An Influence-Driven Feedback System For Preference Similarity Network Clustering Based Consensus Group Decision Making Model

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Abstract

Consensus group decision making (CGDM) allows the integration within this area of study of other advanced frameworks such as Social Network Analysis (SNA), Social Influence Network (SIN), clustering and trust-based concepts, among others. These complementary frameworks help to bridge the gap between their corresponding theories in such a way that important elements are not overlooked and are appropriately taken into consideration. In this paper, a new *influence-driven feedback mechanism* procedure is introduced for a preference similarity network clustering based consensus reaching process. The proposed influence-driven feedback mechanism aims at identifying the network influencer for the generation of advices. This procedure ensures that valuable recommendations are coming from the expert with most similar preferences with the other experts in the group. This is achieved by adapting, from the SIN theory into the CGDM context, an eigenvector-like measure of centrality for the purpose of: (i) measuring the influence score of experts, and (ii) determining the network influencer. Based on the initial evaluations on a set of alternatives provide by the experts in a group, the proposed influence score measure, which is named the σ-*centrality*, is used to define the similarity social influence network (SSIN) matrix. The σ-*centrality* is obtained by taking into account both the endogenous (internal network connections) and exogenous (external) factors, which means that SSIN connections as well as the opinion contribution from third parties are permitted in the nomination of the network influencer. The influence-driven feedback mechanism process is designed based on the satisfying of two important conditions to ensure that (1) the revised consensus degree is above the consensus threshold and that (2) the clustering solution is improved.

Keywords: Consensus, Preference Similarity, Agglomerative Hierarchical Clustering, Social Influence Network, Centrality, Feedback Mechanism.

1. Introduction

Experts in consensus group decision making (CGDM) aim to achieve a common solution with acceptable group agreement. Among the factors that may difficult the required group agreement or consensus level are the experts' background, culture, expertise and motivation. This is because each expert aspires to get his/her opinion to be appropriately considered by the group [1]. New CGDM proposals have been introduced in recent years to deal with this issue,

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among which it is worth mentioning the following: stochastic optimization cost based CGDM [2], multi-stage fluctuation utility consensus procedure [3], correlation consensus measure [4], hesitant based fuzzy CGDM [5], bipolar consensus approach [6], preferential voting consensus models [7], consensus modeling with cost chance constraint under uncertainty [8], multi-stage fluctuation utility consensus measure [3] and consensus approaches with utility preferences and limited budget [9].

An interesting approach of recent development to deal with the CGDM problem is based on the application of clustering methodologies and derived definitions of useful concepts aiming at presenting homogeneity of experts preferences as a whole: internal and external cohesions, cluster consensus, level consensus and global cluster consensus of the group of experts. Within this approach we can highlight the clustering-based consensus reaching processes proposed by Garcia-Lapresta and Perez-Roman [10, 11], and the partition technique applied by Abel et al. [12] that use experts' opinion similarities and dissimilarities in grouping a set of experts into subgroups to reach a compromise in determining the final group solution. Kamis et al. [13, 14] also advanced the use of preference similarity in deriving a clustering-based CGDM by building an undirected weighted preference similarity network between the group of experts based on the 'structural equivalence' concept and the agglomerative hierarchical clustering algorithm.

Recently, researchers have shown an interest in developing CGDM frameworks to effectively cater for decision making problems with large-scale users and real-time communications, analysing structure of opinion exchange, users interaction pattern and trust relations formation. A systematic review of existing approaches on CGDM within a social network context [15–18] was carried out by Herrera-Viedma et al. [19]. The evolution of CGDM in social networks has led to the development of another relevant area of study, *Social Influence Network* (SIN). The *influence* element in SIN plays an important role in CGDM as a key representation of network connection strengths [20], and it has been applied in estimating missing preferences [21], in the evolution of preferences [22] and in the estimation of experts' importance degrees [23, 24].

Most CGDM methods include a *feedback mechanism* when the group consensus level is not acceptable. This mechanism simulates a discussion medium where experts revisit their evaluations based on the recommendations provided by a 'moderator or leader'. Usually, experts who contribute low to consensus are identified and each of them will get an advice on how to change his/her opinion in order to increase the group consensus level, and eventually to achieve an acceptable group agreement. There exist numerous proposed procedures on implementing a feedback mechanism and advice generation in CGDM. Wu and Chiclana [25] introduced a visual information feedback mechanism by providing graphical representations of experts' current status and simulation of future consensus status after their initial evaluations were replaced with the provided recommendation values. The idea of visualizing consensus state before and after the feedback mechanism was also adopted by Liu et al. [26], where advices were generated by means of the implementation of a trust-based induced feedback recommendation system. Similar feedback approaches and advice generation systems in CGDM can be found in Wu et al. [27] and Gong et al. [28].

This study contributes to consensus generation within a social network framework by introducing a new influence-driven feedback mechanism. The concept of eigenvector-like centrality measure from Social Influence Network (SIN) is proposed to be used and implemented within a preference similarity network clustering based consensus group decision making model. A new centrality influence score measure is developed for the purpose of identifying the network influencer, who will act as the leader in a new Similarity Social Influence Network (SSIN) feedback mechanism. The proposed influence-based concept is also utilized in an IOWA [29] based *fuzzy majority* dominance resolution process [30] of the CGDM problem to guarantee that the best alternative of consensus is selected by the majority of the experts as a whole.

The rest of the paper is organized as follows: Section 2 introduces the concept of reciprocal

fuzzy preference relation and its associated intensity preference vector, while Section 3 presents the preference similarity network clustering based consensus reaching process. Section 4 describes the background of social influence theory in decision making context and presents in detail the development of the proposed influence-driven feedback mechanism with its practical implementation steps within a CGDM. An influence-driven resolution process of a CGDM and the general research flow of the proposed model for the influence-based feedback preference similarity network clustering based consensus group decision making problem are provided in Section 5. Conclusions are drawn in Section 6.

2. Preliminaries on reciprocal fuzzy preference relations

In solving a consensus decision making problem with a finite set of feasible alternatives, $\mathbb{A} = \{A_1, A_2, \ldots, A_n\}$ ($n > 2$), a group of experts $E = \{e^1, e^2, \ldots, e^m\}$ uses their knowledge, experience and expertise to express their opinions with the aim of achieving a final decision with sufficient agreement from every group member. When pairwise comparing alternatives, experts assign to each alternative an intensity of preference degree towards any other alternative [31]. Mathematically, this preference representation is modelled using a *reciprocal fuzzy preference relation* (RFPR) on $\mathbb{A}$, i.e. a fuzzy binary relation $\mu_P : \mathbb{A} \times \mathbb{A} \longrightarrow [0,1]$ that associated each pair of alternatives (A_i, A_j) a value $\mu_P(A_i, A_j) = p_{ij}$ verifying the reciprocity property $p_{ij} + p_{ji} = 1$ ($\forall i, j$) and the following interpretation:

$$
\begin{array}{lll}
p_{ij} = 0.5 & \text{if } A_i \text{ and } A_j \text{ are equally preferred (indifference)} & \\
p_{ij} \in (0.5, 1) & \text{if } A_i \text{ is slightly preferred to } A_j & (1) \\
p_{ij} = 1 & \text{if } A_i \text{ is absolutely preferred to } A_j &
\end{array}
$$

An RFPR on a set of alternatives $\mathbb{A}$ can therefore be represented using a matrix of dimension $n \times n$, $P = (p_{ij})$. The set of RFPRs of dimension $n \times n$ will be denoted as $\mathbb{P}_{n \times n}$. Reciprocity property allows the representation of an RFPR in terms of its *intensity preference vector* (IPV) [4], $V \in \mathbb{R}^{n(n-1)/2}$:

$$
V = \left(p_{12}, p_{13}, \ldots, p_{1n}, p_{23}, \ldots, p_{2n}, \ldots, p_{(n-1)n}\right) = \left(v_1, v_2, \ldots, v_r, \ldots, v_{n(n-1)/2}\right). \quad (2)
$$

The set of IPVs from all experts E on the set of alternatives $\mathbb{A}$ will be denoted by $\mathbb{V} = \{V^1, V^2, \ldots, V^m\}$.

Example 1. *"For the purpose of demonstrating our proposed model, we utilize the existing hypothetical example in [22] where eight (8) experts, $E = \{e^1, e^2, \ldots, e^8\}$, express their initial evaluations over six (6) alternatives, $\mathbb{A} = \{A_1, A_2, \ldots, A_6\}$. The RFPR of expert e^1, P^1, and its associated IPV, V^1, are:*

$$
P^1 = \begin{bmatrix}
1 & \mathbf{0.27} & \mathbf{0.64} & \mathbf{0.89} & \mathbf{0.74} & \mathbf{0.82} \\
0.73 & 1 & \mathbf{0.80} & \mathbf{0.92} & \mathbf{0.85} & \mathbf{0.89} \\
0.36 & 0.20 & 1 & \mathbf{0.85} & \mathbf{0.62} & \mathbf{0.74} \\
0.11 & 0.08 & 0.15 & 1 & \mathbf{0.20} & \mathbf{0.27} \\
0.26 & 0.15 & 0.38 & 0.80 & 1 & \mathbf{0.64} \\
0.18 & 0.11 & 0.26 & 0.73 & 0.36 & 1
\end{bmatrix}
$$

$V^1 = (\mathbf{0.27}, \mathbf{0.64}, \mathbf{0.89}, \mathbf{0.74}, \mathbf{0.82}, \mathbf{0.80}, \mathbf{0.92}, \mathbf{0.85}, \mathbf{0.89}, \mathbf{0.85}, \mathbf{0.62}, \mathbf{0.74}, \mathbf{0.20}, \mathbf{0.27}, \mathbf{0.64})$.

The IPV of the rest of experts $\{V^2, V^3, \ldots, V^8\}$ are:

$V^2 = (0.80, 0.85, 0.73, 0.92, 0.89, 0.62, 0.36, 0.85, 0.74, 0.26, 0.80, 0.64, 0.89, 0.82, 0.27);$
$V^3 = (0.69, 0.13, 0.20, 0.36, 0.89, 0.06, 0.10, 0.20, 0.78, 0.63, 0.79, 0.98, 0.69, 0.97, 0.94);$
$V^4 = (0.10, 0.36, 0.69, 0.16, 0.27, 0.83, 0.95, 0.63, 0.76, 0.79, 0.25, 0.39, 0.08, 0.14, 0.65);$
$V^5 = (0.56, 0.15, 0.62, 0.69, 0.93, 0.18, 0.85, 0.60, 0.82, 0.82, 0.93, 0.97, 0.54, 0.82, 0.81);$
$V^6 = (0.17, 0.18, 0.33, 0.45, 0.59, 0.47, 0.61, 0.71, 0.82, 0.60, 0.70, 0.61, 0.62, 0.46, 0.31);$
$V^7 = (0.30, 0.40, 0.45, 0.51, 0.66, 0.53, 0.45, 0.73, 0.68, 0.40, 0.54, 0.66, 0.44, 0.60, 0.37);$
$V^8 = (0.75, 0.75, 0.72, 0.77, 0.82, 0.55, 0.62, 0.59, 0.65, 0.60, 0.46, 0.67, 0.20, 0.44, 0.50).$"

3. Preference similarity network clustering based consensus measurement

In this section, the preference similarity network clustering based consensus decision making procedure used in [14] is referred to, which is described in detail in [13]. Using $\mathbb{V}$, the preference similarity between any pair of experts can be measured and a network structure between the experts in the group E can be created. Formally, a preference similarity measure can be defined as follows:

Definition 1. *"Let $\mathbb{V}$ be the set of IPVs of the group of experts E over the set of alternatives $\mathbb{A}$. A preference similarity measure on E is a fuzzy subset of $\mathbb{V} \times \mathbb{V}$ with membership function $S\colon \mathbb{V} \times \mathbb{V} \to [0,1]$ verifying both the reflexive $[S(V^a, V^a) = 1]$ and the symmetric $[S(V^a, V^b) = S(V^b, V^a)]$ properties."*

As per Definition 1, an indirect preference similarity network, $\mathbb{N} = \langle E, T, \mathbb{S} \rangle$, between the experts (nodes) in E can be created with the preference similarity degree between experts, $\mathbb{S} = (S_1, S_2, \ldots, S_{m(m-1)/2})$, as the set of weights attached to the set of ties between nodes, $T = (t_{12}, t_{13}, \ldots, t_{1n}, t_{23}, \ldots, t_{2m}, \ldots, t_{(m-1)m})$, to express the strength of the similarity relation between the network nodes (experts' preferences). In Social Network Analysis (SNA), two nodes are structurally equivalent if they have the same neighbors and similar characteristics in their social environments [32]. The cosine similarity function can be used to achieve the structural equivalence preference similarity network:

$$S^{ab} = S\left(V^a, V^b\right) = \frac{\displaystyle\sum_{i=1}^{n(n-1)/2} \left(v_i^a \cdot v_i^b\right)}{\sqrt{\displaystyle\sum_{i=1}^{n(n-1)/2} \left(v_i^a\right)^2} \cdot \sqrt{\displaystyle\sum_{i=1}^{n(n-1)/2} \left(v_i^b\right)^2}}. \tag{3}$$

For the purpose of categorizing structurally equivalence experts in the undirected weighted preference similarity network, the agglomerative hierarchical clustering approach is applied as per Algorithm 1 below.

Algorithm 1: Agglomerative hierarchical clustering procedure for preference similarity network

---

begin

1   Apply individual partition for each expert in E:
     $P^m = \{\{e^1\}, \{e^2\}, \ldots, \{e^m\}\} = \{\{C_1\}, \{C_2\}, \ldots, \{C_m\}\}$;
     $i \longleftarrow m$;
     while $i > 1$ **do**

2   Form cluster C_a and C_b in $P^i = \{C_1, \ldots, C_i\}$ with maximal S^{ab};

3   Merge cluster C_a and C_b to cluster C_k;

4   Form new partition P^{i-1} by removing C_a and C_b and adding cluster C_k;
       $i \longleftarrow i - 1$;
     end while

end

A *dendogram* is a convenient visualisation of the hierarchical clustering generated by Algorithm 1. Using horizontally cuts at particular α_l-levels ($l = 2, \ldots, m - 1$) in a dendogram, a partition of the experts into a number of clusters (C_l), with respect to the cosine preference similarity matrix S, is obtained (see Example 2 ending this section).

This implementation of the structural equivalence concept means that experts in the same cluster have strong connections between them and, therefore, they are expected to reach higher cohesiveness among them than with outsider experts. Thus, as described below, measurements of experts' cluster homogeneity according to the internal and external cohesions can be computed and combined to derive a cluster consensus measure.

Let $L = \{\alpha_l \; ; \; l = 2, \ldots, m - 1\}$ be the set of all different α_l-levels and $C_l = \{C_{lk} \; ; \; k = 1, \ldots, l\}$ the set of clusters at the α_l-level, with cardinality $\sharp C_{lk}$.

- The α_l-level *cluster internal cohesion degree* of cluster C_{lk}:

$$\delta_{int}(C_{lk}) = \frac{\displaystyle\sum_{i \in C_{lk}} \sum_{j \in C_{lk}} S^{ij}}{(\sharp C_{lk})^2} \qquad (4)$$

- The α_l-level *cluster external cohesion degree* of cluster C_{lk}:

$$\delta_{ext}(C_{lk}) = \frac{\displaystyle\sum_{i \in C_{lk}} \sum_{j \notin C_{lk}} S^{ij}}{\sharp C_{lk}(n - \sharp C_{lk})} \qquad (5)$$

- The α_l-level *cluster consensus degree* of cluster C_{lk}:

$$\delta_{CC}(C_{lk}) = \frac{\sharp C_{lk} \cdot \delta_{int}(C_{lk})}{n} + \frac{(n - \sharp C_{lk}) \cdot \delta_{ext}(C_{lk})}{n} \qquad (6)$$

- The α_l-level *average cluster consensus degree of the group of experts E* measures the agreement index between experts of all clusters at that α_l-level:

$$\delta_{LC}(l) = \frac{\displaystyle\sum_{k=1}^{l} \delta_{CC}(C_{lk})}{l}. \qquad (7)$$

- The *global cluster consensus degree of the group of experts* E is $\delta_{LC}\left(\hat{l}\right)$ with $\alpha_{\hat{l}}$-level having the maximum of all α_l-level average cluster consensus degrees, which is referred to as the *optimal consensus agglomerative hierarchical clustering level*.

Let the threshold value of sufficient group consensus be set at the value φ. The preference similarity network clustering based consensus reaching procedure is summarised in Algorithm 2 below.

Algorithm 2: Preference similarity network clustering based consensus reaching procedure

   begin
1   At each α_l-level, identify experts belong to its corresponding clusters;
2   For each cluster in C_l, determine $\delta_{int}\left(C_{lk}\right)$ and $\delta_{ext}\left(C_{lk}\right)$;
3   Combine the values of $\delta_{int}\left(C_{lk}\right)$ and $\delta_{ext}\left(C_{lk}\right)$ for each cluster in C_l to obtain the
      value of $\delta_{CC}\left(C_{lk}\right)$;
4   For all α_l-level in L, compute $\delta_{LC}\left(l\right)$;
5   Identify optimum level of the agglomerative hierarchical clustering: $\alpha_{\hat{l}}$-level;
6   **if** $\delta_{LC}\left(\hat{l}\right) \geq \varphi$ **then**
     | proceed to resolution process (Section 5);
      else
     | proceed to feedback mechanism (Section 4.2);
      end if
   end

Example 2 (Continuation of Example 1). *"Applying (3), the following cosine preference similarity matrix is obtained:*

$$\mathbb{S} = \begin{bmatrix} 1 & 0.868 & 0.742 & 0.937 & 0.906 & 0.926 & 0.945 & 0.957 \\ 0.868 & 1 & 0.817 & 0.704 & 0.876 & 0.896 & 0.954 & 0.932 \\ 0.742 & 0.817 & 1 & 0.609 & 0.925 & 0.840 & 0.854 & 0.803 \\ 0.937 & 0.704 & 0.609 & 1 & 0.797 & 0.844 & 0.834 & 0.840 \\ 0.906 & 0.876 & 0.925 & 0.797 & 1 & 0.944 & 0.942 & 0.912 \\ 0.926 & 0.896 & 0.840 & 0.844 & 0.944 & 1 & 0.970 & 0.883 \\ 0.945 & 0.954 & 0.854 & 0.834 & 0.942 & 0.970 & 1 & 0.945 \\ 0.957 & 0.932 & 0.803 & 0.840 & 0.912 & 0.883 & 0.945 & 1 \end{bmatrix}$$

Figure 1 shows the complete graph representation of the undirected weighted preference similarity network between the set of experts, with only a few link weights showed for simplicity reason. The experts' cluster pattern according to the dendogram (Figure 2)) is generated by Algorithm 1 at the optimal α_l-level identified by Algorithm 2 as detailed in Table 1. The optimum global consensus of 0.883 is unique and achieved at the α_7-level. If the consensus threshold were above 0.883 (say $\varphi = 0.945$), then group consensus would be not sufficient and a second round of the consensus process would be required, which is to be preceded by the activation of a feedback process to increase the group consensus. It is noticed that the optimum cluster level results in only 2 experts being grouped in a cluster while the rest are on their own cluster, respectively."
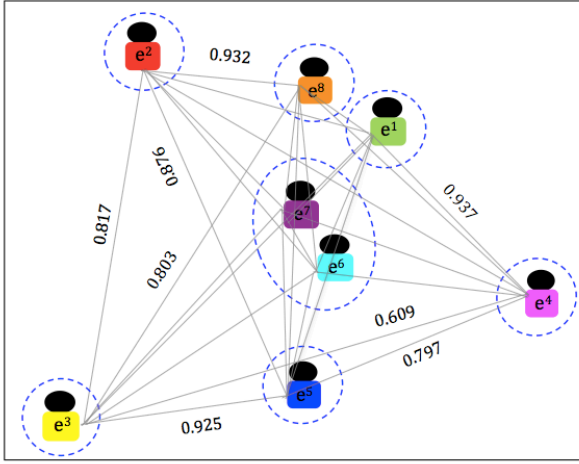
Figure 1: The undirected weighted preference similarity network with experts clustering pattern
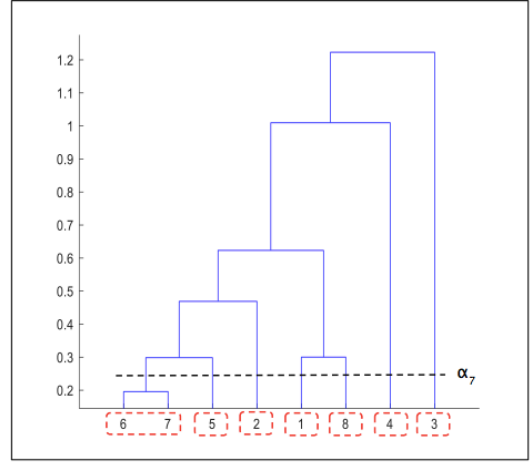

Figure 2: Dendogram generated by Algorithm 1 with the optimal α_l-level from Algorithm 2

Table 1: Cluster internal and external cohesions degrees, cluster consensus and average cluster consensus degrees

α	C	E	δ_{int}	δ_{ext}	δ_{CC}	δ_{LC}
2	1	$e^1, e^2, e^4, e^5, e^6, e^7, e^8$	0.911	0.799	0.897	0.860
	2	e^3	1	0.799	0.824	
3	1	$e^1, e^2, e^5, e^6, e^7, e^8$	0.937	0.828	0.909	0.851
	2	e^4	1	0.795	0.821	
	3	e^3	1	0.799	0.824	
4	1	e^2, e^5, e^6, e^7	0.948	0.871	0.909	0.866
	2	e^1, e^8	0.979	0.887	0.910	
	3	e^4	1	0.795	0.821	
	4	e^3	1	0.799	0.824	
5	1	e^5, e^6, e^7	0.968	0.889	0.919	0.871
	2	e^2	1	0.864	0.881	
	3	e^1, e^8	0.979	0.887	0.910	
	4	e^4	1	0.795	0.821	
	5	e^3	1	0.799	0.824	
6	1	e^5, e^6, e^7	0.968	0.889	0.919	0.877
	2	e^2	1	0.864	0.881	
	3	e^1	1	0.898	0.910	
	4	e^8	1	0.896	0.909	
	5	e^4	1	0.795	0.821	
	6	e^3	1	0.799	0.824	
7	1	e^6, e^7	0.985	0.901	0.922	**0.883**
	2	e^5	1	0.900	0.913	
	3	e^2	1	0.864	0.881	
	4	e^1	1	0.898	0.910	
	5	e^8	1	0.896	0.909	
	6	e^4	1	0.795	0.821	
	7	e^3	1	0.799	0.824	

4. Social influence theory in the CGDM context

We begin this section by reviewing related social influence studies implemented in CGDM frameworks. Our proposed influence-guided feedback mechanism methodology is presented and the second consensus round is then elaborated.

4.1. Review on existing influence methodologies in CGDM

Social influence network (SIN) has been developed continuously since the 1950s by French [33], Harary [34], DeGroot [35] and Friedkin and Johnsen [36]; it is one of the important fields directly linked to group decision making and SNA.

Leading to this correlated direction, recent studies on CGDM with incorporation of social influence theory, derived from social networks, have been introduced. Brunelli et al. [16] addressed consensus evaluation by considering experts' influence strengths in a social network via an eigenvector centrality measure with a fuzzy m-ary adjacency relation approach. In [17], a leadership-based consensus reaching procedure was developed where opinion managers are allowed to give advices and influence the opinion formation from a social network towards achieving consensus. Another study worth mentioning was carried out by Liang et al. [20], where the authors modelled the social influence in terms of experts' tie strengths in a social network, with the number of common members' connections were combined with the number of their direct interactions.

The influence factor is necessary to be implemented in CGDM frameworks in order to ensure the acceptance by the experts of the recommendation advices produced by a feedback mechanism algorithm to increase consensus. By doing so, advices can be generated from the most influential person in the network, which implicitly brings attached a trust statement with it. It has been argued that a feedback mechanism guided by trust relationships is more realistic than the traditional feedback mechanism because the generated advices are more persuasive and acceptable by a group of experts [18, 24, 26, 37].

Perez et al. [22] and Capuano et al. [21] modelled influence processes in GDM by utilising the recursive approach presented in Friedkin and Johnsen [38]. Let E be a set of experts, $W = (w_{ij})_{m \times m}$ a fuzzy adjacency matrix representing experts' relative importance on others' preferences including themselves, and y^1 the initial experts' preferences. After t iterations, the influenced generated preferences will be

$$y^t = AW y^{(t-1)} + (I - A) y^1, \tag{8}$$

where $A = diag(a_{11}, a_{22}, \ldots, a_{mm})$ is the susceptibility of experts to interpersonal influences and I is the $m \times m$ identity matrix. Significantly, the above expression allows to estimate the evolution of experts' preferences iteratively until the process reaches an equilibrium, i.e. $y^\infty = \lim_{t \to \infty} y^t$ exists and it is expressed as follows:

$$y^\infty = (I - AW)^{-1} (I - A) y^1. \tag{9}$$

Capuano et al.'s work [21] provided greater flexibility compared to Perez et al.'s [22]. Capuano's proposed model proved that the complication of defining a numerical level expressing the susceptibility of experts towards influence can be avoided with the estimation of missing preferences been successfully operated and the convergence of experts' preferences achieved.

An alternative influence measure, known as *alpha-centrality*, was introduced by Bonancich and Lloyd [39]. The *alpha-centrality*, denoted here by x, is an eigenvector-like measure that determines expert's status in a network by considering their influence pattern; it is expressed as follows:

$$x = \left(I - \Upsilon \boldsymbol{A}^T \right)^{-1} \boldsymbol{e} \tag{10}$$

where $\boldsymbol{A} = (a^{ij})$ is an adjacency matrix that represents the group's influence pattern, i.e. a^{ij} represents the degree of influence of expert i by expert j; Υ is a scalar describing the relative importance degree of endogenous (internal) versus exogenous (external) effects of experts in a group. Bonacich and Lloyd suggested that Υ should approach $1/\lambda_1$ from below in order to achieve convergence of the solution, with λ_1 being the largest eigenvalue of $\boldsymbol{A}$. Notice that the endogenous factor emerges from connections in the network itself while the exogenous factor, $\boldsymbol{e}$, is external to the network of experts, such as from third party involvements, and that it can affect or change the experts' status. For example, the selection of the best employer will involve peer reviews, which can be presented as a relationship network in a workplace and this criteria is considered as an endogenous effect. In some situation, the selection process involves evaluation from company's top management, which can be seen an exogenous factor in the nomination of the best employer.

4.2. A new social influence-based feedback mechanism

Social influence models focus on the development of interpersonal influence processes, where experts are allowed to manage the conflicting influential preferences by revising and inducing it to behave in a similar way to others for the purpose of achieving group consensus. There exist two main aspects to be explored in social influence theory [38]: (1) how experts change their preferences and influence the other experts; and (2) how to develop structure of social influence models with better configuration and strengthen the interpersonal influence processes.

In a group decision making context, social influence is clarified by changes of attitudes, thoughts, feelings, characters or behaviors of expert(s), if there exists interaction from another expert in the group [20]. This situation allows modification of experts' preferences due to the social influence factor during interactions, discussions or opinion exchange in a network. Experts with strong knowledge, experience, background, trustworthy and motivation are able to act as 'leaders' in the network, in such a way that they can influence the others to move in the similar direction towards group consensus. This can be done by implementing a specific procedure in CGDM, known as 'feedback mechanism module'. Practically, a feedback mechanism must have flexible acceptance of the recommendation advices according to the affordability of the adjustment cost and experts guidance in order to choose the optimal feedback parameter for the purpose of balancing the individual adjustment cost with the group consensus [40].

The proposed novel influence-based feedback mechanism comprises three main phases: (1) identification of expert(s) with low consensus contribution; (2) identification of a network influencer; and (3) generation of advice. It is implemented in two different situations: (a) without exogenous effect; and (b) with exogenous effect. Explanations on this feedback process are presented below.

4.2.1. Identification of expert(s) with low consensus contribution

The global cluster consensus degree, $\delta_{LC}\left(\hat{l}\right)$, is the average cluster consensus degree of the experts at the optimal consensus agglomerative hierarchical clustering $\alpha_{\hat{l}}$-level. At this level, those experts in a cluster $C_{\hat{l}k}$ with $\alpha_{\hat{l}}$-level cluster consensus below the global cluster consensus degree will be identified as contributing low to consensus (below the average). This is formulated as follows:

$$e_{low} = \left\{ e^o \in E \mid e^o \in C_{\hat{l}k} \wedge C_{\hat{l}k} \in C_{low} \right\}, \tag{11}$$

$$C_{low} = \left\{ C_{\hat{l}k} \mid \delta_{CC}\left(C_{\hat{l}k}\right) < \delta_{LC}\left(\hat{l}\right) \wedge \ k = 1, \ldots, \hat{l} \right\}. \tag{12}$$

Experts in e_{low} will be adviced on how to change their opinions in order to increase the consensus level. This is proposed to be done by identifying the network influencer using influence-driven concepts from SIN theory.

4.2.2. Identification of the network influencer

For the purpose of ensuring recommendations or advices perform well in decision making process, we adapt a 'memory-based collaborative filtering' idea [41] into SIN, where historical/initial data of a group of users are used to provide 'valuable recommendations coming from someone who has shared similar history with other people in a group' [42]. According to this concept, we utilize the experts' initial evaluations (Section 2) as historical data to obtain the experts' preference similarity matrix, $\mathbb{S}$, as a criterion to be composed in our proposed SIN, which is constructed by a digraph that links the set of experts E (nodes) with edge connecting expert e^i and e^j, (e^i, e^j), with the influence strength weight of the jth expert over the ith expert.

In our case, the SIN comprises a set of experts, $E = \{e^1, e^2, \ldots, e^8\}$, and a row normalized preference similarity matrix, $\mathbb{S}_\eta = \left(S_\eta^{ij}\right)_{n \times n}$, where S_η^{ij} is the proportion of overall group influence on i that comes from j. Notice that $\mathbb{S}_\eta$ is obtained by taking a row normalization step on the preference similarity matrix (Section 3), $\mathbb{S}$, so that the following property $\sum_{j=1}^n S_\eta^{ij} = 1$ for all $i \in (1, \ldots, n)$ [21] is verified. This property ensures the influence of each expert towards all of his/her peers is 1 in total. Formally, we name our SIN as *Similarity Social Influence Network (SSIN)*, which is visualised for a simple case of 3 expert nodes in Fig. 3 and defined below:

Definition 2. *"A Similarity Social Influence Network (SSIN) is an ordered 3-tuple, $G = \langle E, L, \mathbb{S}_\eta \rangle$, comprising a set of nodes (E), a set of edges (L) or ordered pairs of experts in E, and a set of row normalized preference similarity weights $(\mathbb{S}_\eta)$ attached to L."*
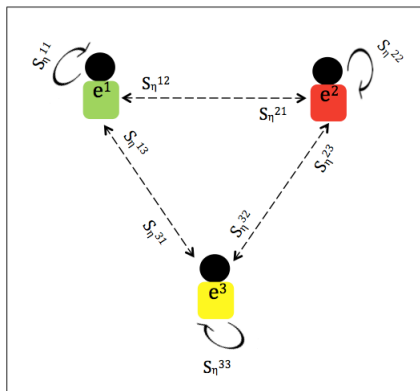

Figure 3: The general representation of SSIN consisting 3-expert nodes

In this study, we make use of the previously described influence measure by Bonacich and Lloyd [39]. It is adapted to our preference similarity network clustering based consensus group decision making model in order to identify the most influential expert of the network. The identified influencer will act as the 'leader' in designing feedback rules with the aim to increase the group consensus level when this is below a satisfactory threshold value. We name our proposed influence measure the σ-*centrality* and defines it formally below:

Definition 3. *"Let $\mathbb{S}_\eta$ be a set of row normalized preference similarity weights in SSIN G, σ the relative importance of endogenous (network connections) versus exogenous (external) effects, and $Z = (z)_{m \times 1}$ a set of individual expert exogenous effect values. Then, the influence score or σ-centrality of experts E, $Y = (y^1, \ldots, y^m)$, is:*

$$Y = \left(I - \sigma \, \mathbb{S}_\eta^T\right)^{-1} Z.$$

In the absence of exogenous effect, Z is set as the unity vector, i.e. the vector with all components equal to 1."

The combination of these two factors, endogenous and exogenous, produce stability in generating experts' influence scores because it is dependent on internal contributions to consensus from the generated preference similarity network and third-party importance evaluations over experts when available. Indeed, the value of Y represents the influence score of each expert, in such a way that both endogenous (internal) and exogenous (external) factors are considered.

Experts who fall in clusters having higher cluster consensus degrees than or equal to the global cluster consensus degree of the group can be classified as belonging to the group of most influential experts in the network. Let $C^*_{\hat{l}k}$ be set of clusters at the optimum clustering $\alpha_{\hat{l}}$-level with cluster consensus indexes, $\delta_{CC}(k)$, above the global cluster consensus degree of the group of experts, $\delta_{LC}\left(\hat{l}\right)$, and $e^{y^*}_{\hat{l}k}$ be the experts belonging to the clusters in $C^*_{\hat{l}k}$. The *identification of possible network influencers* can be formally written as:

$$e^{y^*}_{\hat{l}k} = \left\{ e^y \mid e^y \in C_{\hat{l}k} \wedge C_{\hat{l}k} \in C^*_{\hat{l}k} \right\}, \tag{13}$$

$$C^*_{\hat{l}k} = \left\{ C_{\hat{l}k} \mid \delta_{CC}(k) \geq \delta_{LC}\left(\hat{l}\right) \wedge \ k = 1,\ldots,\hat{l} \right\} \tag{14}$$

Thus, the network influencer, e^*, is the expert with highest influence score among those in $e^{y^*}_{\hat{l}k}$:

$$e^* = \max_{e^y \in C^*_{\hat{l}k}} Y\left(e^y\right). \tag{15}$$

The identified network influencer, e^*, will act as the advisor for the experts who contribute low to consensus on how to change their opinions with the aim to increase and, subsequently, to reach the group consensus threshold level. This is elaborated in the following section.

4.2.3. Generation of advice

As mentioned in Section 4.2.1, for the purpose of increasing the consensus level of the group, an expert e^o in e_{low} is provided (feedback) the following new intensity preference vector $\widetilde{V}^o$:

$$\widetilde{V}^o = (1 - \beta) \cdot V^o + \beta \cdot V^* \tag{16}$$

where V^o is the current IPV of e^o, V^* is the IPV of the network influencer, e^*, and $\beta \in [0,1]$ is a control parameter that can be used to adjust the extent of the change of preferences feedback to the experts in e_{low}.

This section concludes with the below algorithmic representation of the proposed influence-based feedback mechanism procedure.

Algorithm 3: Influence-based feedback mechanism

begin

  1   At $\alpha_{\hat{l}}$-level, identify C_{low}, a set of clusters having less δ_{CC} than $\delta_{LC}\left(\hat{l}\right)$;

  2   List all experts belong to C_{low}, denoted as e_{low};

  3   Construct row normalized preference similarity matrix, $\mathbb{S}_\eta$;

  4   Decide the individual exogenous effect, Z: (Assume Z as a vector of ones if no exogenous effect involved);

  5   Determine σ-*centrality*, Y using Equation (3);

  6   Rank experts according to Y scores in descending order: $R(Y)$;

  7   Identify $C^*_{\hat{l}k}$ as in Equation (14);

  8   List all experts belonging to $C^*_{\hat{l}k}$ (Equation (13));

  9   Identify the network influencer, e^*, by Equation (15);

  10   Generate advice preferences, $\widetilde{V}^o$, using Equation (16).

end

4.3. Consensus reaching procedure with optimal parameter of control

If $\beta = 0$ no changes are recommended and the original preferences of the expert remain unchanged. If $\beta = 1$ then the expert's preferences are completely substituted by those of the network influencer. For the influence-based feedback mechanism to be effective, a control parameter $\beta \in [0,1]$ should be selected to guarantee that the following two conditions are verified:

- the global cluster consensus degree of the group of experts of the second consensus round, $\delta^2_{LC}\left(\hat{l}\right)$ must be greater than or equal the consensus threshold, φ and;

- the optimal agglomerative hierarchical clustering level in second round of consensus process, $\alpha^2_{\hat{l}}$ must be less than the optimal agglomerative hierarchical clustering level of first round, $\alpha_{\hat{l}}$.

The advice generation with $\beta = 0$ produces the first round of consensus solution, and therefore it is not implemented if group consensus is to be achieved. For simplicity, we use discrete values of β from the set $\{0.1, \ldots, 0.9, 1\}$. The first condition above states that sufficient consensus level will be achieved, while the second one is purposely introduced to achieve a better clustered solution (lower number of clusters) after the implementation of the feedback process. Without imposing restrictions to the parameter of control, the above two conditions will be achieved at some extent because when the feedback advices are implemented, the experts will be more similar because the preferences will be closer the network influencer's preferences, which also will have a positive effect on the cohesiveness within clusters. These two conditions are formally presented in the following definition.

Definition 4. *"The revised global cluster consensus degree, $\delta^2\left(\hat{l}\right)$, of the group of experts, E, for the second round of the consensus reaching process satisfies: $\left(\delta^2_{LC}\left(\hat{l}\right) \geq \varphi\right) \wedge \left(\alpha^2_{\hat{l}} < \alpha_{\hat{l}}\right)$."*

The algorithm to find the optimal parameter of control, within the discrete set of values $\{0.1, \ldots, 0.9, 1\}$, with respect to Definition 4 is presented below:

Algorithm 4: The second round of consensus reaching procedure

begin

1   At $\beta = 0.1$, replace V^o with $\widetilde{V}^o$ for all e_{low} in C_{low};

2   Run Algorithm 1 and 2;

3   Identify $\alpha^2_{\hat{l}}$-level and its corresponding $\delta^2_{LC}\left(\hat{l}\right)$;

4   **if** $\left(\delta^2_{LC}\left(\hat{l}\right) \geq \varphi\right) \wedge \left(\alpha^2_{\hat{l}} < \alpha_{\hat{l}}\right)$ **then**
     end second round consensus procedure;
   else
     repeat this algorithm with next discrete β-level;
   end if
   Run next round of consensus procedure;

end

Example 3 (Continuation of Example 2). *"By referring to Table 1, the global consensus degree of the group of experts is 0.883, which is lower than the pre-determined consensus threshold (0.945). It means that the consensus level is insufficient, thus the feedback mechanism is activated. In phase 1 of the proposed influence-based feedback mechanism (Algorithm 3, Step 1 and 2) the experts with low consensus contribution are identified:*

- *Clusters having less cluster consensus degrees than 0.883 are C_3, C_6 and C_7.*

- *Thus, $e_{low} = \{e^2, e^3, e^4\}$.*

In phase 2 of the proposed influence-based feedback mechanism (Algorithm 3, Step 3 until 9), the network influencer is identified by computing the influence score or σ-centrality associated to each expert.

- *The row normalized preference similarity matrix, $\mathbb{S}_\eta$, is computed and its corresponding SSIN is presented in Figure 4 with only some of the row normalized preference similarity weights displayed for simplicity reason.*

$$\mathbb{S}_\eta = \begin{bmatrix} 0.137 & 0.119 & 0.102 & 0.129 & 0.124 & 0.127 & 0.130 & 0.131 \\ 0.123 & 0.142 & 0.116 & 0.100 & 0.124 & 0.127 & 0.135 & 0.132 \\ 0.113 & 0.124 & 0.152 & 0.092 & 0.140 & 0.127 & 0.130 & 0.122 \\ 0.143 & 0.107 & 0.093 & 0.152 & 0.121 & 0.128 & 0.127 & 0.128 \\ 0.124 & 0.120 & 0.127 & 0.109 & 0.137 & 0.129 & 0.129 & 0.125 \\ 0.127 & 0.123 & 0.115 & 0.116 & 0.129 & 0.137 & 0.133 & 0.121 \\ 0.127 & 0.128 & 0.115 & 0.112 & 0.127 & 0.130 & 0.134 & 0.127 \\ 0.132 & 0.128 & 0.110 & 0.116 & 0.125 & 0.121 & 0.130 & 0.138 \end{bmatrix}$$
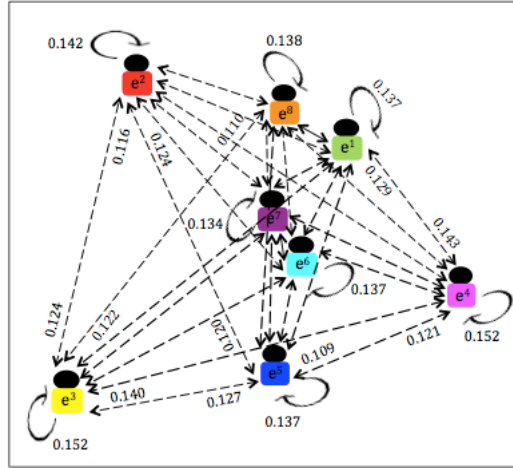

Figure 4: The generated similarity social influence network (SSIN)

- *From $\mathbb{S}_\eta$, the σ value is determined. As mentioned in Section 4.1, the convergence of the solution is assured if σ is selected subject to the constraint $\sigma < \dfrac{1}{\lambda_1}$, where λ_1 is the unique largest eigenvalue of $\mathbb{S}_\eta$. The eigenvalues of $\mathbb{S}_\eta$ are*

$$\{0.998, 0.0726, 0.0313, 0.0158, 0.0055, 0.0021, 0.0007, 0.0013\}$$

and $\lambda_1 = 0.998$. The value $\sigma = 0.9$ is selected to fulfil the above boundary constraint.

- *Whether exogenous (external) effect plays a role in the SSIN is to be established. The following two cases (refer to Table 2) are discussed:*

  - *Without exogenous effect case with Z set as the unit vector. In this case it is*

    $$Y = [0.128, 0.124, 0.117, 0.117, 0.128, 0.128, 0.130, 0.128].$$
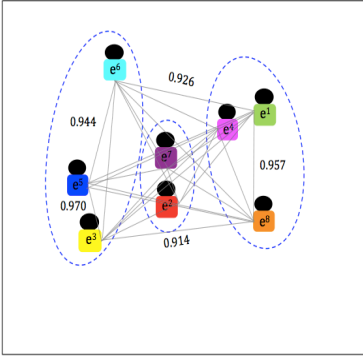
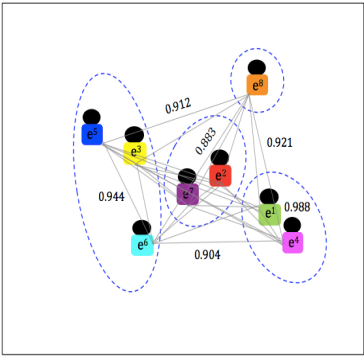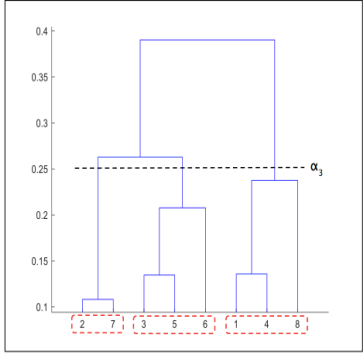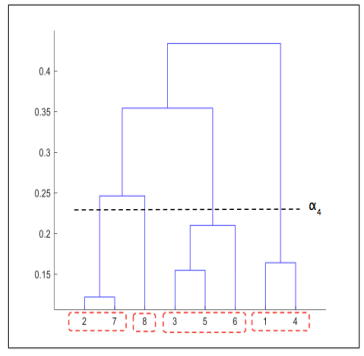 *The network influencer would be e^7.*

13

– *Exogenous effect case with following vector* $Z = [1, 0.7, 0.4, 0.8, 0.9, 0.3, 0.5, 0.1]$. *In this case it is*

$$Y = [0.137, 0.126, 0.113, 0.121, 0.135, 0.122, 0.129, 0.117].$$

The network influencer would be e^1.

Table 2 provides a complete comparative view of the results obtained after the feedback mechanism is implemented with and without exogenous effects, respectively.

Table 2: Comparative results of feedback mechanism with and without exogenous effects in σ-centrality measure

	Without Exogenous Effect	With Exogenous Effect
Network Influencer	e_7	e_1
Feedback preferences	$\widetilde{VP}^2 = (0.50, 0.58, 0.56, 0.67, 0.75,$ $0.57, 0.41, 0.78, 0.70, 0.34,$ $0.64, 0.65, 0.62, 0.69, 0.33)$	$\widehat{VP}^2 = (0.54, 0.75, 0.81, 0.83, 0.86,$ $0.71, 0.64, 0.85, 0.82, 0.56,$ $0.71, 0.69, 0.55, 0.55, 0.46)$
	$\widetilde{VP}^3 = (0.46, 0.29, 0.35, 0.45, 0.75,$ $0.34, 0.31, 0.52, 0.72, 0.49,$ $0.64, 0.79, 0.54, 0.75, 0.60)$	$\widehat{VP}^3 = (0.48, 0.39, 0.55, 0.55, 0.86,$ $0.43, 0.51, 0.53, 0.84, 0.74,$ $0.71, 0.86, 0.45, 0.62, 0.79)$
	$\widetilde{VP}^4 = (0.22, 0.38, 0.55, 0.37, 0.50,$ $0.65, 0.65, 0.69, 0.71, 0.56,$ $0.42, 0.55, 0.30, 0.42, 0.48)$	$\widehat{VP}^4 = (0.19, 0.50, 0.79, 0.45, 0.55,$ $0.82, 0.94, 0.74, 0.83, 0.82,$ $0.44, 0.57, 0.14, 0.21, 0.65)$
Preference similarity network		
Dendogram		
Revised global consensus	0.949	0.946
Optimum β-level	0.6	0.5
Optimum α-level	Level 3	Level 4

The feedback preference values in the case of exogenous effect are in general greater than in the case without exogenous effect. Obviously, the experts preference similarity degrees increase after the feedback mechanism, which is reflected in the higher revised global cluster consensus of the group of experts. In fact, the consensus threshold set in Example 2, $\varphi = 0.945$, is reached in both cases after the feedback process (in the second consensus round): 0.949 with an optimum

parameter of control $\beta = 0.6$ for the case without exogenous effect; and 0.946 with optimum parameter of control $\beta = 0.5$ for the case with exogenous effect.

Table 3 provides the computations for the optimal global consensus degrees and their corresponding optimal cluster α-levels at each β-level for the second round of consensus reaching process for both with and without exogenous effects.

Table 3: Optimal global consensus degrees and their corresponding optimal cluster solutions (α-levels) at each β-level for the second round of consensus reaching process, consisting with and without exogenous effects in σ-centrality influence measure

β-levels	Without exogenous Effect		With exogenous Effect	
	Global Consensus	Optimal α-level	Global Consensus	Optimal α-level
0	0.883	Level 7	0.833	Level 7
0.1	0.895	Level 7	0.899	Level 7
0.2	0.908	Level 7	0.914	Level 7
0.3	0.920	Level 6	0.930	Level 7
0.4	0.933	Level 7	0.939	Level 7
0.5	0.942	Level 6	0.946	Level 4
0.6	0.949	Level 3		

The above results show that in the second consensus round the experts form closer relations with each other than in the first consensus round, and consequently a lower number of clusters are obtained with the optimal clustering level decreasing from the original α_7 to α_3 and α_4 for without and with exogenous effect, respectively. These results confirm the advantage of implementing the proposed σ-centrality influence measure in (i) generating feedback preference values based on the network influencer to enable the group reaching consensus; and (ii)the the nomination of a network influencer does not necessarily rely only on the network connections constructed in SSIN but also on external evaluations from third parties that can be available or desirable to be taken into consideration."

5. Influence-driven resolution process

This section describes the two required phases of the influence-driven resolution process associated to the CGDM problem described in the paper: (1) the fusion phase; and (2) the exploitation phase.

5.1. Fusion phase

The feedback individual preferences derived from the consensus reaching process are aggregated to obtain the group consensus preference relation. Among the many aggregation operators available, the *Ordered Weighted Average* (OWA) operator, introduced by Yager [43], has become one of the well-known fusion techniques applied in decision making area. The definition of an OWA operator is presented below:

Definition 5. *"An OWA operator of dimension n is a mapping $\phi\colon \mathbb{R}^n \longrightarrow \mathbb{R}$, associated with a weighting vector, $W = (\omega_1, \ldots, \omega_n)$, such that $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$, with following expression:*

$$\phi(p_1, \ldots, p_n) = \sum_{i=1}^{n} \omega_i \cdot p_{\varrho(i)}$$

where $\varrho\colon \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ is a permutation function such that $p_{\varrho(i)} \geq p_{\varrho(i+1)}$, $\forall i = 1, \ldots, n - 1$, i.e., $p_{\varrho(i)}$ is the i-th highest value in the set $\{p_1, \ldots, p_n\}$."

The implementation of the concept of majority is desirable when deriving the final group solution to the CGDM problem. An interesting methodology for doing this was developed by Yager [44] with the *quantifier guided linguistic OWA operator*. This methodology allows to implement natural language terms such as *'most of'*, *'some of'*, *'at least one'*, *'as many as possible'* and *'all'* by means of their representation as fuzzy sets on the domain $[0,1]$ with corresponding appropriate increasing monotonic quantifier membership functions $Q\colon [0,1] \to [0,1]$ verifying $Q(0) = 0, Q(1) = 1$ reflecting in general the proportion of criteria/experts satisfied by an acceptable solution [43]. A quantifier guided linguistic OWA operator weights are computed as follows:

$$w_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right), i = 1, \ldots, n. \tag{17}$$

Yager [44] further extended the above approach to allow for different importance degrees associated to criteria or experts to be implemented. In this case, the quantifier guided linguistic OWA operator are computed using the following expression:

$$w_i = Q\left(\frac{\sum_{k=1}^{i} \mu_\varrho(k)}{Z}\right) - Q\left(\frac{\sum_{k=1}^{i-1} \mu_\varrho(k)}{Z}\right) \tag{18}$$

where $Z = \sum_{k=1}^{n} \mu_k$ is the total sum of importance and ϱ is the permutation utilized for the purpose of obtaining the ordering of the values to be fused.

Yager and Filev [29] introduced a more general type of OWA operator, the *Induced OWA* (IOWA) operator, by inducing the reordering step of the first argument variable upon the magnitude of the second variable, *the order inducing variable*, which is mathematically defined as below:

Definition 6. *"An IOWA operator of dimension n is a function $\Phi_W\colon (\mathbb{R} \times \mathbb{R})^n \longrightarrow \mathbb{R}$, to which a weighting vector is associated $W = (\omega_1, \ldots, \omega_n)$ such that $\omega_i \in [0,1]$ and $\sum_i \omega_i = 1$, with following expression:*

$$\Phi_W\left(\langle u_1, p_1\rangle, \ldots, \langle u_n, p_n\rangle\right) = \sum_{i=1}^{n} \omega_i \cdot p_{\varrho(i)}$$

being $\varrho\colon \{1, \ldots, n\} \longrightarrow \{1, \ldots, n\}$ a permutation where $p_{\varrho(i)} \geq p_{\varrho(i+1)}$, $\forall i = 1, \ldots, n-1$, i.e., $\langle u_{\varrho(i)}, p_{\varrho(i)}\rangle$ is the 2-tuple with $u_{\varrho(i)}$ the i-th highest value in the set $\{u_1, \ldots, u_n\}$."

The influence score of each expert, Y, obtained using Definition 3 is proposed in this context as the order inducing variable of the experts' preference evaluations to fuse, $\{p_{ij}^1, \ldots, p_{ij}^m\}$, which leads to the following σ-*IOWA operator*:

Definition 7. *"The σ-IOWA operator of dimension m, Φ_W^σ, is an IOWA operator with the set of influence score of experts in the network, $Y = (y^1, \ldots, y^m)$, as the order inducing variable."*

Thus, denoting by W the weighting vector calculated using Equation (18) with Q a fuzzy linguistic quantifier representing the concept of soft majority desired to implement, the *collective preference relation* derived using the σ-*IOWA* operator, Φ_W^σ, will be

$$p_{ij}^c = \Phi_W^\sigma\left(\langle y^1, p_{ij}^1\rangle, \ldots, \langle y^m, p_{ij}^m\rangle\right). \tag{19}$$

Cleary, the higher the influence score, the more influence an expert has in the network, and consequently the higher the contribution (weight value) of that expert in the fusion process. Indirectly, the implication of less influential experts can be mitigated. This can be achieved by implementing the concept of fuzzy majority via an increasing concave linguistic quantifier Q [45, 46]. Yager [44] proposed the following parameterized family of RIM quantifiers $Q(r) = r^a$, $a \geq 0$ to model the majority concept *'most of'*, which is concave when $a \in [0, 1]$. For illustrative purpose, the value $a = 1/2$ will be used, and the collective preferences represent the degree of "preference of one alternative over another for '*most of*' the influential experts" in the network.

5.2. Exploitation Phase

The exploitation phase of the CGDM process aims to rank the alternatives so that the best one can be identified. The OWA *Quantifier Guided Dominance Degree* (QGDD) [30] is proposed to be applied, which is formally defined as:

Definition 8. *"Let $P^c = \left(p_{ij}^c\right)$ be the collective preference relation of the set of alternatives $\mathbb{A} = \{A_1, A_2, \ldots, A_n\}$. The quantifier guided dominance degree that alternative A_i has over the others, in a fuzzy majority sense, is expressed as below:*

$$QGDD(A_i) = \Phi_Q\left(p_{ij}^c, j = 1, \ldots, n, j \neq i\right) \tag{20}$$

being Φ_Q an OWA operator guided by the linguistic quantifier Q representing the fuzzy majority concept."

The set of alternatives with maximum QGDD will be the solution to the influence-based CGDM problem;

$$\mathbb{A}^{QGDD} = \{A \mid A \in \mathbb{A}, \ QGDD(A) = sup_{A \in \mathbb{A}} QGDD(A)\}. \tag{21}$$

Algorithm 5 shows the consecutive steps of the proposed influence-driven resolution process.

Algorithm 5: Influence-driven Resolution Process

begin

1   Rank $Y = (y^1, \ldots, y^m)$ in descending order, $R(Y)$;

2   Find IOWA weighting vector (Equation (18));

3   Construct the collective preference relation, $P^c = \left(p_{ij}^c\right)$ (Equation (19)) using σ-IOWA operator (Definition 7);

4   Compute $QGDD(A_i)$ (Definition 8);

5   Rank the alternatives and choose the best one (Equation (21)).

end

Example 4 (Continuation of Example 3)**.** *"Table 4 presents the results obtained after the influence-driven resolution process is implemented without and with exogenous effects. The values obtained for the influence scores (in normalized form), IOWA weighting vectors, collective preferences, maximal dominance degrees and final ranking of alternatives are included. The values obtained without exogenous effect are slightly different to the ones obtained with exogenous effect, which indicated that the exogenous effect absolutely affects the resolution procedure final ranking of alternatives."*

Figure 5 provides the general research flow of the proposed influence-driven feedback system for preference similarity network clustering based consensus group decision making model.

Table 4: Exogenous effects on influence based resolution process

|	Without exogenous effect	With exogenous effect
Influence score	$Y = (0.128, 0.124, 0.117, 0.117, 0.128, 0.128, 0.130, 0.128)$	$Y = (0.137, 0.126, 0.113, 0.121, 0.135, 0.122, 0.129, 0.117)$
IOWA Weighting Vector	$W = (0.361, 0.147, 0.113, 0.096, 0.084, 0.074, 0.065, 0.060)$	$W = (0.370, 0.151, 0.112, 0.093, 0.080, 0.072, 0.064, 0.058)$
Collective Preferences	$\begin{bmatrix} 1 & 0.379 & 0.396 & 0.534 & 0.573 & 0.723 \\ 0.582 & 1 & 0.497 & 0.586 & 0.696 & 0.740 \\ 0.574 & 0.510 & 1 & 0.555 & 0.623 & 0.710 \\ 0.461 & 0.394 & 0.427 & 1 & 0.443 & 0.577 \\ 0.430 & 0.371 & 0.406 & 0.556 & 1 & 0.483 \\ 0.346 & 0.343 & 0.356 & 0.474 & 0.502 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0.371 & 0.495 & 0.710 & 0.662 & 0.787 \\ 0.589 & 1 & 0.605 & 0.764 & 0.744 & 0.821 \\ 0.539 & 0.472 & 1 & 0.724 & 0.655 & 0.741 \\ 0.395 & 0.326 & 0.362 & 1 & 0.355 & 0.458 \\ 0.398 & 0.359 & 0.400 & 0.597 & 1 & 0.593 \\ 0.329 & 0.325 & 0.346 & 0.523 & 0.455 & 1 \end{bmatrix}$
Maximal Dominance Degrees	$QGDD = (0.593, 0.665, 0.635, 0.499, 0.487, 0.441)$	$QGDD = (0.676, 0.749, 0.673, 0.406, 0.519, 0.441)$
Ranking of Alternatives	$A_2 \succ A_3 \succ A_1 \succ A_4 \succ A_5 \succ A_6$	$A_2 \succ A_1 \succ A_3 \succ A_5 \succ A_6 \succ A_4$
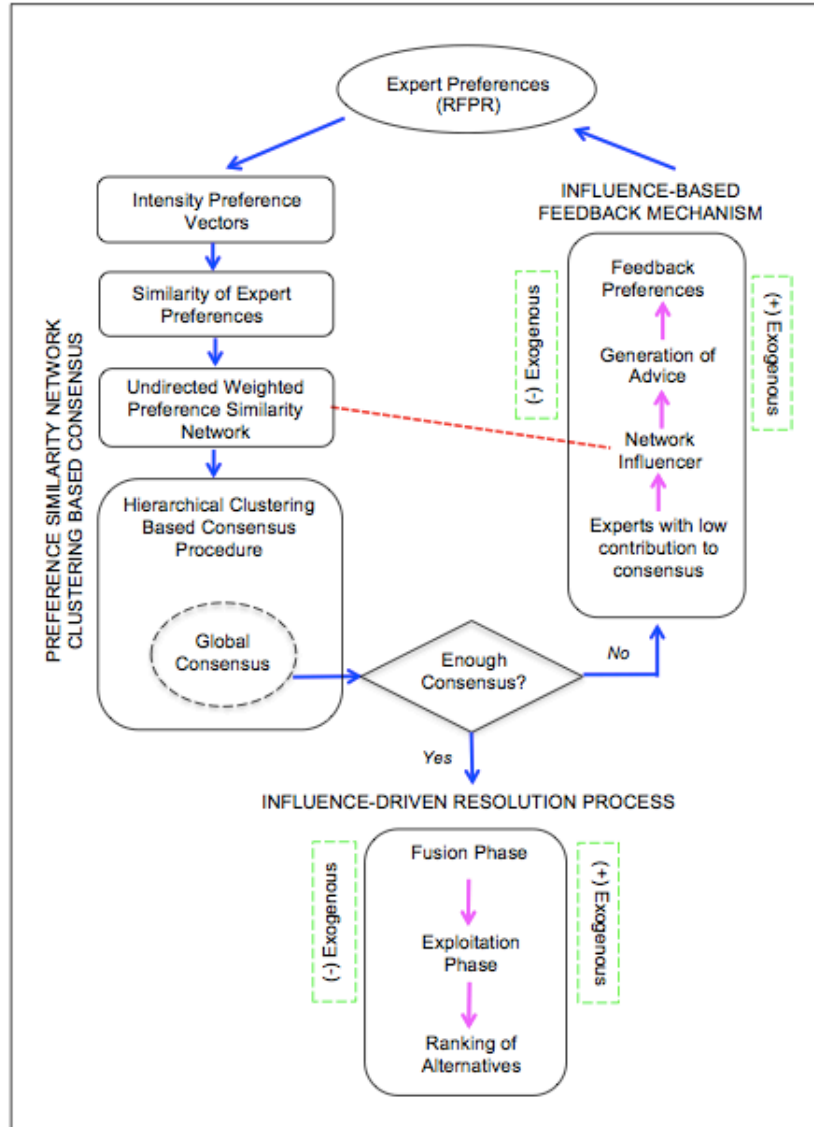
Figure 5: A research flow of the proposed influence-driven feedback system for preference similarity network clustering based consensus group decision making

6. Conclusion

The incorporation of Social Influence Network (SIN) in consensus group decision making (CGDM) models provide new avenues to researchers within this area of study with respect to the introduction of new advanced research decision-making frameworks. We proposed an influence-driven feedback system to be implemented in a preference similarity network clustering based group decision making procedure. Inspired by the introduction of the alpha-centrality eigenvector-like measure by Bonancich and Lloyd [39], we proposed to use this measure in a group decision making context by building an SIN adjacency matrix, based on the preference similarity matrix obtained from the experts' initial evaluations, with the aim of providing feedback and generating advices to experts in a group to support the reaching of consensus. The proposed approach is built on a new influence score measure, called σ-*centrality*, and the convergence of the solution is confirmed when there exists a unique highest eigenvalue in the set of eigenvalues from the row normalized preference similarity matrix. This approach ensures in the first place that valuable recommendations are coming from expert with similar preferences. In addition, both endogenous and exogenous factors are comprised in order to express experts' importance weights based on the internal SSIN connections and third parties evaluations, in such a way that a *fair* nomination of the network influencer is achieved. This influence-driven feedback mechanism positively contributes towards the achievement of sufficient consensus state, with experts with low contribution to consensus being successfully moved closer to each other, following the recommendations that are built from the network influencer's preferences. The feedback process is designed based on the satisfying of two important conditions to ensure that (1) the revised consensus degree is above the consensus threshold and that (2) the clustering solution is improved. Summarizing, the proposed model is an effective CGDM model that considers important elements that are usually overlooked by other existing frameworks.

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