

Grey Self-memory Combined Model for Complex Equipment Cost Estimation

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Abstract

To improve the using rationality of complex equipment cost, this paper presents a novel grey self-memory combined model for predicting the equipment cost. The proposed model can improve the modeling accuracy by means of the self-memory prediction technique. The combined model combines the advantages of the self-memory principle and traditional grey model through coupling of the above two prediction methods. The weakness of the traditional grey prediction model, i.e., being sensitive to initial value, can be overcome by using multi-time-point initial field instead of only single-time-point initial field in the system's self-memorization equation. As shown in the two case studies of complex equipment cost estimation, the novel grey self-memory combined model can take full advantage of the system's multi-time historical monitoring data and accurately predict the system's evolutionary trend. Three popular accuracy test criteria are adopted to test and verify the reliability and robustness of the combined model, and its superior predictive performance over other traditional grey prediction models. The results show that the proposed combined model enriches equipment cost estimation methods, and can be applied to other similar complex equipment cost estimation problems.

Key words: complex equipment cost estimation; grey system model; self-memory prediction technique; combined prediction model

1. Introduction

With the rapid development of science and technology and modernization construction, the sharp expansion tendency of complex equipment are all present in the fields of domestic and overseas engineering and military. For a long time, scientific and systematic research is lacking in the equipment cost management of engineering and military in our country. Therefore, it is of great significance to quantitative analyse the estimation and control of full life circle cost about equipment development, manufacture and use in the demonstration and design of all kinds of complex equipment. Especially for early stage of greater controllability, the equipment cost at all stages of life cycle should be predict accurately and

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reasonably. Consequently, the growth of complex equipment cost could be control effectively, and the efficiency of equipment purchase and utilization benefit of limited budgets could be enhanced scientifically^[1].

Some basic prediction methods, including expert judgement method, engineering method, analogy method and parametric method, etc.^[2,3], are often employed in the equipment cost estimation. But some certain limitations exist in these traditional methods. For example, expert judgement method is easy to be affected by subjective factors including knowledge and experience, and there is lack of the accurate quantitative description for things development. Engineering method is complex, tedious and time-consuming. Analogy method often be employed accompany with experts estimation and has uncertainty. The historical data of sufficient quantity are need in parametric method. Moreover, along with the continuous developing and perfecting of new theories, there have been emerged some intelligent prediction techniques^[4-6] including grey system, neural network and genetic algorithm. They have been broadly utilized in the fields of cost estimation, and the prediction accuracy and efficiency of equipment cost have been improved simultaneously.

Aiming at increasing complexity, uncertainty and chaos of the systems engineering, grey prediction method^[7] can weaken the randomness of original sequence and excavate its inherent law by means of accumulated generating operation. Consequently, the scientific and quantitative prediction could be obtained to reveal the system's future development trend. And the superiority of grey method over conventional statistical methods is that they only require a limited amount of statistical data without knowing their statistical distribution. Especially, it has unique advantages for the short-term prediction of small sample sequence. When estimating the complex equipment cost, there are more uncertain factors and relatively small collected sample size owing to the complexity of equipment system. Therefore the complex equipment cost estimation problem is characteristic of typical grey uncertainty, and appropriate for grey prediction method. It has been used broadly in equipment cost estimation in fields including development cost forecasting, maintenance support cost forecasting, life cycle cost forecasting, etc.^[8-10], and achieved effective prediction outcomes.

Meanwhile, some scholars have optimized the traditional grey prediction method to improve the prediction accuracy of equipment cost, with the help of neural network, genetic algorithm and other intelligent optimization techniques. The compositional modelling for traditional grey prediction method is conducted in this paper considering another prediction technique (the self-memory principle of dynamic system). The self-memory prediction technique has the advantage for adequately remember historical statistics data. By introducing the memory function, the self-memorization prediction equation containing multiple time-point initial fields can be deduced to instead of the single time-point initial field. The limitation of being sensitive to the initial value of traditional models can be overcome. The method has better prediction robustness, and it has achieved ideal and stable predictive effect in fields including meteorology, hydrology, electrical energy and building settlement^[12-14]. However there is no anything exploratory development existing in the field of equipment cost estimation.

The purpose of this paper is to construct a grey self-memory combined model appropriate for the complex equipment cost estimation under the condition of small sample size. The paper is organized as follows. Section 2 provides an overview of traditional grey series model for equipment cost estimation. Section 3 presents the

coupling modeling procedure of grey self-memory combined model for equipment cost estimation and its accuracy test criterion. In Section 4, two case studies of development cost estimation and maintenance support cost estimation are adopted to demonstrate the adaptability and effectiveness of the proposed novel equipment cost estimation model. Finally, some conclusions are drawn in Section 5.

2. Traditional grey series model for equipment cost estimation

The historical data of equipment cost often present some certain change laws under the condition of small sample size. The grey series model system represented by GM(1,1) model, can construct the corresponding grey prediction models in accordance with the different regularity and characteristic of original data sequences. For example, GM(1,1) model is appropriate for the original equipment cost data characteristic of exponential growth, grey Verhulst model is appropriate for the equipment cost data characteristic of sigmoid process with saturation condition, etc. Then GM(1,1) model is selected as the example for reviewing the traditional grey prediction models as follows.

Assume that the original data sequence of equipment cost

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)),$$

where $x^{(0)}(k)$ is the time series data at time k . And the sequence

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n))$$

is the first-order accumulated generation (abbreviated as 1-AGO) sequence of $X^{(0)}$, where $x^{(1)}(k) = \sum_{i=1}^k x^{(0)}(i)$, $k = 1, 2, \dots, n$. Then, the sequence

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), \dots, z^{(1)}(n))$$

is the background value sequence taken to be the mean generation (abbreviated as 1-MGO) of consecutive neighbors of $X^{(1)}$, where $z^{(1)}(k) = \frac{1}{2}(x^{(1)}(k) + x^{(1)}(k-1))$, $k = 2, 3, \dots, n$.

Definition 1^[15] The sequences $X^{(0)}$, $X^{(1)}$ and $Z^{(1)}$ are defined as mentioned above, then the equation

$$x^{(0)}(k) + az^{(1)}(k) = b \quad (1)$$

is called the basic form of the GM(1,1) model, where the parameters a and b are called developing and grey input coefficients respectively. Meanwhile, the first-order differential equation

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \quad (2)$$

is called the whitenization differential equation of the GM(1,1) model.

Theorem 1^[15] Assume that $X^{(0)}$ is the original data sequence of equipment cost, and the sequences $X^{(1)}$ and $Z^{(1)}$ are defined as mentioned

above. If $\hat{R} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}$ is the parameter sequence, and

$$M = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \quad N = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix},$$

then the least square estimate parameter sequence \hat{R} of GM(1,1) model $x^{(0)}(k) + az^{(1)}(k) = b$ can be obtained as

$$\hat{R} = (M^T M)^{-1} M^T N.$$

Theorem 2^[15] Assume that M and N are defined as theorem 1, and the parameter sequence $\hat{R} = \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} = (M^T M)^{-1} M^T N$, then the time response function

of the whitening differential equation (2) of GM(1,1) model is given by

$$x^{(1)}(t) = \frac{b}{a} + \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-a(t-1)}, \quad (3)$$

Then after the equation (3) is discretized, the time response sequence of GM(1,1) model is given by

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a}) e^{-ak} + \frac{b}{a}, \quad k=1, 2, \dots, n-1, \quad (4)$$

and consider the inverse accumulated generation operation

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1 - e^a) \left(x^{(0)}(1) - \frac{b}{a} \right) e^{-ak}, \quad k=1, 2, \dots, n-1,$$

then the simulative sequence $\hat{X}^{(0)}$ of GM(1,1) model can be obtained.

3. Grey self-memory combined model for equipment cost estimation

In this section, GM(1,1) model is still selected as the example for combinatorial optimizing the traditional grey prediction models by introducing self-memory prediction technique. Then the grey self-memory combined model for equipment cost estimation can be construct, and its corresponding modelling process is concluded as follows.

Definition 2 If let $\frac{dx^{(1)}}{dt}$ in the whitening differential equation (2)

be $F(x, \lambda, t)$, then

$$F(x, \lambda, t) = -ax^{(1)} + b \quad (5)$$

3.1 Coupling modeling procedure

Step 1 Determining the self-memory dynamic equation

Definition 3 The whitening differential equation $\frac{dx^{(1)}}{dt} = -ax^{(1)} + b$,

which has been determined by GM(1,1) model, is considered to be the systematic self-memory dynamic equation of the grey self-memory combined prediction model:

$$\frac{dx}{dt} = F(x, \lambda, t), \quad (6)$$

where x is a variable, λ is a parameter, t is time interval series, and $F(x, \lambda, t) = -ax^{(1)} + b$ is the dynamic kernel. Meanwhile, introduce a memory function $\beta(t)$, in the which $|\beta(t)| \leq 1$, and define an inner product in the Hilbert space:

$$(f, g) \equiv \int_{a_0}^{b_0} f(\xi)g(\xi)d\xi \quad (f, g \in L^2). \quad (7)$$

The relation between the local time variation of variable x and the dynamic kernel function $F(x, \lambda, t)$ has been embodied in the self-memory dynamic equation (6).

Step 2 Deducing the self-memory difference-integral equation

Let one time set $T = \{t_{-p}, t_{-p+1}, \dots, t_{-1}, t_0, t\}$, where $t_{-p}, t_{-p+1}, \dots, t_{-1}$ is historical observation time, t_0 is predicted initial time, t is coming prediction time, the retrospective order of the equation is p and time sampling interval is Δt .

Supposing that variable x and memory function $\beta(t)$ are continuous, differentiable and integrable, after applying the above inner product operation (7) into the self-memory dynamic equation (6), the analytic formula of Eq. (6) is obtained as

$$\begin{aligned} & \int_{t_{-p}}^{t_{-p+1}} \beta(\tau) \frac{\partial x}{\partial \tau} d\tau + \int_{t_{-p+1}}^{t_{-p+2}} \beta(\tau) \frac{\partial x}{\partial \tau} d\tau + \dots + \int_{t_0}^t \beta(\tau) \frac{\partial x}{\partial \tau} d\tau \\ & = \int_{t_{-p}}^t \beta(\tau) F(x, \lambda, \tau) d\tau. \end{aligned} \quad (8)$$

The Eq. (8) could be seen as a kind of weighted integral in the which $\beta(t)$ is the weight. For every integral term on the left-hand side of Eq. (8), after integration by parts, applying the median theorem and performing algebra operation, a difference-integral equation is deduced as:

$$\beta_t x_t - \beta_{-p} x_{-p} - \sum_{i=-p}^0 x_i^m (\beta_{i+1} - \beta_i) - \int_{t_{-p}}^t \beta(\tau) F(x, \lambda, \tau) d\tau = 0, \quad (9)$$

where $\beta_t \equiv \beta(t)$, $x_t \equiv x(t)$, $\beta_i \equiv \beta(t_i)$, $x_i \equiv x(t_i)$, and mid-value $x_i^m \equiv x(t_m)$, $t_i < t_m < t_{i+1}$, $i = -p, -p+1, \dots, 0$. Let $x_{-p-1}^m \equiv x_{-p}$ and $\beta_{-p-1} \equiv 0$, Eq. (8) can be converted into

$$\begin{aligned} x_t & = \frac{1}{\beta_t} \sum_{i=-p-1}^0 x_i^m (\beta_{i+1} - \beta_i) + \frac{1}{\beta_t} \int_{t_{-p}}^t \beta(\tau) F(x, \lambda, \tau) d\tau \\ & = S_1 + S_2, \end{aligned} \quad (10)$$

which is called the self-memory equation with the retrospective order p . As the first term S_1 in Eq. (10) denotes the relative contributions of historical data at $p+1$ times to the value of variable x_t , it is defined as the self-memory term. The second term S_2 is the total contribution of the function

$F(x, \lambda, t) = -ax^{(1)} + b$ in the retrospective time interval $[t_{-p}, t_0]$, and it is defined as the exogenous effect term. Equation (10) emphasizes serial correlation of the system by itself, i.e., the self-memory characteristic of the system. Therefore, it is the self-memory prediction equation of the system.

Step 3 Discretizing the self-memory prediction equation

If integral operation is substituted by summation and differential is transformed into difference in Eq. (10), then the mid-value x_i^m is replaced simply by two values of different times, namely $x_i^m = \frac{1}{2}(x_{i+1} + x_i) \equiv y_i$. By taking equidistance time interval $\Delta t_i = t_{i+1} - t_i = 1$, and merging β_i and β_i together, the self-memory equation of discrete form is shown as follows:

$$x_i = \sum_{i=-p-1}^{-1} \alpha_i y_i + \sum_{i=-p}^0 \theta_i F(x, \lambda, i), \quad (11)$$

where the memory coefficients $\alpha_i = (\beta_{i+1} - \beta_i)/\beta_i$, $\theta_i = \beta_i/\beta_i$, and the dynamic kernel function $F(x, \lambda, t) = -ax^{(1)} + b$.

Step 4 Solving the memory coefficients by the least square method

$F(x, \lambda, t)$ is considered as input for the system, and x_i is considered as output for the system. Assume that there are $L(L > p)$ items of historical data, the memory coefficients α_i and θ_i can be estimated by the least square method. Let

$$X_t = \begin{bmatrix} x_{t1} \\ x_{t2} \\ \vdots \\ x_{tL} \end{bmatrix}_{L \times 1}, \quad A = \begin{bmatrix} \alpha_{-p-1} \\ \alpha_{-p} \\ \vdots \\ \alpha_{-1} \end{bmatrix}_{(p+1) \times 1}, \quad \Theta = \begin{bmatrix} \theta_{-p} \\ \theta_{-p+1} \\ \vdots \\ \theta_0 \end{bmatrix}_{(p+1) \times 1},$$

$$Y = \begin{bmatrix} y_{-p-1,1} & y_{-p,1} & \cdots & y_{-1,1} \\ y_{-p-1,2} & y_{-p,2} & \cdots & y_{-1,2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{-p-1,L} & y_{-p,L} & \cdots & y_{-1,L} \end{bmatrix}_{L \times (p+1)},$$

$$\Gamma = \begin{bmatrix} F(x, \lambda, -p)_1 & F(x, \lambda, -p+1)_1 & \cdots & F(x, \lambda, 0)_1 \\ F(x, \lambda, -p)_2 & F(x, \lambda, -p+1)_2 & \cdots & F(x, \lambda, 0)_2 \\ \vdots & \vdots & \ddots & \vdots \\ F(x, \lambda, -p)_L & F(x, \lambda, -p+1)_L & \cdots & F(x, \lambda, 0)_L \end{bmatrix}_{L \times (p+1)},$$

then the discrete self-memory prediction equation (11) can be expressed in matrix form as follows:

$$X_t = YA + \Gamma\Theta. \quad (12)$$

Let $Z = [Y, \Gamma]$, $W = \begin{bmatrix} A \\ \Theta \end{bmatrix}$, then Eq. (12) turns into $X_t = ZW$, thereby

$W = \begin{bmatrix} A \\ \Theta \end{bmatrix}$ is obtained by the least square method:

$$W = (Z^T Z)^{-1} Z^T X_i. \quad (13)$$

Step 5 Solving the grey self-memory combination prediction model

When the memory coefficients α_i and θ_i , which are determined by Eq. (13), are substituted into the discrete self-memory prediction equation (11), the corresponding simulation and prediction values $\hat{x}^{(1)}(t)$ can be carried out. Finally, the simulated and predicted sequence $\hat{X}^{(0)}$ of the grey self-memory combined model can be obtained by the first-order inverse accumulation as follows:

$$\hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), \quad (14)$$

where $t = 1, 2, \dots, n$ and $\hat{x}^{(1)}(0) \equiv 0$.

Step 6 Checking the modeling accuracy

Simulation and prediction accuracy is an important criterion for evaluating prediction models. Accuracy test must be performed to evaluate the reliability and robustness of prediction models before extrapolation and application. Various accuracy test methods to determine whether it is reasonable or not can be used in practical situation. Considering typical grey uncertainty characteristics, the Absolute Percentage Error (APE) at time k and three popular average errors (MSE, AME and MAPE)^[16,17] have been used to compare the accuracy of different prediction models. And the model error analysis can be carry out simultaneously.

3.2 Modeling program procedure

In order to reduce the computational effort of the discrete self-memory prediction equation, the calculation process could be carried out as mentioned above with the help of Matlab software. Meanwhile, the programming procedure for the grey self-memory combined prediction model is shown in Fig. 1.

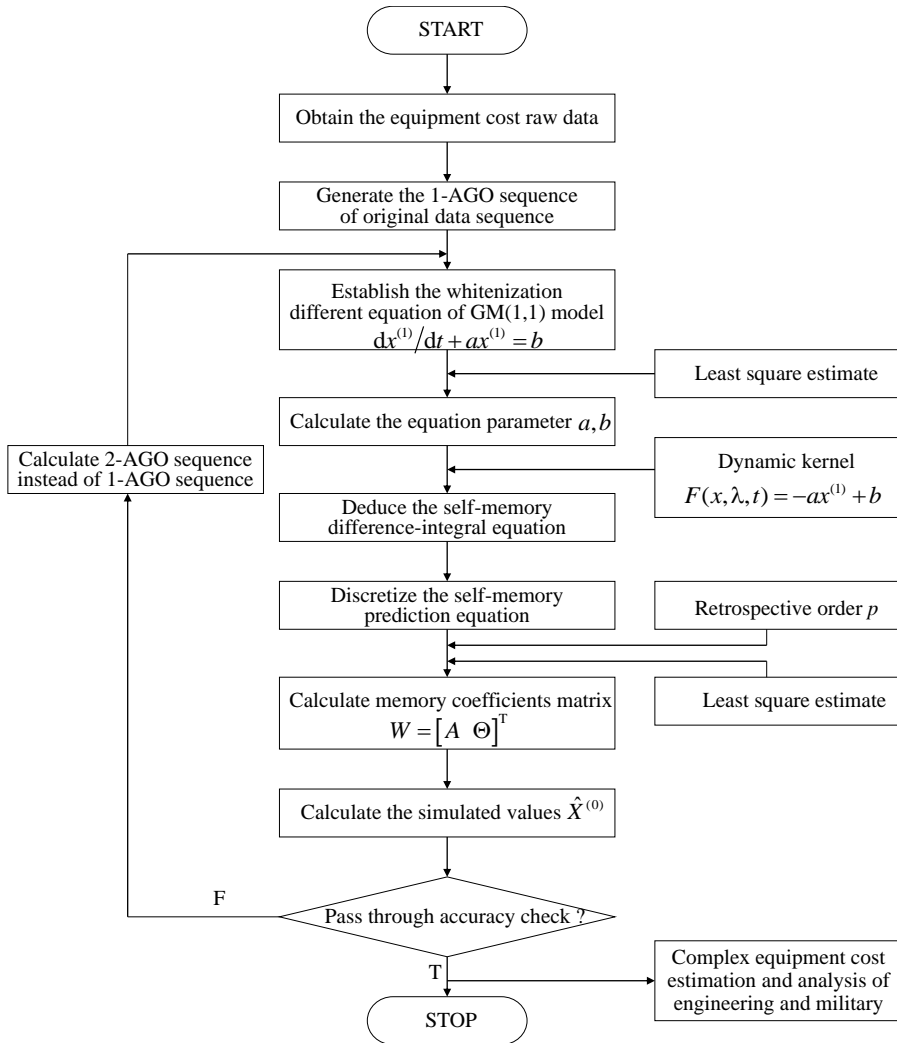


Fig. 1 Programming procedure for the grey self-memory combined prediction model.

4. Case study

In this section, two case studies including the yearly accumulated development cost estimation of torpedo and the maintenance support cost estimation of U.S.military equipment are adopted as study object. The corresponding grey self-memory combined models will be compared with traditional grey models to demonstrate their effectiveness and superiority. And three popular test criterions including MSE, AME and MAPE are adopted to synthetically analyse the modeling errors of different prediction models.

4.1 The yearly accumulated development cost estimation of torpedo

Table 1 lists the yearly accumulated development cost per unit statistical data of a certain type torpedo in China from 1995 to 2004^[18].

Table 1 The yearly accumulated development cost per unit statistical data of torpedo

from 1995 to 2004

Year	1995	1996	1997	1998	1999
Accumulated development cost (10 ⁴ yuan)	496	1275	2462	3487	3975
Year	2000	2001	2002	2003	2004
Accumulated development cost (10 ⁴ yuan)	4230	4387	4497	4584	4663

Then the grey self-memory combined model, which has the memory function for historical statistics data, is established to model and predict the variation tendency of torpedo's yearly accumulated development cost, and compared with the traditional Verhulst model^[18]. When taking the modeling analysis, the accumulated development cost data from 1995 to 2002 are taken as the training samples, and the data from 2003 to 2004 are chosen as the testing sample for prediction test. At first, based on the corresponding statistics data, the whitenization differential equation of traditional GM(1,1) model is formulated as follows:

$$\frac{dx}{dt} = 0.1305x + 2087.5795. \quad (15)$$

Consequently, the differential equation $dx/dt = F(x, \lambda, t)$, which has been determined by Eq. (15), is considered as the dynamic kernel $F(x, \lambda, t)$ of the self-memory equation. Then the self-memory GM(1,1) combined model can be established for torpedo's yearly accumulated development cost estimation. The value of retrospective order is determined as $p = 1$ by trial calculation method under the principle of minimum error of fitting root-mean-square. Then the corresponding self-memory prediction equation of discrete form can be expressed as

$$x_t = \sum_{i=-2}^{-1} \alpha_i y_i + \sum_{i=-1}^0 \theta_i F(x, \lambda, i),$$

where the memory coefficients matrix is

$$W = [\alpha_{-2}, \alpha_{-1}, \theta_{-1}, \theta_0]^T = [07197, 0.2177, -14.5504, 15.0963]^T.$$

Through calculations, the simulation prediction values and their corresponding APE(k) of two compared models, traditional Verhulst model and self-memory GM(1,1) combined model, are presented in Table 2 respectively. Among the table, there is no simulated value of the first time-point as a result of modeling mechanism in Verhulst model. Similarly, there are no simulated values of the first two time-points owing to the retrospective order $p = 1$ in self-memory GM(1,1) combined model.

Table 2 The simulation prediction values and error comparison of the two compared models (Year 1995 to 2004)

Year	Actual cost (10 ⁴ yuan)	Verhulst model		Self-memory GM(1,1) model	
		Simulation prediction cost	APE (%)	Simulation prediction cost	APE (%)
1995	496	—	—	—	—
1996	1275	1167	8.471 %	—	—
1997	2462	2231	9.383 %	2520	2.368 %
1998	3487	3290	5.650 %	3296	5.491 %
1999	3975	3952	0.579 %	4188	5.347 %
2000	4230	4254	0.567 %	4163	1.583 %
2001	4387	4371	0.365 %	4364	0.525 %

2002	4497	4415	1.823 %	4501	0.094 %
2003	4584	4436	3.229 %	4591	0.148 %
2004	4663	4499	3.517 %	4664	0.017 %

From the comparison of simulation prediction results about yearly accumulated development cost, the range of $APE(k)$ for eight simulation training samples is from 0.365% to 9.383% in Verhulst model, and its corresponding average relative error is 3.834%. Moreover, the $APE(k)$ of new-established self-memory GM(1,1) model has been decreased drastically with the help of the self-memory technique, its range is reduced from 0.094% to 5.491%, and the average relative error is reduced to 2.565% significantly. Meanwhile, the new model has obvious superiority for prediction, the single-step and two-step rolling prediction error are reduced significantly from 3.229% to 0.148% and from 3.517% to 0.017%, respectively.

Three accuracy test criteria values (MSE, AME and MAPE) of different accumulated development cost estimation models are listed in Table 3. From the viewpoint of error test, the self-memory GM(1,1) model always shows lower error values than the Verhulst model. It is shown that the self-memory technique further reduce the modeling errors compared with the traditional grey prediction model. Consequently, the self-memory GM(1,1) model has passed the modeling simulation and prediction accuracy test, and promoted the predictive performance markedly. It possesses the better prediction reliability and robustness compared with traditional grey prediction model.

Table 3 Accuracy check of simulation prediction values for different models (Year 1995 to 2002)

Prediction model	MSE	AME	MAPE
Verhulst model	15988.429	97.286	3.834 %
Self-memory GM(1,1) model	15041.333	92.667	2.565 %

4.2 The maintenance support cost estimation of U.S.military equipment

Table 4 lists the maintenance support cost per unit statistical data of U.S.military equipment in the first quarter from 1995 to 2007^[9].

Table 4 The maintenance support cost per unit statistical data of U.S.military equipment in the first quarter from 1995 to 2007

Year	1995	1996	1997	1998	1999	2000	2001
Maintenance support cost (10 ⁶ dollar)	20.37	21.64	21.59	23.99	23.98	25.67	26.50
Year	2002	2003	2004	2005	2006	2007	
Maintenance support cost (10 ⁶ dollar)	27.88	29.55	30.99	31.89	33.92	35.01	

Similarly, the grey self-memory combined model is established for the equipment maintenance support cost estimation, and compared with the traditional GM(1,1) model^[9]. The maintenance support cost data from 1995 to 2004 are taken as the training samples, and the data from 2005 to 2007 are chosen as the testing sample for prediction test. At first, based on the corresponding statistics data, the whitenization differential equation of traditional GM(1,1) model is formulated as follows:

$$\frac{dx}{dt} = 0.0466x + 19.7691, \quad (16)$$

Consequently, the differential equation $dx/dt = F(x, \lambda, t)$, which has been determined by Eq. (16), is considered as the dynamic kernel $F(x, \lambda, t)$ of the self-memory equation. Then the self-memory GM(1,1) combined model can be established for maintenance support cost estimation. The value of retrospective order is determined as $p = 1$ by trial calculation method. Then the corresponding self-memory prediction equation of discrete form can be expressed as

$$x_t = \sum_{i=-2}^{-1} \alpha_i y_i + \sum_{i=-1}^0 \theta_i F(x, \lambda, i),$$

where the memory coefficients matrix is

$$W = [\alpha_{-2}, \alpha_{-1}, \theta_{-1}, \theta_0]^T = [0.0369, 1.0370, 3.5188, -1.8822]^T.$$

Through calculations, the simulation prediction values and their corresponding APE(k) of two compared models, traditional GM(1,1) model and self-memory GM(1,1) combined model, are presented in Table 5 respectively. Similarly, there are no simulated values of the front partial time-points owing to modeling mechanism in two compared prediction models.

Table 5 The simulation prediction values and error comparison of the two compared models (The first quarter of year 1995 to 2007)

Year	Actual cost (10 ⁶ dollar)	Traditional GM(1,1) model		Self-memory GM(1,1) model	
		Simulation prediction cost	APE (%)	Simulation prediction cost	APE (%)
1995	20.37	—	—	—	—
1996	21.64	21.210	1.986 %	—	—
1997	21.59	22.222	2.932 %	21.591	0.004 %
1998	23.99	23.284	2.942 %	23.671	1.329 %
1999	23.98	24.396	1.735 %	24.272	1.218 %
2000	25.67	25.561	0.425 %	25.863	0.750 %
2001	26.50	26.782	1.063 %	26.539	0.149 %
2002	27.88	28.060	0.647 %	28.020	0.501 %
2003	29.55	29.400	0.506 %	29.134	1.408 %
2004	30.99	30.804	0.599 %	30.754	0.760 %
2005	31.89	32.275	1.207 %	32.153	0.823 %
2006	33.92	33.817	0.304 %	33.940	0.060 %
2007	35.01	35.431	1.203 %	35.359	0.997 %

From the comparison of simulation prediction results about equipment maintenance support cost, the range of APE(k) for ten simulation training samples is from 0.425% to 2.942% in traditional GM(1,1) model, and its corresponding average relative error is 1.426%. Moreover, the APE(k) of new-established self-memory GM(1,1) model has been decreased drastically with the help of the self-memory technique, its range is reduced from 0.004% to 1.408%, and the average relative error is reduced to 0.765% significantly. Meanwhile, the new model has obvious superiority for prediction, the single-step and multi-step rolling prediction error are also reduced significantly.

Three accuracy test criteria values (MSE, AME and MAPE) of different maintenance support cost estimation models are listed in Table 6. From the viewpoint of error test, it is shown that the self-memory technique further reduce

the modeling errors of traditional grey prediction model. Therefore, the self-memory GM(1,1) model has passed the modeling simulation and prediction accuracy test, and it possesses the better prediction reliability and robustness.

Table 6 Accuracy check of simulation prediction values for different models
(The first quarter of year 1995 to 2004)

Prediction model	MSE	AME	MAPE
Traditional GM(1,1) model	0.159635	0.343444	1.426 %
Self-memory GM(1,1) model	0.059269	0.204500	0.765 %

In addition, Figures 2 and 4 illustrate the fitting results of the simulation prediction curves obtained by the two compared models with the monitoring curves. And Figures 3 and 5 illustrate their corresponding comparison results of $APE(k)$ distribution. As can be seen from the given figures, the proposed self-memory GM(1,1) model possesses more stable and ideal simulation and prediction effects. It can better deal with the fluctuation situation emerging in original sequence, and better catch the tendency of integral development and individual variation of dynamical system. Moreover, the single-point relative errors of new model are less than traditional model as a whole, and the distribution of errors is more stable. The obvious simulation prediction superiority at later time points fully reflects the new information priority principle of grey system theory.

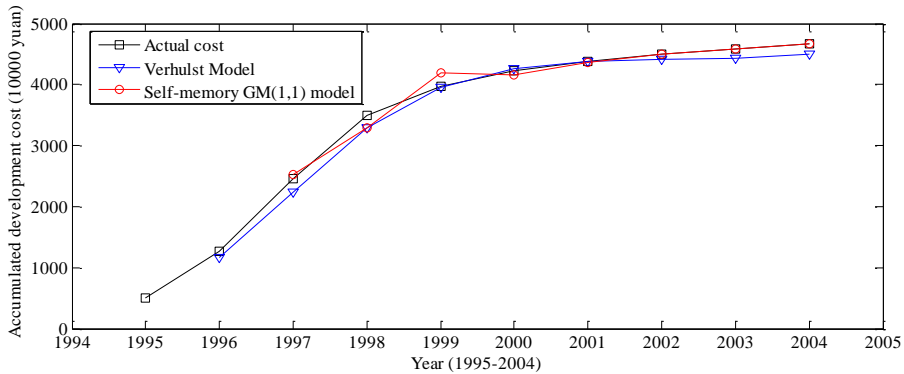


Fig. 2 Comparison of simulation prediction curves between two different grey models.

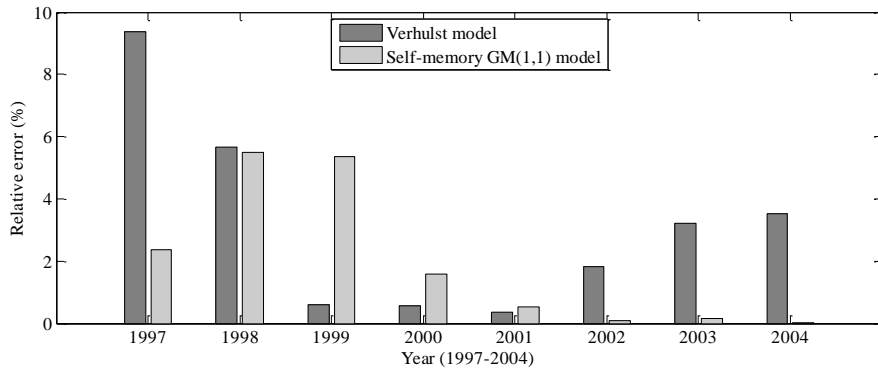


Fig. 3 Comparison of relative error distribution between two grey models' simulation prediction values.

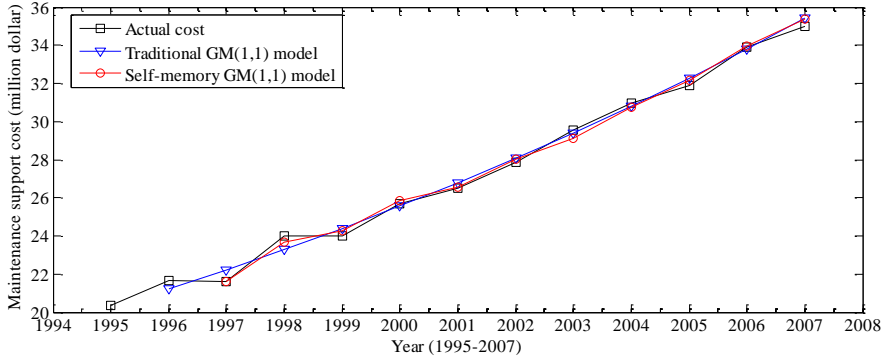


Fig. 4 Comparison of simulation prediction curves between two different grey models.

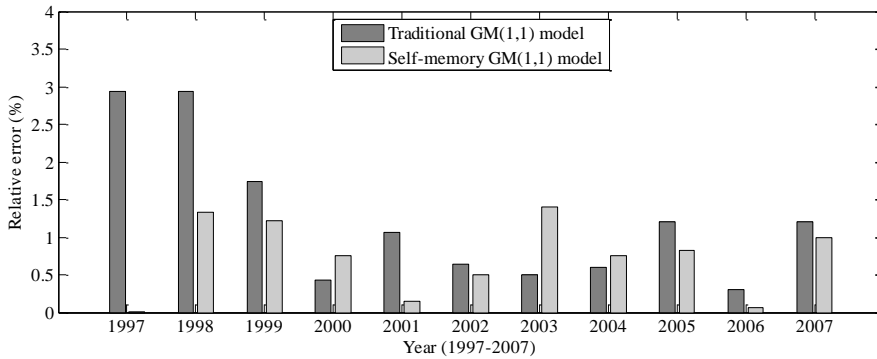


Fig. 5 Comparison of relative error distribution between two grey models' simulation prediction values.

The novel cost estimation model combines the advantages of the self-memory principle and traditional grey model through coupling their prediction methods. It can take full advantage of the system's multi-time historical monitoring data and accurately predict the system's evolutionary trend. Because the traditional grey prediction model essentially belongs to the initial value solving problem of differential equations which only meet the initial condition at one point, i.e., the observed values at one moment. Accordingly, the original dynamical differential equation has the limitation of being sensitive to initial values, and that becomes a disadvantage when historical information is not fully available. The excellent predictive performance of novel cost estimation model lies in that the weakness of traditional grey prediction model, i.e., sensitivity to initial value, can be overcome by using multi-time-point initial field instead of only single-time-point initial field in the system's self-memorization equation.

5. Conclusions and future work

In this paper, complex equipment cost estimation with a limited amount of data has been studied. In order to further promote its prediction accuracy, the self-memory technique is introduced into the traditional grey prediction model. Its excellent predictive performance lies in that only single-time-point initial field is replaced by multiple-time-point initial field. Therefore, it overcomes the weakness of being sensitive to initial values of the traditional grey model and possesses the better prediction robustness. As shown in the case studies of torpedo's development cost and equipment maintenance support cost, the proposed new prediction model

can take full advantage of the systematic historical statistical information and catch tightly the system's tendency of integral development and individual variation. It is worth popularizing and applying to other similar complex equipment cost estimation problems.

However, there are still some problems which should be solved in our future work. Firstly, complex equipment cost estimation often be affected by many factors. If the key factors affecting equipment cost could be analyzed and extracted, it is more reasonable to estimate the complex equipment cost on the basis of the key influence factors. Therefore, if the cost historical data of several key factors are considered as the multi-variable time series, certain multi-variable grey self-memory combined model could be designed to estimate the equipment cost needs further exploration. In addition, as time goes on, new equipment cost data will continually enter into the equipment systems. Therefore, the grey self-memory combined model can be metabolic adjusted basing on new information priority principle.

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