RAIRO - Operations Research

Benchmarking with Network DEA in a Fuzzy Environment

--Manuscript Draft--

Manuscript Number:	ro160205R2
Article Type:	Research article
Full Title:	Benchmarking with Network DEA in a Fuzzy Environment
Short Title:	Benchmarking with Network DEA in a Fuzzy Environment
Section/Category:	Special issue - Fuzzy Data Envelopment Analysis: Recent Developments and Applications
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Manuscript Region of Origin:	UNITED KINGDOM
Keywords:	Internal structure; DEA; Fuzzy sets; Scale and relative efficiency measure
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Author Comments:	

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ro160205R1
Second Revision

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All additions and changes to the original manuscript are highlighted in this revision.

Abstract

Benchmarking is a powerful and thriving tool to enhance the performance and profitabilities of organizations in business engineering. Though performance benchmarking has practically and theoretically developed in distinct fields such as banking, education, health and so on, supply chain benchmarking across multiple echelons that includes certain characteristics such as intermediate measure differs from other fields. In spite of incremental benchmarking activities in practice, there is the dearth of a unified and effective guideline for benchmarking in organizations. Amongst the benchmarking tools, data envelopment analysis (DEA) as a non-parametric technique has been widely used to measure the relative efficiency of firms. However, the conventional DEA models that are bearing out precise input and output data turn out to be incapable of dealing with uncertainty, particularly when the gathered data encompasses natural language expressions and human judgements. In this paper, we present an imprecise network benchmarking for the purpose of reflecting the human judgments with the fuzzy values rather than precise numbers. In doing so, we propose the fuzzy network DEA models to compute the overall system scale and technical efficiency of those organizations whose internal structure is known. A classification scheme is presented based upon their fuzzy efficiencies with the aim of classifying the organizations. We finally provide a case study of the airport and travel sector to elucidate the details of the proposed method in this study.

Keywords: Internal structure; DEA; Fuzzy sets; Scale and relative efficiency measure

1. Introduction

The literature on supply chain management (SCM) showcases the fact that many supply chains fail thanks to poor and inappropriate tools for benchmarking their performance. The supply chain failures can be prevented by the use of integrated and adequate benchmarking approaches in which the performance of several supply chain networks are assessed simultaneously to determine the best practices.

A large volume of research over the past three decades has substantiated that data envelopment analysis (DEA) is a very powerful benchmarking methodology for identifying the relative efficiency of homogeneous decision making units (DMUs). DEA models such as CCR and BCC models exploits the set of efficient observations in input-output space to construct an empirical production frontier (i.e., efficient frontier) and, in turn, obtain efficiencies relative to this frontier (Charnes et al., 1978; Banker et al., 1984). In fact, a production possibility set (PPS) is estimated as the set of all feasible input–output combinations along with satisfying certain axioms. A DMU is said to be relative efficient if one cannot find a point in the PPS that produces more output without a consequent relative increase in inputs, or that consumes less inputs while keeping the outputs unchanged. Contrarily, the DMU is said to be relative inefficient if the amounts of the current inputs can be reduced with the same amounts of outputs or the amounts of the current outputs can be augmented without changing the amounts of inputs.

An evaluated DMU is traditionally examined as a black box that transforms initial inputs consumed into final outputs produced without focusing on the internal structure and mathematical transformation function. However, a production system usually includes the internal operations in which the inputs go through several processes to produce a number of intermediate measures and outputs. The negligence of the network structure in benchmarking for both manufacturing and service sectors often results in a truly misleading analysis.

As reported in the literature throughout, the theoretical development and applications aspect of DEA are fully grown, particularly for precise situations (Cook and Seiford, 2009; Emrouznejad et al., 2008).

There is a certain stream of research in the DEA literature that takes account of the operations of processes and it has been called *network DEA* (Färe and Grosskopf, 2000). The fundamental idea is to think of the production technology of individual processes into the production technology when estimating system efficiency. Many network DEA models have been developed for treating internal network structures (e.g., Agrell and Hatami-Marbini, 2013; Chen et al., 2013; Kao, 2014). The aim of our literature review underneath is not to review all the existing network DEA models but it is need here to point out that many of the developed models require substantial modifications and only suit for a certain network structure.

According to Chen et al. (2 013), the existing network DEA approaches include two groups based on the conventional DEA models. One group entails the multiplier-based network DEA models which measure the overall network efficiency by combining the ratio efficiency of each division in the network using geometric or arithmetic averages. The other group embraces the DEA envelopment models by defining the PPS for each division through the network structure.

Castelli et al. (2010) and Castelli and Pesenti (2014) reviewed the DEA models that have been developed for evaluating the DMUs with known internal structures in which three main categories of models involving (*i*) shared flow models, (*ii*) multilevel models, and (*iii*) network models are introduced with the aim of stating the commonalities and discrepancies between these models.

The shared flow models require to be deployed in situations where DMUs have the network processes with shared input resources either allocated to various processes of operations or considered as a decision variable to maximize the DMU efficiencies as a whole (see e.g., Beasley, 1995; Golany et al. 2006; Wu et al. 2015; Ding et al. 2015). The multilevel models embrace DMUs with independent divisions when additional inputs/outputs are not connected to any of its divisions (see e.g., Cook et al. 1998; Azadeh et al. 2008). The network models are composed of intermediate measures among the divisions. Put differently, the divisions in the network models are interdependent and intermediate measures produced by the preceded division may be consumed as an input by other divisions (see e.g., Prieto and Zofio, 2007; Kao and Hwang, 2008; Chen et al., 2010; Fukuyama and Weber, 2010; Herrera-Restrepo et al., 2016; Despotis et al., 2016). The network DEA models have been initially proposed by Färe and Whittaker (1995) and Färe and Grosskopf (1996) based on the two-stage process and later generalized to multiple processes by Färe and Grosskopf (2000).

Cook et al. (2010) and Agrell and Hatami-Marbini (2013) provided an overview of DEA models for fielding two-stage network structures. Agrell and Hatami-Marbini (2013) zeroed in on performance analysis in SCM, particularly the methodological studies made by way of two-stage models and the related state-of-the-art was categorized into three groups; (*i*) two-stage process

DEA models, (*ii*) game theory DEA models, and (*iii*) bi-level programming. The two-stage models are the special case of multi-stage framework where each DMU is composed of two divisions (see e.g., Chen et al., 2009; Wang and Chin, 2010; Kao and Hwang, 2008; Despotis et al., 2016). The game theory DEA models use the concept of non-cooperative and cooperative games in game theory to treat the network structure of operations (see e.g., Liang et al. 2006; Zha et al. 2010, Du et al. 2011). The final group defined by Agrell and Hatami-Marbini (2013) includes those methods which have been developed based on bi-level programming aiming to evaluate the performance of a two-stage process in decentralized decisions (Wu, 2010).

Recently, Kao (2014) presented a review on network DEA models and introduced two different classifications. One classification has nine categories of models based on efficiency measurement, distance measure and output-input ratio as follows: independent, system distance measure, process distance measure, factor distance measure, slacks-based measure, ratio-form system efficiency, ratio-form process efficiency, game theoretic, and value-based, and the other classification bears on network structures as follows: two-stage, general two-stage, series, parallel, mixed, hierarchical, and dynamic.

Setting aside the internal structure of DMUs, uncertain data in DEA can be classified into *incomplete precise* data and *imprecise* data. The former utilizes probability methods, and the latter utilizes fuzzy set theory to give verbal statements without missing their imprecise characteristics. The majority of management decisions in real-world practice are made in terms of expert's intuitive judgement and are expressed linguistically (e.g., “low delay” and “big delay”). Therefore, it is essential to consider the expert's judgement in the decision-making process by means of linguistic expressions. The values of linguistic variables are not numbers but are words, phrases, or sentences and the theory of fuzzy sets has been developed in the area of decision sciences to qutitatively deal with the linguistic variables in a rational manner (c.f. Bellman and Zadeh, 1970; Zadeh, 1978).

While real-world problems contain qualitative, incomplete, subjective and judgment information, conventional black-box and network DEA models only require crisp data. For instance, separate and incompatible information systems gathering production data in distinct segments of production process may lead to “noise” or measurement errors in the dataset. Given that the DEA approach is sensitive to data fluctuations, the correct consideration of such uncertain information is vital for evaluating accurately the performance of DMUs and, in turn, making appropriate decisions.

To tackle *incomplete precise* and *imprecise* data in DEA, three major approaches including "fuzzy DEA" (see e.g., Sengupta 1992; Hatami-Marbini et al. 2017a, 2017b, 2017c), "interval DEA" (see e.g., Cooper et al., 1999; Toloo et al., 2008, Toloo, 2014; Hatami-Marbini et al. 2014) and "stochastic DEA" (see e.g., Land et al. 1993, Olesen and Petersen 1995, 2016) have dominated the literature. This paper places emphasis on fuzzy DEA approach to conquer the uncertainty in the performance evaluation process.

As per two recent surveys conducted by Hatami-Marbini et al. (2011a) and Emrouznejad et al. (2014), the DEA literature includes multiple approaches for solving fuzzy DEA models, which can be categorised into six groups: the tolerance approach (see e.g., Sengupta 1992), the α-level based approach (see e.g., Saati et al. 2002; Hatami-Marbini et al., 2010; 2011c; 2013; Saati et al. 2013), the fuzzy ranking approach (see e.g., Emrouznejad et al., 2011; Hatami-Marbini et al., 2011b), the possibility approach (see e.g., Lertworasirikul et al., 2003), the fuzzy arithmetic (see e.g., Wang et al., 2009; Hatami-Marbini et al., 2015), and the fuzzy random/type-2 fuzzy sets (see e.g., Tavana et al., 2012, 2014).

Although the above-mentioned discussions show the recent increased interest in the network DEA approach, there exist only few studies examining notion of fuzziness to handle the subjective data. Kao and Liu (2011) and Kao and Lin (2012) developed the fuzzy version of relational two-stage model of Kao and Hwang (2008) and parallel processes of Kao (2009, 2012) to obtain the fuzzy efficiency using a pair of two-level mathematical programs introduced by Kao and Liu (2000). Based upon Kao and Liu (2011) and Kao and Lin (2012), Lozano (2014a, 2014b) proposed the alternative methods for estimating the fuzzy efficiencies of the different processes.

In this paper, we propose a fuzzy network benchmarking model that enables us to treat a general network structure such as supply chain network with multiple stages and multiple levels where the observations are represented by fuzzy numbers. The intermediate measures render the proposed model relational and interdependent. The proposed fuzzy network DEA models in this research are concentrated on fuzzy arithmetic to evaluate the overall system scale and technical efficiency of the firms whose internal structure is known. Besides, we introduce a classification scheme based on overall system scale and technical efficiency to classify the firms. We also present a case study of the airport and travel sector to interpret the application and detailed results of the proposed method.

The rest of this paper is organized as follows. The next section presents the deterministic network relational DEA model developed by Lozano (2011). Section 3 extends the deterministic case to a fuzzy environment using the standard fuzzy arithmetic to conquer fuzziness in observations. Section 4 presents a case study on airport operations to illustrate the way of modelling and benchmarking airport operations as a network system under a fuzzy environment. The paper is concluded in Section 5.

2. Relational network DEA model

Suppose that there is a set of *n* DMUs (supply chains) to be evaluated, each of which encompasses *p* processes denoted by *P*=1,…,*p* where $I(p)$ and $O(p)$ stand for the set of inputs and outputs of the p^{th} process, respectively. Let us the p^{th} process consumes x_{ij}^{p}, i Î $I(p)$, *j*=1,…,*n* to produce y_{kj}^{p}, k Î $O(p)$, *j*=1,…,*n* along with assuming that the total amount of the i^{th} input and k^{th} output of all processes associated with DMU$_j$ (*j*=1,…,*n*) are $x_{ij} = \sum_{p\in P_I(i)} x_{ij}^{p}$ and $y_{kj} = \sum_{p\in P_O(k)} y_{kj}^{p}$ where $P_I(i)$ and $P_O(k)$ are the sets of processes that correspond to input *i* and output *k*. Consider *L* links or intermediate measures between the processes denoted by z_{lj}^{p}, *l*=1,…,*L*, *j*=1,…,*n* that are divided into two different inward and outward sets including $Int^{in}(p)$ and $Int^{out}(p)$ within the network structure, in which the total amount of the intermediate measures of the p^{th} process associated with DMU$_j$ is $\sum_{p\in Int^{in}(l)} z_{lj}^{p}$, *l*=1,…,*L*, *j*=1,…,*n* and $\sum_{p\in Int^{out}(l)} \underline{z}_{lj}^{p}$, *l*=1,…,*L*, *j*=1,…,*n*. We also suppose that $\sum_{p\in Int^{in}(l)} z_{lj}^{p}$ =$\sum_{p\in Int^{out}(l)} \underline{z}_{lj}^{p}$, *l*=1,…,*L*, *j*=1,…,*n* (Lozano, 2011).

The idea of benchmarking used in network production systems (e.g., supply chain) is to estimate a universal underlying technology for comparing the production systems. In what follows, the technology or production possibility set (PPS) is first defined based on the observed data, and then the observed production of a network production system is evaluated relative to the estimated PPS.

$$T_s = \left\{ \left(x^p, y^p, z^p, \underline{z}^p\right) \in \mathbb{R}_+^{I(p)} \times \mathbb{R}_+^{O(p)} \times \mathbb{R}_+^{L} \,|\, y^p \text{ can be produced by } x^p, z^p \text{ and } \underline{z}^p \right\}$$

The PPS of the network production system, denoted by T_s, is the combination of the PPS of all processes, denoted by T_p. Let us initially represent T_p as follows:

$$T_p = \Big\{ \left(x^p, y^p, z^p, \underline{z}^p\right) \in \mathbb{R}_+^{I(p)} \times \mathbb{R}_+^{O(p)} \times \mathbb{R}_+^{L} \,|\, \exists \lambda_j^p \in \phi^p(\zeta) \colon \sum_j \lambda_j^p x_{ij}^p \le x_i^p, \forall i \in$$

$$I(p), \sum_j \lambda_j^p y_{kj}^p \ge y_k^p, \forall k \in O(p),\ \sum_j \lambda_j^p z_{lj}^p \le z_l^p, \forall l \in Int^{in}(p), \sum_j \lambda_j^p z_{lj}^p \ge z_l^p, \forall l \in Int^{out}(p), \Big\}$$

where the T_p set satisfies the minimal extrapolation technologies and the following axioms:

A1.Envelopment: $\left(x_{ij}^p, y_{kj}^p, z_{lj}^p, \underline{z}_{lj}^p\right) \in T_p,\ \forall j$

A2. Free disposability:

- Free disposability of inputs: $\left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p,\ \bar{\bar{x}}^p \geq x^p \Longrightarrow \left(\bar{\bar{x}}^p, y^p, z^p, \underline{z}^p\right) \in T_p$
- Free disposability of outputs: $\left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p,\ \bar{\bar{y}}^p \leq y^p \Longrightarrow \left(x^p, \bar{\bar{y}}^p, z^p, \underline{z}^p\right) \in T_p$
- Free disposability of intermediate measures: $\left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p,\ \bar{\bar{z}}^p \geq z^p \ \ for\ all\ \bar{\bar{z}}^p \in p^{in}(r), \underline{\bar{\bar{z}}}^p \leq \underline{z}^p\ for\ all\ \bar{\bar{z}}^p \in p^{out}(r) \Longrightarrow \left(x^p, y^p, \bar{\bar{z}}^p, \underline{\bar{\bar{z}}}^p\right) \in T_p.$

A3. Convexity: the set T_p is convex if for any two points $\left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p$, $\left(\bar{\bar{x}}^p, \bar{\bar{y}}^p, \bar{\bar{z}}^p, \underline{\bar{\bar{z}}}^p\right) \in T_p$ and any arbitrary weight $0 \leq \lambda \leq 1$, $(1-\lambda)\left(x^p, y^p, z^p, \underline{z}^p\right) + \lambda\left(\bar{\bar{x}}^p, \bar{\bar{y}}^p, \bar{\bar{z}}^p, \underline{\bar{\bar{z}}}^p\right)$ also belongs to T_p.

A4. ζ-returns to scale: $\left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p \Longrightarrow \kappa(x, y) \in T_p,\ \forall \kappa \in \phi^p(\zeta)$ where the $\phi^p(\zeta)$ set identifies the shape of the frontier under the condition of the returns to scale (RTS). In particular, $\phi^p(crs) = \{\lambda_j^p \in \mathbb{R}^+ | \lambda_j^p\ free\}$ and $\phi^p(vrs) = \{\lambda_j^p \in \mathbb{R}^+ | \sum_j \lambda_j^p = 1\}$.

At present, we can define the following PPS for the network production system, T_s, which satisfies the above-mentioned axioms:

$$T_s = \left\{(x_i, y_k) |\ \exists \left(x^p, y^p, z^p, \underline{z}^p\right) \in T_p : \sum_{p \in P_I(i)} x_{ij}^p \leq x_i\ , \forall i, \sum_{p \in P_O(k)} y_{kj}^p \geq y_k, \forall k,\ \sum_{p \in Int^{out}(l)} z_{lj}^p - \sum_{p \in Int^{in}(l)} z_{lj}^p \geq 0\ , \forall l\right\}.$$

The Farrell (1957) input efficiency measure is applied to determine the [input-oriented] technical efficiency of DMU_0 as defined below:

$\theta_0 = \min\{\theta_0 > 0 | (\theta_0 x, y) \in T_s\}$

According to the input efficiency measure, a network production system is classified as *efficient* if $\theta_0 = 1$ and as *inefficient* if $\theta_0 < 1$. Given T_s, the efficiency measure can be calculated for a DMU_0 under evaluation by solving the following linear programming (LP) problem:

$$\begin{aligned}
&\min \theta_0 - \varepsilon\left(\sum_i s_i^- + \sum_k s_k^+ + \sum_l s_l^\#\right) \\
&st.\ \ \sum_{p \in P_I(i)} \sum_j \lambda_j^p x_{ij}^p + s_i^- = \theta_0 x_{i0},\ \ \forall i, \\
&\quad \sum_{p \in P_O(k)} \sum_j \lambda_j^p y_{kj}^p - s_k^+ = y_{k0},\ \ \forall k, \\
&\quad \sum_{p \in Int^{in}(l)} \sum_j \lambda_j^p z_{lj}^p - \sum_{p \in Int^{out}(l)} \sum_j \lambda_j^p \underline{z}_{lj}^p - s_l^\# = 0,\ \ \forall l, \\
&\quad \sum_j \lambda_j^p \in \phi(\zeta),\ \ \forall p,
\end{aligned} \tag{1}$$

$\lambda_j^p \geq 0, \quad \forall j, p,$

$s_i^-, s_k^+, s_l^\# \geq 0, \forall i, k, l.$

where ε is a very small positive number and s_i^-, s_k^+ and $s_l^\#$ are the slack variables indicating input excesses, output shortfalls and intermediate shortfalls, respectively. We note that $\phi(\zeta)$ identifies the shape of the frontier under the condition of RTS. In this study, we concentrate on constant and variable RTS models by utilizing $\lambda_j^p \in \mathbb{R}^+$ and $\sum_j \lambda_j^p = 1$ constraints in lieu of $\sum_j \lambda_j^p \in \phi(\zeta)$ for each p, which these two distinct models are respectively called the CRS and VRS network DEA models, respectively. If an optimal solution θ_0^* of the above LP model satisfies $\theta_0^* = 1$, then DMU_0 is called *efficient*. It is also referred to as "*radial efficiency*". If a value of θ_0^* is less than 1 DMU_0 is called *inefficient* and $(1 - \theta_0^*)$ bespeaks the maximal proportionate reduction of inputs allowed by the PPS, and any more reductions are also associated with nonzero slacks.

The notion of scale efficiency (SE) can be also taken into account in the network structure to measure the depletion from not operating at the optimal scale size. Given the input efficiency of DMU_0 in the CRS and VRS models, we calculate its network SE using the following ratio; $SE_0=\theta_0^*(CRS)/\theta_0^*(VRS)$. The SE_0 measure varies within [0, 1] and it is equal to 1 when DMU_0 is operating at optimal scale size, i.e., the VRS and CRS technologies coincide. When a value of SE_0 is smaller than one, it deduces that the system is not scale efficient.

3. Fuzzy network DEA model

Suppose that we look into the performance evaluation of a network production system where some observations are imprecisely measured, and these imprecise data can be characterized by fuzzy numbers. We note that a fuzzy number is a normal and convex fuzzy subset characterized by a given membership with a grade of between 0 and 1. The functional form of the membership function hinges on a priori information that interprets how each fuzzy variable conceptualizes during a production period. Generally, a trapezoidal fuzzy number, denoted as $\tilde{a} = (a^{(1)}, a^{(2)}, a^{(3)}, a^{(4)})$, is the most widely used fuzzy numbers in practical and theoretical studies with the following membership function (Zimmermann, 1996):

$$\mu_{\tilde{a}} = \begin{cases} \dfrac{x - a^{(1)}}{a^{(2)} - a^{(1)}}, & a^{(1)} \le x \le a^{(2)} \\ 1, & a^{(2)} \le x \le a^{(3)} \\ \dfrac{a^{(4)} - x}{a^{(4)} - a^{(3)}}, & a^{(3)} \le x \le a^{(4)} \\ 0, & Otherwise \end{cases} \tag{2}$$

If $a^{(3)} = a^{(4)}$, then $\tilde{a}$ is called a triangular fuzzy number. We note that a non-fuzzy number a is a special case of the fuzzy number in which $a^{(1)} = a^{(2)} = a^{(3)} = a^{(4)}$.

Let us consider n network production systems (DMUs) to be evaluated with the identical notations as those presented in the preceding section. We assume that for a given process of DMU_j the corresponding observation $\left(x_{ij}^{p}, y_{kj}^{p}, z_{lj}^{p}, \underline{z}_{lj}^{p}\right) \forall i, k, l$ is uncertain and characterized by the trapezoidal fuzzy number $\tilde{x}_{ij}^{p} = \left(x_{ij}^{p(1)}, x_{ij}^{p(2)}, x_{ij}^{p(3)}, x_{ij}^{p(4)}\right) \forall i$, $\tilde{y}_{kj}^{p} = \left(y_{kj}^{p(1)}, y_{kj}^{p(2)}, y_{kj}^{p(3)}, y_{kj}^{p(4)}\right) \forall k$, $\tilde{z}_{lj}^{p} = \left(z_{lj}^{p(1)}, z_{lj}^{p(2)}, z_{lj}^{p(3)}, z_{lj}^{p(4)}\right) \forall l$ and $\tilde{\underline{z}}_{lj}^{p} = \left(\underline{z}_{lj}^{p(1)}, \underline{z}_{lj}^{p(2)}, \underline{z}_{lj}^{p(3)}, \underline{z}_{lj}^{p(4)}\right) \forall l$ where the values of $x_{ij}^{p(1)}$, $y_{kj}^{p(1)}$, $z_{lj}^{p(1)}$ and $\underline{z}_{lj}^{p(1)}$ are positive. In the presence of the fuzzy data, the network DEA model (1) can be re-formulated by the following fuzzy LP model to obtain the fuzzy efficiency measure of DMU_0:

$$\begin{aligned} &\min \tilde{\theta}_0 - \varepsilon\left(\textstyle\sum_i s_i^- + \sum_k s_k^+ + \sum_l s_l^\#\right) \\ &st. \ \textstyle\sum_{p \in P_I(i)} \sum_j \lambda_j^p \tilde{x}_{ij}^p + s_i^- = \theta_0 \tilde{x}_{i0}, \ \ \forall i, \\ &\quad \textstyle\sum_{p \in P_O(k)} \sum_j \lambda_j^p \tilde{y}_{kj}^p - s_k^+ = \tilde{y}_{k0}, \ \ \forall k, \\ &\quad \textstyle\sum_{p \in P^{in}(l)} \sum_j \lambda_j^p \tilde{z}_{lj}^p - \sum_{p \in P^{out}(l)} \sum_j \lambda_j^p \tilde{\underline{z}}_{lj}^p - s_l^\# = 0, \ \ \forall l, \\ &\quad \textstyle\sum_j \lambda_j^p \in \phi(\zeta), \ \forall p, \\ &\quad \lambda_j^p \ge 0, \forall j, p, \\ &\quad s_i^-, s_r^+, s_l^\# \ge 0, \forall i, k, l. \end{aligned} \tag{3}$$

where $\tilde{x}_{i0} = \sum_{p \in P_I(i)} \tilde{x}_{i0}^p = \left(\sum_{p \in P_I(i)} x_{i0}^{p(1)}, \sum_{p \in P_I(i)} x_{i0}^{p(2)}, \sum_{p \in P_I(i)} x_{i0}^{p(3)}, \sum_{p \in P_I(i)} x_{i0}^{p(4)}\right) = \left(x_{i0}^{p(1)}, x_{i0}^{p(2)}, x_{i0}^{p(3)}, x_{i0}^{p(4)}\right)$ and $\tilde{y}_{k0} = \sum_{p \in P_O(k)} \tilde{y}_{k0}^p = \left(\sum_{p \in P_O(k)} y_{k0}^{p(1)}, \sum_{p \in P_O(k)} y_{k0}^{p(2)}, \sum_{p \in P_O(k)} y_{k0}^{p(3)}, \sum_{p \in P_O(k)} y_{k0}^{p(4)}\right) = \left(y_{k0}^{p(1)}, y_{k0}^{p(2)}, y_{k0}^{p(3)}, y_{k0}^{p(4)}\right)$.

The substitution of the trapezoidal fuzzy numbers into model (3) leads to the following model:

$$
\begin{aligned}
&\min\ (\theta_0^{(1)}, \theta_0^{(2)}, \theta_0^{(3)}, \theta_0^{(4)}) - \varepsilon\left(\textstyle\sum_i s_i^- + \sum_k s_k^+ + \sum_l s_l^\#\right)\\
&st.\ \textstyle\sum_{p\in P_I(i)}\sum_j \lambda_j^p \left(x_{ij}^{p(1)}, x_{ij}^{p(2)}, x_{ij}^{p(3)}, x_{ij}^{p(4)}\right) + s_i^- = \theta_0 \left(x_{i0}^{p(1)}, x_{i0}^{p(2)}, x_{i0}^{p(3)}, x_{i0}^{p(4)}\right),\quad \forall i,\\
&\quad \textstyle\sum_{p\in P_O(k)}\sum_j \lambda_j^p (y_{kj}^{p(1)}, y_{kj}^{p(2)}, y_{kj}^{p(3)}, y_{kj}^{p(4)}) - s_k^+ = \left(y_{k0}^{p(1)}, y_{k0}^{p(2)}, y_{k0}^{p(3)}, y_{k0}^{p(4)}\right),\quad \forall k,\\
&\quad \textstyle\sum_{p\in P^{in}(l)}\sum_j \lambda_j^p (z_{lj}^{p(1)}, z_{lj}^{p(2)}, z_{lj}^{p(3)}, z_{lj}^{p(4)}) - \sum_{p\in P^{out}(l)}\sum_j \lambda_j^p \left(z_{lj}^{p(1)}, z_{lj}^{p(2)}, z_{lj}^{p(3)}, z_{lj}^{p(4)}\right) -\\
&\quad -s_l^\# = 0,\ \forall l,\\
&\quad \textstyle\sum_j \lambda_j^p \in \phi(\zeta),\ \forall p,\\
&\quad \lambda_j^p \geq 0, \forall j, p,\\
&\quad s_i^-, s_k^+, s_l^\# \geq 0, \forall i, k, l.
\end{aligned}
\tag{4}
$$

To compute the efficiency measure of the network production system under evaluation denoted by subscript "0" (DMU_0), we require to solve the fuzzy network DEA model (4) subject to the complexity stemming from the notion of fuzziness. As stated earlier, the existing fuzzy DEA literature includes several distinct categories. For the purpose of preserving the specifications of conventional DEA models along with treating the computational burden of existing fuzzy DEA models, the fuzzy arithmetic group might be the most suitable approach to measure the relative efficiency of the DMUs with consideration of the internal complexity of the production process. According to the standard fuzzy arithmetic operations, model (4) can be rewritten by the four DEA models to determine the optimal value of $\theta_0^{(1)}, \theta_0^{(2)}, \theta_0^{(3)}$ and $\theta_0^{(4)}$ individually which is allowed to establish the best fuzzy relative efficiency of DMU_0. We take account of a fixed and unified production frontier for all the DMUs to attain an unbiased and consistent evaluation when calculating $\theta_0^{(1)}, \theta_0^{(2)}, \theta_0^{(3)}$ and $\theta_0^{(4)}$. In this respect, the best production activities of the *n* DMUs came from the uppermost bound of outputs and lowest bound of inputs are equipped with a unified production frontier, which is used in the following four DEA models:

<u>Network DEA model for calculating $\theta_0^{(1)}$</u>

$$
\begin{aligned}
&\min\ \theta_0^{(1)} - \varepsilon\left(\textstyle\sum_i s_i^- + \sum_r s_r^+ + \sum_l s_l^\#\right)\\
&st.\ \textstyle\sum_{p\in P_I(i)}\sum_j \lambda_j^p x_{ij}^{p(1)} + s_i^- = \theta_0^{(1)} x_{i0}^{p(4)},\ \forall i,\\
&\quad \textstyle\sum_{p\in P_O(k)}\sum_j \lambda_j^p y_{kj}^{p(4)} - s_r^+ = y_{k0}^{p(1)},\quad \forall k,\\
&\textstyle\sum_{p\in P^{in}(l)}\sum_j \lambda_j^p z_{lj}^{p(1)} - \sum_{p\in P^{out}(l)}\sum_j \lambda_j^p z_{lj}^{p(4)} - s_l^\# = 0,\ \forall l,
\end{aligned}
\tag{5}
$$

$$\sum_j \lambda_j^p \in \phi(\zeta),\ \forall p,$$

$$\lambda_j^p \geq 0, \forall j, p,$$

$$s_i^-, s_r^+, s_l^\# \geq 0, \forall i, r, l.$$

<u>Network DEA model for calculating $\theta_0^{(2)}$</u>

$$\begin{aligned}
&\min\ \ \theta_0^{(2)} - \varepsilon\left(\sum_i s_i^- + \sum_r s_r^+ + \sum_l s_l^\#\right) \\
&st.\ \sum_{p\in P_I(i)} \sum_j \lambda_j^p x_{ij}^{p(1)} + s_i^- = \theta_0^{(2)} x_{i0}^{p(2)},\ \ \forall i, \\
&\quad \sum_{p\in P_O(k)} \sum_j \lambda_j^p y_{kj}^{p(4)} - s_r^+ = y_{k0}^{p(2)},\ \ \forall k, \\
&\sum_{p\in P^{in}(l)} \sum_j \lambda_j^p z_{lj}^{p(1)} - \sum_{p\in P^{out}(l)} \sum_j \lambda_j^p z_{lj}^{p(4)} - s_l^\# = 0,\ \ \forall l, \\
&\quad \sum_j \lambda_j^p \in \phi(\zeta),\ \forall p, \\
&\quad \lambda_j^p \geq 0, \forall j, p, \\
&\quad s_i^-, s_r^+, s_l^\# \geq 0, \forall i, r, l.
\end{aligned} \tag{6}$$

<u>Network DEA model for calculating $\theta_0^{(3)}$</u>

$$\begin{aligned}
&\min\ \ \theta_0^{(3)} - \varepsilon\left(\sum_i s_i^- + \sum_r s_r^+ + \sum_l s_l^\#\right) \\
&st.\ \sum_{p\in P_I(i)} \sum_j \lambda_j^p x_{ij}^{p(1)} + s_i^- = \theta_0^{(3)} x_{i0}^{p(3)},\ \ \forall i, \\
&\quad \sum_{p\in P_O(k)} \sum_j \lambda_j^p y_{kj}^{p(4)} - s_r^+ = y_{k0}^{p(3)},\ \ \forall k, \\
&\sum_{p\in P^{in}(l)} \sum_j \lambda_j^p z_{lj}^{p(1)} - \sum_{p\in P^{out}(l)} \sum_j \lambda_j^p z_{lj}^{p(4)} - s_l^\# = 0,\ \ \forall l, \\
&\quad \sum_j \lambda_j^p \in \phi(\zeta),\ \forall p, \\
&\quad \lambda_j^p \geq 0, \forall j, p,
\end{aligned} \tag{7}$$

<u>Network DEA model for calculating $\theta_0^{(4)}$</u>

$$\begin{aligned}
&\min\ \ \theta_0^{(4)} - \varepsilon\left(\sum_i s_i^- + \sum_r s_r^+ + \sum_l s_l^\#\right) \\
&st.\ \sum_{p\in P_I(i)} \sum_j \lambda_j^p x_{ij}^{p(1)} + s_i^- = \theta_0^{(4)} x_{i0}^{p(1)},\ \ \forall i, \\
&\quad \sum_{p\in P_O(k)} \sum_j \lambda_j^p y_{kj}^{p(4)} - s_r^+ = y_{k0}^{p(4)},\ \ \forall k, \\
&\sum_{p\in P^{in}(l)} \sum_j \lambda_j^p z_{lj}^{p(1)} - \sum_{p\in P^{out}(l)} \sum_j \lambda_j^p z_{lj}^{p(4)} - s_l^\# = 0,\ \ \forall l, \\
&\quad \sum_j \lambda_j^p \in \phi(\zeta),\ \forall p, \\
&\quad \lambda_j^p \geq 0, \forall j, p,
\end{aligned} \tag{8}$$

$s_i^-, s_r^+, s_l^\# \geq 0, \forall i, r, l.$

Solving models (5)-(8) enables us to acquire the best possible relative fuzzy [overall] efficiency $(\theta_0^{(1)*}, \theta_0^{(2)*}, \theta_0^{(3)*}, \theta_0^{(4)*})$ of DMU_0. The optimal solutions of the above models enable us to present the following definitions to define subjectively the efficient and inefficient DMU_j:

Definition 1. A DMU_j is called *fully efficient* if $\theta^{(1)*} = 1$ using model (5), implies that $\theta^{(1)*} = \theta^{(2)*} = \theta^{(3)*} = \theta^{(3)*} = 1$, a DMU_j is called *very highly efficient* if $\theta^{(2)*} = 1$ using model (6), implies that $\theta^{(2)*} = \theta^{(3)*} = \theta^{(3)*} = 1$, a DMU_j is called *highly efficient* if $\theta^{(3)*} = 1$ using model (7), implies that $\theta^{(3)*} = \theta^{(3)*} = 1$, and finally a DMU_j is called *efficient* if $\theta^{(4)*} = 1$ using model (8).

Definition 2. A DMU_j is called *inefficient* if the optimal value of $\theta^{(4)*}$ derived from model (8). is less than unity.

Given that the decision-makers normally wish to rank the inefficient DMUs resulted from Definition 2, we determine the nearest point associated to each fuzzy efficiency as per the following definition:

Definition 3 (Asady and Zendehnam, 2007). Let $\tilde{\theta} = (\theta^{(1)}, \theta^{(2)}, \theta^{(3)}, \theta^{(4)})$ be a trapezoidal fuzzy number, the nearest point of $\tilde{\theta}$ can be first calculated as follows:

$$M_{\tilde{\theta}} = \frac{\theta^{(2)} + \theta^{(3)}}{2} + \frac{\theta^{(4)} - \theta^{(2)} - \theta^{(3)} + \theta^{(1)}}{4} \tag{9}$$

Then, a larger value of the nearest point (M) shows that DMU_j is preferred. This simple and efficient defuzzification method generates very realistic results against other complicated methods without losing the basic properties[1].

The fuzzy measure of efficiency provided by CRS and VRS network models are known as total technical efficiency (TTE) and pure technical efficiency (PTE). The ratio "TTE /PTE" stands for a fuzzy measure of scale efficiency (SE). Assume that $\tilde{\theta}_0(crs) = (\theta_{crs}^{(1)*}, \theta_{crs}^{(2)*}, \theta_{crs}^{(3)*}, \theta_{crs}^{(4)*})$ and $\tilde{\theta}_0(vrs) = (\theta_{vrs}^{(1)*}, \theta_{vrs}^{(2)*}, \theta_{vrs}^{(3)*}, \theta_{vrs}^{(4)*})$ are the fuzzy efficiencies for the TTE and PTE, respectively, the fuzzy measure of SE_0 for DMU_0 is expressed as follows:

[1] See Asady and Zendehnam (2007) for going through certain mathematical advantages of the above ranking fuzzy number.

$$\tilde{\theta}(SE_0) = \frac{\tilde{\theta}_0(crs)}{\tilde{\theta}_0(vrs)} = \frac{(\theta_{crs}^{(1)*}, \theta_{crs}^{(2)*}, \theta_{crs}^{(3)*}, \theta_{crs}^{(4)*})}{(\theta_{vrs}^{(1)*}, \theta_{vrs}^{(2)*}, \theta_{vrs}^{(3)*}, \theta_{vrs}^{(4)*})} \quad (10)$$

Given that $1/(\theta_{vrs}^{(1)*}, \theta_{vrs}^{(2)*}, \theta_{vrs}^{(3)*}, \theta_{vrs}^{(4)*}) = (\theta_{vrs}^{(4)*}, \theta_{vrs}^{(3)*}, \theta_{vrs}^{(2)*}, \theta_{vrs}^{(1)*})$, equation (10) can be transformed into a multiplication operation as follows:

$$\tilde{\theta}(SE_0) = (\theta_{crs}^{(1)*}, \theta_{crs}^{(2)*}, \theta_{crs}^{(3)*}, \theta_{crs}^{(4)*}) \otimes (\theta_{vrs}^{(4)*}, \theta_{vrs}^{(3)*}, \theta_{vrs}^{(2)*}, \theta_{vrs}^{(1)*}) \quad (11)$$

Dubois and Prade (1978) suggested the following standard approximation to calculate the above multiplication, which is definitely easy and computationally efficient.

$$\tilde{\theta}(SE_0) \cong (\theta_{crs}^{(1)*}.\theta_{vrs}^{(4)*}, \theta_{crs}^{(2)*}.\theta_{vrs}^{(3)*}, \theta_{crs}^{(3)*}.\theta_{vrs}^{(2)*}, \theta_{crs}^{(4)*}.\theta_{vrs}^{(1)*}) \quad (12)$$

Even though the standard approximation associated with the multiplication operation is widely used in the literature, Dubois and Prade (1978) noted that erroneous results are considerably appeared when the spread of the fuzzy number is not small and the membership value is near 1. To acquire the actual product, the multiplication operation can be carried out at each α level. Let us initially define the interval confidence method of $\tilde{\theta}_{0\alpha}(crs)$ and $\tilde{\theta}_{0\alpha}^{-1}(vrs)$ which is $((\theta_{crs}^{(2)*} - \theta_{crs}^{(1)*})\alpha + \theta_{crs}^{(1)*}, -(\theta_{crs}^{(4)*} - \theta_{crs}^{(3)*})\alpha + \theta_{crs}^{(4)*})$ and $((\theta_{vrs}^{(2)*} - \theta_{vrs}^{(1)*})\alpha + \theta_{vrs}^{(1)*}, -(\theta_{vrs}^{(4)*} - \theta_{vrs}^{(3)*})\alpha + \theta_{vrs}^{(4)*})$ for every $\alpha \in [0,1]$ (Kaufmann and Gupta, 1988). Next, the product $\tilde{\theta}(SE_0)$ of two trapezoidal fuzzy numbers $\tilde{\theta}_0(crs)$ and $\tilde{\theta}_0^{-1}(vrs)$ is computed by multiplying the α-levels defined by the interval confidence method.

$$\begin{aligned}\tilde{\theta}(SE_0) &= \tilde{\theta}_0(crs) \otimes \tilde{\theta}_0^{-1}(vrs) \\ &= ((\theta_{crs}^{(2)*} - \theta_{crs}^{(1)*})\alpha + \theta_{crs}^{(1)*}, -(\theta_{crs}^{(4)*} - \theta_{crs}^{(3)*})\alpha + \theta_{crs}^{(4)*}) \times ((\theta_{vrs}^{(2)*} \\ &\quad - \theta_{vrs}^{(1)*})\alpha + \theta_{vrs}^{(1)*}, -(\theta_{vrs}^{(4)*} - \theta_{vrs}^{(3)*})\alpha + \theta_{vrs}^{(4)*})\end{aligned} \quad (13)$$

The lines connecting the endpoints for every $\alpha \in [0,1]$ results in the actual product which is the fuzzy measure of SE_0. Note that if $\tilde{\theta}_{0\alpha}(crs)$ turns out to be precise as $(\theta_{crs}^{(1)*}, \theta_{crs}^{(1)*}, \theta_{crs}^{(1)*}, \theta_{crs}^{(1)*})$, then $\tilde{\theta}(SE_0)$ can be expressed as follows:

$$\tilde{\theta}(SE_0) = ((\theta_{vrs}^{(2)*} - \theta_{vrs}^{(1)*})\theta_{crs}^{(1)*}\alpha + \theta_{vrs}^{(1)*}\theta_{crs}^{(1)*}, -(\theta_{vrs}^{(4)*} - \theta_{vrs}^{(3)*})\theta_{crs}^{(1)*}\alpha + \theta_{vrs}^{(4)*}\theta_{crs}^{(1)*}) \quad (14)$$

The upper and lower limits of interval SE_0 measure varies within [0, 1] in which the lower limit is always smaller than or equal to the upper limit. Therefore, we think of the following definition to provide a classification in terms of the scale efficiency measure of the DMU under evaluation.

Definition 4. Consider the interval SE of DMU_0 derived from (13) for a given α. If the lower limit of $\tilde{\theta}(SE_0)$ is equal to 1, i.e., $\tilde{\theta}(SE_0) = (1, 1)$, then it is called *full scale efficient*, if the upper limit of $\tilde{\theta}(SE_0)$ is equal to 1 and the lower limit of $\tilde{\theta}(SE_0)$ is less than one, then we call it *scale efficient*, and if the upper limit of $\tilde{\theta}(SE_0)$ is less than one, then we call it *scale inefficient*. In fact, a DMU is *full scale efficient* when the network system is completely operating at optimal scale size, and the system is *scale efficient* when the system is partially functioning at optimal scale size.

4. Application

In this section, we exemplify our proposed method by analyzing and benchmarking the airport operations which can be observed as a two-process structure including “Aircraft Movement” and “Aircraft Loading” as the first and second processes, respectively (Gillen and Lall, 1997, 2001; Lozano et al., 2013). The first process uses three inputs; *total runway area* ($I1S_1$), *apron capacity* ($I2S_1$), *number of boarding gates* ($I3S_1$) to generate the *accumulated flight delays* ($O1S_1$) as an undesirable output as well as the *airplane traffic movements* (Inter) as an intermediate measure. The second process consumes two inputs; *number of check-in counters* ($I1S_2$) and *number of baggage belts* ($I2S_2$) and the intermediate measure (*airplane traffic movements*) to produce two outputs; *annual passenger movements* ($O1S_2$) and *cargo handled* ($O2S_2$). The *aircraft traffic movements* as an intermediate measure signifies the number of airplane movements including landings and take-offs of airplanes, which plays a part in providing the service of moving passengers and cargos.

In comparison with the structure proposed by Lozano et al. (2013), we discard the *number of delayed flights* as an undesirable output of the first process since it is highly correlated with the *accumulated flight delays*. The structural pattern is depicted in Figure 1.

------- Insert Figure 1 here-------

We draw special attention to the *accumulated flight delays* which is an unpleasant output derived from the first process. To deal with undesirable outputs, several approaches have been developed in the literature. Dyckhoff and Allen (2001) classified the respective approaches for handling undesirable outputs into three categories: (*i*) taking into account the reciprocal of the undesirable output in which the undesirable output is changed to the desirable one (Scheel, 2001), (*ii*) taking into account a multi-criteria approach in which the undesirable output is modelled as an input (Rheinhard et al, 1999), and (*iii*) employing the translation property in BCC and additive DEA models which implies that a positive scalar is added to the reciprocal additive transformation of the undesirable output (Ali and Seiford, 1990). Simplistically, the approach to treat undesirable outputs is to consider an undesirable output as an input or utilizes the reciprocal (Gomes and Lins, 2008). In this research, we model the undesirable output of the first process (i.e., *accumulated flight delays*) as an input of this process. The dataset for 39 Spanish airports taken from Lozano et al. (2013) is presented in Table 1. To highlight the importance of inescapable uncertainty in the performance analysis, particularly in airport operations, in this section, we extend the dataset reported in Table 1 into an uncertain data setting. We assume that $I1S_1$, $I2S_1$ and $O1S_1$ are not precisely measured due to the uncertainty and subjectiveness. To deal with such uncertainty, we take account of a trapezoidal fuzzy number whose vertex is identical to the deterministic amount with assigning a degree of membership of 1. The precise values of $I1S_1$, $I2S_1$ and $O1S_1$ are therefore substituted with the trapezoidal fuzzy numbers as ($0.85*I1S_1$, $I1S_1$, $I1S_1$, $1.25*I1S_1$), ($0.85*I2S_1$, $I2S_1$, $I2S_1$, $1.25*I2S_1$) and ($0.85*O1S_1$, $O1S_1$, $O1S_1$, $1.25*O1S_1$). These fuzzy numbers in fact are triangular fuzzy numbers due to the equality between the two points at the top of each trapezoidal fuzzy numbers.

------- Insert Table 1 here-------

We calculate the fuzzy efficiencies for every airport using models (5)-(8) under the VRS assumption, as shown in "VRS" column of Table 2. Note that we take the two convexity constraints for the first and second processes into consideration, i.e., $\sum_j \lambda_j^p = 1\ ,\ p = 1{,}2$, to satisfy the VRS assumption. Since the results of models (7) and (8) are equal, $\theta^{(2)} = \theta^{(3)}$ and the approximated efficiency of each airport is a triangular fuzzy number. According to Definition 1, the *Vitoria*, *Saragossa, Madrid Barajas, Girona-Costa Brava, Cordoba* and *Barcelona* airports are classified as *fully efficient* because $\theta^{(1)*}$ is equal to 1, and *Albacete*, *Badajoz, El Hierro* and *La Gomera* airports are classified as *efficient* since $\theta^{(3)*} = 1$. It is also shown in the 5th column of Table 2 in which "F. Eff." and "Eff." stand for *fully efficient* and *efficient* categories, respectively. The airport whose efficiency derived from model (8) is less one is classified as *inefficient*. To provide a ranking for the inefficient airports, we exploit the nearest point whose formula is introduced in Definition 3. The nearest points of inefficient airports are reported in the 5th column of Table 2 and the numbers in parentheses indicate their rankings. Accordingly, *Melilla* is superior among the inefficient airports, followed by *Alicante*, *Leon*, and *Palma de Mallorca* airports, respectively. Interestingly, the *Ibiza* airport is the worst performance in total. The method proposed by Lozano et al. (2013) without taking uncertainty into account eight airports including *Albacete*, *Barcelona*, *Cordoba*, *Girona-Costa Brava*, *Madrid Barajas*, *Palma de Mallorca*, *Saragossa* and *Vitoria* are efficient. Contrary to our approach in this paper, apart from the *Palma de Mallorca* airport which is not efficient anymore, the outstanding seven airports not only remain efficient but also the *Badajoz*, *El Hierro* and *La Gomera* airports are efficient.

------- Insert Table 2 here-------

At present, let us analyze the role of convexity constraints for all processes. In what follows, we zero in on the convexity of Process 1 without regarding convexity constraint for Process 2, i.e., $\sum_j \lambda_j^1 = 1$. The associated results are summarized in the "VRS (Convexity for Process 1)" column of Table 2. Contrary to the VRS case, the fuzzy efficiencies for 59% of inefficient airports are slightly declined in the absence of the convexity constraint of Process 2, and the significant difference bears on the *Madrid Barajas* airport as "F. Eff." which turns out to be inefficient. It is need to point out that *Ibiza* is still the most inferior airport. The convexity of Process 2 is, in turn, considered under the VRS assumption, i.e., $\sum_j \lambda_j^2 = 1$, to evaluate the fuzzy efficiency of the Spanish airports in the presence of a number of fuzzy data embedded in Process 1. This model setting leads to the identical solutions for models (5)-(8), i.e. $\theta^{(1)*} = \theta^{(2)*} = \theta^{(3)*} = \theta^{(4)*}$, as shown in "VRS (convexity of Process 2)" column of Table 2. It is remarked that the *Madrid Barajas* airport is efficient which is the same as the VRS case. Besides, the efficiency of two *Albacete* and *La Gomera* airports are considerably decreased as can be also observed from their ranks reported in Table 2. The last column of Table 2 shows the total technical efficiency (TTE) of the airports under the CRS assumption. Given that the TTE measures are deterministic, we take account of Equation (14) to obtain the interval SE for five different α levels, i.e., α={0,0.25,0.5,0.75,1} as presented in Table 3. Note that *full scale efficient* and *scale inefficient* are denoted by FSE and SIN, respectively, in the last column of Table 3. The last column of Table 3 shows the associated classification in terms of the scale efficiency measure which *Barcelona*, *Cordoba*, *Girona-Costa Brava*, *Saragossa* and *Vitoria* are classified as *full scale efficient* because the lower limit of $\tilde{\theta}(SE_0)$ for all α-levels is equal to 1, meaning that these airports are completely operating at optimal scale size.

------- Insert Table 3 here-------

Given some results may be far from the actual performance especially from the practitioner view, we have need of underlining that our airport benchmarking analysis in this section is not intended to secure an in-depth study and understanding of the performance of Spanish airports, but rather to signify the application of the proposed methodology.

5. Conclusions

Due to the lack of availability of precise input and output data in many real-world applications as well as going beyond the black-box structure of firms, this study has proposed a new fuzzy network DEA model based upon the fuzzy arithmetic to conquer the uncertainty and fuzziness embedded in network structures. We have developed input-oriented fuzzy network DEA models to compute the fuzzy technical and scale efficiencies. Although most network systems in DEA literature are presumed to be simple, i.e., two processes, we have focused on general network production structures which can be the mixtures of series and parallel structures. In addition, a classification framework based on the fuzzy scale and efficiency measures has been introduced to provide a better understanding of a network production systems against other homogeneous systems. Fuzzy efficiency and fuzzy scale measures resulted from the proposed approach are more informative than crisp measures. Put differently, our approach enables us to reflect the real situation and human judgments with the fuzzy values rather than precise number. To illustrate the main steps of the model, we have applied the fuzzy network DEA models to evaluate the performances and scale efficiency measure of 39 airports in which every airport includes the two production processes.

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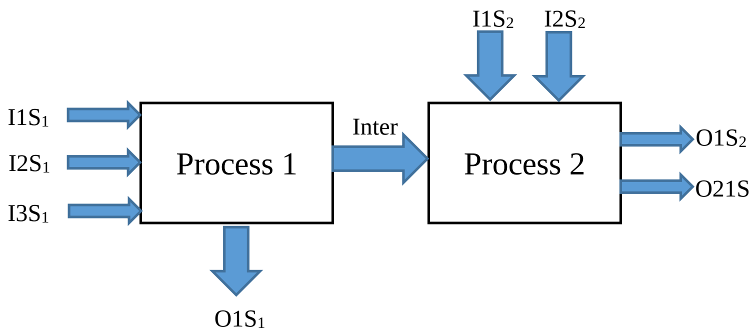

Figure 1. A two-process system

Table 1. Input and output data for the 39 Spanish airports

Airport	$I1S_1$	$I2S_1$	$I3S_1$	Inter	$O1S_1$	$I1S_2$	$I2S_2$	$O1S_2$	$O2S_2$
A Coruña	87300	5	4	17.719	23783.4	10	3	1174.97	283.57
Albacete	162000	2	2	2.113	1376.5	4	1	19.25	8.92
Alicante	135000	31	16	81.097	142445.8	42	9	9578.3	5982.31
Almeria	144000	15	5	18.28	20149.1	17	4	1024.3	21.32
Asturias	99000	7	9	18.371	23893.5	11	3	1530.25	139.47
Badajoz	171000	1	2	4.033	2365.4	4	1	81.01	0
Barcelona	475020	121	65	321.693	645924.6	143	19	30272.08	103996.49
Bilbao	207000	21	12	61.682	80848.2	36	7	4172.9	3178.76
Cordoba	62100	23	1	9.604	254.4	1	0	22.23	0
El Hierro	37500	3	2	4.775	641.6	5	1	195.43	171.72
Fuerteventura	153000	34	10	44.552	72179.7	34	8	4492	2722.66
Girona-Costa Brava	108000	17	7	49.927	100305.6	18	3	5510.97	184.13
Gran Canaria	139500	55	38	116.252	136380.7	86	19	10212.12	33695.25
Granada-Jaen	134550	11	3	19.279	17868.8	12	3	1422.01	66.89
Ibiza	126000	25	12	57.233	152840.1	48	8	4647.36	3928.39
Jerez	103500	9	5	50.551	19292.2	13	3	1303.82	90.43
La Gomera	45000	3	2	3.393	420.7	5	1	41.89	7.86
La Palma	99000	5	5	20.109	8286	13	2	1151.36	1277.26
Lanzarote	108000	24	16	53.375	101685.6	49	8	5438.18	5429.59
Leon	94500	5	2	5.705	7191.5	3	1	123.18	15.98
Madrid Barajas	927000	263	230	469.746	908360	484	53	50846.49	329186.63
Malaga	144000	43	30	119.821	277663.8	85	16	12813.47	4800.27
Melilla	64260	5	2	10.959	2979.6	4	1	314.64	386.34
Murcia	138000	5	5	19.339	24103.1	18	4	1876.26	2.73
Palma de Mallorca	295650	86	68	193.379	501486	204	16	22832.86	21395.79
Pamplona	99315	7	2	12.971	11691.8	4	1	434.48	52.94
Reus	110475	5	5	26.676	18240.8	8	3	1278.07	119.85
Salamanca	150000	6	2	12.45	6626.1	4	2	60.1	0
San Sebastian	78930	6	3	12.282	11184	6	2	403.19	63.79
Santander	104400	8	5	19.198	17842	8	2	856.61	37.48
Santiago	144000	16	12	21.945	34322.3	19	5	1917.47	2418.8
Saragossa	302310	12	3	14.584	19547.6	6	2	594.95	21438.89
Seville	151200	23	10	65.067	51084.9	42	6	4392.15	6102.26

Tenerife North	153000	16	16	67.8	32637	37	5	4236.62	20781.67
Tenerife South	144000	44	22	60.779	110818.9	87	14	8251.99	8567.09
Valencia	144000	35	18	96.795	102719.2	42	8	5779.34	13325.8
Valladolid	180000	7	5	13.002	14760.6	8	2	479.69	34.65
Vigo	108000	8	6	17.934	25593.6	12	3	1278.76	1481.94
Vitoria	157500	18	3	12.225	11585.8	7	2	67.82	34989727

Note: The units of the data are: $I1S_1$ (square meters), $I2S_1$ (no. of stands), $I3S_1$ (no. of gates), Inter (thousand operations), $O1S_1$ (minutes), $I1S_2$ (no. of counters), $I2S_2$ (Number of belts), $O1S_2$ (Thousand passengers), and $O2S_2$ (Tonnes).

Table 2. Efficiencies of the Spanish airports

Airport	VRS				VRS (Convexity for Process 1)				VRS (convexity for Process 2)	CRS
	$\theta^{(1)}$	$\theta^{(2)} = \theta^{(3)}$	$\theta^{(4)}$	Nearest point Classification	$\theta^{(1)}$	$\theta^{(2)} = \theta^{(3)}$	$\theta^{(4)}$	Nearest point Classification	$\theta^{(1)} = \theta^{(2)} = \theta^{(3)} = \theta^{(4)}$	$\theta^{(1)} = \theta^{(2)} = \theta^{(3)} = \theta^{(4)}$
A Coruña	0.488	0.498	0.565	0.512 (15)	0.488	0.498	0.565	0.512 (12)	0.461 (17)	0.388 (19)
Albacete	0.964	0.986	1	Eff.	0.964	0.986	1	Eff.	0.25 (37)	0.016 (39)
Alicante	0.917	0.917	0.917	0.917 (2)	0.77	0.77	0.77	0.770 (5)	0.917 (7)	0.77 (6)
Almeria	0.349	0.363	0.372	0.362 (28)	0.349	0.363	0.372	0.362 (27)	0.241 (38)	0.197 (32)
Asturias	0.517	0.517	0.517	0.517 (14)	0.456	0.456	0.456	0.456 (17)	0.517 (15)	0.456 (14)
Badajoz	0.99	0.996	1	Eff.	0.99	0.996	1	Eff.	0.296 (35)	0.066 (36)
Barcelona	1	1	1	F. Eff.	1	1	1	F. Eff.	1 (1)	1 (1)
Bilbao	0.398	0.398	0.398	0.398 (22)	0.395	0.395	0.395	0.395 (22)	0.398 (24)	0.395 (18)
Cordoba	1	1	1	F. Eff.	1	1	1	F. Eff.	1 (1)	1 (1)
El Hierro	0.964	0.986	1	Eff.	0.964	0.986	1	Eff.	0.312 (33)	0.134 (35)
Fuerteventura	0.448	0.448	0.448	0.448 (17)	0.446	0.446	0.446	0.446 (18)	0.448 (18)	0.446 (15)
Girona-Costa Brava	1	1	1	F. Eff.	1	1	1	F. Eff.	1 (1)	1 (1)
Gran Canaria	0.531	0.531	0.531	0.531 (13)	0.461	0.461	0.461	0.461 (16)	0.531 (14)	0.461 (13)
Granada-Jaen	0.552	0.58	0.598	0.578 (11)	0.552	0.58	0.598	0.578 (10)	0.445 (20)	0.387 (20)
Ibiza	0.339	0.339	0.339	0.339 (29)	0.338	0.338	0.338	0.338 (30)	0.339 (32)	0.338 (27)
Jerez	0.383	0.388	0.394	0.388 (25)	0.377	0.388	0.394	0.387 (25)	0.383 (26)	0.328 (29)
La Gomera	0.964	0.986	1	Eff.	0.964	0.986	1	Eff.	0.212 (39)	0.028 (38)
La Palma	0.398	0.467	0.549	0.470 (16)	0.398	0.467	0.549	0.470 (15)	0.36 (31)	0.329 (28)
Lanzarote	0.397	0.397	0.397	0.397 (23)	0.396	0.396	0.396	0.396 (21)	0.397 (25)	0.396 (17)
Leon	0.883	0.917	0.939	0.914 (3)	0.883	0.917	0.939	0.914 (2)	0.438 (21)	0.135 (34)

Madrid Barajas	1	1	1	F. Eff.	0.748	0.748	0.748	0.748 (7)	1 (1)	0.748 (8)
Malaga	0.645	0.645	0.645	0.645 (7)	0.502	0.502	0.502	0.502 (14)	0.645 (9)	0.502 (12)
Melilla	0.895	0.928	0.949	0.925 (1)	0.895	0.928	0.949	0.925	0.491 (16)	0.275 (30)
Murcia	0.395	0.428	0.504	0.439 (19)	0.395	0.428	0.504	0.439 (19)	0.375 (28)	0.34 (26)
Palma de Mallorca	0.887	0.887	0.887	0.887 (4)	0.752	0.752	0.752	0.752 (6)	0.887 (8)	0.752 (7)
Pamplona	0.832	0.873	0.9	0.870 (6)	0.832	0.873	0.9	0.870 (4)	0.571 (12)	0.357 (22)
Reus	0.613	0.613	0.613	0.613 (9)	0.524	0.524	0.535	0.527 (11)	0.613 (10)	0.524 (10)
Salamanca	0.852	0.881	0.904	0.880 (5)	0.852	0.881	0.904	0.880 (3)	0.279 (36)	0.049 (37)
San Sebastian	0.618	0.637	0.649	0.635 (8)	0.618	0.637	0.649	0.635 (8)	0.365 (30)	0.221 (31)
Santander	0.448	0.448	0.448	0.448 (17)	0.382	0.392	0.398	0.391 (23)	0.448 (18)	0.35 (24)
Santiago	0.38	0.38	0.38	0.380 (26)	0.353	0.353	0.353	0.353 (28)	0.38 (27)	0.353 (23)
Saragossa	1	1	1	F. Eff.	1	1	1	F. Eff.	1 (1)	1 (1)
Seville	0.422	0.422	0.422	0.422 (21)	0.421	0.421	0.421	0.421 (20)	0.422 (23)	0.421 (16)
Tenerife North	0.601	0.601	0.601	0.601 (10)	0.6	0.6	0.6	0.600 (9)	0.601 (11)	0.6 (9)
Tenerife South	0.366	0.366	0.366	0.366 (27)	0.342	0.342	0.342	0.342 (29)	0.366 (29)	0.342 (25)
Valencia	0.543	0.543	0.543	0.543 (12)	0.509	0.509	0.509	0.509 (13)	0.543 (13)	0.509 (11)
Valladolid	0.382	0.391	0.397	0.390 (24)	0.382	0.391	0.397	0.390 (24)	0.302 (34)	0.196 (33)
Vigo	0.426	0.426	0.426	0.426 (2)	0.371	0.371	0.371	0.371 (26)	0.426 (22)	0.371 (21)
Vitoria	1	1	1	F. Eff.	1	1	1	F. Eff.	1 (1)	1 (1)

Table 3. SE of the Spanish airports for different α-levels

Airport	$\widetilde{\theta}(SE)_0$		$\widetilde{\theta}(SE)_{0.25}$		$\widetilde{\theta}(SE)_{0.5}$		$\widetilde{\theta}(SE)_{0.75}$		$\widetilde{\theta}(SE)_1$		Classification
A Coruña	0.189	0.219	0.190	0.213	0.191	0.206	0.192	0.200	0.193	0.193	SIN
Albacete	0.015	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	0.016	SIN
Alicante	0.706	0.706	0.706	0.706	0.706	0.706	0.706	0.706	0.706	0.706	SIN
Almeria	0.069	0.073	0.069	0.073	0.070	0.072	0.071	0.072	0.072	0.072	SIN
Asturias	0.236	0.236	0.236	0.236	0.236	0.236	0.236	0.236	0.236	0.236	SIN
Badajoz	0.065	0.066	0.065	0.066	0.066	0.066	0.066	0.066	0.066	0.066	SIN
Barcelona	1	1	1	1	1	1	1	1	1	1	FSE
Bilbao	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	SIN
Cordoba	1	1	1	1	1	1	1	1	1	1	FSE
El Hierro	0.129	0.134	0.130	0.134	0.131	0.133	0.131	0.133	0.132	0.132	SIN
Fuerteventura	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	SIN
Girona-Costa Brava	1	1	1	1	1	1	1	1	1	1	FSE
Gran Canaria	0.245	0.245	0.245	0.245	0.245	0.245	0.245	0.245	0.245	0.245	SIN
Granada-Jaen	0.214	0.231	0.216	0.230	0.219	0.228	0.222	0.226	0.224	0.224	SIN
Ibiza	0.115	0.115	0.115	0.115	0.115	0.115	0.115	0.115	0.115	0.115	SIN
Jerez	0.126	0.129	0.126	0.129	0.126	0.128	0.127	0.128	0.127	0.127	SIN
La Gomera	0.027	0.028	0.027	0.028	0.027	0.028	0.027	0.028	0.028	0.028	SIN
La Palma	0.131	0.181	0.137	0.174	0.142	0.167	0.148	0.160	0.154	0.154	SIN
Lanzarote	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	SIN
Leon	0.119	0.127	0.120	0.126	0.122	0.125	0.123	0.125	0.124	0.124	SIN
Madrid Barajas	0.748	0.748	0.748	0.748	0.748	0.748	0.748	0.748	0.748	0.748	SIN
Malaga	0.324	0.324	0.324	0.324	0.324	0.324	0.324	0.324	0.324	0.324	SIN
Melilla	0.246	0.261	0.248	0.260	0.251	0.258	0.253	0.257	0.255	0.255	SIN
Murcia	0.134	0.171	0.137	0.165	0.140	0.158	0.143	0.152	0.146	0.146	SIN
Palma de Mallorca	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	0.667	SIN
Pamplona	0.297	0.321	0.301	0.319	0.304	0.316	0.308	0.314	0.312	0.312	SIN
Reus	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	0.321	SIN
Salamanca	0.042	0.044	0.042	0.044	0.042	0.044	0.043	0.043	0.043	0.043	SIN
San Sebastian	0.137	0.143	0.138	0.143	0.139	0.142	0.140	0.141	0.141	0.141	SIN
Santander	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	0.157	SIN
Santiago	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	0.134	SIN
Saragossa	1	1	1	1	1	1	1	1	1	1	FSE
Seville	0.178	0.178	0.178	0.178	0.178	0.178	0.178	0.178	0.178	0.178	SIN

Tenerife North	0.361	0.361	0.361	0.361	0.361	0.361	0.361	0.361	0.361	0.361	SIN
Tenerife South	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125	SIN
Valencia	0.276	0.276	0.276	0.276	0.276	0.276	0.276	0.276	0.276	0.276	SIN
Valladolid	0.075	0.078	0.075	0.078	0.076	0.077	0.076	0.077	0.077	0.077	SIN
Vigo	0.158	0.158	0.158	0.158	0.158	0.158	0.158	0.158	0.158	0.158	SIN
Vitoria	1	1	1	1	1	1	1	1	1	1	FSE

Authors' Response

Benchmarking with Network DEA in a Fuzzy Environment

2nd Revision
ro160205R1
RAIRO - Operations Research

ACKNOWLEDGEMENT

The author would like to thank the anonymous reviewers for their insightful and constructive comments.

All additions and changes to the first revision are highlighted in this revision.

Reviewer

1. **In response to the following general comment made by the reviewer:**
   The authors have addressed a number of queries satisfactorily but the level of English is still poor with many grammatical errors. It is hard to justify acceptance at this stage. For example, some of the added texts highlighted in yellow have hanging sentences and circular reasoning. Please show evidence of improvement.

We have meticulously read the manuscript several times to optimize the accuracy and readability of our document along with resolving all issues with spelling, grammar, punctuation, and word usage.