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Grey GERT Network Model of Equipment Lifetime Evaluation Based on Small Samples

Xiaoyu Yang[1,2*], Zhigeng Fang[1], Liangyan Tao[1]

1.College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, P.R. China
2.Centre for Computational Intelligence, De Montfort University, The Gateway, Leicester, LE1 9BH, UK

Abstract

The reliability evaluation of high reliability and long life equipment is widely concerned in recent decades. Enough failure samples of these kinds of equipment are not easy or economic to obtain in reliability test, in addition, experience information is sometimes inaccurate or uncertainty. To overcome the deficiency in traditional method which requires large numbers of samples, a quantitative analysis model of equipment reliability evaluation is proposed in this paper in view of the few failure data of equipment life tests. GERT network is introduced to describe the kinds of working states of the equipment system and random process of equipment state transition choice after stress impact of single component. Considering the uncertainty and inaccuracy of the statistical data and experience information, the parameters of GERT network are represented by interval grey number. The system equivalent transfer function could be obtained by GERT matrix solving algorithm, and the reliability evaluation of equipment system can be realized. The case study results show that the equipment reliability evaluation Grey-GERT model based on small samples would save much time with little accuracy losing. Besides, the study provides a new thinking for reliability accelerated life test.

Keywords: Systems Reliability Evaluation; GERT Network; Interval Grey Number; Small Sample

1. Introduction

Equipment reliability evaluation relies on the data obtained from reliability test, which are mainly failure data[1]. In aerospace, military, power industry etc. area, equipment is designed with high reliability and long lifetime, results in fewer failure samples for reliability analysis, even using accelerated life test method[2]. For some large expensive complex equipment, the cost of obtaining a large number of life test data is fairly high, or experiment of the whole apparatus is rather difficult. System reliability modeling method is widely used for reliability evaluation of these kinds of equipment, such as the pyramid model, reliability block diagram[3,4], fault tree analysis and Bayesian networks. Reliability or life distribution of the components or

[*] *Corresponding Author: Xiaoyu Yang, College of Economics and Management , Nanjing University of Aeronautics and Astronautics, Nanjing 211106, P.R. China; Email: yangxy@nuaa.edu.cn*

subsystems should be known for these models' analysis. Traditional reliability analyses of components are mainly statistical methods, which need large samples of failure data, related studies have achieved abundant research results in this field, including parameter estimation, life estimation and fitting test and so on.

In view of the new transformation and the shortcoming of traditional reliability test, new methods for reliability tests and new technologies for reliability evaluation are objectively required. On the one hand, reliability enhancement testing (RET) and highly accelerated life test (HALT) have been widely used in engineering test[5,6]. By applying single or integrated environmental stress, higher than the level of its actual working environment stress, to the test products, potential defects of the products may be motivated quickly, and more failure data could be obtained. Scholars had done extensive research work on reliability enhancement test efficiency, test theory, statistical model, and data analysis. Gregg K. Hobbs[7] did lots of research on reliability enhancement test efficiency and test theory and technology. Wayne Nelson[8] focused on the statistical model, testing profile and data acquisition and analysis. Mike Silverman, David Rahe and H. Anthony Chan[9] had made a great deal of effort in the technology and application of intensifying test. Shisong Mao and Lingling Wang[10] summarized some basic methods and specific operation of accelerated life test, providing a good tool for engineering and technical personnel.

On the other hand, the methods of using historical and expertise information to enlarging the sample size and shorten the confidence interval or improve the accuracy of evaluation results were extensively studied, one of the most representative theories is Bayes (Bayesian) estimation methods. VP Savchuk and HF Martz[11] used maximum entropy and maximum posterior risk criteria to fuse the multiple sources of prior information, WU Linli and PAN Guang[12] put forward a method to get the reliability test prior distribution of small sample complex products based on analytic hierarchy process (AHP), ZY Mao[13] established a Bayesian reliability analysis model based on the distribution chart to realize the multiple information fusion. But the distribution form of Bayes prior information is usually chosen with individual bias which makes the method controversial. A digital simulation technology named Bootstrap got the attention of scholars, which completely depend on the sample data. Xiaoyang WANG[14] proposed a bootstrap-based data processing method to facilitate the accelerated life test to improve the precision. JIA Zhanqiang et al[15] put forward a modified Bootstrap and Bayesian Bootstrap sample expansion method to evaluate the real-time performance reliability of small-sample product. WAN Rangxin[16] proposed a performance reliability assessment method based on Bayesian Bootstrap simulation, expanded small samples to large samples by randomly weighted digital simulation and realized parameter estimation.

In the case of incomplete or uncertain information, the road to refined models is not going to work[17]. Instead of expanding the sample size by engineering or simulation means, the study of information mining of the small data and the exploration of equipment failure mechanism are still insufficient. Regarding those history information and experience information as accurate data and ignoring the inaccuracy and uncertainty of the data are actually very risky. Grey system provides solutions to those practical problems with incomplete information or inaccurate data, and the interval grey number provides a promising way to handle small example and poor information problems[18,19]. Many researchers have studied algorithm rules of interval grey number combined with prediction models[20,21], decision making models[22,23], incidence models[24] and network models. Aiqing ruan[25] defined several types of Grey GERT network and showed the its practical meaning, Sifeng

Liu[26] constructed a new G-GERT network model and applied it to macroeconomics economy system, Mingli Hu[27] studied a novel GERT model with grey probability, and applied the model in the rocket launch and restore problem.

Considering the inaccuracy and uncertainty of reliability assessment with small samples for those high reliability and long life pieces of equipment, this paper put forward a reliability evaluation model of grey random network on the basis of equipment failure mechanism. The model describes the equipment failure process under stress impact and realizes equipment reliability evaluation of small samples.

2. The construction thinking of random network model of reliability evaluation

The operational reliability of a piece of equipment is decided by its inherent reliability and the environmental stress impact. The equipment working in the harsh conditions such as high temperature, high pressure, high electromagnetic radiation, seawater corrosion and so on is more likely to break down. Equipment inherent reliability is a kind of ability of resistance the stress impact, so the equipment with higher reliability failed at a lower probability every time after the impact. Theoretically, if we obtain the stress arrival time and failure probability after the stress impact, we can evaluate the equipment reliability. In order to motivate equipment defects quickly, reliability enhancement testing will take higher stress level than its actual working condition, so most of the test will be done in laboratories, stress impact time and strength could be artificial controlled.

During the reliability enhancement testing, integrated environmental stress consists of temperature, humidity and vibration. For stress impact controlled accelerated reliability test, life estimation can be done simply by measuring the failure probability of every impact. If the arrival of environmental impact of the equipment failure is a random process, the impact will be considered a "particle stream", the arrived particle number N(t) in a certain time is subject to Poisson distribution.

Set N(t) for arrived impact number at the moment t, then N (t) has the following properties.

(1) $N(t)\geq 0$.

(2) N(t) is an integer.

(3) N(t) is a monotone nondecreasing function.

(4) When $s<t$, N(t)-N(s) presents the arrived impact number in interval (s, t).

According to the definition of counting process, the impact arrival process $\{N(t), t\geq 0\}$ is a counting process.

The impact arrival process in non-overlap time interval is also a process with independent increments, that is to say for $t_1 < t_2 \leq t_3 < t_4$, $[t_1, t_2)$ and $(t_3, t_4]$ are two non-overlap time intervals, failure times in $[t_1, t_2)$ is $N(t_2)-N(t_1)$, in $(t_3, t_4]$ is $N(t_4)-N(t_3)$, $N(t_2)-N(t_1)$ and $N(t_4)-N(t_3)$ are mutually statistics independent. In engineering practice, the impact arrival process should also meet the common assumption, namely the possibility of two or more impact arrival at the same time is negligible.

Observing the environment impact which affected the equipment reliability is not that easy. But failure data is easier to get, such as when and how many times the failure occurred. The occurrence of failure can be viewed as the external expression of the equipment state transition after repeated stress impact, therefore the observations of failure can be used to substitute the observations of stress impact, and the average lifetime evaluation could be done. Its schematic diagram is as follows.

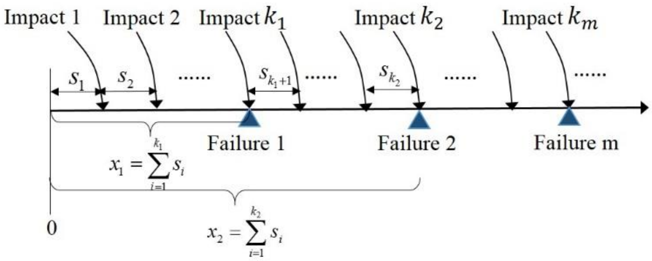

Figure 1 The schematic diagram of equipment failure under impact stress

Above describes the equipment working states from operation to failure under impact stress, at the beginning of the test, all equipment are in operation state, after each shock, the equipment choose at a certain probability maintaining its operation state or transferring to failure state immediately. If the equipment maintains its operation state, it will function well until next impact arrival, then the equipment should make another choice until it breaks down at the moment of impact arrival number k . Do reliability test to a batch of equipment, environmental stress of each equipment can be assumed as same, and lifetime of each equipment is independent identically distributed random variables. Each equipment makes an independent choice when impacts arrive. When failure is observed, actually you observe the results of several impacts. The equipment response of impact stress could be measured approximately by a number of failure data, thus reliability evaluation could be done after calculating the retention time of operation state and corresponding state transition probability. For accelerated reliability test with controlled impact stress, lifetime evaluation can be obtained simply by measuring the equipment's transition probability under each shock.

In order to research conveniently, we need to transfer the above process to a mathematical model. Graphical Evaluation and Review Technique (GERT) model is known for its convenience and flexibility, so here we convert the above process to a GERT network model, called lifetime evaluation Grey GERT (G-GERT) model because the network is used for lifetime evaluation in reliability test and its parameters are represented by interval grey numbers. Basic model building steps are as follows:

Step 1 Construct a RBD of the equipment based on its system structure, than convert it to lifetime evaluation G-GERT model according to the characteristics of the equipment structure or the problem concerned.

Step 2 Determine the distribution function of the components according to expert experience or historical statistics data. Observe the state of field test components, and calculate operation time and state transition probability.

Step 3 Determine the parameter of lifetime evaluation G-GERT model, write the gain matrix of system signal flow graph on the basis of network structure.

Step 4 Reduce and transform the gain matrix of system signal flow graph on the basis of problem objective, calculate the equivalent gain matrix of flow graph.

Step 5 Find the equivalent transfer function of system, and evaluate the lifetime and reliability of the equipment.

3. Construction of Lifetime Evaluation G-GERT Network Model

for Equipment Reliability Test

3.1 Basic element of lifetime evaluation G-GERT network

Equipment failure process can be regarded as a random process, which could be described with Grey-GERT network model, the elementary unit of G-GERT model is defined as follows.

Definition 1 G-GERT network model for equipment reliability evaluation consists of nodes, arrows and state transition flows: the node represents a certain state of the equipment, the arrows are used to describe the transfer relationships between different states, the flows are used for quantitative description of the transfer relationships, and the elementary unit is shown in Figure 2.

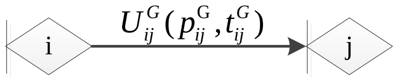

Figure 2 The schematic diagram of elementary unit in G-GERT

In the diagram, U_{ij}^{G} represents the state transition flow from state i to state j of the of the equipment, p_{ij}^{G} is state transition probability; t_{ij}^{G} stands for the time required for state transition, $p_{ij}^{G} \in \left[p_{ij}^{L}, p_{ij}^{U} \right]$ and $t_{ij}^{G} \in \left[t_{ij}^{L}, t_{ij}^{U} \right]$ are interval grey numbers.

Combining the theory of graphical evaluation and review technique, the moment generating function definition of directed arc (ij) in equipment state transition process is given similarly.

Definition 2 In equipment lifetime evaluation G-GERT network model based on failure choice mechanism, assume the time interval t_{ij}^{G} of stress impact arrival according to some probability distribution $f(x)$, then the moment generating function of state transition directed arc (ij) is $M_{ij}(s) = \int_{-\infty}^{\infty} e^{st_{ij}^{G}} f(t_{ij}^{G}) dt_{ij}$.

In order to guarantee the universal of the research, the probability distribution $f(x)$ here can be any form. In reliability evaluation G-GERT model, let p_{ij}^{G} be the probability of being executed of arc (ij) when equipment in state i, thus the equivalent transfer function of arc (ij) is $W_{ij}(s) = p_{ij}^{G} \cdot M_{ij}(s)$.

In the analysis of the lifetime evaluation G-GERT network in this paper, parameters about time from source node to terminal node need to know, for instance, average lifetime of equipment. According to the basic properties of moment generating function, these parameters are easy to get.

Let $W_E(s)$ be the equivalent transfer function of the system, the system equivalent probability p_E^{G} equals the value of equivalent transfer function $W_E(s)$ at $s=0$, namely $p_E^{G} = W_E(s)\big|_{s=0}$, equivalent moment generating function $M_E(s) = \frac{W_E(s)}{p_E^{G}} = \frac{W_E(s)}{W_E(0)}$.

According to the properties of moment generating function, the value of n-degree derivation of moment generating function at $s=0$ is the origin moment of random variable, so the average lifetime of equipment $E(t)$ and the variance of lifetime $V(t)$ are as follows.

$$E(t)=\frac{\partial M_E(s)}{\partial s}\bigg|_{s=0}=\frac{\partial}{\partial s}\left[\frac{W_E(s)}{W_E(0)}\right]\bigg|_{s=0} \tag{1}$$

$$E(t^2)=\frac{\partial^2 M_E(s)}{\partial s^2}\bigg|_{s=0}=\frac{\partial^2}{\partial s^2}\left[\frac{W_E(s)}{W_E(0)}\right]\bigg|_{s=0} \tag{2}$$

$$\begin{aligned}V(t)&=E(t^2)-\left(E(t)\right)^2\\&=\frac{\partial^2}{\partial s^2}\left[\frac{W_E(s)}{W_E(0)}\right]\bigg|_{s=0}-\left\{\frac{\partial}{\partial s}\left[\frac{W_E(s)}{W_E(0)}\right]\bigg|_{s=0}\right\}^2\end{aligned} \tag{3}$$

3.2 Three basic types of lifetime evaluation G-GERT network

In engineering practice, there are variable working states of the equipment and its components, including perfect condition, degradation working state and break down completely. These system working states can be characterized by G-GERT network nodes. For system components in reality, generally only consider two kinds of state, the normal working state and failure working state, for the whole system, all the working states should be considered. There are three basic types of G-GERT network based on the system connection structure.

Definition 3 The converted lifetime evaluation G-GERT network structure for the equipment with two states (operation or failure) in reliability (enhancement) test is shown in Figure 3.

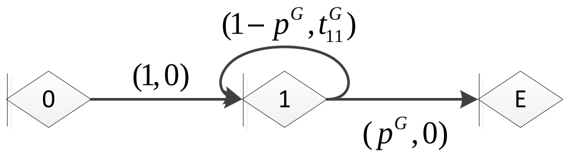

Figure 3 The structure of lifetime evaluation G-GERT network for equipment with two states

The G-GERT network above describes the operational process of equipment from operation state to failure state under impact stress. Node 0 is a virtual node, indicate the beginning of the test, Node 1 represents the perfect condition of equipment, Node E represents the equipment wholly breaking down. If the equipment does not fail when stress impact arrives, it is represented by arrow (11), otherwise, it is arrow (1E). t_{11}^G is operation time for the equipment keeping its perfect condition, $1-p^G$ is the probability of no failure occurred when stress impact arrives. Arrow $(1E)$ stands for the situation that equipment breaks down immediately after impact arrival, the failure probability is p^G. If time interval t_{11}^G of stress impact arrival obeys index distribution, the GERT network in Figure 3 describes a typical homogeneous Poisson process, by changing the distribution time interval t_{11}^{G}, the model can represent varieties of random process.

Definition 4 The lifetime evaluation G-GERT network structure for a series structural system of two components is shown in Figure 4, if the working states of the two components are mutual independence.

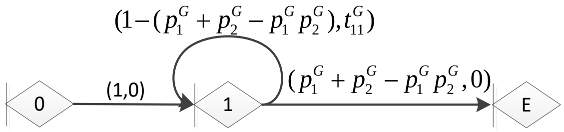

Figure 4 The structure of lifetime evaluation G-GERT network for series structural system of two components

The system show in Figure 4 is working in perfect condition for t_{11}^{G}, p_1^G and p_2^G are failure probabilities of component 1 and component 2 under stress impact. For a series structural system of *N* components, if the working states of each component are mutual independence, the structure of lifetime evaluation G-GERT network is similar to Figure 4.

Definition 5 The lifetime evaluation G-GERT network structure for a parallel structural system of two components is shown in Figure 5, if the working states of the two components are mutual independence.

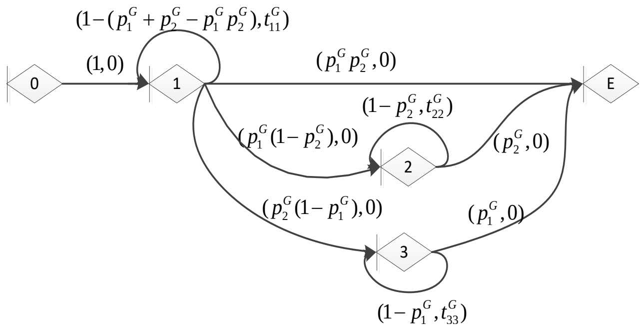

Figure 5 The structure of lifetime evaluation G-GERT network for parallel structural system of two components

The symbolic meaning in Figure 5 is the same as in Figure 4. For a parallel structural system of *N* components, if the working states of each component are mutual independence, the structure of lifetime evaluation G-GERT network is similar to Figure 5, and there are $2^N - 1$ arrows from node 1 to node E, node 2and 3 represent the degradation working states of the system. In a lifetime evaluation G-GERT network, relevant parameters of the system from a certain degradation working state to complete failure can also be obtained.

3.3 Matrix solving algorithm of lifetime evaluation G-GERT network

Using matrix solving algorithm to replace traditional moment generating function make the lifetime evaluation G-GERT model more easier to compute, and according to the network structure, some properties can be derived.

Theorem 1 For a lifetime evaluation G-GERT network with single component (shown in Figure 3), if the expectation time of the equipment in operation state is

$E(t_{11})$, and $0 < p \le 1$ is a constant, then the expectation of equipment life is

$$E(t_E) = \frac{1-p}{p} E(t_{11}) \cdot \qquad (4)$$

Proof: Let $f(t_{11})$ be the probability distribution density function of the time of equipment in operation state, the equivalent moment generating function of the system is

$$M_E = W_E = \frac{p}{1-(1-p)M_{11}},$$

the expectation of equipment lifetime is

$$E(t) = \left.\frac{\partial M_E(s)}{\partial s}\right|_{s=0} = \frac{\partial}{\partial s}\left[\frac{p}{1-(1-p)M_{11}(s)}\right]\Bigg|_{s=0},$$

$$= \left\{\frac{p(1-p)}{[1-(1-p)M_{11}(s)]^2}\cdot\frac{\partial}{\partial s}M_{11}(s)\right\}\Bigg|_{s=0}$$

And for

$$\left[\frac{\partial}{\partial s}M_{11}(s)\right]_{s=0} = \left[\frac{\partial}{\partial s}\int_0^{\infty} e^{st_{11}} f(t_{11})dt_{11}\right]_{s=0} = \left[\int_0^{\infty} t_{11}\cdot e^{st_{11}} f(t_{11})dt_{11}\right]_{s=0}$$

$$= \int_0^{\infty} t_{11} f(t_{11})dt_{11} = E(t_{11})$$

$$M_{11}(s)\big|_{s=0} = \left[\int_0^{\infty} e^{st_{11}} f(t_{11})dt_{11}\right]_{s=0} = \int_0^{\infty} f(t_{11})dt_{11} = 1$$

So

$$E(t) = \left\{\frac{p(1-p)}{[1-(1-p)M_{11}(s)]^2}\cdot\frac{\partial}{\partial s}M_{11}(s)\right\}\Bigg|_{s=0}.$$

$$= \frac{p(1-p)}{[1-(1-p)]^2}E(t_{11}) = \frac{1-p}{p}E(t_{11})$$

Characteristic analysis of G-GERT network self-loops is one of the most difficult parts of solving the model. The network size becomes larger when the network structure is more complicated, easy to make mistakes or omissions. However, matrix expression provides a new solution to GERT network.

Definition 6 Matrix A_s^G is defined as the signal flow graph gain matrix of G-GERT network, if a_{ji}^G represents the transfer function w_{ij}^G of G-GERT from node i to node j.

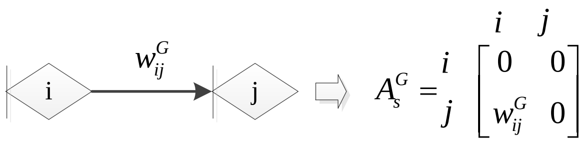

Figure 6 Matrix conversion schematic of G-GERT basic structure

Definition 7 Assume A_s^G is the signal flow graph gain matrix of G-GERT network, then its flow graph gain matrix is $A^G = A_s^G - I = \begin{bmatrix} -1 & 0 \\ w_{ij}^G & -1 \end{bmatrix}$.

Theorem 2 The equivalent transfer function from source node u to any node x_i is $W_E^G = \frac{x_i}{u} = \frac{\sum_{j=1}^{n} A_{ji}^G b_j^G}{\left|A^G\right|}$ in a lifetime evaluation G-GERT network. $\left|A^G\right|$ is the determinant of matrix A^G, A_{ji}^G is algebraic complement of matrix element a_{ji}^G, b_j^G is the gain value of source point u to its operational node j.

Theorem 3 Assume that the ith diagonal element a_{ii}^G in gain matrix A_s^G is nonzero, namely the ith node in G-GERT network has a self-loop, the new signal flow graph gain matrix A_s' obtained by the following matrix transformation is equal to the operation of eliminating the self-loop of the ith node.

$$\begin{cases} a_{ii}' = 0 \\ a_{ki}' = \dfrac{a_{ki}^G}{1-a_{ii}^G}, k = 1,2,\cdots,n, k \neq i \\ a_{kj}' = a_{kj}, k, j = 1,2,\cdots n, and\, k \neq i, j \neq i \end{cases} \tag{5}$$

Theorem 4 Assume that the k th node in G-GERT network do not have a self-loop and the gain matrix of this G-GERT network is $A_{n\times n}$, if the element of matrix A' is $a_{ij}' = a_{ik} \times a_{kj} + a_{ij}$, $k = 1,2,\cdots,n,\ , i \neq k, j \neq k$, cancel the k th row and the k th column in matrix $A_{n\times n}$ at the same time, then the new gain matrix A' obtained by the above matrix transformation is equal to the operation of elimination the k th node in G-GERT network (signal flow graph).Detailed proof of these Theorems refer to reference [28-30].

4. Case study

A type of equipment consists of 8 reliability units, and its reliability block diagram is shown in Figure 7. Unit A is a newly upgraded equipment with reliability test data deficiencies. According to the historical reliability statistics, the life distribution of Unit A is index distribution. Conduct a reliability test with no replacements of 15 new upgraded unit A, the failure data of the complete samples is shown in Table 1, but only 5 failure data was observed within the stipulated time.

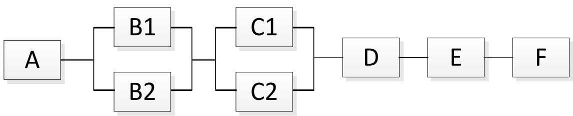

Figure 7 The RBD of the equipment system

Table 1 Life test data of Unit A (unit: h)

770	1450	1589	1775	2257	2325	2829	3398
4835	6405	6540	7292	7509	7517	13934	

Step 1 Construct the system lifetime evaluation G-GERT network based on its RBD, shown in Figure 8.

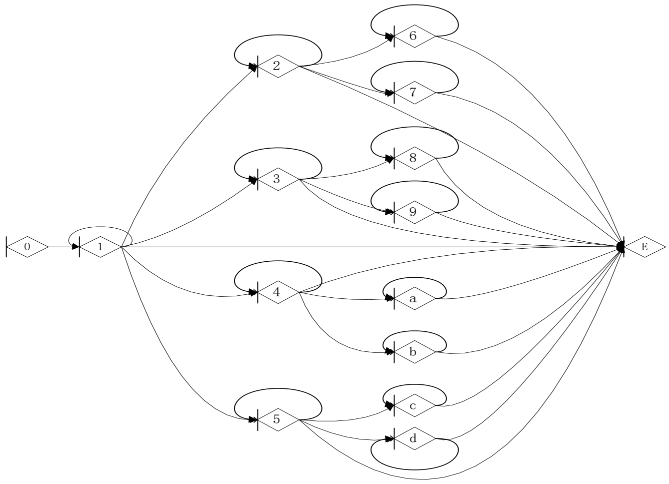

Figure 8 The system lifetime evaluation G-GERT network of the equipment

Step 2 For the case that lacking of the experimental data of unit A, statistical model of lifetime evaluation does not work well， G-GERT network in Figure 3 can be used to estimate the average lifetime of Unit A. The expectation of Unit A lifetime is

$$E(t_A) = \frac{1-p_r}{p_r} \cdot \frac{1}{r} t_r \, , \tag{6}$$

r is the number of observed failure samples, t_r is the failure moment of the r th equipment.

The transition probability value of the Type-Ⅱ censoring life test of exponent distribution is shown in Table 2 by experimentally measurement. The number in first row is the total number of test samples, n is the number of observed failure data.

Table 2 The transition probability value table of the Type-Ⅱ censoring life test of exponent distribution

n	15	20	25	30
2	[0.060,0.067]	[0.044,0.050]	[0.036,0.042]	[0.030,0.035]
3	[0.062,0.069]	[0.046,0.052]	[0.038,0.042]	[0.031,0.035]
4	[0.065,0.071]	[0.049,0.053]	[0.039,0.042]	[0.031,0.035]
5	[0.070,0.074]	[0.050,0.054]	[0.040,0.043]	[0.032,0.036]

By table lookup or simulation experiment, the state transition probability p_r with 15 test samples are obtained, the calculation results are shown in Table 3, θ is the average lifetime of the complete samples.

Table 3 Comparison of life evaluation results of unit A

r	p_r	MLE		Proposed method		θ
		result	error	result	error range	
2	[0.060,0.067]	10535	124.39%	[10032,11399]	113.67%-142.78%	4695
3	[0.062,0.069]	7625.67	62.42%	[7149,7990]	52.26%-70.17%	
4	[0.065,0.071]	6277.25	33.70%	[5797,6373]	23.48%-35.73%	
5	[0.070,0.074]	6082.2	29.55%	[5616,6044]	19.61%-28.72%	

From the viewpoint of error analysis, the G-GERT model shows lower error ranges than maximum likelihood estimation (MLE) in this case. Total testing time of 5 failure samples is only one tenth of the complete samples, but the difference between two results is only 19.61%-28.72%. So in some cases, the G-GERT model for reliability evaluation is even better than MLE method in overall effect, and the testing time and experimental expense could be saved significantly.

Combining historical statistics and calculation of G-GERT network of single component, the operation time and state transition probability of each component of the equipment can be obtained, shown in Table 4.

Table 4 The state parameter table of each component of equipment system

Unit	lifetime	Operation time	transition probability
A	[5616,6044]	10	[0.0017,0.0018]
B1	4500	10	0.0022
B2	4500	10	0.0022
C1	5000	10	0.0020
C2	[4000,4800]	10	[0.0021,0.0025]
D	6000	10	0.0017
E	7500	10	0.0013
F	4000	10	0.0025

Step 3 On the basis of network structure, write the gain matrix of signal flow graph of the G-GERT in Figure 8.

$$A_S^G=\begin{bmatrix} w_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{12} & w_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{13} & 0 & w_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{14} & 0 & 0 & w_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ w_{15} & 0 & 0 & 0 & w_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{26} & 0 & 0 & 0 & w_{66} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w_{27} & 0 & 0 & 0 & 0 & w_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{38} & 0 & 0 & 0 & 0 & w_{88} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_{39} & 0 & 0 & 0 & 0 & 0 & w_{99} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_{4a} & 0 & 0 & 0 & 0 & 0 & w_{aa} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w_{4b} & 0 & 0 & 0 & 0 & 0 & 0 & w_{bb} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{5c} & 0 & 0 & 0 & 0 & 0 & 0 & w_{cc} & 0 & 0 \\ 0 & 0 & 0 & 0 & w_{5d} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{dd} & 0 \\ w_{1E} & w_{2E} & w_{3E} & w_{4E} & w_{5E} & w_{6E} & w_{7E} & w_{8E} & w_{9E} & w_{aE} & w_{bE} & w_{cE} & w_{dE} & 0 \end{bmatrix}$$

Because the diagonal elements in gain matrix of signal flow graph are not zero, we can simplify the matrix by eliminating the self-loop operation (Theorem 3) and eliminating nodes operation (Theorem 4). The reduced matrix by eliminating node 6 to node d is $A_S^{G'}$.

$$A_S^{G'} = \begin{bmatrix} w_{11} & 0 & 0 & 0 & 0 & 0 \\ w_{12} & w_{22} & 0 & 0 & 0 & 0 \\ w_{13} & 0 & w_{33} & 0 & 0 & 0 \\ w_{14} & 0 & 0 & w_{44} & 0 & 0 \\ w_{15} & 0 & 0 & 0 & w_{55} & 0 \\ w_{1E} & a'_{61} & a'_{62} & a'_{63} & a'_{64} & 0 \end{bmatrix}$$

$$a'_{61} = w_{2E} + \frac{w_{26}w_{6E}}{1-w_{66}} + \frac{w_{27}w_{7E}}{1-w_{77}}, a'_{62} = w_{3E} + \frac{w_{38}w_{8E}}{1-w_{88}} + \frac{w_{39}w_{9E}}{1-w_{99}}$$

$$a'_{63} = w_{4E} + \frac{w_{4\text{a}}w_{\text{a}E}}{1-w_{\text{aa}}} + \frac{w_{4\text{b}}w_{\text{b}E}}{1-w_{\text{bb}}}, a'_{64} = w_{5E} + \frac{w_{5\text{c}}w_{\text{c}E}}{1-w_{\text{cc}}} + \frac{w_{5\text{d}}w_{dE}}{1-w_{dd}}.$$

The equivalent transfer of the system function can be obtained by further reduction.

Step 4 Calculate the average lifetime $E(t_E)$ according to the equivalent transfer function of the system W_E.

$$E(t_E) = \left.\frac{\partial M_E(s)}{\partial s}\right|_{s=0} = \left.\frac{\partial W_E(s)}{\partial s}\right|_{s=0} = [1111.81, 1280.25](\text{h})$$

The calculation result of lifetime evaluation G-GERT network is more conservative compared with 1373-1398h calculated by RBD. By simulating the failure process of the equipment under stress impact, the model realizes the evaluation of equipment system, and it takes more randomness and uncertainty into consideration. Reliability evaluation of small samples can't ignore its instability. When evaluation results and empirical information are inaccurate or uncertain, using interval grey number as parameters is closer to reality, and the result of G-GERT is more sensitive than RBD. In addition, average transition time between any working states can be acquired according to the network structure.

5. Conclusions

The equipment reliability evaluation of small samples will become more and more important in the context of complex equipment development and custom manufacturing. In view of small samples and random processes theory, the lifetime evaluation G-GERT model based on equipment state transition choice after stress impact is put forward in this paper. The proposed model could merge history information and field test information, and obtain better life estimation results in the case of few failure data and uncertainty information. Multiple working states could be described by G-GERT model, and considering random properties of system operation, the expectation and variance of reliable life and remaining life can be calculated. The numerical example results show that lifetime evaluation G-GERT network model is feasible and effective.

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