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Observer-based parametric decoupling controller design for a class of multi-variable non-linear uncertain systems

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ABSTRACT

This paper presents a novel decoupling control strategy for Lipschitz multi-variable non-linear uncertain systems. Using the explicit parametric design, an observer-based output feedback controller has been developed with free parameters while the closed-loop system can be further described by transfer function matrix with these free parameters. The coupling effects of the systems would be attenuated if the free parameters are optimised where the performance criterion is given based on the H_∞ norm of the transfer functions. Moreover, the sufficient conditions of stabilization have been obtained for observer, controller and closed-loop system, respectively. Following the procedure of the presented control strategy, an illustrative numerical example is given to demonstrate the effectiveness of the presented control strategy. In addition, the similar design approach has been discussed for filtering problem which is a potential extension of the presented control strategy.

1. Introduction

Decoupling analysis and control are important research topics in many research fields due to the fact that the coupling or interaction phenomenon commonly exists in the complex industrial dynamic processes. Subjected to the coupling effects, the results for the SISO systems cannot be extended to the MIMO systems directly. Therefore, the decoupling controller design becomes more and more significant since 1950s.

Firstly, the decoupling problem described by state-space model was presented by Morgan (1964). After years of development, Falb and Wolovich (1967), Gilbert and Pivnichny (1969) and Descusse, Lafay, and Malabre (1988) gave the answers to this problem gradually. Corresponding developments also appeared in the field of optimal decoupling control (Park, Choi, & Kuc, 2002) and robust decoupling control (Conte & Perdon, 1993). However all of these control methods focus on the linear deterministic systems, thus these results cannot be extended to dynamic systems with non-linear properties, for example the Lipschitz non-linearity. To overcome these existing shortcomings forms the purpose of this paper.

To deal with the control of the state-space models with Lipschitz non-linearity, a plenty of results have been developed. In particular, various non-linear controllers and observers have been proposed by Zhu

and Han (2002) and Ding (2011). Considering the uncertainties of the parameters, the robust non-linear controller has been designed by Wang, Huang, and Unbehauen (1999) and Shen and Tamura (1995). All the mentioned controllers can achieve good performance, however there was no separate decoupling design included in these controllers. Fu and Chai (2007), Zhang, Chai, and Wang (2011), and Chai, Zhang, Wang, Su, and Sun (2011) designed the control structures as the linear controllers with non-linear compensator. Although the linear controllers were easy to implement comparing with the non-linear ones, the compensator always increase computational complexity. Therefore, it is significant to develop a simply control law for the implementation of the complex dynamic system with coupling attenuation.

In this paper, the Lipschitz non-linear uncertain multi-variable systems have been investigated while the decoupling design is very difficult since the decoupling design would affect the stability of the closed-loop systems. To the best of our best knowledge, there is no existing solution to the parametric output-feedback decoupling design for non-linear uncertain systems with the stability analysis. Based upon the investigated non-linear uncertain multi-variable model, the controller and linear observer can be designed and analysed theoretically while the non-linear term can be considered as

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unmodelled dynamics which satisfies the Lipschitz condition. The sufficient conditions are given for the convergence of the observer, the stabilization of the controller and stabilization of the closed-loop system, respectively. Using this controller structure, the design procedure is simplified which reduces the complexity of implementation. Furthermore, the parameters of the controller and observer can be optimised using the parametric state feedback (Roppenecker, 1986; Zhang, Wang, & Wang, 2016) and H_∞ norm based performance criterion while the H_∞ norm can be obtained by numerical algorithms (Belur & Praagman, 2011; Boyd, Balakrishnan, & Kabamba, 1988). Using the presented control strategy, the optimal output feedback control law is obtained and the performance has been verified by the numerical simulation. Basically, the novelties and contributions of this paper can be summarised as follows: (1) a parametric decoupling control strategy has been presented for a class of non-linear uncertain systems; (2) the stability analysis has been developed using the presented control algorithm; (3) a novel performance criterion is given to enhance the system performance with coupling attenuation; (4) an extended parametric filtering approach is also given as an extension of the presented algorithm.

The rest of the paper is organised as follows. In Section 2, the preliminaries are given including the model formulation and control objectives. The optimal decoupling control strategy is developed while the convergence of the linear observer, the stabilization of the parametric state feedback controller and the stabilization of closed-loop non-linear system are analysed in Section 3. Moreover, the parameter optimisation and design procedure are also given in this section. Sections 4 and 5 present the results of numerical simulation and the potential application of filtering problem, respectively. Finally, the conclusions are drawn in Section 6.

2. Preliminaries

2.1. Formulation

Suppose that the complex industrial dynamic process can be modelled by the following non-linear uncertain multivariable systems.

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t) + \phi(x(t), u(t))$$
$$y(t) = Cx(t) \qquad (1)$$

where $x \in \mathbb{R}^m$, $u \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$ are the system state vector, input vector and output vector, respectively. m and n are positive integers while system matrices A, B, C and parameter uncertainties $\Delta A(t)$, $\Delta B(t)$ are of appropriate dimensions. The non-linear term $\phi(x(t), u(t))$ is a vector-valued non-linear function. Assume that the

investigated system model (1) satisfies the following assumptions.

Assumption 2.1: $\phi(x(t), u(t))$ is Lipschitz function with respect to the state x, uniformly in the control u, and there exists a real constant $\gamma_c > 0$ such that the following inequality holds.

$$\|\phi(x_1, u_1) - \phi(x_2, u_2)\| \le \gamma_c \|x_1 - x_2\|$$
$$\phi(0, u(t)) = 0 \qquad (2)$$

Assumption 2.2: The pair (A, B) is controllable and the pair (A, C) is observable.

Assumption 2.3: The admissible parameter uncertainties are of the norm-bounded form

$$[\Delta A(t) \quad \Delta B(t)] = M[\Xi_1(t)N_1 \quad \Xi_2(t)N_2] \qquad (3)$$

In Eq. (3), M, N_1 and N_2 denote the structure of the uncertainties which are known real constant matrices with proper dimensions. $\Xi_1(t)$ and $\Xi_2(t)$ are unknown time-varying matrices which respectively meet the following conditions.

$$\Xi_1^T(t)\Xi_1(t) \le I, \Xi_2^T(t)\Xi_2(t) \le I \qquad (4)$$

Remark 2.1: All the assumptions mentioned above have been widely used in non-linear control. In particular, the parameter uncertainty structure in Eq. (3) has been widely used in the problem of stabilization of uncertain systems (Khargonekar, Petersen, & Zhou, 1990). Moreover, it can represent parameter uncertainty in many physical cases.

The control objective is to develop a new control strategy so that the closed-loop system remains stabilization and the optimal control law should be designed to attenuate the couplings of the investigated non-linear uncertain systems.

2.2. Mathematical preliminaries

The concept of H_∞ norm will be used to optimise the parameters while the H_∞ norm of a transfer function $G(s)$ can be defined as follows.

$$\gamma = \|G(s)\|_{H_\infty} = \sup_{\text{Re}(s) \ge 0} \sigma_{\max}(G(s)) \qquad (5)$$

The physical meaning of the Eq. (5) is that γ represents the maximum amplification gain of the transfer function $G(s)$. Numerically, the value of the H_∞ norm of the closed-loop transfer function should be close to one, similarly, the system inputs cannot affect the system outputs if the H_∞ norm equals to zero.

For the state-space model, there exist various methods to obtain the analytical or numerical solution of the H_∞ norm (Doyle, Francis, & Tannenbaum, 2013). Specifically, the characteristic equation is given by

$$\det\left(\frac{1}{\gamma^2}C^TC(sI-A)^{-1}BB^T+(sI+A)^T\right)=0 \quad (6)$$

which has eigenvalues on the imaginary axis.

In addition, the following lemma (Petersen, 1987) has been recalled here which can be used to analyse the convergence and stabilization of the presented control strategy.

Lemma 2.1: *Given any real constant matrices X and Y with proper dimensions. Then there exists a constant $\xi > 0$, such that the following inequality holds.*

$$X^TY + Y^TX \le \xi X^TX + \xi^{-1}Y^TY \quad (7)$$

3. Control strategy

As mentioned in Introduction, the control strategy can be divided into two parts: observer-based output feedback design and the parametric optimisation for decoupling performance enhancement while the block diagram is given by Figure 1.
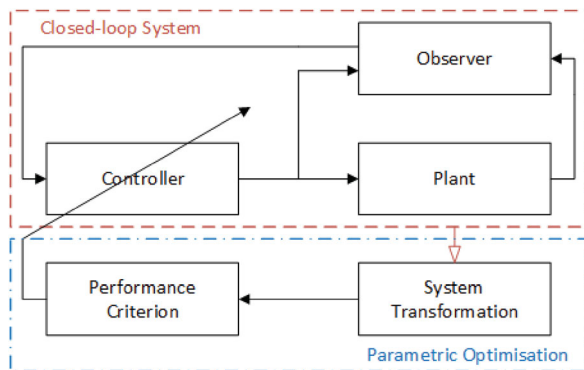
3.1. State feedback design

The linear state feedback controller can be determined by the nominal linear model and the control law is described by

$$u(t) = Kx(t) \quad (8)$$

where the gain matrix K can be obtained by parametric design (Roppenecker, 1986; Zhang et al., 2016). In particular, we have

$$K = [W_1f_1, \ldots, W_mf_m]$$
$$\times [(\lambda_1^*I - A_1)^{-1}B_1f_1, \ldots, (\lambda_m^*I - A_m)^{-1}B_mf_m]^{-1} \quad (9)$$

Figure 1. The block diagram for the presented control strategy.

where modified parameter vectors and closed-loop eigenvalues are denoted by $f_1, \ldots, f_m$ and $\lambda_1^*, \ldots, \lambda_m^*$ which can be considered as free parameters. In the case of a common open-loop and closed-loop eigenvalue, the gain matrix K can be determined by the following equations.

$$A_j = A + v_j^0 w_j^{0T}$$
$$W_j = I - \frac{e_i w_j^{0T}B}{w_j^{0T}b_i} \quad (10)$$
$$B_j = BW_j + v_j^0 e_i^T, \quad j = 1, \ldots, m$$

where v_j^0 and w_j^0 denote the open-loop eigenvectors and eigenrows of A. b_i is the ith column of B. e_i is a unit vector while the ith element is 1. In the other case, no common eigenvalue results in $w_j^{0T}b_i = 0$ which leads to

$$A_j = A$$
$$W_j = I \quad (11)$$
$$B_j = B, \quad j = 1, \ldots, m$$

Substituting control law (8) into the system model (1) yields the closed-loop system:

$$\dot{x}(t) = (A_c + \Delta A_c(t))x(t) + \phi(x(t), u(t)) \quad (12)$$

where $A_c = A + BK$, $\Delta A_c(t) = \Delta A(t) + \Delta B(t)K$. Thus the following lemma can be proposed.

Lemma 3.1: *For the nonlinear uncertain multi-variable system given by (1), with the Assumptions A1–A3 and with the control law given by (8), then there exist three positive constants $\varepsilon_1, \varepsilon_2, \varepsilon_3$, so that the equilibrium $x(t) = 0$ is stabilized if the following matrix inequality has a positive-definite solution $P = P^T > 0$.*

$$A_c^TP + PA_c + \varepsilon_1N_1^TN_1 + (\varepsilon_1^{-1}+\varepsilon_2^{-1})PMM^TP$$
$$+ \varepsilon_2K^TN_2^TN_2K + \varepsilon_3P^TP + \varepsilon_3^{-1}\gamma^2I < 0 \quad (13)$$

Proof: Consider the Lyapunov function candidate as

$$V_c(x) = x^T(t)Px(t), P = P^T > 0 \quad (14)$$

The time derivative of $V_c(x)$ along the trajectories of (12) is given as follows.

$$\dot{V}_c(x) = x^T(t)A_c^TPx(t) + x^T(t)PA_cx(t)$$
$$+ x^T(t)P\phi(x(t), u(t)) + \phi^T(x(t), u(t))Px(t)$$
$$+ x^T(t)\Delta A^T(t)Px(t) + x^T(t)P\Delta A(t)x(t)$$
$$+ x^T(t)K^T\Delta B^T(t)Px(t) + x^T(t)P\Delta B(t)Kx(t) \quad (15)$$

Let ε_1, ε_2 and ε_3 be positive constants, the following matrix inequalities hold using Lemma 2.1.

$$x^T(t)\Delta A^T(t)Px(t) + x^T(t)P\Delta A(t)x(t)$$
$$= x^T(t)(M\Xi_1 N_1)^T Px(t) + x^T(t)pM\Xi_1 N_1 x(t)$$
$$\leq x^T(t)(\varepsilon_1 N_1^T N_1 + \varepsilon_1^{-1} PMM^T P)x(t) \qquad (16)$$

$$x^T(t)K^T\Delta B^T(t)Px(t) + x^T(t)P\Delta B(t)Kx(t)$$
$$= x^T(t)K^T(M\Xi_2 N_2)^T Px(t) + x^T(t)PM\Xi_2 N_2 Kx(t)$$
$$\leq x^T(t)(\varepsilon_2 K^T N_2^T N_2 K + \varepsilon_2^{-1} PMM^T P)x(t) \qquad (17)$$

$$x^T(t)P\phi(x(t),u(t)) + \phi^T(x(t),u(t))Px(t)$$
$$\leq \varepsilon_3 x^T(t)P^T Px(t) + \varepsilon_3^{-1}\phi^T(x(t),u(t))\phi(x(t),u(t))$$
$$\leq x^T(t)(\varepsilon_3 P^T P + \varepsilon_3^{-1}\gamma^2 I)x(t) \qquad (18)$$

Substituting these inequalities into the derivative of $V_c(x)$ with Assumption A1, we have

$$\dot{V}_c(x) \leq x^T(t)(A_c^T P + PA_c)x(t)$$
$$+ x^T(t)(\varepsilon_1 N_1^T N_1 + \varepsilon_1^{-1} PMM^T P)x(t)$$
$$+ x^T(t)(\varepsilon_2 K^T N_2^T N_2 K + \varepsilon_2^{-1} PMM^T P)x(t)$$
$$+ x^T(t)(\varepsilon_3 P^T P + \varepsilon_3^{-1}\gamma^2 I)x(t) \qquad (19)$$

Since $\dot{V}_c(x) < 0$, the proof of Lemma 3.1 is completed. ∎

3.2. Observer design

Using the linear observer to estimate the states of the model (1), the linear observer can be designed based on the nominal linear model.

$$\dot{\hat{x}}(t) = (A - LC)\hat{x}(t) + Ly(t) + Bu(t) \qquad (20)$$

where the estimated state vector can be denoted by $\hat{x}$ and L is pre-specified gain matrix of this observer.

Introducing the error of the estimation by

$$e(t) = x(t) - \hat{x}(t) \qquad (21)$$

and substituting the Eqs. (20)–(21) to system model (1). The closed-loop model can be described by

$$\dot{e}(t) = A_o e(t) + \Delta A_c(t)x(t) + \phi(x(t),u(t)) \qquad (22)$$

where $A_o = A - LC$. Similar to Lemma 3.1, Lemma 3.2 is given as follows.

Lemma 3.2: *For the non-linear uncertain multi-variable system given by* (1), *with the Assumptions* A1–A3 *and with the linear observer given by* (20), *then there exists three positive constants* ε_1, ε_2, ε_3, *so that the estimation error* $e(t)$

converges to zero if the following matrix inequalities have a positive-definite solution $P = P^T > 0$.

$$A_o P + PA_o + \varepsilon_3 P^T P < 0 \qquad (23)$$

$$\varepsilon_1 N_1^T N_1 + (\varepsilon_1^{-1} + \varepsilon_2^{-1})PMM^T P + \varepsilon_2 K^T N_2^T N_2 K$$
$$+ \varepsilon_3^{-1}\gamma^2 I < 0 \qquad (24)$$

Proof: Consider the Lyapunov function candidate as

$$V_o(e) = e^T(t)Pe(t), P = P^T > 0 \qquad (25)$$

The time derivative of $V_o(e)$ along the trajectories of (21) is given by the following equation.

$$\dot{V}_o(e) = e^T(t)(A_o P + PA_o)e(t)$$
$$+ e^T(t)P\phi(x(t),u(t)) + x^T(t)\Delta A^T(t)Px(t)$$
$$+ \phi(x(t),u(t))^T Pe(t) + x^T(t)P\Delta A(t)x(t)$$
$$+ x^T(t)K^T\Delta B^T(t)Px(t) + x^T(t)P\Delta B(t)Kx(t) \qquad (26)$$

Similar to the proof of Lemma 3.1, we have

$$\dot{V}_o(e) \leq e^T(t)(A_o P + PA_o)e(t)$$
$$+ x^T(t)(\varepsilon_1 N_1^T N_1 + \varepsilon_1^{-1} PMM^T P)x(t)$$
$$+ x^T(t)(\varepsilon_2 K^T N_2^T N_2 K + \varepsilon_2^{-1} PMM^T P)x(t)$$
$$+ \varepsilon_3 e^T(t)P^T Pe(t) + \varepsilon_3^{-1}\gamma^2 x^T(t)x(t) \qquad (27)$$

which ends the proof ∎

3.3. Output feedback design

Combining the parametric state feedback controller and the designed observer, the output feedback controller can be obtained for the system (1).

$$u(t) = K\hat{x}(t) \qquad (28)$$

which leads to the closed-loop dynamics as follows.

$$\dot{x}(t) = A_c x(t) + \Delta A_c(t)x(t) + \phi(x(t),u(t)) \qquad (29)$$
$$- (B + \Delta B(t))Ke(t)$$

Furthermore, the stability of the closed-loop control design can be guaranteed by the following theorem.

Theorem 3.3: *For the nonlinear uncertain multi-variable system given by* (1), *with the Assumptions* A1–A3 *and with the control law given by* (28) *using the observer* (20), *then there exists a set of positive constants* ε_i, $i = 1, \ldots, 8$, *so*

that the equilibrium $x(t) = 0$ is stabilized if the following matrix inequalities have positive-definite solution $P_1 = P_1^T > 0, P_2 = P_2^T > 0$.

$$\varepsilon_4 K^T B^T BK + \varepsilon_5 K^T N_2^T N_2 K + A_o P_2 + P_2 A_o + \varepsilon_8 P_2^T P_2 < 0 \tag{30}$$

$$A_c^T P_1 + P_1 A_c + (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_4^{-1} + \varepsilon_5^{-1}) P_1 M M^T P$$
$$+ (\varepsilon_6^{-1} + \varepsilon_7^{-1}) P_2 M M^T P_2 + (\varepsilon_2 + \varepsilon_7) K^T N_2^T N_2 K$$
$$+ \varepsilon_3 P_1^T P_1 + (\varepsilon_3^{-1} + \varepsilon_8^{-1}) \gamma^2 I_1 + (\varepsilon_1 + \varepsilon_6) N_1^T N_1 < 0 \tag{31}$$

Proof: Consider the Lyapunov function candidate as

$$V(x(t), e(t)) = x^T(t) P_1 x(t) + e^T(t) P_2 e(t) \tag{32}$$

The time derivative of $V(x(t), e(t))$ along the trajectories of (29) is shown as follows.

$$\dot{V}(x(t), e(t))$$
$$= x^T A_c^T P x + x^T P A_c x + \phi^T(x, u) P x$$
$$+ x^T P \phi(x, u) + x^T \Delta A^T P x + x^T P \Delta A x$$
$$+ x^T K^T \Delta B^T P x + x^T P \Delta B K x - e^T K^T B^T P x$$
$$- x^T P B K e - e^T K^T \Delta B^T P x - x^T P \Delta B K e$$
$$+ e^T(t)(A_o P + P A_o) e(t) + e^T(t) P \phi(x(t), u(t))$$
$$+ \phi(x(t), u(t))^T P e(t) + x^T(t) \Delta A^T(t) P x(t)$$
$$+ x^T(t) P \Delta A(t) x(t) + x^T(t) K^T \Delta B^T(t) P x(t)$$
$$+ x^T(t) P \Delta B(t) K x(t) \tag{33}$$

Let ε_4 and ε_5 be positive constants, the following matrix inequalities hold using Lemma 2.1.

$$- e^T(t) K^T B^T P x(t) - x^T(t) P B K e(t)$$
$$\leq \varepsilon_4 e^T(t) K^T B^T B K e(t) + \varepsilon_4^{-1} x^T(t) P M M^T P x(t) \tag{34}$$

$$- e^T(t) K^T \Delta B^T(t) P x(t) - x^T(t) P \Delta B(t) K e(t)$$
$$\leq \varepsilon_5 e^T(t) K^T N_2^T N_2 K e(t) + \varepsilon_5^{-1} x^T(t) P M M^T P x(t) \tag{35}$$

Substituting these inequalities into the derivative of $V(x(t), e(t))$ and using Lemma 3.2, we have

$$\dot{V} \leq x^T [A_c^T P_1 + P_1 A_c + (\varepsilon_1 + \varepsilon_6) N_1^T N_1$$
$$+ (\varepsilon_2 + \varepsilon_7) K^T N_2^T N_2 K + \varepsilon_3 P_1^T P_1 + (\varepsilon_3^{-1} + \varepsilon_8^{-1}) \gamma^2 I$$
$$+ (\varepsilon_1^{-1} + \varepsilon_2^{-1} + \varepsilon_4^{-1} + \varepsilon_5^{-1}) P_1 M M^T P_1$$
$$+ (\varepsilon_6^{-1} + \varepsilon_7^{-1}) P_2 M M^T P_2] x$$
$$+ e^T(\varepsilon_4 K^T B^T B K + \varepsilon_5 K^T N_2^T N_2 K$$
$$+ A_o P_2 + P_2 A_o + \varepsilon_8 P_2^T P_2) e \tag{36}$$

which leads to the conditions and the proof has been completed. ∎

3.4. Parametric optimisation

To deal with the coupling effects of the investigated MIMO system, the free parameters of the controller should be optimised. Substituting the feedback gain matrix (9) and the control law (28) to the nominal linear model which is used to design the controller, the linear closed-loop model can be obtained as follows.

$$\dot{x}(t) = A_c x(t)$$
$$y(t) = C x(t) \tag{37}$$

The transfer function matrix of this state space model (37) can be obtained by

$$G(s, \lambda_i^*, f_i) = C(sI - A_c)^{-1} B \tag{38}$$

where the elements of the matrix are transfer functions which can be changed by turning the free parameters of the controller. Furthermore, the Eq. (38) can be expressed by another form as follows:

$$G(s, \lambda_i^*, f_i) = \bar{G}(s, \lambda_i^*, f_i) + \bar{\bar{G}}(s, \lambda_i^*, f_i) \tag{39}$$

where the matrices $\bar{G}$ and $\bar{\bar{G}}$ denote the diagonal matrix and the off-diagonal elements of matrix G, respectively.

The coupling effects should be attenuated if the H_∞ norm of the matrix $\bar{\bar{G}}$ is close to zero, meanwhile the norm of the matrix $\bar{G}$ should be close to one. Therefore, two performance criteria can be proposed as follows.

$$J_1(\lambda_i^*, f_i) = \min \sum_{j=1}^m \sum_{i=1}^m \| G_{ij}^*(s, \lambda_i^*, f_i) \|_{H_\infty}, \quad j \neq i \tag{40}$$

$$J_2(\lambda_i^*, f_i) = \min \sum_{j=1}^m \sum_{i=1}^m \| 1 - G_{ij}^*(s, \lambda_i^*, f_i) \|_{H_\infty}, \quad j = i \tag{41}$$

Comparing (1) and (37), the non-linear dynamic with unmatched time-varying uncertain parameters can also affect the performance of the decoupling design. The probabilistic decoupling (Zhang, Zhou, Wang, & Chai, 2015, 2017) should be considered as a compensation performance criterion while the outputs of the system should be sampled as y_k with sampling instance k. Thus, we have the following criterion.

$$J_3(\lambda_i^*, f_i) = \min \left\| \gamma_J(y_k) - \prod_{i=0}^n \gamma_i(y_{i,k}) \right\| \tag{42}$$

where γ_J and γ_i denote the joint probability density function and the marginal probability density function for each system output y_i, respectively. Moreover, these probability density functions in Eq. (42) can be estimated by kernel density estimation (Zhang & Wang, 2016).

Therefore, the complete performance criterion can be given as follows.

$$J = R_1 J_1(\lambda_i^*, f_i) + R_2 J_2(\lambda_i^*, f_i) + R_3 J_3(\lambda_i^*, f_i) \qquad (43)$$

where real positive R_1, R_2 and R_3 stand for the weights.

Then the optimal free parameter f_i can be obtained by gradient descent once the eigenvalues λ_i^* are pre-specified.

$$f_{i,j+1} = f_{i,j} + \mu \left.\frac{dJ}{df_i}\right|_{f_i=f_{i,j}} \qquad i = 1, \dots, m \qquad (44)$$

where j denotes the optimisation searching iteration index. μ stands for the pre-specified step. Note that the free parametric optimisation would not affect the stability of the closed-loop system design.

Basically, the parametric design supplies more flexibilities with free parameters which enhance the decoupling performance using novel criterion. Moreover, this control design can be further considered as a framework while the performance criterion can be replaced by other design requirements and the stability of the design can be guaranteed. In addition, the presented control strategy can be applied to many practical applications, for instance, the neural interaction attenuation (Zhang & Sepulveda, 2017a, 2017b) can be analysed and designed following the proposed approach.

Remark 3.1: Based on the dual principle, the observer gain matrix can be also obtained using proposed optimisation approach which is discussed in Section 5. Meanwhile, the optimisation operation can also be replaced by multi-objective optimisation algorithms then the weights can be neglected.

Remark 3.2: Only a few elements of the parameter vectors f_i affect the control performance directly. Therefore, in order to determine the free parameters quickly, trial and error method can be used and the performance criterion can verify the manually selected parameters simply.

3.5. Design procedure

The procedure of the proposed control strategy is summarised as follows:

Step 1   Setup the initial free parameters of the controller.
Step 2   Transfer the closed-loop model to the transfer function matrix and develop the expressions of the performance criterion.
Step 3   Use the numerical approach to optimise the performance criterion, by computing the H_∞ norm

and gradient descent, and the optimal parameters are obtained.
Step 4   Update the feedback gain matrix of the control law, and verify it by the conditions of Lemma 3.1 to guarantee stability of the system, if the conditions hold, then go to next step, otherwise, return to Step 1.
Step 5   Obtain the feedback gain matrix of the observer by dual principle and verify it by Lemma 3.2.
Step 6   Verify the optimal parameters by Theorem 3.3, and if the conditions can be satisfied, then complete the procedure, otherwise, return to Step 1.

4. A numerical simulation

In order to illustrate the effectiveness of decoupling control strategy proposed in this paper, a numerical simulation has been carried out.

Consider the parameters of the Lipschitz non-linear uncertain multi-variable systems (1) as follows.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0 & 0.2 & 0 \\ 0.1 & 0 & 0.1 \end{bmatrix}$$

$$\Xi_1(t) = \Xi_2(t) = \sin t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0 & 0 & 0.1 \\ 0 & 0.1 & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$$

$$\phi(x(t), u(t)) = \begin{bmatrix} 0.1 \sin x_1 + 0.1 \sin x_2 \\ 0.1 \sin x_3 \\ 0.1 \sin x_1 + 0.1 \sin x_3 \end{bmatrix}$$

Pre-selecting the free parameters λ_i^* and setting the initial values of f_i, the optimal parameters can be obtained by computing the value of performance criterion.

$$\lambda_1^* = -1, \quad \lambda_2^* = -2, \quad \lambda_3^* = -3, \quad f_1^* = (3 \quad 1)^T$$
$$f_2^* = (9.98 \quad -1.1)^T, \quad f_3^* = (0 \quad 0.97)^T$$

Using the Eq.(9) and the dual principle, both the feedback gain matrix of the controller and the observer can be
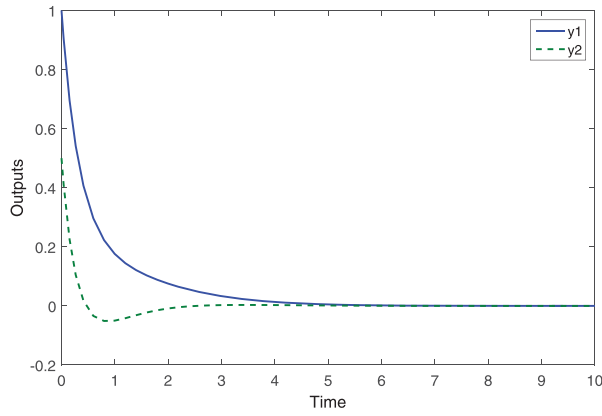
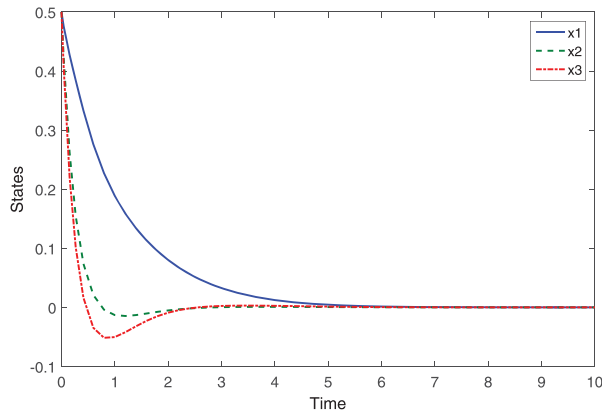**Figure 2.** The outputs of the closed-loop system.

Figure 4. The estimation errors of the observer.

Figure 3. The estimated states.
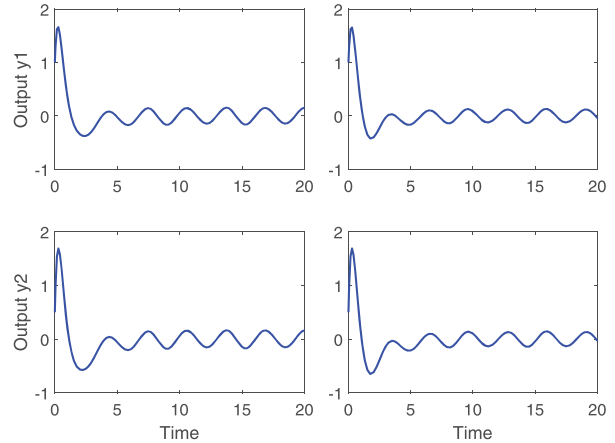
Figure 5. The decomposed outputs with the pulse using K.

developed.

$$K^* = \begin{bmatrix} -4.5231 & -2.8615 & 2.4308 \\ -1.5077 & -1.9538 & 1.4769 \end{bmatrix}$$

$$L = \begin{bmatrix} 27.4089 & -6.9170 \\ -9.2883 & 2.3958 \\ 4.4179 & 4.8795 \end{bmatrix}$$

Substituting the parameters, the closed-loop dynamic can be simulated with the initial states $x_1 = x_2 = x_3 = 0.5$. Then Figures 2–4 show the control performance, the estimated states, and the estimation errors, respectively. From the results, the controller and observer can meet the control objective for the Lipschitz non-linear uncertain multi-variable system (1) while the procedure of designing is simply and easy to implement.

In order to illustrate the decoupling performance, a disturbance sine wave is introduced to the closed-loop dynamic to affect the control signals, and the amplitude, frequency of this wave are 0.1, 2, respectively. Another feedback gain matrix of the controller K is selected to
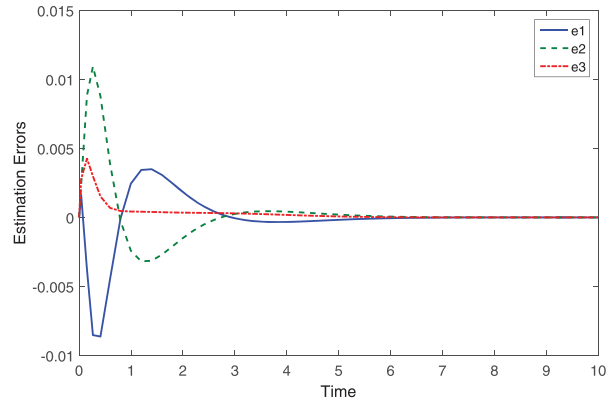
compare with the optimal parameters.

$$K = \begin{bmatrix} -9.2609 & -5.9565 & 5.1739 \\ 19.0870 & 16.6522 & -12.3913 \end{bmatrix}$$

The control performance with the different feedback gain matrices can be showed by Figures 5 and 6. By analysing the results of the simulation, the decoupling performance with the optimal parameters is better and the coupling affects have been attenuated. When the system inputs periodic fluctuate with the given sine wave disturbance, the coupling effects of the system outputs still exist using feedback gain matrix K showing by Figure 5, while the outputs y_1, y_2 seem almost unaffected by the non-diagonal control inputs u_1, u_2, and also the performance of the diagonal outputs become better.

To demonstrate the performance of the presented control strategy, one additional comparison is given in this manuscript while the 2-dimensional PI controller has been used with the parameters as $Kp = [1.5, 1.5]$ and $Ki = [0.1, 0.1]$. Based on the results of Figures 7 and 8, it has been shown that the designed PI controller cannot attenuate the system couplings comparing to the presented
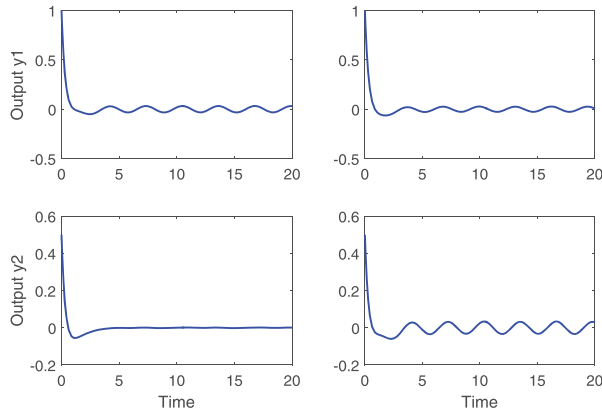
Figure 6. The decomposed outputs with the pulse using K^*.
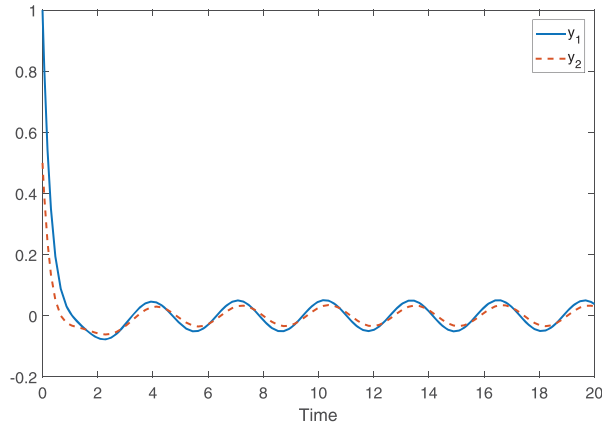
Figure 7. The system outputs with sine-wave disturbance using K^*.
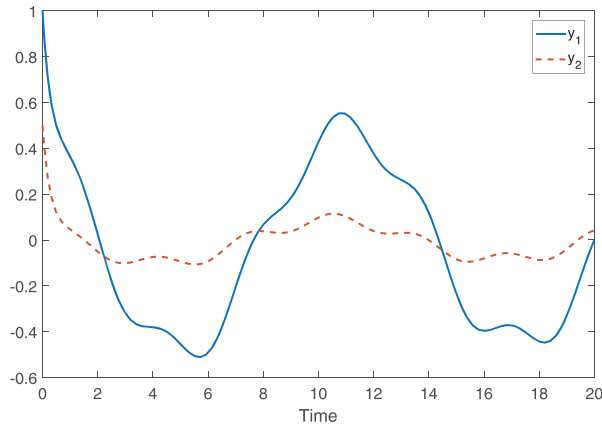
Figure 8. The system outputs with sine-wave disturbance using PI controller

controller. Moreover the transient performance is also deteriorated without optimisation design.

5. Potential extension

Similar to the presented continuous-time control strategy, the discrete-time extension can be obtained while

the model (1) can be transformed as follows:

$$x_{k+1} = (A + \Delta A)x_k + (B + \Delta B)u_k + \phi(x_k, u_k) + \omega_k \tag{45}$$

$$y_k = Cx_k \tag{46}$$

while ω_k denotes the non-Gaussian noise.

Following the main result of this paper, the associate sufficient conditions for convergence and stabilisation can also be developed which are omitted due to the similarity. However, the linear observer should be replaced by filter in order to enhance the performance of the closed-loop system.

The structure of the filter can be selected as the discrete-time format of Eq. (20) as follows:

$$\hat{x}_{k+1} = (A - LC)\hat{x}_k + Ly_k + Bu_k \tag{47}$$

where the parametric observer gain matrix L can be expressed following the results from Roppenecker O'reilly (1989).

$$L = -[f_1, \ldots, f_m] \times [C(\lambda_1^* I - A)^{-1} f_1, \\ \ldots, C(\lambda_m^* I - A)^{-1} f_m]^{-1} \tag{48}$$

where modified parameter vectors and eigenvalues of matrix A_o are denoted by $f_1, \ldots, f_m$ and $\lambda_1^*, \ldots, \lambda_m^*$ which can be considered as free parameters.

Thus, the closed-loop system can be further described by the error dynamics equation,

$$e_{k+1} = A_o e_k + \Delta A_c x_k + \phi(x_k, u_k) \tag{49}$$

while the filter gain L can also be obtained to converge the estimation error using parametric design with free parameters.

To optimise the free parameter for filter gain L, the following criteria should be considered,

$$J_k = H(e_{1:k}) + M(e_{1:k}) \tag{50}$$

where M and H stand for the mean value and entropy, respectively. Motivated by probabilistic decoupling, the randomness of the estimated states can also be attenuated if the probabilistic decoupling among the states have been minimized. Thus we have the complete performance criterion as follows:

$$J_k = R_1 H(e_{1:k}) + R_2 M(e_{1:k}) + R_3 J_{PDF} \tag{51}$$

where R_1, R_2 and R_3 are weights and

$$J_{PDF} = \min \left\| \gamma_J(e_k) - \prod_{i=0}^{m} \gamma_i(e_{i,k}) \right\| \tag{52}$$

Based on the free parameter optimisation, the optimal filter gain L is obtained and therefore the performance of

the closed-loop system is enhanced with the presented control design. Note that the optimisation operation for feedback gain K and filter gain L can be processed at the same time. In practice, the filtering approach can be applied for the battery management problem (Liu, Li, Peng, & Zhang, 2018; Liu, Li, Yang, Zhang, & Deng, 2017; Liu, Li, & Zhang, 2017) while the internal states can be estimated.

6. Conclusion

Decoupling control for multi-variable non-linear uncertain systems have been investigated using the observer-based parametric output feedback design. Combining the H_∞ norm and parametric optimisation, the optimal parameters for decoupling design have been obtained by optimising the presented performance criterion. Meanwhile the theoretical analysis is given to guarantee the robustness, stabilization and convergence of the closed-loop systems. Based on the results of the numerical simulation, the effectiveness of the presented decoupling control strategy has been verified while the control objectives have been achieved. As an extension, the discrete-time extension is also discussed and the associate filter design can be developed following the procedure of the presented algorithm.

Due to the non-linearity of the performance criterion, the parametric optimisation is difficult to obtain the global optimal solution and the local optimal solution is subjected to the initial value. Therefore, intelligent optimisation methods would be used to optimise the parameters which can should be taken into account in the future.

Disclosure statement

No potential conflict of interest was reported by the authors.

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