Title: Consensus measure with multi-stage fluctuation utility based on China's urban demolition negotiation

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Abstract Utility functions are often used to reflect decision makers' (DMs') preferences. They have the following two merits: one refers to the representation of the DM's utility (satisfaction) level, the other one to the measuring of the consensus level in a negotiation process. Taking the background of China's urban house demolition, a new kind of consensus model is established by using different types of multi-stage fluctuation utility functions, such as concave, convex, S-shaped, reversed S-shaped, reversed U-shaped as well as their combinations, to reveal negotiators' dynamic physiological preferences and consensus level. Moreover, the effects of the decision-making budget and the individual compensation tolerance on the consensus level are also discussed in this paper. Compared with previous research, the proposed model takes both the negotiation cost and DM's preference structure into consideration, and most importantly, it is computational less complex.

Keywords Consensus decision making · Utility function · Piecewise linear preference · Consensus level · Linear optimization

1 Introduction

Consensus decision making is a group decision making (GDM) process that aims to achieve an ultimate and mutual agreement by shifting the opinion of each group member (Liao et al. 2016; Cabrerizo et al. 2015; Chen et al. 2015; Herrera et al. 2005). Accordingly, it should be noted that: (1) the decision-making problem under debate is clearly presented, or there is an open discussion to ensure the problem is deterministic; (2) all participants in the GDM are equal; (3) everyone has an opportunity to express his (her) view about the problem, and can fully clarify his (her) decision preference; and (4) every member can produce a high-quality resolution through deep thinking. Solving most consensus problems does not only need to consume costs, but also requires an individual with superb leadership, communication and negotiation skills, who can dominate the GDM process (i.e., a moderator). Generally speaking, the moderator is a figure with authority in the GDM, and he (she) needs to use all possible means (such as policy, law and resource consumption, collectively referred to as budget) to convince each individual decision maker (DM) to achieve a collective opinion, and we call the endeavor the moderator makes as his coordination capability. Despite this, the chances for reaching a complete agreement are rather low due to different individual opinion tolerances and preferences. Therefore, the ultimate goal of consensus decision making becomes to seek an acceptable consensus (i.e.,"soft" consensus) rather than a complete consensus (Herrera et al. 2014; Herrera et al. 2007).

In the "soft" consensus reaching process, participants always show a tolerance range of their own proposed opinions. As a matter of fact, the tolerance range (i.e., opinion tolerances) is the acceptable extent of the opinion discrepancy by group member (Zimbardo 1960; Gong et al. 2016). Relevant research can be found in the literature where the concepts of deviation degree and similarity degree between two linguistic values were defined by Xu (2005); the distance measurement between the individual opinions and the group opinion through comparing the positions of the alternatives by Herrera et al. (2002); the proposal of the ε-consensus to reveal the proximity between expert's opinion and consensus opinion in the study of Ben et al. (2007); and the provision of a consensus scheme for a set of arguments through tightening the range of opinions amongst experts in the literature of Xu (2011). Besides, opinion preference is an unavoidable factor when collecting all members' opinions. In order to describe the preference degree in the economic field, Bernoulli (1954) firstly used the concept of utility. Yet utility theory was truly put forward by Von Neumann and Morgenstern (1947), which laid a solid foundation to later research (Feenstra 2003; Hensher et al. 2016; Charpentier et al. 2016; Gul et al. 2014; Kolbin et al. 2016), and gradually evolved in the hot spot of the decision-making area. However, after fuzzy set theory was proposed (Zadeh 1965), the concept of utility function was gradually transformed into the application of membership function (Pandey et al. 2015; Yadav et al. 2015; Liu 2014; Chang 2008).

Therefore, taking DM's opinion tolerance and preference into account helps to facilitate the decision-making process. However, the key concerns for the moderator focus on two aspects: (1) what extent the consensus can really reach up to, namely the consensus level is the actual issue the moderator cares about; and (2) how much the moderator needs to cost for reaching a consensus, i.e., the negotiation cost. Available research about consensus measure, including qualitative judgment methods (Skinner et al. 2015; Ameyaw et al. 2015; Wortley et al. 2016; McMillan et al. 2016; Singh et al. 2015) and quantitative measurement methods (Xu 2012; Chiclana et al. 2013; Golunska et al. 2014; Zhang et al. 2014; Zhang et al. 2015; Dong et al. 2015; Akiyama et al. 2016), mainly focused on a single distance-based measurement of the consensus level. Yet a new consensus model by introducing multi-stage fluctuation utility function and minimum cost consensus is proposed in this paper, which considers both utility preference and the negotiation cost. Besides, the influencing factors, including the moderator's budget and the individual opinion tolerance, have also been analyzed. Most important, the negotiation on China's urban demolition is used in this paper to make the methodology more specific and pragmatic.

The rest of the paper is organized as follows: Section 2 presents a preliminary including the background of China's urban demolition, the hypotheses for modeling, and the influential factors in the consensus negotiation. The experts approach to utility, whether it is an avert or love attitude to risk, will be mathematically represented and captured with piecewise linear functions, and in particular the concave, convex, S-shaped, reversed S-shaped and reversed U-shaped multi-stage fluctuation linear utility functions are constructed in Section 3. Section 4 presents the mathematical modeling of the consensus model with multi-stage fluctuation linear utility functions. An empirical analysis of China's urban demolition including as well the impact of the two influential factors on the consensus level, the Government's budget and residents' tolerances, is provided in Section 5. Finally, Section 6 summarizes conclusions and future research.

2 Preliminary

2.1 Background

House demolition plays an important part in the reform of China's urbanization, and it mainly involves compensation negotiations between the Government and moved residents. Generally, the Government adopts active remedies to encourage and persuade residents to move: only when the amounts of compensation received by residents satisfy their desired ones, and also meet the Government's expectation, can the psychological satisfaction of both sides reach to a high degree, i.e. consensus is achieved.

In urban relocation negotiations, the Government plays the role of moderator while residents are individual DMs. Naturally, the Government needs to negotiate with residents regarding the compensation price. In such negotiations, consensus is reached only when both sides agree on the price and the expected utilities of compensation are largely satisfied. During the process, the Government must consider the following factors: estate's real value based on considerations such as location; negotiation difficulty, for extreme examples, may become deadlocked owing to residents' low willingness to move; and fixed compensation standards set on Government's assessment of market price.

2.2 Hypothesis

Clearly, to reach the demolition consensus, the Government needs to adopt a policy of different prices for different houses. Also, the Government needs a certain communication budget, which can not only guarantee to convince all the residents, but also can maintain within its own capacity. Simultaneously, residents have different utility preferences within their own expected compensation ranges, and hope to receive maximum compensations. From the Government's perspective, while maximizing expected utility, the price deviation between each resident and the Government should be within the acceptable range of its price interval. Regarding the needs of mathematical modeling, following hypotheses are proposed on the above analysis:

Hypothesis 1 The budget used to reach the consensus on relocation is limited. In other words, the cost to persuade residents to move is predetermined;

Hypothesis 2 The compensation expected by residents and the price proposed by Government are both denoted by uncertain intervals; and different compensation intervals respond to different preference utilities;

Hypothesis 3 The compensation deviations between the residents and the Government satisfy certain restrictions (i.e., residents' price tolerances exist);

Hypothesis 4 Both the Government and the residents expect to achieve an optimal group utility (i.e., a high consensus level).

Actually, these four hypotheses lay the foundation for the following section, which respond to the four factors that affect the consensus negotiation.

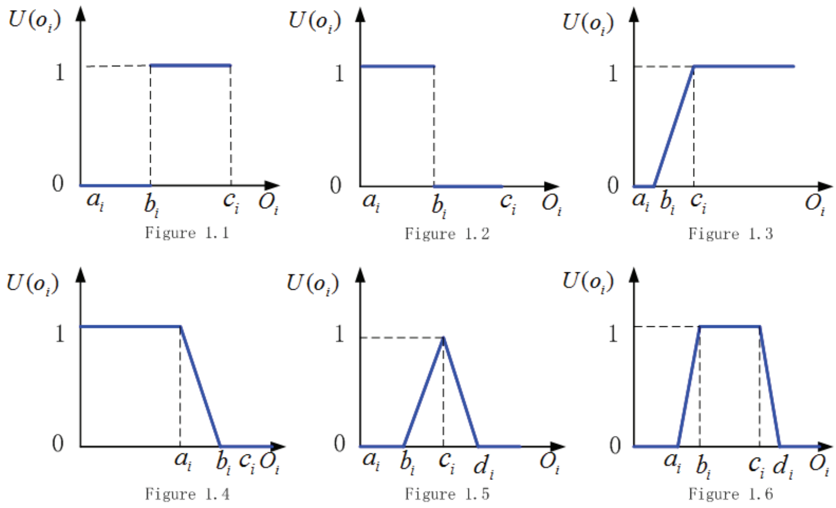

Fig. 1 Basic utility functions

2.3 Factors affecting the consensus negotiation

For the convenience of description, we assume that there are m residents $e_1,\ e_2,\ \ldots, e_m$ in the negotiation process, and their corresponding expected compensation prices are $o_1, o_2, \ldots$, and o_m, respectively. Meanwhile, the Government is represented by G, and its compensation price is denoted as o'. In addition, both the Government and residents are collectively referred as DMs. Thus, consensus negotiation of China's urban demolition mainly involves the following four factors we referred to in Section 2.2:

(1) **Compensation interval.** To reach the demolition consensus, G needs to compensate each e_i ($i \in \{1, 2, \cdots, m\}$) at current market price. In this process, the compensation expression of both sides are usually not a determined or precise number, i.e. a crisp number, but a price range, i.e. an interval. Thus, e_i's desired compensation price is denoted as $o_i = [o_{li},\ o_{ui}]$, where o_{li} represents the lower bound of e_i's compensation price or conservative price, and o_{ui} is the upper bound or optimistic price.
(2) **Compensation budget.** The amount $B = \sum_{i=1}^{m} \omega_i |o_i - o'|$ is used to measure the total budget provided by G when reaching the demolition consensus, i.e. , the total budget employed to persuade all residents to move. In the above expression, $|o_i - o'|$ stands for the absolute deviation of compensation between e_i and G, ω_i denotes the unit compensation assigned to e_i for persuading him (her) to change his (her) compensation price, i.e. its unit effort cost; $\omega_i |o_i - o'|$ naturally indicates the total cost for convincing e_i to move or, in other words, the individual negotiation budget. Clearly, G wishes the total budget to be as small as possible.
(3) **Consensus tolerance.** In the demolition negotiation process, e_i has his (her) own tolerance related to G's compensation price. A resident agrees to move only when the compensation price is within a resident's tolerance, in which case consensus is reached. Here, we assume e_i's tolerance is ε_i, and consequently we have to consider the following constraints $|o_i - o'| \leq \varepsilon_i$ ($\forall i$).
(4) **Decision utility.** DMs may present different psychological preferences during the GDM process due to their educational background, personality and social status, and all these factors may affect the process and quality of the negotiation. Therefore, it becomes extremely necessary to consider all DMs' psychological preferences for reaching a better negotiation consensus.

3 Utility Functions

As it was mentioned in Section 2.3, DM's psychological preference can affect the negotiation process. To illustrate this, an utility function is used to represent a DM's preference structure. Specifically, within the compensation range of o_i, different values might provide different utilities to the DMs, i.e. each value $x \in [o_{li},\ o_{ui}]$ has a specific utility value $U(x) \in [0, 1]$. Assume that $U(x)$ is a linear (piecewise) function with single variable, which satisfies $0 \leq U(x) \leq 1$. When $U(x) \in \{0, 1\}$, we have a binary utility model (see Figures 1.1 and 1.2). $U'(x)$ indicates the derivative of the utility function and represents the change amount of DM's unit compensation utility, so it is the marginal utility of the compensation. Actually, the marginal utility of compensation is the slope of the linear utility function $U(x)$. When $U'(x) > 0$ the DM's utility increases with the growth of compensation as Figure 1.3 illustrates; while $U'(x) < 0$ reveals that the DM's utility decreases with the growth of compensation, which is shown in Figure 1.4. A combination of both traits with $U'(x) > 0$ and $U'(x) < 0$ is given in Figures 1.5 and 1.6.

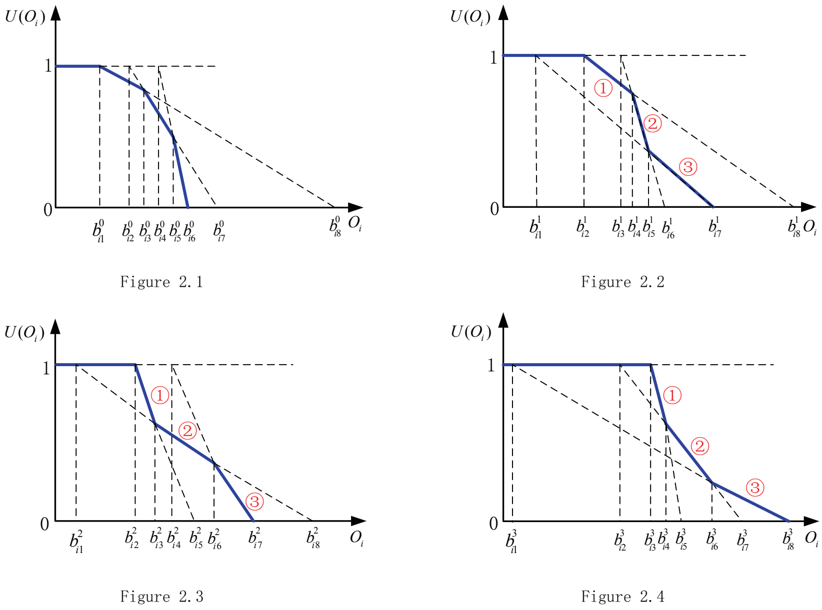

Fig. 2 Piecewise linear utility functions with left-skewed preferences

3.1 Basic types

According to the utility theory (Fishburn 1988), when the marginal utility increases then an individual behaves as a risk lover; conversely, when the marginal utility decreases the individual behaves as a risk averter. Actually, in real decision-making situations, individual's utility usually does not exhibit simple monotone increasing or decreasing trends as listed in Figure 1, but frequently shows a multi-stage fluctuation trait with both risk attitudes of appetite and aversion, which can be simulated by a type of multi-stage fluctuation utility functions as proposed next.

3.2 Proposed types

Multi-stage fluctuation utility functions, based on the above basic types of piecewise linear utility functions of Figure 1, will be used to characterize the DM's psychological fluctuation trait in different decision intervals. Here, three types of multi-stage fluctuation utility functions are provided, namely, left-skewed type, right-skewed type, and middle-skewed type.

3.2.1 Piecewise linear utility functions with left-skewed preference

When a DM's preference is left-skewed type, he (she) prefers the left point of the compensation interval, and the utility decreases with the growth of his (her) compensation. Furthermore, the DM's marginal utility and attitude towards risk differ among their compensation intervals. Here, four kinds of piecewise linear utility functions with left-skewed preferences are discussed.

– Piecewise linear concave utility function with left-skewed preference – PLUFL-1 (Figure 2.1):

$$Case\ I:\ U(O_i) = \begin{cases} 1, & if\ O_i < b_{i1}^0 \\ \frac{b_{i8}^0 - O_i}{b_{i8}^0 - b_{i1}^0}, & if\ b_{i1}^0 \le O_i < b_{i3}^0 \\ \frac{b_{i7}^0 - O_i}{b_{i7}^0 - b_{i2}^0}, & if\ b_{i3}^0 \le O_i < b_{i5}^0 \\ \frac{b_{i6}^0 - O_i}{b_{i6}^0 - b_{i4}^0}, & if\ b_{i5}^0 \le O_i < b_{i6}^0 \\ 0, & if\ O_i \ge b_{i6}^0 \end{cases}$$

PLUFL-1 is a linear concave function, where $b_{ij}^0, i \in M, j = 1, \cdots, 8$ denote the piecewise points of the utility function in the horizontal axis. In subinterval $[b_{i1}^0,\ b_{i3}^0]$, the marginal utility of PLUFL-1 is $a_1 = \frac{1}{b_{i1}^0 - b_{i8}^0}$, while the marginal utility in subintervals $[b_{i3}^0,\ b_{i5}^0]$ and $[b_{i5}^0,\ b_{i6}^0]$ are $a_2 = \frac{1}{b_{i2}^0 - b_{i7}^0}$ and $a_3 = \frac{1}{b_{i4}^0 - b_{i6}^0}$, respectively. Notice that the utility of PLUFL-1 decreases as the DM's compensation increases. When $a_3 < a_2 < a_1 < 0$ it is $|a_1| < |a_2| < |a_3|$, thus the utility of PLUFL-1 quickly decreases with the growth of the compensation, i.e. the marginal utility decreases). Under the preference of PLUFL-1, both the utility and marginal utility decrease. As a result, the DM is considered in this case as a risk averter.

– Piecewise linear reversed S1-shaped utility function with left-skewed preference – PLUFL-2 (Figure 2.2):

$$Case\ II:\ U(O_i)\ =\ \begin{cases} 1, & if\ O_i < b_{i2}^1 \\ \frac{b_{i8}^1 - O_i}{b_{i8}^1 - b_{i2}^1} + \tilde{M} * \delta_1, & if\ b_{i2}^1 \leq O_i < b_{i4}^1 \\ \frac{b_{i6}^1 - O_i}{b_{i6}^1 - b_{i3}^1} + \tilde{M} * \delta_1, & if\ b_{i4}^1 \leq O_i < b_{i5}^1 \\ \frac{b_{i7}^1 - O_i}{b_{i7}^1 - b_{i1}^1} + \tilde{M} * (1 - \delta_1), & if\ b_{i5}^1 \leq O_i < b_{i7}^1 \\ 0, & if\ O_i \geq b_{i7}^1 \end{cases}$$

where $b_{ij}^1, i \in M, j = 1, \cdots, 8$ denotes the piecewise points of utility function in the horizontal axis, $\tilde{M}$ stands for a real number that can be infinite (we take $\tilde{M} = 10^6$ in empirical analysis part), and δ_1 denotes an additional $\{0, 1\}$-binary variable (Yang et al. 1991; Wen et al. 2014). It's easy to find that the value of $U(O_i)$ is under the entire segment, including ① and ②, if $\delta_1 = 0$ holds (namely O_i lies in interval $[0,\ b_{i5}^1]$), while under the segments of ③ if $\delta_1 = 1$ holds (i.e., O_i is within the interval $[b_{i5}^1,\ b_{i7}^1]$) (see Figure 2.2). Actually, the entire segment (including ①, ②) and segment ③ is a kind of relation of union rather than intersection. Meanwhile, in subintervals $[b_{i2}^1,\ b_{i4}^1]$, $[b_{i4}^1,\ b_{i5}^1]$ and $[b_{i5}^1,\ b_{i7}^1]$, the marginal utilities of PLUFL-2 are $b_1 = \frac{1}{b_{i2}^1 - b_{i8}^1}$, $b_2 = \frac{1}{b_{i3}^1 - b_{i6}^1}$ and $b_3 = \frac{1}{b_{i1}^1 - b_{i7}^1}$, respectively. The utility of PLUFL-2 thus decreases as the DM's compensation increases. When $b_2 < b_1 < 0, b_2 < b_3 < 0$, $|b_1| < |b_2|$, it is $|b_3| < |b_2|$. Therefore the utility of PLUFL-2 quickly decreases from subinterval $[b_{i2}^1,\ b_{i4}^1]$ to subinterval $[b_{i4}^1,\ b_{i5}^1]$, i.e. the marginal utility decreases, making the DM to behave as a risk averter; however, the utility of PLUFL-2 slowly decreases from subinterval $[b_{i4}^1,\ b_{i5}^1]$ to subinterval $[b_{i5}^1,\ b_{i7}^1]$, i.e. the marginal utility increases, and the DM acts as a risk lover. In short, under PLUFL-2, the utility decreases as the DM's compensation increases, and the marginal utility tends to be different. In this situation, the DM is first a risk averter and later a risk lover as the compensation moves from the lower to the upper bound.

– Piecewise linear reversed S2-shaped utility function with left-skewed preference – PLUFL-3 (Figure 2.3):

$$Case\ III:\ U(O_i)\ =\ \begin{cases} 1, & if\ O_i < b_{i2}^2 \\ \frac{b_{i5}^2 - O_i}{b_{i5}^2 - b_{i2}^2} + \tilde{M} * (1 - \delta_2), & if\ b_{i2}^2 \leq O_i < b_{i3}^2 \\ \frac{b_{i8}^2 - O_i}{b_{i8}^2 - b_{i1}^2} + \tilde{M} * \delta_2, & if\ b_{i3}^2 \leq O_i < b_{i6}^2 \\ \frac{b_{i7}^2 - O_i}{b_{i7}^2 - b_{i4}^2} + \tilde{M} * \delta_2, & if\ b_{i6}^2 \leq O_i < b_{i7}^2 \\ 0, & if\ O_i \geq b_{i7}^2 \end{cases}$$

where $b_{ij}^2, i \in M, j = 1, \cdots, 8$ represents the piecewise points of the utility function, $\tilde{M}$ stands for an infinite real number, δ_2 denotes an additional $\{0, 1\}$-binary variable. Also, we can find that the value of $U(O_i)$ is under the segment of ①(namely O_i is in interval $[0,\ b_{i3}^2]$ at this point) if $\delta_2 = 1$ holds, while under the entire segment, including ② and ③, if $\delta_2 = 0$ holds (namely O_i lies in $[b_{i3}^2,\ b_{i7}^2]$) (see Figure 2.3). The marginal utility of the subintervals $[b_{i2}^2,\ b_{i3}^2]$, $[b_{i3}^2,\ b_{i6}^2]$ and $[b_{i6}^2,\ b_{i7}^2]$ are $c_1 = \frac{1}{b_{i2}^2 - b_{i5}^2}$, $c_2 = \frac{1}{b_{i1}^2 - b_{i8}^2}$ and $c_3 = \frac{1}{b_{i4}^2 - b_{i7}^2}$, respectively. The utility of PLUFL-3 decreases as DM's compensation increases, and given that $c_1 < c_2 < 0, c_3 < c_2 < 0$ implies $|c_1| > |c_2|, |c_3| > |c_2|$, it is clear that the utility of PLUFL-3 slowly decreases from subinterval $[b_{i2}^2,\ b_{i3}^2]$ to subinterval $[b_{i3}^2,\ b_{i6}^2]$, i.e. the marginal utility increases, meaning the DM behaves as a risk lover; while the utility of PLUFL-3 decreases quickly from subinterval $[b_{i3}^2,\ b_{i6}^2]$ to subinterval $[b_{i6}^2,\ b_{i7}^2]$, i.e. the marginal utility decreases, making the DM to act as a risk averter. In short, for PLUFL-3 the DM's utility decreases with the growth of his (her) compensation, while the marginal utility differs within different subintervals. So, unlike Case II, the DM in this case is first a risk lover and then a risk averter as the compensation moves from the lower to the upper bound.

– Piecewise linear convex utility function with left-skewed preference – PLUFL-4 (Figure 2.4):

$$Case\ IV:\ U(O_i)\ =\ \begin{cases} 1, & if\ O_i < b_{i3}^3 \\ \frac{b_{i5}^3 - O_i}{b_{i5}^3 - b_{i3}^3} + \tilde{M} * \delta_3 + \tilde{M} * \delta_4, & if\ b_{i3}^3 \leq O_i < b_{i4}^3 \\ \frac{b_{i7}^3 - O_i}{b_{i7}^3 - b_{i2}^3} + \tilde{M} * (1 - \delta_3), & if\ b_{i4}^3 \leq O_i < b_{i6}^3 \\ \frac{b_{i8}^3 - O_i}{b_{i8}^3 - b_{i1}^3} + \tilde{M} * \delta_3, & if\ b_{i6}^3 \leq O_i < b_{i8}^3 \\ 0, & if\ O_i \geq b_{i8}^3 \end{cases}$$

PLUFL-4 is a linear convex function, where $b_{ij}^3, i \in M, j = 1, \cdots, 8$ denote the piecewise points in the horizontal axis, $\tilde{M}$ represents a real number that can be infinite, and δ_3, δ_4 are two additional binary variables for representing the union relations of three segments. We believe that the value of $U(O_i)$ is under the segment of ③ if $\delta_3 = 0$ and $\delta_4 \neq 0$ hold, under the segment of ① as well as ③ if $\delta_3 = 0$ and $\delta_4 = 0$ hold, while under the segment of ② if $\delta_3 = 1$ (see Figure 2.4). Marginal utilities of PLUFL-4 are $r_1 = \frac{1}{b_{i3}^3 - b_{i5}^3}$, $r_2 = \frac{1}{b_{i2}^3 - b_{i7}^3}$, and $r_3 = \frac{1}{b_{i1}^3 - b_{i8}^3}$ for the three compensation subintervals $[b_{i3}^3, b_{i4}^3]$, $[b_{i4}^3,\ b_{i6}^3]$, and $[b_{i6}^3,\ b_{i8}^3]$, respectively. Obviously, the utility of PLUFL-4 decreases as the DM's compensation increases, and the marginal utility increases because when $r_1 < r_2 < r_3 < 0$ it is $|r_1| > |r_2| > |r_3|$. In this situation, as the DM's compensation increases, the utility decreases while the marginal utility increases, indicating that the DM is a risk lover.

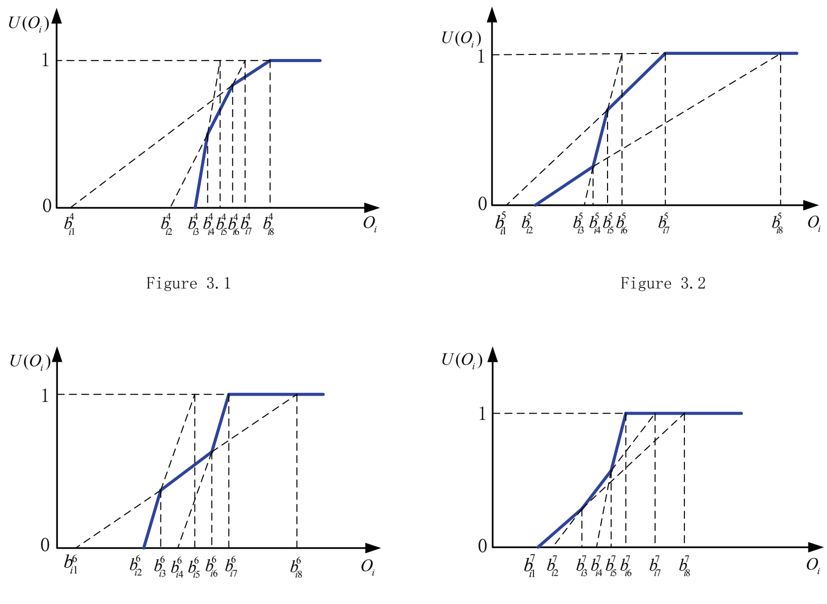

Figure 3.1 Figure 3.2 Figure 3.3 Figure 3.4

Fig. 3 Piecewise linear utility functions with right-skewed preference

3.2.2 *Piecewise linear utility function with right-skewed preference*

When a DM's preference is right-skewed, he (she) prefers the right point of the compensation interval, and accordingly the utility increases with the growth of his (her) compensation. Moreover, DM's marginal utility and attitude towards risk still can differ in different compensation subintervals. Following a similar description and analysis to the one provided before, four kinds of piecewise linear utility functions with right-skewed preference are possible:

– Piecewise linear concave utility function with right-skewed preference – PLUFR-1 (Figure 3.1):

$$Case\ V:\ U(O_i) = \begin{cases} 0, & if\ O_i < b_{i3}^4 \\ \frac{O_i - b_{i3}^4}{b_{i5}^4 - b_{i3}^4}, & if\ b_{i3}^4 \le O_i < b_{i4}^4 \\ \frac{O_i - b_{i2}^4}{b_{i7}^4 - b_{i2}^4}, & if\ b_{i4}^4 \le O_i < b_{i6}^4 \\ \frac{O_i - b_{i1}^4}{b_{i8}^4 - b_{i1}^4}, & if\ b_{i6}^4 \le O_i < b_{i8}^4 \\ 1, & if\ O_i \ge b_{i8}^4 \end{cases}$$

As the compensation increases in PLUFR-1,the utility increases while the marginal utility decreases, which represents the DM as a risk averter.

– Piecewise linear S1-shaped utility function with right-skewed preference – PLUFR-2 (Figure 3.2):

$$Case\ VI:\ U(O_i) = \begin{cases} 0, & if\ O_i < b_{i2}^5 \\ \frac{O_i - b_{i2}^5}{b_{i8}^5 - b_{i2}^5} + \tilde{M} * (1 - \delta_5), & if\ b_{i2}^5 \le O_i < b_{i4}^5 \\ \frac{O_i - b_{i3}^5}{b_{i6}^5 - b_{i3}^5} + \tilde{M} * \delta_5, & if\ b_{i4}^5 \le O_i < b_{i5}^5 \\ \frac{O_i - b_{i1}^5}{b_{i7}^5 - b_{i1}^5} + \tilde{M} * \delta_5, & if\ b_{i5}^5 \le O_i < b_{i7}^5 \\ 1, & if\ O_i \ge b_{i7}^5 \end{cases}$$

The utility in PLUFR-2 increases with the DM's compensation, while the marginal utility differs, making the DM to behave first as a risk lover and then as a risk averter as the compensation price moves from the lower to the upper bound.

– Piecewise linear S2-shaped utility function with right-skewed preference (PLUFR-3, Figure 3.3):

$$Case\ VII:\ U(O_i) = \begin{cases} 0, & if\ O_i < b_{i2}^6 \\ \frac{O_i - b_{i2}^6}{b_{i5}^6 - b_{i2}^6} + \tilde{M} * \delta_6, & if\ b_{i2}^6 \le O_i < b_{i3}^6 \\ \frac{O_i - b_{i1}^6}{b_{i8}^6 - b_{i1}^6} + \tilde{M} * \delta_6, & if\ b_{i3}^6 \le O_i < b_{i6}^6 \\ \frac{O_i - b_{i4}^6}{b_{i7}^6 - b_{i4}^6} + \tilde{M} * (1 - \delta_6), & if\ b_{i6}^6 \le O_i < b_{i7}^6 \\ 1, & if\ O_i \ge b_{i7}^6 \end{cases}$$

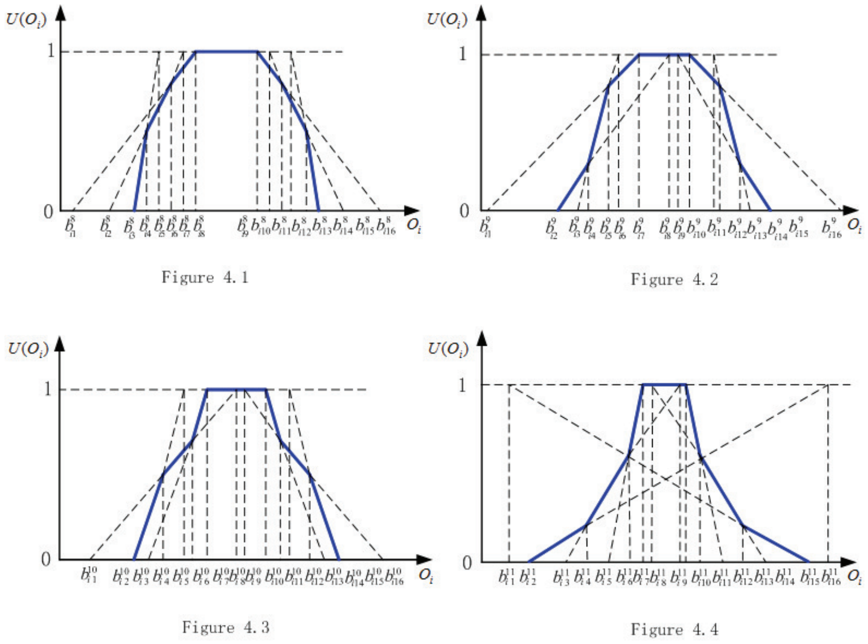

Fig. 4 Piecewise linear utility functions with middle-skewed preference

For PLUFR-3, the utility increases with the DM's compensation, and the changes in marginal utility indicates that this type of DM is first a risk averter and then a risk lover as the compensation moves from the lower to the upper bound.

– Piecewise linear convex utility function with right-skewed preference (PLUFR-4, Figure 3.4):

$$Case\ VIII:\ U(O_i)\ =\ \begin{cases} 0 & if\ O_i < b_{i1}^7 \\ \frac{O_i - b_{i1}^7}{b_{i8}^7 - b_{i1}^7} + \tilde{M} * \delta_7 + \tilde{M} * \delta_8, & if\ b_{i1}^7 \le O_i < b_{i3}^7 \\ \frac{O_i - b_{i2}^7}{b_{i7}^7 - b_{i2}^7} + \tilde{M} * (1 - \delta_7), & if\ b_{i3}^7 \le O_i < b_{i5}^7 \\ \frac{O_i - b_{i4}^7}{b_{i6}^7 - b_{i4}^7} + \tilde{M} * \delta_7, & if\ b_{i5}^7 \le O_i < b_{i6}^7 \\ 1 & if\ O_i \ge b_{i6}^7 \end{cases}$$

For PLUFR-4 both the utility and the marginal utility increase, and therefore the DM is a risk lover.

3.2.3 Piecewise linear utility function with middle-skewed preference

Generally, the ultimate goal of GDM is to acquire collective opinions through multiple negotiations with respect to target alternatives. In such process, DMs always expect to make the most profitable decisions regarding their own interests, and usually tend to display a certain "extreme tendency", preferring either the maximum or minimum values of the compensation interval. However, in practice, negotiation tends to ultimately compromise. In those cases, the DM's compensation turns out to be an interval central located number. Therefore, it is necessary to closely investigate the middle-skewed preference. Given limitations of space, here we only present the preference with symmetrical structure.

– Piecewise linear concave utility function with middle-skewed preference – PLUFM-1 (Figure 4.1):

$$Case\ IX:\ U(O_i)\ =\ \begin{cases} 0, & if\ O_i < b_{i3}^8 \\ \frac{O_i - b_{i3}^8}{b_{i5}^8 - b_{i3}^8}, & if\ b_{i3}^8 \le O_i < b_{i4}^8 \\ \frac{O_i - b_{i2}^8}{b_{i7}^8 - b_{i2}^8}, & if\ b_{i4}^8 \le O_i < b_{i6}^8 \\ \frac{O_i - b_{i1}^8}{b_{i8}^8 - b_{i1}^8}, & if\ b_{i6}^8 \le O_i < b_{i8}^8 \\ 1, & if\ b_{i8}^8 \le O_i < b_{i9}^8 \\ \frac{b_{i16}^8 - O_i}{b_{i16}^8 - b_{i9}^8}, & if\ b_{i9}^8 \le O_i < b_{i11}^8 \\ \frac{b_{i15}^8 - O_i}{b_{i15}^8 - b_{i10}^8}, & if\ b_{i11}^8 \le O_i < b_{i13}^8 \\ \frac{b_{i14}^8 - O_i}{b_{i14}^8 - b_{i12}^8}, & if\ b_{i13}^8 \le O_i < b_{i14}^8 \\ 0, & if\ O_i \ge b_{i14}^8 \end{cases}$$

With PLUFM-1, the DM prefers the intermediate value of compensation interval, that is $[b_{i8}^8,\ b_{i9}^8]$, where the compensation utility of the DM reaches the maximum value and its corresponding marginal utility remains unchanged (that is 0). Besides, the DM's utility shows an increasing tendency in the left section $[b^8_{i3},\ b^8_{i8}]$, and a decreasing tendency in the right section $[b^8_{i9},\ b^8_{i14}]$. Meanwhile, the marginal utility presents a decreasing trend in both sections (for more details refer to PLUFR-1 and PLUFL-1). Obviously, the DM is a risk averter concerning the preference of PLUFM-1.

– Piecewise linear reversed U1-shaped utility function with middle-skewed preference – PLUFM-2 (Figure 4.2):

$$Case\ X:\ U(O_i)\ =\begin{cases}0, & if\ O_i < b^9_{i2}\\ \frac{O_i-b^9_{i2}}{b^9_{i8}-b^9_{i2}}+\tilde{M}*(1-\delta_9), & if\ b^9_{i2}\le O_i < b^9_{i4}\\ \frac{O_i-b^9_{i3}}{b^9_{i6}-b^9_{i3}}+\tilde{M}*\delta_9, & if\ b^9_{i4}\le O_i < b^9_{i5}\\ \frac{O_i-b^9_{i1}}{b^9_{i7}-b^9_{i1}}+\tilde{M}*\delta_9, & if\ b^9_{i5}\le O_i < b^5_{i7}\\ 1, & if\ b^9_{i7}\le O_i < b^9_{i10}\\ \frac{b^9_{i16}-O_i}{b^9_{i16}-b^9_{i10}}+\tilde{M}*\delta_{10}, & if\ b^9_{i10}\le O_i < b^9_{i12}\\ \frac{b^9_{i14}-O_i}{b^9_{i14}-b^9_{i11}}+\tilde{M}*\delta_{10}, & if\ b^9_{i12}\le O_i < b^9_{i13}\\ \frac{b^9_{i15}-O_i}{b^9_{i15}-b^9_{i9}}+\tilde{M}*(1-\delta_{10}), & if\ b^9_{i13}\le O_i < b^9_{i15}\\ 0, & if\ O_i\ge b^9_{i15}\end{cases}$$

A DM with PLUFM-2 prefers the intermediate value of the compensation interval $[b^9_{i7},\ b^9_{i10}]$. Moreover, the DM's utility increases in $[b^9_{i2},\ b^9_{i7}]$, yet decreases in $[b^9_{i10},\ b^9_{i15}]$. The marginal utilities in both $[b^9_{i2},\ b^9_{i7}]$ and $[b^9_{i10},\ b^9_{i15}]$ tend to be complex (for more details see PLUFR-2 and PLUFL-2). In short, the DM shows two different attitudes towards risk under the case of PLUFM-2.

– Piecewise linear reversed U2-shaped utility function with middle-skewed preference – PLUFM-3 (Figure 4.3)

$$Case\ XI:\ U(O_i)\ =\begin{cases}0, & if\ O_i < b^{10}_{i2}\\ \frac{O_i-b^{10}_{i2}}{b^{10}_{i5}-b^{10}_{i2}}+\tilde{M}*\delta_{11}, & if\ b^{10}_{i2}\le O_i < b^{10}_{i4}\\ \frac{O_i-b^{10}_{i1}}{b^{10}_{i8}-b^{10}_{i1}}+\tilde{M}*\delta_{11}, & if\ b^{10}_{i4}\le O_i < b^{10}_{i6}\\ \frac{O_i-b^{10}_{i3}}{b^{10}_{i7}-b^{10}_{i3}}+\tilde{M}*(1-\delta_{11}), & if\ b^{10}_{i6}\le O_i < b^{10}_{i7}\\ 1, & if\ b^{10}_{i7}\le O_i < b^{10}_{i10}\\ \frac{b^{10}_{i14}-O_i}{b^{10}_{i14}-b^{10}_{i10}}+\tilde{M}*(1-\delta_{12}), & if\ b^{10}_{i10}\le O_i < b^{10}_{i11}\\ \frac{b^{10}_{i16}-O_i}{b^{10}_{i16}-b^{10}_{i9}}+\tilde{M}*\delta_{12}, & if\ b^{10}_{i11}\le O_i < b^{10}_{i13}\\ \frac{b^{10}_{i15}-O_i}{b^{10}_{i15}-b^{10}_{i12}}+\tilde{M}*\delta_{12}, & if\ b^{10}_{i13}\le O_i < b^{10}_{i15}\\ 0, & if\ O_i\ge b^{10}_{i15}\end{cases}$$

A DM with PLUFM-3 prefers the compensation value within $[b^{10}_{i7}, b^{10}_{i10}]$. The DM's compensation utility increases in $[b^{10}_{i2},\ b^{10}_{i7}]$, and decreases in $[b^{10}_{i10},\ b^{10}_{i15}]$. Meanwhile, the marginal utilities both in $[b^{10}_{i2},\ b^{10}_{i7}]$ and $[b^{10}_{i10},\ b^{10}_{i15}]$ also turns out to be complex (for more details refer to PLUFR-3 and PLUFL-3). Briefly, the DM in this case also has different attitudes towards risk.

– Piecewise linear convex utility functions with middle-skewed preference – PLUFM-4 (Figure 4.4):

$$Case\ XII:\ U(O_i)\ =\begin{cases}0, & if\ O_i < b^{11}_{i2}\\ \frac{O_i-b^{11}_{i2}}{b^{11}_{i16}-b^{11}_{i2}}+\tilde{M}*\delta_{13}+\tilde{M}*\delta_{14}, & if\ b^{11}_{i2}\le O_i < b^{11}_{i4}\\ \frac{O_i-b^{11}_{i3}}{b^{11}_{i9}-b^{11}_{i3}}+\tilde{M}*(1-\delta_{13}), & if\ b^{11}_{i4}\le O_i < b^{11}_{i6}\\ \frac{O_i-b^{11}_{i5}}{b^{11}_{i7}-b^{11}_{i5}}+\tilde{M}*\delta_{13}, & if\ b^{11}_{i6}\le O_i < b^{11}_{i7}\\ 1, & if\ b^{11}_{i7}\le O_i < b^{11}_{i10}\\ \frac{b^{11}_{i12}-O_i}{b^{11}_{i12}-b^{11}_{i10}}+\tilde{M}*\delta_{15}+\tilde{M}*\delta_{16}, & if\ b^{11}_{i10}\le O_i < b^{11}_{i11}\\ \frac{b^{11}_{i14}-O_i}{b^{11}_{i14}-b^{11}_{i8}}+\tilde{M}*(1-\delta_{15}), & if\ b^{11}_{i11}\le O_i < b^{11}_{i13}\\ \frac{b^{11}_{i15}-O_i}{b^{11}_{i15}-b^{11}_{i1}}+\tilde{M}*\delta_{15}, & if\ b^{11}_{i13}\le O_i < b^{11}_{i15}\\ 0, & if\ O_i\ge b^{11}_{i15}\end{cases}$$

A DM with a PLUFM-4 linear convex function will act as a risk lover.

4 Consensus modeling with multi-stage fluctuation utility function

To achieve a demolition consensus, we need to construct a consensus model that can reflect the negotiation process. As we mentioned in Section 2.1, let o_i denotes e_i's ($i \in M = \{1, 2, \ldots, m\}$) expected compensation, o' represents G's compensation, and $l_i = |o_i - o'|$ denotes the distance deviation between the compensation of e_i and G. G always

Table 1 Combinations of different utility preference for G and e_i

G \ e_i	Left	Middle	Right
Left	L-L	L-M	L-R
Middle	M-L	M-M	M-R
Right	R-L	R-M	R-R

hopes for the deviation to be as small as possible. Different e_is have different interests and, therefore, G deservedly needs to provide them with distinct unit costs to persuade them to change their original requests. Thus, we have the following consensus model:

$$Model\ (1):\ min\ \sum_{i=1}^{n}\omega_i|o_i-o'| \\ s.t.\begin{cases}|o_i-o'|\le\varepsilon_i, i=1,2,\cdots,m\\ o'\in O,\ o_i\in[a_i,b_i], \omega_i\in[0,1]\ i\in M\end{cases} \tag{1}$$

where ω_i is the unit cost assigned to e_i, which is a predetermined constant, o_i represents e_i's compensation within his (her) compensation interval $[a_i,\ b_i]$, O is the feasible set of consensus compensation o', ε_i is the allowable compensation deviation, and $|o_i-o'|\le\varepsilon_i$ denotes the compensation deviation constraint (tolerance) between o_i and o'.

Out of self-interest, each e_i always expects their requests to get enough attention in the negotiation process. Simultaneously, both e_i and G may show distinct utility distributions on the compensation problem. Owing to preference discrepancies, it is difficult to fulfil e_is' preference utilities by only considering the minimum cost as the objective function. Thus, G's budget must be flexible enough to convince all residents to change their original compensation requests. In other words, to satisfy each e_i's optimal utility, G must arrange a sufficient budget to reach the consensus. Hence, to simulate such negotiation process, the utilities are included into the original consensus Model (1) for both e_i and G. Thus, the new consensus decision-making model is constructed as follows:

$$Model\ (2):\ max\ \lambda \\ s.t.\begin{cases}\sum_{i=1}^{m}\omega_i|o_i-o'|=B & (2-1)\\ \lambda\le U(o') & (2-2)\\ \lambda\le U(o_i),\ i\in M & (2-3)\\ |o_i-o'|\le\varepsilon_i,\ i\in M & (2-4)\\ o'\in O,\ o_i\in[a_i,b_i],\ i\in M & (2-5)\end{cases} \tag{2}$$

where λ indicates the group utility; B is the predetermined budget for reaching consensus; $U(o')$, $U(o_i)$ $(i\in M)$ are utility functions for G and e_i. Constraint (2-4) indicates that the allowable compensation deviation between e_i and G is $[o'-\varepsilon_i,\ o'+\varepsilon_i]$, which is regarded as the range e_i is prepared to accept. The utility value λ is the ultimate goal of the negotiation, and to some extent, it can also be considered as a measure of the consensus level.

The solution of Model (1) gives an indication of the minimum budget required to satisfy consensus negotiation subject to the allowed compensation deviation. Unlike Model (1), all DM's expected utilities are further taken into account in Model (2), which can better reflect the real decision-making scenario.

Also, from constraints (2-2) and (2-3) in Model (2), we can see that G's and e_i's utility preferences can influence the consensus level. To detail this, we propose different kinds of consensus models with varied utility preferences. Specifically, G's and e_i's utility preferences are divided into left-skewed, middle-skewed, and right-skewed types (see Table 1). The first row of Table 1 represents the left-skewed, middle-skewed, and right-skewed types of e_i's preference, respectively; similarly, the first column of Table 1 indicates the left-skewed, middle-skewed, and right-skewed types of G's preference, respectively. And the rest stand for the preference combinations for G and e_i. Take "M-R" for example, it represents a combination that G's preference is middle-skewed type and e_i' s preference is right-skewed type. Its economic significance shows that G hopes to spend a moderate compensation price reaching the demolition consensus, and it achieves its highest utility level; and for moved residents, the higher the compensation prices are, the larger their utilities are.

In addition, from Model (2), it is clear that G's total budget B, and e_i's tolerance ε_i both affect the overall decision-making utility, thus may have influence on the consensus level. Therefore, later analysis of the case will focus on the following two aspects: (1) sensitivity analysis on G's budget B; and (2) sensitivity analysis on e_i's tolerance ε_i.

5 Empirical analysis

To illustrate how both G's compensation budget and each e_i's compensation tolerance affect the consensus level, this section is arranged as follows: (i) without considering all the negotiators' psychological preferences, G's minimum budget is first calculated by Model (1). Then, with different preference constraints under the fixed budget, the optimal compensation prices for G and e_i are obtained by Model (2); and (ii), the impact of G's budget and e_i's compensation tolerance are deeply discussed.

Assume the demolition negotiation involves four residents, and their expected compensation intervals are $o_1 = [48,\ 52]$, $o_2 = [50,\ 55]$, $o_3 = [60,\ 65]$, and $o_4 = [62,\ 67]$, respectively. G's predetermined budget interval is $o' = [50,\ 65]$ (unit: ten thousands RMB). To reach a consensus, G must negotiate with every e_i $(i \in 1,2,3,4)$. Let the unit costs to each resident be $\omega_1 = 0.8,\ \omega_2 = 1.5,\ \omega_3 = 1.2$, and $\omega_4 = 0.9$ (unit: thousand RMB). Suppose the allowable compensation deviations (namely the upper limit of the range between G's and each e_i's expected compensations) are $\varepsilon_1 = 10.5,\ \varepsilon_2 = 9,\ \varepsilon_3 = 9.5$ and $\varepsilon_4 = 9.8$, respectively.

5.1 General model and its economic interpretation

Without considering the utilities of G and e_is, an optimized consensus model with minimum cost is built as follows:

$$Model\ (3):\ min\ \phi = \sum_{i=1}^{m} \omega_i(u_i + v_i)$$

$$s.t.\begin{cases} o' - u_1 + v_1 = o_1, o' - u_2 + v_2 = o_2 & (3-1)\\ o' - u_3 + v_3 = o_3, o' - u_4 + v_4 = o_4 & (3-2)\\ |o_1 - o'| \le 10.5, |o_2 - o'| \le 9 & (3-3)\\ |o_3 - o'| \le 9.5, |o_4 - o'| \le 9.8 & (3-4)\\ o_1 \in [48,\ 52], o_2 \in [50,\ 55] & (3-5)\\ o_3 \in [60,\ 65], o_4 \in [62,\ 67] & (3-6)\\ o' \in [50,\ 65] & (3-7)\\ u_i, v_i \ge 0, i = 1,2,3,4 & (3-8) \end{cases}$$

where, $u_i + v_i = |o' - o_i|$ denotes the compensation deviation between G's and each e_i's compensations, and $u_i,\ v_i$ are nonnegative constants assigned to simplify the formula $|o' - o_i|$, satisfying $u_i v_i = 0$. Constraints (3-1) and (3-2) are the linear deviation constraints attached to the equation $u_i + v_i = |o' - o_i|$, which is equivalent to $o' - o_i = u_i - v_i,\ i = 1,2,3,4$. Constraints (3-3) and (3-4) represent the allowable deviations between G's and each e_i's compensations. An optimal solution to Model (3) can be obtained as $X^* = (3\ 0\ 0\ 0\ 0\ 5\ 0\ 7\ 55\ 52\ 55\ 60\ 62\ )^T$. The objective value of Model (3) is $min\ \phi = 14.7$, which represents the minimum negotiation budget for G to reach the consensus. The optimal expected compensation prices for e_i $(i \in 1,2,3,4)$ and G are $o_1 = 52,\ o_2 = 55,\ o_3 = 60,\ o_4 = 62$, and $o' = 55$, respectively. Obviously, Model (3) only considers the optimal total compensation from the perspective of minimum cost, and does not consider the preference utilities of e_is and G. That is, Model (3) is just a concrete example of the general Model (1). In fact, it is unrealistic to set a budget in Model (2) with the solution of Model (1) when taking all the DM's expected utilities into account. Thus, the budget B in Model (4) is fixed as 30 instead of 14.7 obtained by Model (3). Additionally, the compensation preference of each e_i's and G's are represented by PLUFR-1, PLUFR-2, PLUFR-3, PLUFR-4, and PLUFM-1, respectively. Therefore, based on the limited budget, the optimal consensus model with preference restrictions is built as follows:

$$Model\ (4):\ max\ \lambda$$

$$s.t.\begin{cases} 0.8*(u_1+v_1) + 1.5*(u_2+v_2) + 1.2*(u_3+v_3) & \\ +0.9*(u_4+v_4) = 30 & (4-1)\\ o' - u_1 + v_1 - o_1 = 0, o' - u_2 + v_2 - o_2 = 0 & (4-2)\\ o' - u_3 + v_3 - o_3 = 0, o' - u_4 + v_4 - o_4 = 0 & (4-3)\\ o_1 < o' + 10.5, o_1 > o' - 10.5, o_2 < o' + 9, o_2 > o' - 9 & (4-4)\\ o_3 < o' + 9.5, o_3 > o' - 9.5, o_4 < o' + 9.8, o_4 > o' - 9.8 & (4-5)\\ \lambda \le \frac{o_1-48}{1.066666667}, \lambda \le \frac{o_1-47.44}{3.2}, \lambda \le \frac{o_1-42}{10} & (4-6)\\ \lambda \le \frac{o_2-50}{10/3} + \tilde{M}*(1-\delta_1), \lambda \le \frac{o_2-50.7}{0.375} + \tilde{M}*\delta_1, & \\ \lambda \le \frac{o_2-37.5}{17.5} + \tilde{M}*\delta_1 & (4-7)\\ \lambda \le \frac{o_3-60}{10/3} + \tilde{M}*\delta_2, \lambda \le \frac{o_3-59.2}{6} + \tilde{M}*\delta_2, & \\ \lambda \le \frac{o_3-60}{5} + \tilde{M}*(1-\delta_2) & (4-8)\\ \lambda \le \frac{o_4-62}{25/3} + \tilde{M}*\delta_3 + \tilde{M}*\delta_4, \lambda \le \frac{o_4-190/3}{4} + \tilde{M}*\delta_3, & \\ \lambda \le \frac{o_4-64.5}{2.5} + \tilde{M}*(1-\delta_3) & (4-9)\\ \lambda \le \frac{82.5-o'}{24.5}, \lambda \le \frac{65.14-o'}{2.8}, \lambda \le \frac{65-o'}{7/3} & (4-10)\\ \lambda \le \frac{o'-50}{15/8}, \lambda \le \frac{o'-49.02}{5.6}, \lambda \le \frac{o'-39.5}{17.5} & (4-11)\\ o_1 \in [48,\ 52], o_2 \in [50,\ 55], o_3 \in [60,\ 65], o_4 \in [62,\ 67], & \\ o' \in [50,\ 65] & (4-12)\\ u_i, v_i \ge 0, i = 1,2,3,4 & (4-13) \end{cases}$$

In Model (4), constraint (4-1) is the total budget arranged by G to each e_i to reach the consensus. Constraints (4-2) and (4-3) indicate the deviation between e_i's and G's expected compensation prices. Constraints (4-6)-(4-11) are the preferences expressed by each e_i and G. Obviously, Model (4) is just a concrete example of Model (2). The

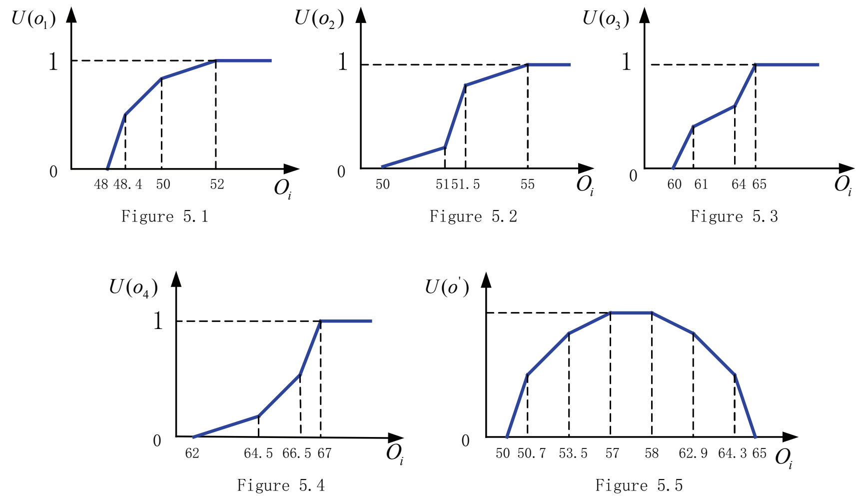

Fig. 5 Compensation preferences of Government and residents

optimal solutions to Model (4) is $X^* = (9.28\ 0\ 7.31\ 0\ 0\ 4.67\ 0\ 6.67\ 60.33\ 51.05\ 53.02\ 65\ 67\ )^T$. The objective value of Model (4) is $max\ \lambda = 0.91$. The optimal compensation prices for each e_i and G are $o_1 = 51.05,\ o_2 = 53.02,\ o_3 = 65,\ o_4 = 67$, and $o' = 60.33$, respectively.

Taking resident e_4 for example (see Figure 5.4), the expected compensation price is the interval of [62, 67], namely his (her) acceptable price for arriving the relocation consensus is [62, 67]. In specific, for e_4, if G offers a compensation price of 62, his (her) utility level is 0; and if the compensation price is offered as 67, then the utility reaches its maximum possible level of 1. e_4's utility increases with the compensation price proposed by G. Meanwhile, his (her) utility level tends to increase quickly with the growth of the price (that is the marginal utility increases). Specifically, the marginal utility of the price subinterval [66.5, 67] exceeds that of [64.5, 66.5], and further exceeds that of [62, 64.5]. In other words, the higher the price G pays, the higher e_4's utility, and the higher e_4's satisfaction. Therefore, e_4 is a risk lover.

Taking G's preference into consideration (see Figure 5.5), it hopes that the compensation price demanded by e_i lies in the middle section of interval [50, 65], (i.e., [57, 58]), where utility level reaches 1 and its marginal utility remains 0. If compensation price proposed by e_i remains in the low price subinterval [50, 57], then G's utility increases, while the marginal utility decreasing because of its low willingness to pay more. In other words, although the utility level in the low price subinterval increases with the price, G is unwilling to spend more on compensation, so its utility increases slowly. Yet in the high price subinterval, G believes e_i's demanding price are too high to be consistent with its own interests. Thus, the utility level decreases, and the value decreases quickly. Based on the analysis in Section 3.2.3, G is a risk averter.

The optimal compensation prices for each e_i are $o_1 = 51.05,\ o_2 = 53.02,\ o_3 = 65$, and $o_4 = 67$, respectively, which all approach the upper value of their own price intervals; as for G, its optimal compensation approaches the intermediate area of its price interval. In general, the group utility on the relocation issue is 0.91, which means that both G and e_i are pleased with the relocation alternative, so the consensus level is relatively high.

5.2 Sensitivity analysis on the consensus model

5.2.1 Sensitivity analysis on budget

Under the premise of all decision preferences and e_i's compensation tolerance fixed, different budgets will affect the consensus level. To describe the influence of G's budget on consensus level, four different utility structures, denoted by P_j $(j \in 1,2,3,4)$(see Appendix I), are provided in this section. Results are shown in Figure 6 and table 2 (see Appendix II).

Maintaining preference and tolerances unchanged, if we adjust budget B, the consensus level (i.e., the overall negotiation utility) substantially shows a first climbing then declining tendency. Specifically, when the budget B is less than 16 (ten thousand yuan), the consensus levels are close to 0 under the given four fixed utility structures, then the group consensus reaches at a extremely low level; as B increases, the consensus levels also increase (each e_i's psychological utility increases with the growth of compensation price, and the total cost is within G's budget); when B is within [25, 29], then consensus levels approach 1, namely arriving at a relatively high consensus level; however, when B continues to increase, the consensus levels decrease all the way until they reache zero, because G's psychological utility reduces with the growing budget, and also the sensitivities of residents' psychological utilities decline.

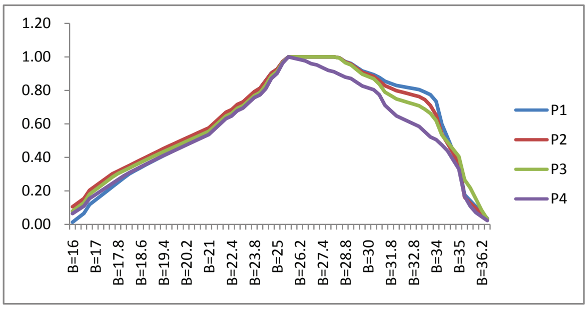

Fig. 6 Effects on the whole utility under different budgets

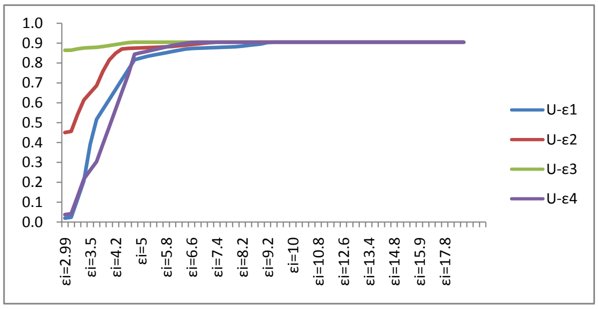

Fig. 7 Effects on the whole utility under different tolerances

5.2.2 Sensitivity analysis on tolerance

Similarly, for each e_i, he (she) has his (her) own acceptable range for G's compensation price (i.e., price tolerance), only when the compensation prices provided by G are within the scope of their psychological endurance, it can be possible to achieve the psychological utility value of e_i. Otherwise, if the compensation price can not meet e_i's psychological utility value, then it does not help to reach the consensus. Therefore, the effects of different price tolerances on the consensus level are discussed in this section, when psychological preference utilities and budget are determined. By only changing one resident's price tolerance at a time, we still use Model (4) to achieve the goal. Results are shown in Figure 7 and Table 3 (see Appendix II), where $U - \varepsilon_i$ denotes the consensus level under e_i's tolerance ε_i.

Keeping utility preference and Government's budget unchanged, the overall negotiation utility increases with the growth of e_i's psychological tolerance ε_i. Thus, results provide a good explanation about the phenomenon that as long as e_i's tolerance increases, it will be much easier for him (her) to accept G's compensation price, in other words, he (she) will be more likely to satisfy.

6 Summary and Conclusions

6.1 Comparisons and possible extensions

The provided consensus proposed model improves previous consensus models because it allows the implementation of both psychological preference and negotiation cost, which makes it more adaptable to real decision making. On the one hand, it has the advantage of reflecting DM's preference when compared with the research of Ben et al. (Ben et al 2007; Ben et al 2009), i.e., the consensus models in Ben et al.'s work evaluated consensus status only from cost point of view, and without considering the DM's psychological preference; indeed, Ben et al.'s model is constructed as follows:

$$\begin{aligned} &min\ \phi = \sum_{i=1}^{m} \omega_i |o_i - o'| \\ &or\ min\ \phi = \sum_{i=1}^{m} \omega_i (o_i - o')^2 \\ &s.t.\ \ |o_i - o'| \leq \varepsilon \end{aligned}$$

On the other hand, few previous studied on utility models take the cost into account. In other words, this type of utility model, previously reported in literature, only indicates the change of DM's psychological preference, but the negotiations cost has not been considered. For example, the present linear utility function (represented by function $u_k(o_k)$) proposed by Yang et al. was built as follows (shown in Figure 1.3):

$$u_k(o_k) = \begin{cases} 1, & if\ o_k \geq c_i \\ \frac{o_k - b_i}{c_i - b_i}, & if\ b_i \leq o_k \leq c_i \\ 0, & if\ o_k \leq b_i \end{cases}$$

And its development of linear utility model is shown as follows (see Figure 1.5):

$$u_k(o_k) = \begin{cases} 0, & if\ O_k \leq b_i \\ \frac{o_k - b_i}{c_i - b_i}, & if\ b_i \leq o_k \leq c_i \\ \frac{d_i - o_k}{d_i - c_i}, & if\ c_i \leq o_k \leq d_i \\ 0, & if\ o_k \geq d_i \end{cases}$$

A more complex model constructed by Chang (Chang 2007) is as follows:

$$u_k(o_k) = \begin{cases} 0, & if\ o_k < x_{i2}^1 \\ \frac{o_k - x_{i2}^1}{x_{i8}^1 - x_{i2}^1} + \tilde{M} * (1 - \delta), & if\ x_{i2}^1 \leq o_k < x_{i4}^1 \\ \frac{o_k - x_{i3}^1}{x_{i6}^1 - x_{i3}^1} + \tilde{M} * \delta, & if\ x_{i4}^1 \leq o_k < x_{i5}^1 \\ \frac{o_k - x_{i1}^1}{x_{i7}^1 - x_{i1}^1} + \tilde{M} * \delta, & if\ x_{i5}^1 \leq o_k < x_{i7}^1 \\ 1, & if\ o_k \geq x_{i7}^1 \end{cases}$$

where $x_{ij}^1, i \in M, j = 1, \cdots, 8$ denotes the piecewise points in the horizontal axis, $\tilde{M}$ represents a real number that can be infinite, and δ are two additional binary variables (see similar Figure 3.2).

Thus, all reported models in the literature are constructed either from the point of view of cost or from the point of view of preference, while the proposed consensus model with multi-stage fluctuations utility function is actually a combination of consensus model regarding cost and multi-stage fluctuations utility function (see Model (2)). Moreover, the proposed model not only considers the negotiation cost for reaching a consensus but also the DM's psychological preference. Therefore, the proposed model is closer to real decision-making situations. Besides, some more extensions of utility model can be found in the research of Chang et al. (Chang and Lin 2009; Chang 2011), and scholars who are interested in these studies can introduce these utility models into a more complex consensus model.

6.2 Summarizing contributions

Solving the consensus negotiation problem regarding urban demolition not only needs to consider the minimum cost provided by G, but also all DMs' preference utilities. To do this, multi-stage fluctuation utility functions (i.e., piecewise linear utility functions), including concave, convex, S-shaped, reversed S-shaped, and reversed U-shaped types, and their combinations, are added into the presented consensus model. The characteristics of the presented model are the following: (1) the total budget used to reach consensus is limited; (2) the compensation tolerance of the moved residents are considered; (3) multi-stage fluctuation utility functions, which is added into our cost-based consensus model, are employed to simulate DMs' behaviors and reflect the consensus level. Moreover, the influence of total budget and moved residents' tolerance on consensus level are investigated via an empirical analysis; and (4) all the models constructed in this paper adopt a linear form, making the models easier to solve.

Although we have obtained some meaningful results, some further work still can be done: (1) this paper assumes that the DM's psychological preference shows a linear utility fluctuation, while in real decision-making situations most people's psychological utilities exhibit non-linear fluctuations. Thus, we envisage the use of non-linear utility function to describe their psychological preferences in future studies; and (2) it was also assumed that there is no interaction among DMs, but only the existence of interaction between the individual DM and the moderator was emphasized. However, in a real decision-making process, interactions may exist among DMs, and they usually behave as various communication modes. Thus, the consensus model based on different communication modes among DMs and multi-stage fluctuation utility preference deserves to be explored in the future.

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Appendix I

Utility structure of P_1 :

$$\begin{cases} \lambda \leq \frac{o_1-48}{1.066666667}, \lambda \leq \frac{o_1-47.44}{3.2}, \lambda \leq \frac{o_1-42}{10} \\ \lambda \leq \frac{o_2-50}{10/3} + \tilde{M} * (1-\delta_1), \lambda \leq \frac{o_2-50.7}{0.375} + \tilde{M} * \delta_1 \\ \lambda \leq \frac{o_2-37.5}{17.5} + \tilde{M} * \delta_1, \lambda \leq \frac{o_3-60}{10/3} + \tilde{M} * \delta_2 \\ \lambda \leq \frac{o_3-59.2}{6} + \tilde{M} * \delta_2, \lambda \leq \frac{o_3-60}{5} + \tilde{M} * (1-\delta_2) \\ \lambda \leq \frac{o_4-62}{25/3} + \tilde{M} * \delta_3 + \tilde{M} * \delta_4, \lambda \leq \frac{o_4-190/3}{4} + \tilde{M} * \delta_3 \\ \lambda \leq \frac{o_4-64.5}{2.5} + \tilde{M} * (1-\delta_3), \lambda \leq \frac{82.5-o'}{24.5}, \lambda \leq \frac{65.14-o'}{2.8} \\ \lambda \leq \frac{65-o'}{7/3}, \lambda \leq \frac{o'-50}{15/8}, \lambda \leq \frac{o'-49.02}{5.6}, \lambda \leq \frac{o'-39.5}{17.5} \end{cases}$$

Utility structure of P_2 :

$$\begin{cases} \lambda \leq \frac{o_1-48}{17/8}, \lambda \leq \frac{o_1-47.84}{3.2}, \lambda \leq \frac{o_1-44}{8} \\ \lambda \leq \frac{o_2-50}{10/3} + \tilde{M} * (1-\delta_1), \lambda \leq \frac{o_2-50.4}{3/4} + \tilde{M} * \delta_1 \\ \lambda \leq \frac{o_2-40}{15} + \tilde{M} * \delta_1, \lambda \leq \frac{o_3-60}{3} + \tilde{M} * \delta_2, \\ \lambda \leq \frac{o_3-59.04}{31/5} + \tilde{M} * \delta_2, \lambda \leq \frac{o_3-60}{5} + \tilde{M} * (1-\delta_2) \\ \lambda \leq \frac{o_4-62}{20/3} + \tilde{M} * \delta_3 + \tilde{M} * \delta_4, \lambda \leq \frac{o_4-62.5}{5} + \tilde{M} * \delta_3 \\ \lambda \leq \frac{o_4-64.5}{2.5} + \tilde{M} * (1-\delta_3) \lambda \leq \frac{82.15-o'}{169/7}, \lambda \leq \frac{65.07-o'}{2.8} \\ \lambda \leq \frac{65-o'}{18/7}, \lambda \leq \frac{o'-50}{15/4}, \lambda \leq \frac{o'-49.72}{5.6}, \lambda \leq \frac{o'-43}{14} \end{cases}$$

Utility structure of P_3 :

$$\begin{cases} \lambda \leq \frac{o_1-48}{17/8}, \lambda \leq \frac{o_1-47.6}{4}, \lambda \leq \frac{o_1-46}{6} \\ \lambda \leq \frac{o_2-50}{10/3} + \tilde{M} * (1-\delta_1), \lambda \leq \frac{o_2-50.1}{9/8} + \tilde{M} * \delta_1 \\ \lambda \leq \frac{o_2-42.5}{12.5} + \tilde{M} * \delta_1, \lambda \leq \frac{o_3-60}{17/6} + \tilde{M} * \delta_2 \\ \lambda \leq \frac{o_3-58.96}{19/3} + \tilde{M} * \delta_2, \lambda \leq \frac{o_3-60}{5} + \tilde{M} * (1-\delta_2) \\ \lambda \leq \frac{o_4-62}{7.5} + \tilde{M} * \delta_3 + \tilde{M} * \delta_4, \lambda \leq \frac{o_4-62.75}{5} + \tilde{M} * \delta_3 \\ \lambda \leq \frac{o_4-65.75}{1.25} + \tilde{M} * (1-\delta_3), \lambda \leq \frac{81.8-o'}{23.8}, \lambda \leq \frac{65-o'}{2.8} \\ \lambda \leq \frac{65-o'}{2.8}, \lambda \leq \frac{o'-50}{15/4}, \lambda \leq \frac{o'-49.3}{7}, \lambda \leq \frac{o'-46.5}{10.5} \end{cases}$$

Utility structure of P_4 :

$$\begin{cases} \lambda \leq \frac{o_1-48}{8/5}, \lambda \leq \frac{o_1-47.88}{2.4}, \lambda \leq \frac{o_1-41}{11} \\ \lambda \leq \frac{o_2-50}{5} + \tilde{M} * (1-\delta_1), \lambda \leq \frac{o_2-50.3}{1.5} + \tilde{M} * \delta_1 \\ \lambda \leq \frac{o_2-47.5}{7.5} + \tilde{M} * \delta_1, \lambda \leq \frac{o_3-60}{11/3} + \tilde{M} * \delta_2 \\ \lambda \leq \frac{o_3-59.3}{6} + \tilde{M} * \delta_2, \lambda \leq \frac{o_3-60.5}{4.5} + \tilde{M} * (1-\delta_2) \\ \lambda \leq \frac{o_4-62}{23/3} + \tilde{M} * \delta_3 + \tilde{M} * \delta_4, \lambda \leq \frac{o_4-62.8}{5} + \tilde{M} * \delta_3 \\ \lambda \leq \frac{o_4-66}{1} + \tilde{M} * (1-\delta_3), \lambda \leq \frac{81.8-o'}{23.8}, \lambda \leq \frac{65-o'}{2.8} \\ \lambda \leq \frac{65-o'}{2.8}, \lambda \leq \frac{o'-50}{2.8}, \lambda \leq \frac{o'-49.79}{4.2}, \lambda \leq \frac{o'-37.75}{19.25} \end{cases}$$

Appendix II

Table 2 Results of the effects on the whole utility under different budgets

P_j \ B	16	16.2	16.4	16.8	17	17.2	17.4	17.6	18	18.2	18.4	18.6	18.8	19	19.2	18.8	19	19.2	19.4
P_1	0.013	0.039	0.066	0.118	0.145	0.171	0.039	0.066	0.118	0.145	0.171	0.197	0.224	0.250	0.276	0.302	0.320	0.339	0.357
P_2	0.105	0.130	0.154	0.204	0.228	0.253	0.278	0.302	0.318	0.335	0.352	0.369	0.385	0.402	0.419	0.436	0.452	0.468	0.484
P_3	0.079	0.104	0.130	0.180	0.206	0.231	0.256	0.282	0.304	0.320	0.337	0.353	0.370	0.386	0.403	0.419	0.434	0.449	0.464
P_4	0.065	0.088	0.110	0.155	0.178	0.200	0.222	0.245	0.267	0.290	0.309	0.326	0.344	0.361	0.378	0.395	0.411	0.427	0.443

P_j \ B	19.6	19.8	20	20.2	20.4	20.6	20.8	21	21.4	21.8	22.2	22.4	22.8	23	23.4	23.8	24	24.4	24.8
P_1	0.427	0.444	0.461	0.479	0.496	0.513	0.530	0.548	0.582	0.617	0.651	0.668	0.703	0.720	0.755	0.791	0.813	0.858	0.904
P_2	0.468	0.484	0.499	0.515	0.530	0.546	0.561	0.577	0.608	0.638	0.669	0.685	0.716	0.731	0.762	0.793	0.813	0.858	0.904
P_3	0.449	0.464	0.480	0.495	0.510	0.525	0.541	0.556	0.586	0.617	0.648	0.663	0.693	0.709	0.739	0.770	0.786	0.832	0.886
P_4	0.427	0.443	0.458	0.474	0.490	0.506	0.521	0.537	0.569	0.600	0.631	0.647	0.679	0.694	0.726	0.757	0.773	0.809	0.871

P_j \ B	25	25.4	25.8	26	26.2	26.4	26.8	27	27.4	27.8	28	28.4	28.8	29	29.4	29.8	30	30.2	30.6
P_1	0.927	0.973	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.972	0.961	0.939	0.916	0.905	0.894	0.877
P_2	0.927	0.973	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.994	0.969	0.957	0.932	0.908	0.895	0.883	0.857
P_3	0.913	0.967	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.993	0.966	0.952	0.924	0.897	0.883	0.870	0.837
P_4	0.902	0.963	1.000	0.992	0.984	0.976	0.960	0.952	0.936	0.919	0.911	0.895	0.879	0.871	0.849	0.826	0.815	0.804	0.774

P_j \ B	31.4	31.8	32.2	32.4	32.6	32.8	33	33.4	33.8	34	34.4	34.6	34.8	35	35.6	35.8	36	36.2	36.4
P_1	0.854	0.842	0.829	0.823	0.817	0.811	0.805	0.790	0.774	0.734	0.599	0.518	0.437	0.357	0.177	0.142	0.106	0.071	0.035
P_2	0.828	0.813	0.798	0.790	0.781	0.772	0.763	0.745	0.709	0.653	0.540	0.483	0.426	0.360	0.163	0.127	0.096	0.064	0.032
P_3	0.789	0.770	0.749	0.739	0.729	0.718	0.708	0.687	0.663	0.620	0.534	0.491	0.448	0.405	0.267	0.218	0.152	0.084	0.032
P_4	0.711	0.679	0.648	0.632	0.616	0.601	0.585	0.553	0.522	0.506	0.475	0.440	0.385	0.331	0.169	0.109	0.071	0.047	0.024

Table 3 Results of the effects on the whole utility under different tolerance

$U-\varepsilon_i$ \ ε_i	2.99	3	3.2	3.4	3.5	3.6	3.8	4	4.2	4.4	4.6	4.8	5	5.2	5.4	5.6
$U-\varepsilon_1$	0.019	0.024	0.116	0.208	0.391	0.517	0.567	0.617	0.667	0.717	0.767	0.817	0.826	0.834	0.840	0.846
$U-\varepsilon_2$	0.450	0.456	0.542	0.614	0.650	0.686	0.758	0.816	0.849	0.871	0.874	0.875	0.876	0.878	0.879	0.880
$U-\varepsilon_3$	0.864	0.864	0.871	0.876	0.877	0.879	0.883	0.888	0.893	0.898	0.903	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_4$	0.037	0.042	0.129	0.217	0.260	0.304	0.392	0.479	0.567	0.654	0.742	0.844	0.852	0.859	0.867	0.874

$U-\varepsilon_i$ \ ε_i	5.8	6	6.2	6.4	6.6	6.8	7	7.2	7.4	7.6	7.8	8	8.2	8.4	8.6	8.8
$U-\varepsilon_1$	0.852	0.858	0.864	0.870	0.873	0.875	0.876	0.877	0.878	0.880	0.881	0.882	0.885	0.889	0.893	0.896
$U-\varepsilon_2$	0.882	0.884	0.887	0.890	0.894	0.897	0.900	0.903	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_3$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_4$	0.881	0.889	0.894	0.899	0.903	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905

$U-\varepsilon_i$ \ ε_i	9.2	9.4	9.6	9.8	10	10.2	10.4	10.6	10.8	11.4	11.8	12.2	12.6	12.9	13	13.2
$U-\varepsilon_1$	0.904	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_2$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_3$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_4$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905

$U-\varepsilon_i$ \ ε_i	13.4	13.8	14	14.4	14.8	15	15.2	15.6	15.9	16.2	16.8	17.4	17.8	18.2	18.6	19.2
$U-\varepsilon_1$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_2$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_3$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905
$U-\varepsilon_4$	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905	0.905