A new type of preference relations: Fuzzy preference relations with self-confidence

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Abstract— **Preference relations are very useful to express decision makers' preferences over alternatives in the process of decision-making. However, multiple self-confidence levels are not considered in existing preference relations. In this study, we propose a new type of preference relations: fuzzy preference relations with self-confidence. A linear programming model is proposed for estimating priority vectors of this new type of preference relations. Finally, two numerical examples are provided to demonstrate the linear programming model, and a comparative analysis is used to show the influence of self-confidence levels on the decision-making results.**

Keywords—Fuzzy preference relations; self-confidence levels; priority vector; linear programming model

I. Introduction

In decision making problems, decision makers often use preference relations to express their opinions over a finite set of alternatives by means of a predefined domain. Based on the use of different domains, three types of preference relations have been widely investigated: fuzzy preference relations [3, 5, 11, 12, 21, 23, 25], multiplicative preference relations [4, 24] and linguistic preference relations [2, 6, 10, 13, 19].

A complete preference relation of dimension n contains n^2 preference elements, each one representing the degree up to which an alternative is preferred to another one [8, 19, 22-25]. Sometimes, decision makers do not show self-confidence on the preference information because of time pressure and limited expertise in regards to the problem domain. In these situations, decision makers may provide their preference information by means of incomplete preference relations, i.e. a preference relation with some of the elements missing or unknown [1, 16, 17, 26, 27].

For a complete preference relation, the decision maker must provide all preference information, and it is assumed that this preference information is provided with same self-confidence level. In an incomplete preference relation, two kinds of self-confidence levels are obviously present: (1) The decision maker is with self-confidence for the provided preference information regarding pairs of alternatives and (2) the decision maker is without self-confidence if preference information regarding pairs of alternatives is not provided.

However, multiple self-confidence levels are possible in theory and also in practice but it has been neglected in existing preference representation formats. Therefore, it would be of great importance to provide decision makers with appropriate mathematical tools to

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enable them to express multiple self-confidence levels when providing their preferences. In this paper, we propose a new type of preference relations by taking multiple self-confidence levels into consideration, and we name them fuzzy preference relations with self-confidence. In this new type of preference relation, an element contains two parts, one corresponds to the actual preference value regarding a pair of alternatives, while the other part, which is modelled using an ordinal linguistic scale, corresponds to the decision maker's self-confidence level associated to the first part or preference value provided. Additionally, a programming model based decision process to derive the priority vector of a set of alternatives assessed via a preference relation with self-confidence is developed.

The rest of this study is organized as follows. Section 2 introduces the basic knowledge regarding fuzzy preference relations and the 2-tuple linguistic ordinal scale model. Section 3 defines a new type of preference relations: fuzzy preference relation with self-confidence. Then, Section 4 proposes a linear programming model to derive the priority vector of a fuzzy preference relation with self-confidence. Following this, Section 5 provides two illustrative examples and a comparative analysis. Finally, Section 6 concludes the study.

II. PRELIMINARIES

In this section, we introduce the basic knowledge regarding fuzzy preference relations and the 2-tuple linguistic ordinal scale model.

A. *Fuzzy preference relations*

Fuzzy preference relations are also called additive preference relations or additive reciprocal preference relations [3, 14, 23]. Let $X=\{x_1,x_2,...,x_n\}$ be a finite set of alternatives, a fuzzy preference relation is defined as follows.

Definition 1 [23]: A fuzzy preference relation P on a finite set of alternatives X is a relation on $X\times X$ that is characterised by a membership function $\mu_P : X\times X \rightarrow [0,1]$, where $\mu_P(x_i,x_j)=p_{ij}$ denotes the degree or intensity of preference of alternative x_i over alternative x_j.

A fuzzy preference relation may be conveniently denoted by an $n\times n$ matrix $P=(p_{ij})_{n\times n}$, with the following interpretation: $p_{ij}=0.5$ indicates indifference between x_i and x_j; $p_{ij}>0.5$ indicates a definite preference for x_i over x_j; $p_{ij}=1$ indicates the maximum degree of preference for x_i over x_j. It is assumed that $p_{ij}+p_{ji}=1$ and $p_{ii}=0.5$, which assures that asymmetry of preferences is verified [5].

Transitive property is a very important concept to verify by preference relations, as this together with asymmetry characterise a crisp weak order on a set of alternatives [12]. For a fuzzy preference relation, let $P=(p_{ij})_{n\times n}$, several transitive properties have been proposed in literature [25]:

(1) Weak stochastic transitivity.

$$p_{ij}\geq 0.5, p_{jk}\geq 0.5 \Rightarrow p_{ik}\geq 0.5 \ \ \forall i,j,k .$$

(2) Strong stochastic transitivity.

$$p_{ij}\geq 0.5, p_{jk}\geq 0.5 \Rightarrow p_{ik}\geq \max(p_{ij},p_{jk}) \ \ \forall i,j,k .$$

(3) Additive transitivity.

$$p_{ij}=p_{ik}-p_{jk}+0.5 \ \ \forall i,j,k .$$

Clearly, the additive transitive condition is stronger than the strong stochastic transitivity, and the strong stochastic transitivity is stronger than the weak stochastic transitivity.

B. *2-tuple linguistic ordinal scale model*

The basic notations and operational laws of linguistic variables in an ordinal scale framework are introduced in [7, 9, 18]. Let $S=\{s_i \mid i=0,1,...,g\}$ be a linguistic term set with odd cardinality. The term s_i represents a possible value for a linguistic variable. The set S is ordered: $s_i > s_j$ if and only if $i > j$.

Herrera and Martínez [18] presented the 2-tuple linguistic ordinal model.

Definition 2 (Herrera and Martínez [18]**):** Let $\beta \in [0,\ g]$ be a number in the granularity interval of the linguistic term set $S = \{s_0,\ ...,\ s_g\}$ and let $i = round(\beta)$ and $\alpha = \beta - i$ be two values such that $i \in [0,\ g]$ and $\alpha \in [-0.5,\ 0.5)$. Then, α is called a symbolic translation, with *round* being the usual *rounding* operation.

Herrera-Martínez's model represents the linguistic information by means of 2-tuples $(s_i,\ \alpha)$, where s_i is a simple term in S and $\alpha \in [-0.5, 0.5)$. This linguistic ordinal model defines a function with the purpose of making transformations between linguistic 2-tuples and numerical values representing their ordinal position within the range $[0, g]$.

Definition 3 (Herrera and Martínez [18]): Let $S = \{s_0,\ ...,\ s_g\}$ be a linguistic term set and $\beta \in [0,\ g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function: $\Delta : [0,\ g] \to S \times [-0.5,\ 0.5)$, where

$$\Delta(\beta) = (s_i, \alpha),\ \text{with} \begin{cases} s_i,\ i = round(\beta) \\ \alpha = \beta - i,\ \alpha \in [-0.5,\ 0.5). \end{cases}$$

In the Herrera-Martínez's model, Δ is a one to one mapping function. For convenience, its range is denoted as $\overline{S}$. Then, Δ has an inverse function $\Delta^{-1} : \overline{S} \to [0,\ g]$ with $\Delta^{-1}((s_i, \alpha)) = i + \alpha$. For notation simplicity, this paper sets $\Delta^{-1}((s_i, 0)) = \Delta^{-1}(s_i)$.

Let $(s_k,\ \alpha)$ and $(s_l,\ \gamma)$ be two 2-tuples. The following ordering can be defined:

(1) if $k < l$, then $(s_k,\ \alpha)$ is smaller than $(s_l,\ \gamma)$.

(2) if $k = l$, then

(a) if $\alpha = \gamma$, then $(s_k,\ \alpha)$ and $(s_l,\ \gamma)$ coincide.

(b) if $\alpha < \gamma$, then $(s_k,\ \alpha)$ is smaller than $(s_l,\ \gamma)$.

III. A NEW TYPE OF PREFERENCE RELATIONS: FUZZY PREFERENCE RELATION WITH SELF-CONFIDENCE

To enable decision makers to characterise their self-confidence levels in a linguistic way, a linguistic terms set $S^{SL} = \{l_0,\ l_1,\ ...,\ l_g\}$ is used. For example, in this paper S^{SL} is set to be

$$S^{SL} = \{l_0 = extremely\ poor, l_1 = very\ poor, l_2 = poor, l_3 = slightly\ poor, l_4 = fair,\ l_5 = slightly\ good, l_6 = good, l_7 = very\ good, l_8 = extremely\ good\}.$$

The decision maker uses linguistic terms in S^{SL} to characterize his/her self-confidence level on the provided preference values. Thus, if $X = \{x_1, x_2, ..., x_n\}$ is a finite set of alternatives, the new type of preference relation with self-confidence is defined as follows:

Definition 4**:** Let P^* be a fuzzy preference relation with self-confidence based on a finite set of alternatives X, shown as follows,

$$P^* = ((p_{ij}, s_{ij})) \qquad (1)$$

where p_{ij} denotes the degree or intensity of preference of alternative x_i over alternative x_j, and $s_{ij} \in S^{SL}$ represents the self-confidence level on the preference value p_{ij}. It is assumed that $p_{ij} + p_{ji} = 1$, $p_{ii} = 0.5$, $s_{ij} = s_{ji}$ and $s_{ii} = l_g$.

Transitive properties of fuzzy preference relations can be used for the new type of fuzzy preference relation at different levels of self-confidence. This is described as follows:

(1) Weak stochastic transitivity at self-confidence level $l \in S^{SL}$.

$p_{ij} \geq 0.5, p_{jk} \geq 0.5 \Rightarrow p_{ik} \geq 0.5$, $\forall i, j, k$ and $s_{ij} \geq l$, $\forall i, j$.

(2) Strong stochastic transitivity at self-confidence level $l \in S^{SL}$.

$p_{ij} \geq 0.5, p_{jk} \geq 0.5 \Rightarrow p_{ik} \geq \max(p_{ij}, p_{jk})$, $\forall i, j, k$ and $s_{ij} \geq l$, $\forall i, j$.

(3)Additive transitivity at self-confidence level $l \in S^{SL}$.

$p_{ij} = p_{ik} - p_{jk} + 0.5$, $\forall i, j, k$ and $s_{ij} \geq l$, $\forall i, j$.

The traditional definition to characterize consistency of preference relations is using a set of pre-established transitive properties [1, 5, 15]. In this paper, a preference relation with self-confidence is considered to be acceptable consistent if it satisfies the weak stochastic transitivity at self-confidence level $l_0 \in S^{SL}$, i.e at the lowest possible self-confidence level.

In this new type of preference relations, the decision maker first provides the preference values regarding pairs of alternatives. The principle of preference value elicitation is similar to the original fuzzy preference relation. Then, the decision maker provides the self-confidence levels associated to the preference values. The self-confidence level is modelled using the linguistic terms set S^{SL}. According to the study presented in Miller [19], it is not difficult for an individual to estimate the self-confidence level.

IV. LINEAR PROGRAMMING MODEL TO DERIVE THE PRIORITY VECTOR OF A FUZZY PREFERENCE RELATION WITH SELF-CONFIDENCE

Let $w=(w_1, w_2, ..., w_n)^T$ be the priority vector of the fuzzy preference relation $P = (p_{ij})_{n\times n}$, where $w_i > 0$, $i = 1, 2, ..., n$, $\sum_{i=1}^{n} w_i = 1$. When the fuzzy preference relation verifies additive transitive property, Tanino shows that [25],

$$p_{ij} = \frac{1}{2}(w_i - w_j) + 0.5, \quad i, j = 1,\ 2,\ ...,\ n \qquad (2)$$

However, in general, fuzzy preference relations are not additive transitive. In these cases,

$$\varepsilon_{ij} = \frac{1}{2}(w_i - w_j) + 0.5 - p_{ij}, \quad i, j = 1, 2, ..., n \qquad (3)$$

is used to measure the error between the preference value p_{ij} and the corresponding additive consistent preference value built with priority vector w. If the fuzzy preference relation is completely consistent, then we have $\varepsilon_{ij} = 0$. For fuzzy preference relation with self-confidence the information deviation associated to error ε_{ij} at self-confidence level s_{ij} ($s_{ij} \in S^{SL}$) can be introduced as follows:

Definition 5 **:** The information deviation associated with an error ε_{ij} at self-confidence level s_{ij} is defined as

$$z_{ij} = |\Delta^{-1}(s_{ij})||\varepsilon_{ij}|, \quad i, j = 1,\ 2,\ ...,\ n \qquad (4)$$

The level of self-confidence s_{ij} in Equation (4) determines the magnification of error ε_{ij}: the larger its value, the larger magnification will be the error ε_{ij} assigned to the corresponding preference value p_{ij}.

In the following, we propose a linear programming model to derive the priority vector of a fuzzy preference relation with self-confidence, which minimizes the sum of information deviation z_{ij} between the decision maker's preference and the priority vector w. In a fuzzy preference relation with self-confidence P^*, since $p_{ij} + p_{ji} = 1$, $s_{ij} = s_{ji}$, we then have

$$z_{ij} = z_{ji}, \quad i,j = 1,2,...,n \tag{5}$$

Therefore, we need to specify only the information deviation z_{ij} for the upper triangular part of the relation P^*. The objective function is as follows,

$$\min z = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} \tag{6}$$

In this way, the linear programming model to obtain the priority vector is constructed as follows,

$$\begin{cases} \min z = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} \\ s.t. \begin{cases} \varepsilon_{ij} = \frac{1}{2}(w_i - w_j) + 0.5 - p_{ij} \\ z_{ij} = |\Delta^{-1}(s_{ij})||\varepsilon_{ij}| \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \end{cases} \tag{7}$$

We use two transformed variables for model (7): $y_{ij} = \varepsilon_{ij}$ and $a_{ij} = \Delta^{-1}(s_{ij})$. In this way, model (7) is transformed into the following linear programming model,

$$\begin{cases} \min z = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} z_{ij} & (a) \\ s.t. \begin{cases} 0.5(w_i - w_j) - y_{ij} = p_{ij} - 0.5, \ i,j = 1,\ 2,\ ...,\ n;\ i<j & (b) \\ z_{ij} - a_{ij} y_{ij} \geq 0,\ i,j = 1,2,...,n;\ i<j & (c) \\ z_{ij} + a_{ij} y_{ij} \geq 0,\ i,j = 1,2,...,n;\ i<j & (d) \\ w_1 + w_2 + ... w_n = 1, & (e) \\ w_i \geq 0, \quad i = 1,\ 2,\ ...,\ n & (f) \\ z_{ij} \geq 0, \ \ i,j = 1,\ 2,\ ...,\ n;\ i<j & (g) \end{cases} \end{cases} \tag{8}$$

In model (8), constraints $(b)-(d)$ guarantee that $z_{ij} \geq |\Delta^{-1}(s_{ij})||\varepsilon_{ij}|$; constraint (e) guarantees that the priority vector is normalized to sum to one; while constraints (f) and (g) guarantee that decision variables w_i and z_{ij} are nonnegative.

V. NUMERICAL EXAMPLES

In this section, we use two numerical examples to demonstrate the linear programming model (8), then we make a comparative analysis to show the influence of self-confidence levels on the decision-making results.

A. *Example 1*

Let P_1^* be a 3×3 fuzzy preference relation with self-confidence.

$$P_1^* = \begin{pmatrix} (0.5, l_8) & (0.3, l_6) & (0.4, l_8) \\ (0.7, l_6) & (0.5, l_8) & (0.2, l_3) \\ (0.6, l_8) & (0.8, l_3) & (0.5, l_8) \end{pmatrix}$$

In the following, we use the linear programming model (8) to obtain the priority vector of P_1^*,

$$\min \quad z = z_{12} + z_{13} + z_{23}$$
$$subject \ \ to$$
$$\begin{cases} 0.5w_1 - 0.5w_2 - y_{12} = -0.2, \\ 0.5w_1 - 0.5w_3 - y_{13} = -0.1, \\ 0.5w_2 - 0.5w_3 - y_{23} = -0.3, \\ z_{12} - 6y_{12} \geq 0, \\ z_{12} + 6y_{12} \geq 0, \\ z_{13} - 8y_{13} \geq 0, \\ z_{13} + 8y_{13} \geq 0, \\ z_{23} - 3y_{23} \geq 0, \\ z_{23} + 3y_{23} \geq 0, \\ w_1 + w_2 + w_3 = 1, \\ w_i \geq 0, \ \ i = 1,2,3 \\ z_{ij} \geq 0, \ \ i,j = 1,2,3 \end{cases}$$

When we solve the model using LINGO, we obtain that $z = 1.2$ and $w = (0.13, 0.53, 0.33)^T$.

B. *Example 2*

We consider a 4×4 fuzzy preference relation with self-confidence.

$$P_2^* = \begin{pmatrix} (0.5,l_8) & (0.5,l_5) & (0.6,l_1) & (0.9,l_3) \\ (0.5,l_5) & (0.5,l_8) & (0.8,l_3) & (0.6,l_8) \\ (0.4,l_1) & (0.2,l_3) & (0.5,l_8) & (0.8,l_7) \\ (0.1,l_3) & (0.4,l_8) & (0.2,l_7) & (0.5,l_8) \end{pmatrix}$$

The linear programming model for the relation P_2^* is

$$\min \ z = z_{12} + z_{13} + z_{14} + z_{23} + z_{24} + z_{34}$$
$$subject \ \ to$$
$$\begin{cases} 0.5w_1 - 0.5w_2 - y_{12} = 0, \\ 0.5w_1 - 0.5w_3 - y_{13} = 0.1, \\ 0.5w_1 - 0.5w_4 - y_{14} = 0.4, \\ 0.5w_2 - 0.5w_3 - y_{23} = 0.3, \\ 0.5w_2 - 0.5w_4 - y_{24} = 0.1, \\ 0.5w_3 - 0.5w_4 - y_{34} = 0.3, \\ z_{12} - 5y_{12} \geq 0, \\ z_{12} + 5y_{12} \geq 0, \\ z_{13} - 1y_{13} \geq 0, \\ z_{13} + 1y_{13} \geq 0, \\ z_{14} - 3y_{14} \geq 0, \\ z_{14} + 3y_{14} \geq 0, \\ z_{23} - 3y_{23} \geq 0, \\ z_{23} + 3y_{23} \geq 0, \\ z_{24} - 8y_{24} \geq 0, \\ z_{24} + 8y_{24} \geq 0, \\ z_{34} - 7y_{34} \geq 0, \\ z_{34} + 7y_{34} \geq 0, \\ w_1 + w_2 + w_3 + w_4 = 1, \\ w_i \geq 0, \ \ i = 1,2,3,4 \\ z_{ij} \geq 0, \ \ i,j = 1,2,3,4 \end{cases}$$

Solving the model by LINGO, we obtain $z = 2.7$ and $w = (0.2, 0.2, 0.6, 0)^T$.

C. Comparative analysis

In this comparative analysis, we study the influence of different self-confidence levels on the decision-making results of the new type of preference relations. There are four preference relations with self-confidence P_3^*, P_4^*, P_5^* and P_6^*. Relations P_3^* and P_4^* have the same preference values but different self-confidence levels with relation P_1^*. Relations P_5^* and P_6^* have the same preference values but different self-confidence levels with relation P_2^*.

$$P_3^* = \begin{pmatrix} (0.5,l_8) & (0.3,l_7) & (0.4,l_4) \\ (0.7,l_7) & (0.5,l_8) & (0.2,l_5) \\ (0.6,l_4) & (0.8,l_5) & (0.5,l_8) \end{pmatrix}$$

$$P_4^* = \begin{pmatrix} (0.5,l_8) & (0.3,l_8) & (0.4,l_8) \\ (0.7,l_8) & (0.5,l_8) & (0.2,l_8) \\ (0.6,l_8) & (0.8,l_8) & (0.5,l_8) \end{pmatrix}$$

$$P_5^* = \begin{pmatrix} (0.5,l_8) & (0.5,l_2) & (0.6,l_5) & (0.9,l_3) \\ (0.5,l_2) & (0.5,l_8) & (0.8,l_6) & (0.6,l_1) \\ (0.4,l_5) & (0.2,l_6) & (0.5,l_8) & (0.8,l_5) \\ (0.1,l_3) & (0.4,l_1) & (0.2,l_5) & (0.5,l_8) \end{pmatrix}$$

$$P_6^* = \begin{pmatrix} (0.5,l_8) & (0.5,l_8) & (0.6,l_8) & (0.9,l_8) \\ (0.5,l_8) & (0.5,l_8) & (0.8,l_8) & (0.6,l_8) \\ (0.4,l_8) & (0.2,l_8) & (0.5,l_8) & (0.8,l_8) \\ (0.1,l_8) & (0.4,l_8) & (0.2,l_8) & (0.5,l_8) \end{pmatrix}$$

Using the linear programming model (8) obtains the decision-making results for each relation. The values of objective function and the priority vectors are presented in the following tables: Table 1 contains the decision-making results for the relations P_1^*, P_3^* and P_4^*; Table 2 contains the decision-making results for the relations P_2^*, P_5^* and P_6^*.

Table 1 The decision-making results for the relations P_1^*, P_3^* and P_4^*

	z	w_1	w_2	w_3
P_1^*	1.2	0.13	0.53	0.33
P_3^*	1.8	0	0.4	0.6
P_4^*	3.2	0.4	0	0.6

Table 2 The decision-making results for the relations P_2^* , P_5^* and P_6^*

	z	w_1	w_2	w_3	w_4
P_2^*	2.7	0.2	0.2	0.6	0
P_5^*	2.77	0.27	0.67	0.06	0
P_6^*	5.6	0.4	0.4	0.2	0

From Table 1 and Table 2, we notice that different self-confidence levels lead to different rankings of alternatives. Thus the self-confidence levels have certain influence on the decision-making results.

VI. CONCLUSION

In this study, we propose a new type of preference relations, which allows decision makers to have multiple self-confidence levels to express their preferences regarding pairs of alternatives. Furthermore, we propose a linear programming model to derive the priority vector of this new type of preference relations. The model is straightforward and easy to understand and formulate, and it can be solved in very little computational time using readily available software such as LINGO. Finally, we use two numerical examples to demonstrate the linear programming model, and we make a comparative analysis to show the influence of self-confidence levels on the decision-making results

In the future, we will investigate the group decision-making problems based on preference relations with self-confidence.

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