An Integrated Inverse Adaptive Neural Fuzzy System with Monte-Carlo Sampling Method for Operational Risk Management

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Abstract

Operational risk refers to deficiencies in processes, systems, people or external events, which may generate losses for an organization. The Basel Committee on Banking Supervision has defined different possibilities for the measurement of operational risk, although financial institutions are allowed to develop their own models to quantify operational risk. The advanced measurement approach, which is a risk-sensitive method for measuring operational risk, is the financial institutions preferred approach, among the available ones, in the expectation of having to hold less regulatory capital for covering operational risk with this approach than with alternative approaches. The advanced measurement approach includes the loss distribution approach as one way to assess operational risk. The loss distribution approach models loss distributions for business-line-risk combinations, with the regulatory capital being calculated as the 99,9% operational value at risk, a percentile of the distribution for the next year annual loss. One of the most important issues when estimating operational value at risk is related to the structure (type of distribution) and shape (long tail) of the loss distribution. The estimation of the loss distribution, in many cases, does not allow to integrate risk management and the evolution of risk; consequently, the assessment of the effects of risk impact management on loss distribution can take a long time. For this reason, this paper proposes a flexible integrated inverse adaptive fuzzy inference model, which is characterized by a Monte-Carlo behavior, that integrates the estimation of loss distribution and different *risk profiles*. This new model allows to see how the management of risk of an organization can evolve over time and it effects on the loss distribution used to estimate the operational value at risk. The experimental study results, reported in this paper, show the flexibility of the model in identifying (1) the structure and shape of the fuzzy input sets that represent the frequency and severity of risk; and (2) the risk profile of an organization. Therefore, the proposed model allows organizations or financial entities to assess the evolution of their risk impact management and its effect on loss distribution and operational value at risk in real time.

Keywords: Monte-Carlo sampling, Integrated adaptive neural fuzzy system, Loss Distribution Approach, Operational Value at Risk, Risk profile, Basel Committee on Banking Supervision

1. Introduction

All organizations face operational risk, since this type of risk refers to the possibility of incurring losses due internal events such as deficiencies, flaws/inadequacies in processes, systems or people or due to external events (Bank for International Settlements, 2016). This means that no operation of an organization is exempt from possible losses. However, for managers and stakeholders, it is important to know when the magnitude of the losses becomes significant for an organization. Only when the magnitude of an operational risk is comprehended, it is possible to prioritize different operational risks.

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Organizations need to manage operational risk to avoid or to mitigate its consequences. The management process includes the measurement of operational risk, which should lead to an understanding of the magnitude of this risk. The Basel Committee on Banking Supervision (BCBS) defined different possibilities for the measurement of operational risk. These definitions treat operational risk measurement from a regulator's perspective. Nevertheless, in addition to complying with the regulator's requirements, financial institutions have also to manage operational risk according to their risk appetite and tolerance.

One issue when measuring operational risk is that operational risk data is not that frequent when compared with other types of risk. Moreover, , according to (Reveiz and León, 2009), the operational risk sources and exposures are more diverse, complex and context-dependent than those typical of other risks, in particular market and credit risk. That is one reason why supervisors require that operational risk measurement includes also qualitative methods, such as scenario analysis, Risk and Control Self Assessments (RCSA) or Key Risk Indicators (Girling, 2013).

The Value at Risk due to operational risk or operational value at risk (OpVaR) is interpreted as the maximum loss that can be expected, given a certain confidence level (α), within a certain period of time. The Loss Distribution Approach (LDA), as defined by Basel II, requires that a financial institution registers continuously all operational risk events and associated losses that occurred in a particular business line and related to a particular risk type, like fraud for example (Mora Valencia, 2010; El Arif and Hinti, 2014; ISO, 2015), to construct an empirical loss distribution (LD), which is subsequently used to estimate the OpVaR. We observe the followoing four development trends in the field of OpVaR that focus on estimating the LD using different methods or computational tools.

- Bayesian risk models to identify the causes, the influence and the relations between a set of factors that define the risk exposure of an organization or financial institution (Lee et al., 2009; Andersen et al., 2012; Figini et al., 2015; Barua et al., 2016).

- Vector models that adapt and learn from operational risk data. These models allow to identify factors, parameters and variables that are relevant to model operational risk. Within this research area, it is worth mentioning support vector machines (SVMs) that integrate various classifiers (Twala, 2010), use multiple agent systems for learning (Yu et al., 2010), use experimental designs for selecting optimal weights (Yu et al., 2011), or apply Bayesian concepts to identify the causes and influence between risk factors (Feki et al., 2012; Yu, 2014).

- Models based on the principles of modeling and simulation of operational risk. We find models that allow to identify qualitatively the variables and parameters that can be used to model operational risk and estimate the OpVaR through the use of ontologies (Ye et al., 2011) or by using data mining techniques (Koyuncugil and Ozgulbas, 2012); autoregressive models for making predictions, based on the evolution of the data, to estimate the OpVaR for both short and medium time predictions (Hernández and Opsina, 2010; Lin and Ko, 2009; Pinto et al., 2011), as well as models that use multivariate distributions based on copulas to obtain the LD (Lopera et al., 2009; Mora Valencia, 2010; Dorogovs et al., 2013; Koliali, 2016).

- Risk models that apply the principles of intelligent computational systems. Operational risk factors, such as those related to fraud, are complex and the data is often of a qualitative nature. Among these models, fuzzy systems stand out as they have demonstrated their effectiveness assessing risk in areas such like aviation or nutritional security (Hadjimichael, 2009). Also, we can find models that estimate risk based on fuzzy neural networks that use different learning schemes (Khashman, 2010; Golmohammadi and Pajoutan, 2011) or linguistic variables (Deng et al., 2011; Mokhtari et al., 2012; Cooper et al., 2014; Mitra et al., 2016).

This paper contributes to the above fourth research area. With regard to measuring operational risk, we identify two ways in literature to approach this task: (1) modelling operational risk as a classification problem, e.g. by using Key Risk Indicators (Reveiz and León, 2009); (2) measuring operational risk in terms of the percentile of the loss distribution (OpVaR) following Basel II. Our work follows this second approach and proposed model's benefits include (i) the possibility of working with qualitative risk data and (ii) the connection of risk measurement (OpVaR) with risk management, based on the risk management matrices. Thus, this paper makes a contribution that goes beyond the mere compliance with standards like Basel II by exploring new ways for operational risk management with the proposal of an Integrated Inverse Adaptive Neural Fuzzy System with Monte-Carlo sampling (IIANFSM) method that identifies the behavior and evolution of operational risk in an organization. The flexible structure of the proposed IIANFISM method can be categorized by the implementation of the following three sub-systems:

- An Integrated Inverse Adaptive Neural Unbalanced Fuzzy System model (IIANUFSm) that identifies the structure and shape of the fuzzy input sets used to represent the frequency and severity of operational risk. Frequency refers to the number of times a risk event has occurred in a period of time, while severity refers to the impact that a particular risk event generated.
- An Integrated Inverse Adaptive Neural Balanced Fuzzy System model (IIANBFSm) to identify the Inherited Risk Matrices (IRMs) that show the risk profile of an organization.
- An Integrated Inverse Adaptive Neural Sampling Fuzzy System model (IIANSFSm) that identifies the evolution of a risk profile, using a Monte-Carlo sampling method for the fuzzy input sets and different Risk Impact Management Matrices (RIMMs) representing a sequence of risk impact, to show the evolution of risk impact management in an organization.

To configure the models, at the beginning (stage zero or start) a loss distribution (LD_MC) of reference is estimated. This is done according to the input variables of frequency and severity of operational risk and in compliance with Basel II definitions for AMA models (Bank for International Settlements, 2010). In a next stage (first stage), the model uses the LD_MC as a reference for the learning process. This is done in order to setup the models structure applying different RIMMs. In a second stage, the assessment of the evolution of risk impact management is carried out by the model. This is done according to the risk profile of the organization and by using Monte-Carlo sampling on the fuzzy input sets and different RIMMs.

Experimental results met expectations as the model was able to identify the risk profile of an organization, integrating three different models in one structure. The model IIANUFSm shows the structure and shape of the fuzzy input sets according to the RIMMs that define the sequence of risk. In the same way, the IRMs obtained by the model IIANBFSm represents the risk profile of an organization. After the learning process and using a neutral RIMM, the results obtained for the third model IIANSFSm revealed that the LDs evolve towards lower values of OpVaR in absence of a learning process due to a better risk impact management in an organization, preserving at all times the structure and shape of the LD distribution. These findings make the model ideal to assess the OpVaR in real time and also its evolution and risk impact management in an organization over time.

In Section 2 the conceptual and theoretical background for modelling the variables of frequency and severity of operational risk, and the estimation of LD by using the Monte-Carlo sampling method will be described. Additionally, the foundation for the estimation of $OpVaR_\alpha$ will be explained. Section 3 presents the IIANFSM method and the behavior with respect to the estimation of the LD. Section 4 reports the experimental results regarding the behavior of the model in terms of the evolution of an operational risk profile. Finally, our main conclusions are drawn in Section 5.

2. Theory

2.1. Operational Risk

The measurement of operational risk was included into the capital adequacy framework, known as Basel II, in 2004, as losses due to cases like Barings Bank and others made it necessary to include operational risk management in the regulation. Accordingly, operational risk has been defined by the Basel Committee on Banking Supervision (BCBS–Basel II) as follows:

> "*the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk*" (Basel Committee on Banking Supervision, 2006, page 144).

Examples complying with this definition include fraud, failure of information systems or processes, external events like earthquakes, etc. Generally, these forms of operational risk are measured per business line and in an aggregated way for the entire business. Risk measured without applying risk impact management measures, i.e. without control process, is known as inherent risk, whereas the remaining risk, after applying risk impact management, i.e. with control process, is known as residual risk.

The variables that allow the characterization of a risk event are (Otero and Veneiro, 2009; Mora Valencia, 2010; El Arif and Hinti, 2014):

- *Frequency*: Number of times a risk event occurs in a period of time (hour, day, week, month, year).
- *Severity*: Amount of loss, expressed in a currency and in a period of time (hour, day, week, month, year), caused by a risk event.

In general, in risk measurement, it is assumed that frequency and severity are independent from each other. Thus, combining a frequency distribution with a severity distribution would lead to an loss distribution (LD) for a particular risk. Due to this independence assumption, neither events with a higher frequency of occurrence nor events with a higher severity pose higher risk. Only the combination of frequency and severity defines the magnitude of risk. Commonly, frequency is modeled using discrete distributions: Poisson or Binomial; while severity is modeled using continuous distributions: Lognormal, Weibull or Generalized Pareto (Otero and Veneiro, 2009; Mora Valencia, 2010; Bank for International Settlements, 2010).

With regard to the losses due to operational risk, these can be of one of the following categories (Fig. 1):

- *Expected losses* (ELs), which are normally of high frequency and low severity.
- *Unexpected losses* (ULs), which correspond to events with lower frequent but higher severity than ELs.
- *Stress losses* (SLs), which are rare and are even less frequent than ULs but with even higher severity. Accordingly, for an organization SLs represent 'extreme losses' that need "to be addressed by suitable measures (disaster and crisis management) and, if appropriate, covered by insurance contracts" (Austrian OeNB and FMA, 2006).

Financial institutions have to internalize the costs of their risk-related behaviors. That is why they have to hold regulatory capital for operational risk. Generally, this capital is determined by the operational value at risk (OpVaR).

2.2. Regulatory Capital for Operational Risk

In general, three different methods are available to determine the value at risk and, eventually, the regulatory capital for operational risk according to Basel II (Otero and Veneiro, 2009):

1. *Basic Indicator Approach (BIA)* to estimate the value at risk ($OpVaR_\alpha$) applying a rate of 15% to the average of gross financial and non-financial income of a financial institution during the three previous years.
2. *Standardised Approach (SA)* divides the operations of a financial entity in eight business lines (Corporate Finance, Trading & Sales, Retail, Commercial, Payment & Settlement, Agency Services, Asset Management, Retail Brokerage). Each line has a beta factor associated that varies between 12% and 15%, calculated with regard to the income generated by the line. The beta factor represents a risk weight for determining OpVaR.
3. *Advanced Measurement Approach (AMA)* allows banks to develop their own operational risk models to quantify OpVaR. Although regulators do not define a particular modeling technique, one common aim of AMA models is to determine the Aggregate LD (ALD) using the distributions of frequency and severity of risk events at a certain confidence level and time horizon. The application of these models in organizations and financial entities usually requires the approval of the government or financial supervisor of a country.

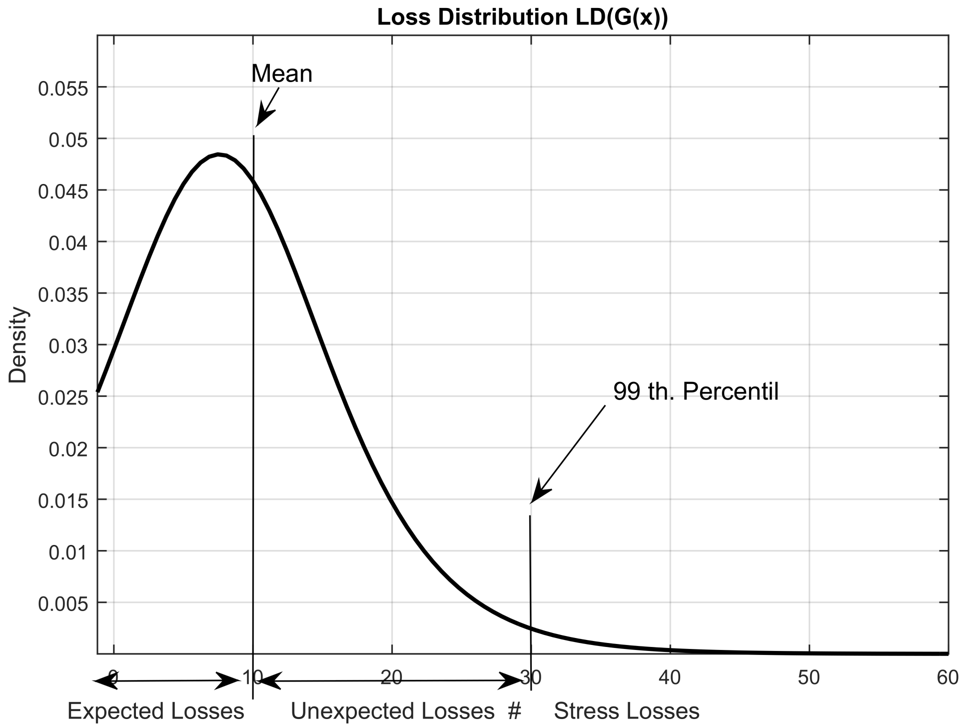


Figure 1: General structure of the loss distribution – LD ($G(x)$)

2.3. Aggregated Loss Distribution (ALD)

The ALD collects historic losses by using a LD matrix (LDM), for which the BCBS–Basel II established the following eight (8) banking business line: corporate finance, trading & sales; retail banking; commercial banking; payment & settlement; agency services; asset management; retail brokerage; and unallocated. Each business line has associated seven (7) risk event types: internal fraud; external fraud; labor relations; clients, products & business practices; damage to physical assets; business disruption and system failures; and execution, delivery & process management. Consequently, for finance institutions, the LDM contains 56 cells corresponding to the 8 business lines and 7 event types and the operational loss will be

$$L = \sum_{k=1}^{56} S_k$$

where S_k is the sum of losses of the k-th cell of the LDM. The LD approach is a method to model the random loss S_k of a particular cell, and it assumes that S_k is the random sum of homogeneous individual losses. The loss can be written as (the subindex k is omitted to simplify) (Mora Valencia, 2010; Bank for International Settlements, 2011):

$$S = \sum_{n=1}^{N(t)} X_n$$

where $N(t)$ is the random number of individual losses for the period $[0, t]$ and X_n is the n-th individual loss. The LD approach is based on the following assumptions (HSBC, 2007; Mora Valencia, 2010):

- *N(t)*: Random variable that indicates the number of events (frequency) that occur in a period of time (hour, day, week, month, year). It follows the loss frequency distribution P and the probability that the number of loss events is equal to n is denoted by $p(n)$.
- X: Random variable that indicates the quantity of losses (severity) caused by risk event in a period of time (hour, day, week, month, year). The individual losses X_n are independent and identically distributed with loss severity distribution F.
- The number of events is independent from the amount of loss events.

Once the probability distributions P and F are chosen, the cumulative distribution function of S, G, can be obtained as:

$$G(x) = \begin{cases} \sum_{n=1}^{\infty} p(n) \cdot F^{n*}(x) & x > 0 \\ \sum_{n=1}^{\infty} p(n) & x = 0 \end{cases} \tag{1}$$

where F^{n*} is the n-fold convolution of F with itself:

$$F^{n*}(x) = Pr\left[\sum_{i=1}^{n} X_i \leq x\right]$$

To obtain the $G(x)$ function the *Panjer Algorithm* or *Monte-Carlo Simulation* is used. In this context, the BCBS–Basel II recommends using a database with 3 to 5 years of operational loss data. However, in comparison with market risk, banks in general have only few operational risk data, so it is also important to use "scenarios" of risk for frequency and severity. This recommendation is made with the possibility to incorporate new real data of risk into the model over time (Bank for International Settlements, 2011).

Additionally and with regard to the loss distribution Otero and Veneiro (2009), referencing the work of Gnedenko (1943), reveal that LDs converge to a Generalized Pareto Distribution, when the number of data increases or the tail of the distribution becomes thinner. Bolancé et al. (2012) also discuss loss distribution modeling and new and more advanced parametric distributions for operational risk, like the the alpha-stable distribution and the g-and-h distribution. However, these authors also report about some flaws of these distributions and ground their work on the generalized Champernowne distribution (GCD), reasoning that GCD, with respect to the tail, converges faster to the "heavytailed Pareto distribution" than the g-and-h distribution.

2.4. Operational Value at Risk ($OpVaR_\alpha$)

The $OpVaR_\alpha$ is the measure of operational risk at the confidence level of α, i.e. the level of losses that is only exceeded with a probability of $1-\alpha$ in $G(x)$ (Fig. 1). The BCBS–Basel II recommends a value of $\alpha = 0.001$. Thus, $OpVaR_\alpha$ is a statistical risk measure that aims to respond to the question: How much can we expect to lose with a certain probability over a certain period of time? Therefore, $OpVaR_\alpha$ is interpreted as the maximum loss that can be expected given a confidence level α and a time horizon (1 year). The term "maximum" has to be interpreted with caution, as losses beyond the OpVaR are possible although not very likely to happen.

Given a LD as the one represented in Fig. 1, the operational risk in the j-th event type for the i-th business line at confidence level of α, $OpVaR(i,j,\alpha)$, is defined as follows (Jobst, 2007):

$$OpVaR(i,j,\alpha) = EL(i,j) + UL(i,j,\alpha) \tag{2}$$

where i is the line of business $(i = 1,2,\ldots,NBL)$; j is the type of risk event $(j = 1,2,\ldots..,NER)$; NBL is the number of business lines; NER is the umber of risk events; $EL(i,j)$ is the expected losses, i.e. the mean of the LDA distribution; $UL(i,j,\alpha)$ is the unexpected losses at a confidence level of α.

The $OpVaR_\alpha$ aggregates the operational risk for the business lines and the operational risk events:

$$OpVaR_\alpha = \sum_{i=1}^{NBL} \sum_{j=1}^{NER} OpVaR(i,j,\alpha).$$

There exist three main methodologies to estimate the $OpVaR_\alpha$: the historical methodology (simulating methodology), the parametric methodology, and the Monte-Carlo methodology (Otero and Veneiro, 2009; Goverment of Canada, 2011).

2.5. Recent developments with regard to operational risk measurement

At the end of 2014, BCBS stated that "the existing set of simple approaches for operational risk – the Basic Indicator Approach (BIA) and the Standardized Approach (TSA), including its variant the Alternative Standardized Approach (ASA) – do not correctly estimate the operational risk capital requirements of a wide spectrum of banks" (Bank for International Settlements, 2016). Thus, practitioners and researchers have shown an increased interest in creating AMA and LDA models for operational risk measurement. In March 2016, BCBS communicated that the "Committee's review of banks' operational risk modeling practices and capital outcomes revealed that the Advanced Measurement Approach's (AMA) inherent complexity, and the lack of comparability arising from a wide range of internal modeling practices, have exacerbated variability in risk-weighted asset calculations, and eroded confidence in risk-weighted capital ratios" (Bank for International Settlements, 2016). Accordingly, BCBS is proposed to remove the AMA from the regulatory framework and put forward instead the idea of a revised operational risk capital framework based on a single non-model-based method for the estimation of operational risk capital. This proposed framework has been termed the Standardized Measurement Approach (SMA) and it "builds on the simplicity and comparability of a standardized approach, and embodies the risk sensitivity of an advanced approach" (Bank for International Settlements, 2016). However, the SMA proposal has been criticized for its intention to remove AMA (Peters et al., 2016). Indeed, it is worth emphasizing that the SMA proposal fails to use a range of data sources, while AMA requires to model operational risk based on different data sources and, consequently, it allows to generate insight for operational risk management. Moreover, at the time of writing, though, the final versions of SMA and Basel IV have not been published yet. This paper makes a contribution to the discussion stressing the fact that AMA and LDA still can be improved and can also be standardized.

3. Methodology

From the perspective of a financial institution there is a need and an opportunity as well to better integrate different elements and aspects of operational risk. On the one hand, an institution always needs to gain and preserve a good understanding of the magnitude of this type of risk by determining the OpVaR for the different business lines and eventually for the entire organization. On the other hand, operational risk has also to be managed, for example via risk management matrices, which generally show risk in terms of a heatmap, as will be detailed in the following subsections.

Financial information is obtained from both public and private institutions as well as from experts with great knowledge and experience. In line with the development trends mentioned in Section 1, it is worth noting that different analytical models to estimate OpVaR work with crisp information. However, there also exist economic factors outside their control that are related to the impact and management of failures in business processes, which include intuition and feelings expressed by experts, in a linguistic form rather than in a numerical one, and that generates uncertainty (Chiclana et al., 2017). This gives ground for the development of models that require the use of fuzzy approaches to properly address linguistic concepts are pervaded with uncertainty to properly explain and estimate OpVaR and its evolution over time. Indeed, in our research the risk impact management of organizations is assumed to follow a sequence of impact that varies from *weak*, *medium* to *strong*, with the RIMMs defining this sequence of risk, in terms of the variables of frequency and severity, will also be described using linguistic labels from the set {*very low, low, medium, high, very high*}, as described in Section 3.1 and Section 3.2, respectively.

One of the most important issues when estimating $OpVaR_\alpha$ is related to the structure (type of distribution) and shape (long tail) of the LD distribution. However, the different available methods to estimate the LD do not allow to integrate risk impact management and the evolution of risk, so the assessment of the effects of risk impact management on LD can take a long time. For this reason, a flexible integrated inverse adaptive fuzzy inference model (IIANFSM) is proposed. This model is characterized by a Monte-Carlo behavior, integrates the estimation of LD and different *risk profiles*, which allows to show how the management of risk of an organization can evolve over time and what its effect is on the LD distribution used to estimate the $OpVaR_\alpha$.

3.1. Data and Degree of Management

A database of 701 records of daily risk events related to the failures of cash machines in a financial entity during the two years period 2009-2010 is used for the analysis and validation of the proposed model. Thus, the data refer to retail banking in terms of the business line and the corresponding risk event that caused the business disruptions is 'system failures'. The recorded risk events, in essence,

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	1	1	1	1	1
	Low	1	2	2	2	2
	Medium	1	2	3	3	3
	High	1	2	3	4	4
	Very High	1	2	3	4	5
		Very Low	Low	Medium	High	Very High

Figure 2: Risk Impact Management Matrix (RIMM).

include the date of occurrence, the number of failed transactions per day and an attributed severity value that captures the associated impact for the organization. Accordingly, the database represents internal risk data only.

In order to achieve a confidence level of 99.9% for the $OpVaR_{\alpha}$, the LD of *reference* (LD_MC) is estimated in accordance with the guidelines given by BCBS–Basel II and, consequently, the database is extended to a total of 1000 records of risk events by applying the Monte-Carlo sampling method (see Section 3.4.1) with regard to frequency and severity (Bank for International Settlements, 2010).

This LD_MC is used by the proposed IIANFSM model as a reference in the learning phase. The evaluation of the behavior of the risk impact management evolution concerning the LD is carried out using a sequence of three risk profiles, which are defined in terms of three RIMMs modeling the following risk impact management sequence or scenarios (E): *E3–weak impact, E2–medium impact, E1–strong impact*. Each of the RIMMs is designed and defined with five rows and five columns, aligned with five qualities (nfs) that define the different linguistic assessment levels for the random (linguistic) input variables of frequency and severity (Chiclana et al., 2017): *very low, low, medium, high, very high*.

The IIANFSM is executed for a total of $k = 500$ cycles in the *learning phase*, using the records of risk events from the database (1000 records). The stop criteria is defined through the difference of rms (root mean square) at instant *k+1* and k verifying $|rms_{k+1} - rms_k| < 5e - 03$ for at least ten cycles of learning.

3.2. Risk Impact Management Matrix (RIMM)

The RIMM is a structure that defines a risk profile or the theoretical foundation for risk impact management with respect to a business line in an organization (Fig. 2). The dimension of RIMM depends on the set of labels that describes the linguistic input variables of frequency and severity (Goverment of Canada, 2011; ISO, 2015), which as mentioned above is set as 5×5 ($nfs \times nfs$) in the present study. Colours are used to show the mixed effect of frequency and severity on risk, where the red color cells indicates very high risk for both frequency and impact, while dark green cells represent very low risk for both frequency and impact. Numbers in cells are also included to indicate the level of risk impact management, ranging from the lowest level of management represented by *"1"* to the highest level of management represented by "5".

In general, having a number nfs of fuzzy sets to define the different linguistic assessment levels for random input variables of frequency and severity, the RIMM will then be represented as:

$$RIMM = \begin{bmatrix} M_{1,1} & M_{1,2} & \cdots & M_{1,nfs} \\ M_{2,1} & M_{2,2} & \cdots & M_{2,nfs} \\ \vdots & \vdots & \vdots & \vdots \\ M_{nfs,1} & M_{nfs,2} & \cdots & M_{nfs,nfs} \end{bmatrix}.$$

		Severity				
	Scenario 1	Very Low	Low	Medium	High	Very High
Frequency	Very Low	1	1	1	1	1
	Low	1	2	2	2	2
	Medium	1	2	3	3	3
	High	1	2	3	4	4
	Very High	1	2	3	4	5

(a)

		Severity				
	Scenario 2	Very Low	Low	Medium	High	Very High
Frequency	Very Low	1	1	2	3	4
	Low	2	2	2	3	4
	Medium	2	3	3	3	4
	High	3	3	4	4	5
	Very High	3	4	4	5	5

(b)

		Severity				
	Scenario 3	Very Low	Low	Medium	High	Very High
Frequency	Very Low	1	2	3	4	5
	Low	2	2	3	4	5
	Medium	3	3	3	4	5
	High	4	4	4	4	5
	Very High	5	5	5	5	5

(c)

Figure 3: Matrices for Risk Impact Management: (a) E1 – Weak Impact (Strong Risk Management); (b) E2 – Medium Impact (Medium Risk Management); (c) E3 – Strong Impact (Weak Risk Management)

In our context, $M_{i,j}$ represents the impact levels of risk in a scale from 1 to 5: [1 (*Very Low*), 2 (*Low*), 3 (*Medium*), 4 (*High*), 5 (*Very High*)]; i represents the frequency levels and j the severity levels

3.3. Sequence of Risk Impact Evolution

To evaluate the general behavior of the IIANFSM model, a sequence of risk or risk profile showing the natural evolution of risk impact management in an organization is defined as in Fig. 3.

3.4. Monte-Carlo Sampling Fuzzy Sets

In order to integrate the behavior of the Monte-Carlo sampling method into the proposed model, the following definitions are required.

***Definition* 1.** Let $X_1, X_2, \ldots.., X_n$ be independent and identically distributed sample from an unknown cumulative distribution function (CDF) $F(x) = P(X \leq x)$. The *Empirical Distribution Function (ECDF)*, also known simply as the *empirical distribution function*, is defined as (Carbone et al., 2016):

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}\{X_i \leq x\}$$

where **1** is the indicator function

$$\mathbf{1}\{X_i \leq x\} = \begin{cases} 1 & if\ X_i \leq x \\ 0 & otherwise. \end{cases}$$

***Definition* 2.** If CDF F is strictly increasing and continuous then $F^{-1}(p) = x$, $p \in [0,1]$, is the unique real number x such that $F(x) = p$. In such a case, this defines the *inverse distribution function or quantile function.*

***Definition* 3.** Let $\{q_1,q_2,\ldots..,q_{nfs}\}$ be the quantiles of a random variable X with CDF *F(x)*. Let $\{XC_{N,1},XC_{N,2},\ldots,XC_{N,nfs}\}$ the normalized location of fuzzy sets that define the different linguistic assessment levels used for this same random variable. Let m be the slope of the correlation line between the quantiles and the normalized location of fuzzy sets. Then, we have:

- When $m=1$, the fuzzy sets that define the different linguistic assessment levels for the random variable are uniformly distributed on the horizontal axis.
- When $m>1$, the fuzzy sets that define the different linguistic assessment levels for the random variable are distributed towards the left side of the horizontal axis.
- When $m<1$, the fuzzy sets that define the different linguistic assessment levels for the random variable are distributed towards the right side of the horizontal axis.

***Definition* 4.** Given $\{q_o,q_1,q_2,q_3,q_4\}$ the quantiles of a random variable with CDF *F(x)*. Then we have that

$$\frac{\partial \mathrm{F}\left(q_2\right)}{\partial \mathrm{x}}\approx \mathrm{m}$$

and when (see Fig. 4):

1. $m=1$: CDF *F(x)* has *asymmetry* $=0$ and the data may come from a centered distribution (balanced fuzzy sets – Fig. 4 (a)).
2. $m>1$: CDF *F(x)* has *asymmetry* >0 and the data may come from a distribution with *long tail* (unbalanced fuzzy sets with tendency to the left side – Fig. 4 (b)).
3. $m<1$: CDF *F(x)* has *asymmetry* <0 and data may come from a distribution with *inverted long tail* (unbalanced fuzzy with tendency to the right side – Fig. 4 (c)).

Compliant with Definition 4, an *a priori* structure and shape of the probability distribution that represent a linguistic variable can be set, leading to the concept of the *loss distribution approach (LDA)* , which, in our case, is estimated by the following proposed fuzzy model.

3.4.1. Monte-Carlo Fuzzy Sampling Method

Let $\{q_1,q_2,\ldots..,q_{nfs}\}$ be the quantiles of a random variable X with CDF $F(x)$, which is also described linguistically via nfs fuzzy linguistic labels with following Gaussian membership functions:

$$u_j\left(x\right)=Exp\left(-\frac{1}{2}\left(\frac{q_j-x}{D_j}\right)^2\right) \tag{3}$$

where

$$D_j=\begin{cases} q_{j+1}-q_{j-1} & j<nfs\\ 2\cdot\left(q_j-q_{j-1}\right) & j=nfs\\ 2\cdot\left(q_{j+1}-q_j\right) & j=0\end{cases} \tag{4}$$

The Monte-Carlo fuzzy sampling function for a uniform random value $p_k\in[0,1]$, $FS\left(p_k\right)$, has the following structure (Peña P and Hernández R, 2016):

$$FS\left(p_k\right)=\sum_{j=1}^{nfs}u_j\left(p_k\right)\cdot F^{-1}\left(q_j\right) \tag{5}$$

where $F^{-1}\left(q_j\right)$ is the *quantile function* for q_j. Thus, the procedure of sampling by using the Monte-Carlo method for a random variable X with CDF $F(x)$ described linguistically via a number of fuzzy linguistic sets is:

1. Normalize fuzzy linguistic sets used to describe the random variable.
2. Generate random number $p_k\in[0,1]$ using a uniform distribution.
3. Compute degrees of membership of random generated value in step 2 with respect to the normalized fuzzy sets j using (3), $u_j\left(p_k\right)$.
4. The sampling value is obtained using (5).
5. Steps 1, 2 and 3 are repeated until 1000 data is reached.

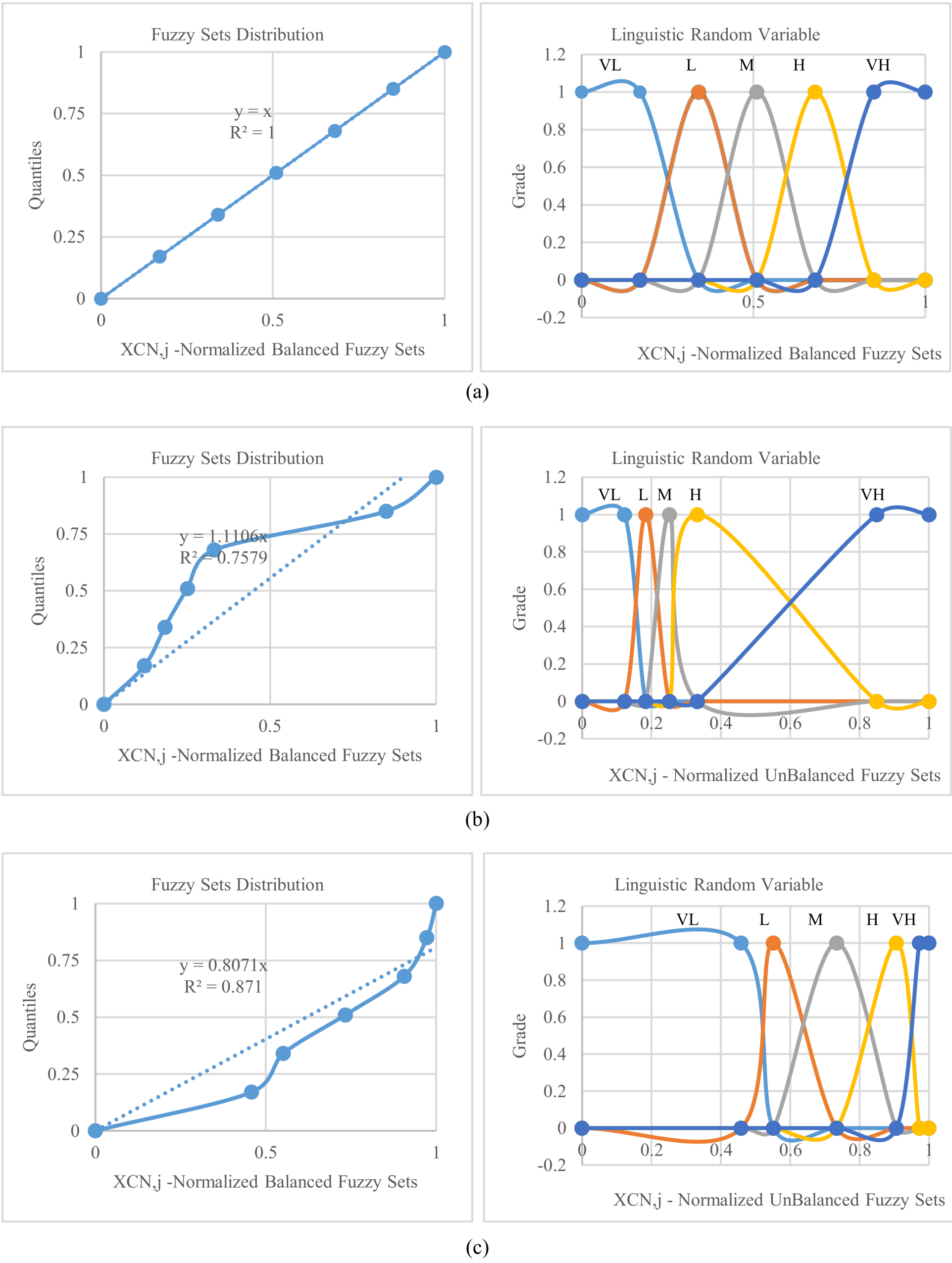


Figure 4: Distribution of fuzzy linguistic labels associated to a random variable

3.5. IIANFSM to Assess the Evolution of OpVaR

3.5.1. IIANFSM General Structure

According to the structure of the basic ANFIS model (Fragiadakis et al., 2014), the structure of the IIANFSM model is denoted and defined as:

$$yr_{ANFIS,k} = \sum_{j=1}^{NR} C_j \cdot \left(\frac{RIMM_j}{SMD}\right) \cdot h_j \cdot x_{s,j} \cdot x_{f,j} \tag{6}$$

where NR is the number of activated rules from the total set of $nfs \times nfs$ possible rules, for the random variables of frequency (f) and severity (s), by sampling values of the random variables frequency and severity at instant k, $x_{f,j}$ and $x_{s,j}$, respectively; k indicates the number of data to estimate the LDA distribution; C_j is the importance of each of the rules for the IIANFSM system as per the matrix, in line with the RIMM structure; $C_{F,j1,j2}$ consisting of the importance of the rules in the IIANFSM system to mitigate the impact generated by the relation between fuzzy set $j_1(=1,2,\dots nfs)$ (Frequency) and fuzzy set $j_2(=1,2,\dots,nfs)$ (Severity):

$$C_{F,j1,j2} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,nfs} \\ c_{2,1} & c_{2,2} & \cdots & c_{2,nfs} \\ \vdots & \vdots & \vdots & \vdots \\ c_{nfs,1} & c_{nfs,2} & \cdots & c_{nfs,nfs} \end{bmatrix};$$

h_j is the antecedent estimate by the rule j, which is computed as:

$$h_j = ux_{f,j1} \cdot ux_{s,j2} \tag{7}$$

where $ux_{f,j1}$ and $ux_{s,j2}$ are the membership values of $x_{f,k}$ and $x_{s,k}$ to the frequency fuzzy set and severity fuzzy set in rule j, respectively, as per the Gaussian membership functions

$$ux_{f,j1} = e^{-\frac{1}{2}\left(\frac{XC_{f,j1}-x_{f,k}}{D_{f,j1}}\right)^2}; \quad ux_{s,j2} = e^{-\frac{1}{2}\left(\frac{XC_{s,j2}-x_{s,k}}{D_{s,j2}}\right)^2}$$

$XC_{f,j1}$ and $XC_{s,j2}$ being the centers of the fuzzy sets $j1$ and $j2$ corresponding to linguistic input variable of frequency and severity, respectively; $D_{f,j1}$ and $D_{s,j2}$ defined as per (4). $RIMM_j$ represents the *j-elements* in the following matrix, $RIMM_{F,j1,j2}$, consisting of the level of management for the combined risk impact of *frequency* and *severity* (see Section 3.2).

$$RIMM_{F,j1,j2} = \begin{bmatrix} M_{1,1} & M_{1,2} & \cdots & M_{1,nfs} \\ M_{2,1} & M_{2,2} & \cdots & M_{2,nfs} \\ \vdots & \vdots & \vdots & \vdots \\ M_{nfs,1} & M_{nfs,2} & \cdots & M_{nfs,nfs} \end{bmatrix}$$

SMD represents the sum of the $RIMM_j$ values of the activated rules: $SMD = \sum_{j=1}^{NR} RIMM_j$.

In accordnace with the general structure, the model can take three different forms to estimate the *LD_MC* or LD of *reference*:

1. The *Integrated Inverse Adaptive Neural Unbalanced Fuzzy System model (IIANUFSm)* allows to identify the structure and shape of the fuzzy input sets using different RIMMs.
2. The *Integrated Inverse Adaptive Neural Balanced Fuzzy System model (IIANBFSm)* allows to identify the Inherent Risk Matrix (IRM) using balanced fuzzy sets and different RIMMs.
3. The *Integrated Inverse Adaptive Neural Sampling Fuzzy System model (IIANSFSm)* uses a Monte-Carlo sampling process for the fuzzy input sets to assess the evolution of operational risk values over time.

Table 1: Fuzzy Inherent Risk Matrix (FIRM)

Frequency labels	Frequency levels	Control Measure	Effectivity C_j's	Average Control	Fuzzy Residual Risk
LB_{j1}	LLB_{j1}	$RIMM_{F,j1,j2}$	$C_{F,j1,j2}$	FCL_{j1}	FRR_j

• LB_{j1} are the impact labels for frequency in accordance with the $j_1 - frequency$ levels ($j_1 = 1,2,3,\ldots,nfs$).

• LLB_{j1} are the level of risk for frequency. These values range from 0.1 to 0.5 in accordance with the labels that are defining this linguistic variable.

• $RIMM_{F,j1,j2}$ is the matrix consisting of the levels of risk impact management according to the combined impact of $j_1 - frequency$ and $j_2 - severity$.

• $C_{F,j1,j2}$ is the matrix of importance of each rule of risk impact management in the IIANFSM structure.

• FCL_{j1} are the fuzzy control levels for $j_1 - frequency$, which is defined using the product t-norm of the values that define the importance of rules ($C_{F,j1,j2}$) and the risk impact management matrix ($RIMM_{F,j1,j2}$): $FCL_{j1} = \sum_{j2=1}^{nfs} RIMM_{F,j1,j2} \cdot C_{F,j1,j2}$

• FRR_{j1} is the fuzzy residual risk, and it represents the ratio between the level of risk for the frequency an the fuzzy average control for the severity: $FRR_{j1} = \frac{LLB_{j1}}{FCL_{j1}}$

• The *risk profile (RP)* indicates the average resulting fuzzy residual risk obtained through the effect of risk impact management: $RP = \dfrac{1}{nfs} \sum_{j1=1}^{nfs} FRR_{j1}$

The adaptive process is defined by the *generalized delta rule* (Rumelhart and Hinton, 1986):

$$XC_{j,i} = XC_{j,i} - \gamma \cdot \frac{\partial rms_k}{\partial XC_{j,i}}; \quad C_j = C_j - \gamma \cdot \frac{\partial rms_k}{\partial C_j}; \quad D_j = D_j - \gamma \cdot \frac{\partial rms_k}{\partial D_j};$$

where γ represents the learning factor; rms_k is the root mean square at *instant k*:

$$rms_k = \sqrt{(yd_k - yr_k)^2}$$

and yd_k is the reference value at instant k that conforms the *LD_MC* distribution. From expressions (6) and (7), the learning rules take the following form:

$$C_j = C_j + \gamma \cdot h_j \cdot \left(\frac{RIMM_j}{SMD}\right) \cdot x_{f,k} \cdot x_{s,k} \tag{8}$$

$$XC_{j,i} = XC_{j,i} + \gamma \cdot C_j \cdot \left(\frac{RIMM_j}{SMD}\right) \cdot x_{f,k} \cdot x_{s,k} \cdot h_j \cdot \left(\frac{x_{k,i} - XC_{j,i}}{D_{j,i}^2}\right) \tag{9}$$

$$D_{j,i} = D_{j,i} + \gamma \cdot C_j \cdot \left(\frac{RIMM_j}{SMD}\right) \cdot x_{f,k} \cdot x_{s,k} \cdot h_j \cdot \left(\frac{(x_{i,k} - XC_{j,i})^2}{D_{j,i}^3}\right) \tag{10}$$

3.5.2. Fuzzy Inherent Risk Matrix (FIRM)

The inherent risk arises from exposure to an uncertainty of probable events or changes in business conditions or the economy in general that can impact the operations of an organization. In this way, the fuzzy inherent risk matrix (FIRM) allows to estimate the residual risk or risk profile after applying risk impact management (measures) in an organization, taking into account the relationship between the normalized exposition of inherent risks and the importance of the rules of management delivered by the IIANFSM that allow to mitigate the risks. This is described in more detail in Table 1.

3.5.3. Fuzzy OpVaR

Let $F(x)$ be the cumulative distribution function (CDF) of a random variable X, and u the value of X in the tail of the distribution, then the probability of X being located between u and $u+y$ $(y>0)$ when x>u is:

$$P[u<X<u+y|x>u]=\frac{P[u<x<u+y]}{P[x>u]}=\frac{F(u+y)-F(u)}{1-F(u)}=F_u(y)$$

$F_u(y)$ represents the right tail of the distribution of probability. According to Gnedenko et al. (Gnedenko et al., 1969), for a wide class of distributions $F(x)$, increasing the u variable leads to $F_u(y)$ to converge to the following *generalized Pareto distribution (GPD)*:

$$G_{\xi,\beta}(y)=1-\left(1+\frac{\xi.(y-u)}{\beta}\right)^{-\frac{1}{\xi}},\ \xi\neq 0$$

where u is a threshold parameter; β is a scale parameter $(\beta>0)$; and ξ is a shape parameter $(\xi\epsilon R)$.

The estimation of $1-F(u)$ can be determined from the empirical data:

$$1-F(u)=P[x>u]=\frac{n_u}{n}$$

$$P[x>u+y]=P[x>u+y|x>u]\cdot P[x>u]=\frac{n_u}{n}\cdot\left[1-G_{\xi,\beta}(y)\right]$$

where u representing a value close to the 95% percentile of the empirical distribution $(u\in R)$; n is the total number of observations that make up the empirical distribution; and n_u is the number of observations of x that are greater than u. Therefore, the estimator of the tail of the CDF $F(x)$, when x is big, can be expressed as follows:

$$F(x)=P[x<u+y]=1-P[x>u+y]=1-\frac{n_u}{n}\cdot\left(1+\xi\cdot\frac{(x-u)}{\beta}\right)^{-1/\xi}$$

According to (2), in order to calculate the *OpVaR* with a confidence level of q it is necessary to solve the following equation:

$$q=1-\frac{n_u}{n}\cdot\left(1+\xi\cdot\frac{(OpVaR-u)}{\beta}\right)^{-1/\xi}$$

Thus, we have that *OpVaR* is:

$$OpVaR=u+\frac{\beta}{\xi}\cdot\left[\left(\frac{n_u}{n}\cdot\frac{1}{(1-q)}\right)^{\xi}-1\right]$$

In line with the fuzzy representation of the LD distribution from Definition 3, the parameters ξ and β can be estimated from:

$$q_j=1-\frac{n_u}{n}\cdot\left(1+\xi\cdot\frac{LD^{-1}(q_j)-u}{\beta}\right)^{-1/\xi}$$

where $LD^{-1}(q_j)$ is the *quantile function* that represents the CDF of the LD distribution for the value q_j that are the different values according to the number of fuzzy sets that define the random variable of interest. In the particular case of using j =1 (Very Low), 2 (Low), 3 (Medium), 4 (High), 5 (Very High), the quantiles for the LD experimental distribution are defined as $q_0, q_{0.25}, q_{0.5}, q_{0.75}, q_1$, respectively.

3.6. Experimental Validation of IIANFSM

For a general validation of the IIANFSM, three stages were taken into account according to each of the forms that integrate the model:

First stage. *Identification of Input Fuzzy Sets – IIANUFSm.* In this stage the model estimates the LD_MC for the sequence of risk (*learning phase*) using unbalanced input fuzzy sets. The model was evaluated using the Index of Agreement (IOA) for the estimation of LD_MC, where it is expected that the structure and shape of the input fuzzy sets for frequency and severity evolve towards distributions of heavy tails (Definition 4).

Second stage. *Identification of FIRM – IIANBFSm.* In this stage the model estimates the LD_MC for the sequence of risk (*learning phase*) using balanced fuzzy input sets. The risk profile is given by the FIRM, where it is expected that the importance of the rules in the IIANBFSm take lower values due to a better risk impact management.

Third stage. *Evolution of Risk Impact Management – IIANSFSm.* In this stage the model estimates the LD_MC using the Monte-Carlo sampling method for linguistic input and by using a neutral RIMM (*learning phase*). In this stage, the model is evaluated according to the evolution experienced by the LD using the sequence of risk that shows the natural evolution of risk impact management in an organization. It is expected that LDs evolve towards distributions with *heavy tails,* lower values of $OpVaR_{99.9\%}$, higher values of coverage with similar structure and shape than the LD_MC of reference.

3.6.1. IC-fingreprint structure

The LD is represented by an *IC*-fingerprint structure that groups the main statistical indices that characterize the $OpVaR_\alpha$ (De Martino et al., 2007; Cooper et al., 2014):

a) $OpVaR_{99.9\%}$: Operational risk value set at the 99.9 percentile on the LD distribution.
b) EL (Expected Losses): This value is represented by the mean of the LD distribution.
c) UL (Unexpected losses): $UL = OpVaR_{99.9\%} - EL$
d) Coverage of EL (CEL): $CEL=\dfrac{EL}{OpVaR_{99.9\%}}$, which indicates the percentage of coverage of EL by the $OpVaR_{99.9\%}$.
e) Coverage of UL (CUL): $CUL=\dfrac{UL}{OpVaR_{99.9\%}}$, whic indicates the percentage of coverage of UL by the $OpVaR_{99.9\%}$.
f) Tail Data (TD): Number of data located between EL and the $OpVaR_{99.9\%}$ value, or number of data located in the tail of the LD distribution.
g) Exposure Grade (EG): $EG = \dfrac{\frac{TD}{ND} \cdot \frac{OpVaR_{99.9\%}+UL}{2}}{OpVaR_{99.9\%}}$, which is the percentage of coverage reached by $OpVaR_{99.9\%}$ with regard to average losses located in the tail of the LD distribution.
h) Insured Value (IV): $IV{=}OpVaR_{99.9\%} \cdot EG$, which indicates the insured value to cover the events of risk at a level of 99.9%.

3.6.2. Learning Phase

In coherence with the general structure defined by the IIANFSM, the behavior of the IIANBFSm is described below through the estimation of the LD_MC in the *learning phase*:

1. It begins with the normalization of data for the frequency (X_f) and severity (X_s) variables, and with the estimation of the LD_MC of reference (yd_k) using expression (1) (Section 2.3) and the Monte-Carlo sampling fuzzy sets method (Section 3.4).
2. The input (Freq.:X_f,Sev.:X_s) is defined using balanced fuzzy sets for labels (as in Fig. 4 (a)). The location and shape of the membership functions of these FSs are provided in Table 2.
3. Before the learning process starts, the vector of importance for each rule ($C_j \in [0,1]$) for the IIANBFSm system is setup, which takes the array structure ($C_{nfs \times nfs}$).

Table 2: Location and shape of the FS's membership functions

	Very Low	Low	Medium	High	Very High
$XC_{N,j,i}$	0	0.25	0.5	0.75	1
$D_{N,j,i}$	0.5	0.5	0.5	0.5	0.5

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	h1	h2	h3	h4	h5
	Low	h6	h7	h8	h9	h10
	Medium	h11	h12	h13	h14	h15
	High	h16	h17	h18	h19	h20
	Very High	h21	h22	h23	h24	h25

Figure 5: Matrix of activated rules linked to input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$

4. To estimate the LD_MC in the learning phase, the IIANBFSm model uses the following neutral RIMM:

$$RIMM = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

After the learning process this matrix may be changed by different RIMMs (E3-E2-E1) with the aim to assess the impact of management on LD.

5. At this point the learning phase begins. Let $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$ be two random values for frequency and severity, respectively, at instant k. Using the corresponding Gaussian membership functions (3) with parameters given in Table 2, the corresponding membership values are computed and shown in Table 3. Despite the fact that the fuzzy sets are defined using Gaussian membership functions, the case study is developed taking the higher values of membership associated with the fuzzy input sets for each linguistic variable. Figure 5 shows the rules activated by input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$:

Table 3: Estimation of membership values for input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$

	Very Low	Low	Medium	High	Very High
$XC_{j,f}$	0	0.25	0.5	0.75	1
$XC_{j,s}$	0	0.25	0.5	0.75	1
$D_{j,i}$	0.5	0.5	0.5	0.5	0.5
$u_{f,j,k}$	0	0.9801	0.9559	0	0
$u_{sj,k}$	0	0	0.9155	0.9968	0

6. According to Fig. 5, using expression (7), we proceed with the computation of antecedent h_j of each activated rule, which is shown in Table 4.
7. According with the structure of IIANBFSm and assuming a set of random numbers for the vector C_j ($C_j \sim N(0,1)$), the output value linked to input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$ is calculated, as shown in Table 5.

Table 4: Antecedent values of activated rules by input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$

h_8	h_9	h_{13}	h_{14}
0.8972	0.9769	0.8751	0.9528

Table 5: Estimation of $yr_{IANBFSm,k}$ linked to input values $x_{f,k} = 0.35$ and $x_{s,k} = 0.71$

h_8	h_9	h_{13}	h_{14}
$RIMM_8$	$RIMM_9$	$RIMM_{13}$	$RIMM_{14}$
1	1	1	1
$SMD = 4$			
C_8	C_9	C_{13}	C_{14}
0.4356	0.2587	0.8795	0.5678
$yd_k = 0.3512$			
$yr_{IANBFISM} = 0.2869$			
$rsm = 0.0643$			

8. Using a value of $\gamma = 0.1$, the structure of the IIANBFSm for the *rule-9* is updated using expressions (8), (9) and (10):

$$C_{9,k+1} = 0.2587 + 0.1 \times \left(\frac{1}{4}\right) \times 0.9769 \times 0.35 \times 0.71$$

$$XC_{4,1,k+1} = 0.75 + 0.1 \times 0.2587 \times \left(\frac{1}{4}\right) \times 0.9769 \times 0.35 \times 0.71 \left(\frac{0.35 - 0.75}{0.5^2}\right)$$

$$XC_{4,2,k+1} = 0.75 + 0.1 \times 0.2587 \times \left(\frac{1}{4}\right) \times 0.9769 \times 0.3 \times 0.71 \times \left(\frac{0.71 - 0.75}{0.5^2}\right)$$

   As per (3), this procedure produces an update of the diameter $D_{j,i}$ of the fuzzy sets used to represent the linguistic input variables.
9. This estimation process continues until $k = 1000$. When $k \leq 1000$, the process moves to step 5.
10. Steps 5 – 9 is repeated until a number of iterations (NI) is reached or $|rsm_{i1+1} - rsm_{i1}| < 5e-03$ for ten consecutively iterations.

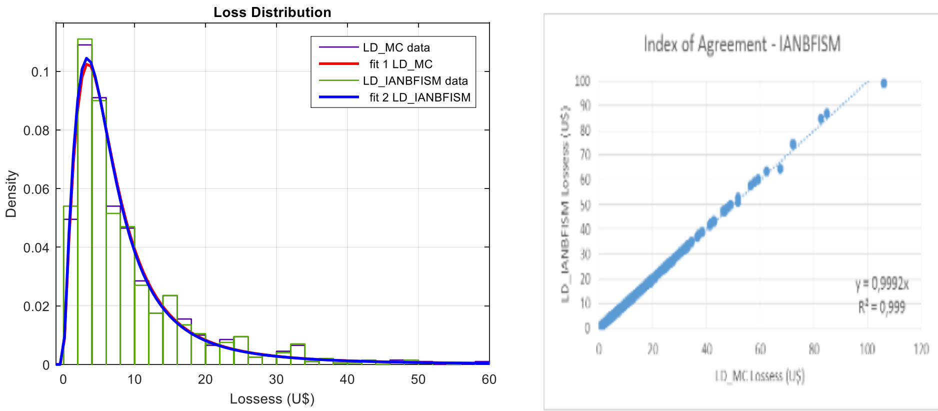
Figure 6 shows that the IIANBFSm reached an IOA close to one with regard to the estimation of LD_MC, which shows the good performance of the model in the *learning phase*. The model reached similar values for $OpVar_{99.9\%}$, tail data and coverage indices ($EL/OpVar_{99.9\%}$, $UL/OpVar_{99.9\%}$) (see Table 6), which show the good performance of the IIANBFSm model in the estimation of LD_MC in the learning phase.

In coherence with this structure, the *learning* adheres to $C_{F,j1,j2}$ (Fig. 7), the importance of the rules for the system according to the RIMM defined by the IIANBFSm. Blue bars show the need to increase the levels of risk impact management, while the red bars show the importance of the rules (negative values) in the sense of mitigating (reducing) frequency and severity for the operational risk through better risk impact management.

4. Experimental Results

4.1. Stage 1: Identification of Fuzzy Input Sets (IIANUFSm)

Table 7 shows the behavior of the model in the learning phase in the estimation of the LD_MC for each risk profile in the sequence of risk. It can be observed that the model reached an IOA close to one regarding LD_MC (G.Pareto), and similar values for the *IC-fingerprint* indices, showing the stability



(a) (b)

Figure 6: LD estimated by the IIANBFSm in the learning phase (a) PDF function (b) IOA chart

Table 6: LD_MC estimated by the IIANBFSm model in the learning phase

	LDA_MC	**LD_Est**
Distribution	G.Pareto	G.Pareto
NLoglik	-3316.700	-3132.300
shape (ξ)	0.176	0.186
scale (β)	7.71078	7.005
threshold (Θ)	-2.20E-15	0.000
Kurtosis	20.311	18.833
Assymmetry	3.608	3.511
EL (Exp.)	9.636	9.611
UL (UnExp.)	96.524	87.822
$OpVar_{99.9\%}$	106.160	97.433
EL/$OpVar_{99.9\%}$	0.091	0.099
UL/$OpVar_{99.9\%}$	0.909	0.901
Tail Data	321.000	319.000

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	0.6811543	0.5796059	0.1545442	0.37932176	-0.32364892
	Low	0.80579218	0.19113157	0.64735186	0.06777605	-0.24965995
	Medium	0.7184421	0.34103079	0.07841213	0.23531615	-0.30477613
	High	0.57782578	0.1159844	0.02470434	-0.57029486	-0.32400807
	Very High	0.29844688	-0.32626445	-0.29062395	-0.40450698	-0.0565972

Figure 7: $C_{F,j1,j2}$ – matrix of importance of inference rules that conform the IIANBFSm

Table 7: LD estimated by the IIANUFSm for scenarios E1-E2-E3

	LDA_MC	**E3**	**E2**	**E1**
Distribution	G. Pareto	G.Pareto	G.Pareto	G.Pareto
AIC	-3136.700	-3101.200	-3048.800	-3130.000
shape (ξ)	0.176	0.212	0.261	0.184
scale (β)	7.1078	6.615	5.972	7.001
threshold (Θ)	-2.20E-15	0.000	0.000	0.000
Kurtosis	20.331	22.837	23.240	17.592
Assymmetry	3.606	3.810	3.944	3.440
$OpVar_{99.9\%}$	106.160	107.524	109.808	97.562
EL (Exp.)	9.638	9.403	9.083	9.600
UL (UnExp.)	96.522	98.121	100.725	87.962
EL/$OpVar_{99.9\%}$	0.091	0.087	0.083	0.098
UL/$OpVar_{99.9\%}$	0.909	0.913	0.917	0.902
Tail data	321.000	310.000	300.000	318.000
Exposure	0.306	0.296	0.288	0.302
Insurance	32.530	31.875	31.580	29.498
IOA	1.0000	0.9991	0.9980	0.9976

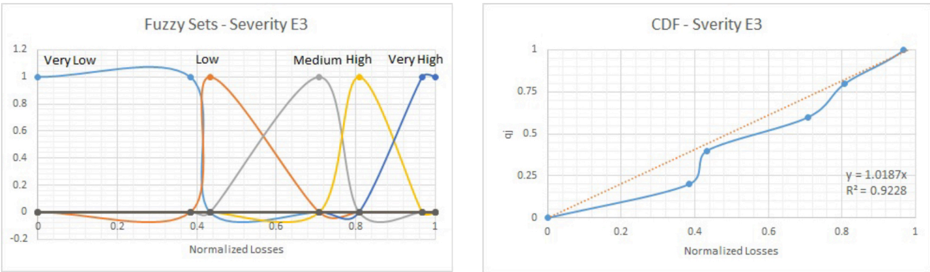
of the model in the estimation of LD_MC for different risk profiles. Figure 8 shows the structure and shape of the obtained input fuzzy sets that represent the random linguistic variable for severity for each risk profile in the sequence of risk E3-E2-E1. From Definition 4, it can be observed that the fuzzy sets evolve towards lower values of loss in line with the sequence of risk (E3-E2-E1), as shown by the slope of the line, which evolves towards higher values of slope, with values above one that determine *a priori* the structure and shape (*heavy tail*) of the distribution for the input variable severity. Clearly, the obtained fuzzy sets in the E1 scenario are located toward the right side of the axis of frequency and severity (Figure 8(a)), indicating the presence of a centered CDF with slope value close to one. For the E2 and E3 scenarios, the obtained fuzzy sets are located toward the left side of the horizontal axis, indicating the presence of long tails with higher losses (Figure 8(b), 8(c)), requiring the attention of risk impact management.

4.2. Stage 2: Identification of a Fuzzy Inherent Risk Matrix (FIRM)

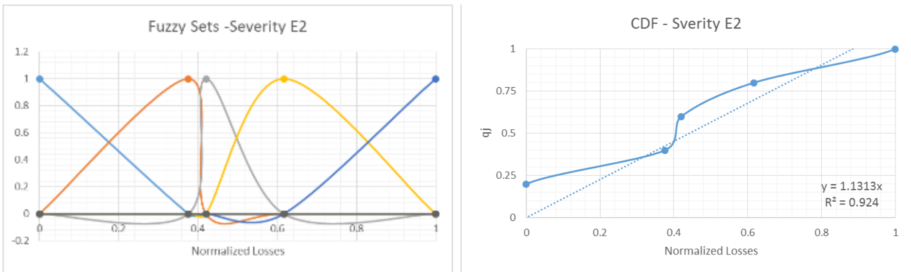
Table 8 shows that the model reached an IOA close to one with regard to the estimation of LD_MC for the sequence of risk, with similar values of $OpVaR_{99.9\%}$, tail data and coverage indices ($UL/OpVar_{99.9\%}$, $EL/OpVaR_{99.9\%}$). At this stage the learning was based on $C_{F,j1,j2}$ because the input sets were represented by balanced fuzzy sets. The model reached negative IOA for $RIMM_{j1,j2}$ above the value of 75%, indicating that risk impact management is required. The importance of the rules of matrix $C_{F,j1,j2}$ where data evolves towards negative values according to the sequence of risk is given in Fig. 9. This figure shows the evolution experimented by the risk profile according to the sequence of risk, with strong levels of management leading towards lower values of risk are required. Fig. 10 shows the structure of FIRM for the RIMM E1, where the risk profile is given for the average of the residual risk for each level of frequency and its level of management for the adjoint severity. As in the Case Study, the blue bars show the need to increase the levels of risk impact management, while the red bars show the importance of the rules (negative values) in the sense of mitigating the frequency and severity for the operational risk through better risk impact management.

Taking the operational value at risk defined by the BCBS–Basel II ($OpVaR_{\alpha}$), the FIRM configures a new indicator to assess the quality of risk impact management in an organization:

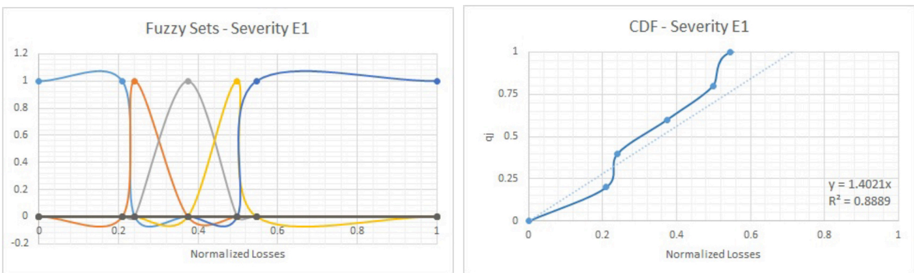
$$OpVaR_{\alpha,ie} = (1 + RP_{ie})OpVaR_{\alpha,ie} \tag{11}$$



(a)



(b)



(c)

Figure 8: Structure and shape of the input fuzzy sets that conform for severity after the learning process in the Stage 1: (a) E3 – Strong Impact (Weak Risk Management); (b) E2 – Medium Impact (Medium Risk Management); (c) E1 – Weak Impact (Strong Risk Management)

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	0.5853287	0.48458725	0.2756972	0.3751047	0.87608528
	Low	0.45481831	0.02941327	0.53535199	0.49034968	0.70496351
	Medium	0.71418828	0.22390389	0.3556366	0.29167801	0.14897402
	High	0.63314921	0.20413367	0.06619806	-0.31423971	-0.18163905
	Very High	0.06276532	0.41548559	0.17827292	-0.41180202	-0.23342417

(a)

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	0.68001211	0.52301264	0.18321574	0.3698585	-0.39183259
	Low	0.91959007	0.21969216	0.65561962	0.06261079	-0.23170303
	Medium	0.668167	0.42221192	0.06957509	0.24084143	-0.26962173
	High	0.61206049	0.12984668	0.02734651	-0.54115534	-0.27702427
	Very High	0.24971083	-0.39586142	-0.24237131	-0.35866952	-0.05323335

(b)

		Severity				
		Very Low	Low	Medium	High	Very High
Frequency	Very Low	0.87780476	0.61729431	0.64365882	-0.41070113	-0.44732311
	Low	0.61378771	0.15329142	0.32459244	-0.34388041	-0.11487402
	Medium	0.38941908	0.22817448	-0.03355768	-0.11783367	-0.29351413
	High	-0.36123511	-0.15580635	-0.47177956	-0.20594387	-0.49818829
	Very High	-0.40179858	-0.22001854	-0.01437018	-0.41870841	-0.06629004

(c)

Figure 9: $C_{F,j1,j2}$ matrices obtained by the IIANBFSm in the learning phase for the sequence of risk (a) E3 – Strong Impact (Weak Risk Management); (b) E2 – Medium Impact (Medium Risk Management); (c) E1 – Weak Impact (Strong Risk Management)

Table 8: LD estimated by the IIANBFSm for scenarios E3-E2-E1

	LDA_MC	E3	E2	E1
Distribution	G.Pareto	G.Pareto	G.Pareto	G.Pareto
NLoglik.	-3316.700	-3122.700	-3121.300	-3132.300
shape (ξ)	0.176	0.194	0.193	0.186
scale (β)	7.71078	6.882	6.878	7.005
threshold (Θ)	-2.20E-15	0.000	0.000	0.000
Kurtosis	20.311	19.865	20.202	18.833
Assymmetry	3.608	3.600	3.636	3.511
EL (Exp.)	9.636	9.546	9.535	9.611
UL (UnExp.)	96.524	89.599	95.710	87.822
$OpVar_{99.9\%}$	106.160	99.145	105.245	97.433
EL/$OpVar_{99.9\%}$	0.091	0.096	0.091	0.099
UL/$OpVar_{99.9\%}$	0.909	0.904	0.909	0.901
Tail Data	321.000	316.000	316.000	319.000
Exposure	35.778	32.999	34.914	32.782
Insurance	0.337	0.333	0.332	0.336
IOA (LD)	1.0000	0.9995	0.9997	0.9995
IOA (RIMM)	1.0000	-0.8301	-0.8091	-0.7710
Risk Profile	1.0000	0.1348	1.1468	1.6762

	Fuzzy Quality of Risk Management					Fuzzy Residual Risk
Fequency	Risk Level	Control Measures	Efectivity Cj's	Average Control	Residual Risk	
Very Low	0.1	1	0.87780476	0.22359557	0.44723605	0.44723605
		2	0.61378771			
		3	0.38941908			
		4	-0.36123511			
		5	-0.40179858			
Low	0.2	2	0.61729431	0.12458706	1.60530309	1.60530309
		2	0.15329142			
		3	0.22817448			
		4	-0.15580635			
		5	-0.22001854			
Medium	0.3	3	0.64365882	0.17462909	1.71792685	1.71792685
		3	0.32459244			
		3	-0.03355768			
		4	-0.04717796			
		5	-0.01437018			
High	0.4	4	-0.41070113	-0.2994135	-1.33594512	-1.33594512
		4	-0.34388041			
		4	-0.11783367			
		4	-0.20594387			
		5	-0.41870841			
Very High	0.5	5	-0.44732311	-0.28403792	-1.76032834	-1.76032834
		5	-0.11487402			
		5	-0.29351413			
		5	-0.49818829			
		5	-0.06629004			
		Risk Profile				0.13483851

Figure 10: Fuzzy Inherit Risk Management given by the IIANBFSm after the learning process for scenario E1

Table 9: LD estimated by the IIANSFSm - Stage 3

	LDA_MC	**LD_IANSFISm**
Distribution	G. Pareto	G.Pareto
AIC	-3708.4100	-3646.8300
shape (ξ)	0.1755	0.1755
scale (β)	7.1078	6.1207
threshold (Θ)	0.0000	0.0000
Kuirtosis	20.3110	17.9468
Assymmetry	3.6080	3.4430
$OpVar_{99.9\%}$	105.1600	78.2328
EL (Exp.)	8.6360	7.4341
UL (UnExp.)	95.5241	70.7987
EL/$OpVar_{99.9\%}$	0.0821	0.0950
UL/$OpVar_{99.9\%}$	0.9179	0.9050
Tail data	321.0000	320.0000
Exposure	0.3078	0.3048
Insurance	3.2370	2.3845
IOA	1.0000	0.9945

where RP_{ie} is the risk profile (or FIRM) given by the IIANBFSm for the *ie-scenario (ie=E1,E2,E3).* According to (11), it can be observed that $OpVaR_{\alpha}$ evolves towards lower values generating a better risk profile until located close to the $OpVaR_{\alpha}$of reference, defined by the BCBS–Basel II. This equation demonstrates that the system can reach the equilibrium in $RP_{ie} = 0$, point from which the positive inherent risk can be covered by a better management of negative inherent risk in a business line or risk event type.

4.3. Stage 3: Sensitivity Analysis (IIANSFSm)

Table 9 shows the IIANSFSm reached an IOA close to one in the estimation of LD_MC at Stage 3, with similar values of tail data and coverage, which demonstrates the stability of the model in the estimation of LD_MC using a neutral RIMM and Monte-Carlo sampling for the linguistic input variables.

Figure 11 shows that the structure and shape of the fuzzy input sets reached values of slope greater than unity, indicating the presence of distributions with heavier tails for frequency and severity, which means that the learning mechanism used by the IIANSFSm is coherent with BCBS–Basel II with respect to the estimation of LD_MC.

After the estimation of the LD_MC by the IIANSFSm using a neutral RIMM and in absence of a learning mechanism, the model carried out the estimation of LD using each of the matrices that conform the sequence of risk (E1-E2-E3). Figure 12 shows the evolution experimented by the LD. The LD estimated by the RIMM_E3 is located close to the LD_MC, which shows that a better risk impact management in organizations or financial entities can meet the standards established by the BCBS–Basel II with regard to this type of risk.

Despite this evolution, as per Table 10 the LD's remained invariant in terms of the structure (heavy tail) and shapes (Generalized Pareto) that define the LD_MC. This shows the stability of the model in estimating the LD in absence of a learning mechanism. This stability is a consequence of only small variations experienced by the IOA, which guarantees long tail distributions of the LDs representing each impact matrix. Equally, in this table, we observe that the value of $OpVaR_{99.9\%}$ increases from 19.669 (U$) for E1 up to 105.008 (U$) for E3. This shows a major value of coverage for the UL with lower data in the tail due to better risk impact management, leading organizations or financial entities to a lower level of risk exposure.

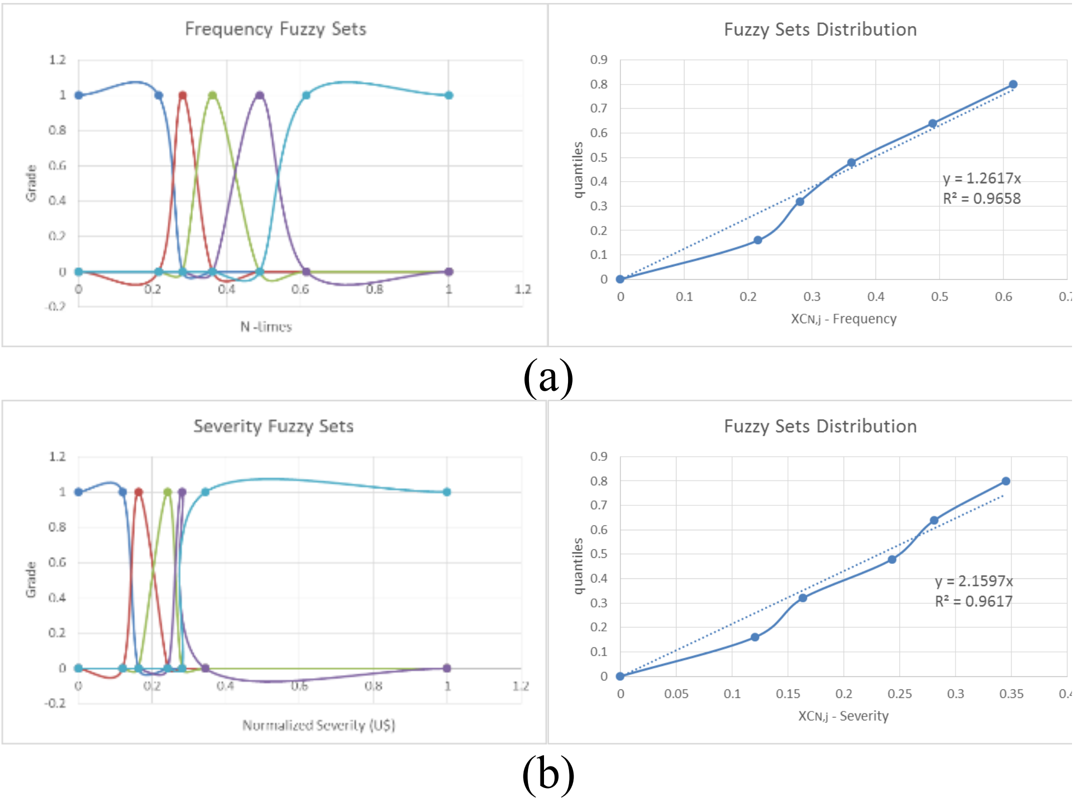


Figure 11: Structure and shape of the fuzzy input sets obtained by the IIANSFSm using a neutral RIMM in the learning phase

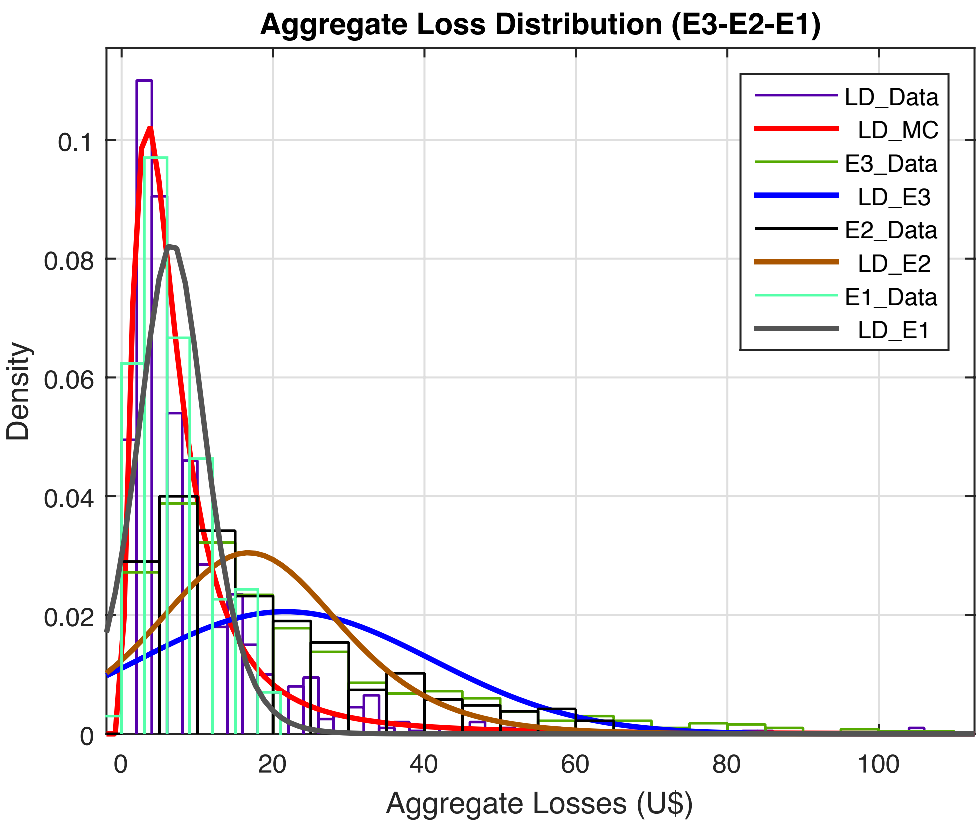


Figure 12: Evolution of LDA distributions in a sequence of risk E1-E2-E3

Table 10: Evolution of the LDA distribution due to the evolution of the scenarios for the proposed sequence of risk

	LD_MC	**LD_E3**	**LD_E2**	**LD_E1**
Distribution	G.Pareto	G.Pareto	G.Pareto	G.Pareto
Nloglik	-3161.000	-4060.700	-4231.600	-3333.700
μ	9.613	21.630	18.401	6.639
σ	10.819	19.384	15.815	8.217
shape (ξ)	0.5931	0.428	-0.321	-0.582
scale (β)	3.5005	0.942	17.820	8.510
threshold (Θ)	4.4423	11.159	13.324	5.153
Kurtosis	20.311	2.691	14.189	160.172
Assymmetry	3.608	1.634	-0.537	-9.498
$OpVar_{99.9\%}$	105.880	105.008	62.294	19.669
EL (Exp.L)	9.613	21.630	18.401	6.639
UL (UnExp.L.)	96.267	83.378	43.893	13.030
EL/$OpVar_{99.9\%}$	0.091	0.206	0.295	0.338
UL/$OpVar_{99.9\%}$	0.909	0.794	0.705	0.662
Tail Data	321.000	364.000	390.000	448.000
Exposure	0.306	0.327	0.332	0.372
Insurance	3.244	3.429	2.071	0.732

5. Conclusions

This paper presents an Integrated Inverse Adaptive Neural Fuzzy system with a Monte-Carlo structure (IIANFSM), which integrates in a single model both the Monte-Carlo sampling method and different RIMMs, which show the behavior in terms of the effect of risk impact management with regard to the LD in real time. The proposed model, in its different forms (IIANUFSm – Integrated Inverse Adaptive Neural Unbalanced Fuzzy System; IIANBFSm – Integrated Inverse Adaptive Neural Balanced Fuzzy System; IIANSFSm – Integrated Inverse Adaptive Neural Sampling Fuzzy System), allows in a learning phase identifying both the random input variables for frequency and severity and the CDFs that permit to identify the risk profile in an organization or FIRM.

The IIANSFSm was tested and evaluated in terms of the impact that different risk profiles generate on the LD in the learning phase. It can be highlighted that the model achieved a good stability in estimating the LD, showing a similar structure (Generalized Pareto) and form (long tail) under the three possible different risk impact management profiles and in absence of a learning mechanism. This characteristic makes the model ideal for evaluating how risk evolves within an organization or financial institution taking into account its risk profile. Thanks to its adaptation capacity, i.e. the capacity to learn from new risk data, the IIANFSM model overcomes the limitations related to a lack of available operational risk event data. The model can constantly monitor the evolution of the risk profile of an organization or financial entity, which reinforces the validity of the fuzzy approach that has been put into practice in this paper.

Referring to the structure of IIANFSM and the BCBC–Basel II definitions, the assessment of the evolution of risk impact management by integrating fuzzy scenarios and databases that represent external losses caused by operational risk events in a region or country is important. If external data were included, it would help to increase the credibility of the model, not only when it is assessed by the governmental or supervising authorities that regulate and monitor operational risk levels of financial institutions, but also for bank or financial institution operational risk managers. With respect to AMA models and loss distribution approaches, this paper contributes in showing that AMA and LDA still can be improved, as with the proposed IIANFSM, with the objective to improve operational risk impact management.

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