Providing Personalized Guidance in Arithmetic Problem Solving

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Abstract. Supervising a student's resolution of an arithmetic word problem is a cumbersome task. Different students may use different lines of reasoning to reach the final solution, and the assistance provided should be consistent with the resolution path that the student has in mind. In addition, further learning gains can be achieved if the previous student's background is also considered in the process. In this paper, we outline a relatively simple method to adapt the hints given by an Intelligent Tutoring System to the line of reasoning that the student is currently following. We also outline possible extensions to build a model of the student's most relevant skills, by tracking user's actions.

Keywords: Personalization, adaptation, Intelligent Tutoring System, Word Problem Solving, Arithmetic teaching

1 Introduction

Developing the students' problem solving skills is a fundamental part of mathematics learning. Word/story problems are commonly used in this context, as a means to promote the student's engagement and provide an adequate framework to practice mathematics skills. The importance of arithmetic word problem is supported by the development of several computer systems focused on this activity, such as HERON [13], Story Problem Solver [12], WORDMATH [10], MathCAL [6] or AnimalWatch [5].

Successful problem solvers construct a model of the situation described in the problem statement, and base their solution plan on this model [8, 14]. We can think of this model as a number of relations between the quantities that explicitly or implicitly appear in the problem statement. When a problem resolution is supervised by a human in a one-to-one situation, direct observation allows the tutor to induce the model that the student has in mind. This allows the tutor to provide contextualized help that is consistent with the student's previous resolution steps. The tutor is also constantly collecting information about the student. This information is generally used to adapt explanations to the student's specific characteristics.

Intelligent Tutoring Systems (ITS) aimed at developing word problem solving skills need also provide personalized guidance. In most cases, the situation described by the problem statement may be modeled in several ways. In this case, the ITS should be able to evaluate the previous user interaction to determine the solution scheme that the student is using, and provide adequate guidance in accordance to this scheme. In this paper, we use a sample problem to illustrate a relatively simple strategy to infer the solution scheme that the student is following.

2 Solution Schemes

When a student reads a problem statement, he/she generally builds a mental scheme of the problem solution. This solution scheme generally includes the student's interpretation of the quantities involved, and a set of relations between these quantities. Lets consider the following problem statement: "A basket contains 60 pieces of fruit, between apples and pears. It has 10 more apples than pears. How may apples are there in the basket?". One possible mental solution scheme (S_1) would be to divide the 60 pieces into two groups of fruits, both with the same number of elements (30); and then mentally transfer half the difference (5) from one group to the other. Another different mental solution scheme (S_2) would consist in mentally setting the 10 extra apples apart; then dividing the remaining pieces of fruit into the two groups; and finally adding the 10 extra apples which were taken apart. Other solution schemes may consider computing the number of apples after computing the number of pears.

Lets suppose that the student has started the resolution by doing the operation $10/2 = 5$, but is finding problems to propose the next operation. If there is a system intervention, it would make little sense that the system suggests the student uses the expression $60 - 10 = 50$. Such a recommendation would very likely cause confusion on the learner. This is because this action belongs to a line of reasoning that is not the one that the student was following. On the contrary, the suggestion $60/2 = 30$ would make more sense, and be in line with the student's solution scheme.

Potentially valid solution schemes can easily be internally represented as a set of quantities and relations between quantities [7]. Table 1 shows such a representation for the solution scheme S_1 above.

Representation	Description	Initial value
Total	Total number of pieces	60
Half_Total	Half the number of pieces	unknown
Excess	Extra number of apples	10
Half_Excess	Half the extra number of apples	unknown
Apples	Number of apples	unknown

Relations
Half_Total=Total/2
Half_Excess=Excess/2
Apples=Half_Total+Half_Excess

Table 1. Quantities (left) and relations (right) in solution scheme S1. Only values for the quantities $Total$ and $Excess$ are known.

This way of representing solution schemes allows any automated system to determine all valid expressions that the student may use. For explanation purposes, lets define an active relation as one that contains a single unknown quantity. In addition, lets define a way to generate a expression from any active relation. This consists in replacing all known quantities by their respective numeric values, and the unknown quantity by the number which makes the resulting expression numerically correct. With these definitions, valid expressions correspond to the ones generated by all active relations, in any solution scheme (and rearranged versions of them).

For example, S_1 has two active relations (the first two relations in Table 1). These generate the expressions $30 = 60/2$ and $5 = 10/2$, respectively. Hence, a student may start solving the problem at hand according to S_1 by using these two expressions (or a rearranged version of them). The use of a different expression may imply a mistake or that the learner is following a different solution scheme.

3 Adaptive Help

Tracking the state of each solution scheme is a key issue to provide adequate help messages that are consistent with the student's current line of reasoning. To this end, every valid learner's input is simultaneously processed in the context of each solution scheme. This is done by comparing the user's input to the expressions generated by the active relations. If an equivalent expression is found, the value of the corresponding unknown quantity is updated, and the relation is removed from the relations table. For example, the expression $10/2 = 5$ as a first user input would match the relation Half_Excess=Excess/2 in S_1, and yield the dynamic state in Table 2. With this method, unknown quantities are solved one at a time. Hence, the number of remaining expressions in each mental solution scheme is always the same as the number of unknown quantities in the scheme.

This simple tracking mechanism allows one to associate the progress of a mental solution scheme with the percentage of relations that have already been used. This simple measures allows an automated system to easily compute the

Representation	Description	value
Total	Total number of pieces	60
Half_Total	Half the number of pieces	unknown
Excess	Extra number of apples	10
Half_Excess	Half the extra number of apples	5
Apples	Number of apples	unknown

Unused Relations
Half_Total=Total/2
Apples=Half_Total+Half_Excess

Table 2. Representation of the dynamic state of S_1 after processing expression $10/2 = 5$. The quantity $Half\_Excess$ has become known, and the relation has disappeared from the corresponding table.

solution scheme that the learner is most likely following, and provide consistent feedback based on his information. For example, lets suppose that a student has already introduced the expression $10/2 = 5$ and asks for a suggestion. S_1 would be pointed out as the most likely solution scheme. Since the only active relation in this scheme is Half_Total=Total/2 (see Table 2), an automated system would be able to recommend the use of the expression $30 = 60/2$. The way this action is translated into a help message may depend on the specific system and/or other information registered as part of the student model. Foe example, the system may simply recommend the user to compute half the number of pieces by using an appropriate expression, and refine the message further if the user is still not able to set it correctly.

To test the effectiveness of this strategy to determine the mental scheme that the student is following, the method has been integrated into an existing ITS [1, 4]. The results obtained have been reported in [3]. To support the relevance of the research, a first study showed that there are significant differences between the help provided by expert and novice teachers. In many cases, novice teachers gave hints which were not consistent with the observable reasoning, according to the previous learner's calculations. In addition, it is shown that the aids provided by the ITS were similar to the ones offered by experts teachers (in 84% of the cases).

4 Further adaptation

Apart from considering the student's current line of reasoning, it is also possible to build a student model out of his/previous interaction with the system. This model can be used to further adapt help messages to the user's needs. For example, a particular learner may have difficulties at using multiplicative relations, and benefit from additional explanations. We are currently working on the definition of an ontology that allows the system to keep track of the most relevant skills in arithmetic problems solving.

A first attempt in this direction was made in [1], in the context of algebra learning. A labeling scheme for relations allowed the ITS to estimate the learner's skills at detecting and expressing certain type of conceptual schemes. Following with this idea, we are working on the definition of an appropriate labeling for an arithmetic context. The intention is that correct inputs, mistakes and help requests can be linked to concrete skills and tracked by an automated system.

In addition, we are currently considering ways of detecting and using the student's affective state to improve learning. An initial discussion was provided in [2]. As a first experiment in this direction, we have prepared a series of exercises that students will need to solve using the ITS. Some of these exercises seek to elicit concrete emotions. For example, a student may get confused if the ITS repeatedly provides hints based on a solution scheme that he/she is not following; or frustrated if right answers are reported as incorrect and suggestions to use relations in non-natural solution schemes are issued. To capture emotional data of interest, we have prepared a modified version of the existing ITS. This new version uses self-reporting at several stages. Before the student starts solving any exercise, he/she has to fill the Attributional Achievement Motivation Scale presented in [11]. This is a self reporting test based on Weiners attributional theory [15], which is used to explain the attributional causes of the academic achievement on a given subject (arithmetic problem solving in our case). The test is composed of 22 items structured in 5 factors, namely interest, task or capacity, effort, exams and the teachers pedagogical capacity. After completing each exercise, the student has to report about his/her affective state (valence and activation). To this end, we have used Self-Assessment Manikins (SAM) [9]. At the end of the series, the student is asked to fill a self-report. Finally, we have included a descriptive self-report that the student has to fill once the entire series of exercises has been completed. Results from this research will be used to built an ITS that provides emotional support, and measure the performance improvement obtained with respect to the original ITS. A first proposal consists in replacing the current help on demand mechanism by a rule-based system that is able to use interaction data to both provide automatic recommendations and adapt the content of the messages, according to the user's affective state.

5 Conclusions

Teaching arithmetic word problem solving is a complex task. Significant differences in tutoring between expert and non-expert teachers have been identified and reported in [3]. One major factor behind these differences is the ability of the teacher to provide feedback that is consistent with the current student reasoning. In this paper, we have described an strategy that makes it possible to transfer this fundamental skill to an automated system. It could be claimed that the system would not be able to handle solution schemes that the system is not aware of. However, this is also the case in human supervision. A human may interpret as incorrect any action that does not match a valid step in the solution schemes that he/she is able to generate.

We have also outlined future improvements aimed at providing a closer behavior to a human expert, by considering both previous interactions and the learner's affective state. We have also described the design of a new experiment to help the integration of affective support into the ITS.

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