Estimation of demand diversity and daily demand profile for offgrid electrification in developing countries

P. Boait a,*, V. Advania, R. Gammona

^aInstitute of Energy and Sustainable Development, De Montfort University, Queens Building, The Gateway, Leicester LE1 9BH, UK.

Abstract

The potential for small self-contained grid systems to provide electricity for currently unserved regions of the developing world is widely recognised. However planning and managing the electrical demand that will be supported, so that a mini-grid system is not overloaded and its available resource is used as fully as possible, is actually more difficult than for a large scale grid system. This paper discusses the mathematical reasons why this is the case, and describes a practical software tool for mini-grid demand estimation and planning that is complementary to the widely used HOMER software. This software tool is made available for download on an open source basis. Finally a conclusion is offered that mini-grid systems should aim to serve at least 50 households so that demand variability is more manageable and economies of scale can be realised.

Keywords: mini-grid; micro-grid; demand diversity; Monte Carlo model

1. Introduction

There remains 18% of the world's population without access to electricity (IEA 2014). Substantial progress has been made through innovations such as the solar home system (Komatsu et. al. 2011), but the full potential of electricity for lifting people out of poverty can only be achieved when it is available at the cost and capacity levels needed for commercial applications such as processing or storage of agricultural produce. The obvious way to drive down the cost and increase the availability of electricity is through the economy of scale provided by some form of grid supply. However, conventional grid connection is not a practical or economic solution for a substantial proportion of this population, particularly in

^{*} Corresponding author. Tel.: +44-116-257-7980; fax +44-116-257-7981; *E-mail address*: p.boait@dmu.ac.uk

Africa (Szabo et al. 2011). Also, arguably the architecture of large scale fossil fuelled generation, accompanied by high voltage high capacity transmission and distribution networks, is no longer universally appropriate given the need to avoid carbon emissions by employing renewable energy sources that are geographically dispersed. The emergence and growth of localised electricity generation and distribution in developed economies reflects this reality (DECC 2014). These arguments make mini- or micro-grids attractive as the way forward for rural electrification (ARE 2011). Such grids will serve a local community and either have no connection to a national grid system at all (hence off-grid) or have a connection that may be either severely limited in capacity relative to the local demand or unreliable. The potential for mini-grids to meet the needs of this unserved population has been shown by many practical demonstration projects (Yadoo and Cruickshank 2012) and start-up enterprises (Access:energy 2015), but large scale rollout of mini-grid technologies has not yet happened.

One of the barriers to exploitation of this potential is the need to sustain a balance between electricity supply and demand, which begins at the planning and design stage of a mini-grid project and then must be achieved continuously in subsequent operation. For a national grid system this is performed by the System Operator¹ (SO). Their role is recognised as critical, and they will expect to invest in a range of costly and sophisticated tools to help them discharge this function. For a mini-grid exactly the same role has to be performed, but with resources scaled down accordingly and often with the additional constraints arising from a remote or rural location. The purpose of this paper is to describe and make available a simple software tool that can assist mini-grid designers and operators in this difficult task. It allows the peak, average, and variability of demand to be predicted from a given population of consumers and appliances, and it presents results in a form that is compatible with the

_

¹ System Operator is the generic name given to the organisation responsible for ensuring a real time balance between supply and demand on a large scale grid by despatching generation or manageable demand.

popular HOMER software package that is widely used for mini-grid research, planning and design (Lambert et al. 2006, Mondal and Denish 2010).

2. Prediction of electricity demand

The aggregate electrical demand² presented at any time to the generator of a mini-grid will be composed of a number of individual loads arising from particular devices and appliances that have been switched on, and will be switched off, at times determined either by a human user or by some automated control responding to the environment of the powerconsuming appliance. While there will be some correlation of operating times for loads with related functions, such as lighting coming on in the evening, as long as the decision taking processes that determine times of operation of each load are independent, the precise population of operating loads at any given time will be uncertain. For a few households with limited electricity consuming devices (perhaps progressing from solar home systems to a shared PV-powered micro-grid) it is likely that at some time all will switched on and the maximum possible demand will be the sum of the loads drawn by all the available appliances. However, this will be unusual, and as the number of power-consuming households and businesses rises, and they start to collect a range of appliances for different purposes, the likelihood of every available appliance being presented simultaneously becomes negligible. The challenge then is to decide what maximum demand can be expected from a given population. The ratio between the maximum demand likely to occur in practice and the total possible demand is known as the diversity factor, which is often expressed as a percentage.

When planning electricity distribution supplied from a conventional grid system, the calculation of diversity has traditionally used a combination of heuristic formulae and

_

² The terms "demand" and "load" are often used interchangeably in electrical engineering. In this paper demand is used to refer to the total electrical power consumed by a set of individual active loads.

engineering judgement. As the classic work by Fred Porges on electricity distribution in buildings (Porges 1974) puts it:

"One can apply a diversity factor to the total installed load to arrive at the maximum simultaneous load. To do this, one needs an accurate knowledge of how the premises are going to be used, which one can get by a combination of factual knowledge and intuition......A general knowledge of life and how buildings are used may be of more help than theoretical principles"

This reflects the difficulty in quantitative characterisation of the aggregate demand expected from a given population of electrical appliances and people. The goal in applying mathematics to this problem must be to guide and clarify the human judgement that is essential to arrive at a design or management decision. This challenge is particularly evident to the designers and operators of a mini-grid in the developing world where the available generating resource is unlikely to match the latent demand and there is a strong incentive to generate cash flow from new consumers and loads. Managers are often under pressure to maximise income to repay capital cost or to meet battery or diesel generator replacement costs. This can easily lead to overload with consequences such as brownouts which reduce consumer confidence in the service (Quetchenbach et. al, 2013) or excessively deep discharge of batteries which results in shorter lifetimes and higher costs.

3. Modelling methods

The central limit theorem provides a useful model of the aggregate demand arising from a population of loads each of whose power consumption expressed as a time series is intermittent and stochastic. It states that the means of n independent samples drawn from any distribution with mean m and standard deviation σ will have an approximately normal distribution with a mean equal to m and a standard deviation equal to σ/\sqrt{n} . This implies that

as the number of electricity-consuming appliances n served by a grid increases, the variability (standard deviation) of their total electricity consumption will decrease by a factor of $1/\sqrt{n}$. Figure 1 illustrates the radical effect of this in practice. It shows the results of multiple simulations of a number of refrigerators or freezers running with a 20% compressor duty cycle over a year, with randomisation of their relative operation as would occur in practice. This is referred to as a Monte Carlo simulation. It was performed in Matlab using the methods described in detail later in the paper. Each round point shows the maximum demand presented at any time during a simulated year of operation by that number of refrigerators, expressed as a fraction of the total demand that would occur if all their compressors operated simultaneously. The much higher proportionate demand presented for numbers below 20 is clear. Each square point shows the standard deviation observed during a simulation, again the much higher variability for lower numbers is evident. This highlights the challenge faced by the SO of a micro-grid serving perhaps 20 homes where the variability of demand is much higher than that faced by a national scale SO.

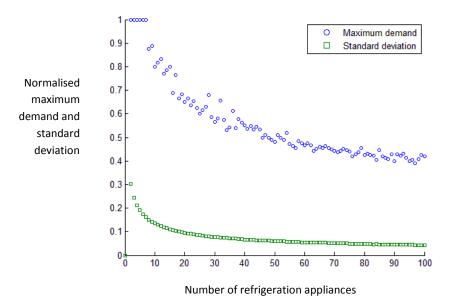


Fig. 1. Effect of number of power-consuming appliances on observed peak demand as a proportion of maximum possible demand

This form of Monte Carlo simulation is the basis of the demand prediction software tool described in this paper. However, before moving onto the detail of the software tool it is useful to review the heuristic methods of demand prediction routinely used in the electricity industry for national scale grids to show why they are unsuitable for micro- and mini-grids. The key parameter normally employed for sizing on-grid distribution network components is "After Diversity Maximum Demand" (ADMD), which is based on a methodology generally attributed to Boggis (1953), who recognised the phenomenon illustrated in Figure 1 and sought to derive simple approximations that could be used by network planners at a time when computer resources for simulation were limited. ADMD is defined as the maximum observed demand per consumer, as the number n of connected consumers, each consuming E_b approaches infinity:

$$ADMD = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{i=n} E_i$$
 (1)

ADMD is usually obtained by measuring demand over a year at a point of aggregation such as a transformer or transmission node and identifying the maximum observed for a particular time of day, then dividing by the number of consumers. Ideally the aggregation is of 1000 or more reasonably homogenous consumers. Over time distribution network operators have accumulated measured ADMD values from a range of network segments and use them to set predicted ADMD values in corporate engineering policy documents with rules for their application in the design of new network extensions – for example Smith (2003). The predicted ADMD (A) is then multiplied by the number A0 of consumers and a diversity-related factor A2 introduced to allow for the smaller population in the extension. Smith (2003) uses the linear approximation proposed by Boggis by assigning a constant A3 the branch distribution network branch, then calculating maximum demand A5 for the branch as:

$$D_m = An + k \tag{2}$$

Boggis (1953) also proposes as alternatives that reflect the asymptotic curve in Figure 1:

$$D_m = An(1 + \frac{k}{n}) \tag{3}$$

or:

$$D_m = An(1 + \frac{k}{\sqrt{n}}) \tag{4}$$

where k is a factor determined from measurement to fit a particular population of consumers.

It can be seen that the large n premise in the definition of ADMD makes it a questionable approach to mini-grid design. None of equations (2) - (4) is easily applicable at the planning stage for a mini-grid because reliable values for A are unlikely to be available, while the k factor is only obtainable by experience and is inherently less accurate for small n. Another issue is the practical need for prediction of maximum demand at different times of day so that the overall daily demand profile and generation resource utilisation can be assessed. This implies a need for multiple values of ADMD and k to create a profile using this method.

McQueen et al. (2004) have shown that a Monte Carlo simulation provides results that are consistent with this conventional method and can provide more accurate predictions of demand, particularly for the small consumer populations typical of mini-grids. They take measured demand profiles and disaggregate them into randomised loads from each consumer. The approach taken for the software tool described here is to simulate the aggregate consumer demand on a bottom up basis from three data elements:

 the population of each main type of electricity-consuming device or appliance available to the prospective or actual consumers;

- the typical load presented by each type;
- an assessment for each device type of the probability that it will be in use at the given time of day.

It is envisaged that the population data will come from a survey conducted during the planning process, or could comprise an initial set of devices such as light fittings that might be supplied as part of the mini-grid introduction. Alternatively it can be an estimate by the system manager based on the number of connected consumers and the typical appliance fit in their homes or workplaces. The loads presented by each type of appliance can be readily obtained by sample measurement or published data. The probability of use p must initially be a judgement which can be clarified over time by observation – this is where the "intuition" mentioned by Porges is needed. The simulation simply takes each device in the population, and at each time interval determines randomly whether it is "on" or "off" with a probability p and power consumed when on E. A binomial distribution of on and off states for each appliance X_i is created over n_t trials (time intervals):

$$X_i \sim bin(n_t, p_i) \tag{5}$$

Then the time sequence of aggregate demand D is simply the sum of these distributions over all N appliances:

$$D = \sum_{i=1}^{i=N} X_i E_i \tag{6}$$

The maximum demand D_m is the maximum value in D within the given number n_t of time intervals. The simulation computes the standard deviation of all the values of D in the set, and the mean demand D_{me} given by:

$$D_{me} = \sum_{i=1}^{i=N} E_i p_i \tag{7}$$

The duration of the simulation set by n_t is significant because as the length of the simulation increases the probability of picking up combinations of loads in the tail of the binomial distribution increases and hence the maximum observed demand rises asymptotically. Figure 2 illustrates the effect of increasing length on a simulation of the evening demand arising from 80 appliances of various types including lighting, refrigeration, and televisions with a total plated demand of 6kW. Each round point plots the maximum demand observed in a single simulation of the length indicated. Five simulations were performed at each length. The increasing average value (indicated by a square point) of the five runs with length and their narrowing spread are evident.

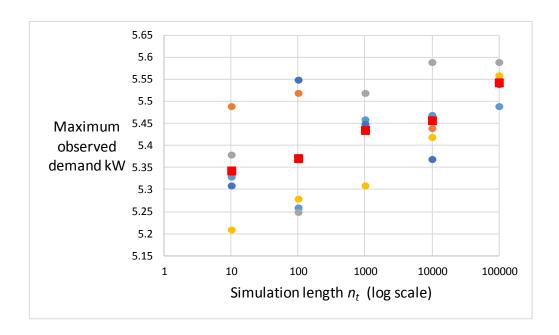


Fig. 2. Effect of increasing simulation length on maximum observed demand

The time interval that is chosen to be represented by a single binomial trial defines the time granularity of the simulation. As Figure 2 shows, for a simulation of length n_t there will always be a risk that a larger n_t representing the same overall time duration T will reveal a

higher maximum demand occurring over the shorter time interval T/n_t . Since a brownout lasting one minute is probably tolerable in a mini-grid if it is infrequent, the simulation tool proposed here takes one minute as the default time granularity. The usefully improved accuracy of 1-minute granularity over half-hourly is confirmed by McQueen et al. (2015). A value of $n_t = 100,000$ then corresponds to about 69 days of operation.

4. The ESCoBox mini-grid load model

An example screen presented by our simulation tool is shown in Figure 3. It is branded ESCoBox as that is the title of one of the sponsoring projects. This project has the goal of helping mini-grid operators to manage demand more effectively and thereby lower the cost and improve the availability of electricity to their consumers. The tool is used as follows. A list of appliance types is presented that is embedded within the tool and aims to cover all the common options. Additional types can be manually added at the bottom of the list. For each appliance type the power it uses when operating is shown in the Power Used column – this is a default typical figure that can be changed manually, and must be entered for new appliance types that are added to the list. The user then enters the number of each appliance type expected to be operating, and selects the expected duty cycle from values between 0.1 and 1.0 offered by a drop-down menu. Since the applicable population of appliances and their probability of use will often change during the day, any evaluation using the model must be associated with a time of day and probably a day of week where there is significant weekday dependency.

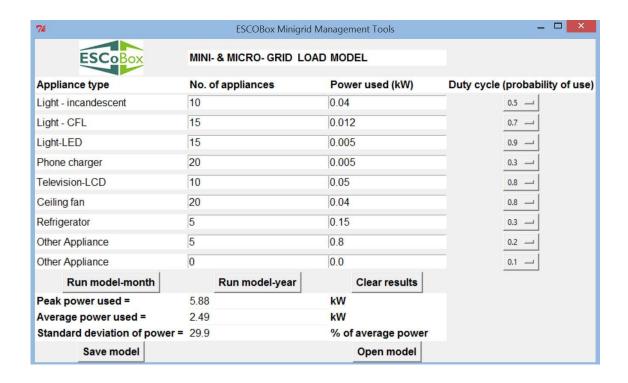


Fig. 3. Example screen of load model

The drop-down value can be interpreted in two ways. Where the appliance is operating continuously, such as a refrigerator, the value is the duty cycle of the compressor that provides the significant load. An irrigation pump filling a tank might similarly operate intermittently under the control of a level switch. Where the appliance is under human control, then the value is the probability that the appliance will be switched on during the time interval being considered. This probability may either reflect the probability of use, such as a light that may or may not be on in the evening, or intermittency of use, such as a hair drier that is employed for a few minutes at a time by a hairdresser.

The "Run model-month" and Run model-year" buttons initiate the simulation with n_t values of 44,640 and 535,680 respectively – these values are the number of minutes in an average month and in a year. Each run returns the observed maximum, mean, and standard deviation of demand. Table 1 shows the time taken to execute simulations for a range of scenarios on a Microsoft Surface laptop computer with i5 processor (1.7Ghz, 4GB RAM)

running Windows 8. Each scenario is based on the appliance types and duty cycles shown in Figure 3, which is intended to represent evening operation of a 20-home micro-grid (with some use of higher power appliances in the "other appliance" category). The 200-home and 2000-home scenarios were the same as that visible in Figure 3, but with appliance populations multiplied by 10 and 100 respectively.

Table 1. Run times in minutes and seconds for a range of mini-grid simulation sizes

Simulation length and size	20-home	200-home	2000-home
Month ($n_t = 44640$)	3s	14s	2m 7s
Year $(n_t = 535,680)$	18s	2m 12s	19m 50s

The ESCoBox load model is coded in Python 2.7. The code is published on an open source basis for inspection and download on GitHub (Boait 2015) with installation instructions. The usual caveat for open source software applies, that it is offered in the hope that it is useful but no assurance of fitness for purpose is given.

5. Use of the ESCoBox mini-grid load model with HOMER

The use of this tool in conjunction with HOMER is illustrated by the data entry screen for HOMER shown in Figure 4. The "Load" table on the left hand side requires average demand for each hour of the day and random variability percentages to be entered in the "Day-to-day" and "Time-step-to-time-step" fields. The effect of these variability values is defined in the HOMER documentation (HOMER 2015) as follows:

"1. For each day, HOMER draws a random number from a normal distribution with mean of zero and standard deviation equal to the daily noise value. That's the 'daily perturbation factor'.

- 2. For each hour, HOMER draws another random number from a normal distribution with mean of zero and standard deviation equal to the hourly noise value. That's the 'hourly perturbation factor'.
- 3. For each hour, HOMER multiplies the unperturbed load value by (one plus the daily perturbation factor for that day plus the hourly perturbation factor for that hour)."

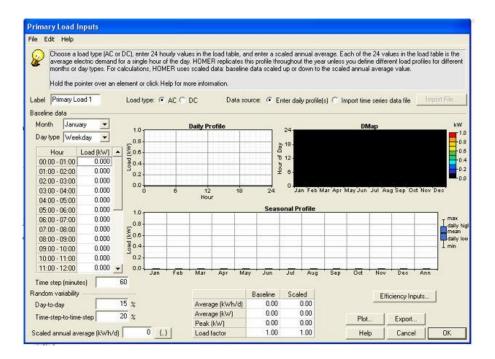


Fig. 4. HOMER load input screen

A normal distribution as employed by HOMER is a good approximation to a binomial distribution as used by the present model, as long as the population X of load-presenting appliances is large enough such that the steps in aggregate demand resulting from an individual appliance turning on or off are not significant. This will be the case for most practical purposes. A more important limitation in the HOMER method is the relatively small number of trials n_t . A randomised value for demand in a given hour in a day is only taken once per day in the simulation, so an n_t of 365 represents a year. As Figure 2 shows,

this may not reveal the maximum demand likely to occur, particularly if, as is assumed in this paper, peaks with a shorter duration than an hour are of interest. However, there is no question that HOMER is effective in illustrating the impact of demand variability. Figure 5 plots the peak demand calculated by HOMER for a range of values entered into the variability fields, using a real-life load dataset from a micro-hydro mini-grid (the Day 2 values shown in Figure 6).

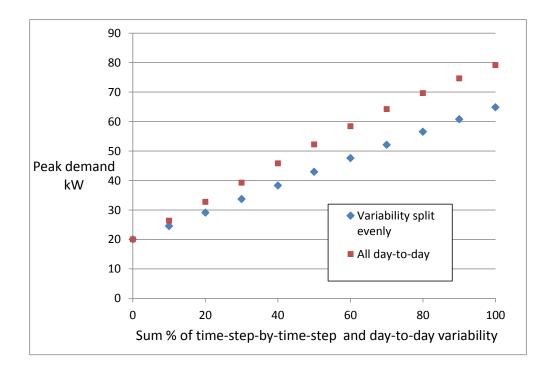


Fig. 5. Effect of HOMER variability factors on peak demand

The way in which HOMER splits the variability into two components means that their combined effect is less than the arithmetic sum of the two standard deviations³, unless one of the components is zero. So, for example, if 50% is entered for both, this gives a total variability of about 71%. The x-axis in Figure 5 indicates the total of both components, and the two plots respectively show the peak demand when the total variability is divided equally between components and when it is all allocated to day-to-day variability and the time-step-to-time-step value is zero.

-

³ The sum of two variables with standard deviations σ_1 and σ_2 has a standard deviation $\sigma_{total} = \sqrt{(\sigma_1^2 + \sigma_2^2)}$.

To use the ESCoBox model with HOMER, the first step is to perform multiple runs of ESCoBox to provide an average daily demand profile for the load column in HOMER. Table 2 shows an example set of 7 ESCoBox models used to provide values for HOMER for each hour of the day. These were generated by multiplying the appliance numbers in Figure 3 by 10 to represent a mini-grid with 200 consuming households and adjusting the duty cycle and power values to reflect likely use at that time of day. These models give a demand profile roughly similar to those shown in Figure 6. It can be seen that standard deviation varies during the day reflecting the different probabilities of appliance use – lower probabilities result in higher standard deviation. Because of the way the HOMER variabilities are combined and the small n_t as described above, there is no exact mapping between ESCoBox standard deviations and HOMER variabilities. So interpreting the set of ESCoBox standard deviation values to provide the two variability values for HOMER requires judgement.

If HOMER is being used to design the engineering aspects of a mini-grid the critical parameter once the average demand profile has been determined is peak demand. So the variability values need to be set in HOMER such that peak predicted by ESCoBox is obtained. However, in general to obtain a realistic model in HOMER values for both day-to-day and timestep variability should be entered. The lowest level of standard deviation seen in the ESCoBox profile provides an estimate of day-to-day variability since all hours of the day then have at least that level of variability. It is then logical to put the highest standard deviation from ESCoBox in the timestep value to ensure the worst-case timestep variability is represented. If this is done for the example in Table 2 HOMER gives a peak demand of 38.17 kW which is a good match for the ESCoBox estimate of peak demand - 38.08 kW at hours 17-19. This close result may not occur in all cases - the two HOMER variabilities should be adjusted if necessary, keeping a realistic balance between them, until the estimates of peak demand match.

Table 2. Use of the ESCoBox model to simulate a demand profile for HOMER

Hour of day	ESCoBox model	Average kW	Peak kW	Standard deviation
0	200-home-9-appliance-night	7.11	9.77	8.00
1	200-home-9-appliance-night	7.11	9.77	8.00
2	200-home-9-appliance-night	7.11	9.77	8.00
3	200-home-9-appliance-dawn	8.00	10.74	7.40
4	200-home-9-appliance-dawn	8.00	10.74	7.40
5	200-home-9-appliance-morning	11.80	22.87	15.20
6	200-home-9-appliance-morning	11.80	22.87	15.20
7	200-home-9-appliance-morning	11.80	22.87	15.20
8	200-home-9-appliance-mid_day	7.15	9.69	8.20
9	200-home-9-appliance-mid_day	7.15	9.69	8.20
10	200-home-9-appliance-mid_day	7.15	9.69	8.20
11	200-home-9-appliance-mid_day	7.15	9.69	8.20
12	200-home-9-appliance-mid_day	7.15	9.69	8.20
13	200-home-9-appliance-mid_day	7.15	9.69	8.20
14	200-home-9-appliance-mid_day	7.15	9.69	8.20
15	200-home-9-appliance-late_afternoon	10.15	16.70	13.80
16	200-home-9-appliance-late_afternoon	10.15	16.70	13.80
17	200-home-9-appliance-evening_peak	24.98	38.02	9.40
18	200-home-9-appliance-evening_peak	24.98	38.02	9.40
19	200-home-9-appliance-evening_peak	24.98	38.02	9.40
20	200-home-9-appliance_late_eve	18.77	27.82	9.60
21	200-home-9-appliance_late_eve	18.77	27.82	9.60
22	200-home-9-appliance-night	7.11	9.77	8.00
23	200-home-9-appliance-night	7.11	9.77	8.00

Once a mini-grid is in operation, the system manager can use their knowledge of the number of appliances in use and their likely duty cycle at each time of day to adjust the data entered into the ESCoBox model so that the peak demand predicted by the model is similar to

the observed peak demand. The manager then has an approximate model of his consumer population. The value of this is that they can use it to predict the effect of taking on more customers by adding their expected appliance use into the model and only accept additional loads that will not cause the peak demand to exceed the capacity of the system, thereby minimising the risk of brownouts or excessive battery discharge. Typically when operation of a new mini-grid has been stabilised so that peak demand is at the maximum that can be supported, the daily profile of demand will be similar to the two daily profiles shown in Figure 6. These are taken from a micro-hydro system in Malawi. This kind of demand profile on a generator-limited system results in a utilisation factor (i.e. the proportion of potential generation that is actually used) of 40-50%. Similar utilisation figures can arise for a photovoltaic-powered system in favourable seasons when there is a surplus of PV power in the middle of the day.

The economic benefit from a mini-grid system can therefore be increased if additional loads can be accepted in the middle of the day between morning and evening peaks. The ESCoBox Load Model can be used to assess whether a given commercial load, such as an intermittently-operating power tool or mill, can be accepted. There will also have to be some means of constraining the operation of such appliances to the mid-day period – the technology to achieve this is also being addressed by the ESCoBox project.

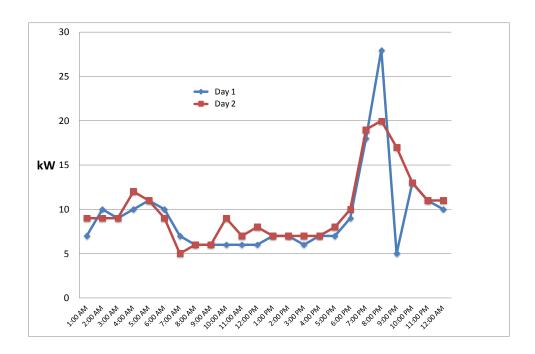


Fig. 6. Daily demand profiles from a micro-hydro system (data courtesy of Practical Action)

Conclusion

For mini- and micro-grids to realise their potential for rural electrification in developing countries they need to be designed and managed so that the service they provide is reliable and economically sustainable. This requires demand and supply to be optimally matched in planning and operation, with an understanding of the peaks in demand that are likely to occur so that their potential to be disruptive to the service provided and to system reliability can be managed effectively. Because the stochastic behaviour of demand is actually less favourable for mini- and micro-grids than it is for a national electricity system, planning and delivering this optimal match is a more difficult engineering and management challenge than is generally recognised. To address it a range of low cost and accessible tools is required for designers and operators to assist them in their task – the popularity of HOMER for system design confirms this need. The ESCoBox Load Model aims to fill another niche by

supporting the prediction and management of demand. It is also the case that, as Figure 1 shows, the variability of aggregate demand reduces quite rapidly as the number of households or businesses served rises. So a mini-grid with numbers of consumers greater than, say, 50, is more likely to be sustainable than one with 20 because of the greater diversity between households and the lower variability of demand it enjoys.

Acknowledgement

The authors would like to thank the Engineering and Physical Sciences Research Council (EPSRC) and Department for International Development for providing the financial support for this study under the ESCoBox (EP/L002566/1) and OASYS (EP/G063826/2) projects.

References

Access:energy (2015). About us http://accessenergy.org/about-us-2/

Alliance for Rural Electrification (2011). Hybrid Mini-grids for rural electrification: lessons learned. http://www.ruralelec.org/38.0.html#c1936

Boait, P (2015). ESCoBox Load Model. https://github.com/peterboait/ESCoBox_Load_Model

Boggis, J (1953). Diversity, bias and balance. Distribution of Electricity, pp. 357-362.

Department of Energy and Climate Change (2014). Community Energy Strategy; Full Report. https://www.gov.uk/government/publications/community-energy-strategy

HOMER Energy (2015). Knowledgebase article 10155 Load noise.

 $\frac{http://support.homerenergy.com/index.php?/Knowledgebase/Article/View/165/63/10155---load-noise}{noise}$

- International Energy Agency (2014). World Energy Outlook 2014 Electricity database.

 http://www.worldenergyoutlook.org/resources/energydevelopment/energyaccessdatabase/
- Komatsu, S., Kaneko, S, Ghosh P.P (2011). Are micro-benefits negligible? The implications of the rapid expansion of Solar Home Systems (SHS) in rural Bangladesh for sustainable development. Energy Policy 39 pp 4022-4031
- Lambert, T, Gilman, P and Lilienthal, P (2006). Micropower System Modeling with HOMER, by Published in: "Integration of Alternative Sources of Energy", by F. Farret and M. Simões.

 Copyright ©2006 by John Wiley & Sons, Inc

 http://homerenergy.com/documents/MicropowerSystemModelingWithHOMER.pdf
- McQueen, D, Hyland, P, Watson, S (2004). Monte Carlo simulation of residential electricity demand for forecasting maximum demand on distribution networks. *IEEE Trans. Power Sys.* 19 (3) pp.1685-1689
- McQueen, D, Hyland, P, Watson, S (2015). Simulation of power quality in residential electricity networks. Int. Conf. on Renewable Energies and Power Quality (ICREPQ'15) La Coruña 25-27 March, 2015 http://www.icrepq.com/pdfs/MCQUEEN440.pdf
- Mondal, A and Denich, M (2010). Hybrid systems for decentralised power generation in Bangladesh. *Energy for Sustainable Development* 14 (2010) 48-55.
- Porges, F, (1989). The design of electrical services for buildings 3rd Edition, p.84, London, Chapman and Hall.
- Quetchenbach, T.G, M.J. Harper, J. Robinson, K.K. Hervin, N.A. Chase, C. Dorji and A. E. Jacobson, "The GridShare solution: a smart grid approach to improve service provision on a renewable energy mini-grid in Bhutan", Environ. Res. Lett. 8 014018 (11pp) 2013.
- Smith, M. (2003) Specification for Planning and Design of Greenfield Low Voltage Housing Estates.

 Scottish and Southern Energy https://www.ssepd.co.uk/WorkArea/DownloadAsset.aspx?id=929

- Szabo, S, Bodis, K, Huld, T and Moner-Girona, M (2011) Energy solutions in rural Africa: mapping electrification costs of distributed solar and diesel generation versus grid extension. Environ. Res Lett. 6 034002(9pp).
- Yadoo, A, Cruickshank, H (2012). The role for low carbon electrification technologies in poverty reduction and climate change strategies: A focus on renewable energy mini-grids with case studies in Nepal, Peru, and Kenya. Energy Policy 42 591-602.