

## A multicriteria model for solving a real cutting layout problem in publishing industry

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### Abstract

A cutting layout problem is analyzed taking into account the need to use standard-sized sheets of material for printing patterns with different size, type and print run. In this paper a multicriteria model is proposed for cutting layout problem and for printing an additional print run in particular. The multicriteria formulation of the cutting layout problem is considered, where the material waste, the number of the nesting patterns and the cutting layout cost are minimized simultaneously. The paper provides an approach to solve the problem using the additive multicriteria estimation. The efficiency of the proposed multicriteria model is tested for a series of computational experiments.

*Keywords:* Multicriteria model; Optimization, Solving strategy; Cutting layout; Nested patterns

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### 1. Introduction

The cutting layout problem is an important for publishing industry problem, see e.g. [1—7]. In this problem it is necessary to place two-dimensional objects in the given area, in a way that one or more criteria are optimized while given set of constraints are satisfied.

Arising in different industrial applications cutting layout problems have a number of specific features associated with the material used, with the technology of its processing, etc. Taking into account those features it is necessary to develop new mathematical models and methods for various cutting layout applications or modify the well-known ones.

One of the areas, where it is traditionally necessary to solve the cutting layout problems, is publishing industry, which is manufacturing printing products — newspapers, books, etc. [6, 7]. Application and implementation of the concrete cutting layout problem requires analysis and consideration of technology features of nesting pattern, paper rolls, paperboard and related materials. The development and realization of effective plans in manufacturing printed products is one of the conditions for a printing company to function effectively.

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The purpose of the present work is constructing a mathematical model and solving the problem of imposing plan formation with the standard sized sheets when printing covers of printed editions, the size, type and print run of which differ.

## 2. Modeling cutting layout problem

Many problems associated with the technical and technological process aspects of the manufacturing printed production have been solved. Automatic layouting items within a 2D container for the pre-printing processes is considered in [6], where the laying out articles in a newspaper automatically was discussed. When compared to the more commonly solved problem of printing blocks of equal size covering all available area without waste of paper, placing stocks of different size on a material to be cut, e.g. covers of books, is practically impossible without paper material waste. However, we can find such a layout for a standard sized sheet where the paper material waste is minimal and ideally absent even if we use stocks with different sizes. Here we will talk about printing hard covers for books of different sizes..

To model the problem formally, we introduce the following concepts. By a nesting pattern we mean the range of the book covers placed on a single printing sheet according to some rule. In other words, it is a range of book covers of specified sizes, which are placed on a sheet of a specified format. Here it is assumed that such a pattern is available. However, finding an optimal pattern is solving a packing optimization problem, e.g. by  $\Phi$ -function method [10] or the heuristic methods [11, 12], not considered in this work. By the cutting layout we mean the set of nesting patterns and the number of prints made using each nesting pattern.

The problem of choosing a cutting layout for printing an additional print run of book covers is complicated by the fact that book covers, which have to be placed on the existing nesting patterns, have different print runs. It can be explained by the different demand for the products being additionally printed.

While positioning the book covers on the sheets of a standard size we have to pay a special attention to the print run. A print run is a number of any printed edition copies in a single output. One should understand the difference between the trial and an additional print runs. The trial print run is used for eg. a new technology approbation, for a product sales analysis, etc.). An additional print run is used for printing editions of a book for the second, third etc. time, after a part or the whole first print run has been sold. In both cases it is necessary to form a layout selecting which nesting patterns to use and to decide how many stocks to print using each of the selected patterns to obtain the set final print run of the printed edition.

Taking into account discussed assumptions the considered problem can be formulated as follows: it is necessary to choose a cutting layout of a standard sized sheet of paper, such a way which allows printing print runs of different quantity using the minimal number of nesting patterns and reduces wasted material to possible minimum (and consequently minimizes expenses for printing book covers).

Let  $I$  be the number of the admissible cutting layouts;

$T$  is the number of the book cover titles;

$P_t, t = 1, 2, \dots, T$  is the number of print runs for the book cover with the title  $t$  to be printed;

$X_i, i = 1, 2, \dots, I$  is a cutting layout;

$x_{jt}^i$  is the number of book cover with the title  $t$  used for nesting pattern  $j$  in the cutting layout  $i$ .

$j = 1, 2, \dots, N^i, N^i$  is the number of nesting patterns used in cutting layout  $i$ ,

$$x_{jt}^i = \begin{cases} d \in \mathbb{N}, & \text{if } j \text{ nesting pattern includes title } t; \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where  $\mathbb{N}$  is the set of the natural numbers.

The total number of the book covers with the nesting pattern  $j$  used for formation of the cutting layout  $i$  can be computed as:

$$Q_j^i = \sum_{t=1}^T x_{jt}^i. \tag{2}$$

The actual print run  $E_t^i$  of the book cover title  $t$  received using all nesting patterns  $N^i$  for a cutting layout  $i$  can be computed as:

$$E_t^i = \sum_{j=1}^{N^i} x_{jt}^i \cdot n_j^i, \quad j = 1, 2, \dots, N^i, \tag{3}$$

where  $n_j^i$  is the number of prints, which should be done using nesting pattern  $j$  in the cutting layout  $i$ ;

$F_k$ ,  $k = 1, 2, \dots, K$  is the format used for printing the given covers, which can be represented by a tuple  $F_k = \{(a_k, b_k) \in R^2\}$ , where  $a_k, b_k$  are the page width and page height for the format sheet  $k$ ;

$O = \{o_1, o_2, o_3, o_4\}$  is the set of margins (left, right, top and bottom) of a sheet.

$C_k$ ,  $k = 1, 2, \dots, K$ , is the cost of the format sheet  $k$ .

The placement area of the sheet is determined with the help of the following rule:

$$S_k = [a_k - (o_3 + o_4)] \times [b_k - (o_1 + o_2)]. \tag{4}$$

$E_t$ ,  $t = 1, 2, \dots, T$  is the linear size of the book cover title  $t$ , which can be presented by a tuple  $E_t = \{(w_t, h_t) \in R^2\}$ , where  $w_t, h_t$  are the bleed width and height of book cover title, respectively.

The effective area of the nesting pattern  $j$  is equal to:

$$S_j^i = \sum_{t=1}^T x_{jt}^i \cdot s_{jt}^i, \tag{5}$$

where  $s_{jt}^i = w_t \cdot h_t$  is the area of the book cover  $t$ ;

The total area of the nesting pattern  $j$  is:

$$(S_j^i)^* = W_j^i \cdot H_j^i, \tag{6}$$

where  $W_j^i, H_j^i$  are the width and heights of the rectangular, which restricts the area covered by the book covers with the nesting pattern  $j$  (see fig. 1).

Suppose that we need to print four editions. It is possible to print the editions using one of the nesting patterns shown on figure 1. The first one consists of the cover with the title c1, two covers with the title c2, one cover with the title c3 and two covers with title c4. The other nesting pattern includes once cover with the title c1 and c2, twice cover with the title c3 and trice cover with the title c4. Therefore the nesting patterns from figure 1 can be designated as follows:

$$X_i = \{x_1^i, x_2^i\} = \{(1, 2, 1, 2); (1, 1, 2, 3)\}. \tag{7}$$

The factor of filling in the nesting pattern  $j$  can be found as:

$$v_1 = \frac{S_j^i}{(S_j^i)^*}. \tag{8}$$

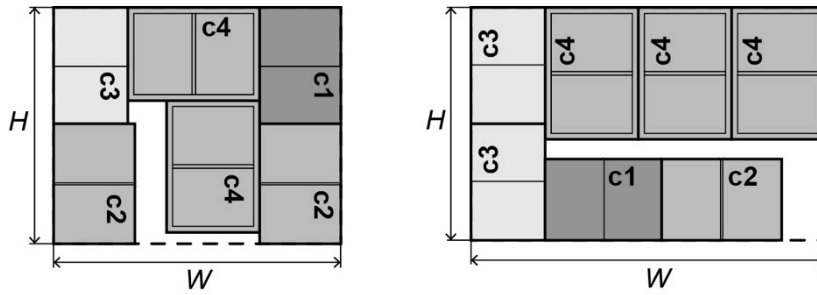


Fig. 1 — Examples of nesting patterns

The factor of filling in the nesting pattern  $j$  for the format sheet  $k$  characterizes the efficiency of using the material. For example we have some nesting pattern that is filled by the covers in the best way but the printing sheet we use to print the nesting pattern has bad factor of filling. It will take a great material waste. To avoid this situation we advise to use factor that is determined by the following relation:

$$v_2 = \frac{(S_j^i)^*}{S_k} \tag{9}$$

Then the feasible solutions area of the cutting layout problem domain  $\Omega$  can be presented as the following tuple of the described above elements:

$$\Omega = \{ \omega \in \Omega \subset R^7 \mid \omega = (X, Q, n, a, b, W, H) \} \tag{10}$$

### 3. Decision evaluation criteria

The efficiency of the cutting layout is evaluated using the following criteria:

- **Minimizing the paper material waste.** In the cutting layout there can be such a nesting pattern, which allows a big amount of paper material waste. However, if the number of prints done using such a nesting pattern is much less than the number of prints made using the nesting patterns with the maximal possible filling of the sheets, the total amount of the material waste using all the nesting patterns in the cutting layout can be satisfying.

Therefore it is necessary to estimate not only the material waste by each nesting pattern but also the total material waste when all nesting patterns are used in the cutting layout. Then we have to choose the criterion  $K_1(\omega)$  as the efficiency criterion of the cutting layout using:

$$K_1(\omega) = \sum_{j=1}^{N^i} (S_k - S_j^i) \cdot n_j^i \tag{11}$$

- **Minimizing the number of nesting patterns.** Each nesting pattern corresponds to the set of film mechanicals, plates and other related expenses, which its use entails. Thus the smaller is the number of the nesting patterns in the cutting layout is, the lower is its cost.

$$K_2(\omega) = N^i \tag{12}$$

where  $N^i$  is the number of the nesting patterns used in the cutting layout  $i$ .

- **Minimizing the cutting layout cost.** The cost of the cutting layout consists of all expenses related to printing the given quantity of prints using all the nesting patterns. With a large print run the cost of the paper on which the product is printed influences to the large extend the printing cost. Therefore, it is necessary to take it into account when forming the cutting layout.

$$K_4(\omega) = \sum_{j=1}^{N^i} x_j^i \cdot C_k \cdot n_j^i, \quad \forall j, k : (S_j^i)^* \leq S_k. \tag{14}$$

- **Minimizing the paper material waste in the cutting layout.** When selecting a cutting layout from those with the same cost, the one with the minimal waste of the paper material is optimal.

$$K_5(\omega) = \sum_{j=1}^{N^i} [(S_k - S_j^i) \cdot n_j^i] \cdot C_k. \tag{15}$$

#### 4. Constraints of the problem

The cutting layout has to satisfy the following constraints:

1. The maximal and the minimal **number of the book covers**, used in the nesting pattern  $j$  of the cutting layout  $i$  for the format sheet  $k$  determine the limits for the possible values of the number of the book covers variable:

$$(Q_j^i)_{\min} \leq Q_j^i \leq (Q_j^i)_{\max}, \tag{16}$$

where

$$(Q_j^i)_{\min} = \left\lfloor \frac{S_k}{\max_t \{S_{jt}^i\}} \right\rfloor, \quad (Q_j^i)_{\max} = \left\lfloor \frac{S_k}{\min_t \{S_{jt}^i\}} \right\rfloor, \quad t = 1, 2, \dots, T, \tag{17}$$

where  $\lfloor \cdot \rfloor$  is the integer part of the real number;

$\min_t \{S_{jt}^i\}$ ,  $\max_t \{S_{jt}^i\}$  are the minimal and the maximal area of all book cover titles  $t$ , correspondingly;  $S_k$  is the useful area of the format sheet  $k$ .

2. The **efficiency** of the nesting pattern  $j$ . When we form the nesting patterns using the covers of different sizes in most of the cases it is impossible to fill the nesting pattern entirely.. Thus, it is recommended to limit the compelled paper material waste left when using the nesting pattern so that the factor of the filling in the nesting pattern  $j$  (8) is not lower than the admissible factor  $k_{allowed}$ :

$$k_1 \geq k_{allowed}. \tag{18}$$

In some cases it is reasonable to use the common efficiency factor of using the nesting patterns for the entire cutting layout, then

$$(k_1)^* \geq k_{allowed}, \tag{19}$$

where  $k_1$  is defined as:

$$(k_1)^* = \frac{\sum_{j=1}^{N^i} \frac{S_j^i}{(S_j^i)^*}}{N^i}. \quad (20)$$

3. The given **print run** for each book cover title and the actual print run received using the optimal cutting layout should be the same:

$$P_t = E_t^i, \forall t = 1, 2, \dots, T \quad (21)$$

Sometimes it is not possible to satisfy this condition due to discrete nature of the problem. Therefore, when forming the feasible solutions area it is desirable to place such a number of book cover copies within the format sheet  $k$ , which is not smaller than the planned one:

$$E_t^i \geq p_k, \quad (22)$$

or we can use the condition of receiving such number of book cover copies that falls in the given intervals:

$$(p_k)_{\min} \leq E_t^i \leq (p_k)_{\max}. \quad (23)$$

4. The technological printing features determine a smallest **number of prints** that makes sense to print on a single nesting pattern  $j$  in the cutting layout  $i$ :

$$n_j^i \geq n_{\text{allowed}}. \quad (24)$$

## 5. Mathematical model

Taking into account constraints (16)—(24) it is necessary to construct a mathematical model of determining the optimal cutting layout for the problem domain  $\Omega$  with feasible solutions defined by (10) for the multiple criteria defined by (11)—(15). This is a discrete multicriteria formulation of the cutting layout optimization problem of the form:

$$\Lambda(\Omega) \rightarrow \underset{\omega \in \Omega}{extr}, \quad (25)$$

where  $\Lambda(\Omega) = \{K_z(\omega)\}$ ,  $z = 1, 2, \dots, Z$ ,  $K_1(\omega) \rightarrow \min$ ,  $K_2(\omega) \rightarrow \min$ ,  $K_3(\omega) \rightarrow \min$ ,  $K_4(\omega) \rightarrow \min$ .  
The solution of the problem (25) can be found by:

$$\omega^0 = \arg \underset{\omega \in \Omega}{extr} \{K_1(\omega), K_2(\omega), \dots, K_4(\omega)\}. \quad (26)$$

Solving the multicriteria problem (25)—(26). can be done by one of the alternative approaches

1. Define the feasible solutions area  $\Omega$  by introducing the initial optimization criteria and constraints. 2. Narrowing the  $\Omega$  area by searching non-dominated or Pareto optimal solutions  $\Omega^c \subset \Omega$  such that:

$$\Omega^c = \{\omega^q, \omega^p \in \Omega \mid \forall p \neq q: \omega^q \prec \omega^p\}, \quad (27)$$

where  $\nless$  means the nondomination (incomparability) of alternatives  $\omega^q$  and  $\omega^p$ , belonging to the Pareto set. All solutions belonging to the Pareto set are better on at least one criterion than any solution not belonging to the set and is not worse on the rest of criteria:

$$\omega^q \succ \omega^v \rightarrow \{ \omega^q, \omega^v \in \Omega \mid \forall q \neq v: \omega_a^q < \omega_a^v \vee \omega_b^q \leq \omega_b^v \}, \tag{28}$$

where  $a \neq b, a, b \in K_z, z = 1, 2, \dots, Z$ .

Among Pareto non-dominated solution, all of which are optimal with respect to the criteria specified, the one should be selected, e.g. by expert.

2. An alternative approach would be to combine all criteria into a single one, e.g. by additive sum operation, which is used in this work. In particular, the weighted sum of the criteria is optimized:

$$\sum_z \lambda_z K_z(\omega) \rightarrow \min_{\omega \in \Omega}, \tag{29}$$

where  $\lambda_z$  is weight of the criterion  $z$  and  $K_z(\omega)$  is the value of the criterion  $z$ . Weights of criteria can be identified by expert.

To solve the cutting layout problem formulated as a single objective discrete integer program, commercial solvers can be applied, which use methods such as branch and bound, see e.g. [13]. Here, the software module written in the C++ language was developed for solving the constructed mathematical model.

### 6. Computing experiments

To test the efficiency of the proposed approach we compared the results of the computational experiments to the results suggested by experts in the publishing and printing field. The initial data set of the book cover sizes and print run sizes are listed in table 1.

Table 1.— The initial data

№	Book cover size	Print run size
1	340x220	3000
2	340x230	4000
3	340x220	3000
4	340x220	6000
5	340x230	4000

The following weight factors for importance of each of the criterion for solution estimation were chosen by the experts:  $a_1 = 0.5; a_2 = 0.15; a_3 = 0.25; a_4 = 0.1$ . The following set of the cutting layout for the given book covers was selected from the set of randomly generated nesting patterns for the analysis (see table 2).

Table 2.— The cutting layout for printing the book covers

	Cutting layout	Sheet Format	Price
1	Nesting Pattern 0: 1 1 1 0 1	500x700	10500
	Nesting Pattern 1: 0 1 0 2 1	500x700	10500
	Nesting Pattern 2: 0 0 0 4 0	450x700	12200
2	Nesting Pattern 0: 4 0 0 0 0	500x700	10500
	Nesting Pattern 1: 0 4 0 0 0	450x700	12200

	Nesting Pattern 2: 0 0 4 0 0	500x700	10500
	Nesting Pattern 3: 0 0 0 4 0	500x700	10500
	Nesting Pattern 4: 0 0 0 0 4	450x700	12200
3	Nesting Pattern 0: 0 1 1 1 1	500x700	10500
	Nesting Pattern 1: 1 1 0 1 1	500x700	10500
	Nesting Pattern 2: 1 0 0 1 0	350x450	2986,5
4	Nesting Pattern 0: 1 0 1 2 0	450x700	12200
	Nesting Pattern 1: 0 2 0 0 2	500x700	10500

Weight factors:  $a_1 = 0.5$ ;  $a_2 = 0.15$ ;  $a_3 = 0.05$ ;  $a_4 = 0.2$ ;  $a_5 = 0.1$

Solution №1580.

Table 3.— The result of using the cutting layout for printing the book covers

Nesting Pattern	Print	Sheet Format	Price	The actual/required print run				
				340x220	340x230	340x220	340x220	340x230
0: 1 0 1 2 0	3000	450x700	12200	3000/3000	0	3000/3000	6000/6000	0
1: 0 2 0 0 2	2000	500x700	10500	0	4000/4000	0	0	4000/4000

Criteria values: Criterion 1 = 0,75; Criterion 2 = 1; Criterion 3 = 0; Criterion 5 = 1.

The obtained solution corresponds to the expert's solution, which shows the effectiveness of the constructed mathematical model of the cutting layout formation for printing book covers. The total time of searching for the optimal solution was less than a second, which allows using the given model in real problem solving for publishing industry. Experiments were conducted on IBM with CPU clock of 2,87 GH and operative memory content of 2 GB.

## 7. Conclusion

The mathematical model constructed in this work, the proposed strategy of forming the optimal cutting layout for printing book covers, and the software module can be used as a part of larger package for the publishing process automation. The computational experiments have shown efficiency of the suggested approach while searching for the optimal cutting layout for printing book covers. The given means can be used in manufacturing departments, which will reduce the printing expenses and increase efficiency of the printing materials usage.

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