Consensus Reaching in Social Network Group Decision Making: Research Paradigms and Challenges[1]

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Abstract: In social network group decision making (SNGDM), the consensus reaching process (CRP) is used to help decision makers with social relationships reach consensus. Many CRP studies have been conducted in SNGDM until now. This paper provides a review of CRPs in SNGDM, and as a result it classifies them into two paradigms: (i) the CRP paradigm based on trust relationships, and (ii) the CRP paradigm based on opinion evolution. Furthermore, identified research challenges are put forward to advance this area of research.

Keywords: Group decision making, Social network, Consensus, Opinion evolution

1. Introduction

Group decision making (GDM) consists of a group of decision makers who express their opinions on a set of alternatives with the aim of choosing the best alternative [1,2]. Traditionally, GDM models focus on two processes: the consensus reaching process (CRP) and the selection process. In a CRP, decision makers might need to modify their opinions, making them more similar or closer, in order to increase the consensus level of the group. This is considered an iterative and dynamic process leading to the final group consensus based solution after multiple rounds of discussions and modifications [3,4,5]. Complete consensus state is not necessary in general and, as a consequence, soft consensus approaches are often employed in CRPs. As

mentioned above, when consensus among decision makers is reached, a selection process is applied usually by fusing the preferences of individual decision makers into a collective preference from which the final ordering of the considered alternatives is derived [6,7].

With the development of information and network technology, a new type of social network group decision making (SNGDM) problems has to be faced today. On the one hand, a social network allows information exchange and communication, and, therefore, provides social relationships among the decision makers [8,9,10]. During their interactions, decision makers with more experience and knowledge may influence other decision makers in the social network. On the other hand, opinion evolution, also called opinion dynamics, describes the process of forming opinions among a group of interactive decision makers [11-21]. Generally, in opinion evolution a decision maker would take into account other decision makers' opinions to form or evolve their own opinions. In a repeated process of interacting, the decision makers will update their opinions, which can lead to a consensus, fragmentation or polarization.

Usually, a consensus based solution in GDM is needed, which increases the satisfaction of decision makers as their opinions are reconsidered to reach an acceptable consensus level in the corresponding CRP. However, in some situation, trust relationships defined by a social network, play a key element in GDM and, as a consequence, consensus models in SNGDM is becoming a hot research topic. To date, many studies have been conducted on consensus in SNGDM, with a recent literature review in SNGDM provided by Herrera-Viedma et al. [140]. However, it is noted that this review does not analyze the CRP paradigms in SNGDM as mentioned above: (1) the CRP paradigm based on trust relationships, and (2) the CRP paradigm based on opinion evolution. This paper aims at filling this literature review gap and will analyze the research challenges to address in future to advance this topic.

The rest of the paper is organized as follows. In Section 2, the basic knowledge regarding the general framework of CRPs and social network analysis for GDM is introduced. The CRP paradigm based on trust relationships is introduced in Section 3, while the CRP paradigm based on opinion evolution is analyzed in Section 4. The research challenges in SNGDM are presented in Section 5. Finally, conclusions are drawn in Section 6.

2. Background

In this section, we introduce some basic knowledge regarding the general framework of CRPs and social network analysis for GDM.

2.1 General CRP

In a CRP, a group of decision makers, $D = \{d_1, d_2, \cdots, d_m\}$, express their preferences over several alternatives $X = \{x_1, x_2, \cdots, x_n\}$, and try to achieve a consensus based solution. The general framework of a CRP is depicted in Fig. 1, which consists of five key elements:
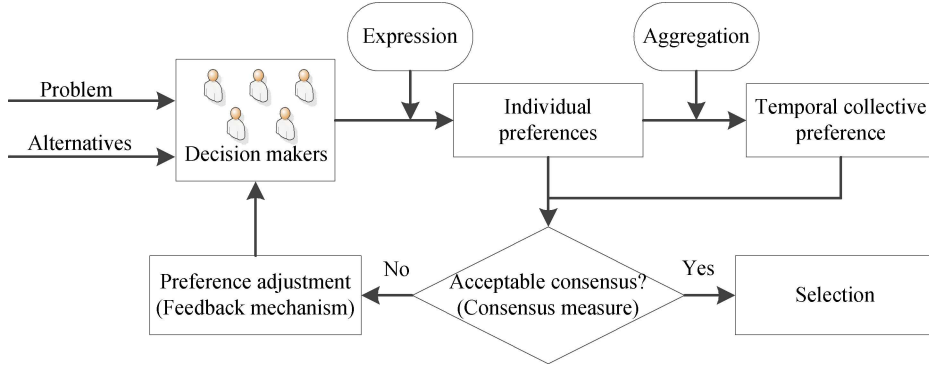

Fig.1. The general framework of a CRP

(1) Preference representation

A preference representation structure is utilized to represent the opinions of decision makers. Decision makers' preferences on a set of alternatives X can be represented in different ways using distinct preference representation structures, such as: utility values [22]; preference orderings [23,24]; additive preference relations [25-27]; linguistic preferences [28-32]; multiple attribute preferences [33,34]; and heterogeneous preference representation structures [35-39]. The additive preference relation (also called fuzzy preference relation) is one of the most widely used preference presentation structures. The additive preference relation of decision maker d_k is usually represented in matrix form $P^k = (p_{ij}^k)_{n \times n}$, where $p_{ij}^k \in [0,1]$ denotes his/her preference degree on the alternative x_i over the alternative x_j, with $p_{ij}^k + p_{ji}^k = 1$ (for all i, j). Generally, the choice of a particular preference representation structure will not change the basic methodology applied during a CRP. Without loss of generality, the additive preference relation will be adopted in this paper to introduce the basic ideas and concepts of CRPs.

(2) Aggregation

An aggregation operator, f, is utilized to aggregate the individual preferences into a collective one, $P^c = (p_{ij}^c)_{n \times n}$, as follows:

$$p_{ij}^c = f_\pi(p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^m) \tag{1}$$

where $\pi = (\pi_1, \pi_2, \cdots, \pi_m)$ is a weighting vector with components verifying $\pi_k \geq 0$ and $\sum_{k=1}^{m} \pi_k = 1$.

Many different aggregation operators (e.g., weighted average (WA), ordered weighted average (OWA), and importance induced ordered weighted average (I-IOWA)) [40,41] have been developed and therefore are possible to be used in the aggregation step of a CRP.

(3) Consensus measure

A consensus measure is a function that measures the similarity degree among decision makers, which is based on the use of a distance function [42,43], in one of the two following ways:

(a) Consensus measure based on distances to the collective preference. The collective preference represents the opinion of the group, and consensus at this level is calculated by measuring the distances between each individual preference relation and the aggregated collective preference relation.

(b) Consensus measure based on pairwise distances between decision makers. Consensus at this level is calculated by measuring the distances between each pair of individual preference relations.

(4) Feedback mechanism

A feedback mechanism aims to support decision makers in increasing the group consensus. It is often based on two kinds of consensus rules:

(a) Identification rule (IR) and direction rule (DR) [44,45]. The first one is used to identify the decision makers who contribute less to consensus, whom are requested to modify their preferences, while the second one is generated to support the identified decision makers on the direction of change of their preferences, which invariably means to go in the direction of getting closer to the collective preference or the preferences of the most trusted/influential/important decision maker(s) within the group.

(b) Optimization-based consensus rule [46-50], which are utilized to minimize the number of adjusted decision makers, alternatives, preference values, and distance between the original and adjusted preferences. The optimal adjusted preferences generated from optimization-based consensus rules are often used as the target values for decision makers to modify their preferences.

(5) Selection

When a predefined consensus threshold value among the group of decision makers is achieved, a selection process is applied to derive a final consensus ranking (from best to worst) of the alternatives. Because there are different preference representation structures available to be used in a decision making problem, different selection approaches are also available. In particular, when dealing with additive preference relations, an approach based on dominance and non-dominance degrees of alternatives is often adopted [51,52].

Table 1. CRPs with different design of key elements

References	Preference structures	Consensus measure	Feedback mechanism
Ben-Arieh et al. [46]; Zhang et al. [49]; Gong et al. [54]	Utility values	Based on distances to the collective preference	Optimization-based consensus rule
Guha et al. [55]; Wu et al. [56]; Parreiras et al. [57]	Multiple attribute preference	Based on distances between experts	IR and DR based consensus rule
Dong et al. [34]; Zhang et al. [58] Kim et al. [59];	Multiple attribute preference	Based on distances to the collective preference	IR and DR based consensus rule
Dong et al. [60]; Palomares et al. [61]; Kacprzyk et al. [62]; Pérez et al. [63]; Quesada et al. [64];	Additive preference relations	Based on distances between experts	IR and DR based consensus rule
Alonso et al. [9]; Herrera-Viedma et al. [45]; Zhang et al. [65]; Cabrerizo et al. [66]; Mata et al. [67]	Linguistic preference relations	Based on distances between experts	IR and DR based consensus rule
Dong et al. [34,37]; Herrera-Viedma et al. [39]; Zhang et al. [68]; Choudhury et al. [69]	Heterogeneous preference representation structure	Based on distances to the collective preference	IR and DR based consensus rule

Considering different combinations of the key elements of a CRP, a number of CRPs have

been developed, with some references to publications where they have been reported given in Table 1.

2.2 Social network analysis for GDM

A social network is a platform where users disseminate information and communicate with each other, which in turn can be used to study relationships among users using what is known as social network analysis. Social network analysis has been widely studied in different fields, such as group decision making [73,74], multiagent systems [53,141] and so on. In this section, we mainly introduce the basic knowledge of social network analysis in GDM.

In SNGDM, a social network is essentially represented as a graph $G(D,E)$, with nodes representing decision makers $D=\{d_1,d_2,\cdots,d_m\}$ and edges E corresponding the social relationships between decision makers. Concepts in social network analysis in GDM are formally described below as Definitions 1-4 [70,83].

Definition 1. A social network is defined by a directed graph $G(D,E)$, where $D=\{d_1,d_2,\cdots,d_m\}$ is the set of decision makers, E is a set of ordered pair of elements of D and edge $(d_i,d_j)\in E$ defines that decision maker d_i directly trusts decision maker d_j.

Definition 2. An adjacent matrix $A=(a_{ij})_{m\times m}$ is used to describe $G(D,E)$, where

$$a_{ij}=\begin{cases}1, & (d_i,d_j)\in E\\0, & (d_i,d_j)\notin E\end{cases}, \tag{2}$$

thus $a_{ij}=1$ denotes decision maker d_i directly trust d_j.

An adjacent matrix as per Definition 2 can only describe whether trust relationship between each pair of decision makers exists or not, with trust strengths among decision makers are not permitted. To overcome this issue, a weighted adjacent matrix is proposed. For notation simplicity, we still use $A=(a_{ij})_{m\times m}$ to denote a weighted adjacent matrix. In a weighted adjacent matrix $A=(a_{ij})_{m\times m}$, $a_{ij}\in[0,1]$ denotes the trust strength from decision maker d_i to decision maker d_j, with the larger the value of a_{ij} the higher the trust decision maker d_i has on decision maker d_j. An example of a weighted adjacent matrix, associated with the directed social network

depicted in Fig.2, is:

$$A = \begin{pmatrix} - & a_{12} & 0 & a_{14} & a_{15} \\ 0 & - & a_{23} & 0 & 0 \\ 0 & 0 & - & a_{34} & 0 \\ 0 & 0 & 0 & - & a_{45} \\ 0 & 0 & a_{53} & 0 & - \end{pmatrix}$$
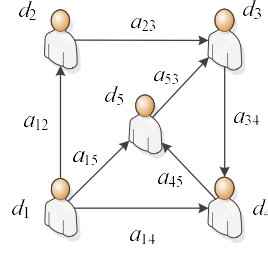

Fig.2. A weighted directed graph

For notation simplicity, adjacent matrix and weighted adjacent matrix are both called adjacent matrix in this paper.

Definition 3. A sequence of edges $(d_i, d_{i_1})(d_{i_1}, d_{i_2}) \cdots (d_{i_{n-1}}, d_j)$ in a social network $G(D, E)$ is called a trust path from decision maker d_i to decision maker d_j, and it is denoted as $d_i \rightarrow d_j$.

Definition 4. The in-degree centrality index $C(d_k)$ of decision maker d_k is defined as:

$$C(d_k) = \frac{1}{m-1} \sum_{i=1, i \neq k}^{m} a_{ik} \tag{3}$$

The in-degree centrality index of a decision maker in a social network $G(D, E)$ reflects the importance degree of this decision maker in the social network [71]. In general, the higher the value of the in-degree centrality index of a decision maker the higher the importance of such decision maker in the social network is, and therefore it is used to compute the weighting vector associated to a group of decision makers in SNGDM models (see Section 3.2). For example, if we set $a_{15} = 0.8$ and $a_{45} = 0.6$ in Fig.2, then the in-degree centrality index of decision maker d_5 can be computed by $C(d_5) = (a_{15} + a_{45}) / 4 = 0.35$.

3. CRP paradigm based on trust relationships in SNGDM

In this section, we introduce the CRP paradigm based on trust relationships in SNGDM.

Specifically, Section 3.1 presents the framework of a CRP paradigm based on trust relationships, Section 3.2 introduces the trust propagation, and Sections 3.3-3.5 review the main results regarding the use of trust relationships in different phases of a CRP.

3.1 The framework of CRP paradigm based on trust relationships

In recent years, the trust relationships among decision makers, described by a social network, are increasingly playing a key role in different phases of CRPs in the SNGDM problem: aggregation [9,75,80,82], incomplete preference values estimation [74,75,77], and feedback mechanism [8,76,79,83]. In the existing research proposals, many CRPs have been studied based on the use of trust relationships, and as such they can be classified as following the CRP paradigm based on the use of trust relationships as depicted in Fig.3.
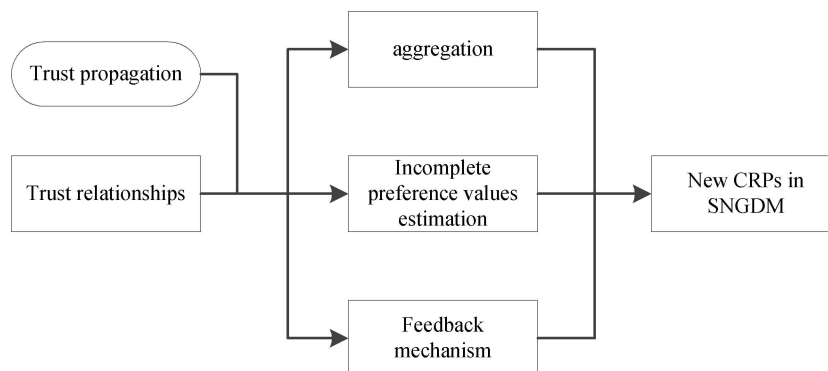

Fig.3. The CRP paradigm based on trust relationships

3.2 Trust propagation

In some situation, a decision maker may not provide trust values on every decision maker, which is illustrated in Fig. 4(a) with no direct trust from decision maker d_1 to decision maker d_3. However, some approaches have been developed to estimate the missing trust values in a social network. For example, Victor et al. [72] and Wu et al. [8,73] proposed to apply a t-norm operator to the trust values in the trust path $(d_1d_2)(d_2d_3)$ to estimate the trust value from decision maker d_1 to decision maker d_3 (see Fig. 4(b)).

In many cases, there might be more than one trust path available between two decision makers. For example, in Fig. 2 there are three trust paths from decision maker d_1 to decision

maker d_3: (1) $(d_1,d_2)(d_2,d_3)$, (2) $(d_1,d_5)(d_5,d_3)$, and (3) $(d_1,d_4)(d_4,d_5)(d_5,d_3)$. Several approaches have been proposed to deal with this situation. Table 2 summarizes the approaches to estimating unknown trust values in a social network.
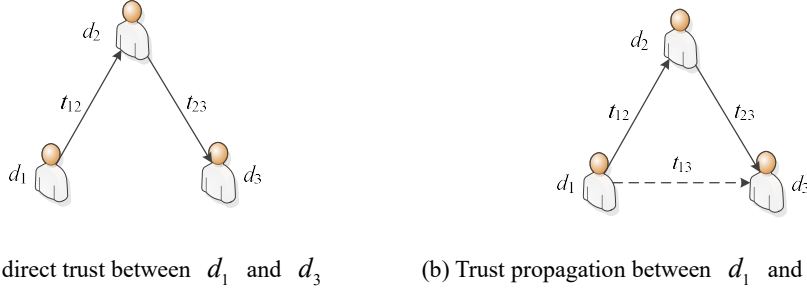

(a) No direct trust between d_1 and d_3 (b) Trust propagation between d_1 and d_3 via d_2

Fig. 4. Trust propagation via a trust path

Table 2. Approaches of estimating unknown trust value

Methods	One or more trust paths	References	Basic idea
The t-norm method	One trust path	Victor et al. [72]; Wu et al. [8,73]	The unknown trust value is calculated based on the t-norm.
Product method	One trust path	Liang et al. [74]; Wu et al. [75]	The unknown trust value is computed as the product of the trust values in the trust path.
Minimum rule	One trust path	Gupta [76]	The unknown trust value is regarded as the minimal direct trust value in the trust path.
Average similarity method	One trust path	Xu et al. [77]	The unknown trust value is calculated based on the distances between collective preference and the individual preferences.
Aggregation	Multiple trust paths	Wu et al. [75]	The unknown trust value is computed by aggregating the estimated trust values associated with each trust path.
Shortest path rule	Multiple trust paths	Wu et al. [8,73]	The unknown trust value is regarded as the estimated trust value corresponding to the shortest trust path.
Maximum rule	Multiple trust paths	Liang et al. [74]; Gupta [76]	The unknown trust value is regarded as the maximum estimated trust value among the trust paths.

Generally, in many CRPs with trust relationships, unknown trust values are estimated first. However, it is noted that there exist some studies (see [9,78-81]) that develop CRPs based on trust relationships with unknown trust values.

3.3 Preferences aggregation driven by trust relationships

As described in Section 2.1, the individual preferences are fused to obtain a collective preference, which requires the application of an aggregation operator with a weighting vector associated to the set of decision makers as its key element. Generally, decision makers' weights are

assumed known beforehand. However, such assumption is unrealistic or improbable in some circumstance. In a social network, decision makers with more importance are often trusted more by other decision makers, which indicates that their weights can be considered higher in SNGDM. Some methods have been proposed to obtain the weights of decision makers from the trust relationships, as mentioned before in Section 2.2, which are subsequently implemented in the preference fusion process.

Indeed, in [9,75,80,82] a WA operator is used to aggregate the individual preferences with weights computed from the trust relationships, while in [8,40,78,81,83] the I-IOWA operator is utilized to fuse the individual preferences, with importance weights derived from the concept of social influence. Liang et al. [74] proposed a linear programming model to determine the collective preference with weights derived from trust relationships among decision makers.
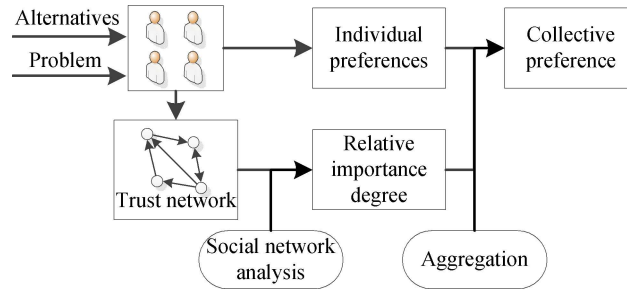

Fig.5. Preferences aggregation driven by trust relationships

Compared with traditional CRPs, the novel CRPs use the techniques of social network analysis to compute decision makers' weights which are subsequently implemented in the aggregation process as per Eq.(1). In SNGDM the preferences aggregation driven by trust relationships is described in Fig.5 and includes a two-step procedure:

(1) *Computing relative importance degree.* As mentioned before, the in-degree centrality index can be interpreted as a measurement of the importance of decision makers in a social network. The relative importance degree λ_k of decision maker d_k is calculated as follows:

$$\lambda_k = \frac{C(d_k)}{\sum_{i=1}^m C(d_i)} \tag{4}$$

(2) *Aggregating the preferences of decision makers.* In this phase, the individual preferences of decision makers are aggregated into a collective one, p_{ij}^c, by using an aggregation operator with the weighting vector, $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_m)$, being the decision makers' relative importance

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degrees:

$$p_{ij}^c = f_\lambda(p_{ij}^1, p_{ij}^2, \cdots, p_{ij}^m) \tag{5}$$

Preferences aggregation driven by trust relationships obtains the weighting vector from the measurement of the importance of decision makers in a social network, which provides with a more solid theoretical foundation and basis for their validity by comparison with the above assumption of being known beforehand.

3.4 Incomplete preference values estimation based on trust relationships

Sometimes, due to limited expertise, domain complexity or pressure in making a decision, a decision maker would find it difficult or even impossible to provide assessment on every alternative. This leads to incomplete preferences, for which various approaches/methods have been proposed (e.g., [84-87]) to complete them by estimating the unknown/missing values. It is reasonable to expect that the more one person trusts another, the more this person will agree with his/her preference. Thus, incomplete preference values of a decision maker in SNGDM are estimated based on the preferences provided by other decision makers he/she trusts using trust relationships-based methods.

Trust values reflect the importance of decision makers' preferences in the incomplete preference values estimation [75]. The missing preference values are estimated using the preference of the decision maker most trusted by the decision maker who provides the incomplete preference [77]. In order to improve the consistency degree, Liang et al. [74] proposed a linear programming method with two objectives: to minimize the distance between the estimated preference and the expert supporters' weighted preferences and to maximize the consistency degree.
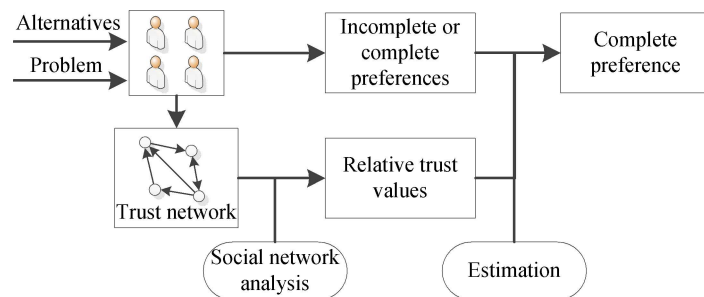

Fig.6. Incomplete preference values estimation based on trust relationships

According to the SNGDM models, the incomplete preference values estimation is carried out in a two-step procedure presented in Fig.6:

(1) *Generating the decision makers' relative trust values from trust relationships*. The relative trust value ϑ_{kl} that decision maker d_k assigns to decision maker d_l is derived via a normalization step:

$$\vartheta_{kl} = \frac{a_{kl}}{\sum_{i=1, i \neq k}^{m} a_{ki}} \tag{6}$$

If both decision makers d_k and d_l express incomplete preferences, then $a_{kl} = 0$ in estimating the unknown value of decision maker d_k. In some models, the maximum method is applied to determine the relative trust values [77], i.e., if decision maker d_k assigns the highest trust value to decision maker d_l, then $\vartheta_{kl} = 1$, and $\vartheta_{ki} = 0$ for $i \neq l$.

(2) *Estimating the unknown values based on the relative trust values*. The incomplete preference value of decision maker d_k, p_{ij}^k, can be estimated based on the rest decision makers' preferences by an aggregation process with weighting vector $\vartheta = (\vartheta_{k1}, \cdots, \vartheta_{k,k-1}, \vartheta_{k,k+1}, \cdots, \vartheta_{km})$:

$$p_{ij}^k = f_\vartheta(p_{ij}^1, \cdots, p_{ij}^{k-1}, p_{ij}^{k+1}, \cdots, p_{ij}^m) \tag{7}$$

In summary, incomplete preference values estimation based on trust relationships estimates the unknown/missing values of a decision maker from the preferences of other decision makers he/she trusts, which is convenient and reasonable. However, the estimation accuracy is closely related to the degree of agreement of their preferences. When the preference of a decision maker is quite different from those he/she trusts, the estimation error is likely to be large and the individual consistency will be ignored to some extent.

3.5 Feedback mechanism guided by trust relationships

In traditional CRPs, decision makers contributing less to consensus are advised to modify their opinions to values closer to the collective preference in an attempt to increase the group consensus level. Considering trust relationships among decision makers, the advice may be more acceptable for the decision maker if it is coming from other decision makers he/she trusts.

Following this idea, some feedback mechanisms guided by trust relationships are developed in SNGDM [8,74,76,79,83]. If a decision maker d_k is an identified expert who is advised to revise his/her preference value, $p_{ij}^{k,t}$, for assessing the pair of alternatives (x_i, x_j) at consensus round t, the feedback process guided by trust relationships is structured as shown in Fig.7 and applied as follows:
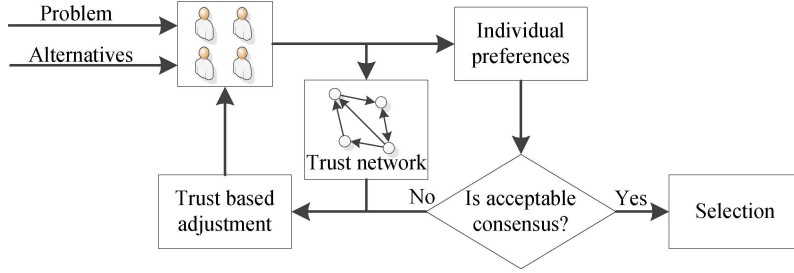

Fig.7. The feedback mechanism guided by trust relationships

(1) Advise decision maker d_k to modify his/her preference value closer to the preference value of decision maker d_l with minimum preference similarity with d_k [74]. The following cases are considered.

Case A: If decision maker d_k directly trust decision maker d_l, the new preference value, $p_{ij}^{k,t+1}$, presented to decision maker d_k is inferred as:

$$p_{ij}^{k,t+1} = (1 - a_{kl}) p_{ij}^{k,t} + a_{kl} p_{ij}^{l,t} \tag{8}$$

Case B: If decision maker d_k do not directly trust decision maker d_l, d_k will judge the social influence of d_l to form his new preference value. Let $\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$ be the set of relative importance degrees of decision makers as described before. Then, the next round preference value presented to decision maker d_k can be inferred as:

$$p_{ij}^{k,t+1} = (1 - \frac{\lambda_l}{\lambda_l + \lambda_k}) p_{ij}^{k,t} + \frac{\lambda_l}{\lambda_l + \lambda_k} p_{ij}^{l,t} \tag{9}$$

(2) Advise decision maker d_k to modify his/her preference value closer to the preference value of decision maker d_l with highest centrality index in the highest consensus level cluster [83]:

$$p_{ij}^{k,t+1} = (1-\sigma)p_{ij}^{k,t} + \sigma p_{ij}^{l,t} \qquad (10)$$

where $\sigma \in [0,1]$ is a parameter to control the degree of advice.

(3) Advise decision maker d_k to modify his/her preference value closer to the collective preference value of the decision makers he/she trusts [79]. Assume that there are r decision makers trusted by decision maker d_k with $p_{ij}^{rc,t}$ being their collective preference value at round t. The next round preference value presented to decision maker d_k will be:

$$p_{ij}^{k,t+1} = (1-\delta)p_{ij}^{k,t} + \delta p_{ij}^{rc,t} \qquad (11)$$

where $(\vartheta_{k1}, \vartheta_{k2} \cdots, \vartheta_{kr})$ is used as the weighting vector in an aggregation operator to compute $p_{ij}^{rc,t}$ and $\delta \in [0,1]$ is a parameter to control the degree of advice.

(4) Advise decision maker d_k to modify his preference value closer to the collective preference value $p_{ij}^{c,t}$:

$$p_{ij}^{k,t+1} = (1-\theta)p_{ij}^{k,t} + \theta p_{ij}^{c,t} \qquad (12)$$

where the collective preference, $p_{ij}^{c,t}$, is computed using trust relationships based weights as proposed in Wu et al. [8], while the parameter $\theta \in [0,1]$ is inferred from the trust relationships among the decision makers as proposed in Gupta [76].

Feedback mechanism guided by trust relationships includes several methods on providing advice, and its relies on the fundamental assumption of advice for a decision maker that comes from trusted decision makers would be more persuasive and acceptable. Evidently, this type of feedback mechanism guided by trust relationships is more realistic than the traditional feedback mechanism.

Notably, in Eqs.(10)-(12), the parameters (e.g. σ, δ and θ) are used to control the degree of advice, and they can be controlled to attenuate the extent of the change required to increase the group consensus. The higher the value of such parameters, the closer the decision maker's modified preference will be to the preference value of reference derived from the trusted experts' preference values. The actual parameter values to implement are inferred from the trust relationships [74,76] or selected to minimize the distance between the original preference value

and the new one that will meet the consensus threshold set in advance by the group of experts [83].

4. CRP paradigm based on opinion evolution in SNGDM

In this section, we review the CRP paradigm based on opinion evolution in SNGDM. Specifically, Section 4.1 introduces the framework of CRP paradigm based on opinion evolution, Section 4.2 introduces the opinion evolution and stable opinions, and Section 4.3 reviews the opinion management strategies for achieving a consensus.

4.1 The framework of CRP paradigm based on opinion evolution

Dating back to 1956, French and John [139] first proposed the basic model of opinion evolution. Since then, opinion evolution models with different evolution rules have emerged, such as the DeGroot model [12,13], Friedkin and Johnsen model [10,11], and bounded confidence model [14,103], among others. Although opinion evolution has been widely studied, we argue that the CRP based on opinion evolution in SNGDM is still at its early stage.

In the CRP paradigm based on trust relationships covered in Section 3, there exists a moderator in the models to guide the change of opinions that derive from the application of the IR and DR rules. However, in reality decision makers' opinions will evolve due to their interaction in a social network. By taking the opinion evolution into account, opinion evolution based consensus models have been developed that differ on the evolution rules they implement [14,15,70]. The framework of the CRP paradigm based on opinion evolution in SNGDM is depicted in Fig. 8, and consists of six key elements: (1) preference representation, (2) social network, (3) opinion evolution, (4) stable opinions, (5) opinion management, and (6) selection. Among these elements, (1), (2) and (6) coincide with those described in Section 2, so we proceed to describe the other three in the following.

(a) *Opinion evolution*. In a social network there exist lots of interactions among decision makers and as a result the opinions of decision makers evolve, which has been widely studied in the discipline of opinion evolution (see Section 4.2).

(b) *Stable opinions*. After several rounds of opinion evolution, the opinions of decision makers will form a stable structure: consensus, polarization, or fragmentation [14,15]. If the

opinions are equal in the stable state, then a consensus is reached. On the other hand, when two or more different opinions exist in the stable state, they indicate polarization and fragmentation, respectively.

(c) *Opinion management*. When the stable opinions are not in a consensus state, opinion management [15,88,89] is used to facilitate consensus, which is described in detail in Section 4.3.
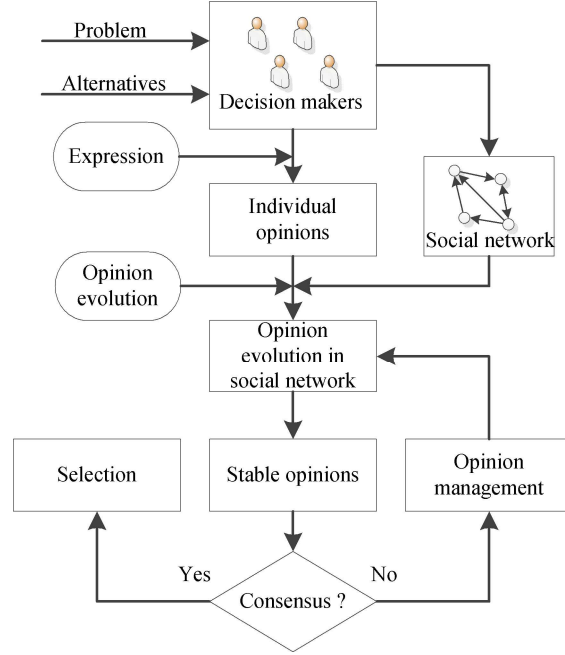

Fig.8. The CRP paradigm based on opinion evolution

4.2 Opinion evolution and stable opinions

In opinion evolution, each decision maker will take into account the opinions of other decision makers. However, they neither simply share nor strictly disregard the opinions of others. Generally, they refer to the opinions of others to a certain extent and modify their own opinions. The opinion evolution process can be formulated as a dynamical process in discrete time, where consensus, polarization, or fragmentation happens in the stable state. A recent survey on opinion evolution can be found in Dong et al. [15].

Let $p^{k,t}$ be the preference value of decision maker d_k at round t. Let w_{kl} be the weight that decision maker d_k assigns to decision maker d_l, where $w_{kl} \geq 0$ and $\sum_{l=1}^{m} w_{kl} = 1$. Then, the opinion evolution process is formulated as follows:

$$p^{k,t+1} = w_{k1}p^{1,t} + w_{k2}p^{2,t} + \cdots + w_{km}p^{m,t} \quad , \quad t = 0,1,2,\cdots \tag{13}$$

or equivalently as:

$$P^{t+1} = W \cdot P^t, \quad t = 0, 1, 2, \cdots \tag{14}$$

where $W = (w_{ij})_{m \times m}$ and $P^t = (p^{1,t}, p^{2,t}, \cdots, p^{m,t})^T$.

On the one hand, the decision makers will form a consensus if $\lim_{t \to \infty} p^{i,t} = c(i = 1, 2, \cdots, m)$ for any $P^0 \in R^n$, where c is called the consensus opinion [14,70]. On the other hand, if there are two or more different opinions at the final stage, then we have a polarization or fragmentation in opinion evolution, respectively. Several opinion evolution models are summarized in Table 3.

Table 3. Some opinion evolution models

Opinion evolution models	Basic idea	References
DeGroot model	The weights do not change across the time or with opinions.	Degroot [12]; Berger [13]; Dong et al. [70]; Han et al. [90]; Lehere [91-93]
Friedkin and Johnsen model	The weights are independent on time, but everyone adheres to his initial opinion to a certain degree.	Pérez et al. [10]; Friedkin et al. [11,94]
Bounded confidence model	The weights depend on opinions.	Hegselmann et al. [14]; Dittmer [95]; Deffuant [96]; AskariSichani et al. [97]; Kurmyshev et al. [98]; Stauffer et al. [99]; Weisbuch [100]; Jalili [101]; Quattrociocchi et al [102]; Deffuant et al [103]; Fortunato [104]; Zollman [105]
Time-variant model	The weights are dependent on time.	Hegselmann et al. [14]; Cohen et al. [106]
Voter model	A decision maker updates his/her opinion based on a randomly selected neighbor.	Frachebourg et al. [107]; Schneider-Mizell et al. [108]; Holley et al. [109]; Basu et al. [110]; Castellano et al. [111]; Castellano et al. [112]; Sood et al. [113]; Diakonova et al. [114]
Majority rule model	The decision makers are randomly divided into several groups and the majority opinion will be voted as the representative of the group.	Chen et al. [115]; Lambiotte [116]; Lanchier et al. [117]
Sznajd model	A decision maker is easier to be persuaded by two or more decision makers who share the same opinion than by a single decision maker.	Stauffer et al. [118]; Rodrigues et al. [119]; Elgazzar [120]; Bernardes et al. [121]

The trust relationships among decision makers in social network, as previously covered, play a particular key role in opinion evolution. Thus, opinion evolution is often studied in a social

network context [90,94]. The DeGroot model [12-14] is considered the "classical" model and it is widely used in opinion evolution. In the following we introduce DeGroot model in a social network context (see Dong et al. [70]), with the other opinion evolution models being similarly analyzed.

Let $A = (a_{ij})_{m \times m}$ be the adjacent matrix associated with $G(D,E)$ as described before. Suppose that the decision maker d_k gives the trust value $\beta_k \in (0,1)$ to his own opinion and assigns $(1 - \beta_k)$ to others' opinions. Then, the weight w_{kl} decision maker d_k assigns to decision maker d_l is computed as [70]:

$$w_{kl} = \frac{(1 - \beta_k)a_{kl}}{\sum_{l=1,l \neq k}^{m} a_{kl}} \tag{15}$$

Let $p^{k,t} \in [0,1]$ be the preference value of decision maker d_k at round t. Thus, the opinion evolution of decision maker d_k is represented as [70]:

$$p^{k,t+1} = \beta_k p^{k,t} + \sum_{l=1,l \neq k}^{m} w_{kl} p^{l,t} \tag{16}$$

Eq. (16) can also be written as:

$$P^{t+1} = W^* \cdot P^t, \quad t = 0,1,2,\cdots \tag{17}$$

where

$$W^* = \begin{bmatrix} \beta_1 & w_{12} & \cdots & w_{1m} \\ w_{21} & \beta_2 & \cdots & w_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \cdots & \beta_m \end{bmatrix} \tag{18}$$

and $P^t = (p^{1,t}, p^{2,t}, \cdots, p^{m,t})^T$.

Because the weights are fixed, Eq.(16) and Eq.(17) are regarded as the DeGroot model, which we call the social network DeGroot (SNDG) model in this paper. Dong et al. in [70] proposed and proved a consensus condition of the SNDG, which are given in Definition 5 and Theorem 1, respectively.

Definition 5 [70]. In $G(D,E)$, opinion leader is defined as the decision maker(s) in the set $D_G^{leader} = \{d_k \mid d_i \to d_k, \text{ for all } d_i \in D / \{d_k\}\}$, and follower is defined as the decision maker(s) in

the set $D_G^{follower} = \{d_k \mid d_k \notin D_G^{leader}\}$.

Example: In Fig. 9(a), there exist trust paths from the other two decision makers to decision maker d_2 and decision maker d_3, respectively, which means that $D_G^{leader} = \{d_2, d_3\}$; however in Fig. 9(b), no one is a leader, i.e., $D_G^{leader} = \varnothing$.
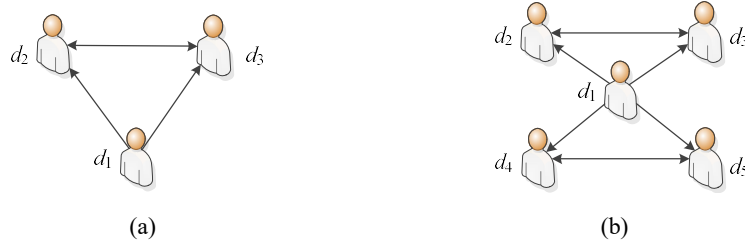

Fig.9. The opinion leaders in a social network

Theorem 1 [70]. All decision makers in the social network $G(D, E)$ can form a consensus under the condition that the set of opinion leaders is nonempty, i.e., $D_G^{leader} \neq \varnothing$, and the consensus opinion c is determined by the original opinions of opinion leaders. i.e.,

$$c = \sum_{d_k \in D_G^{leader}} \omega_k \, p^{k,0}, \text{ where } \omega_k \geq 0 \text{ and } \sum_{d_k \in D_G^{leader}} \omega_k = 1.$$

4.3 Opinion management

The decision makers' opinions will be stable after a few rounds of opinion evolution, although consensus might not be the final outcome. In this case, an opinion management strategy is needed to help decision makers reach consensus. The following several approaches to manage opinions have been developed in the existent references (see Table 4): changing network structure, adjusting opinions, the use of informed decision makers, and the use of media.

In the SNDG model, all the decision makers cannot form a consensus at the social network $G(D, E)$ when $D_G^{leader} = \varnothing$. In Dong et al. [70], an optimization-based model is proposed to add the minimum number of edges to create a new social network $\bar{G}(D, \bar{E})$ with $D_{\bar{G}}^{leader} \neq \varnothing$ and $E \subset \bar{E}$:

$$\begin{cases} \min_{\bar{E}} (\#(\bar{E}) - \#(E)) \\ s.t \quad E \subset \bar{E} \\ \qquad D_{\bar{G}}^{leader} \neq \varnothing \end{cases} \tag{19}$$

In order to solve model (19), a two-step procedure (see Fig. 10) is needed:

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Table 4. Some approaches of opinion management

| Opinion management	Basic idea	References
Changing network structure	Adding edges or nodes into the network to influence the opinion evolution.	Dong et al. [70]; Hegselmann et al. [89]; Han et al. [90]; Han et al. [122]; Han et al. [123]
Adjusting opinions	To persuade the decision makers to modify their opinions toward the desired direction.	Kurz [124]; Ding et al. [125]
Informed decision makers	Informed decision makers pretend to have an opinion similar to others with the purpose that gradually influence the opinions of others to a desired direction.	AskariSichani et al. [97]; Afshar et al. [126]; Fan et al. [127];
Media	To influence decision makers' opinions through the media, so as to achieve the purpose of opinion management.	Mckeown et al. [88]; Quattrociocchi et al. [102]; Crokidakis [128]; Wu et al. [129]; Pineda et al. [130]; Colaiori et al. [131]; Schulze [132];
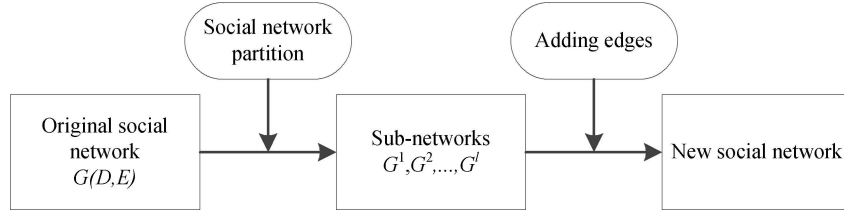
Fig.10. Framework of adding edges

(1) *Social network partition*. A Network Partition Algorithm [70] with the time complexity $O(m^3)$ is proposed to divide $G(D,E)$ into a partition $M = \{G^1(D^1, E^1), \cdots, G^l(D^l, E^l)\}$ whose sub-networks $G^k(D^k, E^k)(k=1,2,...,l)$ satisfy the following three properties:

(a) Completeness: $D = \bigcup_{k=1}^{l} D^k$. It guarantees that all decision makers in $G(D,E)$ are assigned to its sub-networks.

(b) Leadership: For any sub-network $G^k(D^k, E^k) \in M$, the set of opinion leaders is not empty, i.e., $D_{G^k}^{leader} \neq \varnothing$. It means that all sub-networks can form a consensus, respectively.

(c) Disjointness: Let $G^k(D^k, E^k)$ and $G^h(D^h, E^h)$ be any two sub-networks in M, and let $G^{kh}(D^{kh}, E^{kh})$ be the union of them, where $D^{kh} = D^k \bigcup D^h$ and $E^{kh} = \{(d_i, d_j) | (d_i, d_j) \in E; d_i, d_j \in D^{kh}\}$. Then, $D_{G^{kh}}^{leader} = \varnothing$, i.e., the decision makers in the union $G^{kh}(D^{kh}, E^{kh})$ cannot reach a consensus.

(2) *Adding edges*. In Dong et al. [70], a method to create opinion leader(s) in two sub-networks is presented, which is described as Theorem 2.

Theorem 2 [70]. If an edge e is added into the union $G^{kh}(D^{kh}, E^{kh})$ of sub-networks $G^k(D^k, E^k)$ and $G^h(D^h, E^h)$ to obtain $\bar{G}^{kh}(D^{kh}, \bar{E}^{kh})$, then $D_{\bar{G}^{kh}}^{leader} \neq \varnothing$ if and only if $e \in E^{add}$, where $E^{add} = \{(d_i, d_j) \mid d_i \in D_{G^k}^{leader}, d_j \in D^h\} \bigcup \{(d_e, d_r) \mid d_e \in D_{G^h}^{leader}, d_r \in D^k\}$.

Based on Theorem 2, an Adding Edge Algorithm [70] with the time complexity $O(m)$ is developed to construct the new social network $\bar{G}(D, \bar{E})$ in which the decision makers can reach a consensus, and $\min_{\bar{E}}(\#(\bar{E}) - \#(E)) = l - 1$.

5. Summary, critical discussions and new directions

Consensus reaching in a SNGDM problem has become a productive research field in recent years. Two CRP paradigms (based on trust relationships and opinion evolution) have been developed in the SNGDM.

The CRP paradigm based on trust relationships analyzes the effect of the trust relationships in the different aspects of CRPs: incomplete preference values estimation, aggregation, and feedback mechanism. The CRP paradigm based on opinion evolution involves two key elements: opinion evolution and opinion management. Opinion evolution simulates the interaction among decision makers, while opinion management focuses on the design of strategies for facilitating a consensus.

In the two paradigms, there still exist some limitations that need to be highlighted:

(1) We can see that the trust relationships, defined by a social network, play an important role in CRPs in the SNGDM. In the existing SNGDM CRPs, the trust relationships are assumed to be static during the consensus process [8,73-83]. However, in reality the trust relationships between decision makers in a social network will change dynamically which obviously can influence the consensus process [142].

(2) Society and technology trends demand the management of large-scale decision problems [61,64,68] on social networks. However, the presented studies of the CRP paradigm based on trust relationships rarely relate to large-scale network in the SNGDM.

(3) The CRPs based on trust relationships [8,73-76,79,80] follow the classical CRP

framework, in which IR and DR rules are used to adjust the decision makers' original opinions to increase consensus. However, the existing CRPs based on trust relationships rarely take into account the cost of reaching consensus, and also have not analyzed the possibility of reaching consensus via the improvement of trust relationships.

(4) Although opinion evolution has been widely studied and justified, where consensus, polarization or fragmentation are the main focus of these studies [11-15,94-100], we argue that the CRP paradigm based on opinion evolution in the SNGDM context is still at the early stage, and that there are new features still possible to be considered to build a consensus. Particularly, the interactions and opinion evolution among decision makers may be complex, emotional, and unstable in the real world.

(5) We find that most SNGDM studies on the two paradigms [11-14,70,73-83] focus on the theory and methodology, with few SNGDM studies using real data and tackling practical problems.

Thus, future research on this topic can be developed by considering the following directions:

(1) Trust relationships, although playing an important role in the CRPs in SNGDM, are difficult to obtain in real life. Thus, it is necessary to develop approaches to automate the identification of trust relationships among decision makers. Moreover, it would be interesting to further develop CRPs in dynamical social network contexts.

(2) It is necessary to study the large-scale CRP paradigm in SNGDM because many large-scale SNGDM problems exist in our real world. Particularly, complex networks (e.g. Erdos-Rényi random graph [133], the small-world network [134], and the scale-free network [135]) would be a powerful tool to represent the relationships among decision makers.

(3) Generally, CRPs have associated cost/resource constraints [5,46,49,50]. It will be an interesting research direction to develop a CRP model to minimize the cost in social network contexts.

(4) Some decision makers may manipulate the opinions and relationships towards to an established purpose [60,61,70,136-138]. It would be interesting to study the manipulation and non-cooperative behaviors in SNGDM.

(5) Although belonging to a different discipline, opinion evolution is increasingly considered a tool to model SNGDM problems. Thus, it is necessary to further develop a theoretical basis to

build a robust bridge between these different disciplines.

(6) In SNGDM decision makers will not only use heterogeneous preference representation structure [35,39] or different expression domains [31,36, 45,143], but also will measure consensus based on individual satisfaction [68]. Thus, it is necessary to develop new SNGDM CRP models with the consideration of personalities of decision makers.

(7) Because most SNGDM studies concentrate on the theory and methodology, it would be interesting to investigate and validate the developed SNGDM CRPs with real data to arrive at useful real data-driven consensus models to tackle real-world problems.

6. Conclusions

This paper reviews the CRPs in the SNGDM context. Firstly, we introduce the CRP framework and the basic concepts of social network used in GDM. Secondly, we review the CRP paradigm based on trust relationships in which the social relationships are used to estimate unknown/missing preference values, as well as in aggregation and feedback mechanism of the CRP. Thirdly, we review the CRP paradigm based on opinion evolution in which the opinion evolution and opinion management are introduced in detail. Finally, we note some critical comments according to the existing studies and suggest new directions to advance this area of research.

Acknowledgments

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