Performance Comparison of the SGM and the SCM in EMC Simulation

Jinjun Bai, Gang Zhang, Di Wang, Alistair Duffy, *Fellow, IEEE*, and Lixin Wang

Abstract—Uncertainty analysis methods are widely used in today's Electromagnetic Compatibility (EMC) simulations in order to take account of the non-ideality and unpredictability in reality and improve the reliability of simulation results. The Stochastic Galerkin Method (SGM) and the Stochastic Collocation Method (SCM), both based on the generalized Polynomial Chaos (gPC) expansion theory, have become two prevailing types of uncertainty analysis methods thanks to their high accuracy and high computational efficiency. This paper, by using the Feature Selective Validation (FSV) method, presents the quantitative accuracy comparison between the foregoing two methods, with the commonly used Monte Carlo Method (MCM) used as the comparison reference. This paper also introduces SCM into the CST software simulation as an example of performing uncertainty analysis. The advantages and limitations of SGM and SCM are discussed in detail in this paper. Finally, the strategy of how to choose between SGM, SCM, and MCM under different situations is proposed in the conclusion section.

Index Terms— Stochastic Galerkin Method, Stochastic Collocation Method, Uncertainty Analysis, Electromagnetic Compatibility Simulation.

I. INTRODUCTION

Nowadays, the Electromagnetic Compatibility (EMC) community is facing a growing demand for stochastic models which can introduce the uncertainty in reality into simulations. Uncertainty inevitably arises from realistic non-ideality and unpredictability such as tolerances in dimensions, variations in geometry, material defects, and so forth. In order to analyze the effects of uncertainty, many uncertainty analysis methods have been presented in recently studies [1-3].

Among these analysis approaches, the Monte Carlo Method (MCM) is conventionally the first choice for EMC simulations [1, 2]. In MCM, the uncertainty parameters are sampled according to their distributions, consequently a huge number of simulations need to be performed until the final simulation results are obtained. MCM has been proved quite accurate for EMC simulations in existing literatures [1, 2], however at the cost of low computational efficiency. Anyway, thanks to its high accuracy, MCM can be used as the reference for evaluating the performance of other uncertainty analysis methods in terms of precision.

The Method of Moments (MoM) [3] and the Perturbation Method (PM) [4] are two other types of the existing uncertainty analysis methods. MoM uses the first order Taylor series expansions to estimate the mean and uncertainty of the output. However, if the output depends on the input in a nonlinear manner, the accuracy of MoM will be very poor [3]. Although PM tends to be more accurate than MoM, the magnitude of the uncertainties cannot be too large at both the inputs and outputs [4], which greatly limits the application of PM. In a word, conventional uncertainty analysis methods can hardly guarantee high accuracy and high efficiency simultaneously.

Recently, two other types of analysis approaches, the Stochastic Galerkin Method (SGM) and the Stochastic Collocation Method (SCM), have been proposed and applied to EMC simulations [5, 6]. As verified by [7], both of them have been proved to be accurate and computational efficient. Based on the generalized Polynomial Chaos (gPC) expansion theory, both SGM and SCM express the uncertainty analysis results by a polynomial of random variables. SGM is based on the Galerkin process, as a result it is the optimal solution in theory, and has been introduced into the EMC simulation for solving the stochastic Transmission Line Model and the stochastic Maxwell equations [8-11]. In contrast to SGM, SCM is based on the multidimensional Lagrange Interpolation theorem, and is also used to perform uncertainty analysis in the EMC field [12, 13]. The advantage of SCM is that it can be performed without changing the original solver like MCM, therefore the solver can be treated as a "black box" during the uncertainty analysis.

By using the Feature Selective Validation (FSV) method [14, 15], this paper evaluates the performance of SGM and SCM by solving an EMC uncertainty analysis problem put forward in [9]. Furthermore, SCM is introduced into the CST software simulation [16, 17] in order to implement the uncertainty analysis. The advantages and limitations of SGM and SCM are discussed in detail.

The structure of the paper is as follows: Section II introduces the generalized Polynomial Chaos (gPC) expansion theory; Section III discusses the SGM and SCM mechanisms; Section IV validates the algorithm by solving a realistic modeling

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problem; Section V shows the example of applying the uncertainty analysis using a commercial software; Section VI provides a detailed discussion on the differences between the uncertainty analysis methods; and the conclusion part of this paper is presented in Section VII.

II. THE GENERALIZED POLYNOMIAL CHAOS THEORY

Engineering device parameters can be quite uncertain in realistic environment due to the complexity of operating conditions. For example, the heights of the wires inside a bundled cable in a car or an airplane can be quite uncertain because of the natural structure of the bundle or simply because of the movement of the car or the airplane. Consequently it is almost impossible to calculate the crosstalk in such cables by using traditional deterministic EMC simulation methods, since the inputs of these models are no longer fixed values. In this case, uncertainty analysis methods need to be introduced into EMC simulations in order to solve the cable model with uncertain inputs.

The generalized Polynomial Chaos (gPC) expansion theory, as a non-sampling-based method to determine evolution of uncertainty in dynamical system, has been attached much attention to in recent studies thanks to its high accuracy and high efficiency. The core idea of the gPC theory is briefly presented as follows using a transmission line model as an example.

In a lossless transmission line, the current and voltage propagation equations can be written as

$$\begin{cases} \dfrac{\partial}{\partial z}V(z,t) = L \cdot \dfrac{\partial}{\partial t}I(z,t) \\ \dfrac{\partial}{\partial z}I(z,t) = C \cdot \dfrac{\partial}{\partial t}V(z,t) \end{cases}, \tag{1}$$

where L and C are the inductance and capacitance per unit length, and $I(z,t)$ and $V(z,t)$ are the current and voltage at time t and at the position z respectively.

Since the bundle cannot be perfectly stable and the car or the airplane may be moving, the positions of the wires in the bundle cable must be treated as random variables, and such uncertainty can be modeled by a random event θ expressed as

$$\xi(\theta) = \{\xi_1(\theta), \ \xi_2(\theta), \ \cdots, \ \xi_n(\theta)\}, \tag{2}$$

where $\xi(\theta)$ is the random variable space, $\xi_i(\theta)$ is one random variable with its own distribution depending on the random event. In this case, it is possible to use either Gaussian distribution or uniform distribution to model the randomness of the wire heights.

The independence of all the random variables in the random space is an essential prerequisite for applying the gPC theory. Such independence can be supported by the Karhunen-Loeve expansion [7].

The per-unit-length parameters L and C in (1) can also be modulated by the foregoing random event θ. Therefore with all the randomness included, the stochastic transmission line propagation equations can be re-written as

$$\begin{cases} \dfrac{\partial}{\partial z}V(z,t,\xi) = L(\xi) \cdot \dfrac{\partial}{\partial t}I(z,t,\xi) \\ \dfrac{\partial}{\partial z}I(z,t,\xi) = C(\xi) \cdot \dfrac{\partial}{\partial t}V(z,t,\xi) \end{cases}, \tag{3}$$

where $L(\xi)$ and $C(\xi)$ are input parameters modulated by the random variables, and $I(z,t,\xi)$ and $V(z,t,\xi)$ are output parameters influenced by the random inputs. The goal of any uncertainty analysis method is to obtain those outputs.

In the gPC theory, the outputs can be expanded in the polynomial form of the random variables as (4) and (5), with the time variable t and the location variable z omitted for simplicity:

$$V(z,t,\xi) = v_0\varphi_0(\xi) + v_1\varphi_1(\xi) + v_2\varphi_2(\xi), \tag{4}$$

$$I(z,t,\xi) = i_0\varphi_0(\xi) + i_1\varphi_1(\xi) + i_2\varphi_2(\xi), \tag{5}$$

where $\varphi_i(\xi)$ denotes the Chaos Polynomial which is determined by the Askey rule [7] as shown in Table I. The Askey rule illustrates the one-to-one correspondence between the distribution of the random variables and the form of the polynomial. In other words, as long as the distribution of the random variable is given, the Chaos Polynomial $\varphi_i(\xi)$ can be determined. The Chaos Polynomials in (4) and (5) are identical because they are all generated from the same random variables. The coefficients of the polynomial in (4) and (5), i_i and v_i, are to be solved. Admittedly i_i and v_i are functions of z and t, for simplicity z and t are omitted. The core idea of the gPC theory is to express the uncertainty analysis results in the polynomial form of the random variables at the inputs.

TABLE I
THE ASKEY RULE

Random variables	Wiener-Askey chaos	Support
Gaussian	Hermite-chaos	$(-\infty, +\infty)$
Gamma	Laguerre-chaos	$[0, +\infty)$
Beta	Jacobi-chaos	$[0, \ 1]$
Uniform	Legendre-chaos	$[-1, \ 1]$

According to the Askey rule, the polynomials in (4) and (5) are orthogonal to each other, and their relationship is presented as

$$\langle \varphi_i, \ \varphi_j \rangle = \langle \varphi_i^2 \rangle \delta_{ij}, \tag{6}$$

where δ_{ij} is the Kronecker function and can be expressed as

$$\delta_{ij} = \begin{cases} 1 & (i = j) \\ 0 & (i \neq j) \end{cases}. \tag{7}$$

The definition of the inner product is given as

$$\langle \varphi_i, \ \varphi_j \rangle = \int \varphi_i(\xi)\varphi_j(\xi)w(\xi)d\xi, \tag{8}$$

where $w(\xi)$ is the weight function and can be obtained by the joint probability density of the random variables [7]. In the same way, the supports of the integration are the joint supports

of every random variable.

III. THE SGM AND THE SCM MECHANISMS

Both the Stochastic Galerkin Method (SGM) and the Stochastic Collocation Method (SCM) are based on the foregoing gPC theory, and their results can be expressed in the polynomial form.

The theoretical basis of SGM is the Galerkin Process. The outputs parameters in (3), $I(z,t,\xi)$ and $V(z,t,\xi)$, are unfolded by plugging in (4) and (5). When the Galerkin Process is performed on both sides of the stochastic differential equation, an augmented certain differential equation will be obtained using the inner product calculation shown in (8). Traditional EMC simulation methods can be used in this augmented equation. Thus, the coefficients such as i_i and v_i will be acquired. The uncertainty analysis results, such as the expectation or the "worst case" of the output parameters, can be easily obtained by sampling the random variable ξ in (4) and (5). More details of SGM can be found in [6, 7].

As the SCM method is novel in EMC simulations, and it will be described in detail here. The theoretical basis of the SCM is the multidimensional Lagrange Interpolation theorem [12].

The one-dimensional Lagrange Interpolation formula is given as

$$Lag(f)(y) = \sum_{k=1}^{M} f(y_k) Lag_k(y), \qquad (9)$$

$$Lag_k(y) =$$
$$\frac{(y-y_0)(y-y_1)\cdots(y-y_{k-1})(y-y_{k+1})\cdots(y-y_M)}{(y_k-y_0)(y_k-y_1)\cdots(y_k-y_{k-1})(y_k-y_{k+1})\cdots(y_k-y_M)}, \qquad (10)$$

where $f(y)$ is the true value of the function to be solved, $Lag(f)(y)$ is the estimated value of $f(y)$ using the Lagrange Interpolation, $\{y_1, y_1 \cdots y_M\}$ is a set of interpolation points and M is the total number of the points, $f(y_k)$ is true function value at the points, $Lag_k(y)$ is the Lagrange Polynomial and its form is given by (10). It is worth noting that y is similar to the random variable ξ in the uncertainty analysis.

It is obviously seen that the Lagrange Polynomial satisfies

$$Lag_i(y_j) = \delta_{ij}, \qquad (11)$$

where δ_{ij} is the Kronecker function shown in(6).

In the multidimensional Lagrange Interpolation theorem, the interpolation points are in the tensor product form of the one-dimensional interpolation points, whilst the Lagrange Polynomial is also in the tensor product form. More details about the multidimensional Lagrange Interpolation theorem can be found in [12].

The uncertainty analysis result of SCM is given as follows by using the Lagrange Interpolation theorem:

$$\tilde{V}(\xi) = LagV(z,t,\xi) = \sum_{k=1}^{M} V(y_k) Lag_k(\xi), \qquad (12)$$

As shown in (12), the SCM result is in the form of a polynomial of the random variables ξ, like (4) and (5). $\tilde{V}(\xi)$ is the final result of the output parameter $V(z,t,\xi)$ using the Lagrange Interpolation. $V(y_k)$ is the "deterministic" EMC simulation result at the interpolation points y_k. $Lag_k(\xi)$ is the multidimensional Lagrange Polynomial of the interpolation points y_k.

The interpolation points are chosen by the tensor product form of zero points of the Chaos Polynomial. Back to (9), M presents the number of these interpolation points. The orthogonality of the Chaos Polynomial guarantees that such selection is the optimal solution of SCM [12].

The expectation value of SCM is calculated as shown in (13), where the supports of the integration are the joint supports of the random variables according to Table I.

$$E(\tilde{V})(\xi) = \sum_{k=1}^{M} V(y_k) \int_{\Gamma} Lag_k(y) w(y) dy, \qquad (13)$$

Other statistical properties, such as the standard deviation and the worst case value, can be obtained by sampling the random variables in (12).

According to (12), essentially the EMC simulation is simply to calculate $V(y_k)$, consequently there is no need to change the original solver. This is a special advantage of SCM compared with SGM, which makes the realization of SCM easier. However, in SCM, the error in the interpolation calculation inevitably manifests itself in the final analysis results, therefore theoretically the accuracy of SGM is expected to be slightly better than that of SCM.

IV. ALGORITHM VALIDATION

In order to evaluate the performance of the SGM and the SCM approaches, a common EMC uncertainty analysis problem in EMC simulation is presented in this section. The uncertainty model is the crosstalk calculation of the cables with wires that are random in height, which has been mentioned in [9] and [13] as an example.

The analysis results obtained by applying the MCM approach will be treated as the reference data. The results of applying SGM and SCM are compared with the foregoing reference data by using the FSV method, in this way the accuracy of SGM and SCM will be presented clearly in this example. Fig. 1 shows the uncertainty model of the crosstalk calculation problem.

As shown in Fig. 1, the amplitude of the excitation source is $1\,\mathrm{V}$, the radius of the radiating conductor and the disturbed conductor are both $0.1\,\mathrm{mm}$, the horizontal distance between the two conductors is $0.03\,\mathrm{m}$, the length of the two conductors are both $0.5\,\mathrm{m}$. All the loads are $50\,\Omega$. The radiating conductor and the disturbed conductor are surrounded by vacuum. The relative dielectric constant and the relative magnetic permeability of the vacuum are both 1.
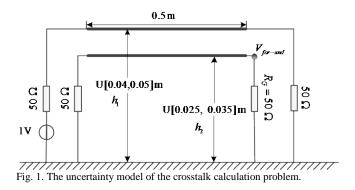
If the heights of the two conductors are uncertain, and the height of the radiating conductor h_1 obeys uniform distribution $U[0.04, 0.05]\,\mathrm{m}$ while the height of the disturbed conductor h_2

obeys uniform distribution $\mathbf{U[0.025, 0.035]m}$, two random variables can be used to model the randomness in height:

$$h_1 = 0.045 + 0.005 \times \xi_1, \tag{14}$$

$$h_2 = 0.03 + 0.005 \times \xi_2, \tag{15}$$

where ξ_1 and ξ_2 both stand for uniform distribution $[-1,\ 1]$.

Fig. 1. The uncertainty model of the crosstalk calculation problem.

The Multiple Conductor Transmission Line Model can be presented to solve the problem in this example, and the electrical parameters such as the inductance matrix will be influenced by the randomness in height. The relationship between the inductance parameters and the heights of the conductors is expressed as

$$L_{12} = \frac{\mu_0}{4\pi} \ln(1 + \frac{4h_1 h_2}{S^2}), \tag{16}$$

$$L_{ii} = \frac{\mu_0}{2\pi} \ln(\frac{2h_i}{r_i}), \tag{17}$$

where L_{12} is for the mutual inductance of the two cables, μ_0 is the permeability of vacuum, S is the separation between the two conductors, L_{ii} is the self-inductance, and r_i is the radius of the conductor.

The uniform distribution is a one-to-one correspondence with the Legendre polynomial according to the Askey rule in Table I. For SGM, the 2rd-order Legendre polynomial is proposed, therefore altogether six polynomials can be acquired according to the gPC theory in [7], which are $\varphi_0(\xi) = 1$,

$\varphi_1(\xi) = \sqrt{3} \times \xi_1$, $\varphi_2(\xi) = \sqrt{3} \times \xi_2$, $\varphi_3(\xi) = \frac{\sqrt{5}}{2} \times (3 \times \xi_1^2 - 1)$,

$\varphi_4(\xi) = 3 \times \xi_1 \times \xi_2$ and $\varphi_5(\xi) = \frac{\sqrt{5}}{2} \times (3 \times \xi_2^2 - 1)$. They are in the tensor product form of the one-dimensional polynomial.

For SCM, the interpolation points y_k are chosen in the tensor product form of the zero points of the Legendre polynomials. The 3th-order polynomial on one-dimensional $\varphi(\xi) = \frac{1}{2} \times (5 \times \xi^3 - 3 \times \xi)$ is chosen. The zero points are

$\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\}$. As the number of the random variables is 2,

the tensor product form is $\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\} \otimes \{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\}$.

The selection of the order of SGM and SCM is discussed here. When the order of the polynomial increases, the results will be more accurate, but the calculation time will also become longer. There exists an appropriate order, and the results of this order are convergent. Namely, when the order increases after this appropriate order, the results will be stable.

Thus, the judgment rule is shown as follow. From low order to high order, the uncertainty analysis is undertaken one by one. When the results of two adjacent orders are much the same, it is proved that the method is convergent. Thus, the relatively higher one of the two orders is the order of the method, and its results are the final results. In our case, as to SGM, the first order results and the second order results are almost same, so we choose second order for SGM. In the similar way, third order is decided for SCM.

The probability density of the crosstalk voltage value at the far end of the disturbed conductor is calculated by MCM, SGM, and SCM respectively. The results generated by MCM are treated as the reference data, and 20000 samples are used in MCM in order to guarantee the convergence.

Fig. 2 and Fig. 3 show the uncertainty analysis results at the single frequency points $2\,\mathrm{MHz}$ and $50\,\mathrm{MHz}$, expressed in the form of probability density. Two different situations are considered: the lossless transmission line, and the loss uniform transmission line with the resistance per unit length R and the conductance per unit length G.

Fig. 2. The probability density of the crosstalk voltage value at 2MHz.
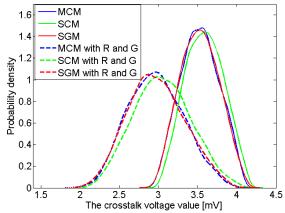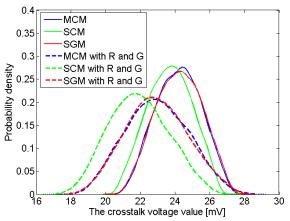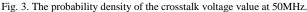

Fig. 3. The probability density of the crosstalk voltage value at 50MHz.

In Fig.2 and 3, the solid line means the lossless transmission result, and the dashed line is the loss uniform result. The blue lines stand for the results of MCM, green of SCM and red of SGM. According to Fig. 2, at $2\,MHz$ both SGM and SCM are seemingly as accurate as MCM. However, as the frequency increases in Fig. 3, the accuracy of SCM falls below that of MCM, whilst the accuracy of SGM is still a close match with that of MCM.

In order to further study the performance of SCM and SGM, the frequency range is extended to the span of $1\,MHz$ - $100\,MHz$. The expectation information, the standard deviation information, and the worst case information are presented instead of the Probability Density Function (PDF) information, as shown in Fig. 4, Fig. 5, and Fig. 6 respectively. These figures contain both the lossless transmission line results (solid lines) and the loss uniform transmission line results (dashed lines).

By using FSV, the Total-GDM values between MCM and SGM, as well as those between MCM and SCM, are calculated as listed in Table II.

TABLE II
THE TOTAL-GDM VALUES

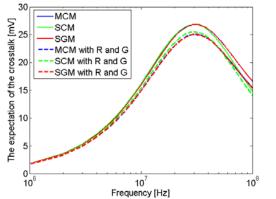| Total-GDM value	Expectation	Standard deviation	Worst case
SGM	0.03	0.22	0.04
SCM	0.13	0.23	0.14
SGM (R and G)	0.03	0.17	0.04
SCM (R and G)	0.12	0.25	0.12

Fig. 4. The expectation of the crosstalk calculation from 1MHz to 100MHz.
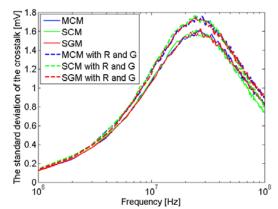

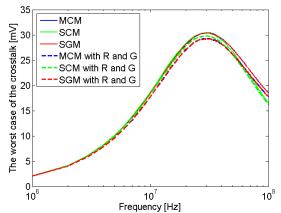Fig. 5. The standard deviation of the crosstalk calculation from 1MHz to 100MHz.

Fig. 6. The worst case of the crosstalk calculation from 1MHz to 100MHz.

FSV has proved its successful applications in credibility evaluation of certainty EMC simulation results [13]. The main idea of FSV is shown in Fig. 7. The Fourier transform is firstly applied to the original data sets, and then DC part, the low- and the high-frequency part are defined. These parts can be used to acquire the values of the Amplitude Difference Measure (ADM) and the Feature Difference Measure (FDM). The Global Difference Measure (GDM) consists of both ADM and FDM, and more details can be found in [14, 15]. Total-GDM, a value which provides a quantitative description in FSV, indicates the validity of simulation results. There exists a one-to-one correspondence between Total-GDM and the qualitative description, as shown in Table III.
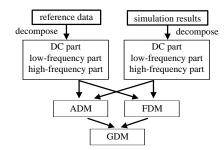

Fig. 7. The main idea of FSV [14].

TABLE III
RELATIONSHIP BETWEEN TOTAL-GDM AND QUANTITATIVE DESCRIPTION [14]

Total-GDM(quantitative)	FSV interpretation(qualitative)
Less than 0.1	Excellent
Between 0.1 and 0.2	Very Good
Between 0.2 and 0.4	Good
Between 0.4 and 0.8	Fair
Between 0.8 and 1.6	Poor
Greater than 1.6	Very Poor

According to Table III, the expectation and the worst case results of SGM are an "Excellent" match with the results of MCM, with the standard deviation being a "Good" or "Very Good" match. In contrast, the expectation and the worst case results of SCM are merely a "Very Good" match with the results of MCM, and the standard deviation is a "Good" match

which is similar to that of SGM.

The solid lines in Fig. 5, which represent the lossless results, show a good match between SCM and MCM yet an even better match between SGM and MCM. Nevertheless, the total-GDM of the standard deviation in Table II shows similar values for "SGM and MCM" (0.22) and "SCM and MCM" (0.23). In order to explain this phenomenon, the ADM value and the FDM value are studied. The ADM of SGM and MCM is 0.0993, and the FDM is 0.1829. In contrast, the ADM of SCM and MCM is 0.0757, and the FDM is 0.1918. Table II and III have proved that SGM is better than SCM in terms of the overall trend, however SCM outweighs SGM in some specific details. Consequently the GDM values of SGM (0.22) and SCM (0.23) are close.

To sum up, although SCM has shown a decent accuracy, it has been demonstrated that SGM is even more accurate and can be as good as MCM. Nevertheless, according to these figures, as the frequency increases the inherent interpolation error of SCM will become significant, which leads to the deterioration of the SCM accuracy at high frequencies compared to that at low frequencies.

Compared between the lossless results and the loss uniform results, the expectation of the former is larger than the latter. The reason is that some energy has been dissipated on the unit-length resistance R and the unit-length conductance G in the loss uniform transmission line, therefore the voltage in far end will be smaller. Meanwhile, the standard deviation of the lossless results is smaller than that in the loss uniform results. The conductance per unit length G is also influenced by the uncertain height parameter in (14) and (15), similar to the unit-length inductance L and unit-length capacitance C.

As for the computational efficiency, the simulation time of SGM is 0.22s while that of SCM is 0.17s. On the contrary, the simulation time of MCM goes up to 394.45s, which means that the computational efficiencies of SGM and SCM are both much better than that of MCM. Furthermore, as the simulation time of a single deterministic run becomes longer, the difference between the MCM and other uncertainty analysis methods will be more significant.

V. THE UNCERTAINTY ANALYSIS USING A COMMERCIAL SOFTWARE

In Section IV, it has been demonstrated that SGM is slightly more accurate than SCM, taking MCM as the reference. However, the advantage of SCM is that the solver requires no modification to perform the calculation, which indicates a better applicability of SCM over SGM under common circumstances.

This section shows an example of performing uncertainty analysis using CST, a commercial software. Since the solver in the software cannot be changed during the uncertainty analysis, SGM is not included in this simulation, and only SCM and MCM are presented.

The model used is similar to that shown in Fig. 1. The height of the radiating conductor h_1 is no longer an uncertain value but a fixed value of 0.04m. The height of the disturbed conductor h_2 still obeys uniform distribution $U[0.025, 0.035]\,m$ .The load R_G of the disturbed conductor becomes the uncertain parameter, obeying the uniform distribution $U[49, 51]\,\Omega$. The results focus on the probability density of the crosstalk voltage value at the far end of the disturbed conductor.

The Cable Studio and the Design Studio in CST are launched. Two parallel cables like Fig. 1 are designed in the Cable Studio (distributed parameter model). A finite PEC (perfect electronic conductor) board is adopted to imitate the ground. In the Design Studio, the loads information is designed, and the sine excitation in 20MHz is set. The cable positions in the Cable Studio can be changed to implement the uncertain input h_2 . The changes of R_G can be realized in the Design Studio.

During the uncertainty analysis process of MCM and SCM, the CST software is treated as a "black box", and multiple runs of deterministic simulations are needed. In this example, 5000 deterministic simulation runs of MCM will be taken in order to ensure the convergence of MCM. Meanwhile, 9 deterministic simulation runs of SCM are executed like the choice in Section IV, and the interpolation points are also $\{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\} \otimes \{-\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}\}$. Fig. 8 shows the results at the single frequency point $20\,MHz$. The blue solid line shows the result of MCM, and the red dashed line shows the result of SCM.

The expectation of the results in MCM is $15.3\,mV$, and that in SCM is $15.2\,mV$ with the error of 0.7%. The standard deviation of the results in MCM is $16.9\,mV$, and that in SCM is $17.3\,mV$ with the error of 2.4%. This indicates that MCM and SCM present very similar accuracy, however SCM requires much less computational effort.
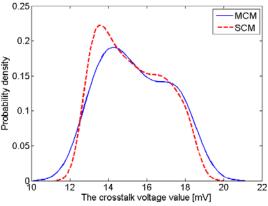

Fig.8. The results of MCM and SCM of the probability density function of the crosstalk voltage at 20MHz, by using CST.

In this section, it is demonstrated that the accuracy of SCM is as good as MCM according the CST simulation. Furthermore, the computational efficiency of SCM outweighs that of MCM. Such advantage in efficiency will be even more magnificent if the uncertainty analysis problems become complicated.

VI.   PERFORMANCE DIFFERENCES BETWEEN SCM, SGM, AND MCM

In this section, the performance differences between SCM, SGM, and MCM are discussed in detail.

In terms of accuracy, MCM is the best uncertainty analysis method so far because it is based on the Weak Law of Large Numbers [5]. SGM is based on the Galerkin Process, and its result is the optimal solution under the order of the Chaos polynomial, as a result SGM is as accurate as MCM. In contrast, the accuracy of SCM is somewhat lower than that of MCM because of the error incurred in the interpolation calculation.

Considering the computational efficiency, both SCM and SGM present similar superiority over MCM, because SCM and SGM only require a limited number of "deterministic" EMC simulations, on the contrary MCM needs thousands of them. The longer each single EMC simulation lasts, the more efficient and thus more desirable SCM and SGM can be.

As far as the applicability is concerned, both SCM and MCM have proved their advantages, because there is no change in the original solver during the uncertainty analysis. SGM, owing to its mechanism and particularly the change in the solver, limits its applicability when the model with uncertainty becomes complicated.

As to the hard-disk storage space, SCM and MCM only need the space for a single EMC simulation at a time, because the cache is cleared and the space is released right after one certain simulation is completed. On the opposite, SGM, since based on the Galerkin Process calculation, occupies much greater storage space because a huge number of projection results need to be stored.

For a High Dimensionality Problem of a random space, the computational efficiency of these three methods will decline anyway when the dimensionality rises up. The influence of the dimensionality upon MCM and SCM is a little larger than SGM, because the amounts of the simulations in MCM and SCM grow exponentially along with the increase of the dimension.

According to the discussions above, the performance comparison of MCM, SGM and SCM is given in Table IV.

TABLE IV
THE PERFORMANCE COMPARISON OF MCM, SGM AND SCM

	MCM	SGM	SCM
Accuracy	good	good	relatively good
Computational efficiency	low	high	high
Hard-disk storage space	little	big	little
Used in software	yes	no	yes
Applicability	good	bad	good
Influence of dimensionality	big	relatively big	big

VII.   CONCLUSION

This paper presented the performance comparison of the Stochastic Galerkin Method (SGM) and the Stochastic Collocation Method (SCM). By using the Feature Selective Validation (FSV) method and comparing with the commonly used Monte Carlo Method (MCM), it has been demonstrated that SGM is as accurate as MCM, and slightly better than SCM. By performing an uncertainty analysis example using CST, it is shown that the applicability of SCM is as good as MCM, and much better than SGM.

According to the discussion in Section VI, the strategy of selecting a suitable uncertainty analysis method is proposed as follow. If a single EMC simulation takes time under minute level, MCM will be the best among the three. If the time of a single simulation is more than hour level and the demand for accuracy is high, SGM should be the most desirable. If the time of a single simulation is more than hour level and the solver is either complicated or not open-sourced, SCM may be the most advisable one.

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