B^{\frac{\alpha}{2\alpha-1}} \left(\frac{\mu}{m_P} \right)^{\frac{1}{2\alpha-1}} , \tag{22}
$$

which depends on the baryon mass μ and number of baryons N_B.

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[1] For a more accurate estimate of the scaling for $R \simeq R_H$ see Ref. [21]. Given the models considered here are still rather unsophisticated and (more importantly) the qualitative nature of the fundamental questions we address, all calculations will remain at the level of order of magnitude estimates.

The evaporation process is governed by the master equation (8), which leads to an evaporation time

$$\tau_{\text{in}\to\text{fin}} \sim \ell_P \left(\frac{M_{\text{in}}}{m_P} \right)^3 \left[1 - \left(\frac{M_{\text{fin}}}{M_{\text{in}}} \right)^3 \right] . \tag{23}$$

In the Hawking picture one usually considers complete evaporation, i.e., $M_{\text{fin}} = 0$, thus leading to the well-known result (2), or more precisely

$$\tau_H \sim \ell_P \left(\frac{M_{\text{in}}}{m_P} \right)^3 . \tag{24}$$

On the other had, we just saw that $M_{\text{fin}} \sim M_0$ in the bootstrapped picture and Eq. (22) yields

$$\frac{M_{\text{fin}}}{M_{\text{in}}} \sim \left(\frac{m_P^2}{\mu\, M_{\text{fin}}} \right)^{\frac{1-\alpha}{2\alpha-1}}$$

$$\sim \left(\frac{m_P^2}{\mu\, M_{\text{in}}} \right)^{\frac{1-\alpha}{\alpha}} . \tag{25}$$

The above ratio must be smaller than one, which requires $N_B > m_P^2/\mu^2$ (equivalent to $M_0/m_P > m_P/\mu$), if $1/2 < \alpha < 1$ or $N_B < m_P^2/\mu^2$ (equivalent to $M_0/m_P < m_P/\mu$) if $0 < \alpha < 1/2$. Under these conditions, we then obtain the evaporation time

$$\tau_\alpha \sim \tau_H \left[1 - \left(\frac{m_P^2}{\mu\, M_{\text{fin}}} \right)^{\frac{3(1-\alpha)}{2\alpha-1}} \right]$$

$$\sim \tau_H \left[1 - \left(\frac{m_P^2}{\mu\, M_{\text{in}}} \right)^{\frac{3(1-\alpha)}{\alpha}} \right] . \tag{26}$$

This expression tells us that $\tau_\alpha < \tau_H$, but the two times differ significantly only provided $\mu\, M_{\text{fin}} \sim m_P^2$. For instance, if we require τ_α equals the Page time $\tau_{\text{Page}} \simeq (7/8)\, \tau_H$ at which $M_0 \sim M_{\text{fin}} \simeq M_{\text{in}}/2$, we obtain

$$\left(\frac{m_P^2}{\mu\, M_{\text{fin}}} \right)^{\frac{1-\alpha}{2\alpha-1}} \simeq \frac{1}{2} \simeq \left(\frac{m_P^2}{\mu\, M_{\text{in}}} \right)^{\frac{1-\alpha}{\alpha}} . \tag{27}$$

For a source made of neutrons with mass $\mu \simeq 10^{-19}\, m_P$, for $1/2 < \alpha < 1$, we find

$$M_{\text{fin}} \simeq 2^{\frac{2\alpha-1}{1-\alpha}} \cdot 10^{19}\, m_P \simeq 2^{\frac{2\alpha-1}{1-\alpha}} \cdot 10^{-19}\, M_\odot , \tag{28}$$

and

$$M_{\text{in}} \simeq 2^{\frac{\alpha}{1-\alpha}} \cdot 10^{19}\, m_P \simeq 2^{\frac{\alpha}{1-\alpha}} \cdot 10^{-19}\, M_\odot , \tag{29}$$

where $M_\odot \sim 10^{38}\, m_P$ is the solar mass.

For the particular case $\alpha = 2/3$, the above expressions become

$$M_{\text{fin}} \simeq 2 \cdot 10^{19}\, m_P \simeq 2 \cdot 10^{-19}\, M_\odot \simeq 0.5\, M_{\text{in}} , \tag{30}$$

which corresponds to an object of final radius

$$R_{\text{fin}} \sim \lambda_B \simeq 10^{19}\, \ell_P \simeq 10^{-16}\, \text{m} . \tag{31}$$

This result is in line with the idea that the breakdown of the semiclassical evaporation should happen at a scale way above the Planck regime, as suggested by the loop-inspired scenario. However, if one tries to put together the corpuscular picture of gravity with the role of matter in the late stages of the evaporation, the expected disruption of the system is found to occur around the Page time [2], that is for $\tau_{\text{Page}} \sim M^3 - (M/2)^3 \sim M^3 \sim \tau_{\text{dep}}$. Yet, this result is more in agreement with the semiclassical scenario.

4. Concluding remarks

In this work, we have tried to put the role of matter in the final stages of a black hole life under the spotlight. To do that, we have compared two quantum models for black holes, namely the corpuscular theory and the black-to-white hole transition. The first thing that we have been able to observe is that, despite emerging from two completely different background pictures, they seem to predict a common scenario, namely that the semiclassical picture will necessarily break down and the system will be disrupted by internal quantum pressure. The key difference between these two pictures resides in the timescale at which this effect should occur. Indeed, at least from the perspective of Loop Quantum Gravity, the black-to-white hole transition is expected after a time $\tau \sim M^2$ from the black hole formation. In the corpuscular picture instead the depletion time scales like the Hawking time, i.e., $\tau \sim M^3$, since there should be no physical reasons, at least in this effective field theory of gravity, for the emergent semiclassical picture to fail before that.

This conclusion makes the phenomenology of the corpuscular scenario more subtle to test from an astrophysical perspective because the typical time scales are close to those predicted by the semiclassical description. Clearly, some different signature is required in order to reveal the quantum nature of black holes and the theoretical investigation of all possible models needs to be continued.

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