COLREGs-Compliant Path Planning for Autonomous Surface Vehicles: A Multiobjective Optimization Approach ⋆

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Abstract: In this paper, a multiobjective optimization framework is proposed for on-line path planning of autonomous surface vehicles (ASVs), where both collision avoidance and COLREGs-compliance are taken into account. Special attention has been paid to situational awareness and risk assessment, particularly when the target ship is in breach of the COLREGs rules defined by the International Maritime Organisation. In order to implement COLREGs, the rules together with physical constraints are formulated as mathematical inequalities. A multiobjective optimization problem based on particle swarm optimization is then solved, the solution of which represents a newly-generated path. It is shown through simulations that the proposed method is able to generate COLREGs-compliant and collision-free paths even for non-cooperative targets i.e. vessels that are in breach of COLREGs.

Keywords: Unmanned surface vehicles, autonomy, collision avoidance, risk assessment, decision making, path planning, multiobjective optimization, COLREGs.

1. INTRODUCTION

1.1 Background

In recent years, the study of autonomous surface vehicles (ASVs) has become an active area of research due to their potential to execute complex missions. One of the basic requirements for ASVs is that they should navigate (by themselves) safely and avoid collisions with any other ships/obstacles or with land mass in the surroundings (Pascoal et al. (2000)). Furthermore, to operate harmoniously with other ships (either manned or unmanned), an ASV should behave in a manner similar to that of other ships in the vicinity. Since all manned craft are required to adhere to the coastguard regulations on prevention of collision at sea (COLREGs) defined by the International Maritime Organisation (IMO) (Cockcroft and Lameijer (2003)), it is imperative to impose COLREGs-compliant behaviour as an integral element of any ASV navigational system.

The current set of COLREGs guidelines dates back to 1972 and have undergone a number of changes over the years. The regulations have been written for manned vessels i.e. for human consumption and are thus not simple to programme or automate. Due to their subjective nature, COLREGs are subject to various interpretations causing uncertainty, which in the worst case can lead to collisions. Minimising collisions between the vessels not only requires consistent understanding of COLREGs but also good seamanship. This paper attempts to incorporate both mariners' interpretation of COLREGs together with (good seamanship) input from experienced navigators.

In practice, depending on mission requirements, other objectives/preferences beside safety may need to be considered when re-planning a path (Ahmed and Deb (2013); McEnteggart and Whidborne (2012)). For ASVs, these typically include generating a path which is optimal as well as smooth without abrupt course changes. Accordingly, in this paper, a multiobjective optimization approach is proposed to address the on-line path planning problem. The requirements of collision avoidance and COLREGs compliance are both incorporated in the proposed approach systematically and an efficient algorithm is proposed to find a feasible path.

1.2 Literature review and motivation

A variety of path planning techniques with consideration of COLREGs have been developed in recent years. Typ-

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ical techniques include artificial potential fields (Naeem et al. (2016)) and heuristic A^* method (Campbell et al. (2014)) developed by the authors' research group, velocity obstacle method (Kuwata et al. (2014)) and Evolutionary algorithms (Szlapczynski (2011)). However, most, if not all of the existing techniques do not scale well to multiple target ships and multiple COLREGs rules such as rules 2, 8, 13-17 in Cockcroft and Lameijer (2003), and usually one objective can be considered only when using these techniques. In addition, to the authors' knowledge, none of the proposed methods consider non-compliant vessels i.e. vessels that are in breach of COLREGs. Indeed, these scenarios pose a serious risk to an ASV in particular when the ownship assumes all other target ships to be COLREGs-compliant which is typically the case with other work in this area. This motivates the research presented in this paper.

Multiobjective optimization is concerned with mathematical optimization problems where multiple objectives are optimized simultaneously. Multiobjective optimization algorithms have been used to analyse and solve problems in many fields of science, engineering and logistics, where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives. Recently, several evolutionary algorithms have been proposed and widely used to solve multiobjective optimization problems (MOPs), of which the multiobjective particle swarm optimization (MOPSO) approach has become a popular choice due to its merits of fast convergence and rather straightforward implementation (Coello et al. (2004)).

1.3 Contributions

The contribution of this paper is threefold: 1) an enhanced situational awareness and decision making method is proposed such that even if the target ship violates COLREGs rules (rule 2b Cockcroft and Lameijer (2003)), the ASV is able to make appropriate decisions to avoid the risk of collision. Note that the majority of reported collisions are due to COLREGs violations and/or incorrect interpretation of COLREGs (Perera et al. (2009); Statheros et al. (2008)); 2) A multiobjective optimization framework is developed for path re-planning, which is flexible and scalable to accommodate multiple target ships and objective functions; 3) A novel and unified representation in the form of mathematical inequalities is proposed for COLREGs rules selection and other ASV constraints, which is rather simple to incorporate in the multiobjective optimization framework for path re-planning.

2. SYSTEM OVERVIEW

A complete process of path planning is composed of a global (off-line) and a local (on-line) path planning modules. Given the destination waypoint, the global path planner generates the desirable path off-line, and presents it as a sequence of waypoints. The on-line path planning module will only be activated if any obstacles are detected between any two given waypoints. When a risk of collision is confirmed, a collision-free and COLREGs-compliant local path (from current location to the closest waypoint) will be generated.

The on-line path planning process is split into three separate sub-processes: situational awareness and risk assessment, COLREGs rules selection and path re-planning. In the following, details of the individual sub-processes will be presented and discussed.

2.1 Situational Awareness & Risk Assessment

This is the critical part of the system, as a failure or incorrect assessment could lead to a catastrophic collision. Note that this part of the system is only activated if a target/obstacle is detected. To assess a risk of collision, the widely-used closest point of approach (CPA) method has been adopted for evaluating if there is a potential collision risk in the near future (Campbell et al. (2014); Bertaska et al. (2015); Kuwata et al. (2014)). Briefly speaking, this method compares the time to closest point of approach (TCPA) and the distance to closest point of approach (DCPA) with prescribed parameters $t_{\max}$ and $d_{\min}$, where $t_{\max}$ and $d_{\min}$ are dependent on the vessel type and also the environment where she is being operated. A risk of collision is deemed to exist and labelled as risk = 1 if

$$0 \le \text{TCPA} \le t_{\max} \quad \text{and} \quad \text{DCPA} \le d_{\min}. \tag{1}$$

However, the existing CPA method only cannot detect whether a target vessel indeed complies with COLREGs rules or not. In fact, *in extremis* caused by non-compliant behaviours of target vessels, the ASV should avoid collision at all costs which may be required by the ordinary practice of seamanship, or by the special circumstances of the case admitted under COLREGs rule 2 on responsibility. Without a proper assessment of the situation, the ASV may continue to follow other COLREGs rules thus failing to make the required evasive manoeuvre in time.

In order to correctly identify non-compliant target vessels, situational awareness based on a modified CPA method is introduced with a compliant/non-compliant *flag* and two additional parameters: t_{safe} and d_{safe}. Typically, $t_{\text{safe}} < t_{\max}$ and $d_{\text{safe}} < d_{\min}$. An urgent risk of collision thus deem to exist and labelled as risk = 2 if

$$0 \le \text{TCPA} \le t_{\text{safe}} \quad \text{and} \quad \text{DCPA} \le d_{\text{safe}}. \tag{2}$$

Now if both (1) and (2) are satisfied, a risk of collision is confirmed and the risk indicator is upgraded to risk = 2.

The situational awareness sub-module is used to distinguish compliant/non-compliant targets, where a criterion based on historic record of collision risks is used. Denoting the current time instant as k, when $\text{risk}(k) = 2$ and if a risk of collision existed at the previous time instant $k-1$, i.e., $\text{risk}(k-1) = 1$, then the target's flag will be set to Flag=*Non-Compliant*; otherwise, all targets are considered as COLREGs-compliant by default. Based on this condition, any target vessel that was required to manoeuvre but did not take an appropriate action in compliance with the COLREGs as a give-way vessel causing an urgent risk (risk = 2), is classified as a non-compliant vessel. This activates the non-compliant behaviour of ASV and an alternate evasive path is generated.

A pseudocode of the situational awareness and risk assessment submodule is given in Algorithm 1.

Algorithm 1 Situation awareness and risk assessment

```
Initialize:
    Set Flag=Compliant ;
while Target vessels detected do
    Read AIS data of ASV and target vessels at time instant $k$ ;
    Calculate TCPA and DCPA ;
    if risk($k$)=1 via equation (1) then
        Return risk($k$)=1 and Flag=Compliant ;
    else if risk($k$)=2 via equation (2) then
        if risk($k-1$) = 0 then
            Return risk($k$)=2 and Flag=Compliant ;
        else if risk($k-1$) = 1 then
            Return risk($k$)=2 and Flag=Non-Compliant ;
        end if
    else
        Return risk($k$) = 0 and Flag=Compliant ;
    end if
end while
```

Table 1. Decision table for encounter rules selection

Risk Flag	0	1	2
Compliant	+	√	√
Noncompliant	NA	NA	×

+: no risks and no rules apply, √: risk exist and certain rule applies, ×: risk exist where rule 2 applies and other rules may be disregarded.

2.2 Decision Making & Rules Selection

Once a collision risk is deemed to exist, the next stage is to determine which COLREGs encounter, i.e., “head-on”, “crossing” or “overtaking”, should be applied. It should be pointed out that COLREGs rules only apply when the target vessels comply with COLREGs rule as well. For instance, consider the scenario that a target ship that is supposed to be overtaking the ASV maintains her course and speed and closes dangerously upon the stern of the ASV. If the ASV also maintains her course and speed as required by COLREGs rule for the case of “being overtaken”, a collision would soon occur. On the contrary, if a non-compliant behaviour of the target ship is realised, the COLREGs rule for the case of “being overtaken” should be superseded and evasion action(s) taken immediately to avoid potential collision.

To fully consider the effect of target vessel’s behaviour on the decision making process, the decision scheme shown in Table 1 is proposed as a reference to COLREGs encounter rule selection. In general, if a risk of collision appears, evasive actions should be considered. The chart in Fig. 1, depicting the COLREGs zones can be used to determine which specific rule should be selected. Given the relative bearing of the target vessel to the ASV, it is uniquely determined which sector it falls in and the COLREGs rule applies accordingly.

Once a risk is confirmed and the decision of Stand-on is made, the ASV contitues its course; while the decision of give-way is made, the next step it to plan a new path by generating alternate waypoints.

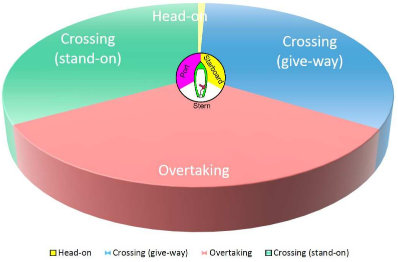

Fig. 1. COLREGs zones for COLREGs rules selection

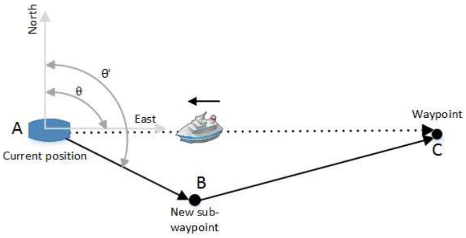

Fig. 2. The illustration of path re-planning

2.3 Path Re-planning

An evasive trajectory can be planned either by generating one or a sequence of sub-waypoints. Here a single evasive waypoint is preferred as it is computationally efficient and hence is well suited for real-time applications. Fig. 2 can be used to illustrate the basic idea of the proposed path re-planning process where a simple 1-1 encounter situation is depicted. At the beginning, the ASV follows the nominal planned path $\overrightarrow{AC}$ from waypoint A towards waypoint C due east with speed v and heading angle θ. At position A, the ASV detects a risk of collision with a target ship, then the path re-planning module generates a new sub-waypoint B. Subsequently, the ASV alters its heading angle from θ to θ' and traverses the new path $\overrightarrow{AB}$ until arriving at sub-waypoint B, returning to the original final waypoint C. Let t denote the time the ASV expends in traversing path segment $\overrightarrow{AB}$.

For path re-planning, several different goals/objectives need to be considered simultaneously: 1) the safety objective is the first and foremost in that any collision risk should be eliminated; 2) the re-planned path should be as smooth as possible and avoid abrupt changes in the course; and 3) the re-planned path should not deviate too much from the original path. This lends itself naturally to a MOP. Besides the above-mentioned objectives, the following constraints should also be considered provided there is sufficient sea-room.

- the minimum acceptable course alteration is not less than 15^0;
- the maximum acceptable course alteration is not bigger than 60^0 except *in extremis*;
- manoeuvres to starboard are favoured over manoeuvres to port;

All the above constraints are not explicitly imposed by COLREGs rules but by good seamanship and/or by economics. In particular, normally a course alteration bigger than 15^0 is large enough to be apparent to other observing vessels (COLREGs rule 8(a)). A course alteration bigger than 60^0 is generally inefficient and hence not recommended, however *in extremis*, if a collision-free path can not be found, such a constraint could be relaxed if necessary.

On the other hand, the selected COLREGs rules in subsection 2.2 also impose constraints on changes in the ASV's course, such as "starboard manoeuvring". The difficulty with COLREGs is that there are no hard constraints provided in the rules and every mariner may interpret the rules in a different manner which had resulted in near misses and in the worst case caused collisions. Automating such rules pose significant challenges especially in unmanned vessels. To overcome such difficulty, the constraints are represented as mathematical inequalities that can be easily incorporated in the optimization framework as follows.

- Obvious course alteration:
$$|\theta' - \theta| \geq 15^0; \tag{3}$$
- Efficiency:
$$|\theta' - \theta| \leq 60^0 \quad \text{if} \quad \text{risk} = 1; \tag{4}$$
- Length of manoeuvres:
$$\underline{t} \leq t \leq \bar{t}; \tag{5}$$
Once a manoeuvre is initiated, the ASV continues at least the minimum duration of time $\underline{t}$, making ASV's decision obvious and predictable to other users of the sea-space. Additionally, the ASV should not continue indefinitely on the new path ensuring minimum possible deviation from the offline trajectory. This is specified by the variable $\bar{t}$, defining the maximum allowable time constraint.
- Manoeuvre to starboard preference:
$$\theta' - \theta \geq 15^0 \tag{6}$$
If a course change is necessary, the ASV should preferably manoeuvre, according to COLREGs rules and under normal circumstances towards the starboard side.

  In order to bias the starboard manoeuvre, the following strategy is adopted in the path re-planning sub-process: first the inequality constraint (6) is imposed and if a solution is found, a sub-waypoint on the starboard side is generated. However, if no feasible solution was found, then the inequality constraint (6) is relaxed and a port side manoeuvre is allowed. For overtaking scenarios, this strategy is particularly useful as the craft is allowed to overtake from either side with proper signalling.

To sum up, the overall process of on-line path planning is depicted in the flowchart of Fig. 3.

3. THE PROPOSED MULTI-OBJECTIVE OPTIMIZATION FRAMEWORK

A general multiobjective framework includes three elements: decision variables, objective functions, and constraints. Constraints are already presented in the previous section, so the other two elements are now introduced in the following.

3.1 The framework for path re-planning

Define the decision variable by the vector x
$$x := [\theta', t]^T, \tag{7}$$
where θ' denote the new relative heading angle of ASV after manoeuvring (i.e. the heading of vector $\overrightarrow{AB}$ relative to $\overrightarrow{AC}$ in Fig. 2) whilst t represents the time required from the current position to reach the newly generated sub-waypoint (from A to B in Fig. 2), respectively.

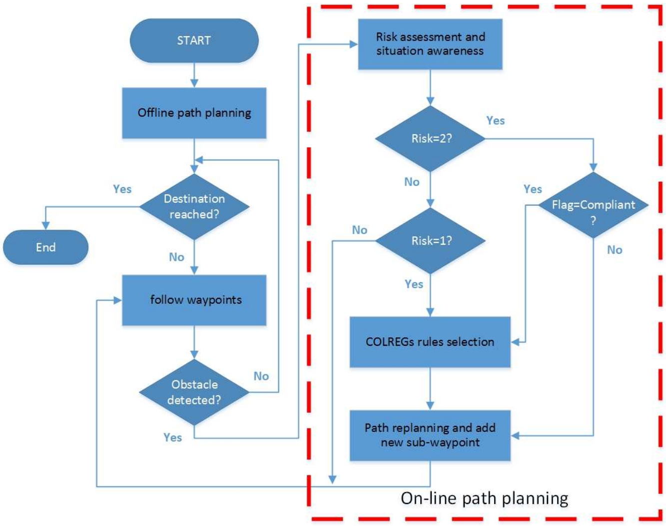

Fig. 3. Flowchart of the path planning process

The three objectives used in path re-planning are formulated as the following mathematical functions:

(1) Safety: this objective is to eliminate the risk of collision, which has the highest priority as safety is the primary concern for all sea-going vessels. Mathematically
$$f(x) = \max_{1 \leq i \leq n} f_i(x), \tag{8}$$
where
$$f_i(x) = \begin{cases} d_{\min} - \text{DCPA}_i(x), & \text{if } \text{DCPA}_i(x) \geq d_{\min}, \\ e^{a(d_{\min} - \text{DCPA}_i(x))/\text{TCPA}_i(x)} - 1, & \text{otherwise} \end{cases}$$
$f_i(x)$ is adapted from Smierzchalski and Michalewicz (2000), $d_{\min}$ is the desirable DCPA of the ASV, $\text{DCPA}_i(x)$ is obtained from the risk assessment submodule on the i^{th} target vessel posing collision risk, and a is a constant scaling parameter.

(2) Path smoothness: this objective function will minimise or prevent any abrupt changes to the modified path. Mathematically, it will attempt to minimise the sum of heading changes in the re-planned path. The objective function is derived by geometry as follows:
$$g(x) = \pi - (\theta' - \theta) - \arctan \frac{vt \sin(\theta' - \theta)}{l_{AB} - vt \cos(\theta' - \theta)} \tag{9}$$
where l_{AB} is the distance from the current position A to the newly generated waypoint B.

(3) Shortest path: this objective function will minimise deviation from the original path thus bringing the ASV back to the originally defined waypoint as soon as feasible.
$$h(x) = vt + l_{BC}, \tag{10}$$
where l_{BC} is the distance from the newly-generated sub-waypoint B to the next waypoint C.

In summary, the path re-planning problem is represented as the following multiobjective optimization problem:
$$\mathcal{P} : \begin{cases} \min \quad F(x) \\ \text{subject to: } (3) - (11), \text{ and } f(x) \leq 0, \end{cases} \tag{11}$$

where the decision variable is given by (7), the objective function $F(x) = [f(x),\ g(x),\ h(x)]^T$, $f(x)$, $g(x)$, and $h(x)$ being defined in (8)-(10), respectively.

3.2 The proposed MOPSO algorithm

For MOPs, typically there exists no solutions that optimize all the objectives simultaneously. Instead, a number of methods/algorithms have been developed to find an approximation of the optimal solutions.

Particle swarm optimization (PSO) is a metaheuristic algorithm that optimizes a problem by iteratively searching in a large space of candidate solutions (Kennedy and Eberhart (1995)). PSO are often well suited for MOPs because 1) ideally there is not any assumption about the underlying objective functions (linear or nonlinear, convex or nonconvex), and objectives and constraints can be easily added, removed, or modified; 2) the swarm-based search can achieve an approximation of a MOP's Pareto front, with each particle representing a trade-off amongst the objectives; and 3) the PSO algorithm is easy to implement and converges with a low computational overhead.

In this paper, the MOP in (11) is solved by using the MOPSO algorithm (Coello et al. (2004)), which, to authors' knowledge has not been applied before in the context of ASV on-line path re-planning incorporating both collision avoidance and COLREGs-compliance.

4. SIMULATION STUDY

In this section, the performance of the proposed on-line path planning method is presented and discussed. The simulations consider a wide range of cases, from single to multiple obstacles, from static to moving targets, from open water to restricted water, and from COLREGs-compliant to non-compliant target vessels. Note that in this paper, OS stands for the ownship i.e the ASV and TG represents target ships.

The same parameter settings are used in all the simulations presented in this paper. For the MOPSO algorithm, an acceptable re-planned path is found with a population of 50 particles typically within $30 - 40$ generations as only minor improvement is observed with an increase in the number of generations.

Figs. 4(a)-(b) depict two similar "overtaking" encounter scenarios in open and restricted waters where the shaded polygons in Fig. 4(b) represents the land mass. By comparing both figures, it is evident that if both starboard and port side manoeuvres are feasible, then the former is preferred (Fig. 4(a) for open water). However, if a starboard manoeuvre is not possible, due to land mass in this example, then a port side manoeuvre will be considered (Fig. 4(b) for restricted water).

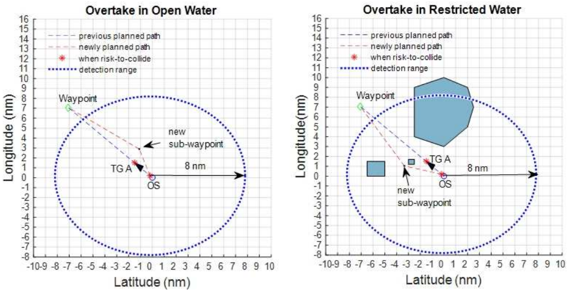

Fig. 4. Overtaking in open water and restricted water

One main advantage of the MOPSO is that it can deal with multiple-vessel encounters simultaneously as illustrated in Fig. 5. There, the ASV has to negotiate 3 moving targets A, B, C and 1 static obstacle D. Figs. 5(a)-(d) depict snapshots showing the original offline and the updated paths as target information is updated and risk of collision is assessed. In Fig. 5(a), TG B presents an "overtaking" scenario, then the collision is avoided by re-planning a local path performing a COLREGs-compliant manoeuvre to the starboard. In Fig. 5(b), on the newly updated path, TG A presents a "crossing" scenario. According to COLREGs, the ASV becomes a give-way vessel to TG A and manoeuvres starboard accordingly. In Fig. 5(c), the static TG D is in the way of the ASV whilst TG C could be a potential risk if the ASV makes only a small alteration to her course. Taking into account of both TGs C and D for path re-planning, the ASV makes a manoeuvre to starboard and thus avoids collision with both TGs C and D simultaneously. Fig. 5(d) depicts the complete re-planned path to the waypoint.

Fig. 6 highlights the advantage of the proposed algorithm by illustrating a scenario where the target ship does'nt obey COLREGs. At the beginning, the ASV detects a collision risk with TG A. From COLREGs, it is determined that the ASV is the stand-on vehicle and the target ship is the give-way vessel and should thus manoeuvre to starboard accordingly. However, through continuous risk assessment, it is found that the target ship maintains the course (stands-on) and hence does not comply with COLREGs rules. When the incorrect behaviour of the target ship is detected and confirmed, the ASV re-plans an alternate evasive path based on Rule 2(b) in the process.

5. CONCLUSION

This paper has proposed a new on-line path planning method for ASVs which generates collision-free and COLREGs-compliant paths using a multiobjective optimization approach based on particle swarm optimization. A key feature of the proposed technique is the incorporation of both compliant and non-compliant target vessels without making any changes to the basic path planner. The proposed method is able to determine the type of encounter in addition to determining whether a target ship complies with the COLREGs. The effectiveness of the proposed algorithm has been validated through simulations showing a range of difficulties encountered at sea. While only some preliminary research results have been reported in this paper, extensive simulations in which practical issues such as measurement noise, disturbance and vehicle dynamics are considered explicitly, are also implemented and will be presented in the future.

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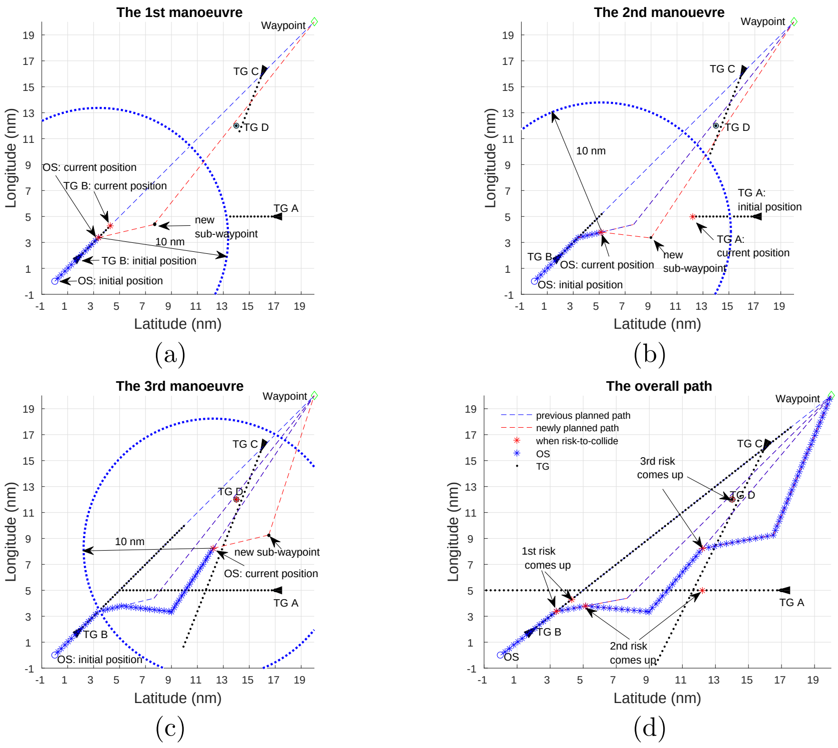

(a)

(b)

(c)

(d)

Fig. 5. Multiple target path re-planning

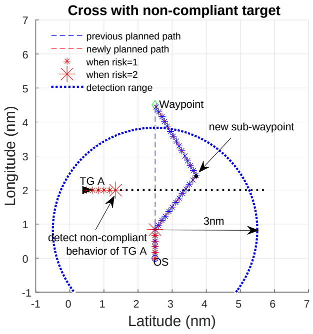

Fig. 6. Scenario of COLREGs non-compliant target

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