Moneyness, Volatility, and
the Cross-Section of Option Returns*

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Abstract

We study the effect of an asset's volatility on the expected returns of European options written on the asset. A simple stochastic discount factor model suggests that the effect differs depending on whether variations in volatility are due to variations in systematic or idiosyncratic volatility. While variations in idiosyncratic volatility only affect an option's elasticity, variations in systematic volatility also oppositely affect the underlying asset's risk. Since moneyness modulates the effects, systematic volatility positively (negatively) prices options with high (low) asset-to-strike price ratios, while idiosyncratic volatility is unambiguously priced. Single-stock call option data support our predictions.

Key words: Asset pricing; option returns; moneyness; total, systematic, and idiosyncratic volatility.

JEL classification: G11, G12, G15.

1 Introduction

Since the 1970s, a large number of studies have examined the effect of an asset's volatility on the expected returns of European options written on the asset. Galai and Masulis (1976), Johnson (2004), Friewald et al. (2014), Lyle (2015), and Hu and Jacobs (2018), for example, present theoretical evidence that the expected return of a European call (put) option decreases (increases) with the volatility of the asset underlying the option. Hu and Jacobs (2018) present empirical evidence supporting these predictions. Despite their empirical success, these studies, however, only take into account the effect that volatility has on an option's elasticity, while ignoring the potential effect of volatility on the expected return of the underlying asset. Thus, these studies implicitly focus on how idiosyncratic volatility prices the cross-section of options, shedding no light on the pricing roles played by systematic or total volatility.

In our paper, we aim to provide a more comprehensive analysis of how an asset's volatility prices European options written on the asset. Using a standard stochastic discount factor model, we show that the systematic and idiosyncratic volatility of the underlying asset have strikingly different effects on the expected option return. For example, while an increase in idiosyncratic volatility unambiguously lowers the expected call option return, an increase in systematic volatility raises that return when the option is in-the-money (ITM), but can lower it when the option is out-of-the-money (OTM). See Figure 1 for a graphical illustration of how option moneyness conditions the relation between expected call option return and systematic (Panel A) or idiosyncratic volatility (Panel B). Conversely, while a higher idiosyncratic volatility unambiguously raises the expected put option return, a higher systematic volatility can raise that return when the option is OTM, but lowers it when the option is ITM. Thus, the effect of total volatility cannot be determined independently from the effects of systematic and idiosyncratic volatility since, ultimately, it is jointly determined by moneyness and the extent to which variations in total volatility are driven by the two volatility components.

While our theoretical analysis considers holding-period returns, the intuition behind our results is perhaps easier to understand from instantaneous returns. Cox and Rubinstein (1985) show that the instantaneous expected return of a call option, $E[\tilde{R}_c^i]$, is

$$E[\tilde{R}_c^i] = R_f^i + \Phi \times \left[E[\tilde{R}^i] - R_f^i \right],$$

(1)

where $E[\tilde{R}^i]$ is the instantaneous expected return of the underlying asset, R_f^i the instantaneous risk-free rate, and Φ the call elasticity, with $\Phi > 1$. Taking the partial derivative with respect
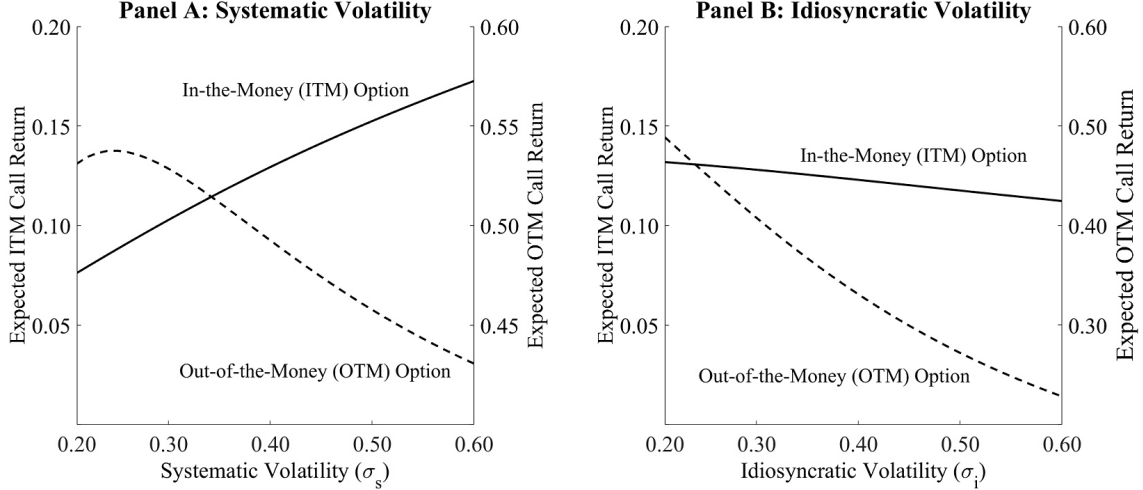
Figure 1: Relations Between Expected Call Option Return and Systematic- and Idiosyncratic-Volatility. The figure plots the relations between expected call option return and systematic (Panel A) and idiosyncratic underlying asset volatility (Panel B) produced by our model. The base case parameters are as follows. The expectations of the log asset payoff and the log stochastic discount factor realization are 0.40 and –0.025, respectively. Systematic and idiosyncratic volatility are 0.40 and 0.20, respectively, while the volatility of the log stochastic discount factor is 0.15. The strike price of the in-the-money (ITM) options is 0.50, while the strike price of the out-of-the-money (OTM) options is 2.50.

to either systematic, σ_s, or idiosyncratic underlying-asset volatility, σ_i, we obtain:

$$\frac{\partial E[\tilde{R}^i_c]}{\partial \sigma_q} = \underbrace{\frac{\partial \Phi}{\partial \sigma_q} \times \left[E[\tilde{R}^i] - R^i_f \right]}_{\text{option effect}} + \underbrace{\frac{\partial E[\tilde{R}^i]}{\partial \sigma_q} \times \Phi}_{\text{underlying asset effect}} \quad , \quad (2)$$

where $\sigma_q \in \{\sigma_s, \sigma_i\}$. Starting with idiosyncratic volatility ($\sigma_q = \sigma_i$), a higher idiosyncratic volatility only affects the option's elasticity (Φ), but not the expected return of the underlying asset ($E[\tilde{R}_i]$). This is because idiosyncratic volatility does not price the underlying asset in a perfect capital-markets world. Galai and Masulis (1976), Lyle (2015), Hu and Jacobs (2018), and others show that a higher idiosyncratic volatility unambiguously lowers a call option's elasticity (i.e., $\frac{\partial \Phi}{\partial \sigma_i} < 0$) and thus its expected return. Turning to systematic volatility ($\sigma_q = \sigma_s$), a higher systematic volatility raises the expected return of the underlying asset, but, identical to a higher idiosyncratic volatility, also lowers the option's elasticity. The ultimate effect of a higher systematic volatility thus hinges on which effect dominates. Our theoretical work shows that, when the option behaves more like the underlying asset (i.e., moneyness is high), the effect on the underlying asset dominates. Conversely, when the option behaves more like an

option (i.e., moneyness is low), the effect on the option's elasticity dominates.[1]

We derive our theoretical results from a two-period, continuous-payoff stochastic discount factor asset pricing model equivalent to Rubinstein's (1976) model. The model assumes that the log future underlying asset payoff and the log future stochastic discount factor realization are bivariate normal, with a negative correlation between them. It is well-known that such models yield the Black and Scholes (1973) European option pricing formulas (see Rubinstein (1976) and Huang and Litzenberger (1987)). To investigate the separate effects of systematic and idiosyncratic volatility, we decompose the log underlying asset payoff into its optimal expectation based on the realization of the stochastic discount factor and a residual. We interpret the volatility of the optimal expectation as the systematic volatility of the underlying asset payoff and the volatility of the residual as its idiosyncratic volatility. While our model relies on stronger assumptions than other models examined in the literature (e.g., Galai and Masulis (1976), Friewald et al. (2014), and Hu and Jacobs (2018)), it is still consistent with the other models.[2] We, however, need to make the stronger assumptions to be able to draw a complete picture of how an asset's volatility affects the expected option return.

We use single-stock options with various moneyness levels to test the volatility predictions of our European option pricing theory. Doing so is complicated by the facts that only American single-stock options are traded on U.S. exchanges, and that Aretz et al. (2017) show that the ability to early exercise an option strongly affects the option's expected return. Merton (1973), however, shows that it is never optimal to early exercise American call options written on assets not paying out cash. The implication is that such American call options are essentially European call options. Thus, we use only American call options written on stocks not paying out cash in our empirical work. In our main tests, we study call option returns from about seven weeks prior to maturity (the end of month $t-1$) to about three weeks prior to maturity (the end of month t). We estimate the market model and the Fama-French (1993)-Carhart (1997; FFC) model over a stock's 60 months of monthly data prior to the end of month $t-1$ to obtain estimates of systematic and idiosyncratic volatility. To study how moneyness conditions the effects of the volatility estimates, we interact the volatility estimates with a moneyness proxy, namely the stock price-to-option strike price ratio at the end of month $t-1$.

Our evidence supports our testable predictions. While systematic and idiosyncratic volatility exert no unconditional effects on the cross-section of call option returns, systematic volatility

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[1]We are indebted to Michael Brennan for providing this explanation of our results.

[2]All models, for example, assume that the future log underlying asset payoff is normal. While this assumption is consistent with our model, our model makes the stronger assumption that the future log underlying asset payoff and the future log stochastic discount factor realization are bivariate normal.

has a significantly negative effect in the subsample of low moneyness options, but a significantly positive effect in the subsample of high moneyness options. For example, the effect of the FFC four-factor systematic volatility estimate rises from –0.51 (t-statistic: –2.75) for options with a moneyness of 0.80 to 0.21 (t-statistic: 3.54) for options with a moneyness of 1.20, with the difference being a highly significant 0.72 (t-statistic: 3.45). In stark contrast, idiosyncratic volatility has a significantly negative effect in the subsample of low moneyness options, but an insignificant effect in the subsample of high moneyness options. For example, the effect of the FFC four-factor idiosyncratic volatility estimate rises from –0.38 (t-statistic: –2.46) for options with a moneyness of 0.80 to –0.09 (t-statistic: –1.63) for options with a moneyness of 1.20, with the difference being a statistically insignificant 0.29 (t-statistic: 1.60).

Our results continue to hold controlling for variables known to price options, including stock and option liquidity proxies (Cao and Han (2013) and Christoffersen et al. (2018)); option mispricing proxies (Stein (1989), Poteshman (2001), and Goyal and Saretto (2009)); and estimates of a stock's variance risk premium and implied risk-neutral moments (Bakshi and Kapadia (2003)). Controlling for firm characteristics known to price stocks (Cao et al. (2018)), however, eliminates the effect of systematic, but not idiosyncratic, volatility at all moneyness levels, consistent with systematic volatility being a function of stock pricing factors related to firm characteristics (e.g., the SMB and HML betas). Our results are also robust to holding the call options to maturity; calculating the volatility estimates from daily instead of monthly data; and allowing for bid-ask transaction costs. Finally, time-series regressions of S&P 500 call option returns on the index's total volatility, which is also its systematic volatility, also yield a significantly positive (negative) relation for ITM (OTM) options.

Our paper adds to a small, but emerging literature identifying factors pricing the cross-section of option returns. Using a stochastic discount factor model based on more general assumptions than ours, Coval and Shumway (2001) show that the expected return of a European call (put) option lies above (below) the risk-free rate of return and increases (increases) with the option's strike price. S&P 500 index option data support these predictions. Using a Black and Scholes (1973) contingent claims framework, Galai and Masulis (1976), Friewald et al. (2014), and Hu and Jacobs (2018) show that the expected return of a European call (put) option decreases (increases) with the idiosyncratic volatility of the asset underlying the option, despite the studies erroneously talking about the effect of total, and not idiosyncratic, volatility. Using a stochastic discount factor model similar to ours, Johnson (2004) and Lyle (2015) confirm these theoretical predictions. Using single-stock option data, Hu and Jacobs (2018) find support for the idiosyncratic volatility predictions. Using the Longstaff and

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Schwartz (2001) framework, Aretz et al. (2017) show that American put options have higher expected returns than equivalent European put options, with the spread positively related to the probability of an early exercise. Using single-stock American put option and synthetic European put option data, they find support for these predictions. Other studies focus on factors pricing the cross-section of delta-hedged option returns (i.e., option returns not driven by underlying asset returns). Goyal and Saretto (2009) show that delta-hedged option returns increase with the ratio of the realized volatility to the implied volatility of the underlying asset, while Cao and Han (2013) show that delta-hedged option returns decrease with underlying asset idiosyncratic volatility. We contribute to this literature by offering refined and more comprehensive theoretical and empirical analyses of the effects of volatility on the cross-section of European option returns, paying particular attention to how option moneyness conditions the separate pricing roles played by systematic and idiosyncratic volatility.

We also add to a large literature studying the effect of volatility on the cross-section of stock returns. On the theoretical front, Merton (1987) and Makiel and Xu (2004) show that idiosyncratic volatility positively prices stocks when investors are unable to hold diversified portfolios. Conversely, Johnson (2004) shows that financial leverage leads idiosyncratic volatility to negatively price stocks. On the empirical front, Ang et al. (2006; 2009) show that historical idiosyncratic volatility negatively prices stocks in U.S. and non-U.S. markets. Bali and Cakici (2008), however, report that Ang et al.'s (2006; 2009) results are not robust to reasonable methodological changes. In addition, Diavatopoulos et al. (2008), Fu (2009), Chua et al. (2010), and Brockman et al. (2012) show that GARCH- or option-implied estimates of expected idiosyncratic volatility positively price stocks. Fink et al. (2012) and Guo et al. (2014), however, claim that the GARCH-based evidence is spurious due to look-ahead bias. Conditioning the effect of historical idiosyncratic volatility on book leverage, Ang et al. (2009) reject Johnson's (2004) claim that leverage produces the negative idiosyncratic volatility pricing. Conditioning on the failure probability, a variable close to our moneyness variable, Song (2008) and Chen and Chollete (2010), however, find evidence supportive of Johnson's (2004) claim. Interpreting a call option as a levered-up stock, we contribute to this literature by offering new evidence on how financial leverage conditions the pricing effects of systematic and idiosyncratic volatility, using assets with much higher financial leverage to run more powerful tests.

We proceed as follows. In Section 2, we investigate the relation between expected European option returns and the volatility of the underlying asset in a simple stochastic discount factor model. In Section 3, we describe our data and variables. We also offer the results from empirical tests studying the relations between systematic and idiosyncratic volatility and call option

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returns. Section 4 offers the results from robustness tests. Section 5 concludes.

2   Theory

In this section, we use a stochastic discount factor model to study the relations between the expected returns of European options and the systematic and idiosyncratic volatility of the assets underlying the options. In Section 2.1, we introduce the model. In Section 2.2, we first derive propositions summarizing the effects of option and underlying asset characteristics (including the volatility variables) on the expected returns of European call options. We next derive similar propositions for the expected returns of European put options.

2.1   Model Assumptions

Consider a two-period, continuous-variables securities market featuring a primitive asset and a continuum of plain-vanilla European call and put options written on that asset. In the absence of arbitrage opportunities, it is well-known that there exists a stochastic discount factor determining the price of the primitive asset according to the Euler equation

$$p = E[\tilde{M} \times \tilde{X}], \tag{3}$$

where p is the price of the primitive asset in the first period, $E[.]$ the expectation operator, $\tilde{M}$ the realization of the stochastic discount factor in the second period, and $\tilde{X}$ the payoff of the primitive asset in the second period, with a tilde indicating a random variable. Denoting the natural log of the stochastic discount factor realization by $\tilde{m}$ and the natural log of the primitive asset payoff by $\tilde{x}$, we can alternatively write the price of the primitive asset as

$$p = E[e^{\tilde{m}+\tilde{x}}]. \tag{4}$$

In line with Rubinstein (1976), we assume that the natural logs of the primitive asset payoff and the stochastic discount factor realization are bivariate normal, with a negative correlation between them. We denote the expected values of the log primitive asset payoff and the log stochastic discount factor realization by μ_x and μ_m, respectively. We further denote the variances of the log primitive asset payoff and the log stochastic discount factor realization by σ_x^2 and σ_m^2, respectively. Under our distributional assumptions, it is well-known that the optimal expectation of the log primitive asset payoff conditional on the stochastic discount

factor realization is given by $\tilde{x}_s \equiv a - b\tilde{m}$, where a and $b > 0$ are parameters. The residual of the log primitive asset payoff is in turn given by $\tilde{x}_i \equiv \tilde{x} - \tilde{x}_s$. Using this decomposition of the log primitive asset payoff, we are able to write the variance-covariance matrix of the log primitive asset payoff and the log stochastic discount factor realization, $\sigma_{x,m}$, as

$$\sigma_{x,m} \equiv \begin{bmatrix} \text{var}(\tilde{x}) & \text{cov}(\tilde{x}, \tilde{m}) \\ \text{cov}(\tilde{x}, \tilde{m}) & \text{var}(\tilde{m}) \end{bmatrix} = \begin{bmatrix} \sigma_x^2 = \sigma_s^2 + \sigma_i^2 & \kappa \sigma_s \sigma_m = -\sigma_s \sigma_m \\ \kappa \sigma_s \sigma_m = -\sigma_s \sigma_m & \sigma_m^2 \end{bmatrix}, \quad (5)$$

where var(.) and cov(.) are the variance and covariance operator, respectively, $\sigma_s^2 = b^2 \sigma_m^2$ and σ_i^2 are the systematic variance and the idiosyncratic variance of the log primitive asset payoff, respectively, and κ is the correlation between the optimal expectation of the log primitive asset payoff and the log stochastic discount factor realization. Because $\text{cov}(\tilde{x}_s, \tilde{m}) = -b\,\text{var}(\tilde{m}) = -b\sigma_m^2$ and $\text{var}(\tilde{x}_s) = b^2\sigma_m^2$, the correlation κ is equal to minus one.[3]

2.2 Model Conclusions

2.2.1 The Expected Return of the Primitive Asset

In our model, the expected return of the primitive asset, $E[\tilde{R}]$, is

$$E[\tilde{R}] = \frac{E[\tilde{X}]}{E[\tilde{M} \times \tilde{X}]} = \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2}}{e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m + \sigma_m^2)}} = \frac{e^{\sigma_s\sigma_m}}{e^{\mu_m + \frac{1}{2}\sigma_m^2}} = R_f e^{\sigma_s\sigma_m}, \quad (6)$$

where $R_f \equiv 1/E[\tilde{M}] = e^{-(\mu_m + \frac{1}{2}\sigma_m^2)}$ is the gross risk-free rate of return. Equation (6) suggests that a higher systematic volatility of the log payoff of the primitive asset, σ_s, increases that asset's expected return. In contrast, neither the expected value, μ_x, nor the idiosyncratic volatility, σ_i, of the log payoff have an influence on the asset's expected return.

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[3]Our main mathematical derivations assume that we can independently change the expected log asset payoff, μ_x, the systematic volatility, σ_s, and the idiosyncratic volatility, σ_i, of the log asset payoff. Since variations in systematic volatility must, however, be ultimately driven by variations in the slope coefficient b, this claim is only true if $\mu_m = 0$ since $\mu_x = a - b\mu_m$. Thus, when $\mu_m \neq 0$, we need to assume that the effects of variations in b on μ_x are offset by simultaneous variations in a. Importantly, however, we show in part (d) of Appendix A that our theoretical conclusions are robust to allowing the expected log asset payoff to change with variations in systematic asset volatility induced through variations in the slope coefficient b.

2.2.2 The Expected Returns of European Call Options

The expected return of a European call option written on the primitive asset, $E[\tilde{R}_c]$, is

$$
\begin{aligned}
E[\tilde{R}_c] &= \frac{E[\tilde{X}_c]}{p_c} = \frac{E[\max(\tilde{X} - K, 0)]}{E[\tilde{M} \times \max(\tilde{X} - K, 0)]} \\
&= \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]}{e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[e^{\mu_x + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m)} N\left[\frac{\mu_x - \sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]},
\end{aligned} \quad (7)
$$

where K is the option's strike price, $\max(.)$ is the maximum operator, and $N[.]$ is the cumulative standard normal distribution. We derive the closed-form solution for the expected option payoff, $E[\tilde{X}_c]$, using the formula for the expectation of a left-truncated lognormal variable (see Ingersoll (1987)). We derive the closed-form solution for the option value, p_c, using an approach mathematically equivalent to the approach in Rubinstein (1976).

Noting that the primitive asset value, p, is $e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m + \sigma_m^2)}$, that the gross risk-free rate of return, R_f, is $e^{-(\mu_m + \frac{1}{2}\sigma_m^2)}$, and that $\mu_x - \sigma_s\sigma_m + \sigma_x^2 = \ln(p) + \ln(R_f) + \frac{1}{2}\sigma_x^2$, it follows that the closed-formed solution for the call option value, p_c, is identical to the Black-Scholes (1973) European call option pricing formula. While the Black and Scholes (1973) contingent claims framework, however, assumes that the primitive asset value is exogenous, our stochastic discount factor framework specifies the dependance between the primitive asset value and the expectation, the systematic volatility, and the idiosyncratic volatility of the asset's payoff. It is this feature of our framework that allows us to separately study the effects of systematic and idiosyncratic volatility and that most distinguishes us from prior research.

The probability that a call option ends up ITM and yields a positive payoff, π_c, is:

$$
\pi_c = \text{Prob}(\tilde{X} > K) = \text{Prob}\left(\frac{\tilde{x} - \mu_x}{\sigma_x} > \frac{\ln K - \mu_x}{\sigma_x}\right) = N\left[\frac{\mu_x - \ln K}{\sigma_x}\right], \quad (8)
$$

which implies that the probability is a monotonic transformation of the ratio of the expected log asset payoff minus the log strike price to the log asset payoff volatility. We shall refer to options with a probability above 50% as ITM options, to options with a probability of 50% as at-the-money (ATM) options, and to options with a probability below 50% as OTM options. In addition, we shall refer to $\mu_x - \ln K$ as the moneyness level of a call option.[4]

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[4]The literature typically defines moneyness as the ratio of the primitive asset's value to an option's strike price. Keeping the log primitive asset payoff volatility and the correlation between the log primitive asset payoff and the log stochastic discount factor realization fixed, our moneyness definition is a positive transformation

Proposition 1 summarizes the relations between the expected European call option return and option- and primitive asset-characteristics in our model:

PROPOSITION 1: *Assuming that the log primitive asset payoff and the log stochastic discount factor realization are bivariate normal, with a negative correlation between them, the expected return of a European call option with strike price K, $E[\tilde{R}_c]$,*

(a) *decreases with the expected log asset payoff, μ_x.*

(b) *increases with the strike price specified by the option, K.*

(c) *decreases with moneyness, defined as the difference between μ_x and $\ln K$.*

(d) *increases (decreases) with the systematic log primitive asset payoff volatility, σ_s, if*

$$(\sigma_x^2/\sigma_s^2)H'[c^*] - H'[\alpha - \sigma_x + \beta] - \left(\alpha - \sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\alpha - \sigma_x + \beta]\big[1 - H'[c^*]\big] > (<) \ 0,$$

where $H(x) \equiv n(x)/N(-x)$ is the hazard function of the normal random variable x, with $n(.)$ the standard normal density function, $H'(x)$ the first derivative of the hazard function with respect to x, $\alpha \equiv (\ln K - \mu_x)/\sigma_x$, $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, and $c^* \in (\alpha - \sigma_x + \beta, \alpha + \beta)$.

(e) *decreases with the idiosyncratic volatility of the log primitive asset payoff, σ_i.*

Proof: See Appendix A.

Part (b) of Proposition 1 is a well-known result. It follows directly from Coval and Shumway (2001) who find the same relation between the expected call option return and the option's strike price in a more general stochastic discount factor model not specifying the joint distribution of the primitive asset payoff and the stochastic discount factor realization. Conversely, the other parts of the proposition are new to the literature. Part (a) suggests that a higher expected log asset payoff, translating into a higher expected asset value at maturity, decreases the expected call option return. Part (c) combines the variables studied in parts (a) and (b) to calculate our definition of an option's moneyness. It suggests that a higher moneyness leads to a lower expected call option return — independent of whether variations in moneyness are attributable to variations in the expected log asset payoff or in the option's strike price.

Part (d) opens up the possibility that the systematic volatility of the log asset payoff has an ambiguous relation with the expected call option return. In line with this possibility, Corollary 1 suggests that the sign of the relation is determined by an option's moneyness.

COROLLARY 1: *Under the assumptions in Proposition 1, the sign of the relation between*

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of that ratio since $\mu_x - \ln K = \ln(p/K) + R_f - \frac{1}{2}\sigma_x^2 - \sigma_{x,m}$ (see Poon and Stapleton (2005)).
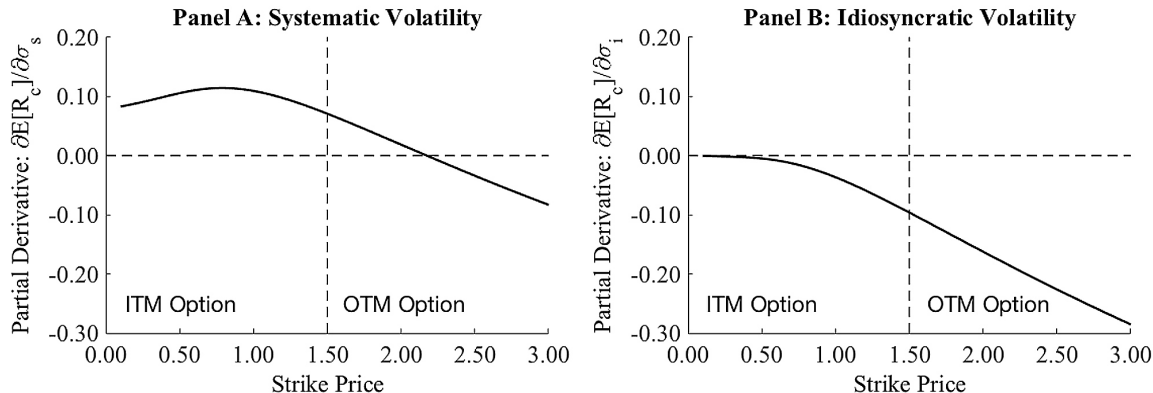
Figure 2: Partial Derivatives of Expected Call Option Return with Respect to Systematic and Idiosyncratic Volatility The figure plots the partial derivatives of the expected call option return with respect to the systematic volatility (Panel A) and the idiosyncratic volatility (Panel B) of the log primitive asset payoff against the strike price. The base case parameter values are as in Figure 1.

the expected call option return, $E[\tilde{R}_c]$, and the systematic volatility of the log asset payoff, σ_s, is positive for ITM and ATM options, but can be negative for OTM options.

Proof: See Appendix A.

Panel A of Figure 2 illustrates Corollary 1, plotting the partial derivative of the expected call option return with respect to the systematic volatility of the log asset payoff against the strike price of the option. The figure confirms that ITM and ATM call options produce a positive relation between the expected call option return and systematic volatility, while OTM call options produce a positive, zero, or negative relation. The figure also suggests that the relation turns negative as the option becomes sufficiently OTM. As discussed before, the intuition behind the ambiguous relation between expected call option return and systematic volatility is that an increase in systematic volatility has two oppositely-signed effects on the expected call option return. The positive effect is that it raises the expected primitive asset return (see Equation (6)); the negative is that it lowers the option's implicit leverage. When the option is ITM, ATM, or mildly OTM, its implicit leverage is low and volatility-insensitive, leading the first effect to dominate. Conversely, when the option is sufficiently OTM, its implicit leverage is high and volatility-sensitive, leading the second effect to dominate.

Part (e) of Proposition 1 suggests that the expected call option return is unambiguously negatively related to the idiosyncratic volatility of the log asset payoff. This result highlights that studies using the Black-Scholes (1973) contingent claims framework to show that "total volatility" has an unambiguously negative effect on expected call option returns do actually

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investigate the effect of idiosyncratic volatility. Corollary 2 suggests that the strength of the idiosyncratic volatility effect depends on an option's moneyness.

COROLLARY 2: *Under the assumptions in Proposition 1, the relation between the expected call option return, $E[\tilde{R}_c]$, and the idiosyncratic volatility of the log asset payoff, σ_i, converges to zero (a negative value) as the option's moneyness converges to infinity (minus infinity).*

Proof: See Appendix A.

Panel B of Figure 2 confirms that the effect of idiosyncratic volatility is unambiguously negative over all moneyness levels. It also shows that the effect converges to zero as we let the strike price go to zero, making the option increasingly similar to the primitive asset.

2.2.3 The Expected Returns of European Put Options

Since our empirical work focuses exclusively on call options, we only briefly discuss how the expected return of a European put option written on the primitive asset, $E[\tilde{R}_p]$, relates to the option- and primitive asset-characteristics in our model. The expected put option return is

$$
\begin{aligned}
E[\tilde{R}_p] &= \frac{E[\tilde{X}_p]}{p_p} = \frac{E[\max(K - \tilde{X}, 0)]}{E[\tilde{M} \times \max(K - \tilde{X}, 0)]} \\
&= \frac{KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]}{e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[KN\left[\frac{\ln K - \mu_x + \sigma_s\sigma_m}{\sigma_x}\right] - e^{\mu_x + \frac{1}{2}(\sigma_x^2 - 2\sigma_s\sigma_m)} N\left[\frac{\ln K - \mu_x + \sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right]},
\end{aligned} \tag{9}
$$

and the probability that the option ends up ITM, π_p, is

$$
\pi_p = \text{Prob}(K > \tilde{X}) = \text{Prob}\left(\frac{\ln K - \mu_x}{\sigma_x} > \frac{\tilde{x} - \mu_x}{\sigma_x}\right) = N\left[\frac{\ln K - \mu_x}{\sigma_x}\right], \tag{10}
$$

where we continue to refer to options with an above 50% probability as ITM options, to options with a 50% probability as ATM options, and to options with a below 50% probability as OTM options, but now define the moneyness level of the option as $\ln K - \mu_x$.

Proposition 2 summarizes the relations between the expected European put option return and the option- and primitive asset-characteristics in our model:

PROPOSITION 2: *Assuming that the log primitive asset payoff and the log stochastic discount factor realization are bivariate normal, with a negative correlation between them, the expected return of a European put option with strike price K, $E[\tilde{R}_p]$,*

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(a) decreases with the expected log asset payoff, μ_x.

(b) increases with the strike price specified by the option, K.

(c) increases with moneyness, defined as the difference between $\ln K$ and μ_x.

(d) increases (decreases) with the systematic log primitive asset payoff volatility, σ_s, if

$$-(\sigma_x^2/\sigma_s^2)H'[c^*] + H'[\sigma_x - \beta - \alpha] - \left(\alpha - \sigma_m \frac{\sigma_i^2}{\sigma_x \sigma_s}\right) H[\sigma_x - \beta - \alpha]\left[1 - H'[c^*]\right] > (<) 0,$$

where $\alpha \equiv (\ln K - \mu_x)/\sigma_x$, $\beta \equiv \frac{\sigma_s \sigma_m}{\sigma_x}$, and $c^* \in (-\beta - \alpha, \sigma_x - \beta - \alpha)$.

(e) increases (decreases) with the idiosyncratic log primitive asset payoff volatility, σ_i, if

$$H'[\sigma_x - \beta - \alpha] - (\alpha + \beta) H[\sigma_x - \beta - \alpha]\left[1 - H'[c^*]\right] > (<) 0.$$

Proof: See the Internet Appendix.

Part (b) of Proposition 2 again follows from the more general theoretical analysis of Coval and Shumway (2001), while the other parts are new to the literature. Part (a) suggests that a higher expected log asset payoff decreases the expected put option return, while part (c) suggests that a higher moneyness increases the expected put option return — independent of whether the higher moneyness is attributable to a higher strike price or a lower expected log asset payoff. Part (d) again opens up the possibility that the relation between expected put option return and systematic log asset payoff volatility is ambiguous. However, in case of put options, we found it much harder to analytically identify the sign of the effect. More specifically, in this case, we are only able to show that, if idiosyncratic volatility is zero ($\sigma_i = 0$), the sign is positive for ATM and OTM options, but positive, zero, or negative for ITM options.[5] Despite that, plotting the effect of systematic volatility on the expected put option return against the strike price, Panel A of Figure 3 suggests, even if idiosyncratic volatility is positive ($\sigma_i > 0$), the sign of the effect of systematic volatility on the expected put option return is negative for sufficiently ITM options, whereas it is positive for sufficiently OTM options.

Interestingly, part (e) of Proposition 2 opens up the possibility that the idiosyncratic volatility of the asset payoff is also ambiguously related to the expected put option return. Consistent with this possibility, Corollary 3 shows that, identical to the sign of the systematic volatility

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[5]The problem is the sum of the first two terms in the inequality in part (d), which is $-(\sigma_x^2/\sigma_s^2)H'[c^*] + H'[\sigma_x - \beta - \alpha]$. While $-H'[c^*] + H'[\sigma_x - \beta - \alpha] > 0$, $(\sigma_x^2/\sigma_s^2) > 1$, raising the value of the negative summand and rendering it impossible to sign the sum. In contrast, in case of call options in part (d) of Proposition 1, $(\sigma_x^2/\sigma_s^2)H'[c^*] - H'[\alpha - \sigma_x + \beta] > 0$, because $(\sigma_x^2/\sigma_s^2) > 1$ and $H'[c^*] - H'[\alpha - \sigma_x + \beta] > 0$.
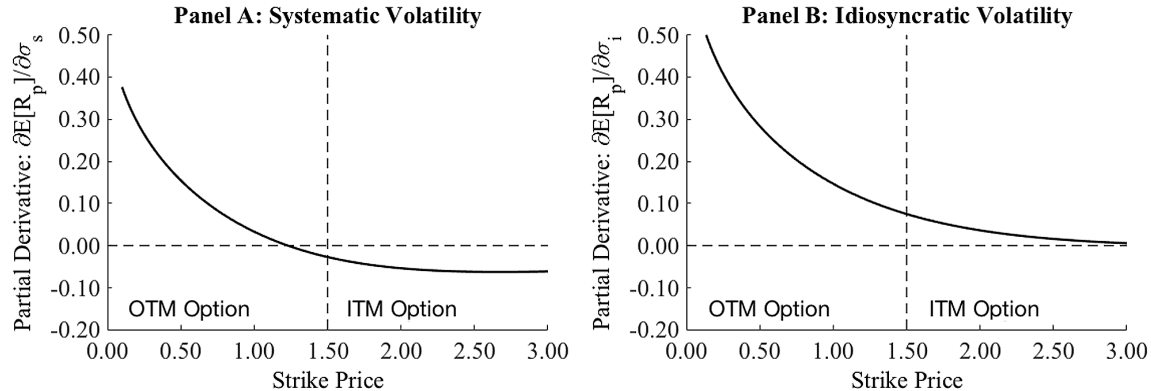
Figure 3: Partial Derivatives of Expected Put Option Return with Respect to Systematic and Idiosyncratic Volatility The figure plots the partial derivatives of the expected put option return with respect to the systematic volatility (Panel A) and the idiosyncratic volatility (Panel B) of the log primitive asset payoff against the strike price. The base case parameter values are as in Figure 1.

effect, the sign of the idiosyncratic volatility effect also depends on option moneyness.

COROLLARY 3: *Under the assumptions in Proposition 2, the relation between the expected put option return, $E[\tilde{R}_p]$, and the idiosyncratic volatility of the log asset payoff, σ_i, is positive for sufficiently OTM options, but can be positive, zero, or negative for ITM and ATM options.*

Proof: See the Internet Appendix.

While Corollary 3 conflicts with Hu and Jacobs' (2018) finding that, in a Black and Scholes (1973) framework, there is an unambiguous positive relation between expected put option return and idiosyncratic volatility, we only ever found numerical examples of a mildly negative relation for extremely deep ITM options. Panel B of Figure 3 supports this claim, showing that the expected put option return converges to a value close, but sometimes ever so slightly below, zero as we let the moneyness of the option converge to infinity.

3 Empirical Tests

In this section, we employ single-stock call option data to test predictions derived from the stochastic discount factor model in Section 2. More specifically, we examine whether expected call option returns (i) decrease with option moneyness; (ii) increase with systematic stock volatility for ITM and ATM options, but decrease with systematic stock volatility for OTM options; and (iii) decrease with idiosyncratic stock volatility. In Section 3.1, we introduce our data. Section 3.2 discusses how we calculate the analysis variables. Section 3.3 offers our main

empirical results on the pricing of moneyness, systematic-, and idiosyncratic-volatility.

3.1 Data

We obtain data on American call options written on single stocks not paying out cash over the options' times-to-maturity from the Ivy DB database provided by Optionmetrics. We only use options written on stocks not paying out cash since Merton (1973) shows that it is never optimal to early exercise such options, rendering them equivalent to European options. While other studies are more relaxed about using American option data in conjunction with European option pricing theories (see, e.g., Carr and Wu (2009), Hu and Jacobs (2018), and Martin and Wagner (2018)), Aretz et al. (2017) show that the mean returns of American put options differ significantly from those of equivalent synthetic European put options.[6] As a result, we refrain from using American options data to study the put option predictions of our model.

We impose standard filters on the call options data. In particular, we exclude options violating well-known arbitrage conditions. Thus, we drop an option if its price does not lie between the underlying stock's price and the maximum of zero and the arbitrage-free value of an equivalent long forward contract. We also exclude options with a zero trading volume, a zero or negative bid price, a bid price above the ask price, and an average bid and ask price below $\$\frac{1}{8}$. We further omit options whose last trade date is not equal to the observation date. To ensure that our sample options are equivalent to European call options, we finally exclude options written on stocks with ex-dividend dates before the options' maturity dates.

We obtain daily and monthly market data on the stocks underlying our sample options from CRSP. We obtain annual accounting data on them from COMPUSTAT. We match the Optionmetrics data and the CRSP-COMPUSTAT data using the six-digit CUSIP. Data on the FFC benchmark factors and the risk-free rate of return are retrieved from Kenneth French's website.[7] Our sample period is January 1996 to August 2014.

3.2 Variable Construction

Our main empirical tests examine the one-month call option return, calculated from the end of month $t - 1$ (about seven weeks prior to maturity) to the end of month t (about three weeks prior to maturity). In robustness tests, we, however, also study the about seven-week

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[6]In addition, Zivney (1991), de Roon and Veld (1996), and McMurray and Yadav (2000) present empirical evidence suggesting American options have significantly higher prices than European options.

[7]The URL address is <https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/>.

to-maturity call option return, calculated from the end of month $t-1$ to the third Friday in month $t+1$. In line with our theoretical work in Section 2, we use an option's moneyness and the systematic and the idiosyncratic volatility of the stock underlying the option to explain the option return. Controlling for the effects of systematic and idiosyncratic volatility, we compute an option's moneyness as the ratio of the underlying stock's price to the option's strike price at the end of month $t-1$ (see footnote (4)). Following an approach similar to Boyer et al. (2010) and Cao and Han (2013), we use the market model or the FFC four-factor model to decompose a stock's historical volatility into systematic volatility and idiosyncratic volatility. To estimate systematic and idisyncratic volatility from the market model, we regress the monthly stock return on the CRSP market index return minus the risk-free rate of return

$$r_{i,t} = \alpha_i + \beta_i^{mkt}(r_{mkt,t} - rf_t) + \epsilon_{i,t}, \tag{11}$$

where $r_{i,t}$ is the return of stock i in month t, $r_{mkt,t}$ the CRSP index return, and rf_t the risk-free rate of return. α_i and β_i^{mkt} are parameters, and $\epsilon_{i,t}$ is the residual. To estimate the variables from the FFC model, we regress the monthly stock return on the FFC benchmark factors

$$r_{i,t} = \alpha_i + \beta_i^{mkt}(r_{mkt,t} - rf_t) + \beta_i^{smb}r_{smb,t} + \beta_i^{hml}r_{hml,t} + \beta_i^{mom}r_{mom,t} + \epsilon_{i,t}, \tag{12}$$

where $r_{smb,t}$ is the return of the SMB (small-minus-big) spread portfolio, $r_{hml,t}$ the return of the HML (high-minus-low book-to-market) spread portfolio, and $r_{mom,t}$ the return of the past eleven-month return (winners-minus-losers) spread portfolio. See Kenneth French's website for more details about these benchmark factors. α_i, β_i^{mkt}, β_i^{smb}, β_i^{hml}, and β_i^{mom} are parameters, and $\epsilon_{i,t}$ is the residual. We estimate both the market and the FFC model using monthly data over the 60 months prior to the end of month $t-1$. In robustness tests, we, however, also follow Hu and Jacobs (2018) and estimate the models using daily data over the month prior to the end of month $t-1$. Using the market or the FFC model, we use the annualized standard deviation of the fitted value over the estimation period as our estimate of systematic volatility and the annualized standard deviation of the residual as idiosyncratic volatility.

3.3 Empirical Results

3.3.1 Descriptive Statistics

In Table 1, we present descriptive statistics on the call options data (Panel A) and the volatility estimates (Panel B). The table indicates that the data contain 280,349 observations, translating
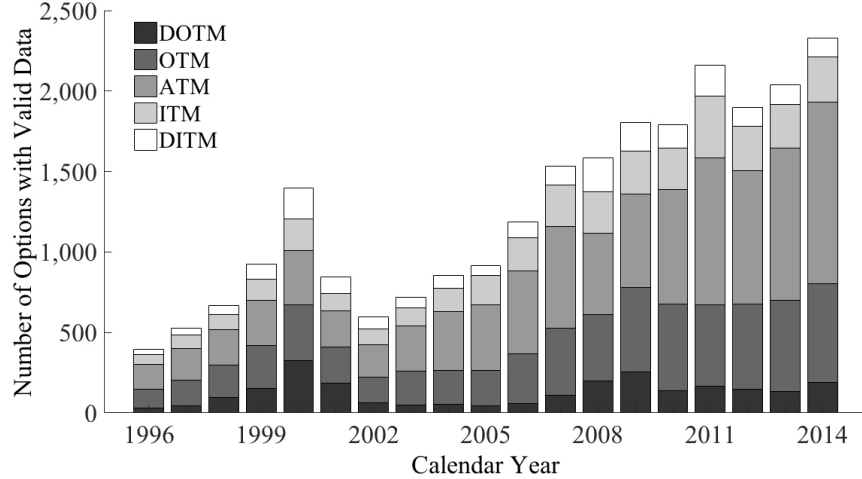
Figure 4: Moneyness Composition of the Options Sample The figure shows the average number of call options with a stock price-to-strike price ratio below 0.85 (DOTM), between 0.85 and 0.95 (OTM), between 0.95 and 1.05 (ATM), between 1.05 and 1.15 (ITM) and above 1.15 (DITM) at the start of the option holding period. We calculate numbers by sample month and then average over months by year.

into an average of 1,257 monthly observations and representing 5,785 stocks. Panel A shows that the mean option return is 14.6% per month, with a standard deviation of 171.3%. Despite the option return not being calculated until maturity, it is still highly right skewed, as suggested by a first percentile of –96.2%, but a 99th percentile of 610.0%. While both the average and the median option are close to ATM, the standard deviation of the moneyness variable, the stock price-to-strike price ratio, suggests that a sizable number of options are significantly ITM or OTM at the start of the option holding period. At the start of the same period, the vast majority of options have between 44 to 53 calendar days (about seven weeks) to maturity, while the average option has an Black-Scholes (1973) implied volatility of 50.6% per annum.

Figure 4 takes a closer look at the moneyness composition of our options sample, showing the average number of deep OTM (moneyness below 0.85), OTM (0.85–0.95), ATM (0.95–1.05), ITM (1.05–1.15), and deep ITM (above 1.15) options by sample year. The figure suggests that the number of option observations rises from a low of about 400 in 1996 to a high of about 2,330 in 2014. On average, about 38% of the option observations falls into the ATM category. Of the remainder, a significantly greater proportion ends up in the two OTM categories (about 38%) than in the two ITM categories (about 24%). The smaller number of ITM compared to ATM and OTM option observations is surprising since ITM options are, in general, more liquid than the others, rendering it more likely that they pass our data filters.

Panel B of Table 1 suggests that the average stock underlying the sample options has a

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historical volatility of about 54% per annum. Using the market model to decompose historical volatility, systematic volatility accounts for an average of about 35% of historical volatility. Using the FFC model, it accounts for an average of about 42%.[8] Thus, in comparison to the CRSP universe, stocks with options written on them tend to have a similar historical, but a higher systematic, volatility, presumably because they are usually bigger stocks.[9] Given also high standard deviations for the systematic and idiosyncratic volatility estimates, with, for example, the standard deviation of the FFC systematic (idiosyncratic) volatility estimate being 20.5% (25.9%) per annum, our data are well suited to disentangle the separate pricing effects of systematic and idiosyncratic volatility for the cross-section of call option returns.

3.3.2 The Pricing of Moneyness, Systematic-, and Idiosyncratic-Volatility

In Table 2, we present the results from Fama-MacBeth (FM; 1973) regressions of call option returns over month t on subsets of moneyness, systematic volatility, idiosyncratic volatility, and interactions between the variables at the start of that month. We employ FM regressions in our empirical work since portfolio sorts would require us to form triple-sorted portfolios based on moneyness, systematic-, and idiosyncratic-volatility to test our predictions. Forming these portfolios is, however, difficult because first, the sample options are skewed toward ATM and OTM options (see Section 3.3.1 and Figure 4) and, second, the systematic and idiosyncratic volatility variables share a high average cross-sectional correlation of 0.649.

Panel A offers parameter estimates and t-statistics (in square parentheses).[10] We employ the market model to estimate the volatility variables in columns (1) to (3) and the FFC model in columns (4) to (6). In columns (1) and (4), we regress call option returns on uninteracted moneyness, systematic-, and idiosyncratic-volatility. In columns (2) and (5), we allow moneyness to linearly condition the effects of systematic and idiosyncratic volatility. Specifically, we model the conditional effect of each volatility variable, $\gamma_{\text{VolVar}|\text{Money}}$, as

$$\gamma_{\text{VolVar}|\text{Money}} = \gamma_{\text{VolVar}} + \gamma_{\text{Money}\times\text{VolVar}} \times \text{Money}_{i,t}, \tag{13}$$

where VolVar is systematic or idiosyncratic volatility, γ_{VolVar} the estimate on the volatility variable, $\gamma_{\text{Money}\times\text{VolVar}}$ the estimate on the moneyness-volatility interaction, and $\text{Money}_{i,t}$ an

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[8]To calculate the systematic volatility proportions, we compute the ratio of systematic volatility squared to total volatility squared at the observation level and then average over observations.

[9]The average stock contained in the CRSP database has an annualized historical volatility of about 51% over the January 1996 to August 2014 sample period, with market-model systematic volatility accounting for 26% of that historical volatility and FFC-model systematic volatility for 35%.

[10]We consistently calculate t-statistics using Newey-West (1987) standard errors.
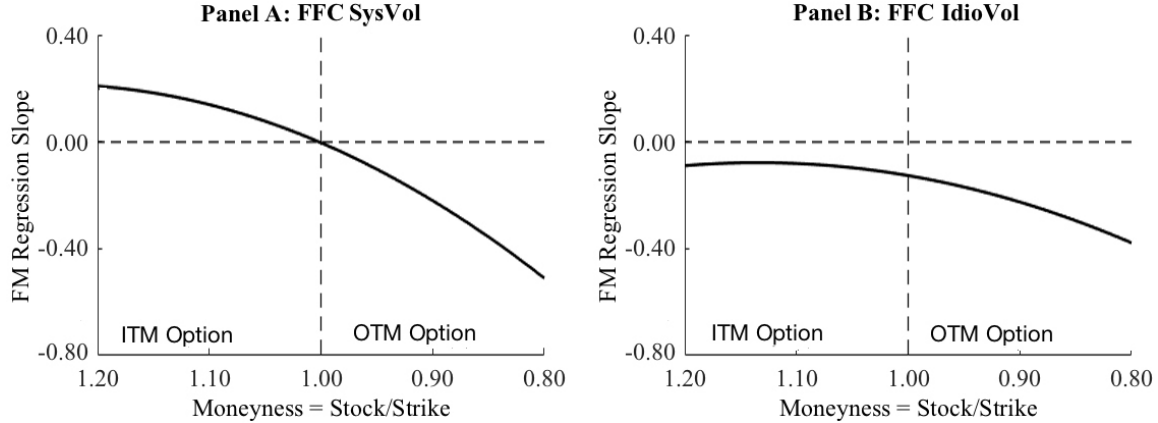
Figure 5: Empirical Effects of Systematic- and Idiosyncratic-Volatility on Call Option Returns Conditional on Moneyness The figure plots the effects of systematic- (Panel A) and idiosyncratic-volatility (Panel B) on the cross-section of call option returns against an option's moneyness. Systematic and idiosyncratic volatility are estimated using the FFC model. The conditional effects of the volatility variables are calculated by plugging the FM regression estimates in column (6) of Panel A in Table 2 into Equation (14) and by letting option moneyness vary from 1.20 (deep ITM) to 0.80 (deep OTM).

option's moneyness level. In columns (3) and (6), we finally allow moneyness to non-linearly condition the volatility effects, consistent with the patterns observed in Figure 2

$$\gamma_{\text{VolVar}|\text{Money}} = \gamma_{\text{VolVar}} + \gamma_{\text{Money} \times \text{VolVar}} \times \text{Money}_{i,t} + \gamma_{\text{Money}^2 \times \text{VolVar}} \times \text{Money}_{i,t}^2, \qquad (14)$$

where $\gamma_{\text{Money}^2 \times \text{VolVar}}$ is the estimate on the interaction between moneyness-squared and the volatility variable, and $\text{Money}_{i,t}^2$ the squared moneyness variable. Panel B offers the implied values and t-statistics of the conditional volatility effects at moneyness levels of 0.80, 0.90, 1.00, 1.10, and 1.20 calculated from Equation (14). The panel also reports the differences in the conditional volatility effects across the extreme moneyness levels (H–L).

Table 2 strongly supports our model predictions. Panel A suggests that while moneyness has an unambiguous and highly significant negative effect on the cross-section of call option returns, the uninteracted effects of systematic and idiosyncratic volatility are insignificant, independent of whether we use the market or the FFC model to decompose volatility (see columns (1) and (4)). Interacting the effects of the volatility variables with moneyness, the remaining columns, however, suggest that both volatility effects strongly and usually significantly increase with moneyness. Using the linear interactions of the market model volatility estimates in column (2) for example, a 0.10 increase in moneyness raises the effect of systematic volatility by 19.7% per month (t-statistic: 3.47), while it raises the effect of idiosyncratic volatility by 8.5% (t-statistic:

2.29). Calculating the volatility effects at different moneyness levels, Panel B suggests that both the market- and the FFC-model systematic volatility estimates have a significantly negative effect on OTM, but a significantly positive effect on ITM, options, with significant spreads across the most extreme ITM and OTM options. Conversely, both idiosyncratic volatility estimates have a significantly negative effect on OTM, but an insignificant effect on ITM, options, with insignificant spreads across the most extreme ITM and OTM options. Figure 5 graphically shows the conditioning influence of moneyness on the effects of systematic (Panel A) and idiosyncratic volatility (Panel B) estimated using the FFC model.[11]

Although the evidence in this section strongly supports our main model predictions, a small deviation from our theory is that while our model predicts that ATM call option returns are positively related to systematic volatility (see Corollary 1), our evidence suggests no relation between these variables (see Panel B in Table 2). Notwithstanding, we later show that the effect of systematic volatility is downward biased in our main tests due to negative jump risk premia in call option returns. Controlling for these premia, the effect of systematic volatility will become significantly positive for ATM and ITM options (see Section 3.3.5).

3.3.3 Controlling for Option and Underlying Stock Liquidity

In this section, we investigate whether our results are sensitive to controlling for option and underlying stock liquidity. In line with the limits to arbitrage arguments of Shleifer and Vishny (1997) and Pontiff (2006), Garleanu et al. (2009) develop a theoretical model in which an option can only be imperfectly delta-hedged, with the hedging imperfection increasing with the volatility of the underlying asset. In the model, market makers demand extra compensation for selling options on more volatile assets, decreasing the expected returns of these options. Since the extra compensation is, however, only required for net positive demand options, we control for option demand using the ratio of an option's open interest to the dollar trading volume of the underlying stock at the end of month $t-1$ (Bollen and Whaley (2004)). We also control for the bid-ask spread scaled by the midpoint price at the end of month $t-1$ since the higher extra compensation should widen the bid-ask spread. We finally control for the liquidity of the underlying stock since it is more costly to rebalance delta hedges involving illiquid assets. We

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[11]To offer more non-parametric evidence on the conditioning role of moneyness on the volatility effects, we have also run FM regressions excluding interaction terms on subsamples based on option moneyness at the start of the option holding period. The first subsample contains options with a moneyness below 0.975, the second those with a moneyness between 0.975 and 1.025, and the third those with a moneyness above 1.025. For each subsample, we then regress the option return over month t on uninteracted versions of moneyness, systematic volatility, and idiosyncratic volatility at the end of month $t-1$. The results from the subsample FM regressions align with those from the FM regressions featuring interaction terms.

proxy for stock illiquidity using the ratio of a stock's absolute daily return to its daily dollar trading volume averaged over the twelve months prior to month t (Amihud (2002)).

Table 3 shows that controlling for an option's open interest, its bid-ask spread, and the liquidity of the underlying stock does not change how systematic and idiosyncratic volatility price the cross-section of call options. In particular, Panels A and B suggest that systematic volatility continues to have a significantly positive effect on ITM, but a significantly negative, effect on OTM options. Conversely, idiosyncratic volatility continues to have a significantly negative effect on OTM, but an insignificant effect on, ITM options. In addition, Panel A also suggests that, of the control variables, the option bid-ask spread is significantly negatively related to call option returns (t-statistics around –5.60). In contrast, neither the option open interest nor stock liquidity have any explanatory power for call option returns.

3.3.4 Controlling for Option Mispricing Factors

We next examine whether mispricing in the options market explains our results. Stein (1989) and Poteshman (2001) offer evidence suggesting that investors overpay for options written on underlying assets whose volatility has recently increased. To account for this effect, we include the change in an underlying stock's volatility calculated from daily data over one calendar month from month $t - 2$ to month $t - 1$ in our tests. Using the ratio of underlying stock volatility calculated from daily data over month $t - 1$ to Black-Scholes (1973) option implied volatility at the end of that month to calculate an option's value statistic, Goyal and Saretto (2009) show that high-ratio (cheap) options have higher returns than low-ratio (expensive) options. To account for this effect, we also add that ratio to our tests. Following Cao and Han (2013), we finally add the change in Black-Scholes (1973) option implied volatility over the option holding period to control for the correction of volatility-related mispricing.

Panels A and B of Table 4 show that controlling for the change in underlying stock volatility, the historical-to-implied volatility ratio, and the change in implied volatility does not change how systematic and idiosyncratic volatility price call options. Consistent with prior studies, Panel A also shows that the change in underlying stock volatility is significantly negatively, while, at least when not controlling for the change in implied volatility, the historical-to-implied volatility ratio is significantly positively related to call option returns. Finally, the change in implied volatility is significantly positively related to call option returns.

3.3.5 Controlling for the Variance Risk Premium and Risk-Neutral Moments

We next investigate whether our results are sensitive to the underlying stock's volatility risk premium and its third and fourth risk-neutral return moments. Assuming that volatility evolves stochastically over time, Bakshi and Kapadia (2003) offer theoretical evidence that option returns contain a component positively related to the volatility risk premium of the underlying asset. They further show that this component is positively related to underlying asset volatility in many option models with stochastic volatility (see, e.g., Heston (1993)). Thus, the volatility estimates used in our tests could possibly partially capture volatility risk premia. To control for stock i's volatility risk premium at the end of month $t-1$, $\text{VRP}_{i,t-1}$, we follow Bali and Hovakimian (2009) and Carr and Wu (2009) and estimate that premium as

$$\text{VRP}_{i,t-1} = \sqrt{\text{Realized Variance}_{i,t-1}} - \sqrt{\text{Implied Variance}_{i,t-1}}, \tag{15}$$

where Realized Variance$_{i,t-1}$ is the sum of stock i's squared daily log returns over month $t-1$ multiplied by twelve, and Implied Variance$_{i,t-1}$ is Britten-Jones and Neuberger's (2000) model-free estimate of stock i's annualized implied variance at the end of month $t-1$. See Appendix B for more details about the calculation of the implied variance estimate.

Allowing for the possibility of jumps in the underlying asset value, Bakshi and Kapadia (2003) show that the option return also contains a component positively related to the jump risk premium of the underlying asset. Since jumps with a negative mean value and a sufficiently high intensity induce left skewness and excess kurtosis into the return distribution of an asset, we follow these authors in using a stock's third (skewness) and fourth (kurtosis) risk-neutral moments to control for its jump risk premium. Other reasons for adding the two higher moments are that Conrad et al. (2013) show that they affect the pricing of stocks with options written on them, and that Boyer and Vorkink (2014) show that skewness negatively prices stock options. Following Bakshi et al. (2003), we calculate stock i's risk-neutral skewness, $\text{RNS}_{i,t-1}$, and risk-neutral kurtosis, $\text{RNK}_{i,t-1}$, at the end of month $t-1$ as

$$\text{RNS}_{i,t-1} = \frac{e^{r\tau}W_{i,t-1,\tau} - 3\mu_{i,t-1,\tau}e^{r\tau}V_{i,t-1,\tau} + 2\mu_{i,t-1,\tau}^3}{[e^{r\tau}V_{i,t-1,\tau} - \mu_{i,t-1,\tau}^2]^{3/2}}, \tag{16}$$

and

$$\text{RNK}_{i,t-1} = \frac{e^{r\tau}X_{i,t-1,\tau} - 4\mu_{i,t-1,\tau}e^{r\tau}W_{i,t-1,\tau} + 6e^{r\tau}\mu_{i,t-1,\tau}^2 V_{i,t-1,\tau} - 3\mu_{i,t-1,\tau}^4}{[e^{r\tau}V_{i,t-1,\tau} - \mu_{i,t-1,\tau}^2]^2}, \tag{17}$$

where r is the risk-free rate, τ the return period, $\mu_{i,t-1,\tau} = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V_{i,t-1,\tau} - \frac{e^{r\tau}}{6}W_{i,t-1,\tau} -$

$\frac{e^{r\tau}}{24}X_{i,t-1,\tau}$, and $V_{i,t-1,\tau}$ $(W_{i,t-1,\tau})$ $[X_{i,t-1,\tau}]$ the value of a volatility (cubic) [quartic] contract paying out the squared (cubed) [quartic] log return at the end of the return period. See Appendix B for more details about the calculation of the values of the contracts.

Table 5 shows that controlling for the volatility risk premium and the higher moments does not materially change how systematic and idiosyncratic volatility price call options, despite the inclusion of these controls reducing sample size by more than 20% (see Panel A). A noteworthy small change, however, is that controlling for the higher moments raises the effect of systematic volatility across all moneyness levels, so that the effect becomes significantly positive for ATM, but insignificant for OTM, options (see Panel B). Importantly, the significantly positive effect for ATM options is more consistent with Corollary 1 and Figure 2 than the insignificant effect found in our main tests. Turning to the control variables, Panel B supports Carr and Wu's (2009) and Driessen et al.'s (2009) conclusion that single-stock options are not significantly related to the volatility risk premium. In addition, it also supports Bali and Murray's (2013) and Boyer and Vorkink's (2014) conclusion that skewness has a negative (t-statistics about –2.25), while kurtosis has a positive (t-statistics about 4.00), pricing effect on call options.

3.3.6 Controlling for Stock Characteristics

Most stock pricing models implicitly assume that the stochastic discount factor realization, $\tilde{M}$, is linear in the returns of spread portfolios sorted on stock characteristics such as market size (Banz (1981)), the book-to-market ratio (Rosenberg et al. (1985) and Fama and French (1992)), return momentum (Jegadeesh and Titman (1993)), asset growth (Cooper et al. (2008)), and profitability (Fama and French (2006) and Novy-Marx (2013)). See, for example, the FFC model, Hou et al.'s (2015) Q-theory model, or Fama and French's (2015) five-factor model. Under this assumption, a stock's systematic volatility is a function of its beta exposures to the spread portfolios and, assuming that the beta exposures proxy for the stock characteristics (see Davis et al. (2000) and Fama and French (2016)), the stock characteristics themselves. Thus, if our systematic volatility estimates efficiently captured a stock's exposure to the stochastic discount factor, then controlling for stock characteristics known to price stocks is likely to mitigate the ability of systematic, but not idiosyncratic, volatility to explain call option returns.

Table 6 strongly supports this hypothesis. Controlling for the underlying stocks' size, book-to-market ratio, momentum, asset growth, and profitability,[12] the effect of systematic volatility

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[12]In line with this literature, we calculate market size as the log of the product of common shares outstanding and the share price in June of year t; the book-to-market ratio as the log of the ratio of the book value of equity at the end of the fiscal year in calendar year $t - 1$ to the product of common shares outstanding and

becomes insignificantly negative for OTM options and, at best, mildly significantly positive for ITM options. Also, the difference in the effect between the most extreme ITM and OTM options becomes insignificant. In contrast, the effect of idiosyncratic volatility remains significantly negative for OTM, but insignificant for ITM, options, with the difference between the most extreme ITM and OTM options continuing to be significant (see Panels A and B). Turning to the stock characteristics, Panel B suggests that only the book-to-market ratio has a consistent and significantly positive pricing effect on call options (t-statistics around 2.10), while the effects of size and profitability are only significantly positive in a subset of the models. The only weak ability of the stock characteristics to price call options is likely due to the fact that only the about 1,000 largest U.S. stocks have options written on them, and that these stocks usually do not produce strong stock characteristic premia (see Fama and French (2008)).

4 Robustness Tests

In this section, we present the results from several robustness tests. In Section 4.1, we repeat our main tests using the returns of call options held until maturity. In Section 4.2, we repeat our main tests using volatility estimates derived from daily data over the calendar month prior to the start of the option holding period. In Section 4.3, we incorporate bid and ask transaction costs into our main tests. In Section 4.4, we use an alternative time-series methodology to find out how moneyness conditions the effect of systematic volatility on European S&P 500 call option returns. Motivated by our prior results, we always control for the bid-ask spread and the change in underlying stock volatility in all FM regressions in this section.[13]

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the share price at the end of that calendar year; asset growth as the log gross change in total assets from the end of the fiscal year in calendar year $t-2$ to the end of the fiscal year in calendar year $t-1$; and profitability as the ratio of sales net of costs of goods sold, selling, general, and administrative expenses, and interest expenses to the book value of equity at the end of the fiscal year in calendar year $t-1$, where the book value of equity is total assets minus total liabilities plus deferred taxes (zero if missing) minus preferred stock (zero if missing). We use the calculated values from July of calendar year t to June of calendar year $t+1$ in the FM regressions. In contrast, we calculate return momentum as the compounded stock return over the period from month $t-12$ to $t-2$, using the calculated value in month t in the FM regressions.

[13]While also significant in Section 3, we do not control for the change in implied volatility and the higher risk-neutral moments in the robustness tests. We avoid doing so because the change in implied volatility is calculated over the option holding period, thus inducing a look-ahead bias, while the inclusion of the higher risk-neutral moments would lead to a significant reduction in sample size.

4.1 Held-to-Maturity Call Option Returns

In line with a large literature, we calculate option returns over a period ending before the options' maturity dates to avoid confounding effects arising from option settlement procedures. Despite the confounding effects, other studies, for example, Goyal and Saretto (2009) or Hu and Jacobs (2018), however, study the returns of options held until maturity. Since our model in Section 2 actually makes predictions about the expected returns of options held until maturity, we now follow these studies and rerun our FM regressions using such returns. More specifically, we now calculate the call option return as the ratio of the maximum of the difference between the end-of-day stock price and the strike price and zero on the third Friday in month $t + 1$ (the maturity date of the options) to the call option price at the end of month $t - 1$.

In Table 7, we present the results from repeating the FM regressions allowing moneyness to non-linearly condition the effects of the volatility variables in columns (3) and (6) of Table 2 on held-to-maturity call option returns, controlling, however, for the bid-ask spread and the change in underlying stock volatility. To conserve space, this table as well as Tables 8 and 9 only show the effects of the market- (Panel A) and FFC-model (Panel B) systematic and idiosyncratic volatility estimates at different moneyness levels, using a table design equivalent to the design used in the second panels in Tables 2 to 6. Table 7 suggests that held-to-maturity call option returns produce similar systematic and idiosyncratic volatility effects as the one-month returns used in our main tests. Systematic volatility continues to have a negative, although this time insignificant, effect on OTM call options and a positive and significant effect on ITM call options, with the spread in the effect across the most extreme moneyness options, however, being less significant. Conversely, idiosyncratic volatility continues to have a significantly negative effect on OTM, but an insignificant effect on ITM, call options, with the spread in the effect across the most extreme moneyness options being highly significant.

4.2 Daily Data Volatility Estimates

In a related paper, Hu and Jacobs (2018) find a negative relation between total stock volatility and the cross-section of ATM single-stock call option returns.[14] While they, however, use daily data over the month prior to the option holding period to calculate their volatility estimate,

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[14]The negative relation found in their empirical work is consistent with our results since first, total stock volatility is predominately driven by idiosyncratic volatility (see Table 1) and, second, systematic (idiosyncratic) volatility has an insignificant (significantly negative) effect on ATM call options in the absence of higher moment controls (see Table 2). To more directly replicate their results, we have also run FM regressions of one-month ahead call option returns with a 0.95–1.05 moneyness on total stock volatility calculated using 60 months of monthly data. Doing so, we also find a significantly negative relation (t-statistic: –3.76).

we use monthly data over the 60 months prior to the same period to calculate ours. We do so since separately calculating systematic and idiosyncratic volatility involves the estimation of two (market model) or five (FFC model) parameters. Following the common practice to add Dimson (1979) correction terms to estimations of the market- or FFC-model on daily data would further raise the number of parameters, with it becoming four and 13, respectively, in case of two lags. Given that the average month features no more than 22 daily observations, with many being zero-return observations, we thus believe that it is more sound to estimate systematic and idiosyncratic volatility using monthly instead of daily data.

Despite our reservations about using daily data over the month prior to the option holding period to calculate our volatility variables, Table 8 shows that doing so without using Dimson (1979) correction terms does not greatly change our conclusions. While market-model systematic volatility is now only weakly positively related to ITM call option returns (t-statistic: 1.73; see Panel A), the relation between FFC-model systematic volatility and these returns remains positive and highly significant (t-statistic: 2.84; see Panel B). Also interestingly, the negative relations between both systematic and idiosyncratic volatility and OTM call option returns are now much more significant, with t-statistics usually below minus five. As a result, the spreads in the systematic and idiosyncratic volatility effects across the most extreme ITM and OTM options tend to be more significant, too, with t-statistics usually above five.

4.3 Bid-and-Ask Price Call Option Returns

Consistent with other studies, we use option returns calculated from bid-ask midpoint prices in our main tests, arguing that the midpoint price is more reflective of an option's true value than either the bid or the ask. Despite that, Santa-Clara and Saretto (2009) show that option trading costs greatly exceed stock trading costs. Supporting their evidence, Goyal and Saretto (2009) and Cao and Han (2013) find that accounting for bid-ask transaction costs greatly diminishes the profitability of their option trading strategies, suggesting that real investors would find it hard to reap the excess profits discovered by them. Since our volatility estimates are positively correlated with bid-ask spreads,[15] accounting for bid-ask transaction costs could also diminish the positive relation between systematic volatility and ITM call option returns found in our tests. To study the effect of bid-ask transaction costs, we thus next repeat our main FM regressions using option returns calculated as the ratio of the difference between the midpoint price and a fraction S of the bid-ask spread at the end of month t to the sum of the

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[15]The average cross-sectional correlation between either the market- or FFC-model systematic (idiosyncratic) volatility estimate and the log bid-ask spread is about 0.05 (0.15).

midpoint price and the fraction S of the bid-ask spread at the end of month $t-1$. To account for variations in transaction costs across different investor types, we set S equal to 0.00, 0.10, 0.25, and 0.50, with $S = 0.50$ implying investors trade at the bid and the ask price.

Table 9 suggests that accounting for bid-ask transaction costs does indeed attenuate the relations between the volatility estimates and call option returns.[16] In spite of that, the changes in the systematic and idiosyncratic volatility effects at the different moneyness levels are, however, small. For example, the effect of FFC-model systematic volatility on call options with a 1.20 moneyness changes only modestly from 21% per month (t-statistic: 2.87) at zero transaction costs ($S = 0.00$) to 18% (t-statistic: 2.58) at high transaction costs ($S = 0.50$). Even when trading takes place at the bid and the ask price ($S = 0.50$), systematic volatility continues to be significantly positively (negatively) related to ITM (OTM) call option returns. Conversely, idiosyncratic volatility is still more negatively related to OTM than to ITM call option returns, although the effects are now more significant for the higher moneyness options.

4.4 Time-Series Regressions

We finally offer evidence on how moneyness conditions the effect of systematic volatility on call option returns using an alternative methodology, namely time-series regressions run on European S&P 500 call option returns. To this end, we first create four S&P 500 call option subsamples based on option moneyness at the end of month $t-1$. The first subsample features the contract with a moneyness closest to 0.80 (OTM), the second the contract with a moneyness closest to 1.00 (ATM), the third the contract with a moneyness closest to 1.20 (ITM), and the fourth the contract with a moneyness closest to 1.40 (DITM). We next calculate the returns of the chosen options over month t. We finally run subsample-specific time-series regressions of the S&P 500 call option return on the total volatility of the S&P 500 calculated from monthly data over the 60 months prior to the option holding period. Given that idiosyncratic volatility is diversified away in broad stock indexes, the total volatility of the S&P 500 is identical to the systematic volatility of the index. To ensure identical sample periods, we run each time-series regression on the period from October 2001 to August 2014 (155 observations).[17]

Supporting the conclusions obtained from our FM regressions, Table 10 suggests that sys-

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[16]While the table only reports results for the FFC-model volatility estimates, our conclusions are identical for the market-model estimates. The market-model results are available from us upon request.

[17]The October 2001–August 2014 sample period is the longest consecutive period for which there is at least one mutually exclusive option contract within each moneyness category. Running the time-series regressions on the longest possible consecutive or non-consecutive period for each moneyness category or using a more standard sample period (as, e.g., January 2002–August 2014) does not materially change our results.

tematic volatility is significantly positively related to the returns of the ATM and ITM S&P 500 call options, but significantly negatively to the returns of the OTM options. To be more specific, a 0.10 increase in index volatility lowers the return of the OTM option by 37.2% per month (*t*-statistic: –2.39), but raises the return of the ITM option by 21.2% per month (*t*-statistic: 3.01). Interestingly, however, the table further suggests that the effect of systematic volatility is not monotonically related to moneyness. While the effect strongly increases from the OTM to the ATM option, it slightly decreases from the ATM to the ITM or the DITM option, without, however, becoming zero or negative again. While we do not want to overemphasize this result, we note that the result is consistent with the pattern in Panel A of Figure 2.

5   Conclusion

We use a stochastic discount factor model equivalent to Rubinstein's (1976) model to study how an asset's volatility affects the expected returns of European options written on the asset. We show that the effect of volatility depends crucially on an option's moneyness and the extent to which variations in volatility are attributable to variations in systematic or idiosyncratic volatility. While variations in idiosyncratic volatility only affect an option's elasticity, variations in systematic volatility also oppositely affect the expected return of the underlying asset. The ultimate effect of variations in systematic volatility thus depends on whether the effect on the option's elasticity or on the underlying asset's expected return prevails. Our work suggests that the elasticity effect prevails for options with more non-linear payoffs, while the underlying asset effect prevails for options with more linear payoffs. For example, ITM and ATM call options produce a positive systematic volatility effect, while OTM call options can produce a negative effect. In contrast, all call options produce a negative idiosyncratic volatility effect.

We use call options written on single stocks not paying out cash over the options' maturity time to evaluate our model's predictions. Using FM regressions modelling the conditioning role of moneyness on the pricing effects of systematic and idiosyncratic volatility using interaction terms, our empirical work strongly supports our predictions. Controlling for the liquidity of the options or the underlying stocks, mispricing in the options market, the variance risk premium and the third and fourth higher moments of the underlying stocks does not materially change these conclusions. Our conclusions are also robust to calculating option returns until maturity; estimating the volatility variables using daily data over the month prior to the option holding period; incorporating bid-ask transaction costs; and using an alternative time-series methodology on S&P 500 call options with different moneyness levels.

Appendix A: Proofs

This appendix proves Proposition 1 and Corollary 1 and 2. We offer the proofs of Proposition 2 and Corollary 3 in the Internet Appendix. Although the correlation between the conditional expectation of the log asset payoff, $\tilde{x}_s$, and the log stochastic discount factor realization, $\tilde{m}$, is consistently equal to minus one, we will nonetheless denote that correlation by κ in our proofs, simplifying some of the mathematical arguments made below.

Proof of Proposition 1:

(a) The partial derivative of the expected call option return, $E[\tilde{R}_c]$, with respect to the expected log payoff of the primitive asset, μ_x, is given by

$$\frac{\partial E[\tilde{R}_c]}{\partial \mu_x} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial \mu_x} = \frac{(\partial E[\tilde{X}_c]/\partial \mu_x)p_c - (\partial p_c/\partial \mu_x)E[\tilde{X}_c]}{p_c^2}. \tag{A.1}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{X}_c]}{\partial \mu_x} = e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] > 0, \tag{A.2}$$

and

$$\frac{\partial p_c}{\partial \mu_x} = e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] > 0. \tag{A.3}$$

Defining $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in (7), (A.2), and (A.3) into (A.1), we obtain

$$\begin{aligned} \frac{\partial E[\tilde{x}_c]}{\partial \mu_x} &= \frac{1}{p_c^2}\left[e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right]\left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - Ke^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\right. \\ &\quad \left. - \left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)e^{z_3} N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right] \tag{A.4} \\ &= -\frac{Ke^{z_1 + z_2}}{p_c^2}\left[N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right. \\ &\quad \left. - e^{\kappa\sigma_s\sigma_m} N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]. \tag{A.5} \end{aligned}$$

Because $e^{z_1 + z_2} > 0$ and $p_E^2 > 0$, the sign of the partial derivative with respect to the expected log primitive asset payoff depends on the sign of the term in the outer square parentheses in Equation (A.5). To obtain a negative relation between the expected call option

return and the expected log primitive asset payoff, it must be the case that

$$N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] \quad > \quad e^{\kappa\sigma_s\sigma_m} N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]. \quad \text{(A.6)}$$

Since $N[\cdot] > 0$, the last inequality is equivalent to

$$\frac{N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]} \quad > \quad e^{\kappa\sigma_s\sigma_m} \frac{N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}. \quad \text{(A.7)}$$

We now note that, if $\kappa = 0$, $\kappa\sigma_s\sigma_m = 0$ and Inequality (A.7) becomes an equality. Because only the right-hand side of the inequality depends on the correlation between the log asset payoff and the log stochastic discount factor realization, the inequality would hold if the right-hand side were monotonically decreasing with decreases in κ to minus one.

The natural logarithm of the right-hand side is

$$\kappa\sigma_s\sigma_m + \ln\left(N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) - \ln\left(N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right). \quad \text{(A.8)}$$

Taking the partial derivative of (A.8) with respect to κ and rearranging, we obtain

$$\sigma_s\sigma_m\left[1 - \left(\frac{n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]} - \frac{n\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}\right)/\sigma_x\right] \quad \text{(A.9)}$$

$$= \quad \sigma_s\sigma_m\left[1 - \left(\frac{n\left[-\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right)\right]} - \frac{n\left[-\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{N\left[-\left(-\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right)\right]}\right)/\sigma_x\right], \quad \text{(A.10)}$$

where the last equality follows from the symmetry of the normal distribution. Using the definition for the hazard function of the normally distributed random variable x, which is $H(x) = n(x)/N(-x)$, we are able to rewrite the right-hand side of (A.10) as

$$\sigma_s\sigma_m\left[1 - \frac{H\left[-\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x} + \sigma_x\right] - H\left[-\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{\sigma_x}\right], \quad \text{(A.11)}$$

or, after applying the mean-value theorem

$$\sigma_s\sigma_m\left(1 - H'[x^*]\right), \quad \text{(A.12)}$$

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where $x^* \in (-(\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K)/\sigma_x, -(\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K)/\sigma_x + \sigma_x)$. Freeman and Guermat (2006) show that $H'[x] < 1$, implying that the right-hand side of Inequality (A.7) monotonically decreases with decreases in κ. Setting κ to minus one thus ensures that the term in the outer square parentheses in (A.5) is positive, in turn proving that the expected call option return decreases with the expected log payoff of the primitive asset.

(b) The partial derivative of the expected call option return with respect to the strike price is

$$\frac{\partial E[\tilde{R}_c]}{\partial K} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial K} = \frac{(\partial E[\tilde{X}_c]/\partial K)p_c - (\partial p_c/\partial K)E[\tilde{X}_c]}{p_c^2}. \tag{A.13}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{X}_c]}{\partial K} = -N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] < 0, \tag{A.14}$$

and

$$\frac{\partial p_c}{\partial K} = -e^{\mu_m + \frac{1}{2}\sigma_m^2}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] < 0. \tag{A.15}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (7), (A.14), and (A.15) into (A.13), we obtain:

$$\begin{aligned}
\frac{\partial E[\tilde{R}_c]}{\partial K} &= \frac{1}{p_c^2}\left[-N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\left(e^{z_3}N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] - Ke^{z_2}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\right. \\
&\quad \left. -\left(-e^{z_2}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\left(e^{z_1}N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\right] \tag{A.16} \\
&= \frac{e^{z_1 + z_2}}{p_c^2}\left[N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right]N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] - \right. \\
&\quad \left. e^{\kappa\sigma_s\sigma_m}N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]N\left[\frac{\mu_x + \sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right], \tag{A.17}
\end{aligned}$$

or, alternatively, $-\frac{1}{K}\frac{\partial E[\tilde{R}_c]}{\partial \mu_x}$. Thus, the partial derivative of the expected call option return with respect to the strike price has the opposite sign of the partial derivative of that return with respect to the expected log primitive asset payoff. It thus follows from part (a) of Proposition 1 that the expected call option return and the strike price are negatively related.

(c) Defining moneyness as the difference between the expected log primitive asset payoff and the strike price, $\psi(\mu_x, K) \equiv \mu_x - \ln K$, the total derivative of moneyness with respect to the

expected log primitive asset payoff and the strike price is given by

$$d\psi(\mu_x, K) = \frac{\partial\psi(\mu_x, K)}{\partial\mu_x}d\mu_x + \frac{\partial\psi(\mu_x, K)}{\partial k}dK = d\mu_x - dK/K. \tag{A.18}$$

The total derivative of the expected call option return with respect to the expected log primitive asset payoff and the strike price is given by

$$
\begin{aligned}
dE[\tilde{R}_c] &= \frac{\partial E[\tilde{R}_c]}{\partial\mu_x}d\mu_x + \frac{\partial E[\tilde{R}_c]}{\partial K}dK &\text{(A.19)} \\
&= \frac{\partial E[\tilde{R}_c]}{\partial\mu_x}d\mu_x - \frac{1}{K}\frac{\partial E[\tilde{R}_c]}{\partial\mu_x}dK &\text{(A.20)} \\
&= \frac{\partial E[\tilde{R}_c]}{\partial\mu_x}(d\mu_x - dK/K), &\text{(A.21)}
\end{aligned}
$$

where we use $\frac{\partial E[\tilde{R}_c]}{\partial\mu_x} = -\frac{1}{K}\frac{\partial E[\tilde{R}_c]}{\partial K}$ in the last equality (see the proof of part (b) of Proposition 1). As $\partial E[\tilde{R}_E]/\partial\mu_x < 0$, a higher moneyness decreases the expected call option return.

(d) The partial derivative of the expected call option return, $E[\tilde{R}_c]$ with respect to the systematic volatility of the log primitive asset payoff, σ_s, is given by

$$\frac{\partial E[\tilde{R}_c]}{\partial\sigma_s} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial\sigma_s} = \frac{(\partial E[\tilde{X}_c]/\partial\sigma_s)p_c - (\partial p_c/\partial\sigma_s)E[\tilde{X}_c]}{p_c^2}. \tag{A.22}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{X}_c]}{\partial\sigma_s} = \sigma_s e^{\mu_x + \frac{1}{2}\sigma_x^2}N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x}Kn\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0, \tag{A.23}$$

and

$$\frac{\partial p_c}{\partial\sigma_s} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[(\sigma_s + \kappa\sigma_m)e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]}N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x}Kn\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]. \tag{A.24}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (7), (A.23), and (A.24) into (A.22) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$,

and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain

$$
\begin{aligned}
\frac{\partial E[\tilde{R}_c]}{\partial \sigma_s} &= \frac{1}{p_c^2}\left[\left(\sigma_s e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x} Kn\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\right.\\
&\quad \times \left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - Ke^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) - \\
&\quad \left(\left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right) \cdot \left((\sigma_s + \kappa\sigma_m)e^{z_3}\right.\right.\\
&\quad \left.\left.\left. \times N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_s}{\sigma_x} Ke^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\right)\right].
\end{aligned} \quad (A.25)
$$

The sign of the partial derivative is positive if and only if

$$
\begin{aligned}
&\left(\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + Kn\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right]\right.\\
&\left. -Ke^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) > \left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\\
&\left((\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + Ke^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right), \quad (A.26)
\end{aligned}
$$

and negative if and only if the inequality holds with the opposite inequality sign.

Because $E[\tilde{X}_c] \equiv e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0$ and $p_c \equiv e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] -$ $Ke^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] > 0$, we can divide the inequality first by the right term in the product on the left-hand side of the inequality and then second by the left term in the product on the right-hand side without changing the inequality sign. The result is

$$
\frac{\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + Kn\left[\frac{\mu_x - \ln K}{\sigma_x}\right]}{e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x - \ln K}{\sigma_x}\right]} > \frac{(\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + Kn\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - KN\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]} \quad (A.27)
$$

Dividing the numerator and the denominator of the right-hand side of inequality (A.27) by $e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right]$, the right-hand side becomes

$$
\frac{(\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s}) + e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] / N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right]}{1 - e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] / N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right]}. \quad (A.28)
$$

We now note that $e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]$ can be rewritten as:

$$e^{-(\mu_x+\frac{1}{2}\sigma_x^2+\kappa\sigma_s\sigma_m-\ln K)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\mu_x+\kappa\sigma_s\sigma_m-\ln K)^2}{\sigma_x^2}}$$

$$=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\frac{(\mu_x+\kappa\sigma_s\sigma_m-\ln K)^2+2(\mu_x+\kappa\sigma_s\sigma_m-\ln K)\sigma_x^2+\sigma_x^4}{\sigma_x^2}}=n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\quad\text{(A.29)}$$

Using (A.29), we can write (A.28) as:

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\cdot N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right)/\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\cdot N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\right)}\quad\text{(A.30)}$$

Using the definition for the hazard function of the normally distributed random variable x, $H(x)=n(x)/N(-x)$, we can write the right-hand side of Inequality (A.27) as:

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}.\quad\text{(A.31)}$$

We now note that, if $\kappa=0$, then $\kappa\sigma_s\sigma_m=0$ and Inequality (A.27) is an equality. Because only the right-hand side of the inequality depends on the correlation between the log primitive asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa=-1$ if the right-hand side were monotonically increasing in κ. Conversely, the inequality with the opposite inequality sign would hold if the right-hand side were monotonically decreasing in κ. Defining $\alpha\equiv(\ln K-\mu_x)/\sigma_x$ and $\beta\equiv\frac{\sigma_s\sigma_m}{\sigma_x}$, we write the right-hand side of (A.27) as

$$\frac{(\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s})+H\left[\alpha-\sigma_x-\beta\kappa\right]}{1-H\left[\alpha-\sigma_x-\beta\kappa\right]/H\left[\alpha-\beta\kappa\right]}.\quad\text{(A.32)}$$

The partial derivative of (A.32) with respect to κ is proportional to:[18]

$$\beta\left[(\sigma_x^2/\sigma_s^2)-H'[\alpha-\sigma_x-\beta\kappa]\right]\left[1-H[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa]\right]+\beta\left[\sigma_x+\kappa\frac{\sigma_x\sigma_m}{\sigma_s}+H[\alpha-\sigma_x-\beta\kappa]\right]$$

$$\times\quad\left[-H'[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa]+H[\alpha-\sigma_x-\beta\kappa]H'[\alpha-\beta\kappa]/H[\alpha-\beta\kappa]^2\right].\quad\text{(A.33)}$$

Multiplying by $\frac{1}{\beta}>0$ and $H[\alpha-\beta\kappa]>0$, adding and subtracting α and $\beta\kappa$ inside the third

_____

[18]The partial derivative is (A.33) divided by $(1-H\left[\alpha-\sigma_x-\beta\kappa\right]/H\left[\alpha-\beta\kappa\right])^2$.

main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields:

$$\left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]\left[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\right] + H[\alpha - \sigma_x - \beta\kappa]\left[H[\alpha - \sigma_x - \beta\kappa]\right]$$

$$- (\alpha - \sigma_x - \beta\kappa) + \alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right]\left[-(H[\alpha - \sigma_x - \beta\kappa] - (\alpha - \sigma_x - \beta\kappa)) + (H[\alpha - \beta\kappa] - (\alpha - \beta\kappa))\right] \quad \text{(A.34)}$$

$$= \left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]\left[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\right] + H[\alpha - \sigma_x - \beta\kappa]\left[H[\alpha - \sigma_x - \beta\kappa]\right]$$

$$- (\alpha - \sigma_x - \beta\kappa) + \alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right]\left[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa] - \sigma_x\right]. \quad \text{(A.35)}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives:

$$\left[\frac{\sigma_x^2}{\sigma_s^2} - H'[\alpha - \sigma_x - \beta\kappa]\right]H'[c^*] + H'[\alpha - \sigma_x - \beta\kappa]\left[H'[c^*] - 1\right] + \left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\alpha - \sigma_x - \beta\kappa]\left[H'[c^*] - 1\right]$$

$$= \frac{\sigma_x^2}{\sigma_s^2}H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\alpha - \sigma_x - \beta\kappa]\left[1 - H'[c^*]\right], \quad \text{(A.36)}$$

where $c^* \in (\alpha - \sigma_x - \beta\kappa, \alpha - \beta\kappa)$. Since κ is equal to minus one, a positive (negative) value for the right-hand side of Equality (A.36) implies that the expected call option return increases (decreases) with the systematic volatility of the log primitive asset payoff.

In the derivation of (A.36), we assume that we are able to increase the systematic volatility of the log primitive asset payoff, σ_x, without changing its expected log payoff, μ_x. However, as we argue in footnote 3, this is, strictly speaking, only true if $\mu_m = 0$. If we instead vary σ_x through varying b while accounting for the effect of b on μ_x, Equation (A.36) becomes

$$\frac{\sigma_x^2}{\sigma_s^2}H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s} - \frac{\mu_m\sigma_x}{\sigma_s\sigma_m}\right)H[\alpha - \sigma_x - \beta\kappa]\left[1 - H'[c^*]\right]. \quad \text{(A.37)}$$

Since $\frac{\mu_m\sigma_x}{\sigma_s\sigma_m} > 0$, (A.37) is larger than the right-hand side of Equation (A.36), making it more likely that systematic volatility has a positive effect on the expected call option return.[19]

(e)   The partial derivative of the expected call option return, $E[\tilde{R}_c]$, with respect to the idiosyncratic volatility of the log primitive asset payoff, σ_i, is given by

$$\frac{\partial E[\tilde{R}_c]}{\partial \sigma_i} = \frac{\partial E[\tilde{X}_c]/p_c}{\partial \sigma_i} = \frac{(\partial E[\tilde{X}_c]/\partial\sigma_i)p_c - (\partial p_c/\partial\sigma_i)E[\tilde{X}_c]}{p_c^2}. \quad \text{(A.38)}$$

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[19]The detailed derivation of (A.37) is available from the authors upon request.

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{X}_c]}{\partial \sigma_i} = \sigma_i e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0, \tag{A.39}$$

and

$$\frac{\partial p_c}{\partial \sigma_i} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[\sigma_i e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right]. \tag{A.40}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (7), (A.39), and (A.43) into (A.38) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain

$$\begin{aligned}
\frac{\partial E[\tilde{R}_c]}{\partial \sigma_i} =& \frac{1}{p_c^2}\left[\left(\sigma_i e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\right. \\
& \times \left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) - \\
& \left(\left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right) \cdot \left(\sigma_i e^{z_3}\right.\right. \\
& \left.\left.\left. \times N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + \frac{\sigma_i}{\sigma_x} K e^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right)\right)\right]. \tag{A.41}
\end{aligned}$$

The sign of the partial derivative is negative if and only if

$$\begin{aligned}
&\left(\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right)\left(e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right]\right. \\
&\left. - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right) < \left(e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]\right) \\
&\left(\sigma_x e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + K e^{z_2} n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]\right). \tag{A.42}
\end{aligned}$$

Because $E[\tilde{X}_E] \equiv e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right] > 0$ and $p_c \equiv e^{z_3} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K e^{z_2} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right] > 0$, we can again divide by the right term in the product on the left-hand side of the inequality and the left term in the product on the right-hand side without changing the sign of the inequality. The result of these operations is

$$\frac{\sigma_x e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x - \ln K}{\sigma_x}\right]}{e^{z_1} N\left[\frac{\mu_x + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x - \ln K}{\sigma_x}\right]} < \frac{\sigma_x e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] + K n\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}{e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - K N\left[\frac{\mu_x + \kappa\sigma_s\sigma_m - \ln K}{\sigma_x}\right]}. \tag{A.43}$$

Dividing the numerator and the denominator of the right-hand side of inequality (A.43) by $e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]$, the right-hand side becomes

$$\frac{\sigma_x+e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}. \tag{A.44}$$

Using (A.29), we can write (A.44) as

$$\frac{\sigma_x+n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\cdot N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\right)/\left(n\left[\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]\cdot N\left[\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]\right)} \tag{A.45}$$

Using the definition for the hazard function of the normally distributed random variable x, $H(x)=n(x)/N(-x)$, we can write the right-hand side of Inequality (A.43) as:

$$\frac{\sigma_x+H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]}{1-H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[-\frac{\mu_x+\kappa\sigma_s\sigma_m-\ln K}{\sigma_x}\right]}. \tag{A.46}$$

We again note that, if $\kappa=0$, then $\kappa\sigma_s\sigma_m=0$ and Inequality (A.43) is an equality. Because only the right-hand side of the inequality depends on the correlation between the log primitive asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa=-1$ if the right-hand side were monotonically decreasing in κ. Defining $\alpha\equiv(\ln K-\mu_x)/\sigma_x$ and $\beta\equiv\frac{\sigma_s\sigma_m}{\sigma_x}$, we are able to write the right-hand side of (A.43) as

$$\frac{\sigma_x+H\left[\alpha-\sigma_x-\beta\kappa\right]}{1-H\left[\alpha-\sigma_x-\beta\kappa\right]/H\left[\alpha-\beta\kappa\right]}. \tag{A.47}$$

The partial derivative of (A.47) with respect to κ is proportional to[20]

$$-\beta H'[\alpha-\sigma_x-\beta\kappa]\big[1-H[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa]\big]+\beta\Big[\sigma_x+H[\alpha-\sigma_x-\beta\kappa]\Big]$$
$$\times\quad\Big[-H'[\alpha-\sigma_x-\beta\kappa]/H[\alpha-\beta\kappa]+H[\alpha-\sigma_x-\beta\kappa]H'[\alpha-\beta\kappa]/H[\alpha-\beta\kappa]^2\Big]. \tag{A.48}$$

Multiplying by $\frac{1}{\beta}>0$ and $H[\alpha-\beta\kappa]>0$, adding and subtracting α and $\beta\kappa$ inside the third

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[20]The partial derivative is (A.48) divided by $(1-H\left[\alpha-\sigma_x-\beta\kappa\right]/H\left[\alpha-\beta\kappa\right])^2$.

main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields

$$-H'[\alpha - \sigma_x - \beta\kappa]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\big] + H[\alpha - \sigma_x - \beta\kappa]\Big[H\left[\alpha - \sigma_x - \beta\kappa\right]$$

$$-(\alpha - \sigma_x - \beta\kappa) + \alpha - \beta\kappa\Big]\big[-\big(H[\alpha - \sigma_x - \beta\kappa] - (\alpha - \sigma_x - \beta\kappa)\big) + \big(H[\alpha - \beta\kappa] - (\alpha - \beta\kappa)\big)\big] \quad \text{(A.49)}$$

$$= \quad -H'[\alpha - \sigma_x - \beta\kappa]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa]\big] + H[\alpha - \sigma_x - \beta\kappa]\Big[H\left[\alpha - \sigma_x - \beta\kappa\right]$$

$$-(\alpha - \sigma_x - \beta\kappa) + \alpha - \beta\kappa\Big]\big[H[\alpha - \beta\kappa] - H[\alpha - \sigma_x - \beta\kappa] - \sigma_x\big]. \quad \text{(A.50)}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives

$$-H'[\alpha - \sigma_x - \beta\kappa]H'[c^*] + H'[\alpha - \sigma_x - \beta\kappa]\big[H'[c^*] - 1\big] + (\alpha - \beta\kappa)\,H[\alpha - \sigma_x - \beta\kappa]\big[H'[c^*] - 1\big]$$

$$= \quad -H'[\alpha - \sigma_x - \beta\kappa] - (\alpha - \beta\kappa)\,H[\alpha - \sigma_x - \beta\kappa]\big[1 - H'[c^*]\big], \quad \text{(A.51)}$$

where $c^* \in (\alpha - \sigma_x - \beta\kappa, \alpha - \beta\kappa)$. If $(\alpha - \beta\kappa) \geq 0$, then (A.51) is negative since $H[.] > 0$, $H'[.] > 0$, and $(1 - H'[.]) > 0$. If $(\alpha - \beta\kappa) < 0$, we use $H'[x] = H[x][H[x] - x]$ to write

$$-H'[\alpha - \sigma_x - \beta\kappa] - (\alpha - \beta\kappa)\,H[\alpha - \sigma_x - \beta\kappa]\big[1 - H'[c^*]\big] \quad \text{(A.52)}$$

$$= \quad H[\alpha - \sigma_x - \beta\kappa]\left(-\big(H[\alpha - \sigma_x - \beta\kappa] - (\alpha - \sigma_x - \beta\kappa)\big) - (\alpha - \beta\kappa)\big[1 - H'[c^*]\big]\right), \text{(A.53)}$$

which has the same sign as

$$-\big(H[\alpha - \sigma_x - \beta\kappa] - (\alpha - \sigma_x - \beta\kappa)\big) - (\alpha - \beta\kappa)\big[1 - H'[c^*]\big] \quad \text{(A.54)}$$

$$< \quad -H[\alpha - \sigma_x - \beta\kappa] + \alpha - \sigma_x - \beta\kappa - \alpha + \beta\kappa \quad \text{(A.55)}$$

$$= \quad -H[\alpha - \sigma_x - \beta\kappa] - \sigma_x < 0, \quad \text{(A.56)}$$

where the first inequality follows from Freeman and Guermat's (2006) result that $(1 - H'[.])$ is bounded by zero and one. Thus, the expected call option return unambiguously decreases with the idiosyncratic volatility of the log primitive asset payoff.

Proof of Corollary 1:

The sign of the relation between the expected call option return and the systematic volatility of the log primitive asset payoff is determined by the sign of the sum

$$\frac{\sigma_x^2}{\sigma_s^2}H'[c^*] - H'[\alpha - \sigma_x - \beta\kappa] - \left(\alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\alpha - \sigma_x - \beta\kappa]\big[1 - H'[c^*]\big], \quad \text{(A.57)}$$

with the proof of part (d) of Proposition 1 suggesting that a positive (negative) sign suggests a positive (negative) relation between that return and systematic volatility.

Given that we define a call option's moneyness as $(\mu_x - \ln K)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ is negatively related to moneyness, and ITM (ATM) [OTM] calls have an α value below (equal to) [above] zero. Noticing that $\sigma_x^2 \geq \sigma_s^2$ and $H'[c^*] > H'[\alpha - \sigma_x - \beta\kappa]$ (since $c^* > (\alpha - \sigma_x - \beta\kappa)$ and $H[.]$ is convex), the sum of the first two terms in (A.57) is positive. Thus, if $\left(\alpha + \kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}\right) \leq 0$, as is the case for ITM ($\alpha < 0$), ATM ($\alpha = 0$), and slightly OTM calls ($0 < \alpha < -\kappa\sigma_m \frac{\sigma_i^2}{\sigma_x\sigma_s}$), (A.57) is positive, and the expected returns of such options increase with systematic volatility. That the expected returns of sufficiently OTM can decrease with systematic volatility can be shown using a numerical example (as, e.g., the examples shown in Figures 1 and 2).

Proof of Corollary 2:

The sign of the relation between the expected call option return and the idiosyncratic volatility of the log primitive asset payoff is determined by the sign of the sum

$$-H'[\alpha - \sigma_x - \beta\kappa] - (\alpha - \beta\kappa)\, H[\alpha - \sigma_x - \beta\kappa]\big[1 - H'[c^*]\big], \tag{A.58}$$

with the proof of part (e) of Proposition 1 suggesting that a negative (zero) sign produces a negative (zero) relation between that return and idiosyncratic volatility.

Given that we define a call option's moneyness as $(\mu_x - \ln K)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ is negatively related to moneyness, and ITM (ATM) [OTM] calls have an α value below (equal to) [above] zero. Letting α go to minus infinity, the call option moves perfectly ITM, while $H[.]$ and $H'[.]$ converge to zero, $(1 - H'[.])$ to one, and $-(\alpha - \beta\kappa)$ to plus infinity. Thus, the first term in (A.58) converges to zero. In principle, the second term could converge to any number between zero and plus infinity. However, since the relation between the expected call option return and idiosyncratic volatility is unambiguously negative, it must converge to zero. Thus, the expected returns of perfectly ITM call options are unrelated to idiosyncratic volatility.

Letting α go to plus infinity, the call option moves perfectly OTM, while $H[.]$ converges to plus infinity, $H'[.]$ to one, $(1 - H'[.])$ to zero, and $-(\alpha - \beta\kappa)$ to minus infinity. Thus, the first term in (A.58) converges to minus one, while the second term converges to a number between zero and minus infinity. The implication is that the expected returns of perfectly OTM call options are negatively related to idiosyncratic volatility.

Appendix B: Implied Volatility and Higher Moments

Consistent with Bakshi and Kapadia (2000), Britten-Jones and Neuberger (2000) show that stock i's model-free implied risk-neutral variance at the end of month $t-1$ can be replicated using OTM European call and put options written on the stock

$$\text{Implied Variance}_{i,t-1} = \int_{K=0}^{F} \frac{2P(K)}{K^2}dK + \int_{K=F}^{\infty} \frac{2C(K)}{K^2}dK, \tag{B.1}$$

where $C(.)$ and $P(.)$ are the prices of the call and put options, respectively, K the strike price, and F the stock's forward price, with the derivatives sharing a common maturity date.

Bakshi and Madan (2000) and Bakshi et al. (2003) show that the values of the quadratic (V_{t-1}), cubic (W_{t-1}), and quartic contracts (X_{t-1}) can be replicated using

$$V_{i,t-1} = \int_{S_{i,t-1}}^{\infty} \frac{2(1 - \ln(K/S_{i,t-1}))}{K^2}C(K)dK + \int_{0}^{S_{i,t-1}} \frac{2(1 + \ln(S_{i,t-1}/K))}{K^2}P(K)dK, \tag{B.2}$$

$$\begin{aligned} W_{i,t-1} = & \int_{S_{i,t-1}}^{\infty} \frac{6\ln(K/S_{i,t-1}) - 3(\ln(K/S_{i,t-1}))^2}{K^2}C(K)dK \\ & - \int_{0}^{S_{i,t-1}} \frac{6\ln(S_{i,t-1}/K) + 3(\ln(S_{i,t-1}/K))^2}{K^2}P(K)dK, \end{aligned} \tag{B.3}$$

$$\begin{aligned} X_{i,t-1} = & \int_{S_{i,t-1}}^{\infty} \frac{12(\ln(K/S_{i,t-1}))^2 - 4(\ln(K/S_{t-1}))^3}{K^2}C(K)dK \\ & + \int_{0}^{S_{t-1}} \frac{12(\ln(S_{i,t-1}/K))^2 + 4(\ln(S_{i,t-1}/K))^3}{K^2}P(K)dK, \end{aligned} \tag{B.4}$$

where $S_{i,t-1}$ is stock i's price at the end of month $t-1$.

We approximate the integrals in (B.1), (B.2), (B.3), and (B.4) as follows. We use a cubic regression model of Black and Scholes (1973) implied volatility on strike price and time-to-maturity to create a smoothed implied volatility surface for each stock on the last trading day of each month. We next use the fitted values from that model to calculate 1,000 interpolated implied volatility estimates, with a strike price-to-stock price ratio ranging from 0.0001 to three (in equal increments) and a time-to-maturity of one month. We then plug the interpolated implied volatility estimates into the Black and Scholes (1973) formula to obtain the option prices $C(K)$ and $P(K)$. We finally use the trapezoidal approximation together with $C(K)$ and $P(K)$ to calculate Implied Variance$_{i,t-a}$, $V_{i,t-1}$, $W_{i,t-1}$, and $X_{i,t-1}$.

Similar to others, we use American option data to calculate the integrals. We only calculate the integrals for stocks with at least two traded call options with a delta greater than 0.50 and two traded put options with a delta smaller than –0.50 at each point in time.

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Table 1
Descriptive Statistics
The table shows descriptive statistics on the call options (Panel A) and the volatility estimates (Panel B) used in our tests. The descriptive statistics include the mean, the standard deviation, and the first, 25th, 50th, 75th, and 99th percentiles. The call return is calculated from about seven weeks to maturity (end of month $t-1$) to about three weeks to maturity (end of month t). Moneyness is the stock price-to-strike price ratio; implied volatility the Black and Scholes (1973) implied volatility; and days-to-maturity the number of calendar days until maturity, all calculated at the start of the option holding period. Total volatility is a stock's historical volatility calculated from monthly data over the 60 months prior to the start of the option holding period. SysVol is the volatility of the fitted value from a stock-specific regression of the stock's return on the excess market return (Market) or the excess market return, SMB, HML, and MOM (FFC). IdioVol is the volatility of the residual. The regressions are estimated using monthly data over the 60 months prior to the start of the option holding period.

	Mean	Standard Deviation	Percentiles				
			1	25	50	75	99
Panel A: Call Option Data							
Call Return	0.15	1.70	−0.96	−0.68	−0.29	0.40	6.10
Moneyness	0.99	0.15	0.68	0.92	0.98	1.05	1.43
Implied Volatility	0.51	0.26	0.15	0.33	0.44	0.62	1.37
Days-to-Maturity	50	2	44	50	50	51	53
Panel B: Volatility Estimates							
Total Volatility	0.54	0.31	0.15	0.34	0.47	0.67	1.61
SysVol (Market)	0.25	0.17	0.01	0.13	0.21	0.32	0.80
IdioVol (Market)	0.47	0.29	0.07	0.28	0.41	0.58	1.45
SysVol (FFC)	0.32	0.21	0.06	0.18	0.27	0.39	1.08
IdioVol (FFC)	0.43	0.26	0.04	0.25	0.38	0.54	1.26

Number of Observations: 280,349
Number of Unique Stocks: 5,785

Table 2
Fama-MacBeth Regressions

The table shows the results from FM regressions of the call option return over month t on subsets of moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), and interactions between the moneyness terms and the volatility terms at the end of month $t-1$. Columns (1) to (3) use volatility estimates obtained from the market model; columns (4) to (6) use estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. Panel A shows coefficient estimates, the mean R-squared, and the number of observation per FM regression; Panel B shows the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates						
Money	−2.16	−3.30	−18.74	−2.17	−3.29	−18.58
	[−17.03]	[−12.14]	[−7.39]	[−17.05]	[−12.29]	[−7.62]
SysVol	−0.04	−2.04	−6.61	−0.03	−1.96	−5.52
	[−0.50]	[−3.29]	[−2.11]	[−0.39]	[−3.75]	[−2.17]
IdioVol	−0.04	−0.87	−3.60	−0.04	−0.68	−3.55
	[−0.97]	[−2.22]	[−2.03]	[−0.85]	[−1.40]	[−1.50]
Money × SysVol		1.97	11.28		1.91	9.23
		[3.47]	[2.00]		[3.91]	[1.99]
Money × IdioVol		0.85	6.13		0.66	6.13
		[2.29]	[1.87]		[1.44]	[1.40]
Money^2			7.47			7.40
			[6.49]			[6.72]
Money^2 × SysVol			−4.70			−3.71
			[−1.86]			[−1.78]
Money^2 × IdioVol			−2.64			−2.70
			[−1.76]			[−1.34]
Constant	2.27	3.39	11.33	2.28	3.38	11.24
	[15.50]	[11.62]	[8.14]	[15.53]	[11.76]	[8.35]
Mean R^2	0.05	0.07	0.09	0.05	0.06	0.09
Obs (in 1,000s)	273.10	273.10	273.10	273.10	273.10	273.10

Table 2
Fama-Macbeth Regressions (cont.)

Panel B: Moneyness-Specific Volatility Premia

| | Moneyness | | | | | |
	0.80	0.90	1.00	1.10	1.20	H–L
SysVol (Market)	−0.59	−0.27	−0.03	0.11	0.16	0.75
	[−2.80]	[−2.43]	[−0.47]	[1.73]	[2.20]	[3.07]
IdioVol (Market)	−0.39	−0.22	−0.11	−0.05	−0.04	0.34
	[−3.03]	[−3.79]	[−3.35]	[−1.31]	[−0.90]	[2.20]
SysVol (FFC)	−0.51	−0.22	0.00	0.14	0.21	0.72
	[−2.75]	[−2.38]	[−0.04]	[2.25]	[3.54]	[3.45]
IdioVol (FFC)	−0.38	−0.23	−0.13	−0.08	−0.09	0.29
	[−2.46]	[−3.41]	[−3.14]	[−1.68]	[−1.63]	[1.60]

Table 3

Fama-MacBeth Regressions Controlling for Stock and Option Liquidity

The table shows the results from FM regressions of the call option return over month t on subsets of moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and liquidity control variables at the end of month $t-1$. Columns (1) to (3) use volatility estimates obtained from the market model; columns (4) to (6) use estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. The liquidity control variables are: StockIlliquidity (the Amihud (2002) stock illiquidity proxy); OptionInterest (option open interest-to-stock dollar trading volume); and OptionBid-Ask (option bid-ask spread-to-option price midpoint). Panel A shows coefficient estimates, the mean R-squared, and the total number of observation per FM regression; Panel B shows the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates						
Money	-18.90	-18.74	-18.80	-18.74	-18.67	-18.74
	$[-7.45]$	$[-7.16]$	$[-7.20]$	$[-7.69]$	$[-7.49]$	$[-7.53]$
Money^2	7.56	7.57	7.62	7.49	7.54	7.59
	[6.58]	[6.38]	[6.45]	[6.82]	[6.72]	[6.79]
SysVol	-6.80	-6.12	-6.30	-5.75	-5.26	-5.46
	$[-2.20]$	$[-1.93]$	$[-2.01]$	$[-2.28]$	$[-2.06]$	$[-2.16]$
Money $\times$ SysVol	11.72	10.20	10.61	9.72	8.60	9.02
	[2.10]	[1.77]	[1.87]	[2.12]	[1.83]	[1.95]
Money^2 $\times$ SysVol	-4.95	-4.14	-4.37	-3.97	-3.37	-3.58
	$[-1.98]$	$[-1.60]$	$[-1.72]$	$[-1.93]$	$[-1.57]$	$[-1.70]$
IdioVol	-3.61	-3.80	-3.79	-3.52	-3.77	-3.73
	$[-2.04]$	$[-2.13]$	$[-2.14]$	$[-1.50]$	$[-1.63]$	$[-1.62]$
Money $\times$ IdioVol	6.18	6.71	6.68	6.08	6.79	6.70
	[1.89]	[2.04]	[2.04]	[1.40]	[1.58]	[1.57]
Money^2 $\times$ IdioVol	-2.65	-2.99	-2.97	-2.66	-3.11	-3.06
	$[-1.78]$	$[-1.98]$	$[-1.98]$	$[-1.34]$	$[-1.56]$	$[-1.56]$

Table 3
Fama-MacBeth Regressions Controlling for Stock and Option Liquidity (cont.)

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates (cont.)						
StockIlliquidity	−0.01		0.00	−0.01		0.00
	[−0.88]		[−0.05]	[−0.79]		[0.04]
OptionInterest		−0.22	−0.52		−0.26	−0.56
		[−0.25]	[−0.66]		[−0.30]	[−0.71]
OptionBid-Ask		−0.07	−0.06		−0.07	−0.06
		[−5.84]	[−5.44]		[−5.98]	[−5.56]
Constant	11.30	11.11	11.12	11.23	11.06	11.09
	[8.26]	[7.72]	[7.83]	[8.47]	[8.03]	[8.15]
Mean R^2	0.10	0.10	0.10	0.10	0.09	0.10
Obs (in 1,000s)	273.10	271.42	271.42	273.10	271.42	271.42

Panel B: Moneyness-Specific Volatility Premia

	Moneyness					
	0.80	0.90	1.00	1.10	1.20	H–L
SysVol (Market)	−0.61	−0.29	−0.06	0.08	0.14	0.75
	[−2.89]	[−2.74]	[−0.95]	[1.35]	[2.04]	[3.09]
IdioVol (Market)	−0.34	−0.18	−0.08	−0.03	−0.04	0.30
	[−2.73]	[−3.01]	[−1.94]	[−0.65]	[−0.75]	[1.94]
SysVol (FFC)	−0.54	−0.24	−0.02	0.13	0.21	0.75
	[−2.92]	[−2.71]	[−0.36]	[2.10]	[2.90]	[3.49]
IdioVol (FFC)	−0.33	−0.18	−0.09	−0.07	−0.10	0.23
	[−2.19]	[−2.70]	[−2.21]	[−1.27]	[−1.49]	[1.27]

Table 4
Fama-MacBeth Regressions Controlling for Mispricing Factors

The table shows the results from FM regressions of the call option return over month t on subsets of moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and mispricing control variables at the end of month $t-1$. Columns (1) to (3) use volatility estimates obtained from the market model; columns (4) to (6) use estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. The mispricing control variables are: Total-to-ImpVol (historical volatility calculated from daily data over month $t-1$ scaled by Black-Scholes (1973) implied volatility at the end of that month); ΔTotalVol (the change in historical volatility calculated from daily data over one month from month $t-2$ to month $t-1$); and ΔImpVol (the change in Black-Scholes (1973) implied volatility from the end of month $t-1$ to the end of month t). Panel A shows coefficient estimates, the mean R-squared, and the total number of observation per FM regression; Panel B shows the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates						
Money	−20.24	−21.47	−21.45	−20.16	−21.23	−21.13
	[−7.50]	[−7.33]	[−7.44]	[−7.48]	[−7.18]	[−7.26]
Money^2	8.20	8.71	8.71	8.17	8.61	8.58
	[6.64]	[6.51]	[6.62]	[6.65]	[6.38]	[6.47]
SysVol	−7.40	−8.23	−8.45	−7.78	−8.12	−8.36
	[−2.30]	[−2.44]	[−2.54]	[−2.69]	[−2.67]	[−2.79]
Money × SysVol	12.84	14.36	14.84	13.58	14.11	14.62
	[2.22]	[2.36]	[2.47]	[2.55]	[2.51]	[2.64]
Money^2 × SysVol	−5.47	−6.14	−6.38	−5.80	−5.96	−6.23
	[−2.11]	[−2.25]	[−2.37]	[−2.39]	[−2.31]	[−2.46]
IdioVol	−3.42	−3.65	−3.51	−2.48	−2.68	−2.47
	[−1.63]	[−1.76]	[−1.69]	[−0.87]	[−0.94]	[−0.87]
Money × IdioVol	5.72	6.01	5.74	4.09	4.34	3.94
	[1.47]	[1.55]	[1.48]	[0.77]	[0.82]	[0.74]
Money^2 × IdioVol	−2.43	−2.49	−2.36	−1.75	−1.83	−1.63
	[−1.35]	[−1.39]	[−1.32]	[−0.72]	[−0.74]	[−0.66]

(continued on next page)

Table 4
Fama-MacBeth Regressions Controlling for Mispricing Factors (cont.)

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates (cont.)						
Total-to-ImpVol	0.10		−0.01	0.09		−0.01
	[3.28]		[−0.36]	[3.10]		[−0.48]
ΔTotalVol	−0.18		−0.09	−0.17		−0.08
	[−5.87]		[−2.51]	[−5.33]		[−2.16]
ΔImpVol		0.59	0.60		0.57	0.58
		[7.11]	[7.00]		[6.87]	[6.81]
Constant	12.11	12.83	12.80	12.06	12.69	12.62
	[8.21]	[8.02]	[8.11]	[8.16]	[7.84]	[7.91]
Mean R^2	0.10	0.11	0.11	0.10	0.11	0.11
Obs (in 1,000s)	267.32	262.16	262.13	267.32	262.16	262.13

Panel B: Moneyness-Specific Volatility Premia

	Moneyness					
	0.80	0.90	1.00	1.10	1.20	H–L
SysVol (Market)	−0.66	−0.26	0.01	0.15	0.17	0.83
	[−2.93]	[−2.37]	[0.16]	[2.29]	[2.24]	[3.24]
IdioVol (Market)	−0.43	−0.26	−0.13	−0.06	−0.02	0.41
	[−3.08]	[−4.28]	[−3.58]	[−1.25]	[−0.46]	[2.39]
SysVol (FFC)	−0.65	−0.25	0.03	0.19	0.21	0.86
	[−3.22]	[−2.63]	[0.50]	[2.67]	[2.84]	[3.77]
IdioVol (FFC)	−0.36	−0.24	−0.16	−0.11	−0.09	0.27
	[−2.03]	[−3.40]	[−3.31]	[−1.81]	[−1.35]	[1.31]

Table 5
Fama-MacBeth Regressions Controlling for Risk-Neutral Moments

The table shows the results from FM regressions of the call option return over month t on subsets of moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and risk-neutral moment control variables at the end of month $t-1$. Columns (1) to (3) use volatility estimates obtained from the market model; columns (4) to (6) use estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. The risk-neutral moments are: the variance risk premium (historical volatility calculated using daily data over month $t-1$ scaled by model-free implied volatility at the end of that month); ImpliedSkew (risk-neutral skewness calculated at the end of month $t-1$); and ImpliedKurtosis (risk-neutral kurtosis calculated at the end of month $t-1$). Panel A shows coefficient estimates, the mean R-squared, and the total number of observation per FM regression; Panel B shows the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

| | Volatility Decomposition Model | | | | | |
| | Market Model | | | FFC Model | | |
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates						
Money	−19.80	−20.78	−20.69	−19.69	−20.70	−20.52
	[−7.09]	[−7.42]	[−7.37]	[−7.04]	[−7.42]	[−7.30]
Money^2	8.03	8.48	8.45	7.96	8.43	8.35
	[6.31]	[6.66]	[6.61]	[6.29]	[6.69]	[6.57]
SysVol	−7.11	−7.27	−7.48	−5.57	−5.81	−5.93
	[−1.83]	[−1.87]	[−1.93]	[−1.67]	[−1.75]	[−1.78]
Money × SysVol	13.16	13.54	13.95	10.21	10.78	11.00
	[1.82]	[1.87]	[1.93]	[1.64]	[1.74]	[1.78]
Money^2 × SysVol	−5.94	−6.13	−6.32	−4.52	−4.82	−4.92
	[−1.77]	[−1.83]	[−1.90]	[−1.58]	[−1.69]	[−1.73]
IdioVol	−2.69	−3.15	−2.98	−2.90	−3.36	−3.12
	[−1.23]	[−1.43]	[−1.35]	[−1.06]	[−1.24]	[−1.14]
Money × IdioVol	4.13	5.06	4.75	4.44	5.36	4.90
	[1.01]	[1.23]	[1.15]	[0.87]	[1.06]	[0.96]
Money^2 × IdioVol	−1.61	−2.04	−1.90	−1.76	−2.17	−1.95
	[−0.85]	[−1.07]	[−0.99]	[−0.75]	[−0.93]	[−0.83]

Table 5
Fama-MacBeth Regressions Controlling for Risk-Neutral Moments (cont.)

| | Volatility Decomposition Model | | | | | |
| | Market Model | | | FFC Model | | |
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates (cont.)						
VarRiskPremium	−0.03		−0.02	−0.02		−0.02
	[−0.87]		[−0.73]	[−0.67]		[−0.53]
ImpliedSkew		−0.09	−0.08		−0.08	−0.08
		[−2.30]	[−2.22]		[−2.28]	[−2.21]
ImpliedKurtosis		0.11	0.11		0.11	0.11
		[4.20]	[4.06]		[4.01]	[3.88]
Constant	11.84	11.94	11.89	11.80	11.92	11.83
	[7.75]	[7.72]	[7.67]	[7.66]	[7.71]	[7.59]
Mean R^2	0.10	0.10	0.10	0.09	0.10	0.10
Obs (in 1,000s)	210.21	210.21	210.21	210.21	210.21	210.21

Panel B: Moneyness-Specific Volatility Premia

| | Moneyness | | | | | |
	0.80	0.90	1.00	1.10	1.20	H–L
SysVol (Market)	−0.37	−0.04	0.15	0.22	0.16	0.53
	[−1.39]	[−0.37]	[1.98]	[2.73]	[1.98]	[1.81]
IdioVol (Market)	−0.39	−0.24	−0.12	−0.05	−0.01	0.38
	[−2.46]	[−3.57]	[−3.26]	[−1.05]	[−0.19]	[2.06]
SysVol (FFC)	−0.28	−0.01	0.15	0.22	0.19	0.47
	[−1.26]	[−0.14]	[2.24]	[2.94]	[2.78]	[1.89]
IdioVol (FFC)	−0.44	−0.29	−0.17	−0.09	−0.05	0.40
	[−2.42]	[−3.80]	[−3.63]	[−1.55]	[−0.77]	[1.86]

Table 6
Fama-MacBeth Regressions Controlling for Stock Characteristics
The table shows the results from FM regressions of the call option return over month t on subsets of moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and stock characteristic controls at the end of month $t-1$. Columns (1) to (3) use volatility estimates obtained from the market model; columns (4) to (6) use estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. The characteristics are: size (log market size at the end of June of calendar year t), momentum (the compounded stock return over months $t-12$ to $t-2$); Book-to-Market (the log of the ratio of a stock's book value from the fiscal year end in the prior calendar year to its market size at the end of the prior calendar year); asset growth (the change in the log asset value from the fiscal year end in calendar year $t-2$ to the fiscal year end in calendar year $t-1$); and profitability (the ratio of sales minus COGS, SG&A expenses, and net interest to the book value of equity, all from the fiscal year end in the prior calendar year). Panel A shows coefficient estimates, the mean R-squared, and the total number of observation per FM regression; Panel B shows the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates						
Money	−19.56	−19.49	−19.95	−19.51	−19.23	−19.91
	[−7.08]	[−7.23]	[−7.19]	[−7.35]	[−7.41]	[−7.50]
Money^2	7.90	7.84	8.08	7.87	7.71	8.07
	[6.31]	[6.41]	[6.44]	[6.56]	[6.58]	[6.74]
SysVol	−2.81	−2.33	−2.68	−2.22	−1.64	−2.14
	[−0.83]	[−0.69]	[−0.79]	[−0.74]	[−0.55]	[−0.72]
Money × SysVol	4.93	4.00	4.64	3.96	2.73	3.78
	[0.80]	[0.65]	[0.75]	[0.72]	[0.50]	[0.70]
Money^2 × SysVol	−2.12	−1.61	−1.97	−1.67	−1.00	−1.56
	[−0.76]	[−0.58]	[−0.69]	[−0.67]	[−0.40]	[−0.64]
IdioVol	−5.89	−6.10	−6.32	−6.23	−6.38	−6.62
	[−3.22]	[−3.25]	[−3.38]	[−2.53]	[−2.59]	[−2.69]
Money × IdioVol	10.17	10.50	11.05	10.64	10.92	11.50
	[3.01]	[3.02]	[3.21]	[2.33]	[2.39]	[2.53]
Money^2 × IdioVol	−4.35	−4.51	−4.74	−4.52	−4.69	−4.93
	[−2.80]	[−2.82]	[−3.00]	[−2.15]	[−2.23]	[−2.36]

Table 6
Fama-MacBeth Regressions Controlling for Stock Characteristics (cont.)

	Volatility Decomposition Model					
	Market Model			FFC Model		
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: FM Regression Estimates (cont.)						
Size	0.01		0.02	0.01		0.02
	[1.49]		[1.95]	[1.27]		[1.80]
Momentum	−0.02		−0.02	−0.02		−0.02
	[−1.00]		[−1.25]	[−0.83]		[−1.06]
Book-to-Market		0.02	0.02		0.02	0.02
		[2.12]	[2.34]		[1.96]	[2.19]
AssetGrowth		−0.03	−0.03		−0.03	−0.03
		[−1.24]	[−1.18]		[−1.34]	[−1.22]
Profitability		0.02	0.02		0.02	0.02
		[2.03]	[1.45]		[2.04]	[1.46]
Constant	11.51	11.73	11.66	11.52	11.59	11.66
	[7.73]	[7.91]	[7.84]	[8.02]	[8.10]	[8.16]
Mean R^2	0.10	0.10	0.12	0.10	0.10	0.11
Obs (in 1,000s)	236.30	234.45	233.46	236.30	234.45	233.46

Panel B: Moneyness-Specific Volatility Premia

	Moneyness					
	0.80	0.90	1.00	1.10	1.20	H–L
SysVol (Market)	−0.22	−0.10	0.00	0.05	0.06	0.28
	[−0.96]	[−0.83]	[−0.06]	[0.61]	[0.65]	[1.02]
IdioVol (Market)	−0.51	−0.21	−0.01	0.10	0.11	0.62
	[−3.82]	[−3.21]	[−0.25]	[2.15]	[1.90]	[4.00]
SysVol (FFC)	−0.12	−0.01	0.07	0.13	0.14	0.26
	[−0.56]	[−0.06]	[1.17]	[1.78]	[2.04]	[1.07]
IdioVol (FFC)	−0.58	−0.27	−0.06	0.05	0.07	0.65
	[−3.49]	[−3.51]	[−1.30]	[1.03]	[1.09]	[3.46]

Table 7
Robustness Test: Held-To-Maturity Option Returns
The table shows the results from FM regressions of the held-to-maturity call option return on moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and two control variables at the end of month $t-1$. The held-to-maturity call option return is calculated from the end of month $t-1$ to the maturity of the option in month $t+1$. Panel A considers volatility estimates obtained from the market model; Panel B considers estimates obtained from the FFC model. See the caption of Table 1 for more details about the moneyness and the volatility variables. The control variables are the option bid-ask spread and the change in the underlying stock's volatility. See the captions of Table 3 and 4 for more details about the controls. The table reports the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

			Moneyness			
	0.80	0.90	1.00	1.10	1.20	H–L
Panel A: Market Model Volatility						
SysVol	−0.42	−0.18	0.02	0.18	0.30	0.73
	[−1.14]	[−0.96]	[0.16]	[1.59]	[2.54]	[1.77]
IdioVol	−0.66	−0.39	−0.20	−0.09	−0.06	0.60
	[−3.38]	[−4.02]	[−3.27]	[−1.51]	[−0.85]	[2.69]
Panel B: FFC Model Volatility						
SysVol	−0.23	−0.07	0.08	0.22	0.34	0.57
	[−0.69]	[−0.39]	[0.77]	[1.98]	[3.08]	[1.53]
IdioVol	−0.82	−0.48	−0.26	−0.15	−0.16	0.66
	[−3.35]	[−4.18]	[−3.44]	[−1.95]	[−1.81]	[2.39]

Table 8
Robustness Test: Daily Data Volatility Estimates
The table shows the results from FM regressions of the call option return over month t on moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and two control variables at the end of month $t-1$. See the caption of Table 1 for more details about the moneyness variables. SysVol is the volatility of the fitted value from a stock-specific regression of the stock's return on the excess market return (Panel A) or the excess market return, SMB, HML, and MOM (Panel B), while IdioVol is the volatility of the residual from these regressions. The regressions are estimated using daily data from the month prior to the option holding period. The control variables are the option bid-ask spread and the change in the underlying stock's volatility. See the captions of Table 3 and 4 for more details about the controls. The table reports the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Moneyness					
	0.80	0.90	1.00	1.10	1.20	H–L
Panel A: Market Model Volatility						
SysVol	−0.82	−0.45	−0.17	0.02	0.13	0.95
	[−5.19]	[−5.06]	[−2.81]	[0.39]	[1.73]	[5.01]
IdioVol	−0.67	−0.39	−0.19	−0.07	−0.04	0.64
	[−7.23]	[−7.61]	[−5.02]	[−1.82]	[−0.90]	[5.88]
Panel B: FFC Model Volatility						
SysVol	−0.95	−0.52	−0.19	0.05	0.19	1.14
	[−5.51]	[−5.52]	[−2.98]	[0.79]	[2.84]	[5.58]
IdioVol	−0.43	−0.27	−0.17	−0.11	−0.12	0.31
	[−3.72]	[−4.46]	[−3.47]	[−2.12]	[−2.12]	[2.11]

Table 9
Robustness Test: Transaction Costs
The table shows the results from FM regressions of the bid-ask adjusted call option return on moneyness (Money), moneyness-squared (Money^2), systematic volatility (SysVol), idiosyncratic volatility (IdioVol), interactions between the moneyness terms and the volatility terms, and control variables at the end of month $t-1$. The bid-ask adjusted call option return is calculated as the ratio of the midpoint price minus a fraction S of the bid-ask spread at the end of month t to the midpoint price plus a fraction S of the bid-ask spread at the end of month $t-1$. To be concise, the table only reports results obtained from the FFC model volatility estimates. See the caption of Table 1 for more details about the moneyness and the volatility variables. The control variables are the option bid-ask spread and the change in the underlying stock's volatility from month $t-2$ to month $t-1$. See the captions of Table 3 and 4 for more details about the controls. The table reports the marginal effects of the volatility components at moneyness levels ranging from 0.80 to 1.20. Plain numbers are estimates, while numbers in square parentheses are t-statistics calculated using Newey-West (1987) standard errors.

	Trading Spread	Moneyness					
	(S)	0.80	0.90	1.00	1.10	1.20	H–L
FFC SysVol	0.00	−0.55	−0.25	−0.02	0.13	0.21	0.76
		[−2.97]	[−2.73]	[−0.38]	[2.07]	[2.87]	[3.55]
	0.10	−0.51	−0.23	−0.01	0.13	0.20	0.71
		[−2.89]	[−2.57]	[−0.25]	[2.08]	[2.79]	[3.47]
	0.25	−0.46	−0.20	0.00	0.13	0.19	0.65
		[−2.76]	[−2.35]	[−0.06]	[2.09]	[2.69]	[3.34]
	0.50	−0.39	−0.16	0.01	0.12	0.18	0.57
		[−2.54]	[−1.98]	[0.24]	[2.14]	[2.58]	[3.14]
FFC IdioVol	0.00	−0.33	−0.18	−0.10	−0.07	−0.11	0.22
		[−2.15]	[−2.74]	[−2.46]	[−1.50]	[−1.75]	[1.20]
	0.10	−0.31	−0.17	−0.10	−0.08	−0.11	0.19
		[−2.08]	[−2.73]	[−2.60]	[−1.69]	[−1.90]	[1.09]
	0.25	−0.28	−0.17	−0.10	−0.09	−0.12	0.15
		[−1.97]	[−2.72]	[−2.81]	[−1.99]	[−2.16]	[0.90]
	0.50	−0.23	−0.15	−0.11	−0.11	−0.14	0.09
		[−1.80]	[−2.72]	[−3.17]	[−2.51]	[−2.60]	[0.58]

Table 10
Robustness Test: Time-Series Regressions
The table shows the results from time-series regressions of European S&P 500 call option returns over month t on the index's volatility at the end of month $t-1$. At the end of each month $t-1$, we classify four S&P 500 call options with a maturity date in month $t+1$ as out-of-the-money (OTM), at-the-money (ATM), in-the-money (ITM), and deep in-the-money (DITM). The OTM option is the option whose moneyness is closest to 0.90, the ITM option the option whose moneyness is closest to one, the ITM option the option whose moneyness is closest to 1.20, and the DITM option the option whose moneyness is closest to 1.40. We estimate the index's volatility using monthly data over the 60 months prior to the end of month $t-1$. Plain numbers are estimates. The numbers in square parentheses are t-statistics. The table also reports the R-squared (R^2) and the number of monthly observations (Obs).

	S&P 500 Stock-to-Strike Price Ratio			
	OTM	ATM	ITM	DITM
	0.70–0.90	0.90–1.10	1.10–1.30	1.30–1.50
	(1)	(2)	(3)	(4)
Total Volatility	−3.72	5.23	2.12	1.44
	[−2.39]	[2.55]	[3.01]	[3.01]
Constant	0.16	−0.90	−0.32	−0.22
	[0.62]	[−2.73]	[−2.85]	[−2.81]
R^2	0.04	0.04	0.06	0.06
Obs	155	155	155	155

Internet Appendix: Additional Proofs

This Internet Appendix offers proofs of Proposition 2 and Corollary 3.

Proof of Proposition 2:

(a) The partial derivative of the expected put option return, $E[\tilde{R}_p]$, with respect to the expected log primitive asset payoff, μ_x, is given by

$$\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} = \frac{\partial E[\tilde{x}_p]/p_p}{\partial \mu_x} = \frac{(\partial E[\tilde{x}_p]/\partial \mu_x)p_p - (\partial p_p/\partial \mu_x)E[\tilde{x}_p]}{p_p^2}. \tag{IA.1}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{x}_p]}{\partial \mu_x} = -e^{\mu_x + \frac{1}{2}\sigma_x^2} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right], \tag{IA.2}$$

and

$$\frac{\partial p_p}{\partial \mu_x} = -e^{\mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]. \tag{IA.3}$$

Defining $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ and substituting the numerator and denominator of the term on the right-hand side of the third equality in (9), (IA.2), and (IA.3) into (IA.1), we obtain

$$
\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} &= \frac{1}{p_p^2}\left[-e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\left(e^{z_2}KN\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)\right. \\
&\quad \left. +e^{z_3}N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right] \tag{IA.4} \\
&= \frac{Ke^{z_1+z_2}}{p_p^2}\left(-N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. \\
&\quad \left. +e^{\kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right). \tag{IA.5}
\end{aligned}
$$

Because $e^{z_1+z_2} > 0$ and $p_p^2 > 0$, the sign of the partial derivative with respect to the expected log primitive asset payoff depends on the sign of the term in the outer square parentheses in Equation (IA.5). The proof of part (a) of Proposition 1 shows that

$$N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] > e^{\kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right], \tag{IA.6}$$

implying that the expected put option return decreases with the expected log asset payoff.

(b) The partial derivative of the expected put option return with respect to the strike price is

$$\frac{\partial E[\tilde{R}_p]}{\partial K} = \frac{\partial E[\tilde{x}_p]/p_p}{\partial K} = \frac{(\partial E[\tilde{x}_p]/\partial K)p_p - (\partial p_p/\partial K)E[\tilde{x}_p]}{p_p^2}. \tag{IA.7}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{x}_p]}{\partial K} = N\left[\frac{\ln K - \mu_x}{\sigma_x}\right], \tag{IA.8}$$

and

$$\frac{\partial p_p}{\partial K} = e^{\mu_m + \frac{1}{2}\sigma_m^2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]. \tag{IA.9}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (9), (IA.8), and (IA.9) into (IA.7), we obtain

$$
\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial K} &= \frac{1}{p_p^2}\left[N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]\left(e^{z_2}KN\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)\right.\\
&\quad\left. -e^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right] \tag{IA.10}\\
&= \frac{e^{z_1+z_2}}{p_p^2}\left(N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right.\\
&\quad\left. -e^{\kappa\sigma_s\sigma_m}N\left[\frac{\ln K - \mu_x}{\sigma_x}\right]N\left[\frac{\ln K - \mu_x - \sigma_x^2 - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right) \tag{IA.11}\\
&= \frac{e^{z_1+z_2}}{p_p^2}\left(-\frac{1}{K}\frac{\partial E[\tilde{R}_p]}{\partial\mu_x}\right), \tag{IA.12}
\end{aligned}
$$

which, since $\frac{\partial E[\tilde{R}_p]}{\partial\mu_x} < 0$ (see the proof of part (a) of Proposition 2), implies a positive relation between the expected put option return and the strike price.

(c) The total differential of the expected put option return, $E[\tilde{R}_p]$, with respect to the expected log primitive asset payoff, μ_x, and the strike price, K, is given by

$$
\begin{aligned}
dE[\tilde{R}_p] &= \frac{\partial E[\tilde{R}_p]}{\partial\mu_x}d\mu_x + \frac{\partial E[\tilde{R}_p]}{\partial K}dK \tag{IA.13}\\
&= \frac{\partial E[\tilde{R}_p]}{\partial\mu_x}d\mu_x - \frac{1}{K}\frac{\partial E[\tilde{R}_p]}{\partial\mu_x}dK \tag{IA.14}\\
&= -\frac{\partial E[\tilde{R}_p]}{\partial\mu_x}\left(\frac{1}{K}dK - d\mu_x\right), \tag{IA.15}
\end{aligned}
$$

where we use $\frac{\partial E[\tilde{R}_p]}{\partial K} = -\frac{1}{K}\frac{\partial E[\tilde{R}_p]}{\partial \mu_x}$ in the second equality (see the proof of part (b) of Proposition 2). Given that we define a put option's moneyness as the difference between the log strike price and the expected log primitive asset payoff, $(\ln K - \mu_x)$, the total differential of moneyness with respect to the expected log primitive asset payoff and the strike price is

$$d(\ln K - \mu_x) = \frac{\partial(\ln K - \mu_x)}{\partial \mu_x}d\mu_x + \frac{\partial(\ln K - \mu_x)}{\partial K}dK = \frac{1}{K}dK - d\mu_x, \qquad \text{(IA.16)}$$

which implies that, since $\frac{\partial E[\tilde{R}_p]}{\partial \mu_x} < 0$ (see the proof of part (a) of Proposition 2), the expected put option return increases with moneyness.

(d) The partial derivative of the expected put option return, $E[\tilde{R}_p]$ with respect to the systematic volatility of the log primitive asset payoff, σ_s, is given by

$$\frac{\partial E[\tilde{R}_p]}{\partial \sigma_s} = \frac{\partial E[\tilde{x}_p]/p_p}{\partial \sigma_s} = \frac{(\partial E[\tilde{x}_p]/\partial \sigma_s)p_p - (\partial p_p/\partial \sigma_s)E[\tilde{x}_p]}{p_p^2}. \qquad \text{(IA.17)}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{x}_p]}{\partial \sigma_s} = \frac{\sigma_s}{\sigma_x}Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_s e^{\mu_x + \frac{1}{2}\sigma_x^2}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right], \qquad \text{(IA.18)}$$

and

$$\frac{\partial p_p}{\partial \sigma_s} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[\frac{\sigma_s}{\sigma_x}Kn\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - (\sigma_s + \kappa\sigma_m)e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right]. \qquad \text{(IA.19)}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (9), (IA.18), and (IA.19) into (IA.17) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain

$$
\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial \sigma_s} = {} & \frac{1}{p_p^2}\left[\left(\frac{\sigma_s}{\sigma_x}Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_s e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right. \\
& \times\left(Ke^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2 - \ln K}{\sigma_x}\right]\right) - \\
& \left(\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \times \left(\frac{\sigma_s}{\sigma_x}Ke^{z_2}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \right.\right. \\
& \left.\left.(\sigma_s + \kappa\sigma_m)e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right)\right)\right]. \qquad \text{(IA.20)}
\end{aligned}
$$

The sign of the partial derivative is positive if and only if

$$
\left(Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right] \right)\left(Ke^{z_2} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right.
$$

$$
\left. -e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_x\sigma_m - \sigma_x^2}{\sigma_x}\right] \right) > \left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right] \right)
$$

$$
\left(Ke^{z_2} n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_3} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] \right) \quad \text{(IA.21)}
$$

and negative if and only if the inequality holds with the opposite inequality sign.

We can divide the inequality by the right term in the product on the left-hand side of the inequality and by the left term in the product on the right-hand side without changing the sign of the inequality. The result of these operations is

$$
\frac{Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1} N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]} > \frac{Kn\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]} \quad \text{(IA.22)}
$$

Dividing the numerator and the denominator of the right-hand side of inequality (IA.22) by $e^{z_1 + \kappa\sigma_s\sigma_m} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]$, the right-hand side becomes

$$
\frac{e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)} n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] / N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{e^{-(z_1 + \kappa\sigma_s\sigma_m - \ln K)} N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] / N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] - 1}. \quad \text{(IA.23)}
$$

We now note that $e^{-(z_1 + \kappa\sigma_x\sigma_m - \ln K)} n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]$ can be rewritten as

$$
e^{-(\mu_x + \frac{1}{2}\sigma_x^2 + \kappa\sigma_s\sigma_m - \ln K)} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\ln K - \mu_x - \kappa\sigma_s\sigma_m)^2}{\sigma_x^2}}
$$

$$
= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(\ln K - \mu_x - \kappa\sigma_s\sigma_m)^2 - 2(\ln K - \mu_x - \kappa\sigma_s\sigma_m)\sigma_x^2 + \sigma_x^4}{\sigma_x^2}} = n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] \quad \text{(IA.24)}
$$

Using (IA.24), we can write (IA.23) as

$$
\frac{n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] / N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{\left(n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] \cdot N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] \right) / \left(n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] \cdot N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right] \right) - 1} \quad \text{(IA.25)}
$$

Using the definition for the hazard function of the normally distributed random variable x,

$H(x) = n(x)/N(-x)$, we can write the right-hand side of Inequality (IA.22) as

$$\frac{H\left[\frac{\mu_x + \kappa\sigma_x\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{H\left[\frac{\mu_x + \kappa\sigma_x\sigma_m + \sigma_x^2 - \ln K}{\sigma_x}\right] / H\left[\frac{\mu_x + \kappa\sigma_x\sigma_m - \ln K}{\sigma_x}\right] - 1}. \tag{IA.26}$$

We now note that, if $\kappa = 0$, then $\kappa\sigma_s\sigma_m = 0$ and Inequality (IA.22) is an equality. Because only the right-hand side of the inequality depends on the correlation between the log primitive asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa = -1$ if the right-hand side were monotonically increasing in κ. Conversely, the inequality with the opposite inequality sign would hold if the right-hand side were monotonically decreasing in κ. Defining $\alpha \equiv (\ln K - \mu_x)/\sigma_x$ and $\beta \equiv \frac{\sigma_s\sigma_m}{\sigma_x}$, we write the right-hand side of (IA.22) as

$$\frac{H\left[\sigma_x + \beta\kappa - \alpha\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})}{H\left[\sigma_x + \beta\kappa - \alpha\right] / H\left[\beta\kappa - \alpha\right] - 1}. \tag{IA.27}$$

The partial derivative of (IA.27) with respect to κ is proportional to[21]

$$\beta\left[H'[\sigma_x + \beta\kappa - \alpha] - (\sigma_x^2/\sigma_s^2)\right]\left[H[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - 1\right] + \beta\left[H\left[\sigma_x + \beta\kappa - \alpha\right] - (\sigma_x + \kappa\frac{\sigma_x\sigma_m}{\sigma_s})\right]$$

$$\times \quad \left[-H'[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] + H[\sigma_x + \beta\kappa - \alpha]H'[\beta\kappa - \alpha]/H[\beta\kappa - \alpha]^2\right]. \tag{IA.28}$$

Multiplying by $1/\beta > 0$ and $H[\beta\kappa - \alpha] > 0$, adding and subtracting $\beta\kappa$ and α inside the third main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields

$$\left[H'[\sigma_x + \beta\kappa - \alpha] - \frac{\sigma_x^2}{\sigma_s^2}\right]\left[H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]\right] + H[\sigma_x + \beta\kappa - \alpha]\left[H\left[\sigma_x + \beta\kappa - \alpha\right]\right.$$

$$\left. -(\sigma_x + \beta\kappa - \alpha) - \alpha - \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right]\left[-\left(H[\sigma_x + \beta\kappa - \alpha] - (\sigma_x + \beta\kappa - \alpha)\right) + (H[\beta\kappa - \alpha] - (\beta\kappa - \alpha))\right] \tag{IA.29}$$

$$= \left[H'[\sigma_x + \beta\kappa - \alpha] - \frac{\sigma_x^2}{\sigma_s^2}\right]\left[H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]\right] + H[\sigma_x + \beta\kappa - \alpha]\left[H\left[\sigma_x + \beta\kappa - \alpha\right]\right.$$

$$\left. -(\sigma_x + \beta\kappa - \alpha) - \alpha - \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right]\left[-H[\sigma_x + \beta\kappa - \alpha] + H[\beta\kappa - \alpha] + \sigma_x\right]. \tag{IA.30}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives

$$\left[H'[\sigma_x + \beta\kappa - \alpha] - \frac{\sigma_x^2}{\sigma_x^2}\right]H'[c^*] + H'[\sigma_x + \beta\kappa - \alpha][-H'[c^*] + 1] - \left(\alpha - \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\sigma_x + \beta\kappa - \alpha][-H'[c^*] + 1]$$

$$= -\frac{\sigma_x^2}{\sigma_s^2}H'[c^*] + H'[\sigma_x + \beta\kappa - \alpha] - \left(\alpha + \kappa\sigma_m\frac{\sigma_i^2}{\sigma_x\sigma_s}\right)H[\sigma_x + \beta\kappa - \alpha][1 - H'[c^*]], \tag{IA.31}$$

---

[21]The partial derivative is (IA.28) divided by $(H\left[\sigma_x + \beta\kappa - \alpha\right] / H\left[\beta\kappa - \alpha\right] - 1)^2$.

where $c^* \in (\beta\kappa - \alpha, \sigma_x + \beta\kappa - \alpha)$. Given that κ is equal to minus one, a positive (negative) value of (IA.31) implies that the expected put option return increases (decreases) with the systematic volatility of the log primitive asset payoff.

(e) The partial derivative of the expected put option return, $E[\tilde{R}_p]$ with respect to the idiosyncratic volatility of the log primitive asset payoff, σ_s, is given by

$$\frac{\partial E[\tilde{R}_p]}{\partial \sigma_i} = \frac{\partial E[\tilde{x}_p]/p_p}{\partial \sigma_i} = \frac{(\partial E[\tilde{x}_p]/\partial \sigma_i)p_p - (\partial p_p/\partial \sigma_i)E[\tilde{x}_p]}{p_p^2}. \tag{IA.32}$$

The partial derivatives on the right-hand side of the second equality are given by

$$\frac{\partial E[\tilde{x}_p]}{\partial \sigma_i} = \frac{\sigma_i}{\sigma_x}Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_i e^{\mu_x + \frac{1}{2}\sigma_x^2}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right], \tag{IA.33}$$

and

$$\frac{\partial p_p}{\partial \sigma_i} = e^{\mu_m + \frac{1}{2}\sigma_m^2}\left[\frac{\sigma_i}{\sigma_x}Kn\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \sigma_i e^{\mu_x + \frac{1}{2}[\sigma_x^2 + 2\kappa\sigma_s\sigma_m]}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right]. \tag{IA.34}$$

Substituting the numerator and denominator of the term on the right-hand side of the third equality in (9), (IA.33), and (IA.34) into (IA.32) and using $z_1 \equiv \mu_x + \frac{1}{2}\sigma_x^2$, $z_2 \equiv \mu_m + \frac{1}{2}\sigma_m^2$, and $z_3 \equiv \mu_x + \mu_m + \frac{1}{2}(\sigma_x^2 + 2\kappa\sigma_s\sigma_m + \sigma_m^2)$ to simplify the notation, we obtain

$$\begin{aligned}
\frac{\partial E[\tilde{R}_p]}{\partial \sigma_i} = {} & \frac{1}{p_p^2}\left[\left(\frac{\sigma_i}{\sigma_x}Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_i e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\right. \\
& \times \left(Ke^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2 - \ln K}{\sigma_x}\right]\right) - \\
& \left(\left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \times \left(\frac{\sigma_i}{\sigma_x}Ke^{z_2}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. - \right. \\
& \left.\left.\sigma_i e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right)\right].
\end{aligned} \tag{IA.35}$$

The sign of the partial derivative is positive if and only if

$$\begin{aligned}
& \left(Kn\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - \sigma_x e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right)\left(Ke^{z_2}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right]\right. \\
& \left. - e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_x\sigma_m - \sigma_x^2}{\sigma_x}\right]\right) > \left(KN\left[\frac{\ln K - \mu_x}{\sigma_x}\right] - e^{z_1}N\left[\frac{\ln K - \mu_x - \sigma_x^2}{\sigma_x}\right]\right) \\
& \left(Ke^{z_2}n\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m}{\sigma_x}\right] - \sigma_x e^{z_3}N\left[\frac{\ln K - \mu_x - \kappa\sigma_s\sigma_m - \sigma_x^2}{\sigma_x}\right]\right), \tag{IA.36}
\end{aligned}$$

and negative if and only if the inequality held with the opposite sign.

We can again divide the inequality by the right term in the product on the left-hand side of the inequality and the left term in the product on the right-hand side without changing the sign of the inequality. The result of these operations gives us

$$\frac{Kn\left[\frac{\ln K-\mu_x}{\sigma_x}\right]-\sigma_x e^{z_1}N\left[\frac{\ln K-\mu_x-\sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K-\mu_x}{\sigma_x}\right]-e^{z_1}N\left[\frac{\ln K-\mu_x-\sigma_x^2}{\sigma_x}\right]}>\frac{Kn\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]-\sigma_x e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]}{KN\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]-e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]}. \quad \text{(IA.37)}$$

Dividing the numerator and the denominator of the right-hand side of inequality (IA.37) by $e^{z_1+\kappa\sigma_s\sigma_m}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]$, the right-hand side becomes

$$\frac{e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-\sigma_x}{e^{-(z_1+\kappa\sigma_s\sigma_m-\ln K)}N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-1}. \quad \text{(IA.38)}$$

Using (IA.24), we can re-write (IA.38) as

$$\frac{n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]/N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]-\sigma_x}{\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]\cdot N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]\right)/\left(n\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m}{\sigma_x}\right]\cdot N\left[\frac{\ln K-\mu_x-\kappa\sigma_s\sigma_m-\sigma_x^2}{\sigma_x}\right]\right)-1}\quad \text{(IA.39)}$$

Using the definition for the hazard function of the normally distributed random variable x, $H(x)=n(x)/N(-x)$, we can write the right-hand side of Inequality (IA.37) as

$$\frac{H\left[\frac{\mu_x+\kappa\sigma_x\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]-\sigma_x}{H\left[\frac{\mu_x+\kappa\sigma_x\sigma_m+\sigma_x^2-\ln K}{\sigma_x}\right]/H\left[\frac{\mu_x+\kappa\sigma_x\sigma_m-\ln K}{\sigma_x}\right]-1}. \quad \text{(IA.40)}$$

We now note that, if $\kappa=0$, then $\kappa\sigma_s\sigma_m=0$ and Inequality (IA.37) is an equality. Because only the right-hand side of the inequality depends on the correlation between the log primitive asset payoff and the log stochastic discount factor realization, the inequality would hold for $\kappa=-1$ if the right-hand side were monotonically increasing in κ. Conversely, the inequality with the opposite inequality sign would hold if the right-hand side were monotonically decreasing in κ. Defining $\alpha\equiv(\ln K-\mu_x)/\sigma_x$ and $\beta\equiv\frac{\sigma_s\sigma_m}{\sigma_x}$, we write the right-hand side of (IA.37) as

$$\frac{H\left[\sigma_x+\beta\kappa-\alpha\right]-\sigma_x}{H\left[\sigma_x+\beta\kappa-\alpha\right]/H\left[\beta\kappa-\alpha\right]-1}. \quad \text{(IA.41)}$$

The partial derivative of (IA.41) with respect to κ is proportional to[22]

$$\beta H'[\sigma_x + \beta\kappa - \alpha][H[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - 1] - \beta[H[\sigma_x + \beta\kappa - \alpha] - \sigma_x]$$

$$\times \quad [H'[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - H[\sigma_x + \beta\kappa - \alpha]H'[\beta\kappa - \alpha]/H[\beta\kappa - \alpha]^2]. \tag{IA.42}$$

Multiplying by $1/\beta > 0$ and $H[\beta\kappa - \alpha] > 0$, adding and subtracting $\beta\kappa$ and α inside the third main expression, using the relationship $H'[x] = H[x][H[x] - x]$, and rearranging yields

$$H'[\sigma_x + \beta\kappa - \alpha][H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]] - H[\sigma_x + \beta\kappa - \alpha][H[\sigma_x + \beta\kappa - \alpha]$$

$$-(\sigma_x + \beta\kappa - \alpha) - \alpha + \beta\kappa][(H[\sigma_x + \beta\kappa - \alpha] - (\sigma_x + \beta\kappa - \alpha)) - (H[\beta\kappa - \alpha] - (\beta\kappa - \alpha))] \tag{IA.43}$$

$$= \quad H'[\sigma_x + \beta\kappa - \alpha][H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha]] - H[\sigma_x + \beta\kappa - \alpha][H[\sigma_x + \beta\kappa - \alpha]$$

$$-(\sigma_x + \beta\kappa - \alpha) - \alpha + \beta\kappa][H[\sigma_x + \beta\kappa - \alpha] - H[\beta\kappa - \alpha] - \sigma_x]. \tag{IA.44}$$

Dividing by $\sigma_x > 0$, using the mean-value theorem, and $H'[x] = H[x][H[x] - x]$ gives

$$H'[\sigma_x + \beta\kappa - \alpha]H'[c^*] - H'[\sigma_x + \beta\kappa - \alpha][H'[c^*] - 1] + (\beta\kappa - \alpha)H[\sigma_x + \beta\kappa - \alpha][1 - H'[c^*]]$$

$$= \quad H'[\sigma_x + \beta\kappa - \alpha] - (\alpha - \beta\kappa)H[\sigma_x + \beta\kappa - \alpha][1 - H'[c^*]], \tag{IA.45}$$

where $c^* \in (\beta\kappa - \alpha, \sigma_x + \beta\kappa - \alpha)$. Since κ is equal to minus one, a positive (negative) value for the right-hand side of Equation (IA.45) implies that the expected put option return increases (decreases) with the idiosyncratic volatility of the log primitive asset payoff.

Proof of Corollary 3:

The sign of the relation between the expected put option return and the idiosyncratic volatility of the log primitive asset payoff is determined by the sign of

$$H'[\sigma_x + \beta\kappa - \alpha] - (\alpha - \beta\kappa)H[\sigma_x + \beta\kappa - \alpha][1 - H'[c^*]], \tag{IA.46}$$

with the proof of part (e) of Proposition 2 showing that the relation is positive (negative) [zero] if the sum in (IA.46) is positive (negative) [zero].

Given that we define a put option's moneyness as $(\ln K - \mu_x)$, $\alpha \equiv \frac{\ln K - \mu_x}{\sigma_x}$ increases with moneyness, and ITM (ATM) [OTM] calls have an α value above (equal to) [below] zero. Since $H'[.] > 0$, sufficiently OTM $((\alpha - \beta\kappa) < 0)$ put options thus produce a positive relation

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[22]The partial derivative is (IA.42) divided by $(H[\sigma_x + \beta\kappa - \alpha]/H[\beta\kappa - \alpha] - 1)^2$.

between the expected put option return and idiosyncratic volatility. Numerical examples reveal that ITM and ATM puts can produce a weakly negative effect.