Consensus modeling with probability and cost constraints under uncertainty opinions

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Abstract: Goal programming is often applied into uncertain group decision making to achieve the optimal solution. Exiting models focus on either the minimum cost (guaranteeing negotiation budget) or the maximum utility (improving satisfaction level). This paper constructs a stochastic optimization cost consensus group decision making model adopting the minimum budget and the maximum utility as objective function simultaneously to study the negotiation consensus with decision makers' opinions expressed in the forms of multiple uncertain preferences such as utility function and normal distribution. Thus, the proposed model is a generalization of the existing cost consensus model and utility consensus model, respectively. Furthermore in this model, utility priority coefficients cause acceptable budget range and chance constraint shows the probability of reaching consensus. Differing from previous optimization models, the proposed model designs a Monte Carlo simulation combined with Genetic Algorithm to reach an optimal solution, which makes it more applicable to real-world decision making.

Keywords: Group decision making; Cost consensus; Uncertain chance constraint; Normal distribution; Utility function; Goal programming priority

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1   Introduction

Group Decision Making (GDM) is a process that consists of more than one person participating in both the decision analysis and the final decision choice [1, 2], which allows for the expression of collective wisdom and opinion. GDM seeks a solution through a process in which decision makers (DMs) achieve a unified opinion, to a certain extent, by gradually changing their own views [3]. The unified opinion is called group consensus. In some decision making systems, moderators (with superior leadership, communication, negotiation, and interpersonal skills) represent collective interests and help the group reach a consensus. The negotiations and consultations with individual DMs generate costs, including economic compensation and resource consumption [4]. Cost consensus is everywhere in life. Aimed at this topic, Ben-Arieh et al. [5, 6] and Gong et al. [4, 7] proposed a serious of cost consensus models:

- Ben-Arieh et al. suggested to reach a consensus at a minimum cost. Then, they developed consensus models with opinion deviation threshold or specified budget constraint.

- Gong et al. introduced cost consensus models with maximum utility of the whole GDM process under the limited budget. They explored different utility preference functions in particular.

The models mentioned above can measure cost or utility in the process of negotiations and deal with the consensus decision making well. However, they all work from just one perspective. Furthermore, the budget is usually not fixed, it will float according to the practical situation. Hence, in this paper, a stochastic optimization group cost consensus decision making (SOCCGDM) model is proposed to guarantee the minimum cost range and maximum DMs' satisfaction utility with uncertain opinions. The proposed SOCCGDM model has the following features:

(1) Apart from fuzzy numbers [8, 9, 10, 11] and utility functions [12, 13] used to measure DMs' preferences, such as the linear utility functions including triangle utility functions [14], S-shape utility functions [15], and trapezoidal utility functions [16], the nonlinear utility functions constructed by Gong et al. [7] and André and Riesgo [17], this paper proposes normal distribution to simulate the moderators' and individual DMs' preferences in case of inaccessible fuzzy numbers and utility functions; the distribution law of mathematical statistics can better explain the inherent characteristics of the preferences when it is difficult to measure DMs' satisfaction level precisely.

(2) Based on the previous studies mentioned above [4, 5, 6, 7], to study the cost consensus group decision making (CCGDM) in a more systematic and complete way, the objective function is composed of the minimum budget and the maximum utility simultaneously. Moreover, the priority of utility is designed to cause acceptable cost range.

(3) In the constraints, the random constrained budget optimization is implemented to measure the probability of reaching consensus. Stochastic optimization refers to the optimization problem with random factors [18]. Scholars have analyzed decision making problems with uncertain random programming [19, 25, 26]. In terms of getting solutions, Chen et al. [22] investigated the chance-constrained model and achieved the deterministic equivalents based on the uncertainty theory; Liu et al. [23] provided uncertain multiobjective programming model and turned it to crisp one via inverse uncertainty distributions; while Abdelaziz et al. [24] converted the multi-objective stochastic program into a deterministic one by combining two models.

(4) However, in the proposed model a Monte Carlo algorithm is used to achieve the optimal solution to a random consensus model, and random conditions can be dealt with by using this useful tool instead of being transformed into certain ones as mentioned above. Differing from the common optimization technique [2, 8, 22], the Monte Carlo simulations ensures that the costs of a negotiation between a moderator and individual decision makers reflect the real-world decision-making needs to reach a consensus within a certain budget.

The reminder of this paper is constructed as follows: Section 2 introduces the cost consensus models proposed by Ben-Arieh & Easton and the improved model considering individual DMs' preference utility suggested by Gong. On the basis of the DMs' different available preferences, four new general kind of cost consensus models with random distribution and preference utility in are introduced in Section 3. The proposed consensus models view the minimum negotiation cost and maximum utility as goals; and the consensus reaching with probability and preference characteristics as constraints. Taking the SOCCGDM problem into the Grains to Greens Program (GTGP) in China as an example, Section 4 builds a cost consensus model based on negotiation cost and preferences (Section 4.1), then economically analyzes the model results (Section 4.2). The conclusions are presented in Section 5.

2 Problem description

Ben-Arieh et al.'s [5, 6] minimum cost consensus model relies on the basic hypotheses that there is a moderator representing the group's interests, whose optimal opinion is the group consensus and individual DMs represent different interest groups, there is a discrepancy between their opinions with consensus, and the moderator will generate a particular costs to contribute to consensus. It is preferable to minimize the total cost, and therefore their model becomes:

$$
\begin{aligned}
&Min \ \phi = \sum_{i=1}^{m} \omega_i \cdot |o_i - o'| \\
&s.t. \left\{ \ o' \in O \right.
\end{aligned}
\tag{1}
$$

where m is the number of individual DMs, ω_i is the unit cost that the moderator d' pays to the individual d_i to obtain the optimal consensus, o_i is the opinion supplied by individual d_i, O is the feasible set of consensus opinions o', $|o_i - o'|$ measures the distance or deviation of opinions between o' and o_i, $\omega_i \cdot |o_i - o'|$ is the expense d' pays to d_i with $\sum_{i=1}^{m} \omega_i \cdot |o_i - o'|$ being the total cost to achieve group consensus.

Suppose that the DMs' preferences are intervals. The minimum cost consensus group decision making (CCGDM) based on interval preference relations is then constructed as follows [4]:

$$Min \; \phi = \sum_{i=1}^{m} \omega_i \cdot |o_i - o'|$$
$$s.t. \begin{cases} o_i \in [a_i, b_i], i \in M \\ o' \in O \end{cases} \tag{2}$$

where M represents the collection of all individuals in the GDM system, $M = \{1, 2, \ldots, m\}$; a_i and b_i are the lower and upper bounds of the interval preference given by individual d_i, respectively.

Models (1)–(2) focus on the minimum cost to obtain the optimal consensus only from the perspective of the moderator's interests, though this model is likely to sacrifice individuals' interests, decreasing their willingness to change their original opinion. An improved model would fully consider individual DMs' preference utility as well in order to guarantee both individuals' value in the GDM and the moderator's expectations of the individuals' final opinion, and so the DMs' optimal utility would replace the minimum cost as the goal of consensus . For this purpose, the moderator would encourage individuals to change their original opinions through a looser budget instead of minimizing expenses. The improved model was introduced in [4, 7]:

$$Max \; \lambda$$
$$s.t. \begin{cases} \sum_{i=1}^{m} \omega_i \cdot |o_i - o'| = B & (3-1) \\ \lambda \leq U_1(o_i) & (3-2) \\ \lambda \leq U_2(o') & (3-3) \\ o_i \in [a_i, b_i], i \in M, o' \in O, \lambda \in [0,1] & (3-4) \end{cases} \tag{3}$$

where λ $(0 \leq \lambda \leq 1)$ is the DMs' utility value, and the higher value of λ the higher utility level and a better match between the GDM result and the DMs' subjective preferences, so it should be as large as possible; B is the total budget determined in advance; $U_1(o_i)$ and $U_2(o')$ are the utility functions of d_i and d', respectively; a_i and b_i are the lower and upper bounds of the interval preference given by individual d_i, respectively. Constraint (3-1) is the budget description; constraints (3-2) and (3-3) are utility characterizations of d_i and d', respectively.

Model (3) has been studied using different utility functions to represent the DMs' preference, among with we can cite: concave, intermediate, convex, S- and inverted S-types. In this paper, we

will only use the right and left partial membership function to represent DMs' utility as represented in Figure 1. Here, we assume that the moderator's opinion preference o' obeys the left membership function and indicates that the decision opinion is biased toward the lower limit of range. In this case, a smaller value is better. o_i obeys the right membership functions, indicating that the decision opinion is biased toward upper limit of range where a higher value is better.
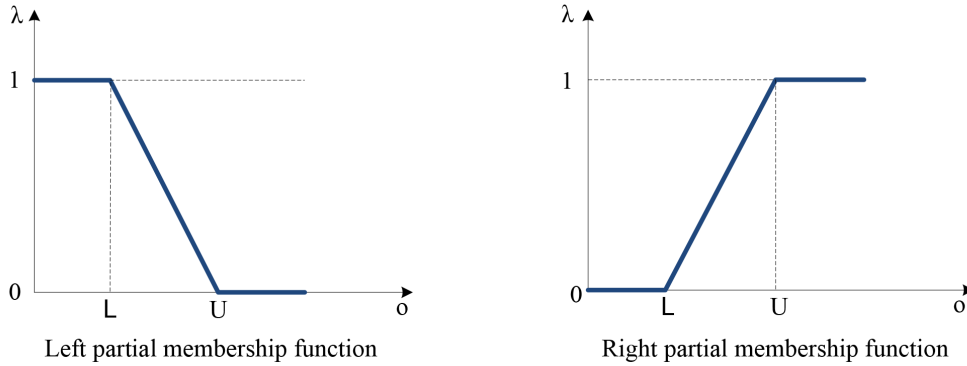

Figure 1: Preference Utility Functions

In an uncertain environment, using utility functions to measure DMs' preference is difficult, so we introduce the probability distribution in mathematical statistics to grasp the internal data features of opinions. This paper thus builds SOCCGDM models that consider random distribution and preference utility based on the DMs' differing preference relation forms.

3 Consensus modeling with random distribution and preference utility

The models proposed by Ben-Arieh and Easton [5, 6] and Gong et al. [4, 7] consider consensus from the perspective of cost or utility, so generalized models in this paper are constructed to analyze these two aspects simultaneously. Also, the budget is presented as intervals instead of fixed number.

We assume that the budget for consensus negotiations is β and the probability of cost consumption is α. Introducing this concept means mastering the probability of consensus under a specific budget, and then promoting efficient consensus attainment. The different DMs all adopt utility functions to describe preferences under uncertainty environments. In this model, we also consider circumstances where the utility functions and preference relations are not accessible, and build the SOCCGDM model with random distribution constraints and the probability of achieving consensus.

3.1 Moderator's budget

3.1.1 Situation one: Consensus modeling with preference utility of individuals' opinions and random distribution of moderator's opinions

Assume that o' obeys the normal distribution [20], o_i are all linear right membership functions. The SOCCGDM model is constructed as follows:

$$Min \ (\beta - P \cdot \lambda)$$

$$s.t. \begin{cases} Pr\{\sum\limits_{i=1}^{m} \omega_i \cdot |o_i - o'| \le \beta\} \ge \alpha & (4-1) \\ \lambda \le \frac{o_i - min\{o_i\}}{max\{o_i\} - min\{o_i\}} & (4-2) \\ o_i \in [a_i, b_i], i \in M & (4-3) \\ o' \sim N(\mu, \sigma^2) & (4-4) \\ 0 \le \lambda \le 1 & (4-5) \end{cases} \qquad (4)$$

where constraint (4-1) states that the consensus cost shall not exceed β below the probability α. In other words, the probability that the total cost $\sum\limits_{i=1}^{m} \omega_i \cdot |o_i - o'|$, which the moderator pays to the individuals to change their opinions, being no more than β is at least α. Constraint (4-2) is the utility constraint: the individual DMs' opinions o_i are between a_i and b_i (see constraint (4-3)), and individual DMs always aim to have the consensus as close to b_i as possible, with the individuals' preference computed via their right utility function. In constraint (4-2), the greater the λ value is, the closer the o_i is biased toward the right end of the interval value. Constraint (4-4) is the distribution of the moderator's opinion o'. Here, we suppose that o' obeys the normal distribution with average μ and variance σ^2. Constraint (4-5) provides the range of the utility value.

In Model (4), the optimal solution maximizes utility value λ and minimizes negotiation cost β. Consequently, the objective function $Min \ (\beta - P \cdot \lambda)$ (namely, $Min \ \beta + Max \ P \cdot \lambda$) states that to reach consensus, the budget is as small as possible for the moderator and satisfies all individual DMs' utilities as large as possible, simultaneously. P, utility adjustment coefficient ($P \ge 1$; $P = 1$, 10, 100, 1000 and 10000 in this article), ensures the maximum utility reaching, is introduced to show that considering the DMs' utility satisfaction can contribute to harmonious consensus reaching even though the minimum budget stands a good chance of sacrificing. Under this circumstance, the scope of cost budget $[Min \ \beta_{p=1}, Min \ \beta_{p>1}]$ is formed with the probability α. The following subsections omit the explanations.

3.1.2 Situation two: Consensus modeling with preference utility of moderator's opinions and random distribution of individuals'opinions

Assume that o' is a linear left membership function, and o_i obeys the normal distribution. The corresponding SOCCGDM model is constructed as follows:

$$Min \ (\beta - P \ \lambda)$$

$$s.t. \begin{cases} Pr\{\sum\limits_{i=1}^{m} \omega_i |o_i - o'| \le \beta\} \ge \alpha & (5-1) \\ \lambda \le \frac{o'_u - o'}{o'_u - o'_l} & (5-2) \\ o' \in [o'_l, o'_u] & (5-3) \\ o_i \sim N(\mu_i, \sigma_i^2), i \in M & (5-4) \\ 0 \le \lambda \le 1 & (5-5) \end{cases} \qquad (5)$$

where constraint (5-2) is a utility constraint. The consensus opinion o is between o'_l and o'_u (see constraint (5-3)). The moderator always aims for a consensus closer to o'_l. We express the moderator's preference by constructing the left utility function, indicating that higher values of λ will show a closer bias of o' towards the left end of the interval value. Constraint (5-4) is the distribution of the individual DMs' opinions o_i; here, we suppose that o_i obeys the normal distribution with average μ_i and variance σ_i^2.

3.2 Opinion deviations between the individual decision makers and the moderator

We can use decision-making deviation, $|o_i - o'| \le \varepsilon (i \in M)$, to measure the distance range between DMs' opinions o_i and the moderator's consensus opinion o'. A smaller value for this deviation represents a higher degree of consensus. In this section, we construct relative models with deviation ranges based on the models above to ensure a final result of group consensus.

3.2.1 Situation one: Consensus modeling with preference utility of individual's opinions and random distribution of moderator's opinions

Based on Section 3.1.1, we construct the SOCCGDM model with opinion deviation as follows:

$$
Min\ (\beta - P\ \lambda)
$$

$$
s.t. \begin{cases}
Pr\{\sum\limits_{i=1}^{m} \omega_i|o_i - o'| \leq \beta\} \geq \alpha & (6-1) \\
\lambda \leq \frac{o_i - min\{o_i\}}{max\{o_i\} - min\{o_i\}} & (6-2) \\
o_i \in [a_i, b_i], i \in M & (6-3) \\
o' \sim N(\mu, \sigma^2) & (6-4) \\
|o_i - o'| \leq \varepsilon_i, i \in M & (6-5) \\
0 \leq \lambda \leq 1 & (6-6)
\end{cases} \tag{6}
$$

Constraints in Model (6) are the same as those in Model (4) with the addition of constraint (6-5), with states that the moderator controls the deviation within a certain range.

3.2.2 Situation two: Consensus modeling with preference utility of moderator's opinions and random distribution of individuals' opinions

Based on Section 3.1.2, we construct the SOCCGDM model with deviating opinions as follows:

$$
Min(\ \beta - P\ \lambda)
$$

$$
s.t. \begin{cases}
Pr\{\sum\limits_{i=1}^{m} \omega_i|o_i - o'| \leq \beta\} \geq \alpha & (7-1) \\
\lambda \leq \frac{o'_u - o'}{o'_u - o'_l} & (7-2) \\
o' \in [o'_l, o'_u] & (7-3) \\
o_i \sim N(\mu_i, \sigma_i^2), i \in M & (7-4) \\
|o_i - o'| \leq \varepsilon_i, i \in M & (7-5) \\
0 \leq \lambda \leq 1 & (7-6)
\end{cases} \tag{7}
$$

As above, constraints in Model (7) are the same as those in Model (5) with the addition of the moderator control of the deviation within a certain range constraint (7-5).

4 Numerical examples

The Grains to Greens Program (GTGP) is an ecological construction project in China with a large investment and high degree of mass participation. A reasonable compensation standard is the key to stimulate farmers' voluntarily participation in the GTGP. This compensation includes grain subsidies, seed planting subsidies, living allowances, and so on (unit: acre). However, in practice, the "one price"

policy, the phenomenon of reverting to the original cultivated land, all expose the irrationality of the subsidy policy. The subsidy has become an incentive for farmers to return farmland, and is the decisive factor that ensures a successful negotiation. Narrowing farmers' disagreements and reaching consensus on GTGP requires that the local government should consume a certain negotiation "cost" (manpower, material resources, working expenses, and so on). The local authority wants to control the "cost" within the local finance capability, and expects the greatest possibility to reach consensus. In addition, farmers' own financial situations, land cultivation (land and crops), characteristics, and behavioral preferences lead them to have different preferences. During the negotiation process, the government's preferences are affected by macroscopic arrangements and appropriation budget. Therefore, the local government and farmers form an uncertain group consensus system based on the GTGP subsidy. Here, we define the local government as the moderator and farmers as individual DMs. Both groups combined are the DMs in the decision making system. Thus, we construct the group consensus model under different circumstances with the preferences of the local government and farmers expressed with utility functions and distribution characteristics. The aim is to optimize the budget and ensure the satisfaction of most DMs. Furthermore, the local government can roughly grasp the negotiation results before consultation by introducing the probability of cost, and then make a reasonable plan for fiscal expenditures ahead of time, which helps to reach a consensus efficiently.

Assume that a GTGP negotiation includes local government d' and 4 farmers d_1, d_2, d_3, d_4. The unit cost of each farmer is $\omega_1 = 0.5$, $\omega_2 = 0.2$, $\omega_3 = 0.8$ and $\omega_4 = 0.4$, respectively. The anticipated cost consumption of local government is β and the opinion deviation ranges between the farmers and the local authority is $\varepsilon_1 = 1.1$, $\varepsilon_2 = 1.3$, $\varepsilon_3 = 0.9$ and $\varepsilon_4 = 1.5$, respectively.

4.1   Numerical models of Grains to Greens Program

Based on Section 3.1.1, assume that the opinion of local government o_i obeys the normal distribution with average value 18 and variance value 3. The farmers' opinions are described using the linear right partial membership functions of Figure 2 in the intervals: $o_1 = [17, 20], o_2 = [16, 21], o_3 = [22, 24]$, and $o_4 = [18, 23]$, respectively. The cost consensus model based on local government's opinion distri-

bution is:

$$Min(\ \beta - P\ \lambda)$$

$$s.t. \begin{cases} Pr\{0.5|o_1 - o'| + 0.2|o_2 - o'| + 0.8|o_3 - o'| + 0.4|o_4 - o'| \le \beta\} \ge \alpha \\ \lambda \le \frac{o_1 - 17}{20 - 17}, \lambda \le \frac{o_2 - 16}{21 - 16}, \lambda \le \frac{o_3 - 22}{24 - 22}, \lambda \le \frac{o_4 - 18}{23 - 18} \\ o_1 \in [17, 20], o_2 \in [16, 21], o_3 \in [22, 24], o_4 \in [18, 23] \\ o' \sim N(18, 3) \\ 0 \le \lambda \le 1 \end{cases} \quad (8)$$
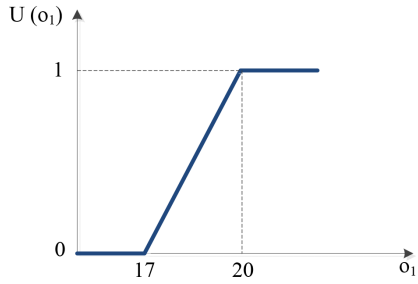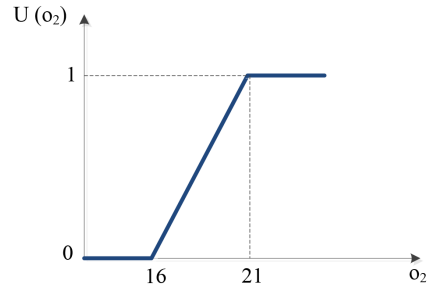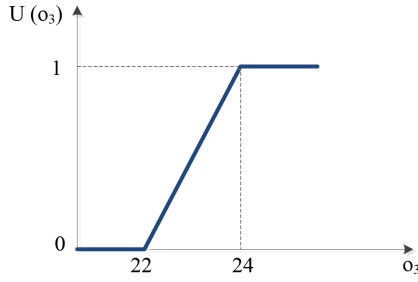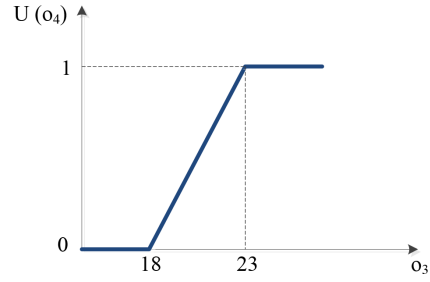

Figure 2: Farmers' Preference Utility Functions

Based on Section 3.1.2, assume that the local government's opinion o' is a linear right partial membership function on the interval $o' = [16, 20]$, as shown in Figure 3, and that the farmers' opinions obey the normal distributions: $o_1 \sim N(18, 2), o_2 \sim N(17, 3), o_3 \sim N(23, 1)$, and $o_4 \sim N(20, 2)$, respectively. The cost consensus model is based on farmers' opinions distribution:

$$Min(\ \beta - P\ \lambda)$$

$$s.t. \begin{cases} Pr\{[0.5|o_1 - o'| + 0.2|o_2 - o'| + 0.8|o_3 - o'| + 0.4|o_4 - o'|] \le \beta\} \ge \alpha \\ \lambda \le \frac{20 - o'}{20 - 16} \\ o' \in [16, 20] \\ o_1 \sim N(18, 2), o_2 \sim N(17, 3), o_3 \sim N(23, 1), o_4 \sim N(20, 2) \\ 0 \le \lambda \le 1 \end{cases} \quad (9)$$
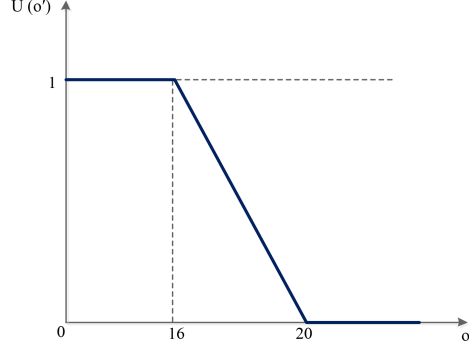
Figure 3: The Local Government's Preference Utility Function

Based on Section 3.2.1, the cost consensus model with the assumed opinion deviation range using the same data as above is:

$$Min(\ \beta - P\ \lambda)$$

$$s.t. \begin{cases} Pr\{0.5|o_1 - o'| + 0.2|o_2 - o'| + 0.8|o_3 - o'| + 0.4|o_4 - o'| \leq \beta\} \geq \alpha \\ \lambda \leq \frac{o'-17}{20-17}, \lambda \leq \frac{o'-16}{21-16}, \lambda \leq \frac{o'-22}{24-22}, \lambda \leq \frac{o'-18}{23-18} \\ o_1 \in [17,20], o_2 \in [16,21], o_3 \in [22,24], o_4 \in [18,23] \\ o' \sim N(18,3) \\ |o_1 - o'| \leq 1.1 \quad or \\ |o_2 - o'| \leq 1.3 \quad or \\ |o_3 - o'| \leq 0.9 \quad or \\ |o_4 - o'| \leq 1.5 \\ 0 \leq \lambda \leq 1 \end{cases} \quad (10)$$

Based on Section 3.2.2, the cost consensus model with the assumed opinion deviation range using the same data as above is:

$$Min \ (\beta - P\,\lambda)$$

$$s.t. \begin{cases} Pr\{0.5|o_1 - o'| + 0.2|o_2 - o'| + 0.8|o_3 - o'| + 0.4|o_4 - o'| \leq \beta\} \geq \alpha \\ \lambda \leq \frac{20 - o'}{20 - 16} \\ o' \in [16, 20] \\ o_1 \sim N(18, 2), o_2 \sim N(17, 3), o_3 \sim N(23, 1), o_4 \sim N(20, 2) \\ |o_1 - o'| \leq 1.1 \quad or \\ |o_2 - o'| \leq 1.3 \quad or \\ |o_3 - o'| \leq 0.9 \quad or \\ |o_4 - o'| \leq 1.5 \\ 0 \leq \lambda \leq 1 \end{cases} \quad (11)$$

In the deviation constraint of Model (11), we use the extraction relationship ("or") to illustrate the difficulty of completely satisfying each individual DM's interests in the GDM. In fact, partially satisfying individual DMs' can also lead to a consensus.

4.2 Analysis of the consensus model

We can obtain the Pareto optimal solution of Models (8)-(11) using Monte Carlo (stochastic simulation) and the Genetic Algorithm [21], as well as the budget scope $[Min \ \beta_{p=1}, Min \ \beta_{p>1}]$ under different probabilities. In the process of test, we reckon that when P is 1000 or 10000, the model results are nearly similar to the ones when P is 100, so we define the scope is $[Min \ \beta_{p=1}, Min \ \beta_{p=100}]$. Tables (1)-(4) below show the results for Models (8)-(11) ($P = 1000, 10000$ are not listed), respectively.

From the results, we can conclude that in Model (8) and Model (10), when the priority of λ is not adjusted ($P = 1$), the models meet $Min\beta$ and $Max\lambda$ simultaneously in the objective function. The individual DMs' (farmers) opinions are close to the lower limits of intervals. According to the individual DMs' right partial membership functions, the value of λ is low, and in this case the costs are relatively low. When $P > 1$, that is, sacrificing the minimum cost to improve farmers' satisfaction degrees. The individual DMs' opinions can be close to the upper limits of their corresponding intervals, so the value of λ is relative high, and in this case the costs are relatively high. In Model (9) and Model (11), when $P = 1$, the moderator's (local government) opinion is close to the right end of its interval. According to the moderator's left partial membership functions, the value of λ is low. When $P > 1$, the moderator's (local government) opinion is close to the left end of its interval, so the value of λ is relatively high. For the moderator, although the budget (when $P > 1$) is higher than the minimum budget (when $P = 1$), it still reflects the small value of the cost, the moderator is agreed by higher

Table 1: Results for Model (8)

α	P	o_1	o_2	o_3	o_4	o'	$Max\lambda$	$Min\beta$
0.8	1	17.2677	16.6258	22.0406	18.1521		0.0082	5.2547
	10	19.6828	20.4524	23.8463	22.4769		0.8677	10.2467
	100	19.9805	20.8292	23.9627	22.8942		0.963	10.9852
0.9	1	17.1222	16.4142	22.1603	18.237		0.0308	6.517
	10	19.8365	20.6778	23.8679	22.7833		0.9198	12.0444
	100	19.9362	20.7862	23.8955	22.9516		0.9353	12.2597
0.95	1	17.2437	16.628	22.2306	18.2269		0.0413	7.7727
	10	19.8947	20.6421	23.8979	22.6788		0.8989	13.0355
	100	19.8782	20.8751	23.9821	22.9614		0.9533	16.1305
0.98	1	17.0721	16.8711	22.1971	18.0521		0.0049	8.9262
	10	19.9436	20.7945	23.8696	22.7449		0.9271	14.535
	100	19.9267	20.8608	23.9651	22.8418		0.9657	14.9215
0.99	1	17.2403	16.5278	22.164	18.1657		0.0079	9.6577
	10	19.8845	20.7419	23.9288	22.481		0.8942	15.2581
	100	19.8611	20.9855	23.9444	22.7729		0.9443	16.3608

satisfaction degree.

Generally, through the computation of the four models, we can find that: the utility value increases obviously along with the chances of utility priority P; minimum cost also increases accordingly (when P reaches higher level, the variation of $Min\beta$ and $Max\lambda$ are tiny). Along with the obvious better chance of reaching a consensus (i.e. 0.8-0.9), the minimum budget increases accordingly. Low probability is corresponding to the low cost, however in the high probability range, because the consensus budget has reached a relative high level, the increased probability do not always lead to higher costs, even though there is an increase, it will not be obvious. From the results, the government can grasp the probability of reaching consensus under different budgets. In addition, the proposed approach provides the acceptable cost range, allowing the expected concession to be at the disposition of the government.

5   Conclusion

In researching CCGDM, the uncertainty of the decision-making environment, DMs' changeable behavior preference and other complex factors can contribute to cost exceeding the budget range (interval) with a certain probability. Therefore, introducing the probability measure of consensus into

Table 2: Results for Model (9)

α	P	o_1	o_2	o_3	o_4	o'	$Max\lambda$	$Min\beta$
0.8	1					19.6608	0.081	5.413
	10					16.0127	0.9964	9.45
	100					16.0009	0.9996	9.4816
0.9	1					19.5963	0.0984	5.8788
	10					16.0241	0.9911	9.9193
	100					16.002	0.9994	10.0077
0.95	1					19.5941	0.0963	6.2825
	10					16.0087	0.9971	10.3904
	100					16.0013	0.9987	10.3924
0.98	1					19.5804	0.0916	6.6871
	10					16.0203	0.9945	10.8248
	100					16.0014	0.9992	10.3735
0.99	1					19.624	0.0817	6.913
	10					16.0161	0.9901	11.0479
	100					16.0003	0.9992	11.2277

Table 3: Results for Model (10)

α	P	o_1	o_2	o_3	o_4	o'	$Max\lambda$	$Min\beta$
0.8	1	19.9274	20.1499	22.0214	20.395		0.0082	3.3328
	10	19.8576	20.7504	23.8603	22.6463		0.9281	5.8324
	100	19.9599	20.7623	23.8732	22.7982		0.9244	5.9333
0.9	1	19.9872	20.2184	22.2029	20.4016		0.0683	3.7951
	10	19.9168	20.6812	23.9649	22.793		0.935	6.2244
	100	19.9997	20.9651	23.9408	22.9454		0.9604	6.5953
0.95	1	19.9597	20.3252	22.0262	20.4551		0.0003	3.7932
	10	19.9877	20.7395	23.9308	22.9021		0.9353	6.3976
	100	19.968	20.8308	23.968	22.8645		0.9533	6.4977
0.98	1	19.9975	20.1798	22.1253	20.3727		0.0517	3.9715
	10	19.9803	20.7496	23.954	22.8388		0.9418	6.5208
	100	19.8832	20.7692	23.9249	22.9328		0.9525	6.6299

0.99	1	19.9779	20.2007	22.1173	20.4063		0.0569	3.9837
	10	19.9097	20.7916	23.9138	22.7172		0.9367	6.5557
	100	19.9886	20.9726	23.9695	22.8889		0.9771	6.6046

Table 4: Results for Model (11)

α	P	o_1	o_2	o_3	o_4	o'	$Max\lambda$	$Min\beta$
0.8	1					19.9265	0.0127	5.1132
	10					16.0019	0.9954	9.0859
	100					16.0001	0.9999	9.1463
0.9	1					19.5505	0.0902	5.6445
	10					16.0596	0.9797	9.454
	100					16.0013	0.9992	10.7531
0.95	1					19.6424	0.0852	6.0313
	10					16.012	0.9953	9.9224
	100					16.0031	0.9991	10.0707
0.98	1					19.618	0.08	6.3215
	10					16.0234	0.9924	10.3518
	100					16.0015	0.9993	10.4701
0.99	1					19.6584	0.053	6.6013
	10					16.0759	0.9796	10.448
	100					16.0014	0.9994	10.4577

the decision making system can explain the process and results better. Furthermore, it is difficult to measure DMs' preference precisely in some uncertain CCGDM. The distribution law of mathematical statistics can better explain the inherent characteristics of the preference relations. So this paper considered the utility function and normal distribution to simulate the moderators' and individual DMs' preference relations. It is worth noting that complete consensus is difficult to achieve, and opinion deviation ranges are thus proposed to measure consensus level. This model not only considers the optimal cost budget, but also the utility maximization. The numerical simulation indicates that the budget increases along with the chances of reaching a consensus on the whole. In addition, utility value increases obviously along with the chances of utility priority, minimum cost also increases accordingly. The SOCCGDM model has following contributions:

(1) Introducing the probability into the budget constraint, the model considers simultaneously the op-

timal cost budget and the utility maximization. It extends both Ben-Arieh et al.'s [5, 6] minimum cost consensus model and Gong et al.'s [4] maximum utility model.

(2) The model adopts interval cost-optimized value $[Min\ \beta_{p=1}, Min\ \beta_{p>1}]$ to determine the upper and lower bounds of the negotiation cost. Compared with the rigid budget model, it is a better means to improve the resilience of budget in the context of satisfying numerous DM's utility.

(3) The model uses Monte Carlo simulation to achieve the optimal solution to a random consensus model, which is more in line with the real-world decision-making.

This paper takes GTGP negotiation as the background application and constructs a cost consensus model demonstrated with a numerical example that minimizes a local government's budget and maximizes farmers'/ local government's utility with the probability of budget realization and farmers'/ local government's preferences as constraints. The case study shows that the proposed model has good applicability. It can be applied in consensus negotiations with probability of achievement and uncertain opinions between moderator and individual DMs.

Uncertain expression forms of opinions' preferences is difficult to simulate accurately in real-word decision making, and as such it is a limitation of this work that deserves future work on determining best realistic preference representation formats. Also, in future, we plan to consider the communication and trust between individual DMs in the process of negotiation. The social network should be applied into consensus decision making [27, 28, 29] because individuals will rely on or accept the opinions from people close to them or with similar interests. So, the complicated social trust relationship will be considered and implemented to handle the preferences forming in SOCCGDM.

Acknowledgements

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