

A Proportion-Based Selection Scheme for Multi-objective Optimization

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Abstract—Classical multi-objective evolutionary algorithms (MOEAs) have been proven to be inefficient for solving multi-objective optimizations problems when the number of objectives increases due to the lack of sufficient selection pressure towards the Pareto front (PF). This poses a great challenge to the design of MOEAs. To cope with this problem, researchers have developed reference-point based methods, where some well-distributed points are produced to assist in maintaining good diversity in the optimization process. However, the convergence speed of the population may be severely affected during the searching procedure. This paper proposes a proportion-based selection scheme (denoted as PSS) to strengthen the convergence to the PF as well as maintain a good diversity of the population. Computational experiments have demonstrated that PSS is significantly better than three peer MOEAs on most test problems in terms of diversity and convergence.

I. INTRODUCTION

Many real world application problems are multi-objective optimization problems (MOPs), which include multiple conflicting objectives that must be optimized simultaneously. In the evolutionary multi-objective optimization (EMO) community, multi-objective optimization evolutionary algorithms (MOEAs) have been demonstrated to be effective in solving these problems [1]-[4]. So far, many different EMO algorithms have been developed, such as Pareto-based methods, e.g., NSGA-II [5], SPEA2 [6], decomposition-based approaches, e.g., MOEA/D [7], and indicator-based approaches [8, 9], e.g., HypE [10], SMS-EMOA [11]. However, Pareto-based algorithms more or less lose selection pressure to the PF in the optimization process when solving problems having more than three objectives, i.e., many-objective optimization problems (MaOPs). As a result, the whole performance of MOEAs can be affected by the decrease of convergence.

Pareto-based approaches compare solutions according to their dominance relation and density. The nondominated individuals are considered as primarily selected solutions. However, the Pareto-based dominance relationship has encountered great difficulties in MaOPs when the number of objectives increases [12, 13, 14]. Therefore, some researchers have focused on modifying the dominance relation to provide sufficient selection pressure towards the PF. Many improved methods have been proposed, such as SPEA2+SDE [15], ϵ -dominance [16, 17], and fuzzy Pareto dominance [18-24].

Indicator-based approaches use a single performance indicator to guide the search during the evolutionary process. The indicator-based EA (IBEA) [25] is a pioneer in this group. Recently, the hypervolume [26], Two_Arch2 [27] and S metric selection evolutionary algorithm [28] have been proposed. Indicator-based approaches have been demonstrated to be effective in balancing convergence and diversity due to their good theoretical properties. Nevertheless, the computational cost of the used metrics, e.g., hypervolume, grows exponentially with an increase in the number of objectives [29].

Decomposition-based methods decompose a problem with multiple objectives into a set of single-objective subproblems, which are then optimized simultaneously using evolutionary algorithms. The diversity of population is maintained by a set of pre-defined well-distributed reference points. The perpendicular distance between the individual and the reference line is usually used in reference vector-based decomposition methods. Reference lines are obtained by connecting reference points and the origin. Consequently, the convergence speed is affected by the perpendicular distance-based method to some degree, although these methods can balance convergence and diversity.

To handle these problems, the perpendicular distance must be replaced with the Euclidean distance to strengthen the selection pressure. In this paper, we propose a proportion-based selection scheme (PSS) for multi-objective optimization, where those solutions having the most approximate proportion can be obtained, thereby maintaining the diversity of the population. The center point (CR) of the reference set is calculated using the average value of all reference points on each objective. The distance from the reference point to the center point is also uniformly-distributed. Therefore, we can use the proportion to find uniformly-distributed Pareto optimal solutions. The proportion-based method is different from Pareto-based approaches and indicator-based approaches, but it is similar to reference point-based MOEAs.

The remainder of this paper is organized as follows. Section II presents the background information on reference point-based algorithms. The proposed PSS is described in detail in Section III. Experimental studies on well-known test problems are carried out in Section IV. Finally, conclusions are drawn in Section V.

II. PRELIMARIES

In this section, basic definitions in this paper are first given. Then, we briefly introduce the original MOEA/D and NSGA-III.

A. Basic Definitions

Perpendicular distance, euclidean distance, direction vector and proportion are defined as follows:

Definition 1: The distance between an individual and the centroid line is called the perpendicular distance, where a centroid line is defined by joining the centroid point with the origin.

Definition 2: The distance from an individual to the centroid is called the euclidean distance.

Definition 3: \vec{AP} is the direction vector of individual P , where A is the projective point of individual P on the centroid line.

Definition 4: The perpendicular distance of each individual is calculated, denoted by D . The proportion P of each individual is described as follows:

$$P_j = \frac{D_j}{\max(D_1, D_2, \dots, D_n)}, \quad (1)$$

where j is the j _th individual of the population.

B. MOEA/D and NSGA-III

The key idea of MOEA/D is to use a predefined set of well-distributed weight vectors to maintain the diversity of the new population. Each subproblem can find the best solution by using the aggregate function in the population. The collection of the solution on each well-distributed weight vector can be viewed as an approximation of the true PF. Generally, there are three aggregation functions: the weighted sum, Chebyshev, and penalty-based boundary intersection (PBI) function in MOEA/D. Let us take the PBI as an example. We suppose

Algorithm 1 Framework of PSS

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1:  $\Lambda \leftarrow \text{GenerateReferencePoints}()$ 
2:  $P_0 \leftarrow \text{InitializePopulation}()$ 
3:  $RP \leftarrow \text{ComputeProportion}(\Lambda)$ 
4:  $RV \leftarrow \text{ObtainVector}(\Lambda)$ 
5:  $t \leftarrow 0$ 
6: while the termination criterion is not met do
7:    $Q_t \leftarrow \text{MakeOffspringPopulation}(P_t)$ 
8:    $S_t \leftarrow P_t \cup Q_t$ 
9:    $SP \leftarrow \text{ComputeProportion}(S_t)$ 
10:   $SV \leftarrow \text{ObtainVector}(S_t)$ 
11:   $\kappa \leftarrow \text{Associate}(S_t, \Lambda, RP, RV, SP, SV)$ 
12:   $P_{t+1} \leftarrow \text{Niching}(\kappa)$ 
13: end while

```

Algorithm 2 GetCentriod(Population S_t)

Require: Population S_t

Ensure: C (The centroid of reference set or population set)

```

1: for  $i = 1$  to  $m$  do
2:    $C_i \leftarrow 0$ 
3:   for  $j = 1$  to  $n$  do
4:      $C_i \leftarrow C_i + S_t[j][i]$ 
5:   end for
6:    $C_i = C_i/n$ 
7: end for

```

w_1, w_2, \dots, w_n is a set of evenly spread weight vectors, then each subproblem is optimized respectively by:

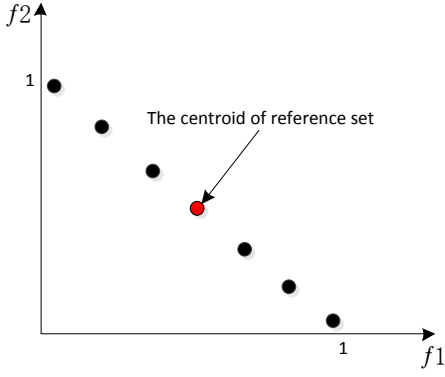
$$PBI(x, w_j) = d_1 + \theta d_2$$

where $j = 1, 2, \dots, N$, $w_j = (w_{j1}, w_{j2}, \dots, w_{jm})^T$, d_1 is the distance from the individual to the origin, d_2 is the perpendicular distance between the individual in the population and each of the reference lines w_j , and θ is the penalized parameter. Each individual x^* of the population can find a reference vector according to the PBI value. MOEA/D can work well for the curve shape friendly of the Pareto-optimal front.

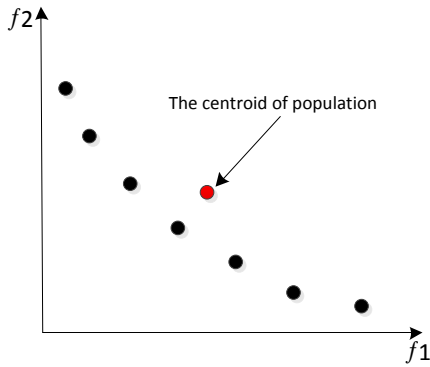
The NSGA-III framework is similar to the original NSGA-II except in its selection mechanism, and the diversity of a population is maintained by a set of well-distributed reference vector. Further details of NSGA-III can be found in references [30, 31].

III. PROPOSED PSS ALGORITHM

The framework of the proposed PSS algorithm is described in Algorithm 1. In line 1, we use Das and Denis's systematic approach [32] to generate a set of N reference points, denoted as $\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_N\}$. λ_j ($j = 1, 2, \dots, N$) is an m -dimensional vector for an m -objective problem. Line 2 denotes that the initial population P_0 with N members is randomly produced. The proportion of the reference point (RP) is initialized in line 3. The centroid point of the reference set is calculated in Algorithm 2. The perpendicular distance of each reference point from the centroid line is calculated. Then, the proportion is the value of perpendicular distance to maximum value of perpendicular distance. In line 4, the perpendicular direction (RV) from the projection point of reference point



(a) reference set



(b) solution set

Fig. 1. The centroid points.

to reference point is initialized. Lines 6-13 are iterated until the termination criterion is satisfied. The offspring population Q_t , having N members, was created from P_t by using the recombination operator and mutation operator in line 7. Then, the new population S_t is the combination of population P and Q in line 8. Lines 9-10 denote that each individual's proportion and direction vector can be obtained in the same way as Lines 3-4. In line 11, the function associate is used to split the members in S_t into a set of N niche, $\kappa = \{\kappa_1, \kappa_2, \dots, \kappa_N\}$. In addition, each reference point is allocated to a niche. In Line 12, the new population, P_{t+1} , is filled with individuals produced by the niche operator. In this paper, proportion is the key concept in PSS.

In the following sections, the important procedures of PSS are described in detail.

A. The Centroid of Reference Set and Population Set

In Fig. 1 (a), CR is the centroid of the reference set and N_r is the size of the reference set. Then, each objective value

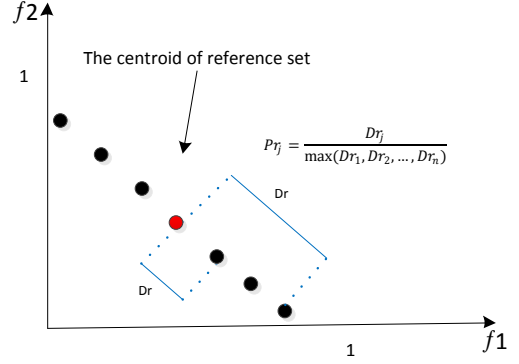


Fig. 2. The proportion of the reference point.

of CR is calculated by the following formula:

$$CR(i) = \frac{1}{N_r} \sum_{j=1}^{N_r} R_{ji}, \quad (2)$$

where $i = (1, 2, \dots, m)$, and m is the number of objectives, $j = (1, 2, \dots, N_r)$, and R_{ji} is the value of the j -th reference point in the i -th objective.

Similarly, in Fig. 1 (b), CS is the centroid of the population set, and N_s is the size of the population set. Then, each objective value of CS is obtained by the following formula:

$$CS(i) = \frac{1}{N_s} \sum_{j=1}^{N_s} S_{ji}, \quad (3)$$

where S_{ji} is the value of j -th individual of the population in the i -th objective. The procedure of calculating the centroid of the population or reference set is presented in Algorithm 2.

B. The Proportion of Reference Set and Population Set

We define a centroid line corresponding to the centroid point by joining the centroid point with the origin. The perpendicular distance (denoted as Dr) of each reference point is calculated. Finally, the proportion of each reference point in the reference set is computed by the following formula:

$$Pr_j = \frac{Dr_j}{\max(Dr_1, Dr_2, \dots, Dr_n)}, \quad (4)$$

where Pr_j is the proportion of the j -th reference point to the centroid point. Dr_j is the perpendicular distance from the reference point to the centroid point of the reference set. This is illustrated in Fig. 2.

Similarly, in Fig. 3, the proportion of each individual in the solution set (Ps) is computed by the following formula:

$$Ps_j = \frac{Ds_j}{\max(Ds_1, Ds_2, \dots, Ds_n)}, \quad (5)$$

where Ds_j is the perpendicular distance from the j -th individual to the centroid point of the solution Set. The procedure is presented in Algorithm 3.

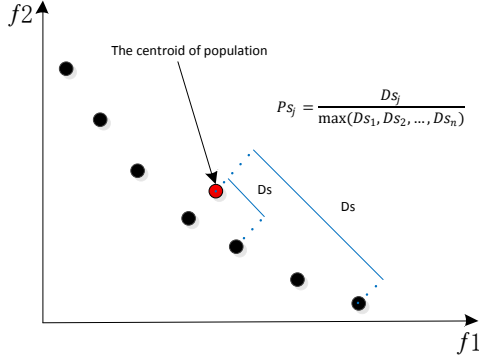


Fig. 3. The proportion of solutions in the population.

Algorithm 3 ComputeProportion(Population S_t)

Require: The Population S_t

Ensure: Proportion set (P)

- 1: $C \leftarrow \text{GetCentriod}(S_t)$
 - 2: **for** $i = 1$ to N **do**
 - 3: $D_i =$ Perpendicular distance between the reference point (or individual) and C
 - 4: **end for**
 - 5: **for** $i = 1$ to N **do**
 - 6: $P_i = D_i / \max(D_1, D_2, \dots, D_n)$
 - 7: **end for**
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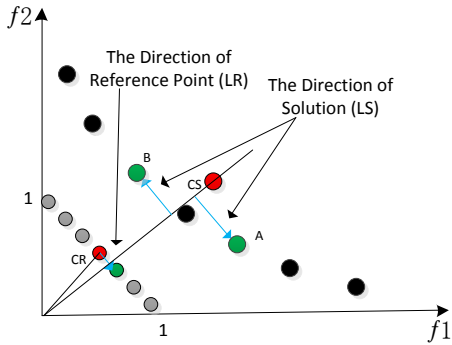


Fig. 4. The direction of the reference point and population set.

C. The Direction of Reference point and Solution point

In Fig. 4, suppose solution A, B is similar to reference point r in that they have similar proportions. A is a best solution for the reference point r , but B is a bad one. Therefore, we use the cosine similarity to select the individual in the population, the bigger the value of cosine similarity, the greater the selected chance that the individual is chosen. we define the direction vector of each reference point ($\vec{D}r$) by joining the reference point with the centroid line of the reference set, the direction vector of each individual in the population ($\vec{D}s$) by joining

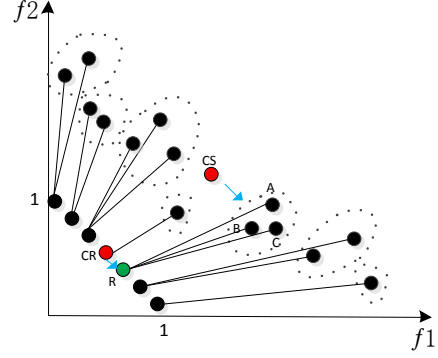


Fig. 5. Association of population members with reference points is illustrated.

Algorithm 4 Associate(R, S_t, RP, RV, SP, SV)

Require: R (Reference Set), S_t (Population Set), RP (Proportion of Reference point), RV (Direction Vector of Reference point), SP (Proportion of Individual), SV (Direction Vector of Individual), Nr (Size of the Reference Set), Ns (Size of the population set)

Ensure: κ

- 1: **for** $i = 1$ to Nr **do**
 - 2: $\kappa_i \leftarrow 0$
 - 3: **for** $j = 1$ to Ns **do**
 - 4: κ_i select the individual that make $RP_i - SP_j$ minimum and cosine maximum, simultaneously
 - 5: **end for**
 - 6: **end for**
-

the individual with the centroid line of the population. The value of cosine is computed by two direction vectors:

$$\cos \alpha = \frac{\vec{D}r \vec{D}s}{|\vec{D}r| |\vec{D}s|} \quad (6)$$

D. Association Operation

After the proportion was computed based on the centroid point, next we need to *associate* each population member with a reference point. The reference point whose proportion and cosine is equal to a population member in the objective space is considered associated with the population member. This is illustrated in Fig. 5. For a reference point R , individuals A, B and C are selected because they have the approximation of proportion and cosine according to the proportion and cosine of R . The procedure is presented in Algorithm 4. Each member of the reference point will associate with one or more individuals in the population.

E. Selection Mechanism

Each reference point could be associated with one or multiple solutions in the population according to the proportion and cosine values of the reference point. Thus, we select an individual with the shortest Euclidean distance between the reference point and associated individual. The results are presented in Fig. 6. For a reference point R , the individual B

TABLE I
THE POPULATION SIZE

objective(m)	dividsion(H)	MOEA/D(N)	PSS(N)
3	91	91	91
5	210	210	210
8	156	156	156
10	275	275	275
15	135	135	135

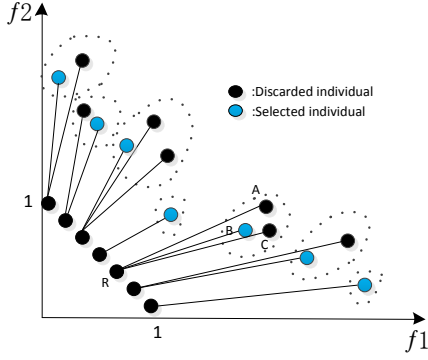


Fig. 6. Niche preservation.

Algorithm 5 Niching(R, κ)

Require: R (Reference Set), κ , N_r is the number of reference set,
 N_s is the number of population set

Ensure: P_{t+1}

```

1:  $k = 0$ 
2: for  $i = 1$  to  $N_r$  do
3:   if  $\text{size}(\kappa_i) == 1$  then
4:      $P_{t+1} = P_{t+1} \cup s$  ( $s \in \kappa_i$ )
5:   end if
6:   if  $\text{size}(\kappa_i) \geq 1$  then
7:      $P_{t+1} = P_{t+1} \cup s$  ( $s \in \kappa_i$  &&  $s$  have the shortest Euclidean distance)
8:   end if
9:    $k++$ 
10: end for
11: while  $k < N_s$  do
12:    $P_{t+1} = P_{t+1} \cup s$  ( $s \in \kappa_{\text{Random}()}$ ) &&  $s$  have the shortest Euclidean distance)
13: end while

```

is selected because it has the shortest Euclidean distance from R , but individuals A and B are discarded. The procedure is presented in Algorithm 5.

IV. EXPERIMENTAL STUDIES

A. Experimental Setup

The five test instances DTLZ1-5 [33] are used as test functions. In each instance, there were 3, 5, 6, 8, 10, and 15 objectives. These test problems have a variety of characteristics and different PFs. In order to evaluate the performance of algorithms, we used the generational distance (GD) [34], inverted generational distance (IGD) [35] metrics to evaluate the proposed method. We obtained the final points in the objective space and called them set A; P^* is a set of uniformly distributed points along the PF. The metrics GD and IGD are defined as follows:

$$GD(A, P^*) = \frac{1}{|A|} \sqrt{\sum_{j=1}^{|A|} d_j^2} \quad (7)$$

$$IGD(A, P^*) = \frac{1}{|P^*|} \sqrt{\sum_{j=1}^{|P^*|} \tilde{d}_i^2}, \quad (8)$$

where $d_i(\tilde{d}_i)$ is the Euclidean distance between the i -th member in the set $A(P_*)$ and its nearest member in the Set $P_*(A)$. GD can measure the convergence of the obtained solutions. IGD is a comprehensive indicator, which evaluates the convergence and diversity of solutions.

We compared three other original algorithms MOEAs, SPEA2+SDE, IBEA and MOEA/D to PSS. The population size N in each algorithm is listed in Table I, where $N = C_{H+m-1}^m - 1$ and H is the number of divisions considered along each objective axis. The population size of the original SPEA2+SDE is 100. The Wilcoxon ranksum test [36] was carried out to indicate the significance between different results at the 0.05 significance level.

B. Comparison Results

In this subsection, the comparison results among PSS, SPEA2+SDE, IBEA and MOEA/D are presented. The mean GD and IGD values of each compared algorithm for DTLZ1-5 problems are provided in Table II, where the best performance is shown in bold.

From Table II, PSS is better than SPEA2+SDE, IBEA and MOEA/D-PBI on most DTLZ1 except 8- and 10-objective instances. Additionally, for DTLZ2, PSS outperforms SPEA2+SDE, IBEA and MOEA/D on all instances. For DTLZ3, PSS is a little worse than SPEA2+SDE and IBEA. PSS is better than SPEA2+SDE, IBEA on DTLZ4, DTLZ5. However, PSS is little worse than MOEA/D-PBI on DTLZ5. The GD metric purely presents the convergence of an algorithm, it indicates that PSS has the best convergence performance among the four algorithms.

PSS performed better than SPEA2+SDE, IBEA and MOEA/D on most tested problems in terms of the IGD metric. It outperformed SPEA2+SDE, IBEA and MOEA/D on DTLZ4 except for problems with 15 objectives. As for DTLZ1, only PSS was worse than SPEA2+SDE and IBEA on 5-objective and 8-objective instances. In addition, PSS had the smaller IGD value than SPEA2+SDE, IBEA and MOEA/D in DTLZ2, DTLZ3, DTLZ4, DTLZ5 on most objective instances. Since the IGD metric can reflect the comprehensive performance of algorithms in terms of convergence and diversity, it can be concluded that PSS shows the best convergence and diversity on most problems.

TABLE II
RESULTS OF THE GD AND IGD VALUES FOR FOUR ALGORITHMS WITH VARYING NUMBER OF OBJECTIVES m , WHERE THE AVERAGE OVER 30 INDEPENDENT RUNS IS SHOWN

Problems	m	GD				IGD			
		PSS	SPEA2+SDE	IBEA	MOEA/D-PBI	PSS	SPEA2+SDE	IBEA	MOEA/D-PBI
DTLZ1	3	2.097425E-4	2.498871E-4‡	2.151756E-4†	2.132740E-4‡	1.870439E-2	2.091488E-2‡	2.404065E-2‡	1.871996E-2‡
	5	1.825690E-3	2.140118E-3‡	1.795267E-3	1.852401E-3‡	6.180973E-2	6.015305E-2	6.474986E-2‡	6.194315E-2‡
	8	3.822195E-3	4.745083E-3‡	4.047640E-3‡	3.804577E-3	1.091299E-1	9.712204E-2‡	9.699570E-2	1.089212E-1‡
	10	4.749049E-3	6.023592E-3‡	5.200348E-3‡	4.741181E-3	9.958144E-2	1.152702E-1‡	1.116092E-1‡	9.647347E-2
	15	4.327403E-3	7.654410E-3‡	6.822330E-3‡	5.078567E-3‡	1.620598E-1	1.427370E-1‡	1.296625E-1	1.401267E-1‡
DTLZ2	3	2.446136E-6	2.379738E-4‡	6.349680E-5‡	3.647608E-5‡	5.009597E-2	7.222258E-2‡	8.333383E-2‡	5.015115E-2‡
	5	2.193702E-5	4.342490E-4‡	2.824714E-4‡	2.501673E-4‡	1.569619E-1	1.838502E-1‡	1.854146E-1‡	1.582530E-1‡
	8	3.375279E-5	4.147951E-4‡	5.951524E-4‡	3.777484E-4‡	4.466976E-1	4.663982E-1‡	4.710770E-1‡	4.512431E-1‡
	10	1.268159E-4	3.344294E-4‡	6.844971E-4‡	9.913873E-4‡	4.285472E-1	5.608170E-1‡	5.562808E-1‡	4.221368E-1
	15	1.117240E-4	3.555315E-4‡	7.645812E-4‡	5.017912E-4‡	6.495836E-1	6.816699E-1‡	6.689002E-1‡	6.764107E-1‡
DTLZ3	3	1.296890E-2	2.408237E-4	5.313469E1‡	4.374955E-4‡	1.325862E-1	7.077331E-2‡	2.221019E2‡	5.066953E-2
	5	4.506600E-3	5.619237E-4‡	4.078149E-4	6.126329E-4‡	1.753776E-1	1.872975E-1‡	1.828076E-1‡	1.592967E-1
	8	1.892800E-3	6.235349E-4‡	8.203467E-4‡	3.264915E-4	4.503561E-1	4.821155E-1‡	4.889935E-1‡	7.273809E-1‡
	10	5.189649E-3	7.992748E-4	2.027073E-2‡	1.536804E-3‡	4.261625E-1	5.929791E-1‡	5.838285E-1‡	6.810744E-1‡
	15	1.789564E-3	8.062062E-4‡	9.103727E-3‡	2.550453E-4	6.564342E-1	7.207506E-1‡	7.124582E-1‡	1.004235‡
DTLZ4	3	1.659870E-6	1.488738E-4‡	1.512529E-3‡	8.394296E-5‡	5.012796E-2	2.671447E-1‡	5.966096E-1‡	5.029633E-2‡
	5	2.522474E-5	4.152005E-4‡	1.143932E-3‡	1.373303E-4‡	1.576663E-1	3.316153E-1‡	2.539851E-1‡	5.090315E-1‡
	8	3.385892E-5	5.664861E-4‡	5.563174E-4‡	8.892509E-5‡	4.558352E-1	4.813135E-1‡	4.720429E-1‡	7.129017E-1‡
	10	1.897215E-4	5.300003E-4‡	4.659565E-4‡	1.320534E-4	4.434634E-1	5.645314E-1‡	5.543840E-1‡	7.292015E-1‡
	15	3.206397E-5	5.389029E-4‡	1.302247E-3‡	1.167799E-4‡	6.566471E-1	6.823312E-1‡	6.772528E-1‡	9.082683E-1‡
DTLZ5	3	6.461373E-4	3.649849E-5‡	1.263669E-6	5.184335E-2‡	2.666319E-2	8.122719E-3	1.144353E-2‡	3.198290E-2‡
	5	1.315031E-2	5.415597E-2‡	6.448064E-2‡	1.383002E-2‡	3.005594E-2	2.867739E-2	5.979775E-2‡	2.924760E-2‡
	8	1.663207E-2	5.910784E-2‡	7.500306E-2‡	1.495193E-3	6.453904E-2	1.256837E-1‡	5.042064E-2	6.724457E-2‡
	10	1.095402E-2	6.590729E-2‡	8.104358E-2‡	1.070695E-3	4.451275E-2	1.457291E-1‡	6.667566E-2‡	5.018808E-2‡
	15	6.629625E-3	7.129200E-2‡	8.113172E-2‡	6.811994E-9	1.528460E-1	1.536967E-1‡	6.292434E-2	1.537017E-1‡

‡ and † indicate PSS performs significantly better than and equivalently to the corresponding algorithm, respectively.

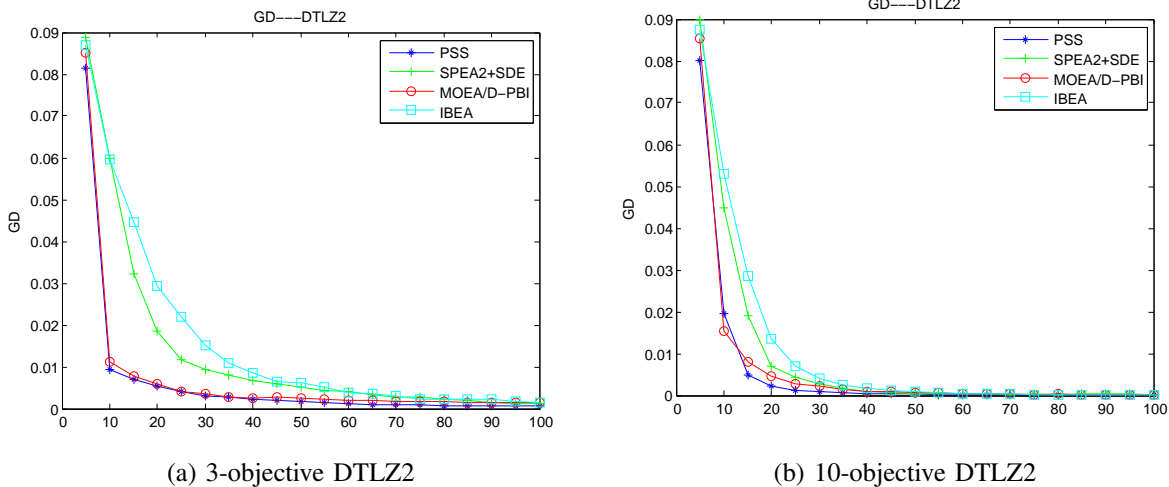


Fig. 7. The convergence regarding the GD metric of algorithms for DTLZ2 with 3-objective and 10-objective instances.

To verify the selection pressure toward PF, the polyline charts were drawn using the GD metric on 3-objective and 10-objective DTLZ2 instances. The four average GD values were obtained by evaluating the former 100 generations of population, respectively. The four polyline, asterisk sign, plus sign, dot sign and rectangular sign, represent the PSS, SPEA2+SDE, MOEA/D and IBEA, respectively. In Fig. 7, for the DTLZ2 problem with 3 objectives, the convergence of PSS is better than SPEA2+SDE, IBEA and MOEA/D after

about 15-th generation, which shows that PSS has a faster convergence speed on some problems. In addition, for the 10-objective DTLZ2 problem, PSS also has better convergence performance than MOEA/D, IBEA and SPEA2+SDE.

The PSS had better performance than MOEA/D, IBEA, SPEA2+SDE on most problems. One of the important reasons for the PSS is due to their selection criterion that can improve the convergence of the algorithm. The selection mechanism in PSS uses the shortest Euclidean distance between the individual of the solution and reference point to produce selection

pressure toward PF. In addition, the proportion method was used to maintain the diversity of the population.

V. CONCLUSION

This paper has proposed the PSS algorithm for multi-objective optimization. PSS aims to improve the selection pressure to maintain the convergence of the population in the selection mechanism. To attain this goal, the proportion-based selection mechanism is introduced into the proposed methods. From comparison results, PSS had the best performance compared to MOEA/D, IBEA and SPEA2+SDE on most instances. In the future, we will apply PSS to more practical problems.

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