A Comparative Study of Fuzzy Logic Controllers for Autonomous Robots

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Abstract

This paper presents the results of applying interval and generalised type-2 fuzzy logic to robot control. Statistical analysis indicates that the type-2 controller provides a smoother more consistent and accurate path in a wall following problem.

1 Introduction

Autonomous robots have to operate in uncertain environments relying on imprecise information:

The operating conditions causing these environmental uncertainties may also change while the robot is performing a task. The challenge of building a control system that works satisfactorily is these conditions is significant:

- The sensor readings that provide the robot with a perception of the environment contain noise. The level of sensory noise can change with the operating conditions, and

- the actuators that implement control decisions taken by the robot may do so erroneously.

Hagras [9] singled out three sources of uncertainty that impact on the performance of a Fuzzy Logic Controller (FLC) for robot navigation.

- Uncertainties from the sensor readings that provide the inputs to the FLC. Sensor readings are often noisy, the noise level varying according to the conditions of observation.

- Uncertainties in effect of a control action output from the FLC. These result from a variation in the coupling of actuators characteristics with environmental conditions.

- Ambiguity in the meanings of the words used to label the fuzzy sets. These are caused by inter expert variation, a lack of knowledge or a inability to express knowledge with the accuracy required for a fuzzy set definition.

To improve the performance of a robot under such uncertainties Hagras[9] proposed the use of an interval type-2 FLC, with interval type-2 fuzzy rules utilising interval type-2 fuzzy sets in the antecedents and consequents. Type-2 interval fuzzy sets have membership grades that are crisp interval sets bounded in $[0, 1]$. In a series of experiments an interval type-2 FLC outperformed a type-1 FLC when performing the task of obstacle avoidance, edge following and goal seeking.

Interval type-2 fuzzy set require practitioners to assume that the uncertainty associated with a value in a fuzzy set has a uniform distribution. The sources of uncertainty identified earlier rarely take uniform distributions. We believe this impacts on the performance of interval type-2 FLC. To model non-uniform distributions of uncertainty requires general type-2 fuzzy sets[1]. Type-2 fuzzy sets have membership grades that are type-1 fuzzy numbers bounded in $[0, 1]$. Each fuzzy number gives a distribution of the possibility for each value in the set. For example a sonar reading of 488mm may have a membership grade of *about*

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[1]In this paper we refer to general type-2 fuzzy set as type-2 fuzzy sets. To avoid confusion the word interval is always included when referring to interval type-2 fuzzy sets.

0.8 in the type-2 fuzzy set *too close*. The same sonar reading may have a membership grade in the equivalent interval type-2 fuzzy set of *between 0.75 and 0.85*.

Introducing this extra modelling capacity has a huge impact on the computational burden of type-2 fuzzy processing. To date the level of this burden has prevented type-2 fuzzy logic from being applied to any control applications. Previous and upcoming work by the authors [1, 5, 4, 8] has shown how this burden can be significantly reduced subject to some limitations in the system design parameters. These works have enabled us to be the first to present a type-2 fuzzy logic controller. This paper contrasts the performance of this novel type-2 FLC with comparable interval type-2 and type-1 FLCs. To compare the relative performance of the controllers we designed an edge following experiment allowing for a statistical analysis to be performed on the results.

2 Type-2 Fuzzy Logic

Type-2 fuzzy logic [11, 12, 17, 25] is an emerging fuzzy technology with a sound theoretical base and a growing number of applications [10, 13, 9, 21, 14, 18, 20, 27, 7]. To date all of the control applications of type-2 fuzzy logic have only used interval type-2 fuzzy logic. We now present an overview of the important operations type-2 and interval type-2 fuzzy logic.

Fundamental to type-2 fuzzy logic is the concept of the type-2 fuzzy set[28, 26]. Type-2 fuzzy sets have membership grades that are type-1 fuzzy numbers bounded in $[0,1]$. Each fuzzy number gives a distribution of the uncertainty associated with each value in the set. The type-2 fuzzy set is defined below.

Definition 1 *A type-2 fuzzy set $\tilde{A}$ is characterised by a membership function μ that maps elements from a domain X to type-1 fuzzy numbers in* $[0,1] \times [0,1]$.

A type-2 fuzzy set can be viewed as a fuzzy set with fuzzy membership grades, each grade being a type-1 fuzzy number bounded in $[0,1]$. This means not only is there a vagueness about where the sets boundary is, but there is also an uncertainty about the position of this vague boundary.

The logical operations used in type-2 fuzzy logic extend those defined for type-1 fuzzy logic. The extension principle [28] is used to derive type-2 fuzzy set operations from their type-1 equivalents. The union or join ($\sqcup$) of two discrete type-2 fuzzy sets $\tilde{A}$ and $\tilde{B}$ both over X is given by:

$$\mu_{\tilde{A} \sqcup \tilde{B}}(x) = \sum_i \sum_j \mu_{\tilde{A}}(x, u_i) \vee \mu_{\tilde{B}}(x, w_j) / u_i \vee w_j \quad (1)$$

The intersection or meet ($\sqcap$) of two discrete type-2 fuzzy set $\tilde{A}$ and $\tilde{B}$ is given by:

$$\mu_{\tilde{A} \sqcap \tilde{B}}(x) = \sum_i \sum_j \mu_{\tilde{A}}(x, u_i) \star \mu_{\tilde{B}}(x, w_j) / u_i \vee w_j \quad (2)$$

In both of these equations $x \in X$, $\star$ is a t-norm such as minimum or product and $\vee$ is a t-conorm generally taken to be maximum.

The join and meet operations are significantly more computationally complex than the type-1 union and intersection. Karnik and Mendel [16] and Coupland and John [1, 5, 4] have redefined the join and meet operations in more computational efficient ways by eliminating redundancy in the operations. Reducing the computational cost of these operations has had a significant impact on their usefulness, particularly their use in FLCs.

In order to calculate a crisp output from any type-2 fuzzy logic system the type-2 set must be type-reduced [17, 15]. Type-reduction arrives at a type-1 fuzzy set that is the possibility distribution of the centroid of the type-2 fuzzy set. This involves finding every possible type-1 fuzzy set that could be embedded in the type-2. The number of embedded sets in a type-2 fuzzy set is given by the equation below.

$$n = \prod_{i=1}^{N} M_i \quad (3)$$

where n is the number of embedded sets in a type-2 fuzzy set with N discrete points in the primary membership function and M_i discrete point in the domain of the secondary membership function at the ith point.

The details of type-reduction methods used to implement the type-2 fuzzy logic controller investigated in this work will be reported elsewhere. The computational problems of type-2 fuzzy logic

have to date stifled this technologies development. The alternative technology of interval type-2 fuzzy logic has become far more widely used. This technology is now discussed.

Interval type-2 fuzzy logic [19] can viewed as a computational efficient approximation of type-2 fuzzy logic. Interval type-2 fuzzy sets have secondary membership functions that are interval sets, sets defined by two boundary points with all points in between having a membership grade of one. Set theoretic operations for interval type-2 fuzzy sets can be defined in terms of the upper and lower boundaries. The union or join ($\sqcup$) of two interval fuzzy sets $\tilde{A}$ and $\tilde{B}$ is given by:

$$\mu_{\tilde{A} \sqcup \tilde{B}}(x) = [\underline{\mu_{\tilde{A}}}(x) \vee \underline{\mu_{\tilde{A}}}(x), \overline{\mu_{\tilde{A}}}(x) \vee \overline{\mu_{\tilde{A}}}(x)] \quad (4)$$

The intersection or meet ($\sqcap$) of two interval fuzzy sets $\tilde{A}$ and $\tilde{B}$ is given by:

$$\mu_{\tilde{A} \sqcap \tilde{B}}(x) = [\underline{\mu_{\tilde{A}}}(x) \star \underline{\mu_{\tilde{A}}}(x), \overline{\mu_{\tilde{A}}}(x) \star \overline{\mu_{\tilde{A}}}(x)] \quad (5)$$

where $\star$ is minimum, $\vee$ is maximum, both $\tilde{A}$ and $\tilde{B}$ are over X and $x \in X$. These operations are far less complex than the type-2 equivalent and are approximately double the complexity of the type-1 operations. Not only is the inferencing process of the interval system less complex that type-2, defuzzification is also far less computationally expensive.

2.1   Discussion

We have seen two technologies that seek to improve the performance of fuzzy logic systems. Type-2 fuzzy logic which seeks to model the uncertainties that surround the membership functions of fuzzy sets. Interval type-2 fuzzy logic which provides an efficient and useful approximation of type-2 fuzzy logic. To date type-2 systems have only been applied in areas where processing cost, although always relevant, has not been at the heart of the application. To date there have been no control applications using generalised type-2 fuzzy logic. The iterative method has been a significant catalyst in the development of interval applications. Non-control applications of interval technology include video classification [20], agents in an intelligent environment [6] and signal processing [18]. Recently interval fuzzy control systems have begun to be explored. Hagras

[9] has done extensive work on mobile robot control. His work shows how individual robot behaviours can be implemented using interval systems. A methodology for integrating these behaviours using interval system is also explored. Figueroa *et al* [7] have also looked at robot tracking and navigation in the context of robotics soccer games. Other control applications include a liquid level process controller [27], steel production modelling [24] and embedded interval systems [21, 22, 23].

The development of embedded type-2 fuzzy logic controllers will be a significant step forward. Only when well developed and tested hardware becomes available will large scale industrial applications be possible. Coupland and John's work on geometric fuzzy logic systems [1, 2, 5, 4, 3] seeks to create opportunities for the development of type-2 hardware systems.

Our thesis is that type-2 system will perform more robustly and consistently than interval systems in real-world environments where modelled uncertainty is crucial. The next section explores the design of three robot FLCs, the interval type-2 and type-2 FLC being based on an initial type-1 FLC. These FLCs will be used to compare the performance of the three fuzzy technologies in a robot control application.

3   Mobile Robot Controller Design

In this paper we are designing a FLC to navigate a mobile robot around a curved edge obstacle maintaining a distance of 0.5 metres between to the centre of the robot and the obstacle at all times. The robots initial position puts the obstacle at a right angle to the robots left wheel at a distance of 0.5m to the centre of the robot. The robot is facing the correct direction to begin navigation of the obstacle. This position is the start point of the ideal path that should be taken by the robot around the obstacle. The task of the FLC is essentially to minimise the deviation from this path. The robot need not be concerned with avoiding other obstacles or any other dynamic element of the environment. The task is simply to move around the curved obstacle, following the ideal path as closely as possible.

We arrived at a fuzzy rule base by relating the

rules to human knowledge about how to follow the ideal path. We chose this route as when have no training data available and no existing PID controller to build upon. The robot that system was deployed on is the commercially available pioneer 2 robot from ActivMedia. The robot has an array of eight sonar sensors to the front and two wheels that may be driven independently. We have chose to have four inputs to the FLC. These are the angle (θ_1) and distance (d_1) of the shortest sonar reading from all eight sensors, the angle (θ_2) and distance (d_2) of the shortest sonar reading from all middle four sensors. The only output from the system is the change in direction (δh) of the robots heading. The robots speed is kept at a constant 0.1ms^{-1}.

The distinct tasks are being performed by these rules:

1. When too far from the edge turn toward the edge.

2. When too close to the edge turn away from the edge.

3. When directly facing the wall turn to the right.

These tasks combine to give the single behaviour of edge following.

The simplicity of the rule base the lends itself to type-2 FLC implementation where the computation levels are a critical factor.

The first rule base to be developed was the type-1 rule base. The rules were based on human knowledge about the task. The rules and membership functions were tuned by hand with the use of a robot simulator. The control system was run many times on the simulator. With each simulated run small adjustments were made to the rules. This continued until the controller gave satisfactory and robust performance.

To arrive at a set of interval type-2 and type-2 rules we took the type-1 rule base and substituted in interval type-2 and type-2 fuzzy sets. The lower bound and upper bounds were set symmetrically around the type-1 functions. The type-2 membership functions were based on a combination of the type-1 and interval type-2 fuzzy sets.

The interval set gave the FOUs of the type-2 sets, that is the supports for the secondary membership functions. Each secondary was defined as a triangular membership function with the type-1 fuzzy sets provide each secondary with a point at unity. For illustrative purposes an example type-1 fuzzy set *correct* is given Figure 1, the interval version of *correct* is given in Figure 2 and the type-2 version in Figure 3

4 Experimental Methodology

In the previous section we explored the design and implemented three robot FLC based on type-1, interval type-2 and type-2 fuzzy logic. In this section we design an experiment to compare the performance of these controllers. We take great care to ensure the results can show statistical significance.

4.1 Tracking the Robots Position

To get meaningful results about the ability of a robot to navigate around the curved obstacle we need to know the path the robot took around the obstacle. In order to track the robots position we mounted a camera directly above the obstacle. We placed a red light emitting diode on the top of the robot at its centre point. The Experiments were conducted in a darkened laboratory. The pixel that corresponded to the location of the LED was consistently identified as the brightest pixel in the frame. This established that the tracking system could consistently identify the position of the robot in frame as a x,y co-ordinate.

In order to find the position of the robot relative to a path we had to find the position of the ideal path within the camera frame. To do this the robots ideal path was scribed on the laboratory floor. This gave us a method for tracking the robots position relative to an ideal path. To calculate the deviation from the ideal path, that is the error *e* of the robots position at point *P* we took the shortest distance between the ideal path and the point *P* in a straight line. To find the point on the ideal path closest to *P* we used an expanding circle technique. A circle with a radius of zero is placed at *P*. The radius of the circle is increased until a point on the circle intersects with a point on the ideal path. The radius of the circle at this point

is the error *e* in the robots position relative to the ideal path.

4.2 Statistical Methods and Data Analysis

Our intention in conducting these experiments is to shown any statistically significant difference between the three controllers. Each experimental run gives a number of discrete data points which make up the path of the robot over the individual run. Each of these data points has an associated error, the amount of deviation from the ideal path. For each run we will take the square root of the mean of the square of the error associated with each data point, the *RMSE*. This gives a single measure of the FLC performance for that particular run. An initial set of 20 experimental runs were conducted using the type-1 FLC. The standard deviation of the *RMSE* amongst these 20 runs was 0.98. To determine the sample size *n* required for the experimental results to be significant we used the following equation

$$n = \frac{2(Z_{\alpha/2} + Z_{\beta})^2 \sigma^2}{E^2} \qquad (6)$$

Where Z_{α} and Z_{β} are the type-1 and type-2 error levels respectively, σ is the standard deviation within the samples and *E* is resolution we want to calculate error too. Using the Bonferroni method we determined $Z_{\alpha/2}$ to be 2.67 and Z_{β} to be 1.28. The standard deviation of the *RMSE* from the initial 20 runs gave the value for *sigma* of 0.98. *E* has a value of 1 since that is highest possible resolution, a single pixel. This gives a rounded value for *n* of 29. We took the decision to repeat each experiments 50 times as this was above this number but not a significant empirical burden.

4.3 Measures Taken During the Experiments

We performed the experiments in a sequential order as to minimize any effect of the battery voltage or any possible time related performance variation. At the end of each run the recorded path was visually checked for any obvious outlying data points. Before each run took place the robot was placed carefully in the start position. The floor was marked to ensure that the robot began in the same position and was facing the same direction for every run.

5 Results

The paths from the fifty experiments from controllers 1,2 and 3 are depicted in Figures 4, 5 and 6 respectively. An initial visual comparison would suggest that the type-2 performed most consistently. The interval controller had a wide but consistent spread. The type-1 controller had spread of paths somewhere between the two with a few paths quite far outside the main spread. It is difficult judge the error of the controllers visually, although the type-2 path appear more tightly packed than the other two.

Table 1 gives the median values and the average ranking the three controllers. No statistically significant conclusions can be drawn from these rankings. However the median positions and mean rankings do point to the type-2 controller having the best performance, followed by the interval type-2 controller and then the type-1 controller. This performance ranking is identical to the ordering of the *RMSE* of the FLC. Looking at consistency of performance both the test for equal variances and the values of $\sigma RMSE$ suggest that the type-1 and type-2 FLC were equally consistent. The interval type-2 FLC had a less consistent performance.

It is important to compare the outcomes that are suggested by the statistical comparison with those give by a visual comparison of the results in Figures 4, 5 and 6. The statistics suggest that FLC performance is ranked type-2, then interval type-2 and then type-1. The path depictions support this conclusion. The statistics suggest that the type-1 and type-2 FLC were equal in the consistency of performance. This is not immediately clear from the visual comparison. Take into account that the type-1 FLC gave the worst performance. A view can be taken that the type-1 FLC made more errors, however these errors were made consistently. The type-2 interval FLC gave a middling performance, but on occasionally made significant errors. This relates well to the visual paths. To summarise these points:

- The type-2 FLC performed consistently well.

- The interval type-2 FLC performed quite well, but was a little inconsistent.

- The type-1 FLC performed relatively badly, but was consistent in this level of error.

These findings are supported by a visual inspection of the paths taken and by a statistical analysis of those paths.

6 Conclusion

In this paper we have designed and compared three FLC using type-1, interval type-2 and type-2 fuzzy logic. The FLC were given the task of following the edge of a curved wall. We have designed a robust empirical method for comparing the performance of these controllers in this task.

The results of this experiment showed that the type-2 controller gave the best performance, then the interval type-2 and then the type-1 FLC. This suggests that interval type-2 fuzzy logic outperforms type-1 logic when faced with uncertainties, confirming the results in [9]. We also suggest that general type-2 fuzzy logic outperforms interval type-2 in the face of significant uncertainties, such as those presented by robot control. We believe this is due to the non-uniform distribution of uncertainty model by type-2 fuzzy sets.

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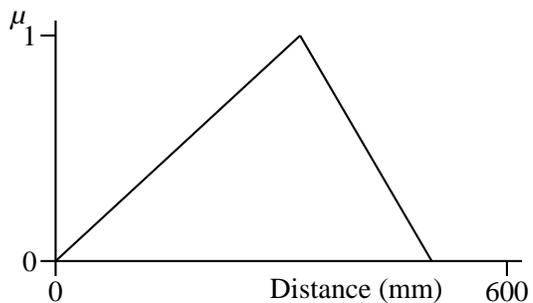
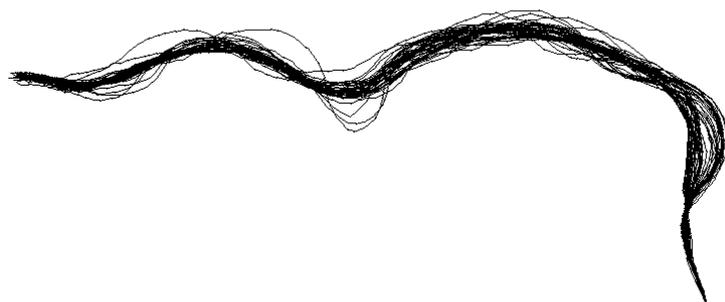Figure 1: The Type-1 Fuzzy Set *correct*.

Figure 2: The Interval Type-2 Fuzzy Set *corr̃ect*.
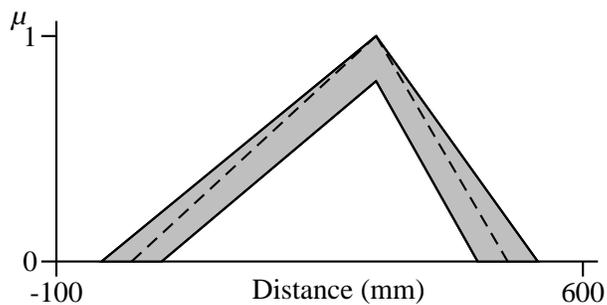

Figure 3: The Type-2 Fuzzy Set *corr̃ect*.

Controller	1	2	3
Median	13.392	11.961	9.802
Average Rank	113.3	84.2	29.0

Table 1: The Median and Average Rank of the Three Controllers from the Kruskal-Wallis Test Procedure.

Figure 4: Paths Taken By the Type-1 Fuzzy Controller.

Figure 5: Paths Taken By the Interval Type-2 Fuzzy Controller.

Figure 6: Paths Taken By the Type-2 Fuzzy Controller.