An Investigation of Orthogonal Wavelet Division Multiplexing Techniques as an Alternative to Orthogonal Frequency Division Multiplex Transmissions and Comparison of Wavelet Families and Their Children

Ph.D Thesis

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This thesis is submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Electronics and Communications Engineering

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To the Soul of my Mother

What I know is not what others knew; what they will know is not what I know! They wrote what they knew and I am writing what I know: in wish that they will write what they will know…

…For the sake of Humanity

Strictly speaking, Fourier series and Wavelet transforms are a series of pure mathematical functions, equations and derivations that are tediously strenuous to comprehend by non-mathematicians and specialists. We shall pick whatever lies within the scope of this research and expound it in an engineering mathematics subsistence.

DEDICATION

This Thesis is dedicated to the soul of my mother (may Allah have mercy upon her and forgive her) who were proudly and very closely supporting me that I even still see, hear and feel her echo which has not been truant before and during the conduct of this thesis and will not be truant forever.

I also jointly dedicate this thesis to my great father who has been always beside me in good and bad days ultimately supporting, encouraging and rewarding me ethically, physically and financially; Thank you for all the prayers and the unconditional faith in me.

May Allah Bless Him.

To the light of my life, my wife

For all of her everlasting love and support, thank you for always being there.

To the memory of my two Suns; Miral and Mohammad

Who inspired me to work harder and smarter throughout this research.

May Allah Bless them all.

DECLARATION

I declare that the research described in this thesis is original work undertaken by me for the degree of Doctor of Philosophy, at the Centre for Electronics and Communications Engineering (CECE), School of Engineering, at De Montfort University, United Kingdom. No part of the material described in this thesis has been submitted for any award of any other degree or qualification in this or any other university or college of advanced education.

This thesis has been submitted to De Montfort University as partial fulfilment for the degree of Doctor of Philosophy from the School of Engineering.

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Abdullah S. Almuttiri

13/04/2016

ABSTRACT

Recently, issues surrounding wireless communications have risen to prominence because of the increase in the popularity of wireless applications. Bandwidth problems, and the difficulty of modulating signals across carriers, represent significant challenges. Every modulation scheme used to date has had limitations, and the use of the Discrete Fourier Transform in OFDM (Orthogonal Frequency Division Multiplex) is no exception. The restriction on further development of OFDM lies primarily within the type of transform it uses in the heart of its system, Fourier transform. OFDM suffers from sensitivity to Peak to Average Power Ratio, carrier frequency offset and wasting some bandwidth to guard successive OFDM symbols.

The discovery of the wavelet transform has opened up a number of potential applications from image compression to watermarking and encryption. Very recently, work has been done to investigate the potential of using wavelet transforms within the communication space. This research will further investigate a recently proposed, innovative, modulation technique, Orthogonal Wavelet Division Multiplex, which utilises the wavelet transform opening a new avenue for an alternative modulation scheme with some interesting potential characteristics.

Wavelet transform has many families and each of those families has children which each differ in filter length. This research consider comprehensively investigates the new modulation scheme, and proposes multi-level dynamic sub-banding as a tool to adapt variable signal bandwidths. Furthermore, all compactly supported wavelet families and their associated children of those families are investigated and evaluated against each other and compared with OFDM.

The linear computational complexity of wavelet transform is less than the logarithmic complexity of Fourier in OFDM. The more important complexity is the operational complexity which is cost effectiveness, such as the time response of the system, the memory consumption and the number of iterative operations required for data processing. Those complexities are investigated for all available compactly supported wavelet families and their children and compared with OFDM.

The evaluation reveals which wavelet families perform more effectively than OFDM, and for each wavelet family identifies which family children perform the best. Based on these results, it is concluded that the wavelet modulation scheme has some interesting advantages over OFDM, such as lower complexity and bandwidth conservation of up to 25%, due to the elimination of guard intervals and dynamic bandwidth allocation, which result in better cost effectiveness.

LIST OF PUBLICATIONS

1. Abdullah S. Almuttiri and Scott L. Linfoot, "*Orthogonal Wavelet Division Multiplex as a Modulation Scheme for Digital Television*" in 2013 IEEE International Conference on Consumer Electronics - Berlin (ICCE-Berlin), 8-11 September 2013, Berlin, Germany, 2013, pp. 136-139.

2. Abdullah S. Almuttiri and Scott L. Linfoot, "*An Advanced Modulation Scheme For Mobile Broadcasting*" in IEEE 17th Mediterranean Electrotechnical Conference, 13-16 Arpil 2014, Beirut, Lebanon, 2014, pp. 33-38.

3. Abdullah S. Almuttiri and Cristian Serdean, "*A Novel Transmission Scheme for Wireless Communications Using Wavelet Transform*", the 8th Saudi Scientific Conference in the UK at Imperial College London, 31st Jan to 1st Feb 2015, Imperial College Press.

4. Abdullah S. Almuttiri and Cristian Serdean, "*Comparison Of Compactly Supported Wavelet Families As A Modulation Scheme In Wireless Transmission*", International Journal of Wavelets, Multiresolution and Information Processing (IJWMIP), Currently under review.

5. Abdullah S. Almuttiri, "*A Transmission Scheme for Communications Systems Using Wavelet Transform and Comparison with Fourier Transform*" European Journal of Scientific Research, vol. 137, no. 3, March 2016.

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LIST OF ACRONYMS

2G	Second Generation.
3GPP	Third Generation Partnership Project.
ADCHWT	Analytical Discrete Cosine Harmonic Wavelet Transform.
ASK	Amplitude Shift Keying.
AWGN	Additive White Gaussian Noise.
BBC	British Broadcasting Corporation.
BER	Bit Error Rate.
BPSK	Binary Phase Shift Keying.
CDCHWP	Coded Discrete Cosine Harmonic Wavelet Packet.
CDMA	Code Division Multiple Access.
CWT	Continuous Wavelet Transform.
DAB	Digital Audio Broadcasting.
DCT	Discrete Cosine Transform.
DECT	Digital Enhanced Cordless Telecommunications.
DFE	Decision Feedback Equaliser.
DFT	Discrete Fourier Transform.
DFFT	Discrete Fast Fourier Transform.
DS-CDMA	Direct Sequence Code Division Multiple Access.
DSP	Digital Signal Processing.
DVB-T	Digital Video Broadcasting Terrestrial.
DMWT	Discrete Multi-Wavelet Transform.

DPSK	Differential Phase Shift Keying.
DWT	Discrete Wavelet Transform.
EM	Electro Magnetic.
GSM	Global System for Mobile.
FEC	Forward Error Correction.
FFT	Fast Fourier Transform.
FDMA	Frequency Division Multiple Access.
FH-CDMA	Frequency Hopped Code Division Multiple Access.
HP	High Priority.
HPF	High-Pass Filter.
IEEE	Institute of Electrical and Electronics Engineering.
ICI	Inter-Cell Interferrence.
ICI	Inter-Carrier Interference.
IFFT	Inverse Fast Fourier Transform.
IoT	Internnet of Things.
ISI	Inter-Symbol Interference.
RS	Reed Solomon.
RF	Radio Frequency.
LAN	Local Area Network.
LE	Linear Equaliser.
LMS	Least Mean Squares.
LTE	Long Term Evolution.
LP	Low Priority.

LPF	Low-Pass Filter.
MAN	Metropolitan Area Network.
MCM	Multi-Carrier Modulation.
MSE	Mean Square Error.
MRA	Multi Resolution Analysis.
MIMO	Multiple-Input Multiple-Output.
MPEG	Moving Picture Expert Group.
NDFT	Nonlinear Discrete Fourier Transform.
OFDM	Orthogonal Frequency Division Multiplexing.
OFDMA	Orthogonal Frequency Division Multiple Access.
OWDM	Orthogonal Wavelet Division Multiplexing.
OWDMA	Orthogonal Wavelet Division Multiple Access.
OWSS	OWDM Spread Spectrum.
PAPR	Peak to Average Power Ratio.
QAM	Quadrature Amplitude Modulator.
QoS	Quality of Service.
SER	Symbol Error Rate.
SNR	Signal to Noise Ratio.
SSP	Secure Signal Processing.
STFT	Sort Time Fourier Transform.
SOW	Statement of Work.
TDMA	Time Division Multiple Access.
WBS	Work Breakdown Structure.

W-CDMA	Wideband Code Division Multiple Access.
WiMAX	Worldwide Interoperability for Microwave Access.
WPM	Wavelet Packet Modulation.
WP-OFDM	Wavelet Packet Based Orthogonal Frequency Division Multiplex.
ZF	Zero Forcing.
ZP	Zero Padding.

Short Names of Wavelet Families

Haar	haar
Daubechies	db*N*
Symlets	sym*N*
Coiflets	coif*N*
Biorthogonal	bior*Nr*.*Nd*
Reverse Biorthogonal	rbio*Nd*.*Nr*
Discrete Meyer	dmey

Wavelet families' children are also referred to as families' member/s.

CHAPTER 1: INTRODUCTION

Objectives:

- To give an introduction to the research.
- To present the motivation and problem formulation to this research.
- To list the research specifications, aims and objectives and scope of this research.
- To list the research questions.
- To give an insight of thesis structure.

1.1 OVERVIEW

Wireless and wired communication systems are now pervasive and include functions from sending signals to opening a garage door; to fourth generation mobile phones that can cope with a broad range of multimedia traffic. Although wireless communication systems are similar to wired systems, there are two main differences. Firstly, in wireless communications, systems channels are varied and often unknown and for each link, there are different propagation losses. Secondly, they are inherently multiple access mediums; therefore, there are more users competing for a limited band of frequencies.

Wireless communications systems started to develop rapidly at the beginning of the 1960s, whereby a revolutionary wireless modulation technique was brought to life by several researchers: Orthogonal Frequency Division Multiplexing (OFDM). Saving the bandwidth resource and squeezing as much information down a channel as possible was the main driving force behind the discovery and introduction of this modulation technique. OFDM was revealed at that time to resolve one of the vital issues of wireless communication systems, which suffered from several environmental issues, such as - multipath propagation. This will be further discussed in greater details in Chapter 2.

OFDM utilises the Discrete Fourier Transform (DFT) within the communication system. It employs the Inverse Fast Fourier Transform (IFFT) for the transmitter and the Fast Fourier Transform (FFT) for the receiver. Issues started to arise with regards to this new modulation technique as wireless communication systems became more advanced and complicated. Cyclic prefix and guard intervals insertion between OFDM symbols was introduced to resolve some of these issues. However, this has not been sufficient to keep up with the advancing wireless systems and modulation techniques.

Over the last three decades, the wavelet transform has been introduced to the data compression algorithms, watermarking and delicately for image and video compression. This is due to the need for further development from the Fourier transform. It is used as an algorithm to analyse the data by transforming the signals from time to time-frequency domains, rather than the frequency domain used in the Fourier transform. Subsequently, a greater development was introduced by researchers for further development of the wavelet transforms. Presently, there are several wavelets which are used in wavelet transform which vary in performance, e.g. Symlets, Coiflets and Daubechies.

Utilisation of the wavelet transform in wireless communication systems can possibly be achieved in various applications. There were several proposals from researchers to use Discrete Wavelet Packet Modulation (DWPM) for signals modulation, suggesting that it performs better than FFT in OFDM systems. Kattoush [1] suggested that DWPM performs better than OFDM under different channel conditions. Furthermore, Nikokar [2] looked at the computation complexity of DWPM, with focus on Haar wavelet and suggested that it is less complex than IFFT in OFDM. Further detailed discussion is presented in the critical review, Chapter 3.

Recently, there has been a proposal introduced by Linfoot *et al.*, [3] to use Orthogonal Frequency Division Multiplexing (OWDM) as an alternative modulation technique for OFDM. In the proposal, a Digital Video Broadcasting - Terrestrial (DVB-T) system was considered as a standard for the testing simulation with a Moving Picture Expert Group (MPEG) frame stream. The system which was tested considered a signal without error detection, correction coding, interleaving and channel coding. Additionally, only two levels of filters banks were examined and only the Haar wavelet was conducted at that research.

The purpose of this research has been to further investigate this brand new modulation technique, OWDM, based on the wavelet transform and assess its viability as an alternative to OFDM. The theoretical study and practical simulation is conducted, compared and evaluated against the state-of-the-art modulation technique, which is OFDM. The complexity of each of OWDM and OFDM has been taken into consideration.

This research is a further investigation of the research associated with this brand new proposed modulation technique, OWDM, as an alternative to OFDM. An in-depth investigation of computational, space and time complexities has been conducted. Additionally, for validation of theories, comprehensive experimental implementations of not only all available wavelet families and families' members, but also exploring different levels of filter banks for each of those families and members have been conducted. Technical discussion, evaluation, results validation and a conclusion of the overall results and recommendations will also be presented at the end of this research.

What has been briefly mentioned in the previous paragraphs will be discussed in greater details in the literature review Chapter 2, and the critical review Chapter 3.

1.2 MOTIVATION AND PROBLEM FORMULATION

During the last decade, the demand on communication bandwidth has grown rapidly. Let alone the incredibly increasing traffic rate due to the increasing demand on multimedia broadcasts which tends to have huge packet size and mass packets transmission that occupies a significant amount of the bandwidth. Not only that, but also the dramatically increasing utility of Internet of Things (IoT), nowadays, which also adds in a tremendous congestion into the traffic. Therefore, less complex systems for faster data rate transmission, less space complex systems for reliable and minimal latency tolerance transmission are demanded more than ever before to cope with the advancement of applications and traffic size. Moreover, with the current state-of-the-art technology, which wastes part of the spectrum to prevent interference between consecutive symbols, it is also increasingly demanded to find a feasible, practical and more effective alternative technology that can utilise this wasted bandwidth. Apart from that, the trade-off between computational, time and space complexities need to be addressed to find the most effective system in terms of computational time and space complexities in order to find the most cost effective and optimal latency tolerance system. Those will be explained in greater details in Chapter 5, Chapter 6 and Chapter 7, thereafter, an evaluation will be presented with evidence in Chapter 8.

One of the main motivations for this research is to investigate the advantages that using wavelets transform as an alternative to Fourier transform in OFDM systems has to offer. Furthermore, the potential advantages over OFDM in terms of various range of complexities need to be investigated. We also need to investigate the elimination of guard intervals, which is used to separate OFDM symbols, from OWDM system. Since OFDM, the current state-of-the-art technology, wastes up to 20% of the spectrum to guard consecutive OFDM symbols; hence, this area of research is significant since there is possibly no need to use guard intervals in OWDM. More importantly, the usability and suitability for dynamic sub-band allocation in asymmetric wavelet filters banks rather than the fixed length of the sub-bands interval allocation in the OFDM system and wavelet packet transform are to be investigated. What is more, in some signals, particularly in the time domain, the signal may contain directly relevant information of the action and behaviour of the signal in the frequency domain at that particular point of time, therefore, the analysis of the signal in the time-frequency domain is necessarily required to know the feature and behaviour of the signal and, therefore, to obtain a significantly better approximation of the signal. This research will investigate this added feature and will also examine this transmission technique using flexible filters banks for flexible bandwidth allocation, not only for some wavelet families but also for all compactly supported wavelet families and their members. This research will further investigate the computational complexity and the many claims that said that wavelet transform is less complex than Fourier transform, especially when it is used as transmission scheme as proposed by [3] [4] [5] and [6]. These have drawn the attention to facilitate a further research and propose a further development to the system that makes use of the wasted bandwidth by eliminating the guard interval and flexible and dynamic bandwidth allocation. All these will be discussed in more details in Chapter 2, Chapter 3 and Chapter 6.

As will be discussed in Chapter 2 and Chapter 5, in wavelet transform, there exists a number of wavelet families/basis and even families' members, known as the family's children, that can be utilised in OWDM, e.g. Haar, Coiflets, Symlets, Daubechies, Biorthogonal, Reverse Biorthogonal and Discrete Meyer. Each of those families has a series of different members or children each of which has different coefficients and, therefore, different computational complexity, computational time and memory space requirements. Hence, they display different behaviours, time responses and memory requirements when used in the proposed system. All those families and families' members will need to be investigated and assessed as used in the proposed system and compared not only among each other's performance but also against the state-of-the-art OFDM's performance as well; this is the main research question. The proposed system, OWDM, will be designed inline and equivalent to the state-of-the-art OFDM system and DVB-T standard will be the baseline. Since OWDM is still being researched and not standardised, the system will be designed and parameterised in an equivalent design of OFDM, so that both systems can be compared and evaluated against each other in a like-for-like basis.

Since the demand for wireless communications has increased very dramatically through the last decade, bandwidth resources are being filled up very quickly and becoming scarcely available. This research can potentially resolve the issue of finding a more effective modulation scheme in terms of bandwidth usage, complexity and better performance, which can perform similar or better than the current state-of-the-art scheme. Previously, an issue of transmitting signals in more efficient way arose and has been resolved by increasing the parallel subcarriers in the OFDM system; nevertheless, this will use more bandwidth and will only partially resolve the issue. What is more, OFDM wastes some of the used bandwidth since guard intervals are required to separate OFDM symbols apart with a null interval of time before or after each OFDM symbol in the OFDM frame to mitigate Inter Symbol Interference (ISI). However, for this research, the approach will be different; by using wavelet transform to modulate the Quadrature Phase Shift Keying (QPSK), Differential Phase Shift Keying (DBSK), Binary Phase Shift Keying (BPSK) or Quadrature Amplitude Modulation (QAM) symbols, guard interval is not needed and hence, bandwidth is more efficiently used. More effectively, the design of the proposed system is allocating bandwidth to different users more dynamically upon demand and hence, significantly results in more efficient bandwidth utilisation. A wide theoretical comparison between OFDM and OWDM will be conducted to support the experimental results. Experiments will be facilitated for the two techniques and the less complex and most efficient technique with the highest performance will be analysed and recommended at the end.

Additionally, as will be discussed in Chapter 2 and Chapter 5, this research is being conducted to facilitate further research into the recently proposed OWDM alternative modulation technique by Linfoot *et al.,* [3]. However, to the best of this author’s knowledge, all reviewed research on the proposed techniques of wavelet packet modulation, even those studies which were published during the conduction of this research and up to January 2016, simply discuss the mathematical computational complexity with some added discussion of eliminating the guard interval. However, this research aims to address all practical complexities, including computational, time consumption, memory space requirements for all the available wavelets. The recently proposed OWDM just lightly touches the computational complexity and tested one family member (Haar wavelet) with just two levels of filters banks without feasibility or practicality investigation of the proposal in terms of time complexity and memory space occupation needed during processing. Additionally, investigate this theory not just for one particular family but all available wavelet families and families’ members. There are many wavelets families and families’ members available and their use and suitability as part of the OWDM filter banks needs to be investigated comprehensively. Therefore, in this research, various wavelets with different levels of decomposition and reconstruction levels will be examined and analysed. Secondly, it has been proposed by Linfoot *et al*., [3] that OWDM is less complex and more flexible than OFDM. Thus, this research will also include an investigation of the simplicity and processing time for both, OFDM and OWDM systems.

Moreover, the demands on bandwidth resources for both military and public services has driven scientists, researchers and the industry to find new modulation and transmission techniques that can save more bandwidth, which could then be used by other occupiers/users without affecting the existing occupiers. OWDM does not need a guard interval to protect consecutive symbols; which could lead to potential savings in bandwidth resources if OWDM proves to be successful and is adopted and implemented by the industry [3].

1.3 RESEARCH SPECIFICATIONS

This research has several specifications that are to be taken into account throughout the design and implementation stages. The research is focusing on investigating the usability and suitability of a brand new modulation technique, OWDM, which will be benchmarked against the current state-of-the-art technology. The complexity of the new modulation scheme and existing technical specifications need to be taken into account. The following are the main specifications:

- Orthogonal Frequency Division Multiplex (OFDM) baseline with 16-QAM.
- Orthogonal Frequency Division Multiplex (OFDM) baseline with 64 QAM.
- Orthogonal Wavelet Division Multiplex (OWDM), 11 levels of filter banks to match IFFT length in OFDM.
- Orthogonal Wavelet Division Multiplex (OWDM), 11 levels of filter banks with 16-QAM and 64QAM.
- Implementing all systems over the following channels environments:
    - Additive White Gaussian Noise (AWGN) channel.
    - Rician, mobile environment.
    - Rayleigh, severe mobile environment with various range of Doppler shifts, 50 Hz, 100 Hz and 200 Hz.
- Assessing the usability and suitability of all compactly supported wavelet families and their associated members: Haar, Coiflets, Daubechies, Symlets, Biorthogonals, Reverse Biorthogonals and Discrete Meyer.

1.3.1 AIMS AND OBJECTIVES

The main aim of this research is to carry out a detailed investigation of this brand new technique, using wavelet transform instead of Fourier transform in OFDM systems, with various depths and dynamic bandwidth allocation for OWDM. Another aim is assessing the performance and suitability of all available compactly supported wavelet families and families' members. Moreover, it is directed to researchers who are interested in modulation techniques for wireless communications in order to draw their attention to this technology and facilitate further research. Finally, this research is considered to be the first step in a larger research for determining the feasibility, usability and practicality of this brand new modulation technique.

The objectives of this research are as follows:

1. Answer the yet open research question of the feasibility of using wavelet transform as an alternative to Fourier transform in OFDM systems.
2. To identify any differences between OFDM and OWDM as modulation and transmission techniques.
3. To analyse the advantages and disadvantages of each technique.
4. To conduct a comparative analysis between OWDM and OFDM to measure their performance, power efficiency, complexity and cost.
5. To conduct simulations for each technique and each wavelet families and families' members to investigate and evaluate the proposal.
6. To address the computational, time and space complexity of the OWDM system for various wavelet families with wide range of vanishing moments and filter symmetries.
7. To investigate and evaluate the performance of all compactly supported wavelet families and families' members.

1.3.2 SCOPE

This research can be split into four main phases. The first phase contains the feasibility study, which is considered to be the most important stage. A full assessment of the entire research has been conducted to determine whether the research is achievable. It is important to include an analysis of the available resources during this stage, because this will be crucial for the decision-making as to whether to proceed to the next phase.

Conducting a feasibility study is considered of significant importance as it ensures the conservation of the financial and time resources of the author and greatly reduces the risk of research failure later on during the planning and implementation stages. A full evaluation and recommendations will be provided at the end of this study.

The second phase is the planning stage, in which, the research details and processes will be planned and explained. An in depth literature review and critical reviews will be conducted to ensure that the viability and contribution to knowledge is stands within this increasingly researched area. Following the conduct of the feasibility and its approval, this phase has started.

The third stage is the simulation and implementation stage, in which, the simulation of OFDM and OWDM will be conducted. Furthermore, a full analysis, evaluation and technical discussion of the results will be provided.

The final stage of the research combines two phases into one. The first phase is to update the literature and critical reviews for an additional insurance that the research question and contributions to knowledge remain tangible and still stand; alternatively, adjust the research to add more tangible contributions to knowledge, should anything occur. The second phase is the writing up and submission stage, whereby writing up, editing, formatting and submission of the research will take place. During this stage, the entire research will be at a closing stage. In addition, there will be a set of recommendations and the possibility for future work. The research will be submitted on 11th April 2016.

1.3.3 BENEFITS

The main benefit is to document an insight into a brand new modulation technique for future use, to facilitate further research with regards to this technology. It will contribute to knowledge and the research community by providing a reference for using wavelet transform instead of Fourier transform in OFDM systems with added flexibility and bandwidth conservation. It will also contribute to the knowledge by filling the research gap regarding which wavelet family and families' members perform better than not just other wavelets but also against Fourier transform which is used currently in the state-of-the-art OFDM. It will also answer the yet open research question regarding the various kind of complexities in hardware and software when practically use this modulation scheme, including computational, space in memory and time consumption complexities. It assesses the suitability of wavelet families and families' members to be used as an alternative to Fourier transform in OFDM systems. It also investigates the currently unanswered questions of the complexity of wavelet transform as opposed to Fourier transform. Additionally, researchers who are interested in modulation schemes for wireless communications could facilitate this research to inform their research and facilitate further studies.

1.4 RESEARCH QUESTIONS

The research questions are:

- Question 1: Is the current IFFT modulation technique used in OFDM system is sufficiently using the bandwidth?

If the answer is No, this poses a new question:

- Question 2: Would using wavelet transform instead of IFFT in OFDM system be more sufficient and conserve some bandwidth?

If the answer is Yes, this leads to another question:

- Question 3: Has this modulation scheme been previously proposed?

If the answer is Yes, another question arises from this:

- Question 4: Have all available wavelet families and families' members been investigated in terms of their computation, time and space complexities?

If the answer is No, the following question arise:

- Question 5: Which wavelet family and family's member outperform IFFT with lower computation, time and space complexities?

- Question 6: Can the current wavelet modulation be further developed to contribute to a cost-effective system?

If the answer is Yes, another question arises:

- Question 7: Can this modulation be practically implemented and how much could the complexity be?

Another question nested in this question is:

  - Which wavelet family and family's member could outperform other families and their associated members and also IFFT?

In the specifications section, next, the approach to answer these questions is presented, which is expanded in the methodology Chapter 4.

1.5 INDICATION OF RESEARCH SUCCESS

The indications of success of this research are:

- All research questions must be answered clearly.
- A study shows how wavelet filters bank, the proposed system, can be utilised in a methodology that could save some more bandwidth.
- A study shows how the proposed system could advance the knowledge and the research in this field.
- A comprehensive investigation of wavelets families and their associated members through a set of multiple experimental works.
- Answer to the research questions indicated in the motivation and problem formulation section, 1.2.

1.6 THESIS STRUCTURE

This thesis has been built in nine chapters where each of them discusses phase of the research throughout the life of this research. These chapters are as follows:

1. Chapter 1: is the introduction of the thesis where the research introduction, research specifications, aims and objectives, benefit and contribution to knowledge, motivation and problem formulation (research questions) are discussed.

2. Chapter 2: is the literature review of the state-of-the-art modulations technologies that are used in telecommunications for signal modulation and transmission.

Enabling technologies in wireless telecommunications of both non-multiple access and multiples access techniques are reviewed.

3. Chapter 3: is the critical review of what has been done and is being done in this specific research area. Wavelet modulation techniques and multiwavelet, that were proposed previously and are being researched currently, are discussed in detail and used to inform this research.

4. Chapter 4: is the research methodology where the the methodology followed in completing this research is discussed. Research requirements, technical approach, resources, risk analysis and alternatives and work breakdown structure are all discussed in this chapter.

5. Chapter 5: thoroughly explores Time-Frequency analysis techniques and further analysis linked to this research regarding wavelets families and families' members are also thoroughly discussed. The differences and additions to that of the researches that addressed similar problems are also addressed in this chapter and highlights of the currently research questions are provided.

6. Chapter 6: discusses the proposed system. The design of the baseline system for comparison, OFDM, is provided. Also the design of the proposed system and the validation of the design are discussed in this chapter.

7. Chapter 7: is where the systems designed and described in the previous chapter, Chapter 6, are implemented and all results and technical observations are provided in this chapter.

8. Chapter 8: all the results are technically discussed and analysed. Critical analysis, comparison and validation of the different wavelet families and families' members are analysed and compared against each other. Also, the proposed system and the baseline state-of-the-art system are analysed and compared against each other.

9. Chapter 9: provides a conclusion of the whole research. Additionally, the research question and contribution to knowledge are revisited. Future work and research are presented at the end of this chapter.

1.7 SUMMARY

In summary, the research specifications including aims and objectives that aimed to be met at the end of this research were discussed. The scope and contribution to knowledge which include the benefit from carrying out this research were addressed. Thereafter, motivation and problem formulation, which also includes the research questions, were presented. Finally, the thesis structure and outline was presented.

CHAPTER 2: LITERATURE SURVEY

Objectives:

- To give an introduction to field of research.
- To present enabling technologies in communications systems.
- To give a literature review of the state-of-the-art multiple carrier systems.
- To give a literature review of the state-of-the-art single access systems.
- To give a literature review of the state-of-the-art single access systems.

2.1 INTRODUCTION

Wireless communication systems are becoming increasingly pervasive and include, for example, from basic functions, such as - sending a signal to an automatic gate, to more sophisticated functions, such as mobile devices that can cope with a large amount of multimedia traffic. According to Cisco [7], data traffic in the mobile networks globally increased by 74% just in 2015, due to the introduction of hundreds of connected devices and applications through the Internet of Things (IoT) and increasing popularity of 4G. This massive growth reached a peak at the end of 2015 with a booming 3.7 exabytes per month, equivalent to $3.7*10^{12}$ gigabytes. The number of devices added into the global connection was 563 million, just during 2015. The average data consumption (not voice), in the same year, rapidly soared to 23 MB per month per smartphone compared to 16 MB in 2014. Regarding PCs' traffic on the mobile network, in 2016, each of those PCs generated 2.9 times traffic more than the average smartphone in the same period. The projected growth in mobile data traffic for the next 5 years still and will be growing rapidly, as figures from the leading networking company Cisco [7] shows. Figure 2.1 shows the projected data growth rate per month from 2015 to 2020, Exabyte=$1*10^{12}$. Although wireless communication systems have much in common with wired systems, there are two key distinct differences. Firstly, for wireless systems, channels are varied and often unknown, exhibiting propagation losses for each link. Secondly, they operate in an essentially a multiple access environment, with multiple users using the same frequencies.

Therefore, the interference between multiple users is something that needs to be managed as a primary design objective in order to meet Quality of Service (QoS) requirements. With multiple access wired systems all users experience the same propagation losses, and access is coordinated by a master controller, which is unlike wireless systems. Because costs are incurred

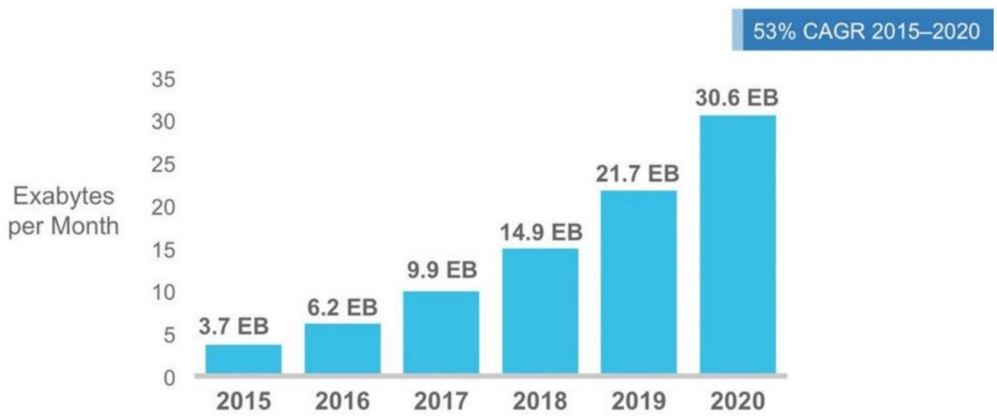

Figure 2.1: Projected Data Growth 2015-2020 [7]

in both sharing and monitoring the channel conditions, it is imperative to understand how to control resources. Apart from this, multi-access channel suffers from high volume of interference, high usage of bandwidth resources and natural white noise.

In this chapter, a literature review of the state-of-the-art technologies currently employed in communications systems will be presented and discussed.

2.2 SCOPE OF THE LITERATURE SURVEY

The literature survey will review the use of many wireless communications systems and more specifically, the modulation techniques that are used on them, i.e. TDMA, FDMA and OFDM. There will also be a review of the transforms and multiplexing techniques which are used on those systems, i.e. Non-linear Discrete Fourier transform (NDFT) and Discrete Fourier Transform (DFT). Specifically, there is a focus on the N-parallel symbols that are modulated by the Inverse Fast Fourier Transform (IFFT) in the transmitter and the Fast Fourier Transform (FFT) in the receiver. Moreover, there is a review of the technique proposed by Heiskala and Terry [8] to modulate the Quadrature Amplitude Modulator (QAM) symbols using the wavelet transform and a theoretical comparison is made with OFDM. Akansu and Agirman [9] say that the subcarriers can be modulated in the OFDM system using NDFT; however, this may be complex. This technique will be reviewed here.

Moreover, there will be a comparison between conventional techniques used in cellular systems and the aforementioned modulation techniques. The conventional techniques include Code Division Multiple Access (CDMA), Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA). Moreover, there will also be a review of the systems that could employ such techniques.

2.3 WIRELESS AD-HOC NETWORK SYSTEMS

Wireless ad-hoc network systems are essentially networks that are decentralised. They comprise of mobile and static nodes and are characterised by the absence of a pre-existing infrastructure, which may include access points, routers or base stations and a centralised administrative function. The nodes have wireless transmitters and receivers and communicate through distributed channels.

Since node transmitters have a limited range, they use multi-hopping relay between intermediate nodes in order to communicate with the receiving node that is located out of range. Thus, each network node is effectively a transmitter, receiver and router. This means that an ad-hoc network can self-organise and self-configure dynamically and nodes create the required routing without the need for an infrastructure or administration [10]. There have been numerous routing protocols designed for ad-hoc network systems proposed over the last 10 years. The proposal of using wavelets transform in this research can be extended to wireless ad-hoc network systems.

2.4 CELLULAR NETWORK SYSTEMS

A wireless cellular network system is centralised and essentially has a pre-existing infrastructure and is administrated centrally. The coverage area is subdivided into smaller areas and cells, in order to manage coverage and traffic. Communication between the nodes in such networks only takes place in the cells, specifically, with base stations situated in the middle of a cell or where three cells intersect. Thus, in cellular network systems, the range of communication is limited to the cell radius, unlike the aforementioned ad-hoc network systems. Therefore, multi-hopping and routing protocols used in ad-hoc network systems are not used in cellular network systems, where the transmission ability can be economised.

The Inter-Cell Interference (ICI) is compatible with multiple access schemes. Unfortunately, problems associated with the ICI may occur even though it is limited by the cell radius. Lee and Messerschmitt [11] state that it is possible to control ICI through various frequency reuse patterns. The ICI can be restricted by using a limited amount of frequency bands for each cell. Because of the random nature of wireless channels, due to shadowing and fading, frequency re-use only provides a statistical bound on ICI.

There have been numerous proposed strategies and algorithms for controlling ICI, which has resulted in an improved cellular network capacity; these strategies include radio resource allocation, power control and frequency reuse factor control.

2.5 MULTI-USER WIRELESS SYSTEMS

In wireless communication systems, there are often multiple users located within each other’s transmission ranges which may result in severe interference. Therefore, there is a need for protocols that allocate users of multi-user wireless systems sufficient radio resources for their QoS requirements. The protocols used for multi-user access in wireless systems are an important area of research due to the demands for wireless communications that need increased QoS. There have been investigations of numerous multiple access schemes with a view to reduce interference and improve network capacity. Several well-known multiple access schemes include: Code Division Multiple Access (CDMA), Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA) and Orthogonal Frequency Division Multiple Access (OFDMA).

2.6 ENABLING TECHNOLOGIES

This section presents the technology that already exists as well as newly developed and proposed technology. This section is divided accordingly:

1) Non Multiple Access Techniques.
2) Multiple Access Techniques.

2.6.1 NON MULTIPLE ACCESS TECHNIQUES

This section discusses non multiple access modulation techniques and transform. There are numerous transforms and modulation techniques used within communication systems since the development of analogue signals to the digital systems. However, the interest in this research is to look at the most recent and closest modulation techniques and transforms to what is being investigated in this research. Discrete Fourier Transform (DFT) and its application in modulation and signal transmission is the baseline for comparison in this research. Therefore, the scope of this section is divided into three sections, the current state-of-the-art modulation system and two associated transforms which are used within the system, as follows:

1) Discrete Fourier Transform (DFT).
2) Nonlinear Discrete Fourier Transform (NDFT).
3) Orthogonal Frequency Division Multiplex (OFDM).

2.6.1.1 DISCRETE FOURIER TRANSFORM AND FAST FOURIER TRANSFORM

The DFT is one of the fundamental transforms in Digital Signal Processing (DSP). It is mainly the discrete form of Fourier Transform (FT), which is a wild mathematical subject that will not be discussed deeply within this research, however the interest is the discrete form of FT. Fourier analysis is fundamentally concerned with functions representations as a multiple sums of trigonometric functions; it can also be seen as a series of periodic functions. The history of FT is two century ago when Fourier, in 1807, proposed that arbitrary functions can possibly be represented by trigonometric series, he proved that later with his famous Fourier series formula [12]. Following this discovery, fast advancements of technology driven applications led to the discovery of a fast computational formula of Fourier transform, known as FFT, by several proposals from various scientists, however, the most used algorithm was proposed by Cooley and Tukey who developed jointly a FFT in 1965 for IBM Company to utilise in high speed computers [13].

Technically, FT generally has a computation complexity of N^2 for any signal that contains N number of samples. This computational complexity is too high for hardware to compute, store and look up; however, the discovery of FFT is driven by reduction of computation complexity of FT. Cooley-Tukey [14] Radix-2 FFT Algorithm reduces computational complexity from $O(N^2)$ to $O(Nlog\,N)$, this logarithmic complexity will be discussed in further details in Chapter 5. The relationship between Continuous Fourier Transform (CFT) and DFT is that DFT is an approximation of CFT for discrete functions. Given a signal with a sequence of $[x_n]$, DFT express this sequence as a sequence of $[X_k]$ complex numbers. Those complex numbers represents the phase and amplitude of sinusoidal components of the input signal. Reconstructing the original function is possible through inversing this method, but with the requirement of truncating the sequence of $[x_n]$ over a finite interval and be a sample of a continuous function. A DFT of signal x can be defined by:

$$X(\omega_k) = \sum_{n=0}^{N-1} x(t_n) e^{-i\omega_k t_n} \ , \qquad k = 0, 1, 2, \ldots, N-1 \tag{2.1}$$

Where $\omega_k = \frac{2\pi}{NT}$ and $t_n = nT$ where T is the sampling intervals. We can also say that $e^{-i\omega_k t_n} = e^{-2i\frac{nk}{N}}$.

Now the sequence signal $X(\omega_k)$, in Equation 2.1, is in the frequency domain and it can be converted back into time domain by performing Inverse DFT (IDFT) which can be calculated by the following formula[1]:

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(\omega_k) e^{2\pi i \frac{nk}{N}} , \qquad n = 0, 1, 2, \ldots, N-1 \tag{2.2}$$

Radix-2 Cooley-Tukey algorithm of computing FFT considers the length of arbitrary integer composite size of $N = N_1 N_2$, the algorithm rewrites the DFT recursively with smaller DFT sizes of N_1 and N_2; this is primarily to reduce the computational complexity. Since the mathematical proofs of these formulas are not in the interest of this research, they are not presented here; however, it can be found in Cooley and Tukey main paper [14].

Since the discovery of OFDM in 1968, when Chang and Gibby [15] proposed the use of Fourier transform to orthogonally multiplexing signals for transmission, IFFT and FFT have been utilised to multiplex signals in the transmitter and reconstruct them in the receiver, respectively. We will explore this in more details later in this chapter in the OFDM section.

2.6.1.2 NON-LINEAR DISCRETE FOURIER TRANSFORM (NDFT)

As an extension of DFT, NDFT is a discrete transform employed in Fourier analysis. NDFT transforms a function from the time domain to the frequency domain. DFT has limitations, since the input needs to be discrete and the associated non-zero values only have a finite duration. The input to the NDFT is a combination of real and complex numbers [9], ideal for computational processing. According to Akansu and Agirman [9], the DFT is for a sequence of N complex numbers X_0 to X_{n-1} and can be shown as:

$$X_k = \sum_{n=0}^{N-1} x_n \, e^{-\frac{2\pi i}{N} kn} \qquad k = 0, \ldots, N-1 \tag{2.3}$$

Where: i = imaginary unit and $e^{(\frac{2\pi i}{N})}$ is a primitive Nth root of unity.

[1] Stoer and Bulirsch [**120**] demonstrate mathematical analytic proof of why this formula is the best reconstruction formula for DFT (equation 2.1).

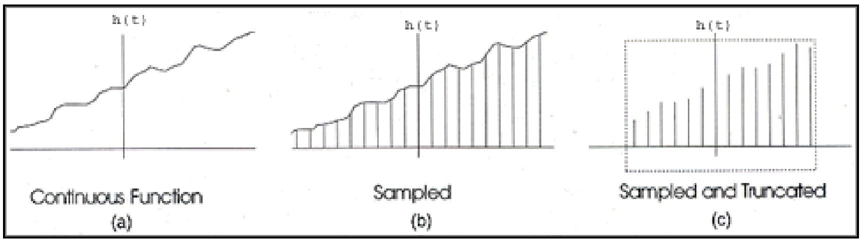

Figure 2.2: Data Sampling and Truncation in NDFT [17]

In terms of NDFT functionality, Equation 2.4 below represents the generalisation of NDFT:

$$\gamma(x,t) = -E1 + \sum_{j=1}^{N} 2\mu j(x,t) - E2j - E2j + 1 \tag{2.4}$$

Randy [16] indicated that an advantage of employing NDFT is that it is suitable for sampling, such as - truncating or data sampling [17]. Unfortunately, ringing occurs in the samples while using this application, which is referred to as Gibb's Phenomenon. Fortunately, however, this ringing can be resolved by increasing the number of data samples, as shown in Figure 2.2. NDFT has been developed by employing interpolation methods [18].

The NDFT of any given sequence, i.e. $x(n)$, of length N can be defined as**:**

$$X(z_k) = X(z)|_{z=z_k} = \sum_{n=0}^{N-1} x[n] z_k^{-n} \quad , \quad k = 0, 1, \dots, N-1 \tag{2.5}$$

Where X(z) is the z-transform of x[n], and $z_1, z_2, \dots, z_{N-1}$ are distinct points found arbitrarily in the z-plane. Equation 2.5 can be shown in matrix form as $X = D\,x$,

Where X is:

$$\underline{X} = \begin{bmatrix} X(z_0) \\ X(z_1) \\ \vdots \\ X(z_{N-1}) \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \tag{2.6}$$

and D is:

$$D = \begin{bmatrix} 1 & z_0^{-1} & z_0^{-2} & \dots & z_{N-1}^{-2} \\ 1 & z_1^{-1} & z_1^{-2} & \dots & z_1^{-(N-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & z_{N-1}^{-1} & z_{N-1}^{-2} & \dots & z_{N-1}^{-(N-1)} \end{bmatrix} \quad (2.7)$$

If the *N* sampling points $z_0, z_1, \dots, z_{N-1}$ are distinct, then *D* is non-singular, and therefore, the inverse of NDFT can be determined by $x = D_{N-1} X$.

In signal modulation, however, NDFT is rarely used and often replaced by the fast form of DFT, FFT. This is due to the computational complexity of NDFT and the non-linearity nature of it. Nevertheless, NDFT is still used in some applications, such as acoustic signal processing applications, more specifically, in musical instruments. Further details of the complexity of Fourier transform and how it is computed in machine language will be discussed in Chapter 5.

2.6.1.3 ORTHOGONAL FREQUENCY DIVISION MULTIPLEX

OFDM is a multiplexing technique that compresses a signal through the channel with the minimum amount of Inter-Symbol Interference (ISI). The technology was introduced in 1968 by Chang and Gibby [15] as a practical transmission technique. The motivation for this research was to discover a more robust and effective modulation technique; as well as a modulation scheme to respond to the demands to receive signals while being mobile. Currently, in addition to voice services, there is limited bit rate resources for mobile users of multimedia. OFDM has saved approximately 50% of the bandwidth compared to conventional modulation techniques at that time, i.e. TDMA, FDMA, CDMA etc; Figure 2.3 shows the saving in bandwidth that OFDM made.

Throughout that last twenty years OFDM has increasingly developed and become a popular technique for wireless modulation and transmission. The technique has been used in many world class wireless communication standards, for example, in Digital Video Broadcasting Terrestrial (DVB-T) [19], Digital Audio Broadcasting (DAB) [20], IEEE 802.16a Metropolitan Area Network (MAN) and the IEEE 802.11a Local Area Network (LAN) standards [21].

For the channel in a mobile environment there are several signals for each user which arrive at the receiver at different times; referred to as multipath fading. This is because of obstacles and the geographical nature of the wireless environment. In order to resolve the multipath issue

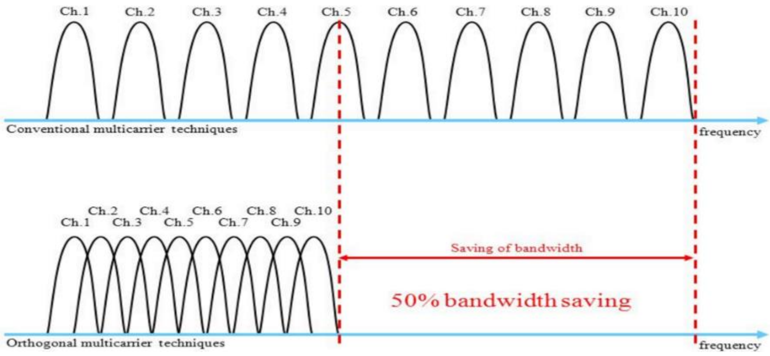

Figure 2.3: How much bandwidth could OFDM save?

a complex equalizer is required at the receiver; however, this may take more processing time as the rate of data increases. Ramjee [22], says that OFDM solves the multipath problem without requiring a complex equalizer at the receiver.

OFDM is plagued by the problem of ISI. However, this problem is mitigated through dividing the bandwidth into N non-overlapping narrow subcarriers and then modulating the signals. The N subcarriers are frequency multiplexed and are sent simultaneously in order to create the OFDM symbol, illustrated in Figure 2.5. Through the addition of a guard interval at the start or at the end of the OFDM information symbols, delay spread result would then appear as a multiplication in the Fourier domain; and thus requires perfect synchronisation between the receiver and the transmitter. Additionally, guard intervals prevent ISI [23]. These N subcarriers, which are a conduit for the information signals, have an orthogonal relationship with one another; thus, no interference takes place between the subcarriers; this is is illustrated in Figure 2.4.

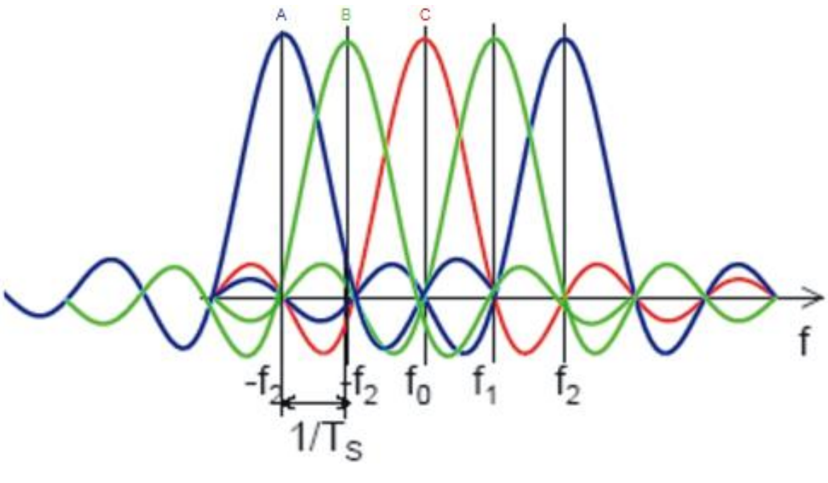

Figure 2.4: OFDM Carrier Spectrum, Orthogonality Principle [**135**]

According to the orthogonality principle when there is an orthogonal relationship between the carriers, the peak of one carrier coincides with the trough of the next carrier. This can be seen by looking at the timing of signals "A" and "B" in Figure 2.4. This principle is also applicable to the consecutive carriers, thus the space between two successive carriers is represented as $1/T_s$. This method of organisation ensures that there will be no collisions or interference between the carriers.

Ramjee [22] reports that the OFDM method of transmission effectively conserves bandwidth use by approximately half. In the frequency domain the step response of the OFDM subcarriers is a sinc function represented as $\frac{sin(x)}{x}$. As a consequence, overlapping in the frequency domain takes place. Moreover, the overlap does not result in interference because of the orthogonal of the carriers [24].

The orthogonality of OFDM carriers, depicted in Figure 2.4 above, is mathematically calculated as:

$$\frac{2}{T_s}\int_{kT}^{(k+1)T} sin(mf_0t).\sin(nf_0t).dt = \begin{cases}1, if\ m = n \\ 0, if\ m \neq n\end{cases} \quad (2.8)$$

The orthogonality is achieved when $f_0 = \frac{1}{T_s}$ where T_s is OFDM symbol duration. One of the primary conditions of achieving orthogonality is that every carrier covers an integer number of periods (cycles) during OFDM symbol time (T_s). The carriers are always time-limited by a window; the sinc function of OFDM spectrum corresponds to a sinusoid carrier multiplied by this time-limited window. Additionally, at the central frequency of any given carrier, all other carriers cross zero, given those carriers are separated by a space of $\frac{1}{T_s}$.

FFT is used by the receiver in OFDM to convert the time waveform back to frequency domain. As a result, discrete frequency samples are picked up by the FFT in accordance to the carrier peaks. For all of the other carriers that pass through zero, there is no consideration of the interference between the subcarriers.

FFT is sensitive to the following changes:

- Number of integer cycles during any symbol period.
- Number of integer cycles that separate subcarriers.

During this process, the amplitude and phase values remain constant.

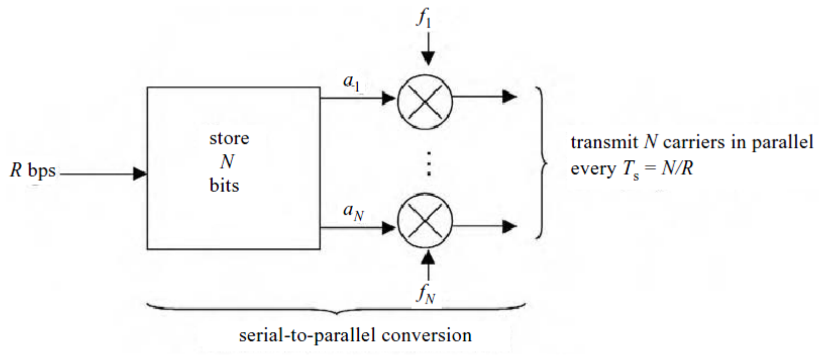

Figure 2.5: Generator of OFDM symbols [25]

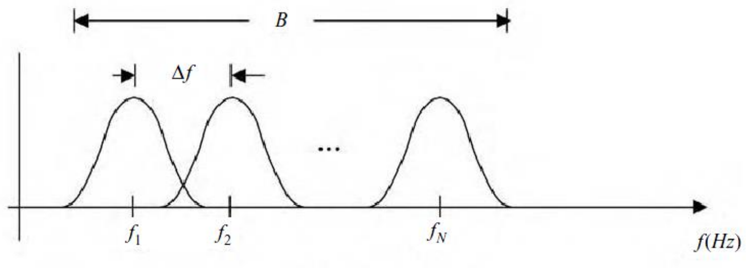

Figure 2.6: OFDM spectrum [25]

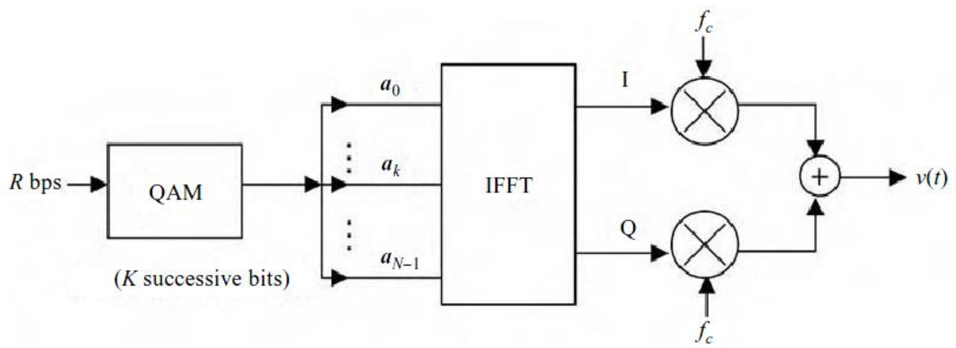

Figure 2.7: OFDM Subcarriers Generation [25]

The subcarriers are generated by OFDM by computing the IFFT of the incoming binary stream, supposing R bps. For this process, a bandwidth of BT = R(1+r) is required, where r represents the roll-off factor for Nyquist shaping. As a result, N bits are stored during an interval of T_s = N/R (OFDM symbol interval). Each N bit is used to modulate the carriers which are subsequently transmitted over the long interval of T_s simultaneously, as shown in Figure 2.6 [25].

The OFDM signal generation, in Figure 2.5, is achieved mathematically as:

$$S_s(t) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j(\omega_n t+\varphi_n)} \ , \ for \ \ t \in [kT,(k+1)T] \tag{2.9}$$

The transmitted data symbol which corresponds to the input symbol from modulation scheme, i.e. QAM, QPSK, BPSK,…etc, is $X[n] = A_n.e^{j\varphi_n}$. The signal generated from Equation 2.9 corresponds to one OFDM symbol with time T.

If the signal $S_s(t)$ is sampled at an interval of T_s seconds, then:

$$S_s(kT_s) = \frac{1}{N}\sum_{n=0}^{N-1} A_n e^{j\varphi_n} e^{jn\Delta\omega kT_s} \ , \ for \ \ k = 0,1,\ldots,N-1 \tag{2.10}$$

T_s is the sampling time and it must be equal to the duration of the serial symbol to be transmitted.

The OFDM process which includes generating the subcarriers and their outputs is shown in Figure 2.7. The formula which represents this process is the following:

$$a(n) = \sum_{k=0}^{N-1} a_k \, e^{\frac{j2\pi kn}{N}} \qquad\qquad n = 0,1,\ldots,N-1 \tag{2.11}$$

Where: $a(n)$ is the IDFT of ak; and ak: k = 1,…,N: successive bits stored.

If K < N successive binary digits be stored, generating one of 2^k possible QAM signals. Each signal corresponds to a complex number ak. It can be seen that Equation 2.11 represents the IFFT [25], and can be evaluated using FFT.

From Equations 2.9 and 2.10, the IFFT can be calculated by this formula:

$$g(kT_s) = \frac{1}{N}\sum_{n=0}^{N-1} G\left(\frac{n}{NT_s}\right) e^{\frac{jn2\pi k}{N}} \tag{2.12}$$

Equations 2.10 and 2.12 are equivalent if: $A_n . e^{j\varphi_n} = G\left(\frac{n}{NT_s}\right)$ and $\Delta\omega = \frac{2\pi}{T}$. The OFDM transmitted symbols can be seen as complex valued, frequency domain samples.

The discrete form of OFDM symbol can be generated by applying IFFT onto the data sequence of a given signal to be transmitted, as depicted in Figure 2.8.

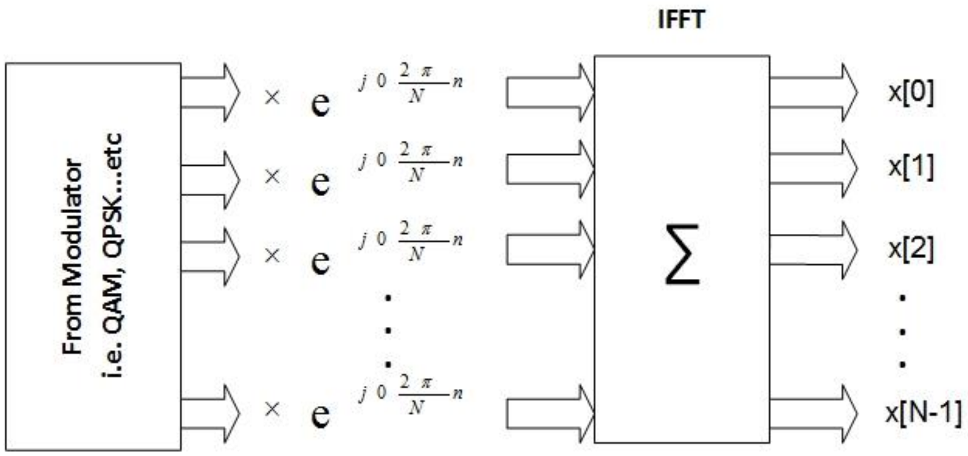

Figure 2.8: IFFT Calculation Clock in OFDM System

The formula to simplify the input data symbols from the modulator and calculating IFFT is the following:

$$x[n] = \sum_{k=0}^{N-1} X[k]. e^{jk\frac{2\pi}{N}n} , \qquad n = 0, 1, \ldots, N-1 \tag{2.13}$$

The orthogonal carriers in this equation are the complex exponential $e^{jk\frac{2\pi}{N}n}$, where N is the total number of multiple carriers used in transmission.

Finally, although OFDM has advantages it also has a number of disadvantages. According to Ramjee [22] these advantages and disadvantages are as follows:

- **Advantages:**
    - Bandwidth is used both sufficiently and efficiently.

- It is more resistant to frequency selective fading because the channel is divided into narrowband flat fading sub-channels.
- It is easier to equalise the channel than to use the adaptive equaliser in single carrier systems.
- Sampling time offsets offers less sensitivity than single carrier systems.
- Offers protection and is more robust against impulsive noise and Inter-Carrier Interference (ICI).
- ISI can be mitigated by guard intervals.
- Some lost symbols can be recovered, because of frequency selectivity, through interleaving and channel coding.

- **Disadvantages:**
    - OFDM is sensitive to amplitude change; although this is mitigated by using linear Radio Frequency (RF) amplifiers with high Peak to Average Power Ratio (PAPR), it consumes more power and is costly.
    - Because of the leakage of DFT, there is high sensitivity to carrier frequency offset and drift.
    - There is a relatively high PAPR in the time domain signal.
    - High level of sensitivity to the Doppler Effect.
    - It wastes some of the bandwidth due to the need for guard interval, unoccupied period to mitigate ISI between successive OFDM symbols.

Since OFDM is the baseline comparison for the proposed system in this research, the critical review chapter and OFDM and OWDM chapter will also explore further about OFDM system and its usability in communications application context. Additionally, in the analyses and comparison chapter, there will be a detailed discussion and an evaluation of the performance of OFDM and OWDM systems.

2.6.2 MULTIPLE ACCESS SCHEMES

Multiple access schemes were developed due to the rapid increase of bandwidth occupancy. Over the last four decades, multiple access communication systems have gone through cascaded developments from dividing time domain or frequency domain to assignment of distinctive code to each user. The core development in such scheme was the multiplexing technique, with minor adjustments to correlated encoding and data mapping. Unlike single access scheme, multiple access schemes enable multiple users to share the same spectrum at the same time without interference.

Since the discovery of Fourier transform, late 17th century, scientists were eager to utilise Fourier transform in communication context. In 1968, Change and Gibby [15] proposed the OFDM system, which is a single access technique, as a result of further development from Frequency Division Multiplexing (FDM). However, in the late 20th century, multiple access version of OFDM was discovered and in the early 21st century, this revolutionary discovery was first brought to life through mobility mode of IEEE 802.16 standard, commonly named as WiMAX [21]. Thereafter, it was deployed in other current state-of-the-art technologies, such as - Long-Term-Evolution (LTE), commonly referred to as 4G. In the following three subsections, the main three multiple access schemes will be discussed.

2.6.3 FREQUENCY DIVISION MULTIPLE ACCESS

Lee [26] says that the radio resource is divided by FDMA into frequency bands that do not overlap. These bands are distributed among the users of a FDMA system, which means that there is very little interference between users that have a well-designed filtering. Moreover, FDMA is employed in first generation wireless systems such as - mobile phone systems and Digital European Cordless Telephones (DECT).

The advantages and disadvantages of FDMA are the following [26]:

- **Advantages:**
  - Because FDMA is implemented in narrow band systems ISI is low and therefore, there is minimal or no equalisation needed.
  - System complexity and synchronisation overheads are low in comparison to TDMA.
- **Disadvantages:**
  - Expensive BandPass Filters (BPF) are needed.
  - Duplexers are needed because transmission and reception operate simultaneously.
  - Tight RF filtering is needed to reduce the adjacent channel interference.

2.6.4 TIME DIVISION MULTIPLE ACCESS

TDMA is technically which works by dividing the radio resource into non-overlapping time slots. Users in a TDMA system are allocated non-overlapping time slots, resulting in no interference between the users; while at the same time, the same frequency bands can be shared by numerous users.

TDMA has been utilised in second generation (2G) wireless systems for example, mobile communications interim standard IS-136, Global system for Mobile (GSM) communications; personal digital cellular systems and DECT standards for wireless phones. TDMA is also used for combat-net radio systems and satellites.

Lee [26] presents the advantages and disadvantages of TDMA as follows:

- **Advantages:**
  - Battery life is prolonged because transmission is not continuous and can be switched off when not being used.
  - The hand-off process is simple because it can listen in to other base stations during idle time slots.
  - Duplexers are not needed because different time slots for transmission and reception can be used.
  - Because there are different numbers of time slots per frame, bandwidth can be supplied to different users when required.
- **Disadvantages:**
  - Adaptive equalisation is often needed because users are only given short time slots and transmission rates are often high.
  - The guard time has to be minimised.
  - High time overhead synchronisation is needed because of the burst transmission.

2.6.5 CODE DIVISION MULTIPLE ACCESS

CDMA assigns distinct codes to users who share the network radio resource, which includes time slots and frequency bands. CDMA uses spread spectrum technology, together with a coding scheme. The code is a pseudo noise sequence characterised by a low cross correlation and the signal presents as noise similar to that of non-intended receivers. Numerous users can share the same radio resource without much coordination required, since the interference is spread over a large spectrum and only affects other users with background noise. CDMA systems fall into the following categories: Direct Sequence CDMA (DS-CDMA) and Frequency Hopped CDMA (FH-CDMA).

CDMA can be found in second Generation (2G) and third Generation (3G) wireless systems, including - Wideband Code Division Multiple Access (W-CDMA), United States Narrowband Spread Spectrum (IS-95), Third Generation Partnership Research (3GPP) and CDMA2000 (3GPP2). Lee [26] presents the advantages and disadvantages of CDMA as follows:

- **Advantages:**
  - Because of a soft capacity limit CDMA, the number of users is not limited; this is despite the fact that an increase in the number of users is met with a corresponding gradual degradation of system performance.
  - CDMA significantly reduces multipath fading because the signal is distributed over a large spectrum.
  - Soft hands-off can be carried out, where each user can be monitored by a number of different base stations and less users drop out in comparison to other hands-off procedures.
- **Disadvantages:**
  - Self-jamming occurs as the sequence spread of different users is not precisely orthogonal in nature.
  - The near-far problem may occur at a CDMA receiver if an unwanted user has a more easily detected power in comparison to a desired user.

2.6.6 ORTHOGONAL FREQUENCY DIVISION MULTIPLE ACCESS

Orthogonal Frequency Division Multiple Access (OFDMA) is a multi-user version of OFDM. In general, this technique was developed with the combination of two modulation techniques together - FDMA, described in Section 2.6.3 and TDMA, described in Section 2.6.4. Multiuser facility is achieved by the assignment of subsets of subcarriers to users individually, as illustrated in Figure 2.9, allowing for a simultaneous low data rate for a large number of users. A number of subcarriers can be assigned individually to various users, to help error coding and data rate control for each user. The OFDMA spectrum is similar to the CDMA spectrum, in which, users can obtain various data rates through the use of spreading codes.

The main application that OFDMA is used within is for LAN based on 802.16 MAN standards. The mathematical formula for OFDMA is as follows [27]:

$$V(t) = \sum_{0}^{N-1} l\, n(t) \sin(2\pi t) \tag{2.14}$$

This is the formula for the summation of all the signals that are transmitted through. OFDMA divides the bandwidth resource into overlapped but orthogonal frequency bands. The orthogonal frequency bands, not necessarily single and adjacent, are exclusively allocated to users and users transmit signals on the assigned subcarriers. The data of each user is divided into several parallel data streams and modulated on the multiple subcarriers allocated to the user in an OFDMA system. Due to orthogonality, there is almost no interference caused between users with well-designed filtering and acceptable frequency offset. Moreover, differing numbers of subcarriers can be allocated to users depending on their QoS requirements.

OFDMA is popularly used in the Fourth Generation wireless systems of wideband communications such as - WiMAX; 3GPP; Long Term Evolution (LTE), also known as 4G; Evolved Universal Terrestrial Radio Access, known as E-UTRA and a candidate access method for the IEEE 802.22 Wireless Regional Area Networks, also known as WRAN.

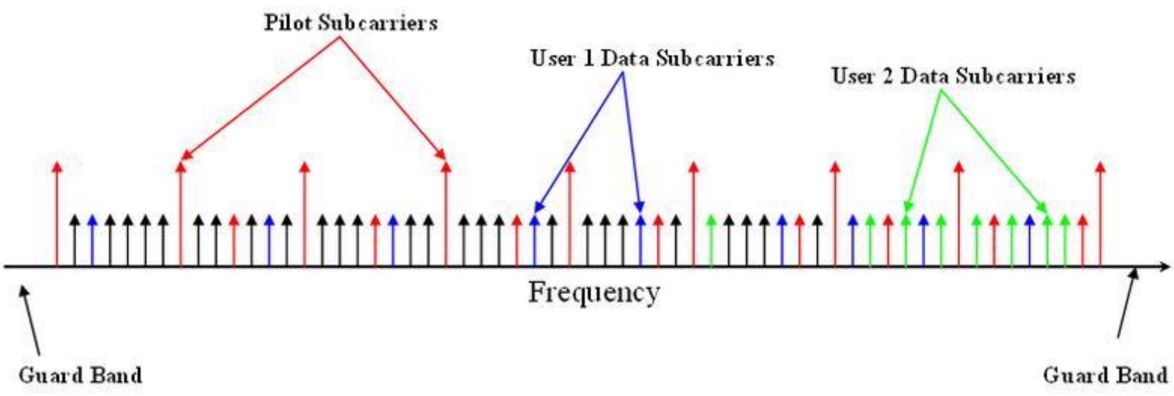

Figure 2.9: Subsets of Carriers for Individual Users for OFDMA [27]

According to Heiskala and Terry [8], the advantages and disadvantages of OFDMA are as follows:

- **Advantages:**
  - As OFDMA is a two dimensional multiple access, which means time and frequency, multiple users have the facility of individual sub-channels for multiple access, which helps to reduce interference; and ICI is kept to minimum. This is achieved by using selective fading techniques of the signal and having receiver simplicity.
  - High frequency efficiency can be achieved since overlapped frequency bands are allocated for users.
  - Simple equalisation is possible since the subcarriers are narrow enough to be assumed flat.
  - ISI can be rapidly reduced easily by guard intervals since the symbol duration on each subcarrier can be longer than for single-tone modulation schemes.
  - Implementation is efficient using IFFT and FFT.
  - Radio resources can be allocated easily to users on demand since a different number of subcarriers can be allocated to different users.
  - The capacity of the channels has increased. OFDMA allows a number of different users to transmit over different portions of the spectrum, thereby increasing the capacity.
  - Scheduling improvement: for the same reason that the capacity has increased, the scheduling of multiusers has also increased.
  - High spectral efficiency, for example, the bit error rate is more efficient within the fading environment.
  - The sub-channels are flexible, giving the following benefits:
    - Variable bandwidths are supported through a scalable structure.
    - Beam forming can be performed through continuous assignment and adaptive modulation code.
    - Pseudo-random permutation and partial usage of sub-channels can be performed for frequency diversity.

- **Disadvantages:**
  - Performance is sensitive to the frequency offset caused by the Doppler shift or frequency synchronisation in order to maintain the orthogonality among the subcarriers.
  - Carrier frequency error can occur due to the narrow subcarrier spacing. Tight synchronisation is required between users for FFT within the receiver.
  - The efficiency is reduced due to a high PAPR. It can be calculated using the following formula [28]: $PAPR = \frac{|x(t)|2}{Pavg}$
  - Large amplitude variations increase the in-band noise when the signal goes through amplifier non-linearity.
  - Linear transmitter circuitry is required due to the high PAPR.
  - Efficiency is reduced by guard intervals.

Single antenna transmitters/receivers which have size, cost or hardware limitations for multiple antennas equipment in a multiuser environment can share their antennas and generate virtual Multiple Input Multiple Output (MIMO) systems with their cooperative actions, called cooperative communication [29]. Since the work of Cover and Gamal [29] on the concept of relaying, many cooperation techniques have been proposed, such as - decode and forward and amplify and forward [30]; compress and forward signalling [31]; coded cooperation; space-time coded cooperation; collaborative beam-forming; asynchronous cooperation and cross-layered approaches.

Cooperative communication within ad-hoc networks increase the QoS between nodes, since the MIMO systems have the advantage of the single input single output systems in the achievable rate, bit error rate, block error rate or outage probability by their spatial multiplexing gain.

Accordingly, cooperative transmitters and/or receivers can communicate through a longer distance than a non-cooperative transmitter and receiver can with the same quality. Equivalently, a smaller number of relay nodes can guarantee the same quality by their cooperative actions within the same region. Since asymptotic results indicate that cooperation does not change non-scalability [32], there remains the question regarding by which constant factor can cooperation boost capacity.

This is an attempt to answer whether it is worth trying to devise practical cooperative schemes. In some scenarios that are considered, the answer is clearly no, while in others, there may be significant benefits.

Whilst it is well-known that optimal MIMO schemes can improve capacity in collocated transmitter and receiver sets; in ad-hoc networks, there is a cost for cooperation; in the form of the exchange of information among the cooperatively transmitting nodes and receiving nodes, there is an additional infrastructure cost to supply synchronism, which is not considered here. Thus, we may get little benefit from cooperation due to the geographic spread of possible partners as compared to the distance to be traversed. Conversely, in a multi-hop route, one link may limit overall capacity and improving it may have a substantial benefits.

Power control plays a critical role in improving the performance of communication systems. Effective transmitter power control increases the overall network capacity in many situations. More transmitters can communicate simultaneously with their intended receivers in a network and their batteries can live longer since effective power control keeps transmitters from spending redundant power; whilst guaranteeing required link qualities if they are feasible in the network.

In the papers by Grandhi *et al.,* [32] and Cover and Gamal [29], centralised power control algorithms were studied. Thus, an eigenvalue problem for the optimal transmitting power was resolved. The optimal power vector was found as an eigenvector associated with a positive eigenvalue of a scaled cross-link gain matrix.

Distributed asynchronous power control algorithms were proposed by Foschini and Miljanic [33] and Mitra [34], due to the computational complexity and difficulty of gathering overall information of a network in centralised algorithms; the distributed algorithms were thus considered very attractive. Many interesting distributed power control algorithms with realistic constraints have also been proposed. Considering an upper limit of transmitting power, Cover and Gamal [29] and Grandhi *et al*. [32] proposed a distributed algorithm which protected active links with realistic call arrivals.

Centralised algorithms require added infrastructure and suffer from latency and network vulnerability. Distributed algorithms are inescapably iterative and the convergence speed is one of the most important criteria along with the stability. The question thus naturally arises as to which algorithm improves the network capacity more. The answer must be dependent on the capability of the central infrastructure of a network system and the relative latencies of centralised control and distributed iterations. If a trade-off between the latency from the centralisation and distributed iterations is investigated, then the minimum overall latency can be obtained and it will provide the answer to improve the network capacity.

In this section, a cooperative power control algorithm and an analysis of its convergence, as proposed by Foschini and Miljanic [33] and Mitra [34], is discussed. Only a single group of certain links may be capable of cooperating or multiple groups of links may be capable of cooperating independently. The cooperative groups share information on transmitting power, locally measured interference and cross-link gains. In real wireless network systems, it is not always possible to figure out the complete cross-link gains, rather the power and interferences, especially in ad-hoc network systems.

The cooperative power control algorithm, proposed by Foschini and Miljanic [33] and Mitra [34], here works well for a generalised scenario, with locally-cooperating multiple independent groups sharing incomplete information of cross-link gains. The enhancement on the convergence speed of the cooperative algorithm is proved with eigenvalue analysis.

Since the demand for mobile communications is rapidly increasing nowadays, OFDMA and MIMO OFDMA systems are being researched to develop the next generation of mobile communications. Nevertheless, utilising asymmetrical wavelets, as proposed in this research, in MIMO system yet present a challenging issues of synchronisation, channel estimation and equalisation. There will be an attempt to investigate the use of wavelets in OFDMA system making use of the current LTE standard, which uses OFDMA in the downlink stream.

2.7 SUMMARY

In summary, a survey of communications and telecommunications systems were conducted in this chapter. The techniques used among those systems were not only investigated, but the chapter also reviewed their strengths and weaknesses. There were a series of unstoppable developments of modulation schemes and telecommunications systems that addressed and solved whatever weaknesses or issues the predecessor systems suffered from, since the first built communications system till this moment. The solutions proposed in previous researches related to modulation techniques were employed in different applications and approaches than the proposed application and approach of this research. Additionally, most of those literatures did not address the flexibility of bandwidth allocation; they just addressed the computation complexity of what were proposed and some of other mitigations of interference and noise reduction.

There will be a comparison of previous results with the experimental results from the present research at the end of this research. The theoretical notions derived from other works could be used to assist the author to get appropriate mathematical derivations that can be used in the experiment of this research, for example, from the studies by Lee [26] and Rappaport [35]. This chapter is a literature review of the state-of-the-art technologies employed currently in communications systems. In the following chapter, there will be a critical literature survey of the proposals and techniques that were closest in similarity to that of this research with critical comparison and evaluation.

CHAPTER 3: CRITICAL REVIEW OF WAVELET AS A MODULATION TECHNIQUE

Objectives:

- To present a critical review discrete Wavelet Packet Transform.
- To give a literature review of the state-of-the-art proposals of Wavelet Transform in signal transmission context.
- To present the current open research questions surrounding Wavelet Transform utilisations in signal transmission.
- To compare and contrast Discrete Wavelet Transform with Discrete Cosine Transform.
- To reveal the gap in the current knowledge and describe how this research differ and advance current knowledge.

3.1 INTRODUCTION

In this chapter, a critical review of researches that previously suggested the utilisation of DWT as modulation technique for signalling and to transmit signals over channels will be presented. A critical evaluation and discussion is carried out and compared against what this research is proposing. Various research papers have already proposed the usage of DWT as modulation technique for signal transmission for different applications and over various channel conditions. Most of those have claimed that it is a sufficient alternative to OFDM with proving that it is less computationally complex and faster in processing time. Others proved that guard intervals are needed due to the usage of filter banks to separate signals and prevent adjacent signals from overlapping. More detailed discussions are provided within this chapter in the next few sections.

This brand new modulation scheme for signal transmission for communications systems, mainly OFDM systems, was first discussed and patented in 2006 [36] and since then, scientists and researchers have been attracted by this field to investigate this new modulation scheme in various communications applications. In this chapter, a critical review of the latest researches from 2006 until the submission of this research will be discussed (March, 2016). However, a considerable number of research questions thus far are yet to be answered concerning this particular field of research.

3.2 DISCRETE WAVELET AS A MODULATION TECHNIQUE

DWT became increasingly popular in Digital Signal Processing (DSP) and Digital Image Processing (DIP) since the start of overseeing the effectiveness of its computational complexity against DFT in 1960s. In DIP Joint Photographer Experts Group (JPEG) committee announced JPEG2000 standard [37] in 2000, with the state-of-the-art wavelet transform replacing the previously used Discrete Cosine Transform (DCT). Since the early 2000s until now, many scientists and researchers investigated this area and until now, there are numerous researches suggesting to utilise DWT in DSP for signalling and transmission techniques.

Wavelet Packet Division Multiplexing (WPDM) was first researched by Wong *et. al.* [38] in 2000, in their mathematically oriented paper and they suggested the utilisation of wavelet packet modulation in impulsive and Gaussian noise environments. He claimed that WPDM is a high capacity, robust and flexible multiple signal transmission scheme, where the message to be transmitted is waveform multiplexed into wavelet packet basis functions. They derived a mathematical expression of the probability of error for WPDM technique in the presence of Gaussian and impulsive noise. They proved that WPDM can provide a sufficient robustness against Gaussian and impulsive noise sources than Time Division Multiplexing (TDM) and OFDM. Since these mathematical formulas are not directly related to what this research is proposing, they will not be included here, but they can be found in [38]. They concluded their derivation with that WPDM waveforms overlap in time and hence, the energy of any burst of impulsive noise will be dispersed over multiple bits, therefore, any strong noise that causes an error in single bit, in TMD, could be efficiently dispersed in WPDM and hence no error occurs. In the summer of 2006, Jain and Meyers were granted a patent named “Communication System Using Orthogonal Wavelet Division Multiplexing (OWDM) and OWDM-Spread Spectrum (OWSS) Signalling” from the United States Patent Office [36].

They proposed an OWDM system that utilises wavelet transform in the form of DWPM which uses a bank of filters comprising of HPF and LPF, as in Figure 3.1, for the decomposition in the transmitter. The process in the synthesis stage is reversed to reconstruct the original signal in the receiver side, as in Figure 3.2. The input stream of r(n) is equally distributed between the two filters HPF and LPF, then down sampling occurs to maintain the data length. When an input symbol from the modulation scheme, i.e. QPSK, QAM etc, is entered into the filter banks, A_i is spread over equally and up sampled, to maintain the data length, then it is passed into HPF and LPF filters through multiple levels and at the last level, into a summation to combine all pulses together, called supersymbol. However, in the receiver side, the process is reversed and the received supersymbol is entered into the same length, but reversed, filters banks to redistribute the signal, with down sampling after each level to maintain the date length. Then at the last level, y_i symbols are reversed and entered into demodulation same as that used in the transmitter.

In summary of this patent [36], filters banks were oriented equally, in which, signal spectrum will be similar to that of OFDM, however, with the advantage of analysing signals in time-frequency domain rather than just the frequency domain in OFDM. Another interesting advantage that the authors claimed is that the computational complexity is less than that for FFT in OFDM system. More about complexity of DWT and DFT will be discussed in Chapter 5.

Since the patent did not include much investigation of the applicability of DWPT in real communications systems applications with direct comparisons of performance and throughout against the currently used OFDM systems, scientists and researchers targeted this area to further investigate the efficiency of DWPT as a modulation scheme. Many scientists and researchers discussed the usability and efficiency of DWPT and compared it to FFT in OFDM. Some proved the claims of complexity reduction made earlier, by Jain and Meyers in their patent and some went to investigate the elimination of guard intervals. Since there are tens of papers that discussed this apart from data relevance and space management, most relative and significant researches are discussed in the following paragraphs and sections.

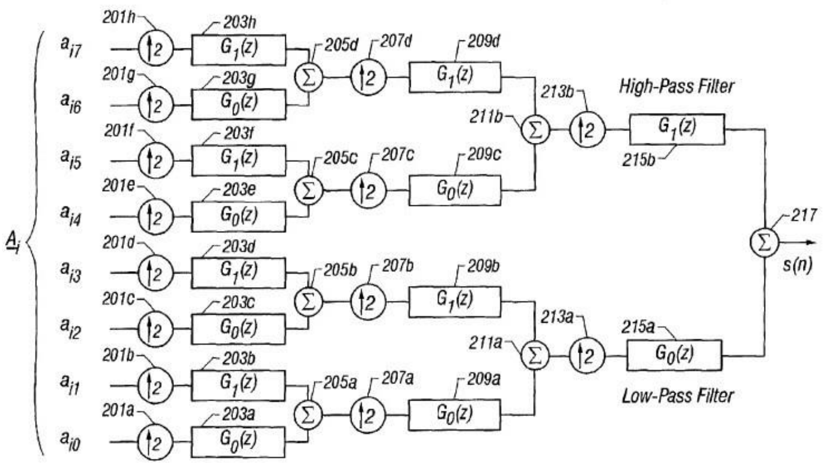

Figure 3.1: DWPT Synthesis Filters Banks [36]

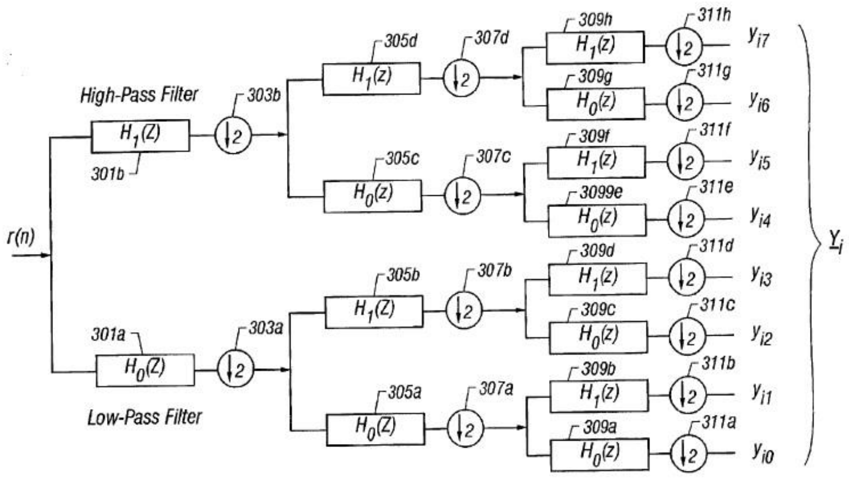

Figure 3.2: DWPT Reconstruction Filters Banks [36]

Rana *et. al.* [39], in 2008, proposed the utilisation of DWPT to estimate SNR in OFDM systems. They proposed a DWPT SNR estimator that performs the estimation process after the FFT process. The proposed estimator works by considering the different noise power levels over OFDM subcarriers. The main idea of their proposal was that they combined FFT and DWPT together within the receiver to estimate SNR in OFDM systems more accurately; however, they claimed that the computational complexity of both schemes combined is $O(Nlog_2N)$ where O is the complexity of wavelet family used and $Nlog_2N$ is the complexity of Fourier transform. Although this paper proposes the utilisation of DWPT in applications not directly of interest of this research, the commonality is that the complexity of DWPT is proved to be the same as the Jain and Meyers patent [36] suggests.

Abdullah and Hussain [4], in 2009, studied DWT-OFDM performance and compared it with FFT-OFDM. Although they called their proposal DWT-OFDM, however, they used WPT which is a replica of Jain and Meyer's patent [36]. Their paper was mainly a review of some wavelets' performance and compared to FFT-OFDM. They found that DWT-OFDM significantly outperformed FFT-OFDM, especially when using biorthogonal with 5 vanishing moments at synthesis and reconstruction filter banks and reverse biorthogonal with 3 vanishing moments at both synthesis, in the transmitter and reconstruction, in the receiver, filters banks.

In the same year, though, Zbydniewski and Zieliński [40] proposed wavelet-OFDM and circular wavelet-OFDM to be used in high speed communications over power lines. They examined the performance of circular wavelet-OFDM which uses guard intervals in wavelet based OFDM and compared it to that of wavelet based OFDM without guard interval and also against the standard OFDM. The application they simulated is high speed power line communications system. Their main objectives were to measure PAPR effect since the conventional OFDM is sensitive to PAPR. They considered AWGN, coloured noise and narrowband interference with and without coloured noise in their channel models. They concluded their study by stating that conventional OFDM system perform better than waveletbased OFDM and circular wavelet based OFDM in the presence of both or either coloured noise and narrowband interference. However, in long impulsive disturbances, wavelet and circular wavelet based OFDM have an advantage over conventional OFDM. Wavelet based

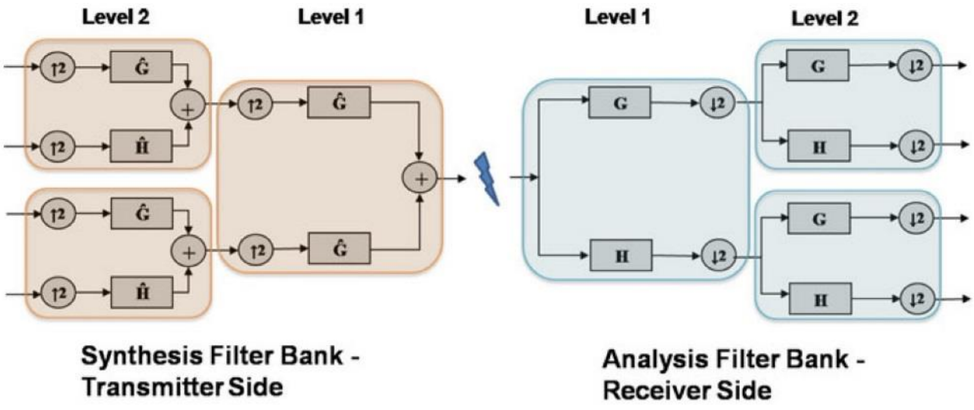

Figure 3.3: WPMCM, Lakshmanan *et. al.* [41] Proposal

OFDM has simple equaliser and perform better than the other two systems not only under coloured noise but also under impulsive disturbances [40].

In 2011 Lakshmanan *et. al.* [41] investigated the sensitivity of DWPM for time synchronisation error. They claimed that Wavelet Packet Multicarrier Modulation (WPMCM) is a strong candidate that could replace the well-known OFDM. However, their proposed system is a replica of Jain and Meyers's patent [36], Figure 3.3 shows their proposed system. Nevertheless, they answered one of the questions that concerns any communication system, particularly multicarrier wireless communication systems. The vulnerability of WPMCM to time-synchronisation error was addressed and compared to that of OFDM. Time synchronisation error, in OFDM systems, is modelled by taking the received data samples and shifting them to right or left by a time offset of $t_{\hat{\varepsilon}}$ where the sign of $t_{\hat{\varepsilon}}$ determine the direction of shift. It is well known that time offset is the main cause of ISI and ICI in any multicarrier systems; Figure 3.4 illustrates time offset. They claimed that one of the main differences between WPMCM and OFDM is timing errors. Although the length of symbols in WPMCM

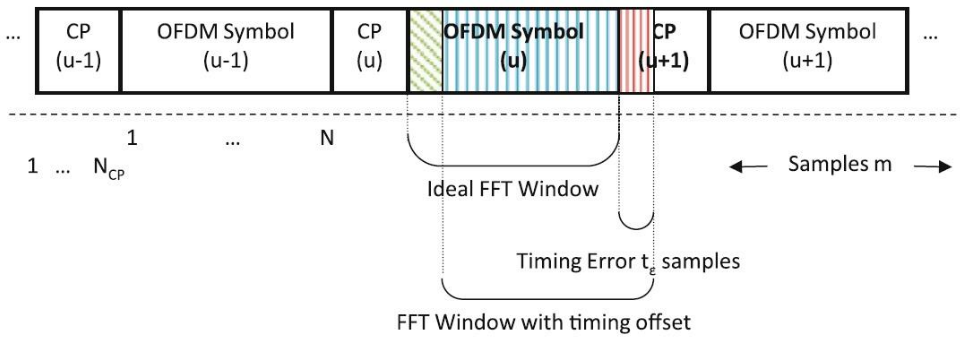

Figure 3.4: Timing offset shift to the right [41]

is determined by the wavelet family used, generally it is significantly longer than that in OFDM. This symbol length, however, does not excessively expand the frame size since it allows symbols to overlap with each other without causing ISI. However, to mitigate ISI in OFDM, Cyclic Prefix (CP) is used before or after each OFDM symbol, as illustrated in Figure 3.4. This CP will significantly increase the performance of OFDM systems, given that the time offset is no longer than the used CP and the direction of time offset is toward the CP associated with that particular OFDM symbol. They concluded their research with that WPMCM shares most of the characteristics of orthogonal multi carrier systems, but with interesting advantages of guard interval elimination and reduced complexity; but on the other hand, it is highly sensitive to timing offset synchronisation. They recommended that WPMCM is a relatively brand new modulation or multiplexing scheme that still has a significant number of questions yet to be answered before it can be seen into practice [41].

Channel equalisation is one way of dramatically reducing time synchronisation error and still one of the questions that need to be answered when using DWPM, DPMCM and WPM-OFDM modulation techniques. Of late, in 2011 Bajpai *et. al.* [42] proposed a methodology for channel equalisation for WPM by minimisation of peak distortion. They proposed a time domain equaliser to compensate for distortions occurring in the channel and minimising ISI, when transmitting WPM subcarriers. According to them, assuming a data sequence of a length of N, $x[n] = [x_0\ x_1\ x_2\ x_3 \dots x_{N-1}]$ is transmitted through the channel and the channel has an impulse response of length L_c with coefficients $c[n] = [c_0\ c_1\ c_2\ c_3 \dots c_{L_c-1}]$, the received signal $y[n]$ can be expressed as: $y[n] = x[n] * c[n] + \eta[n]$, where $*$ is the convolutional operator and $\eta[n]$ is AWGN. Assuming that the impulse response of the channel is represented by c_n and the impulse response of the equaliser is e_n, a single equivalent filter of q_n can be defined as [42]:

$$q_n = \sum_{j=-\infty}^{\infty} e_j c_{n-j} \tag{3.1}$$

There are two possible scenarios: the first when the equaliser has infinite number of taps and the second when the equaliser has finite number of taps. They looked at both scenarios. When the equaliser has an infinite number of taps, the output at the k number of sampling instance could be expressed as [42] [43]:

$$\hat{I}_k = q_0 I_k + \sum_{n \neq k} I_n q_{k-n} + \sum_{j=-\infty}^{\infty} e_j \, \eta_{k-j} \tag{3.2}$$

Where I_k is the desired symbol scaled by a factor of q_0, the first summation represent ISI and the second summation is AWGN. The peak value of the distortion $\Omega(e)$ is defined by [42] [43]:

$$\Omega(e) = \sum_{n=-\infty, n \neq 0}^{\infty} |q_n| = \sum_{n=-\infty, n \neq 0}^{\infty} \left| \sum_{j=-\infty}^{\infty} e_j c_{n-j} \right| \tag{3.3}$$

Where $\Omega(e)$ is the function of the equaliser's tap weights. An equaliser with infinite taps is possible by selecting the tap weights by choosing $\Omega(e) = 0$, which will entirely eliminate ISI. For this scenario, the tap weights can be determined by [42]:

$$q_n = \sum_{j=-\infty}^{\infty} e_j \, c_{n-j} = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{3.4}$$

However, in the frequency domain this expression, 3.4, can be written as:

$$Q(f) = E(f)\, C(f) = 1 \quad \text{or can be} \quad E(f) = 1/C(f) \tag{3.5}$$

It is derived from that for the entire elimination of the ISI, the equaliser must be the inverse of the channel filter; and that is why the peak distortion criterion is sometimes referred to as zero forcing equalization. In the case of equaliser with finite taps of $2M$+1, when $e_j = 0$ for $|j| > M$ the convolution of c_n with e_n is equal to zero outside the period $-M \leq n \leq M + L_c - 1$, that is the same as $q_n = 0$ for $n < -M$ and $n > M + L_c - 1$, where L_c is the length of the channel. When normalizing q_0 to unity, the peak distortion is [42]:

$$\Omega(e) = \sum_{n=M, n \neq 0}^{M+L_c-1} |q_n| = \sum_{n=M, n \neq 0}^{M+L_c-1} \left| \sum_{j=-\infty} e_j c_{n-j} \right| \tag{3.6}$$

When the equaliser has $2M$+1 adjustable parameters, there are some non-zero values at q_n and hence, ISI cannot be entirely eliminated and there will always be some residual remaining, even when optimal coefficients are chosen [42]. They concluded their paper by stating that their proposed equaliser performed with significantly better performance as compared to FFT-OFDM. Their proposed equaliser focused on removing ISI, but not ICI.

In March 2013, Kanti and Rai [44] did a comparative study of using two Daubechies wavelets members (db1, Haar) and (db2) over DVB-T systems. The filter bank they used was a replica of Jain and Meyer's [36] patent. However, their simulation results showed that both (db1 and db2) wavelets outperformed OFDM, with db1 having the best performance among all. Their system has neither error detection and correction code nor interleaving and there was no mention of the usage of guard intervals.

During the conduct of this research, a paper published on the 21st March 2014 by Liu and Hua [45] came to the attention of the author of this thesis. Liu and Hua [45] proposed a novel approach to compute DWT by computing the first order moments without multiplications. They claimed that their algorithm is novel and simpler to compute than the existing methods of DWT computation. They concluded their research by claiming that DWT can be computed by Discrete First Order Moment (DFOM) and with just addition and without multiplications. Since this research uses filter banks to compute DWT and is not interested in other methods, this is not explored further; but this can be used in future works, the details of Liu and Hua algorithm can be found in [45].

Following that research paper, in May in the same year 2014, another research paper was published by Mohanty and Ramavath [46] trying to address one of the open questions about DWT OFDM based modulation scheme. Since PAPR is one of the major issues of MCM systems, i.e. OFDM, they investigated PAPR reduction in DWT OFDM based systems. They proposed a computation of PAPR derived from that already used in OFDM and performed an experimental analysis of their proposal. The "Wavelet Commanding Technique", as they called it, has a PAPR boundary condition when in wavelet based OFDM, which can be a function of wave shape given by [46]:

$$PAPR \ \leq N \max \left| \Psi_{k,p}^{syn}(t - 1N \right|^2 \tag{3.7}$$

In OFDM, in a complex basis function, the upper bound of PAPR is $\leq N$ and hence, in the MCM with fixed number of N subcarriers, the PAPR values depends on the nature of the used bases [46]. Therefore, there exists a possibility of minimising PAPR by choosing the wave shape $\Psi_{k,p}^{syn}(t)$ that best reduces the $\max\left|\Psi_{k,p}^{syn}(t - 1N\right|$ [47]. They concluded their proposal by claiming that their companding wavelet scheme for PAPR minimisation in OFDM system has provided a significantly better performance than conventional FFT based OFDM. Additionally, they claimed that the companding wavelet scheme has lower implementation complexity and lower constraints on subcarriers [46].

During the process of updating the critical review of this research, a paper was published in December 2014 by Gutierrez *et. al.* [48], who designed a systematic transmitter and receiver architecture for flexible filter banks for MCM. In short, they replicated Jain and Meyers' patented system [36], nevertheless, the flexibility they claimed in their research is defined by the usage of multiple filter banks instead of one filter bank to modulate signals and assigning signals to the sub-bands, paths through filters, that are desired but keep the unwanted sub-bands unoccupied. However, this type of sub-banding will still use those bands in which no signal was assigned and give it null carrier, which in communication principle, is still occupied and will be transmitted with a null value. The difference between this research and Gutierrez *et. al.* [48] is that the added flexibility of filter banks in this research will not waste the sub-band and will allocate the sub-bands dynamically according to the demand and desire.

Less than two months right after the last research paper, in July in the same year 2014, a research article was published by Suma *et. al.* [49] in the IET Communications Journal. They were also addressing the open research question of PAPR in DWT OFDM based. The method they proposed was for reducing PAPR by utilising Coded Discrete Cosine Harmonic Wavelet Packet Transform (CDCHWPT). The performance of their proposed technique significantly

Table 3.1: Processing Simplicity and Computational Complexity of DCHWP as proposed by [59]

OFDM system	processing steps and simulation time (for simulation condition in Table 1)	computational complexity in terms of multiplications
DWPT-IDWPT	decimation/ interpolation/filtering (depends on decomposition level)	$LN\ \mathrm{Log}_2 N$ (L = length of filter)
Haar	1.34 s	
DB4	1.43 s	
DCHWPT – with DCT/IDCT	only grouping of coefficients, no decimation, interpolation, filtering 0.2 s	$N/2\ \mathrm{Log}_2 N$

outperformed the conventional Fourier based OFDM system with 13 dB and 32 dB required to achieve 0.001 BER, respectively. They also compared coded and uncoded DCHWP OFDM based systems together, with 6 dB and 11 dB respectively required to achieve 0.0001 BER. They also claimed that their proposed DCHWP OFDM based system is not only simpler in processing efficiency, but also simpler in computational complexity as compared to conventional OFDM. The processing and computational simplicity of their proposed system is summarised in Table 3.1 [49]. However, the wavelet filter banks they were altering to compute DCHWP is similar to Jain and Meyers' [36] patented system, but with some additional DCT computation for added PAPR mitigation and with just Haar and db4 examined, unlike the proposal of this thesis, where an added flexibility to the filters banks is investigated with the use of all wavelet families and families' members under various channel conditions.

Having scoured the scholar libraries in the process of overviewing what has updated in or around this research field, in February 2015, a further research article was published by Suma *et. al.* [50]. This article is connected with the research conducted earlier in DCHWT based OFDM by the same authors, which was discussed earlier in this section. However, what is new in this article is that the authors employed new filter banks trees and they called it "Analytical Discrete Cosine Harmonic Wavelet Transform (ADCHWT)". In their proposed system, they employed dual tree complex wavelets, depicted in Figure 3.5. Although their proposed system outperformed conventional OFDM system it is, however, well known that complex wavelet has a very high memory usage due to high coefficients storage and as the subcarriers increases, the memory needed to store and buffer the data will dramatically increase as well. Additionally, it is apparent from their system diagram, in Figure 3.5, that multiple multiplications and

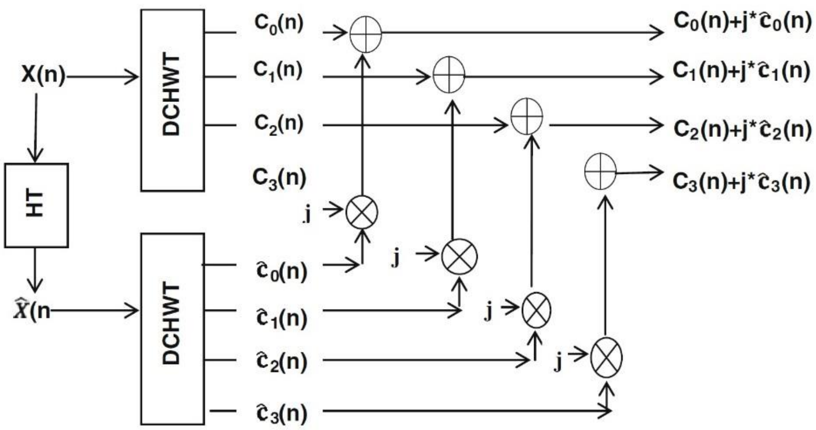

Figure 3.5: ADCHWT Proposed by Suma *et. al.* [50]

additions can cause latency in the systems, as transmission systems need real time communications with almost zero tolerance latency. Nevertheless, this is not the case with this thesis, where a simple wavelet transform with adaptive filters banks are used, alongside examining all available wavelet families and families' members performance under different channel conditions.

3.3 WAVELET TRANSFORM FOR ENCRYPTION (SECURITY)

With the dramatic increase in wireless communications usage across the world, the threat of bypassing security and cybernetic crimes has also increased. The need for more sophisticated security technologies and encryption is recently more demanded than ever before. There are many security protocols and methods in practice these days and others still in the research and development stage. A secure communication system is defined by any point-to-point, point-to-multipoint or multipoint-to-multipoint communication where no other party can intercept or have any means of access to the content being exchanged between the intended users. There are many tools and techniques across various layers to achieve secure communication, e.g. encryption, steganography or identity-based communication.

One of the recently proposed encryption methods is using DWT and Multi-Resolution Analysis (MRA). This area of research, however, was intended to be researched within this thesis; nevertheless, in the beginning of 2012, a research paper was published by Zheng and Huang [51] had a similar idea of what had been planned. Secure Signal Processing (SSP) is basically doing the DSP in the encrypted domain where metadata or watermarking is embedded into the transmitted data for added security and privacy. There are many cryptosystems around nowadays; however, homomorthic cryptosystem [52] proposed an encryption system as an encryption function which permits ciphertext operations without decrypting it into plaintext. Zheng and Huang [51] used this system as baseline for their proposed system. Since the filter coefficients are real numbers in wavelet filter banks, in the encrypted domain some suitable integer coefficients needs to be used instead. According to Mallat [53], in his algorithm, DWT can be recursively defined as:

$$a_j(k) = \frac{1}{\sqrt{2}} \sum_{l \in \mathbb{Z}} h_d(2k - l) a_{j-1}(l) \tag{3.8}$$

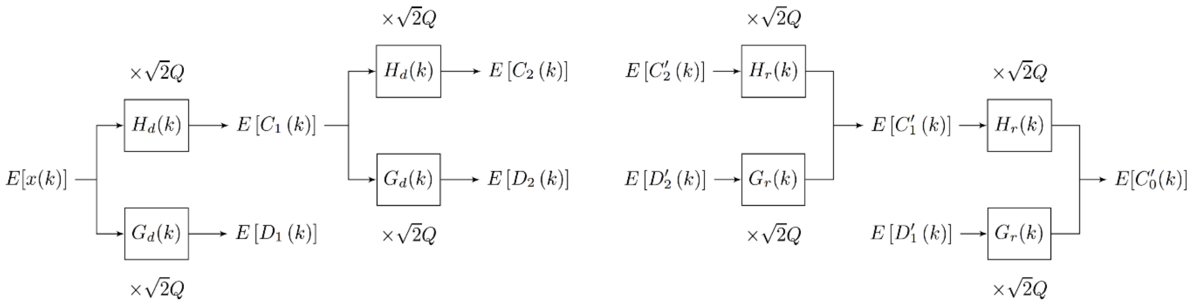

Figure 3.6: DWT and IDWT in the Encrypted Domain [51]

$$d_j(k) = \frac{1}{\sqrt{2}} \sum_{l \in \mathbb{Z}} g_d(2k - l) a_{j-1}(l) \tag{3.9}$$

Where $h_d(k)$ and $g_d(k)$ are the LPF and HPF decomposition coefficients, respectively; j=1, 2, ..., N is the level of decomposition filter bank and $a_j(k)$ and $d_j(k)$ are the approximation and details coefficients, respectively.

By quantizing $h_d(k)$ and $g_d(k)$ these suitable integer coefficients can be obtained and the quantization operation can be expressed as:

$$H_d(k) = \lfloor Q h_d(k) \rceil \tag{3.10}$$

$$G_d(k) = \lfloor Q g_d(k) \rceil \tag{3.11}$$

Where Q is the scaling factor and can be obtained from the application used and $\lfloor . \rceil$ is the rounding function. The recursive integer approximation for DWT can be expressed as:

$$A_j(k) = \sum_{l \in \mathbb{Z}} H_d(2k - l) A_{j-1}(l) \tag{3.12}$$

$$D_j(k) = \sum_{l \in \mathbb{Z}} G_d(2k - l) A_{j-1}(l) \tag{3.13}$$

Where j = 1, 2, …, N and $A_0(l)$ is equal to $x(l)$.

Since all previous computations are multiplications and additions, Equations 3.11 and 3.12 can both be computed in the encrypted domain by utilising homomorphic properties for further analysis of these equations and derivations, please see [51]. Their proposed system is depicted in Figure 3.6.

They concluded their research by claiming that DWT in the encrypted domain will significantly expand the plain value of signals. They found out that one of the limitations of using this technique is the large storage and computational overhead.

In the meantime, from 2012 onwards, there have been many other proposals of utilising DWT for security applications and also employing two dimensional DWT to encrypt data; this field of research is still in its infancy and has many research questions yet to be answered.

3.4 MULTIWAVELETS MODULATION TECHNIQUE

Multiwavelets transform is sometime referred to as two dimensional wavelet transform, since it uses two dimensional set of filters coefficients in a matrix form. As opposed to wavelet transform, which has a single wavelet and a single scaling function, multiwavelets has a set of multiple wavelets and multiple scaling functions. Since the continuous form of wavelet transform is unrealistic in DSP, the discrete form of wavelet transform is used here in multiwavelet forms. Discrete MultiWavelet Transform (DMWT) was around since 2001, when Martin and Bell [54] proposed Multiwavelets for image compression in DIP. In this section, a critical review of the recently researched topic of using multiwavelets in DSP, and in particular, as a modulation technique as an alternative to OFDM is discussed and evaluated against the investigation of this research.

In 2010 and again in 2013, Kattoush *et. al.* [5] and [55] have investigated the performance of wavelets and mutliwavelets in OFDM systems over different channel conditions. The main contribution of their research is the proposal of Discrete Multi-Wavelets Transform (DMWT) to be used in OFDM systems as an alternative to IFFT and FFT. Although wavelet transform has single associated scaling $\emptyset(t)$ and a single wavelet $\psi(t)$ functions, multiwavelets have multiple scaling and wavelet functions. The set of scaling functions could be written as: $\emptyset(t) = [\emptyset_1(t)\ \emptyset_2(t)\ \ \emptyset_3(t) \dots\ \emptyset_{\mathrm{r}}(t)]^{\mathrm{T}}$, where $\emptyset(t)$ is the multiscaling function. The wavelet function is, likewise, defined from the set of wavelet functions $\psi(t) = [\psi_1(t)\ \psi_2(t)\ \ \psi_3(t) \dots\ \psi_{\mathrm{r}}(t)]^{\mathrm{T}}$, when r = 1, $\psi(t)$ is the single wavelet function. Whereas if r ≥ 2 $\psi(t)$ is the multiwavelets function and $\emptyset(t)$ is the multiscaling function. As claimed by Kattoush *et. al.* [5] the researched multiwavelets thus far are just for r = 2.

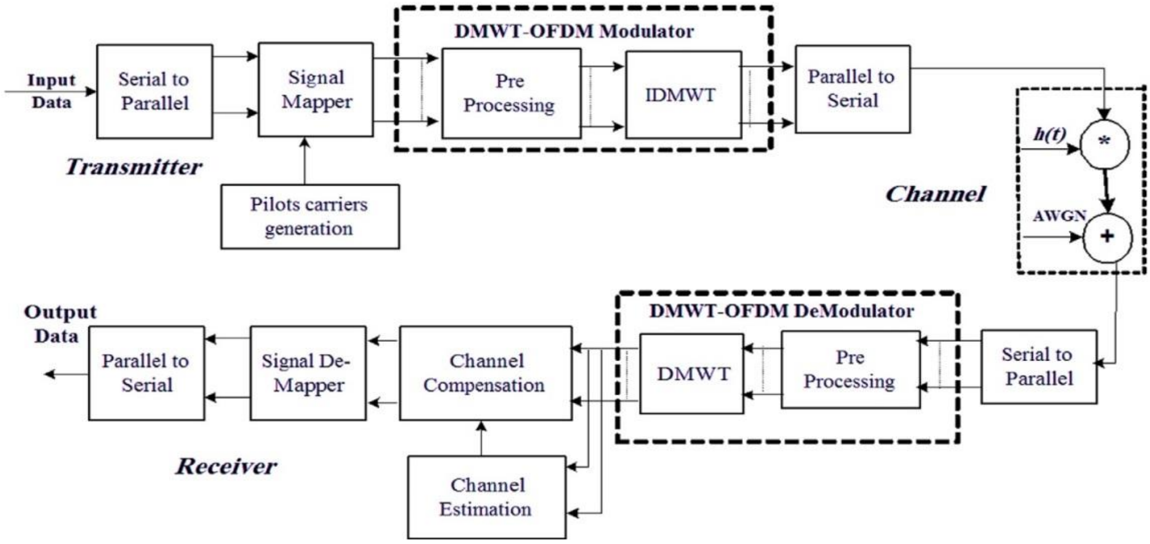

Figure 3.7: Multiwavelets OFDM System Proposal by Kattoush *et. al.* [5]

According to Martin and Bell [54], multiwavelet scaling $\emptyset(t)$ and wavelet $\psi(t)$ functions are:

$$\phi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} H_k\, \emptyset(2t-k) \tag{3.9}$$

$$\phi(t) = \sqrt{2}\sum_{k=-\infty}^{\infty} G_k\, \emptyset(2t-k) \tag{3.10}$$

H_k and G_k are each $n\ x\ n$ matrix representing the filters' coefficients. Since those matrices are two dimensional, hence, there is a degree of freedom as opposed to the conventional wavelet transform. The freedoms that could be acquired are useful to embed more useful properties in the system [5], i.e. symmetry, high order of approximation and orthogonality. However, the well-established quantization operations do not operate with multiwavelets as well as they do with wavelets. The proposed system of Kattoush *et. al.* [5] is depicted in Figure 3.7. In Fourier based OFDM, the transmitted signal can be expressed as:

$$f_F(t) = \frac{1}{\sqrt{2^D r}} \sum_{d=0}^{2^D-1} \sum_{i=1}^{r} n_k e^{j2\pi kt/T}, \quad 0 \le t \le T \tag{3.11}$$

Where n_k are the data to be transmitted, N is the number of subcarriers and T is the data time duration. To replace FFT in OFDM with the proposed DMWT the data to be transmitted can be expressed as:

$$f_W(t) = \frac{1}{\sqrt{2^D r}} \sum_{d=0}^{2^D-1} \sum_{i=1}^{r} n_{d,i} \emptyset_{d,i}(t), \quad 0 \leq t \leq T \tag{3.12}$$

As a result of this expression, the average normalized ICI and ISI power will both be decreased as the number of subcarriers increases. They concluded their study by claiming that a DMWT OFDM system uses half the bandwidth as opposed to a FFT OFDM system. Their proposed system achieved much lower BER than FFT OFDM and DWP OFDM.

However, the method of filter banks in their research is a replica of what had been patented by Jain and Meyers [36] and researched by Nikookar and Lakshmanan [2] and Divakaran [56]. Their novel contribution was the utilisation of multiwavelets. In this research, however, the proposed system and the approach are totally different and more flexible than what was proposed in that paper. Additionally, they did not mention the computational complexity, in conventional wavelet it is $O(N)$, the time complexity which requires the system to transmit and receive all transmitted data and the space complexity which is required to store computed elements during calculation of multi scaling and multi wavelet functions. These complexities will be studied in more detail and compared to this research context in Chapter 5 and Chapter 8.

3.5 MULTIPLE ACCESS WAVELET OFDM

Multiple access communications is the current state-of-the-art technology used in the downlink physical layers of both LTE standard [57] and the IEEE 802.16m (WiMAX) [21]. Therefore, it is important to think forward ahead of the rapidly advancing telecommunications systems and demands. Hence, the flexibility of wavelet transform has been research in multiple access context to further expand the investigation of DWP based OFDM and now in OFDMA. Very recently and towards the completion of this thesis, two articles were published about the utilisation of DWP in multiple access systems. In the summer of 2014, Mahapatra *et. al.* [58] proposed an Orthogonal Wavelet Division Multiple Access (OWDMA) processor architecture for the use in LTE-Advanced in wireless radio over fibre systems over heterogeneous networks. Their definition of the proposed system is that OWDMA is a replica OFDMA, but with utilising wavelet transform to separate sub-bands contents. Since there are two stages to be implemented in parallel so that the final dilation and scaling coefficients are interdependent on predict and update outputs for all stages, hence, there will be a delay which will affect the overall

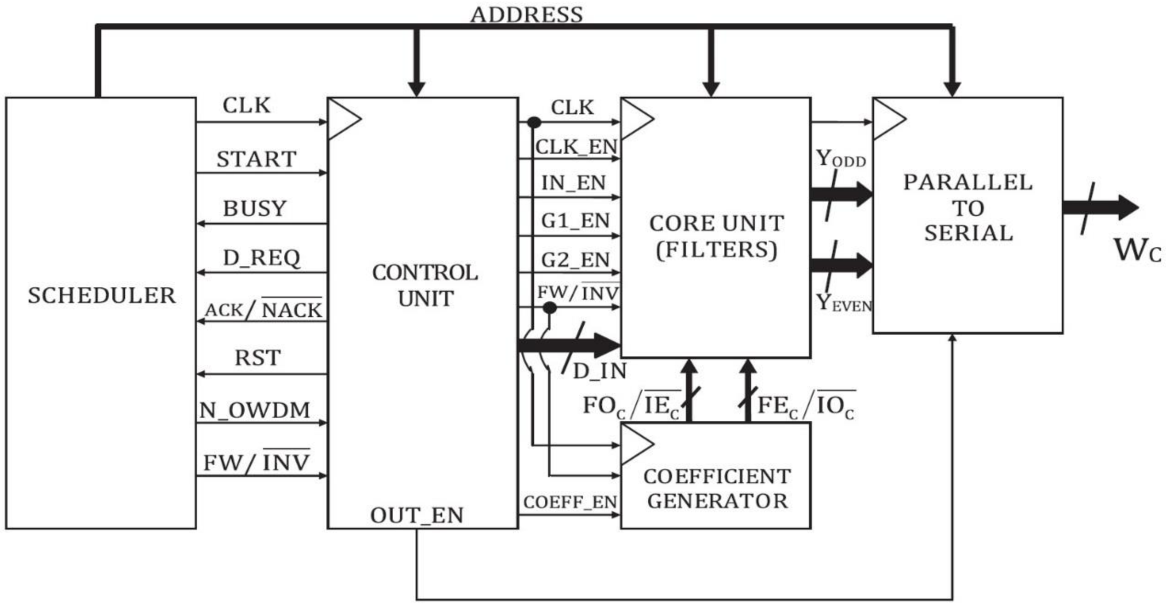

Figure 3.8: OWDMA Hardware Structure as Proposed by [58]

throughput. In their proposed system structure, they took into account the multiplicative filters' coefficients storage in a hardware that will reduce the number of multiplication operations.

Their proposed general hardware system is depicted in Figure 3.8 and the filters core unit, in which they used 9-taps and 7-taps filter structures, is depicted in Figure 3.9, where Δ is a delay by 1 unit. It is apparent from Figure 3.9 that delay, multiple multiplication and addition operators are most likely to cause a delayed signal transmission and it will be unrealistic for transmission systems where synchronisation and real time transmission is mandatory and any

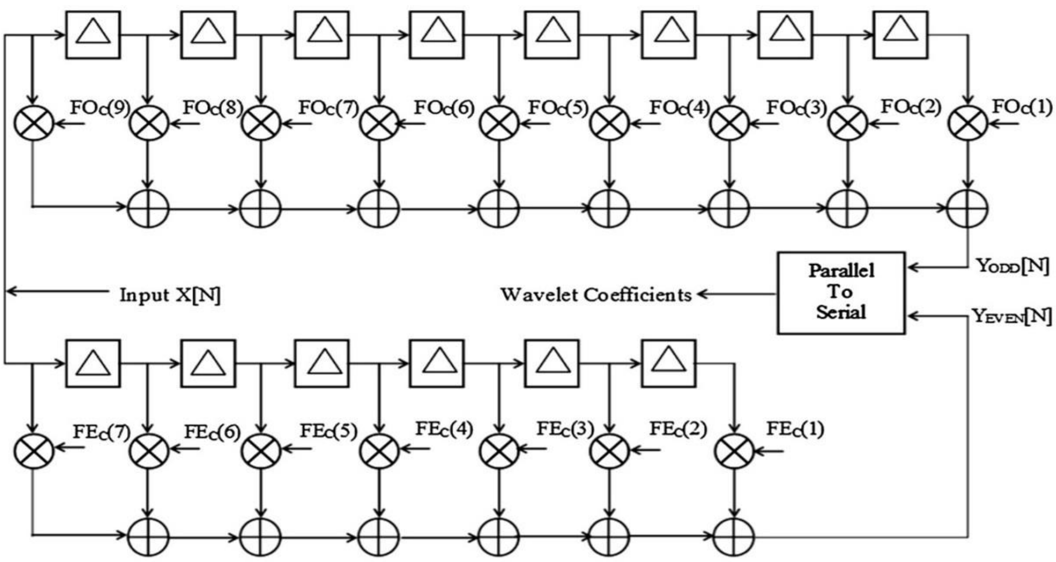

Figure 3.9: OWDMA Core Filter Unit as Proposed by [58]

Table 3.2: OWDMA Hardware Processing time as per [**58**] Proposal

Technique	Architecture	N (size of computation)	No. of CLB slices	No. of LUT slices	BRAM	DSP slices	Time (μs)
OFDMA processing	R2	256	842	650	3	3	4.18
	R2L	1,000	1,025	839	3	3	31.58
	R2		1,106	882	3	6	18.63
	R2L	2,000	1,137	882	5	3	66.74
	R2		1,082	952	5	6	39.41
Proposed OWDMA processing	Parallel and pipelined	256	614	355	3	32	0.91
	Parallel and pipelined	1,000	1,338	1,042	3	32	3.61
	Parallel and pipelined	2,000	1,160	848	5	32	7.26

delay cannot be tolerated. However, their system is only proposed for deployment over Radio-Over-Fibre (ROF) systems; additionally the multiple delays, multiplications and addition operations add more delay in processing time, latency and complexity to the system. They concluded their research by claiming that OWDMA hardware processing time is significantly less than of OFDMA, the summary of their conclusion is presented in Table 3.2. Having said that, although the system structure and sub-banding algorithm in this thesis differs from the [58] system and the intended application is also different, designing a hardware architecture for the proposed system in this thesis can be considered in future work.

3.6 DISCRETE COSINE WAVELET TRANSFORM

Very recently and towards the last year of this research, two researches emerged from the same authors in February 2015 in two different well-known journals. A proposal of a new OFDM based on Discrete Cosine Harmonic Wavelet Transform (DCHWT) system was published by Suma *et. al.* [59] in Springer Wireless Personal Communication Journal and the other research was published in the same year in Sadhana Journal by Springer. Although both topics discussed DCT and the utilisation of this transform in OFDM systems, there were some differences between the two studies. In the first paper, they proposed a DCHWT OFDM system and the second paper was more of an analytical study of the same proposal. Their proposal was mainly based on utilising the energy compaction property of DCT to provide lower leakage in subcarrier, they named their system DCHWT based OFDM. DCHWT had been previously used in image processing but only in their research, in 2015, it was proposed to be used as an alternative to OFDM. By letting $x(p), p = 0,1,2,\ldots,L-1$ be the desired modulated symbols mapped on N number of subcarriers, the transmitted signal for DCHWT based OFDM is expressed as [59]:

$$x_s[n] = \sum_{k=0}^{N-1} \sum_{p=0}^{L-1} x_k\,(p)\psi_{sk}(n - pN) \qquad (3.13)$$

For N number of subcarriers and at the kth decomposition level the mother wavelet $\psi_{sk}(n)$ is cosine modulated sinc function. The PAPR of DCHWT OFDM can be computed by [59]:

$$PAPR = 10\log_{10} \frac{NMax_{0\le n\le NL-1}\{|x_s(n)|\}^2}{E\{|x_s|^2\}} \qquad (3.14)$$

Where the average power is given by:

$$P_{av(DCHWT)} = \frac{1}{N}\sum |x_s(n)|^2 \qquad (3.15)$$

Where x_s the DCHWT OFDM transmitted signal. The average power becomes [59]:

$$P_{av(DCHWT)} = \frac{1}{N}\sum_{K}\sum_{P} |\cos(\omega_0 n)\sin c(\,\omega_c(n - pN))|^2 \qquad (3.16)$$

And the peak power is [59]:

$$P_{av(DCHWT)} = N * max \sum_{K}\sum_{P} |\cos(\omega_0 n)\sin c(\,\omega_c(n - pN))|^2 \qquad (3.17)$$

The Bit Error Rate (BER) of OFDM when QAM is used with N number of subcarriers and M modulation levels can be calculated by [59]:

$$BER = 1 - \frac{\sum_{k=1}^{N}\left(1 - P_{k,\sqrt{M}}\right)^2)}{N} \qquad (3.18)$$

Where $P_{k,\sqrt{M}}$ is:

$$P_{k,\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\,(\sqrt{\frac{3\,H_k\,E_{av}}{(M-1)N_0}} \qquad (3.19)$$

Where $H_k = H_0, H_1, H_2 \ldots, H_{N-1}$ the N number of coefficients of the channel, E_{av} is the average DCHWT OFDM QAM symbol power and N_0 is the noise power.

After testing BPSK, QPSK and 16-QAM modulators and only Haar wavelet, they concluded their study by stating that DCHWT based OFDM remarkably outperformed conventional OFDM in terms of BER performance. In term of PAPR performance, it showed a significant improvement over conventional OFDM; however, it showed only slightly improved performance over conventional wavelet based OFDM. Having said that, the main system that they built is similar to that of Jain and Meyers in their patent [36], however, with slightly different blocks that were added for the Inverse Discrete Cosine Transform (IDCT) and DCT processes. Although this research paper was critically reviewed because it has some similarity in terms of time-frequency domain signal analysis; however, there are significant differences with this research in the design of the system, the processing of multiplexing, symbol modulation, subcarrier modulation and distribution.

3.7 SUMMARY

In conclusion of this chapter, as a further evidence that wavelet transform can be used as an alternative to OFDM, scientists and researchers are actively looking at this hot topic during the conduction of this thesis and also beyond, as discovered previously in this chapter. DWT, as a modulation technique for signal transmission, was reviewed, critically discussed and compared to this research. In DWP modulation cases, it was found that DWT offers better BER vs SNR and better system throughput than FFT in OFDM. However, different wavelet families and families' members have varying performance. No comprehensive research thus far has investigated all wavelet families and families' members, neither in DWP modulation nor in the proposed system in this research.

Some researchers called this scheme “DWPT” [36], others called it wavelet based FFT [39] [42], wavelet packet based OFDM “WP-OFDM” [60] [61], DWT-OFDM [4] or even Wavelet Packet Multicarrier Modulation “WPMCM” [41]. Whatever it is called, the main principle is that the filter banks that are used to synthesis and reconstruct signals are symmetrical and similar to that used by Jain and Myers [36] in their patent. However, in this research, the approach is different, filters banks are asymmetrically oriented for a more flexible system, with flexible bandwidth allocation, no guard interval used, different channel conditions investigated and full system from energy dispersal, Forward Error Correction (FEC), convolutional coding and interleaving to symbol mapping and channel coding. Furthermore, the synthesis filters banks in the transmitter is differently oriented and distributed than what the previously described researches considered. The synthesis filters in the transmitter starts with single level of LPF and HPF, then increases in pairs as the number of levels increases. In the receiver, however, the opposite operation is carried out, where the signal to be reconstructed is entered from the highest level and goes through to the lowest level before it is entered into the demodulation (demapping) process.

Some researchers have also investigated multiwavelets based OFDM where they built systems based on multiwavelet rather than the conventional wavelet. The difference between multiwavelet and wavelet transforms is that in multiwavelet, there are multiple associated wavelets and scaling functions, rather than a single wavelet and a single scaling function in conventional wavelet transform. Although the increased number of wavelets and scaling functions can achieve a higher approximation power over the conventional wavelet transform [62], this comes at a cost of significantly higher memory usage and added complexity and latency, which cannot be tolerated by many transmission systems where real time transmission is required. Nonetheless, these types of schemes can be used in image processing and watermarking, but certainly not for transmission systems.

Nonetheless, there are still a considerable number of research questions thus far yet to be answered concerning this particular field of research, before OWDM or Asymmetric OWDM could be used in practice. These include the wavelet families' performance, families' members' performance; which types of wavelets families and families' members are best performing in this modulation scheme; how much difference will it make for different communications applications; Most importantly, what if there is a comprehensive investigation of all wavelet families and families' member that can support orthogonality and can perform outstandingly against OFDM while adding a novel technique to make the system more flexible, efficient and cost effective in term of bandwidth loss. All those are part of the research questions that still yet to be answered and will be the main consideration of this research.

Therefore, this research has widened the investigation to include not only all the wavelet families but also all the families' members, which will be later discussed in the critical analysis chapter. More importantly, this research also proposes a novel filters banks orientation in order to make the system more effective and dynamic for bandwidth allocation and hence, more cost-effective and bandwidth-efficient. In the next chapter, the methodology and approach of this research is presented.

CHAPTER 4: RESEARCH METHODOLOGY

Objectives:

- To describe the methodology followed to answer the research questions.
- To present the technical flow diagram and requirements.
- To gather the resources needed to pursue this research.
- To describe and analyse the risk factors and alternatives.
- To present the time plan of the research.
- To breakdown the phases and tasks of this research.

4.1 INTRODUCTION

Having succeeded in the feasibility study approval, the research will now enter the planning stage to plan for the subsequent implementation and methodology. In this chapter there will follow a deep discussion in detail regarding what was briefly presented in the feasibility study. A Work Breakdown Structure (WBS) will be presented in order to break down the research into phases to easily fulfil them. Each of these phases will be further broken down into groups of tasks. Thereafter, a Statement of Work (SOW) and methodology for the implementation of each of these tasks will be presented. A resource list needed for the research also will be presented alongside the risks assessment and alternative plan. Finally, a conclusion and recommendations will be made at the end of this chapter.

4.2 REQUIREMENTS

In this section the main requirements of the research will be discussed as a step towards achieving the feasibility of the research. There are three levels of requirements associated with this research. These levels are described below, starting firstly with the highest priority and taking a top down approach in meeting those requirements.

4.2.1 ESSENTIAL (MUST HAVE)

This section will include the essential infrastructure needed for the research and must, therefore, be achieved by the end of the implementation stage. The highest priority requirements for this particular research are as follows:

1) Building a group of simulation codes that test OFDM systems according to DVB-T standard to be the baseline comparison.
2) Building a simulation for Haar wavelet.
3) Building a simulation for Daubechies wavelet family and all its associated members.
4) Develop a group of simulation models and codes to test the hypothesis of OWDM for all of those wavelet families and families' members against OFDM and also themselves.
5) Run the simulations and collect the results for OFDM and OWDM, different families and different families' members.
6) Write up the results into the research report.

4.2.2 RECOMMENDED (PREFERABLE)

There are some recommended requirements which would be preferable if achieved by the end of this research. These requirements are as follows:

1) Analyse arbitrary number of decomposition levels where both LPF and HPF are decomposed to have wider bands.
2) Building a simulation for Symlets wavelet family and all its associated members.
3) Building a simulation for Coiflets wavelet family and all its associated members.
4) Building a simulation for Biorthogonal wavelet family and all its associated members.
5) Building a simulation for Reverse Biorthogonal wavelet family and all its associated members.
6) Comparing the performance of different wavelet families together in OWDM system.
7) To publish the results and write up a paper in academic journal or conference.

4.2.3 DESIRED (NICE TO HAVE)

The desired requirements that would be very nice to have in the research are as follows:

1) To incorporate a fully dynamic simulation model that allows the user to choose various wavelet families and various depths to test all the different wavelet families with arbitrary number of levels of decomposition using one piece of software.
2) To implement the simulation onto FPGA integrated circuit to broadcast the signal physically, upon resource availability.
3) To establish a quick comparison between symmetric and dynamic (asymmetric) sub-bands allocation.
4) Analyse a multiple access OWDM, OWDMA with asymmetrical filters banks.
5) Build a simulation for LTE-A standard as a baseline comparison with the proposed system, OWDM.
6) Propose and examine the feasibility of MIMO OWDM using multifilter banks with cognitive radio technique.

4.3 TECHNICAL APPROACH

The technical and methodology of this research will involve several stages. Each of these stages is a combination of several tasks that need to be completed in order to fulfil the requirements of that particular stage. If they succeed in fulfilling the requirements of all these stages, then the overall requirements of the entire research will be successfully satisfied.

The flow of this research reflects the procedure of investigating a brand new modulation technique through comparison with the most recent technique. In the event of proving that this technique is valid, then this research may be taken further; and implementation of these two different modulation techniques, OFDM and OWDM, may be seen in an actual real-life scenario. These areas are represented as phases in this research. The following flowchart, Figure 4.1, presents the main phases and the flow of this research as follows:

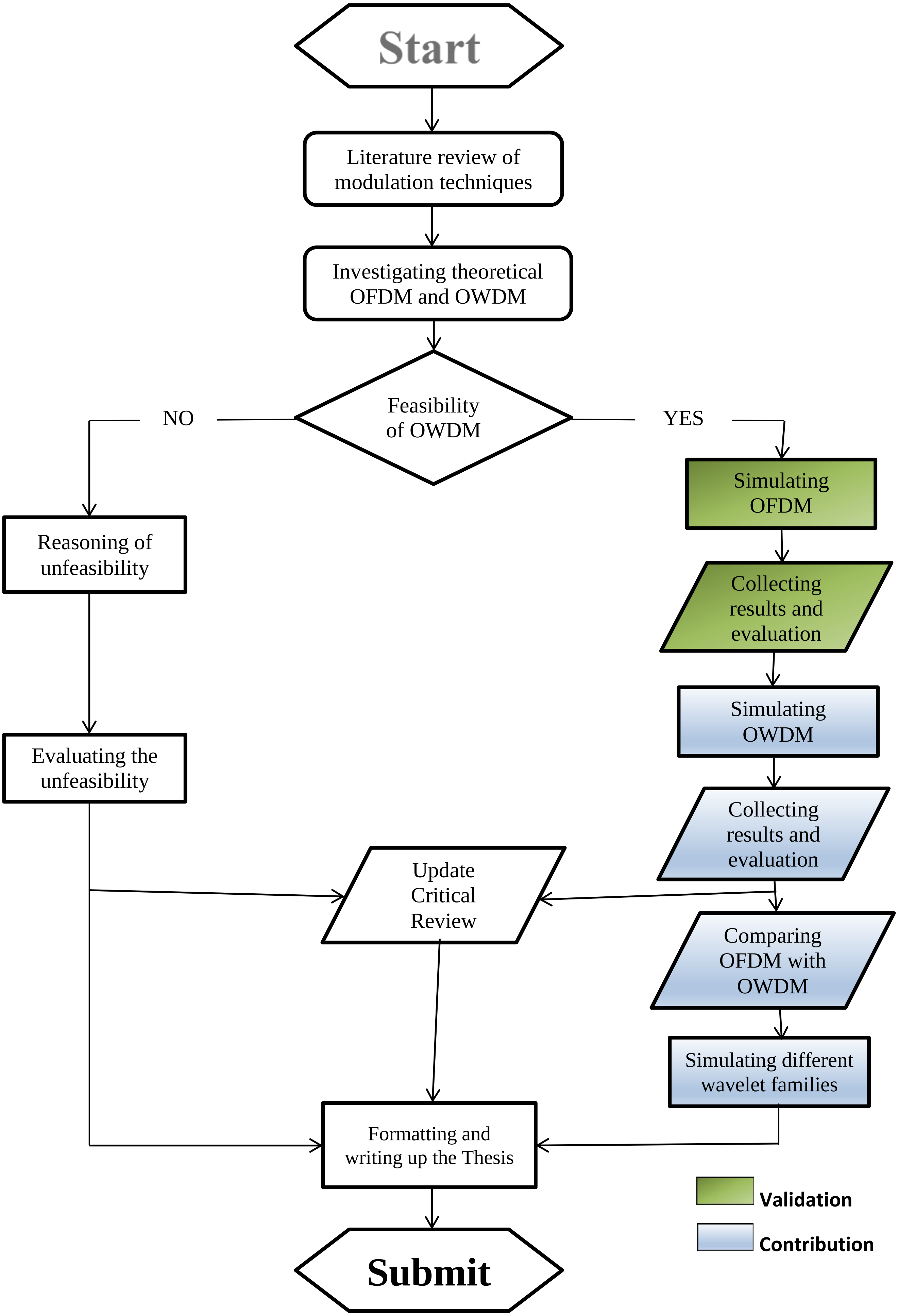

Figure 4.1: Research Flowchart

1) Literature review has been conducted by reading any relevant previous work and analysing the weakness and strength of each book, journal articles or conference papers. Literature review will take up to three weeks to be fulfilled.
2) The previous work that was done on OFDM and OWDM theoretically will be analysed, taking into consideration any previous implementation of any kind of OWDM, if any is out there.
3) In third phase of the research, there was a decision taking phase, to see whether OWDM, as a modulation technique, is feasible. If no, then continuing with next phase through the left branch (4 and 5) is the choice. If yes, then we will go to phase 6 through the right branch, which indeed was the chosen path.
4) If the answer was No, then reasoning for the unfeasibility of OWDM will be investigated theoretically.
5) Then, the unfeasibility of OWDM will be validated and evaluated. After this phase, we will go to phase 11.
6) Simulating OFDM by using MATLAB command to create OFDM .m code was the first step in this path (this is a validation process).
7) Then collecting the results and evaluate them was right after the simulation (this is a validation process).
8) Next, simulating OWDM by using MATLAB command to create OWDM .m code was started (This is a contribution process).
9) Then a period of time to collect the results of OWDM and evaluate them was given (this is a contribution process).
10) Thereafter, joining steps 7 and 9 to compare both systems OFDM with OWDM, in terms of their performance through different environments (this is the novelty and contribution to knowledge).
11) Writing up the report and formatting of the thesis was given about six months.

The technical approach will be expressed in detail for each single task throughout the planning stage if this feasibility study is approved for further processing. The main comparison of systems shall be carried out in terms of performance of BER vs SNR for each individual system and across all systems. Additionally, packet transmission rate, impulse response of filters, filters' coefficients and symbol mappers' performance are compared across systems. Since SNR is most important measure of communications systems' reliability and robustness against noise, it has been chosen as the main measure of performance in this research. SNR is the ratio of wanted signal power to the unwanted noise power. Since noise varies and cannot be predefined, an estimator is needed in any communications system and, therefore, the estimator used in this research shall follow the standardized DVB-T communication system in ETSI EN 300 744 standard [19].

4.4 RESOURCES

There have been various resources used to fulfil the objectives of this research. There are three types of resources which have been required in order to complete this research. These include:

- Material resource.
- Financial resource.
- Human resource.
- Time resource.

4.4.1 MATERIAL RESOURCES

Despite the lack of material reference resources needed for this research, there will be a consultation session with the supervisor of the research regarding any enquiries. Different resources will be used for the purposes of this research which include:

- Books.
- Journal articles.
- Conference papers.
- Published theses.
- Experiment resources if available.
- The MATLAB environment to simulate and examine all modulators.
- Laboratory to utilise FPGA and transceiver to broadcast signal.
- The Microsoft products, Vision, Project, Word, One Note, Excel, PowerPoint and Publisher to create flowcharts.

4.4.2 HUMAN RESOURCES

The human resources that are needed in this research are listed below:

- The Author who will conduct the research.
- The Supervisor who will supervise the research and provide technical support to the author.

There may be an examiners involved in the examination of the research at a later time in the event of the research successfully conducted.

4.4.3 FINANCIAL RESOURCES

The financial resource needed is insignificant as the University will provide the laboratory should the author need to carry out any examinations or experimental analysis. Nevertheless, there is a need to purchase a student's version of the MATLAB interpretation environment for the cost of £92, which has been already paid by the author's financial supporter. This is for the utilisation of the programme and to create the needed codes and simulations at the author's designated pace. The financial resource in this research was not considered to be vital, despite the fact that the university has provided whatever has been required. There has been a need to purchase three programmes to enable the author to work at his own pace. The required programmes and their cost are presented in Table 4.1.

Table 4.1: Financial Resources

	Source	Cost
MS Word 2013	Through DMU msdnaa	Free
MS Visio 2013	Through DMU msdnaa	Free
MS Project	Through DMU msdnaa	Free
MATLAB and Simulink 2013	MathWorks	£920
	Total	£920

4.4.4 TIME RESOURCES

The time resource for this research, at this stage, is planned to be as Gantt chart for the entire duration. There will be further developments to the time plan at the next stage where further details and duration for each single task will be demonstrated. This research officially started on 1st April 2012 and is expected to be handed over on 1st April 2015. The time is divided into the phases of this research, so that each phase has its own time to fulfil the requirements of that particular phase. Furthermore, each phase is broken down into tasks and the time allocated of the phase is divided between these tasks of this particular phase. In this stage, the time allocation for the main phases of the research is represented in the Gantt chart, Figure E.1 in the Appendix E and detailed time plan breakdown time allocation for individual tasks is shown in Figure E.3 in Appendix E.

The focus of the implementation associated with this research relates to three areas of focus as follows:

- Research focus.
- Code creation and analysis.
- Data collection and analysis.

The time that was given for the research focus was 12 months in total, followed by the transfer examination. Meanwhile, the time given for the codes and models building was eighteen months which overlaps with the research focus by four months. Other simulation models/codes will be built for different wavelet families and families' members with fourteen months time frame, due to the enormous literature review and critical analysis to find the nature and environment of these types of families. The last 12 months has been assigned for literature review update and thesis writing and formatting. Please refer to **Error! Reference source not ound.** for the full time resource plan.

4.5 RISKS AND ALTERNATIVES

As with any research, there are some risk factors which may affect the process or the procedures associated with the research at any stage during its lifecycle. Therefore, it is necessary to effectively mitigate every potential risk and analyse them according to the percentage and the likelihood of that risk. All possible occurrences have been considered in order to assess the risks that the research could encounter. The risks associated with this research have been ranked according to a scale of N/10 where, N is the probability or the impact out of 10. The following Table 4.2 shows the risk assessment associated with the research:

Table 4.2: Risk factors table

Risk types \ probability & Impact	Likelihood	Impact	Action required
Resignation, retirement or relocation of the Research Supervisor	1/10	8/10	Monitor
Failure to meet the deadline of the research	2/10	7/10	Monitor
Lack of material resource	7/10	6/10	Mitigate
Natural disasters	2/10	7/10	Monitor
Requirement changed at later stage	4/10	5/10	Monitor
Simulation did not match the hypothesis	4/10	8/10	Mitigate
Results are not as expected to be	6/10	7/10	Mitigate
Time frame is not enough to analyse the codes against the hypothesis	5/10	8/10	Mitigate

For risks with a critical impact (over 5/10), or with a high likelihood (over 5/10) a full plan has been designed to allow swift recovery in the case of their occurrence. The lack of material resources has 7/10 likelihood and 6/10 impact, this has occurred in during the conduct of this research where one of the main software platform, MATLAB and Simulink, was not available for research. Therefore, to mitigate this on time and before any further complications occurs, the author invested £920 for a full version of MARLAB and Simulink including the appropriate needed toolboxes and block sets. Furthermore, unfortunately for this research, the relocation of the supervisor to another institute has the least likelihood of 1/10 with very high impact of 8/10 but it has occurred. This has impacted the author since there were uncertainty period of 8 months where no adequate support and supervision were in place. A mitigation of this made possible by the collaboration the second supervisor who took over as a main supervisor and continued with the same research, however, some alterations were introduced.

4.5.1 EXTERNAL INFLUENCES

Every research will have external influences outside of the control of the executor/s. These influences may occur at any level and at any time during the research lifecycle. Sometimes these influences could last for the entire duration of the research lifecycle; some of which could have a major impact. This would depend on how powerful this external influence actually was and the time of its occurrence. This research has been deemed protected from one of the most powerful external influences i.e. ethical issues. Although, for additional protection, an ethical approval to conduct this research has been acquired. Thus, the research is not affected by any external interference with regards to ethical issues, due to the fact that it had gained approval from the ethical committee throughout the completion of this research. However, the research was still exposed to some potential external influences, such as - competition from other researchers. These influences were resolved by providing the research a unique stamp with regards to the implementation of the OWDM system in unique way.

It was considered to give the OWDM code a user friendly interface which would enable the user to modify just one or two lines in order to examine different wavelets and different levels of filter bank. This would enable the research to examine several specifications of the OWDM system in a lesser time frame. Additionally, if any other OWDM systems had appeared during the execution of this research, then a very tiny change in the code would support another OWDM system that could be analysed and published.

4.5.2 INTERNAL INFLUENCES

Alternative strategies for this research have been planned and designed. A full alternative plan was in existence, should the research have fallen behind schedule. Using MATLAB code has also been useful for this research and it has been planned for implementation. However, using SIMULINK to build one or two systems would have been used as an alternative plan. If time had been limited, it would have been considered to ask for the allowed extension time for the handing over of the research; but this would have been unlikely to happen unless a vital risk had occurred unexpectedly.

4.6 WORK BREAKDOWN STRUCTURE

The Work Breakdown Structure (WBS) is used to breakdown the phases of a large research into smaller tasks and subtasks, wherever needed. This simplifies the execution of the research and enables the author to build up the research and handle each task individually and precisely for more control and management purposes. This also provides a straightforward way to monitor and control the research and to ensure all the requirements are satisfied and met. There are several methods involved in designing a suitable WBS. However, the most suitable approach for this research is the top to bottom approach. The full WBS, which has been followed in order to fulfil the requirements of this research is presented in Figure E.2 in Appendix E.

This WBS has divided the research into four main phases: feasibility study, planning, implementation and handover and research closure. Each phase of these has been divided into a second level of tasks, represented by the tasks which need to be individually fulfilled to fulfil the phase. Level three is represented by subtasks inside bigger tasks, in which the requirements of each of these subtasks in this level need to be satisfied to fulfil the requirements of each task of the upper level.

4.7 SUMMARY

In summary, the technical approach is designed with the likelihood of adding certain tasks to any phase in this research, wherever needed. The final approach and methodology of the research will be presented at the planning stage where the finalisation of the methodology and the approach to fulfil the requirements of each task will be described.

In conclusion, with regards to the entire feasibility study associated with this research in Chapter 1, Chapter 2 and Chapter 3, there are two questions that have remained throughout this preparatory stage on behalf of the author. These questions are: “Does anyone really need to have this research implemented?” and “Has this research already been implemented?”

From the personal perspective of the author, this research has not previously been implemented in exactly the same investigative manner as in this approach. Nevertheless, the wavelet transform has existed previously, although it was used for other applications, e.g. data compression and image compression. It has recently been proposed to be used as a modulation technique for Wireless Communications but no real work has been facilitated on this so far. Thus, the author believes that this research is unique and, therefore, worthy of implementation and further investigation.

The concept of using Orthogonal Wavelet Division Multiplex as a modulation technique is a unique idea. Therefore, the author has strongly recommended the perusal of the research and proceeding on to the next stage. It is also recommended that the research is sufficient to meet the requirements with the resources that are presently available.

CHAPTER 5: TIME-FREQUENCY ANALYSIS AND COMPUTATIONAL COMPLEXITIES

Objectives:

- To describe the computational complexity and cost of operations.
- To analyse the computational complexity of Short-Time Fourier Transform.
- To analyse the computational complexity of Short-Time Fourier Transform.
- To describe the complexity of all available compactly supported wavelet families and their associated children.
- To describe and analyse the computational cost for machinery operations.
- To compare and contrast the complexities of wavelet transform with Fourier transform.
- To identify which wavelet family and child has the least complexity cost.

5.1 INTRODUCTION

Short Time Fourier Transform (STFT) was discovered in 1946 by Dennis Gabor [63] to solve the problem of knowing the behaviour and feature of any given signal in the time domain by analysing the frequency domain at specific given time. STFT is mainly used to obtain the frequency and phase of a local section of a given signal as it changes in time domain. The process of computing STFT is to divide the signal in time domain into shorter equal length segments, then to calculate the normal Fourier transform for each of those segments individually. The resulting Fourier spectrums of each of those segments can then be plotted as a function of time.

Over the last three decades, the wavelet transform has become increasingly popular in many application areas, including signal processing, image processing and communications. The wavelet transform is the direct result of the efforts to improve upon the key shortcomings of Fourier transforms. Unlike Fourier transforms, which are only localised in frequency, wavelets are localised in both time and frequency offering true multiresolution signal analysis. Many wavelet families and wavelet children have been developed over time, offering quite a wide choice of wavelets for different applications. Wavelet Transform has many types of transforms and signal analysis. It is divided into Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). As CWT is unfeasible due to the extreme cost associated with its deployment, as it computes the wavelet transform for an infinite signal harmonic, DWT is implementable and feasible for signal analysis and it is of our interest in this research. DWT utilises discrete-time signal analysis through filter banks with either Finite Impulse Response (FIR) filters or Infinite Impulse Response (IIR) filters. These combinations of filter banks achieve time-frequency signal analysis by performing shifting and scaling functions throughout these filter banks.

Nevertheless, DWT is not the only tools, thus far, could perform time-frequency signal analysis. STFT which has been discovered back in 1946 by a scientist named “Dennis Gabor” [63] has the ability to perform such analysis but with a different technique. The technique used in STFT is called “windowing”, which analyses a small part of the signal at each time. However, this technique has some drawbacks and some application restrictions.

In this chapter, the complexity of each of STFT, DWT and the different wavelet families will be discussed. Additionally the performance, functioning mechanism, advantages and disadvantages and most importantly, the computation and algorithmic data structure complexity of each transform will also be discussed. We will also see the importance of time-frequency domain analysis and why wavelet transform is way more popular than STFT in this particular type of transform.

5.2 STFT AND DWT COMPUTATIONAL COMPLEXITIES

Complexity in operational mathematics, computational cost, in this research interest, is often referred to as a notion of O or what is known as the Big O complexity. This notion describes the limitation on the behaviour of any function when the argument resorts toward a particular value or sometimes toward infinity. However, this notation is not the only complexity notation known in mathematics, but only one of the members of the large notations family popularly called among mathematicians and engineers “Bachmann-Landau” notation [64] [65]. In machine language or often computer science, the big O notation is a method of classifying algorithms by their response, i.e. data processing response time or memory space requirements, to the changes in the input size [66]. The real world machinery is a resource limited entities and so algorithms are the same. The efficiency of any algorithm is characterised and measured by how much resources it consumes:

- Time Resource: Measures of how much time algorithm takes to solve a task.
- Complexity Efficiency: Measures how much it costs based on how many statements (iterations) counts to solve a particular task.
- Space Efficiency: Measures how much memory an algorithm needs to solve tasks.

The big O notation characterises operational functions according to their growth rates so that a range of functions all commonly have the same growth rate can be represented using the same O notation, no matter how differently they operate. For instance, $O(N \log N)$ complexity can be computed or compiled, using any of the following algorithmic operations: FFT, heapsort, quicksort or mergesort. The myth of this notation - why is it referred to as an O? Because of the fact that it refers to the order of the function, since the order of any function is in fact the growth rate of that function.

The data structure for logarithmic complexity, which applies to DFFT computations as seen earlier in this section, is the measure of computation (often execution time or memory occupation size) logarithmically bounded by the data size to be computed, i.e $O(N \log N)$ where N is the data size. In machine language (compilation) terminology, it is eventually done by appending a sequence of n data entries in the memory. Whenever new data needs to be written, instead of looking for a location in the memory for it, appending it to the log and update the indexing entry and append this update to the log, this is also known as B-Trees [67]. To visualise this notion of recursive tree indexing and searching, the following code is compiled to produce Figure 5.1:

```
if (n <= 1) /* base case */
  return n;
else /* recursive caseS */
  return f(n-1) + f(n-2);
```

For the sake of an example, n data size is chosen to be 7, variable a1=200 and variable a2=130. Figure 5.1 is the recursive tracing result of such an input. In the next section, DWT operation and its computational complexity the Big O notion will be revisited, as it is applicable to DWT.

While DFT has a complexity of $O(N \log N)$, DWT however, has a complexity of $O(N)$ where N is the data size [68]. In the lowest level of understanding, this notion of O complexity for machine language is that a constant time $O(1)$ operation is incurious of input size as can be seen in Figure 5.2. Those operations could be any of the following: insert any element to any array, assign a value to any variable, determine if any binary value is odd or even, retrieve a value from dictionary or retrieve an element from any array. For instance, if a new value is to be inserted into memory, inserting an element to a stack or declaring a variable value all those statements take a constant time independent of that operation. In the case of $O(N)$ complexity, the concern of this research, the time consumption is linear and that is why $O(N)$ is the so called "linear complexity". It basically performs a loop of N times operations of

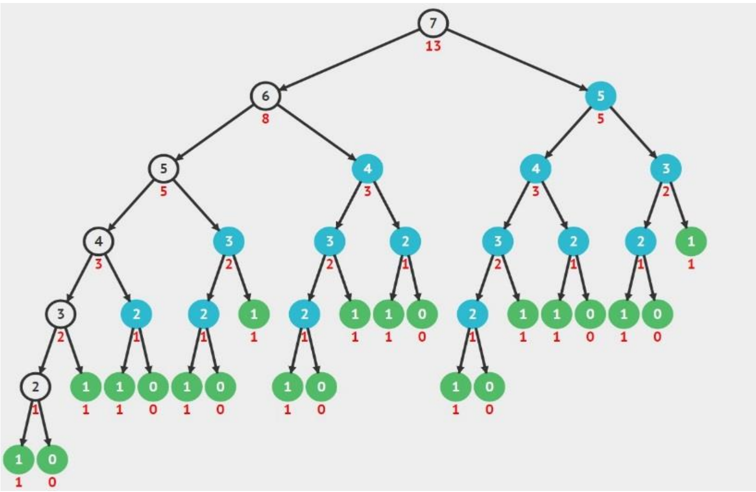

Figure 5.1: Recursive B-Tree for n=7, a1=200 and a2=130

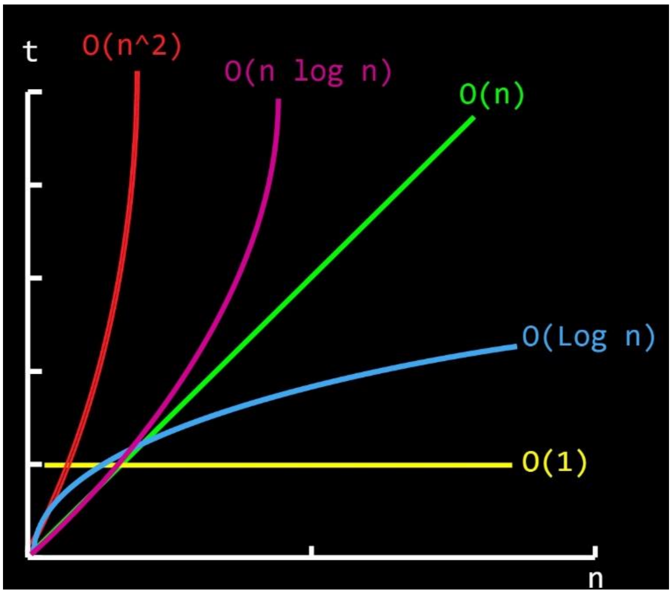

Figure 5.2: Complexities Chart [**146**]

$O(1)$ complex tasks and consequently, the total required time will be $N * O(1)$ which equals to $O(N)$. A comparison of different used sorting algorithms which have $O(1)$, $O(N)$ and $O(N \log N)$ complexities can be seen in Table 5.1 and the growth rates corresponding to $O(N)$, $O(N \log N)$ and N^2 time complexities in seconds and minutes for different n data sizes can be seen in Table 5.2. From the time complexity, we can clearly see that N^2, applicable to NDFT, grows rapidly as the data rate grows and this is an evidence that NDFT is not feasible to be implemented in transmission systems as stated in NDFT Section 2.6.1.2 in Chapter 2.

It can be seen from Figure 5.2 that for $O(n \log n)$ complexity (purple) as the processing dataset (n) grows the needed time (or number of operations) for processing exponentially increases. Whereas for $O(n)$ complexity (green) as the dataset grows the data processing time linearly increases. To illustrate this more, processing time was measured against real data (bits) summarised in Table 5.2. The most used sorting algorithms with their space and time complexities are summarised in Table 5.1.

Table 5.1: Space and Time Complexities of three major used algorithms

Sorting Algorithm	Space Complexity	Time Complexity
Heapsort	$O(N)$	$O(N \log N)$
Mergesort	$O(N)$	$O(N \log N)$
Quicksort	$O(\log N)$	$O(N \log N)$

Table 5.2: Time Complexity for different data sizes

n f(n)	$O(N)$	$O(N \log N)$	N^2
1000	1 ns	9.966 ns	1 ms
10,000	10 ns	130 ns	100 ms
100,000	0.10 ms	1.67 ms	10 sec
1,000,000	1 ms	19.93 ms	16.7 min
10,000,000	0.01 sec	0.23 ms	1.16 days
100,000,000	0.10 sec	2.66 sec	115.7 days
1,000,000,000	1 sec	29.90 sec	31.7 years

5.3 SHORT TIME FOURIER TRANSFORM

Short-Term Fourier Transform (STFT) was first introduced in 1946 when Dennis Gabor [63] utilised the discrete form of Fourier transform to analyse signals in time-frequency domain. He suggested that a fixed time-limited window can be used to analyse a portion of a sinusoidal signal at a time by applying Fourier transform at that specific time. This window can then slide over the signal to compute the localised DFT for the whole signal. This maps the signal into two domains at once, time-frequency domain. This transform, however, is different from conventional Fourier transform due to the fact that conventional Fourier transform transforms the signal from time to frequency domain; nevertheless, STFT transforms the signal from time to time-frequency domain [63]. The main purpose of time-frequency analysis is to better represent the signal in time-frequency domain and hence, a better understanding and analysis of the signal's behaviour.

STFT is a special case of Fourier Transform (FT) where it performs the transform, often fast DFT, for a narrow window over short period of time and sweep over particular frequency. Thus, it creates map of signal which has two dimensional time and frequency function. This compromise in time and frequency provides information about when and at what specific frequencies a signal event occurs. Nevertheless, the limitation of window size has limited the precision of this analysis as it occurs within window that is limited to specific time. Figure 5.3: illustrates how the window is scanned over time and the output of this digital process.

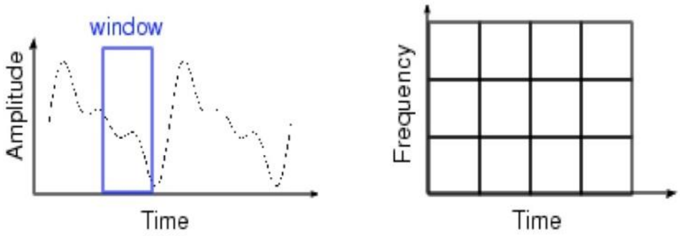

Figure 5.3: STFT Window Sliding over Signal and Time-Frequency Representation

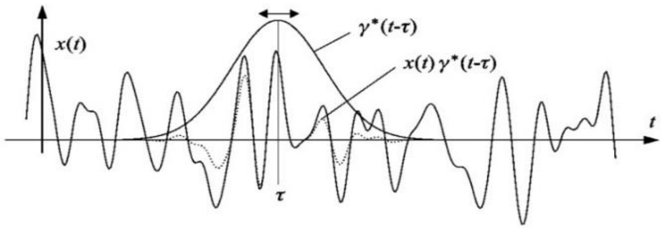

Figure 5.4: Windowing a Signal Spectrum, STFT

Moreover, when choosing a particular window size for the time it cannot be changed and will stay the same size for all frequencies. For instance, when width of window used to analyse low-frequencies signal, there would not be an issue; however, when reaching high-frequencies signals, the accuracy of signal analysis will drop dramatically and there will be a need to change the window size which is practically not possible. Most of digital signals require more flexible analysis approaches to do such analysis.

Figure 5.3: and Figure 5.3 illustrate how a STFT window is sliding over the signal and representation in time-frequency domain. This analysis can solve some of the limitations of conventional Fourier transform, as it can provide the information of the frequency and at which time that a specific signal event occurred. However, there is still a limitation even when using STFT, which is the size of the sliding window which determines the precision of the obtained information [63]. Additionally, the size of the sliding window cannot be changed once it has been chosen and it will remain the same for all frequencies of the signal. Complex signals require a much more flexible approach to analyse signal in time-frequency domain.

As said earlier that STFT is a time limited window that computes the normal DFT within that time and slide over the spectrum to compute DFT for the required signal. As known that DFT is derived from the Fourier series [69] [70] which is a summation of sines and cosines over a limited period. To illustrate this, one can find the Fourier series of a function defined on $[-L, L]$ then:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n\, cosnx\left(\frac{\pi}{L}\right) + b_n\, sinnx(\frac{\pi}{L}) \tag{5.1}$$

Where a_n and b_n are constants and they are:

$$a_n = \frac{1}{L}\int_{-L}^{L} dx\, f(x) \cos nx\left(\frac{\pi}{L}\right) \qquad \text{and} \qquad b_n = \frac{1}{L}\int_{-L}^{L} dx\, f(x)\, sin\, nx\left(\frac{\pi}{L}\right) \tag{5.2}$$

In general, *f* is a complex value and both a_n and b_n are complex numbers.

STFT can be mathematically obtained by multiplying the signal $\boldsymbol{x(t)}$ with the analysis window of $\boldsymbol{\gamma^*(t-\tau)}$, then compute the conventional Fourier transform for this particular period. The transform of the windowed signal is:

$$\mathcal{F}_x^{\gamma} = \int_{-\infty}^{\infty} x(t)\, \gamma^*(t-\tau)\ e^{-j\omega t}\, dt. \tag{5.3}$$

What $\gamma^*(t-\tau)$ does is that it ignores any part of $x(t)$ that falls outside of the window. Then Fourier transform is implemented over this particular part of the signal inside the local window. Figure 5.3: illustrates the process of windowing a signal [71]. This is, however, is for continuous time signals which mostly suits audio signals analysis.

To decompose a signal using STFT, as in Equation 5.1, the signal must be able to be reconstructed to recover the signal, as shown in Equation 5.4. From Equation 5.1 $x(t)$ can be reconstructed by the following equation [71]:

$$x(t) = \sum_{m=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} \mathcal{F}_x^{\gamma}(mT, k\omega_\Delta)\ g(t-mT)\ e^{jk\omega_\Delta t} \tag{5.4}$$

Where $F_x^{\gamma}(mT, k\omega_\Delta)$, m , k ∈ Z of the STFT are the coefficients of the series expansion of the signal $x(t)$. It is observed that the functions set utilized to reconstruct the signal are built from the modulated and shifted versions of the same $g(t)$ [71]. Moreover, perfect reconstructions can be also obtained. For more details and derivations please refer to [71].

In discrete time signals, however, due to the critical sampling for $T\omega_\Delta = 2\pi$ and equal analysis and synthesis, it is not possible to obtain good time and frequency resolutions.

Whenever the time window $\gamma(t) = g(t)$ allows perfect reconstruction, then one of Δ_t or Δ_ω will be infinite. This notion of relationship is widely known as “Balian-Low theorem” [72]. Balian-Low theorem shows that it is possible to construct an orthonormal STFT where the window is differentiable and compactly supported [72].

Therefore, when computing STFT mathematically for any given discrete time signal $x(n)$, we substitute the integration in Equation 5.2, STFT for continuous time signal equation with summation [72]:

$$\mathcal{F}_x^\gamma(m, e^{j\omega}) = \sum_n x(n)\, \gamma^*(n - mN)\, W_M^{kn} \tag{5.5}$$

It is assumed that the sampling rate of the signal is higher than the rate of calculating the spectrum by a factor of $N \in \mathbb{N}$. Here γ^* and g denote the analysis and synthesis windows, respectively. The frequency ω is normalized to the sampling frequency. Due to the limitation of STFT, a further development from Fourier type transform was needed and as a result, wavelet transform was invented and this will be discussed in the next section.

Since STFT utilises the fast form of DFT computation, hence it has the same computational complexity of Discrete Fast Fourier Transform (DFFT) of $O(N \log N)$. This kind of computational complexity is hardware driven and is simply defining how the data structure of such a transform is operating in hardware environments, how it indexes and searches for any stored data during operations. Those operations are directly determined, nowadays, by the actual performance of cache or CPU pipeline optimisation. Let $x_0, x_1, \ldots, x_{N-1}$ be a set of complex numbers, their DFT can be computed by this formula [73]:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k \frac{n}{N}} \qquad k = 0, 1, \ldots, N-1 \tag{5.6}$$

To calculate this formula, it requires $O(N^2)$ operations with N number of outputs of X_k and each single output requires a sum of N number of terms. What is good is that FFT is defined by any operation that can compute the same result with $O(N \log N)$ complexity [73], fortunately, in machine language this can be done by other operations, i.e heapsort, quicksort and mergesort, as discussed in the previous Section 5.2 . According to Johnson and Frigo [73], the O here denotes the upper bound whereas the lower bound complexity is thus far not proved to be known. The same result of formula (5.4), however, can be computed by the well-known Radix-2 by Cooley and Tukey [14] with $\left(\frac{N}{2}\log_2 N\right)$ complex multiplications with eliminating trivial operations, multiplications by 1 and $N\log_2 N$ complex additions. The issues that still stands, generally, are counting the exact operations of FFT and proving the lower bounds of the complexity [74].

5.4 DISCRETE WAVELET TRANSFORM

The wavelet transform was born from the need to further develop the Fourier transform. Wavelet transform is used widely in digital image processing and digital video processing; it was deployed in MPEG2000 standard due to its proven accuracy in image analysis. Furthermore, wavelet analysis is used also in several other applications, such as - medical applications, watermarking, electromechanical machines and sustainable energy applications. However, it has not been used as a wireless/wired communication transmission technique due to the lack of researches and resources. Wireless and broadcasting communications are our interest in this research to examine the performance of wavelets in such applications.

Unlike the Fourier transform, where the sinusoidal wave repeats itself for infinity, the wavelet transform is located only within a finite domain and zeroes anywhere else. Hence, it is possible in the wavelet transform to model changes in frequency by changing the scale of the time domain. Additionally, modelling the changes in time could be achieved by shifting the position of the wavelets; therefore, the frequency and location of the frequency could be modelled. Consequently, this transform is called the time-frequency domain [75]. The wavelet transform converts the signal from the time domain into the time-frequency domain and is situated within a finite domain with zeroes anywhere; conversely, the Fourier transform is different where there is infinite repetition of the sinusoidal wave. Therefore, it is possible to model changes in the frequency in the wavelet transform through a change of the scale of the time domain. Moreover, the changes can be modelled in time by changing the position of the wavelets; thus, it is possible to model both the frequency and its location; giving rise to its name the time-frequency domain [75].

Within wavelet transform, there are two kinds of transforms, namely; the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). CWT is considerably costly due to convolutions performed at all positions. On the other hand, DWT is not that costly because the signal data, which needs processing, is stored discretely. Additionally, the Nyquist sampling theorem shows that with a discrete data signal, the highest frequency which can be modelled is half that of the sampling frequency [75]. Calculation of DWT and IDWT can be achieved using filter banks comprised of LPF and HPF. These filters are FIR filters with variable lengths and different coefficients, dependent on the wavelet family and the family member being used. As shown in Figure 5.5, every LPF is subdivided into a further level of HPF and LPF. Each LPF or HPF is treated as an individual signal and can be traced along the subdivision (sub-bands). These signals do not interfere with each other because of the scaling feature.

There are many methods of computation to obtain the wavelet transform, however, in 1989 and 1990s Mallet [53] and Daubechies [76] respectively introduced the concept of multirate and multiresolution analysis or basically wavelet transform. In 1986 and 1990, Smith and Barnwell [77] and Vaidhyanathan [78] proposed perfect reconstruction and multirate and filter banks, respectively. Multirate filter banks are primarily used in this research and more adaptive and flexible filter banks orientation is proposed.

The dilation Equation 5.7, also called the refinement equation, acts as a starting point from which to determine the scaling function φ and the wavelet function ψ :

$$\frac{1}{2}\varphi\left(\frac{x}{2}\right) = \sum_{n\in Z} w_n \varphi(x-n) \tag{5.7}$$

The DWT and Inverse Discrete Wavelet Transform (IDWT) filters are closely related to the sequence $(w_n)n \in Z$. Whenever φ satisfies the following conditions:

1. It is compactly supported.
2. The sequence w_n is finite.
3. It could act as a filter.

Then W (normalized) is the scaling filter and it is a FIR filter with a $2N$ length and it has a coefficients' sum of 1 of a low-pass filter with norm of $\frac{1}{\sqrt{2}}$ [75]. While there are many wavelet families, what make a good wavelet family candidate are three characteristics: compactly supported, smoothness and orthogonality. In CWT, any given finite signal is continuously represented throughout a family of frequencies bands, often called space function, $L^2(R)$. For example, if any signal that could be represented for every frequency band of [f, $2f$] for all frequencies $f > 0$, then the original signal can easily be reconstructed by performing a decent integration for all resulted frequencies bands.

The sub-bands' frequencies are a scaled versions of sub-band at band 1 and these sub-bands are situations that are generated by shifting one generating function of $\psi(t)\ in\ L^2(R)$. For more illustration, take the frequency band [1, 2], the $\psi(t)$ function is expressed as [75]:

$$\psi(t) = 2sinc(2t) - sinc(t) = \frac{\sin(2\pi t) - \sin(\pi t)}{\pi t} \tag{5.8}$$

Equation 5.8 is what is known as mother wavelets and from this, we can obtain several wavelet families of functions that are nested as family members. For the same example, the sub-band of scale of a or sub-frequency [1/a, 2/a] is generated by functions that are often referred to as child wavelets or family's member:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{\mathrm{e} - \mathrm{b}}{a}\right) \tag{5.9}$$

Where a is positive and represents the scale and b is a real number and represents the shift. Since CWT is impractical and is impossible to analyse signals by computing all wavelets coefficients due to the infinite scales and translations; hence, DWT is used instead to approximate signals [75]. In DWT the wavelet child is rather expressed as:

$$\psi_{m,n}(t) = a^{-m/2}\psi(a^{-m}t - nb) \tag{5.10}$$

Where $a > 0$ and $b > 0$, the discrete subset consists of all points $(a^m - na^m b)$ with m and n in $\mathbb{Z}$. Now the reconstruction of *x* signal is possible by this formula [53]:

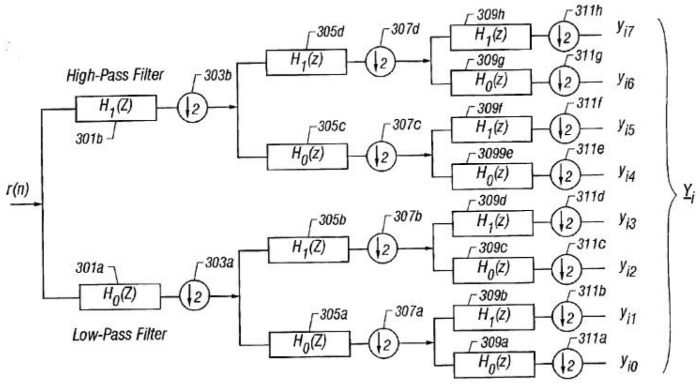

Figure 5.5: Filters Bank (Reconstruction) [**36**]

$$x(t) = \sum_{m\in\mathbb{Z}}\sum_{n\in\mathbb{Z}}\left(x,\ \psi_{m,n}\right).\ \psi_{m,n}(t) \quad (5.11)$$

Where the functions $\{\psi_{m,n} : m, n\ \in \mathbb{Z}\}$ form an orthonormal basis of $L^2(R)$.

To illustrate more, we take this signal and fix the appropriate function $h(x)$:

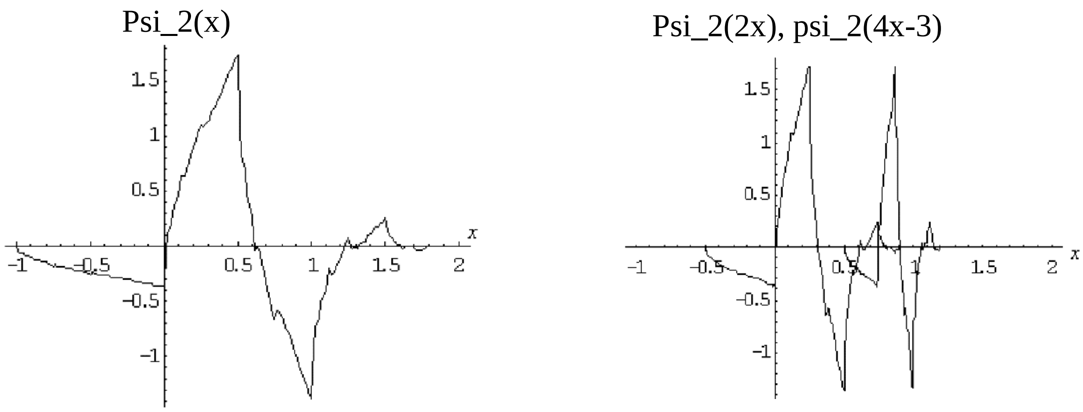

Figure 5.6: h(x) (left) and h(2x) and h(4x-3) (right)

Then all possible translations of this signal can be formed by integers and all possible stretching by powers of 2, this can written as: $h_{jk}(x) = 2^{\frac{j}{2}}h\left(2^j x - k\right)$ where $2^{\frac{j}{2}}$ is a normalized constant. For h(2x) and h(4x-3), let $c_{jk} = \int dx f(x)h_{jk}(x)$, if h is properly chosen, then the f can be calculated from c_{jk} by [69]:

$$f(x) = \sum_{j,k} c_{jk}\ h_{jk}(x) \quad (5.12)$$

Note that h_{jk} has a width order of 2^{-j} and is centralised around $k2^{-j}$ as shown in Figure 5.6.

Now in contrast with STFT, Equations 5.1 and 5.2, these functions of wavelet analysis, Equation 5.10, are easier to manage than Fourier. The advantages over STFT is that:

- ✓ The coefficients c_{jk} are all real numbers.
- ✓ For the high frequencies (j large) then the functions $h_{jk}(t)$ have a good localisation and they becomes thinner as j approaches ∞, as in shown in Figure 5.6. Hence short duration lived duration, in x, high frequency components can be seen from wavelet analysis, however, not from STFT.

The use of this transform in communication systems to modulate signals has been recently patented [36]. Orthogonal Wavelet Division Multiplex (OWDM) has been proposed as an alternative to OFDM and used to separate the sub-bands of the bandwidth and allocate them dynamically with different widths. This is claimed to offer a lower computational complexity in comparison to OFDM, as well as being more flexible [3]. Moreover, OWDM is different from OFDM, which uses FFT where complexity is marginally higher than DWT. In the next section, wavelet families and their properties will be explored and we will see how those applicable families are utilised within the core of this research.

5.4.1 WAVELET FAMILIES

As discussed in the previous section, from the mother wavelet Equation 5.6, a number of subset of members of that mother wavelet can be computed using Equation 5.7. There are number of mother wavelet functions each representing a family of wavelets and each of those families have nested members that are often referred to as “child wavelets”. As seen in the dilation Equation 5.5, whenever φ is compactly supported, the sequence w_n is finite and could act as a filter, then W can be seen as scaling filter, and it is a FIR filter with a $2N$ length and this filter has a coefficients’ sum of 1 of a low-pass filter with norm of $\frac{1}{\sqrt{2}}$. Since there are many wavelets and complex wavelets families are already implemented in mathematics, those relevant to this research will be discussed. Those relevant wavelet families in this research are: Haar (Daubechies 1), Daubechies, Symlets, Coiflets, Biorthogonal, Reverse Biorthogonal and Discrete Meyer. However, in this section the mathematical functions of mother wavelet of those families and how their associated members is located will be discussed. A summary of all properties of those families that are compatible for the use as a modulation scheme are presented in Table 5.3, we will also refer to this table in later chapters.

Table 5.3: Compactly Supported Wavelet Families Properties [18] [53] [69] [79]

Wavelet Family / Property	Haar	db*N*	sym*N*	coif*N*	bior*Nr.Nd*	rbior*Nr.Nd*	dmey
Arbitrary regularity		✓	✓	✓	✓	✓	
Compactly supported orthogonal	✓	✓	✓	✓			✓
Compactly supported biothogonal					✓	✓	✓
Symmetry	✓				✓	✓	✓
Asymmetry		✓					
Close to symmetry			✓	✓			
Arbitrary number of vanishing moments		✓	✓	✓	✓	✓	
Vanishing moments for φ				✓			
Existence of φ	✓	✓	✓	✓	✓	✓	
Orthogonal analysis	✓	✓	✓	✓	✓	✓	

Biorthogonal analysis	✓	✓	✓	✓	✓	✓	
Exact reconstruction	✓	✓	✓	✓	✓	✓	✓
FIR filters	✓	✓	✓	✓	✓	✓	✓
Continuous transform	✓	✓	✓	✓	✓	✓	
Discrete transform	✓	✓	✓	✓	✓	✓	✓
Fast algorithm	✓	✓	✓	✓	✓	✓	✓
Explicit expression	✓				For Spline	For Spline	
FIR-based approximation							✓

5.4.2 HAAR WAVELETS

This family of wavelets were named after Alfred Haar [79] who invented DWT and used his famous formula (Equation 5.13). Haar wavelet is a special case of Daubechies wavelet (Baubechies 1) and it essentially works by taking any input data represented by 2^n numbers and pairing those values up then storing the difference and passing the summation. The process is then recursively repeated by pairing the summations up to supply the next scale, which leads to have $2^n - 1$ differences and finally a summation. The Haar wavelet is known as the simplest possible wavelet family [80] for its technical advantage of being discrete and thus not differentiable. The Haar's mother wavelet function $\psi(t)$ can be written as [79]:

$$\psi(t) = \begin{cases} 1 & 0 \leq t < \frac{1}{2}, \\ -1 & \frac{1}{2} \leq t < 1, \\ 0 & otherwise \end{cases} \tag{5.13}$$

The scaling function $\varphi(t)$ of this wavelet member can be written as [79]:

$$\varphi(t) = \begin{cases} 1 & 0 \leq t < 1, \\ 0 & otherwise \end{cases} \tag{5.14}$$

For every pair *n*, *k* of integers in the $\mathbb{Z}$ then the Haar mother function $\psi_{n,k}$ can be defined in the real **R** by [79]:

$$\varphi_{n,k}(t) = 2^{\frac{n}{2}}\psi(2^n t - k), \quad t \in \mathbf{R} \tag{5.15}$$

This operation is compactly supported within the right open interval of $I_{n,k} = [k2^{-n}, (k+1)2^{-n})$, however, it vanishes outside this interval. The Haar wavelet is a compactly supported, orthogonal and symmetrical wavelet as will be seen in the results from Chapter 8. For more details on Haar wavelets and mathematical derivations of the Haar wavelets, please refer to [79] [81] and [69]. The computational complexity of the Haar is all three categories of complexities are anticipated to be less than FFT in all cases including computational complexity, which is $O(N)$ operations, time and space consumption, as we will see later in Chapter 7 and Chapter 8.

5.4.3 DAUBECHIES WAVELETS

The Daucechies wavelets were named after Ingrid Daubechies [69] who formulated a series (family) of mathematical functions for DWT analysis in time-frequency domain. This wavelet family and subsequent families that were built on this basis are orthogonal, discrete and compactly supported. The naming scheme of this family follows: db*n* where *n* is the number of vanishing moments, sometimes it is named with D*x* where *x* is number of taps. Daubechies wavelet family is the most used DWT which uses recurrence relations in order to breed finer discrete samplings of an implied mother wavelet function and thus a cascaded scales of resolutions that each resolution is double that of the previous scale [69]. As said in the previous section, Haar wavelet is a special case of Daubechies which is identical to (db1). Daubechies is the base wavelet for Symlet family and a part of Biorthogonal and Reverse Biorthogonal. Regarding this family is purely mathematical derivations, nonetheless, what overlaps with the context of this research will be picked; however, if required more information can be found in [69] and [76].

The construction of this family can be achieved by LPF (scaling filter) for scaling sequence and HPF for the wavelet sequence will be normalised to acquire a sum = 2 and a sum of squares = 2. It can also be normalised to $\sqrt{2}$, depending on the application, in which case the sequences and all their shifts are orthonormal, given that the shifts are by an even number. The general formula of scaling sequence for orthogonal DWT with an approximation sequence of A is expressed as [69]:

$$a(Z) = 2^{1-A}(1+Z)^{A}p(Z) \tag{5.16}$$

Where *N*=2*A*, *p*(1)=1 and degree (*p*)=*A*-1, then the orthogonality can be expressed as [69]:

$a(Z)a(z^{-1}) + a(-Z)a(-Z^{-1}) = 4$ or can equally $(2-X)^{A}P(X) + X^{A}P(2-X) = 2^{A}$ with Laurent polynomial $X \coloneqq \frac{1}{2}\,.(2 - Z - Z^{-1})$ generates all symmetric sequences and $X(-Z) = 2 - X(Z)$; where $P(X)$ is for the symmetric Laurent polynomial $P\big(X(Z)\big) = p(Z)p(Z^{-1})$ where $X\big(e^{iw}\big) = 1 - \cos(\omega)$ and $p\big(e^{j\omega}\big)p\big(e^{-j\omega}\big) = \big|p\big(e^{j\omega}\big)\big|^{2}$ P always takes $\geq$ 0 value within the segment [0, 2]. Orthogonality equation has just one minimal solution for each A:

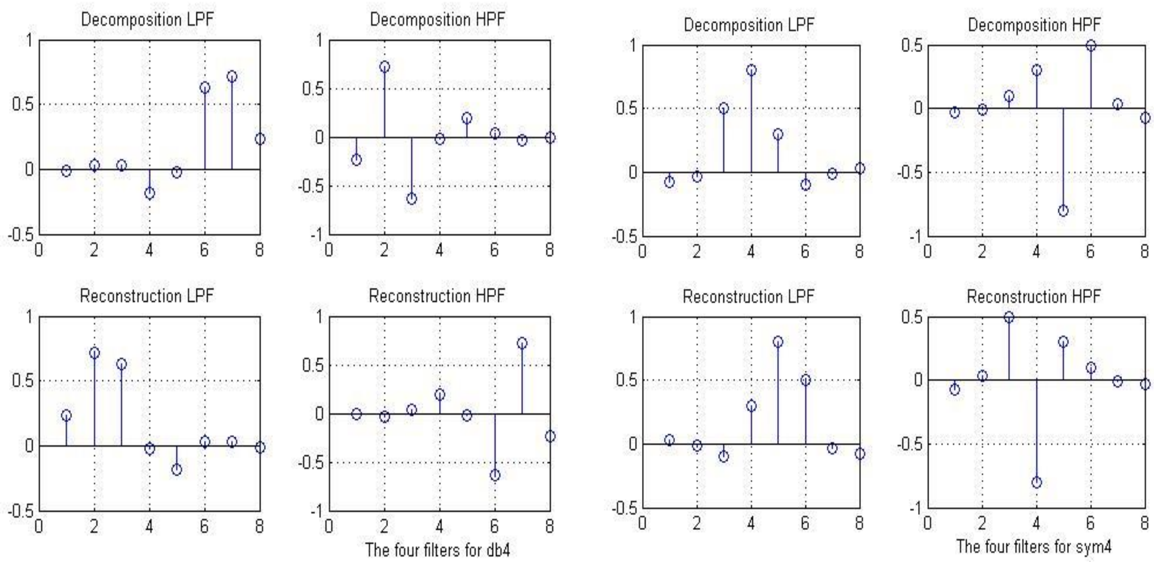

Figure 5.7: db4 (left) and sym4 (right) filters' impulse response

$$P_A(X) = \sum_{k=0}^{A-1} \binom{A+k-1}{A-1} 2^{-2} X^k \tag{5.17}$$

This is essentially dividing the orthogonality Equation 5.13 for each A in the ring of truncated power series in X.

However, the orthogonality Equation 5.13 is antisymmetric $\approx X = 1$ and therefore has this general solution $X^A(X-1)R((X-1)^2)$ with R being polynomial with real coefficients then the sum is: $P(X) = P_A(X) + X^A(X-1)R((X-1)^2)$ shall be ≥ 0 within the interval [0, 2], the P within the interval [0, 2] is bounded by4^{A-r}. In Daubechies family, the pair of filter used should be quadrature mirrors, and calculating c_i number of coefficients for those quadrature mirror filters, for a quadrature mirror filter order 4:

$$c_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_1 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_2 = \frac{1+\sqrt{3}}{4\sqrt{2}}, c_3 = \frac{1+\sqrt{3}}{4\sqrt{2}} \tag{5.18}$$

$$h(z) = h_{-2}z^2 + h_{-1}z + h_0 + h_1 z^{-1}, \qquad \tilde{h}(z) = h_1 z + h_0 + h_{-1}z^{-1} + h_{-2}z^{-2}$$

$$g(z) = -h_1 z^2 + h_0 z - h_0 z + h_{-2}z^{-1}, \qquad \tilde{g}(z) = h_{-2}z - h_{-1} + h_0 z^{-1} - h_1 z^{-2}$$

In this case, in Equation 5.15 c_0 will be replaced by h_{-2}, c_1 by h_{-1}, c_2 by h_0 and c_3 by h_1. Figure 5.6 illustrates the locations of those scaling and wavelet filters which shows the relationship between decompositions and reconstructions calculations.

To illustrate the myths of math in this family with numbers, the scaling coefficients for db4 to db9 are calculated in Appendix A.1, Table A.1, for space preservation only those were chosen and contrasted with Symlets, whose coefficients table is in Appendix A.2, Table A.2. The Symlets family are a newer version of Daubechies family but with improved symmetry and this is why it is often referred to as "symmetrical daubechies". Hence, it follows the same naming pattern of Daubechies, sym$n$ where n is the number of vanishing moments, or sometimes Sx where x is number of taps. To illustrate this difference in symmetry, db 4 and sym 4 filters impulse responses are presented in Figure 5.5. It is apparent from Figure 5.5 that the symmetry has indeed improved. Since Symlets family is an improved version of Daubechies and therefore they have the same mother wavelet but the detailed differences can just be seen in detailed mathematical functions, hence this will not be discussed mathematically in details here but can be found in [69] and [76]. The coefficients of Symlets wavelet are shown in Table A.2. To illustrate the computation and reconstruction using those filters coefficients, we decompose one level of filters and reconstructed it again using the illustration of one level in Figure 5.6. In Figure 5.6 h is the LPF and g is HPF in the analysis filter, and $\tilde{h}$ is LPF and $\tilde{g}$ is HPF in the reconstruction filters bank. Given that all those wavelet filters satisfies the perfect reconstruction condition: $h(z)\tilde{h}(z) + g(z)\tilde{g}(z) = 2\,,\ h(z)\tilde{h}(-z) + g(z)\tilde{g}(-z) = 0$. The scaling filters scale by some factors $\alpha\ and\ \beta$ and shifts by an even integers of $2j\ and\ 2k$ $h'(z) = \alpha z^{2j}h(z),\ g'(z) = \beta z^{2k}g(z)$ and $\tilde{h}'(z) = \alpha^{-1}z^{-2j}\,\tilde{h}(z),\ \tilde{g}'(z) = \beta^{-1}z^{-2k}\,\tilde{g}(z)$ this preserves the condition of perfect reconstruction. It is also possible to produce a valid wavelet when exchanging the filters $h\ and\ g$ with $\tilde{h}$ and $\tilde{g}$.

For any compactly supported FIR wavelet transform, it can be expressed with lifting scheme [76]. Lifting scheme is described with a series of predict and update filters, as depicted in Figure 5.8, where $p_1, p_2, \dots p_m$ denotes predict filters and $u_1 u_2, \dots, u_m$ denote update filters. Following the filtering steps, x_e is multiplied by $K_s\ and\ x_0$ is multiplied by K_d. For the inverse

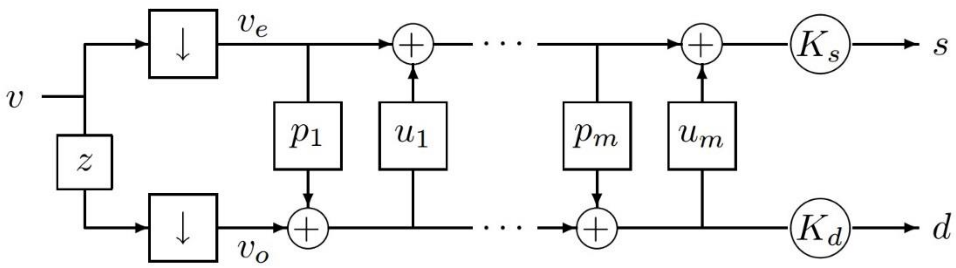

Figure 5.8: Lifting Scheme Predict and Update Filters [76]

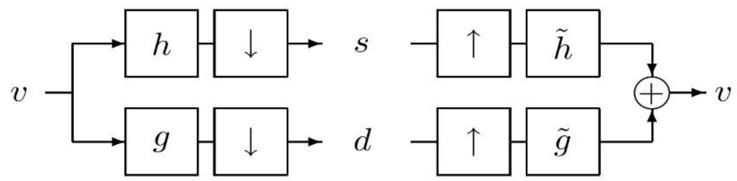

Figure 5.9: One level decomposition and reconstruction

transform, the scaling factors K_s *and* K_d are reversed and the additions are changed to subtractions with reversing the filtering steps.

Daubechies family is seen to be best computed wavelets and thus it was the bases not only for Coiflets and Symlets but also a part of Biorthogonal and Reverse Biorthogonal wavelet families. The differences between those families and the original Daubechies is mainly the symmetry of filter taps, locations of vanishing moments for scaling and wavelet functions, the number of coefficients and their values. Hence, all wavelet families and families' members will be revisited and discussed in the results and analysed in Chapter 7 and Chapter 8 within the context of this research; nevertheless, for in depth mathematical functions building derivations, look at [76] [81] [69] [79] and [82], however, an overview with their coefficients is presented here. Symlets 2, 3, 4, 5, 6, 7 and 8 coefficients are presented in the Appendix A, Table A.2.

Coiflets family was developed by Ingrid Daubechies [69] when Ronald Coifman requested to develop a scaling functions with vanishing moments. The wavelets of this family are near symmetrical and will be discussed in details in Chapter 7 and Chapter 8. It follows a different naming pattern. The wavelet functions have $\frac{N}{3}$ vanishing moments and a scaling function of $\frac{N}{3-1}$. Table A.3, in Appendix A.3, presents the Coiflets scaling coefficients for all available members (5), the wavelet coefficients are computed from the scaling coefficients by $B_k = (-1)^k C_{N-1-k}$, where k represents the coefficient index, B denotes the wavelet coefficient, C is the scaling function and N is the wavelet member index or number of coefficients for that members. All three tables Daubechies, Symlets and Coiflets were generated using this formula:

$$h(z) = \sum_k h_k z^{-k}, \quad g(z) = zh(-z^{-1}), \quad \tilde{h}(z) = h(z^{-1}), \quad \tilde{g}(z) = g(z^{-1}) \qquad (5.19)$$

5.4.4 BIORTHOGONAL AND REVERSE BIORTHOGONAL WAVELETS

Biorthogonal and Reverse Biorthogonal wavelets are rather different than the families mentioned previously. These wavelet have a rather unique feature to them whereby their associated wavelet function is invertible. Therefore, biorthogonal wavelet families are designed to have more margin of freedom than orthogonal wavelets. One of the most important factors is the possibility of constructing symmetrical wavelet function which, of course, reduces the time complexity that systems need for computation. Figure 5.10 shows the symmetry of bior6.8. Another feature is that this family has two scaling functions, rather than one as in orthogonal families. Accordingly, two wavelet functions are also associated with Biorthogonal families and those functions generate multiple multiresolution analysis. The two scaling and wavelet functions are referred to $\emptyset, \widetilde{\emptyset}$ and $\psi, \tilde{\psi}$ consecutively. As a result, the naming pattern for this family is different than others and they follow:

- Biorthogonal, bior*Nr*.*Nd*, where *Nr* is reconstruction order and *Nd* is the decomposition order, i.e. bior2.6 has 2 vanishing moments in the reconstruction and 6 vanishing moments in the decomposition filters bank.
- Reverse Biorthogonal, rbio*Nd*.*Nr*, which is clearly the reverse of decomposition and reconstruction filters, i.e. rbio2.6 has 2 vanishing moments in the decomposition and 6 in the reconstruction filters bank.

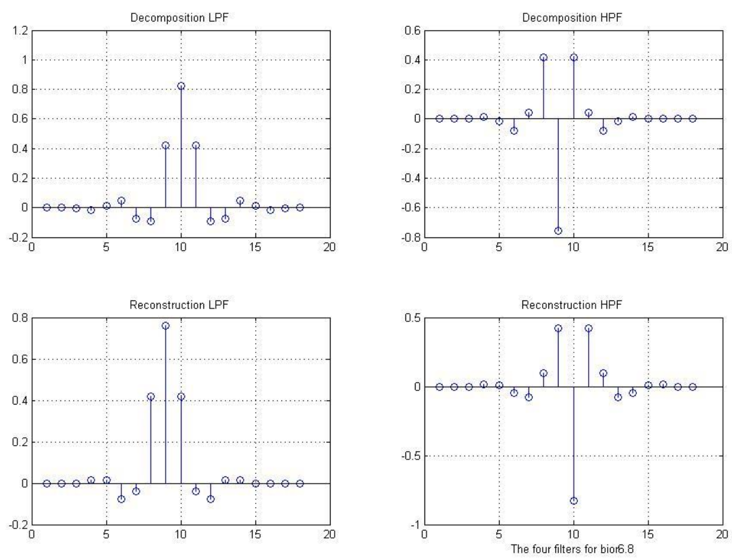

Figure 5.10: Bior6.8 Filters Symmetry

The scaling sequence in this family must satisfy the biorthogonality condition, which is [70]:

$$\sum_{n\in\mathbb{Z}} a_n \tilde{a}_{n+2m} = 2.\,\delta_{m,0} \tag{5.20}$$

Subsequently, the wavelet functions sequence can be obtained from:

$$b_n = (-1)^n \tilde{a}_{M-1-n} \qquad (n = 0, 1, \ldots, N-1)$$

$$\tilde{b}_n = (-1)^n \tilde{a}_{M-1-n} \qquad (n = 0, 1, \ldots, N-1) \tag{5.21}$$

The computed scaling coefficients for Biorthogonal is can be found in Table A.4 in Appendix A. A further analysis of these wavelets in the context of this research will be discussed in Chapter 7 and Chapter 8.

5.5 ORTHOGONAL WAVELET DIVISION MULTIPLEX

Using the DWT, OWDM modulates signals over a specific level of filter banks and is dependent on the number of required subcarriers. Because of the sampling theorem and the way scale convolutions are calculated in wavelet transform, the first path, picked up at a single sample point, is not required. On the other hand, the smallest scale, which is more important for consideration, is where the wavelet picks up two sample points. Thus, this process is achievable by using a series combination of orthogonal filters comprising LPF and HPF filters

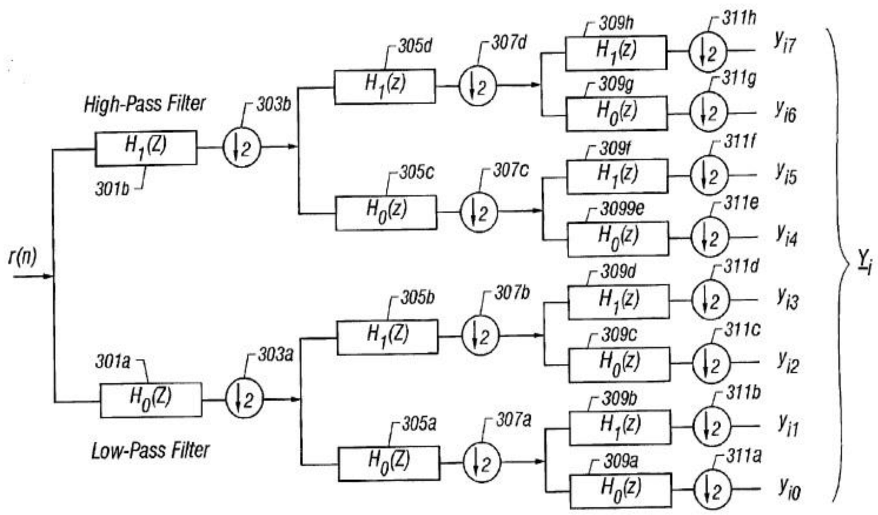

Figure 5.11: Symmetric OWDM (OWSS) [36]

which together form the filters bank, as illustrated in Figure 5.3, in the figure $G_1(Z)$ is HPF and $G_0(Z)$ is LPF. The LPF is comprised of the resulting scaling function, whereas HPF is comprised of the resulting wavelet function [36].

As illustrated in Figure 5.12, each LPF is further divided into HPF and LPF, each of which is considered an individual signal and is traceable along the subdivision (sub-bands). Moreover, due to the scaling feature, these signals will not interfere with each other. The application employed for the bandwidth availability determines the amount of filter bank levels which are calculated for each single split of the LPF and HPF. The OWDM communication system is comprised of two parts: the channel interface and the synthesis section. The former is comprised of a filter bank inputs; and an output that supplies the OWDM signal [36].

Each of the inputs receives a symbol related to the super-symbol, selected from a modulation scheme. An OWDM is generated by the synthesis section signal which is comprised of weighted OWDM pulses; each weighted pulse represents a symbol taken from the super-symbol. OWDM pulses are also employed together with an OWDM Spread Spectrum, referred to as the OWSS which allows wireless channels to operate using an equaliser. This is illustrated in Figure 5.11.

In reference to Hassan [83], the above mentioned filters are mostly single lowpass filters (ho(i)), such as:

$$\begin{aligned} hi(i) &= -1I\ ho(L-1-i) \\ go(i) &= ho(L-1-i) \\ gi(i) &= -(-1)iho(i) \end{aligned} \tag{5.22}$$

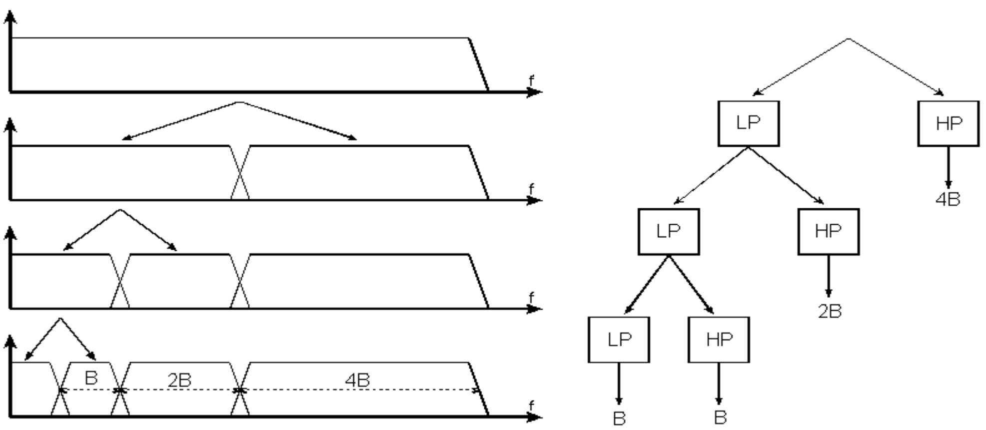

Figure 5.12: Asymmetric frequency splitting using Wavelet Transform [68]

Where L is the length of the filter and $0 \leq i \leq L-1$.

OWDM is premised upon the concept of orthogonal multipulse signalling [11]. Lee and Messerschmitt [11] presented that the pulses $\varphi m(t)$, $m = 0, 1, \ldots, M-1$, form an orthonormal set over a certain time frame and each pulse is orthogonal to itself shifted by the non-zero integer multiples of a certain interval T. Each basis pulse $\varphi m(t)$ is able then to create a 'virtual' channel to carry the am symbol. The vector of symbols $A = [a_0, a_1, \ldots, a_{M-1}]T$ is referred to as a supersymbol and the interval $T = MT_s$ as the supersymbol/block interval, where T_s is the basic symbol interval. Thus, the base band transmitted signal becomes the following:

$$S(t) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m(t - iT) = \sum_{i=-\infty}^{\infty} A_i^T\ \varphi(t - iT) \tag{5.23}$$

Symbol and block timing extraction are conducted at the receiver. The signal is correlated with $\varphi(t - iT)$ in order to detect the nth super symbol at time iT and time $iT + \tau$, where the optimum timing phase is denoted by τ [11]. Because of complementary metal oxide semiconductor and large scale integration, it is easier to carry out signal processing techniques in the discrete time domain. From this point, discrete time formulation of orthogonal multipulse signalling will be employed. The discrete time orthogonal multipulses are $\varphi m(n)$, $m = 0, 1, \ldots, M-1$; subsequently, the corresponding transmitted orthogonal multipulse signal is represented as follows:

$$s(n) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m\,(n - iM) = \sum_{i=-\infty}^{\infty} A_i^T\ \varphi(n - iM) \tag{5.24}$$

In order to make it more convenient and simple, the variable t will be used to denote both the continuous time variable and the discrete time sample index. Moreover, M and T will be employed interchangeably to represent the block length. The following are specific advantages to using orthogonal multipulse signalling:

- Possibility of dynamic separation of sub-bands.
- Potential to be less sensitive to multipath fading.
- No need for guard intervals [3].
- Possibility of reaching channel capacity and counter selective fading.
- Potential for multiplexing at the physical layer.

5.6 PRACTICAL WAVELET TRANSFORM (FAST WAVELET TRANSFORM)

The advances in communications and telecommunications systems are rapidly increasing in the requirement for processing time and data rate size. The drive force is also that DWT, as discussed in the complexity Section 5.3 in this chapter, which has a linear complexity as opposed to the logarithmic complexity of DFT. This is obviously less, however, as the data grows it takes longer time and requires larger memory to compute those wavelets with higher vanishing moments, as in Daubechies wavelets formulas, because of the large number of coefficients involved and thus the number of multiplication and/or additions depends on the sorting algorithms used. Since the demand is rapid and hardware nowadays are capable of handling such high data rate in an even faster speed, the need for development of practical algorithms that can handle such demand in a faster way than ever before dramatically increases. Fortunately, a genius group of scientists from Yale University, Beylkin *et. al.* [84], built and algorithm that can compute DWT in an unprecedented speed and they called it “Fast Wavelet Transform (FWT) or Time-Invariant DWT (TI-DWT)” as also expanded on by Mallat [53].

While the DWT is very sensitive to the signal alignment in time [70], FWT is time-invariant. Not just that but it also optimises the computational speed requirements of the state-of-the-art machines and real time systems. FWT is essentially a mathematical algorithm, built by Mallat [70], to compute DWT faster by turning any signal in the time domain into a sequence of coefficients based on orthonormal basis of finite wavelets. Additionally, this transform can also be readily expanded to multidimensional signals such as images.

In a finitely generated orthogonal MRA, selecting J sampling scale with 2^J sampling rate per unit, the input signal f can be projected onto the space V_J by computing the scaling products by [84]:

$$s_n^{(J)} = 2^J \langle f(t), \varphi(2^J t - n) \rangle \tag{5.25}$$

Where φ is the scaling function of any particular wavelet transform; practically by any decent sampling proceedings with the condition that this signal is largely oversampled, the following equation is the orthogonal projection of the signal of the best approximation of the original signal in the space V_J:

$$P_J[f](x) = \sum_{n \in \mathbb{Z}} s_n^{(J)} \varphi(2^J x - n) \tag{5.26}$$

What characterises MRA is its scaling resulted sequence, which is:

$$a = (a_{-N}, \ldots, a_0, \ldots, a_N), \text{or as its Z} - \text{transform } a(z) = \sum_{n=-N}^{N} a_n z^{-n} \tag{5.27}$$

The wavelet sequence of this scaling sequence is:

$$b = (b_{-N}, \ldots, b_0, \ldots, b_N), \text{or as its Z} - \text{transform } b(z) = \sum_{n=-N}^{N} b_n z^{-n} \tag{5.28}$$

The advantage of wavelets is that some of those coefficients will be zero and thus, will lead to reduction in size. From the resulted wavelet sequence one can compute the wavelet coefficients $d_n^{(k)}$ or at least some range of *k=M, . . ., J-1* without necessity for approximating the integrals within the corresponding scalar products. Those coefficients can be directly computed starting from $s^{(J)}$ with the use of convolution and decimation operators. We will see how these computations is done in practical operations in this research's context in the next Chapter 6, and revisit it again in the results analysis Chapter 7.

As shown previously in DWT section is that the DWT of a discrete signal $x = [x(0), x(1), \ldots, x(N-1)]^T$ is the procedure of computing the coefficients [84]:

$$W_\varphi[j_0, k] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m]\varphi_{j_0,k}[m] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j_0}{2}} \varphi[2^{j_0} m - k], \quad \text{for all } k \tag{5.29}$$

$$W_\psi[j, k] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m]\psi_{j_0,k}[m] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j}{2}} \psi[2^{j} m - k], \text{for all } k \;\&\; j > j_0 \tag{5.30}$$

Where the scaling and wavelet functions for W_φ and W_ψ are respectively:

$$\varphi_{j,k}[m] = 2^{\frac{j}{2}} \varphi\left[2^j m - k\right] \quad \text{and} \quad \psi_{j,k}[m] = 2^{\frac{j}{2}} \psi\left[2^j m - k\right] \tag{5.31}$$

Recalling that the scaling and wavelet functions are both could be expanded on the basis of the scaling functions for the next higher resolution:

$$\varphi[m] = \sum_{l} h_{\varphi}[l]\sqrt{2}\varphi[2m - l] \quad \text{and} \quad \psi[m] = \sum_{l} h_{\psi}[l]\sqrt{2}\psi[2m - l] \tag{5.32}$$

Now from that we can obtain a fast algorithm to obtain $W_{\varphi}[j_0, k]$ and $W_{\psi}[j, k]$ of different scales of j. First, consider that scaling function $\varphi[m]$, by replacing m by $2^j m - k$ (which is scaled by 2^j and translated by *k*), we get [84]:

$$\varphi[2^j m - k] = \sum_{l} h_{\varphi}[l]\sqrt{2}\varphi[2(2^j m - k) - l] = \sum_{l} h_{\varphi}[l]\sqrt{2}\varphi[2^{j+1}m - 2k - l] \tag{5.33}$$

We then let *n=2k+l* or in another word, *l=n-2k*, the above Equation, 5.18, can be rewritten as:

$$\varphi[2^j m - k] = \sum_{n} h_{\varphi}[n - 2k]\sqrt{2}\varphi[2^{j+1}m - n] \tag{5.34}$$

The wavelet function can likewise be expressed as:

$$\psi[2^j m - k] = \sum_{n} h_{\psi}[n - 2k]\sqrt{2}\varphi[2^{j+1}m - n] \tag{5.35}$$

Since this wavelet function, in Equation 5.20, is identical to the one used in the DWT, Equation 5.15, then we can replace the right hand side of the latter equation to be [84]:

$$\begin{aligned} W_{\psi}[j, k] &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j}{2}} \psi[2^j m - k] \\ &= \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j}{2}} \left[\sum_{n} h_{\psi}[n - 2k]\sqrt{2}\varphi[2^{j+1}m - n] \right] \end{aligned} \tag{5.36}$$

Which is equal to:

$$\sum_{n} h_{\psi}[n - 2k] \left[\frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j+1}{2}} \varphi(2^{j+1}m - n) \right] \tag{5.37}$$

Now, the expression inside the bracket is the wavelet transform for the coefficients of scale $j + 1$, so it can be expressed as:

$$W_{\psi}[j+1,k] = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} x[m] 2^{\frac{j+1}{2}} \varphi\left[2^{j+1}m - n\right] \tag{5.38}$$

Thus, there are a recursive relationship between the wavelet transform coefficients of consecutive scales levels of j and $j+1$, and therefore we can write:

$$W_{\psi}[j,k] = \sum_{n} h_{\psi}[n-2k] W_{\varphi}[j+1,n] \tag{5.39}$$

This is also true to the scaling function which can be expressed as:

$$W_{\varphi}[j,k] = \sum_{n} h_{\varphi}[n-2k] W_{\varphi}[j+1,n] \tag{5.40}$$

Now, in contrast to the discrete convolution, which is expressed as:

$$y[k] = h[k] * z[k] = \sum_{n} h[k-n] x[n] \tag{5.41}$$

We can observe that the wavelet coefficients $W_{\psi}[j,k]$ and the scaling coefficients $W_{\varphi}[j,k]$ at any level of j can be obtained from the coefficients $W_{\psi}[j+1,k]$ and $W_{\varphi}[j+1,k]$ at the $(j+1)$, the scale by convolution with time reversed h_{ψ} or h_{φ} and up or down sampling to obtained every other samples in the convolution. Therefore, we can write this as:

$$W_{\psi}[j,k] = h_{\psi}[-n] * W_{\varphi}[j+1,n]\Big|_{n=2k,k\leq 0} \tag{5.42}$$

$$W_{\varphi}[j,k] = h_{\varphi}[-n] * W_{\varphi}[j+1,n]\Big|_{n=2k,k\leq 0} \tag{5.43}$$

From those two Equations, 5.27 and 5.28, overall wavelet and scaling coefficients, $W_{\psi}[j,k]$ and $W_{\varphi}[j,k]$ consecutively, of a given signal x can be found recursively from the coefficients $W_{\varphi}[j,k]$ and $W_{\varphi}[j,k]$ at the highest resolution level of j=J with a maximum details. For instance, N data samples $x[m](m = 0, 1, \ldots, N-1)$ directly sampled from the signal $x(t)$. Therefore, as a member of V_J space these discretely sampled signals can be approximated as a linear combination of the scaling basis functions of $\varphi_{J,k}[m]$, as [84]:

$$x[m] = \sum_{k} W_{\varphi}[J,k]\varphi_{J,k}[m] \tag{5.44}$$

Now, if one can let the k^{th} basis function be a unit impulse at the k^{th} sampling time, for example $\varphi_{J,k}[m] = \delta[k-m]$ and likewise for the i$^{\text{th}}$ component of a unit vector e_j in the N dimensional vector space is $e_{ij} = \delta[i-j]$, then the k^{th} coefficients $W_{\varphi}[J,k]$ is the same as the k^{th} samples of the function $x(t)$, i.e. $W_{\varphi}[J,k] = x(k)$, from which the wavelet and scaling coefficients of the lower scale $j < J$ can be obtained by the subsequent filters bank.

The Inverse Fast Wavelet Transform (IFWT) is achievable, likewise of Fast Forward Wavelet Transform (FFWT), by obtaining the coefficients W_{ψ} and W_{φ} from a given function of $x(t)$ can be implemented by the analysis filters bank. Whereas the inverse IFWT reconstruct the function $x(t)$ from the coefficients W_{ψ} and W_{φ} can be built by the synthesis filters bank, as shown in Figure 5.13.

Therefore, the computational complexity and cost for FWT is simply the convolution computed in each of the filters. Since in the convolution the number of data samples are halved after each subsampling, thus the total computation complexity is:

$$O\left(N + \frac{N}{2} + \frac{N}{4} + \frac{N}{8} + \frac{N}{16} + \ldots + 1\right) = O(N) \tag{5.45}$$

When this is executed in parallel, which in wavelet transform possible, this computation complexity becomes linear with $O(N)$ complexity, as explained in Section 5.2. In contrast with $O(NlogN)$ complex operations of FFT, this wavelet linear complexity is less. The wavelet operations can also be performed in parallel, as Equation 5.45 shows, rather than sequentially as in FFT. Therefore, FWT is promising replacement of FFT due to multiple advantages which includes:

- Analysis of signals in time-frequency domain which allow better localisation of signals at a particular desired frequency.
- Less computational cost of number of operations.
- Less number of computational iterations than that of FFT.
- Faster computation for those wavelet families' members who has least number of vanishing moments.

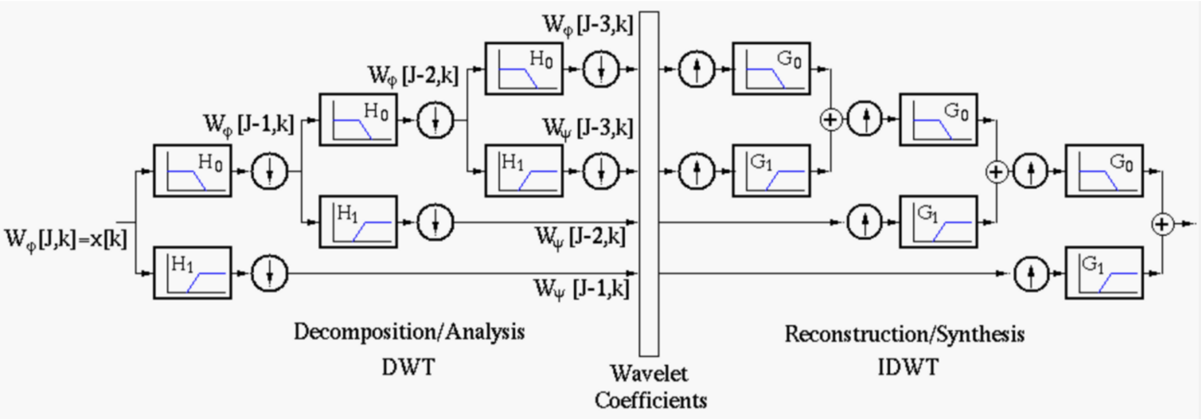

Figure 5.13: FWT and IFWT from DWT and IDWT Filters Bank [**85**]

- Possible reduction in space complexity compared to FFT with least number of vanishing moments.

In engineering perspective, hardware restricts the efficiency but not the standard or theory. Therefore, less expensive machines with maximum tolerated efficiency are always desired in engineering and vendors perspectives. We will see how these computations are performed in practical operations in the context of this research in the next Chapter 6, and revisit it again in the results analysis Chapter 7.

5.7 SUMMARY

In this chapter, we explored and analysed the differences between the mostly used time-frequency signal analysis techniques. We then applied DWT in a communication system context and saw how this transform can be very useful for modulation of signals in signal transmission systems. Then we looked at the complexity of each of those transforms, how they operate in hardware context, CPU and memory consumption and more importantly, the cost associated with data buffering and storage. Data structures of both complexities, $O(N)$ and $O(NlogN)$ were reviewed and compared against each other. It is found that wavelet transform has a complexity of $O(N)$, which allows a linear operation. On the other hand, however, DFT has been found to have a complexity of $O(NlogN)$ which is more complex than that of wavelets; however, in hardware data structure, the complexity of the FFT operation is similar to heapsort, quicksort or mergesort operations, which are well-established. The practicality of using DWT in real time transmission systems was evaluated. In short, the Big-O notation represents the complexity factor of operations needed to complete a task. A rule of thumb is that there is always a trade-off between time complexity and space complexity. Very often, algorithms with $O(N \log N)$ time complexity would have $O(\log N)$ space complexity, as we saw with quicksort algorithm, which present an issue with processing extremely large datasets which is always the case for real time transmission systems, where a recursive function runs on the system often with limited stack memory.

STFT is primarily DFFT operation, however, with a time limited window that it is restricted by and that window sweeps over the signal to compute STFT for the entire signal. Since this operation contains multiple DFT operations, proportional to the window size, it adds to the complexity of the system and certainly to the latency, which is often not tolerable in real time transmission systems. However, this is traded off with the advantage that it analyses the signal in time-frequency domain, which gives a much better approximation and resolution of the signal compared to the conventional DFT, which transforms the signal from time to frequency domain. Furthermore, STFT is restricted by the window size which cannot be changed once it has been set and this is the main reason which drove researchers and scientists to further develop transforms that are more flexible and adaptive to many applications.

This drive ended up with the discovery of the wavelet transform in both CTW and DWT. Since CTW is significantly costly due to the huge memory needed to store infinite number of coefficients, DFT is used instead. The primary advantage of DFT over STFT is that it performs multiresolution analysis with more flexibility and varying depths, thanks to the wavelet and scaling functions. Apart from that, it also has the possibility of analysing and approximating signals using multiple wavelets and scaling functions, as used in multiwavelets. However, since multiwavelet is not yet feasible to be used for transmission schemes due to its added complexity of computation and thus, memory size and latency occurrence, the conventional wavelet family is considered in this research.

To the best of this author's knowledge, there has not been any considerable investigation of wavelet based OFDM in fully standardised system that comply with the current industrial standard. No error detection and correction, encoding or interleaving were included, instead most of these research examined the signal from mapper onwards, which is much easier to simulate but not practical. Additionally, there are no previous comprehensive investigations or studies carried out on a complete system for all available compactly supported wavelet families and their associated members. Moreover, most of the critically surveyed researches did not consider reliable data sizes in their investigations, whereas other researches did not declare the data size they investigated, which indicate either unreliability of the used data size or its unavailability. Also, the system examined was a replica of Jain and Meyers' patent [36] which uses symmetrical filter banks. Therefore, this research shall consider the full DVB-T system and comply with the DVB-T standard [19] and propose a dynamic bandwidth allocation using asymmetrical filter banks. Moreover, not only all available wavelet families will be investigated, but also all their associated members shall be investigated. The data size examined in this research shall be ranging from 1000 frames with 96 bits per frame to 100k frames, which is big enough to transmit 6 minutes HD video.

FWT is the fast algorithm that can compute DWT faster than DWT itself and this was to adapt the rapid increase in demand to not only higher data rate communications systems but also faster and reliable processing times that real time systems demand, with less stack memory usage and less cost involvement. The less computational cost of FWT of only $O(N)$ than the FFT of $O(NlogN)$ make hardware more efficiently running in term of computational complexity and cost. Nevertheless, this is not the case for all wavelet families and their members where the high number of filters taps induces higher number of multiplications and additions operations which in turn adds to the space complexity and thus larger memory is needed which means more cost. While FWT is less complex than FFT, however, FFT is less complex in space requirement than FWT. However, this is not always the case for all wavelet families and members, where Haar, with just two filters taps, can operate within the same space required for FFT operations. In the context of this research, these type of complexities will be analysed during the analysis of results and a conclusion will be drawn and contrasted with what we discovered in this chapter.

CHAPTER 6: THE DESIGN OF THE PROPOSED SYSTEMS

Objectives:

- To describe the system's specifications.
- To describe the baseline system, Orthogonal Frequency Division Multiplex.
- To describe standard used for parameters and results validation.
- To design the proposed system compatibly with the used standard.
- To specifically list the theoretical and practical methodology of operation of the proposed system.

6.1 INTRODUCTION

This chapter will present the system design of the baseline OFDM system that will be built to be the baseline for comparison with the proposed system. Since the comparison has to be significant and reliable, one of the most successful standards that uses OFDM, ETSI 300 744 [19], has been adopted in this research. However, other standards that utilises OFDM are possible to be used as we will discuss further in the conclusion Chapter 9. One of the main reasons that DVB-T standard is adopted for this research is that it broadcasts videos in which no tolerance for any latency and video multimedia tends to be the largest kind of data transmission and heaviest traffic. Therefore, the proposed system could use the bandwidth more efficiently and subsequently reduce the associated cost, more of that will be discussed in Chapter 7 and Chapter 8. The design of the proposed system will also be presented in this chapter with detailed discussion, results and analysis to be presented in Chapter 7 and Chapter 8. This chapter is an implementation of the first phase of work described in the technical approach, Section 4.3, in the methodology, Chapter 4.

6.2 SYSTEMS SPECIFICATIONS

This phase of the research is the most time consuming phase during this research. It has two main sub-phases, one of which has two further sub-tasks, and the other has five. The main two tasks in this phase are as follows:

- The OFDM system.
- The OWDM system.

The specifications of both systems are mainly determined by the adopted DVB-T standard [19] which is the baseline of comparison with the proposed system in this research. However, the IFFT in OFDM is replaced by the proposed filters bank for OWDM and subsequent modifications shall be carried out to make both systems are like-for-like comparable. The technical approach is mentioned earlier in Chapter 4, Section 4.3, and the Flow chart in it will be followed through the right branch as OWDM has been deemed feasible for implementation. The technical approach phase required the time-scale of 11 months to fulfil its requirements. In the following sections, these two systems' specifications will be explained, planned and designed for implementation. Both designed systems are simulated and implemented in Chapter 6 with results analysis in Chapter 7.

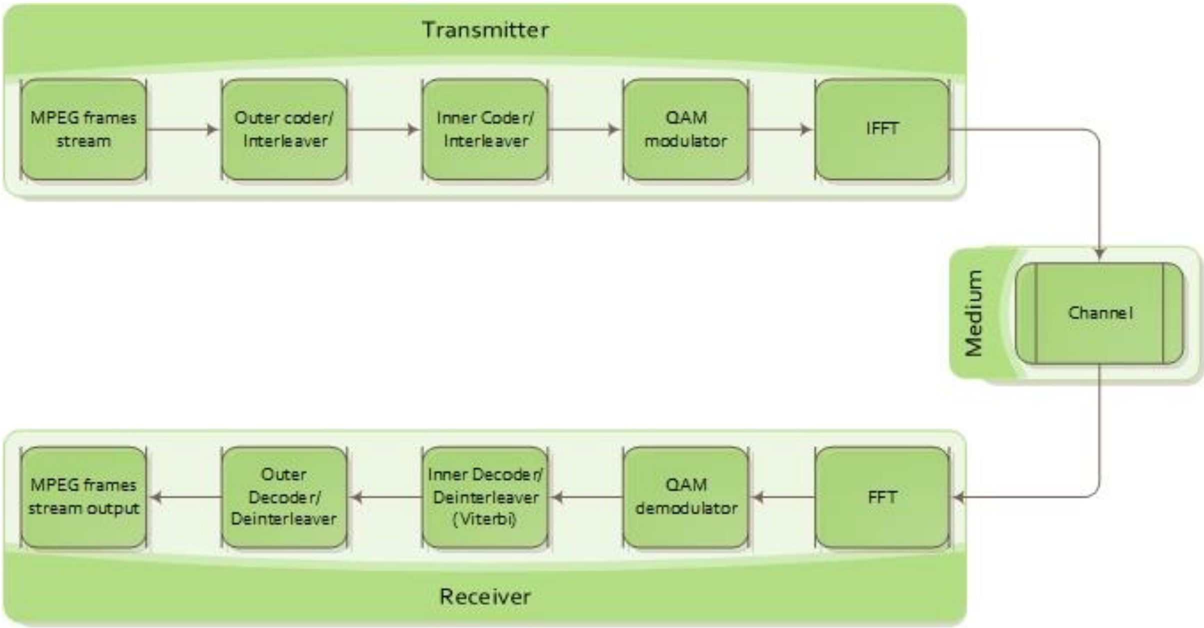

Figure 6.1: OFDM System Transceiver as in [**19**]

6.3 THE OFDM SYSTEM

The DVB-T standard [19] will be designed to compare and evaluate the proposed system. The OFDM system, as generally described in Section 2.6.1.3 in Chapter 2, is the core element of the DVB-T standard which will be designed and thoroughly described in this section. The full system block diagram is depicted in Figure 6.1. As clearly seen from Figure 6.1 the system consists of three core parts, as follows:

- Transmitter to transmit the signal through the channel.
- Channel through which the signal will travel.
- Receiver where the signal will be received and decoded.

6.3.1 THE OFDM TRANSMITTER

The transmitter is considered to be able to handle the MPEG frame streams. Therefore, there has been a need to design a full system to grant a full recovery to the transmitted signal at the receiver side. This transmitter contains the following processes, which will be explained individually at a later stage:

1) Source signal.
2) Outer coder/ interleaver.
3) Inner coder/ interleaver.

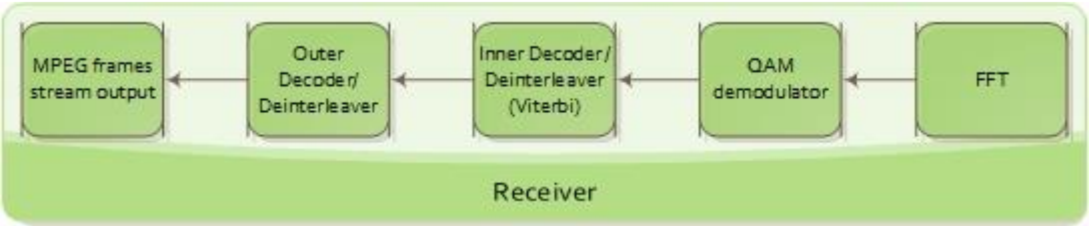

Figure 6.2: OFDM transmitter [**19**]

4) Quadrature Amplitude Modulator (QAM).
5) Inverse Fast Fourier Transform (IFFT).

The transmitter is depicted in Figure 6.2.

6.3.1.1 SOURCE SIGNAL

The source signal used in this research has been considered for use is in the DVB-T standard [19], in the Moving Picture Expert Group (MPEG-2) format. The number of frames which are used ranges from 100 frames to 10000 frames which can be enough to transmit 5.7 minutes video with 30 frames/second resolution. Each of these frames has a size of 96 bits. The video packet transmission is probably one of the most sensitive transmissions to the Signal to Noise Ratio (SNR) which drives to the packet loss during transmission.

6.3.1.2 OUTER CODER/INTERLEAVER

In the case of DVB-T, the outer coder/interleaver can apply two different types of signal processing consecutively onto the input data. The first is Reed Salomon (RS) coding, which is responsible for the detection and correction of errors in the data. For instance, this is achieved by taking the 188 bytes of data and adding 16 bytes to it to have, in total, 204 bytes as an output. Each packet has some parity symbols to protect the data symbols from loss. The parity symbols are distributed between packets depending on the importance of the packets/frames. This rule is known as priority coding.

The second process after outer coding, is outer interleaving of the signal by the outer interleaver, which is a convolutional interleaver based on the Forney approach. The interleaving process is applied onto the error protected packets from the outer coder. This is achieved by composing the data into 12 branches each with a depth of 17 bytes. The process of interleaving and de-interleaving is depicted in Figure 6.3 [19].

The convolutional interleaving of each of these 12 branches works similar to a shift register and is based on the rule of first-in first-out. Each branch, represented by j has a depth of $j * M$ cells where M = 17 = N/I, where N = 204. The switches of input and output must be fully synchronised [19].

6.3.1.3 INNER ENCODER/INTERLEAVER

This process consists of two consecutive processes. Firstly, the inner encoder, which is based on the puncturing convolutional encoder system which relies on a modulo-2 system coding. This process is called "channel coding". The purpose of this process is to improve the channel capacity by adding redundant information to the data. The convolutional coder is always operating on a serial data stream. The outer interleaver is used to rearrange transmitted data sequences in such a method that makes data less susceptible to errors; outer interleaver and deinterleaver is depicted in Figure 6.3.

The inner encoder is based on the Forward Error Correction (FEC) system. It works as a shift register, by taking the input stream bit by bit and always starts at zero for all bit delay. After this, the first four bits are fed into an XOR gate, after which the system will clock each certain time. Each clock represents 1 bit, by one digit and the XOR will provide an output bit for every single clock of the system.

These fed bits are chosen to be the first, third, fourth, sixth and seventh to have the second output of the coder. This process is depicted in Figure 6.4. There are three essential parameters that specify the data being encoded; these are the number of input bits, number of output bits and number of stages of the shift register. In Figure 6.4, the number of shift stages is six. The

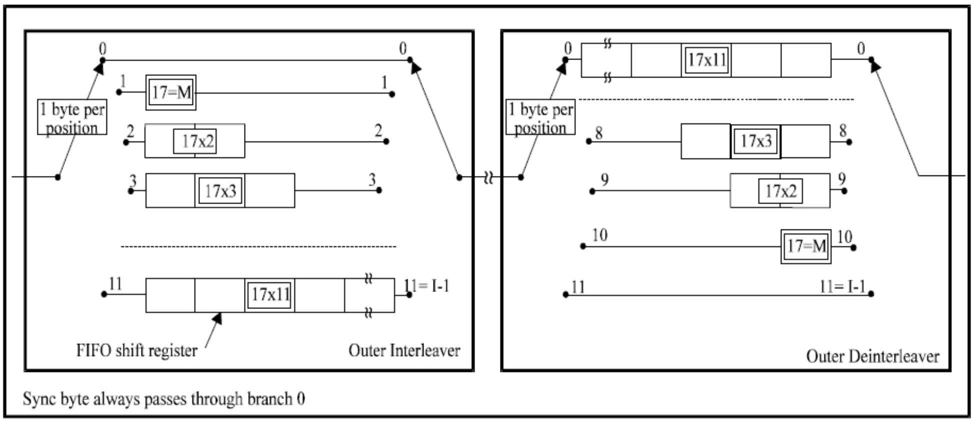

Figure 6.3: Interleaving and deinterleaving process in DVB-T [**19**]

Trellis diagram is used in association of the convolutional coder to represent the event sequence of the data and the tracing of the data bits. The Trellis diagram differs from one code to another and is unique.

Table 6.1: ETSI EN 300 744 DVB-T puncturing pattern for possible code rate [19]

Code Rates r	Puncturing Pattern	Transmitted sequence (after P/S conversion)
1/2	X: 1 Y: 1	X1 Y1
2/3	X: 1 0 Y: 1 1	X1 Y1 Y2
3/4	X: 1 0 1 Y: 1 1 0	X1 Y1 Y2 X3
5/6	X: 1 0 1 0 1 Y: 1 1 0 1 0	X1 Y1 Y2 X3 Y4 X5
7/8	X: 1 0 0 0 1 0 1 Y: 1 1 1 1 0 1 0	X1 Y1 Y2 Y3 Y4 X5 Y6 X7

Figure 6.4 shows the convolutional coding as used in [19], has been also used in this research but with a different code rate. This convolutional coder must be flushed out by feeding it zero bits before starting any encoding process. The reason for this is to guarantee that there are no previously encoded data inside the coder. In addition, it is required to flush out again at the end of each encoding process to guarantee that all input bits are thrown out of the coder.

Secondly, the inner interleaving is the process right after inner coding. The purpose of interleaving is to improve the performance of the inner coder. It has a complex procedure for each mode of transmission, e.g. hierarchal or non-hierarchal. The relationship between the inner coder and inner interleaver is illustrated in Figure 6.5. Moreover, it has two types of interleaving, as follows:

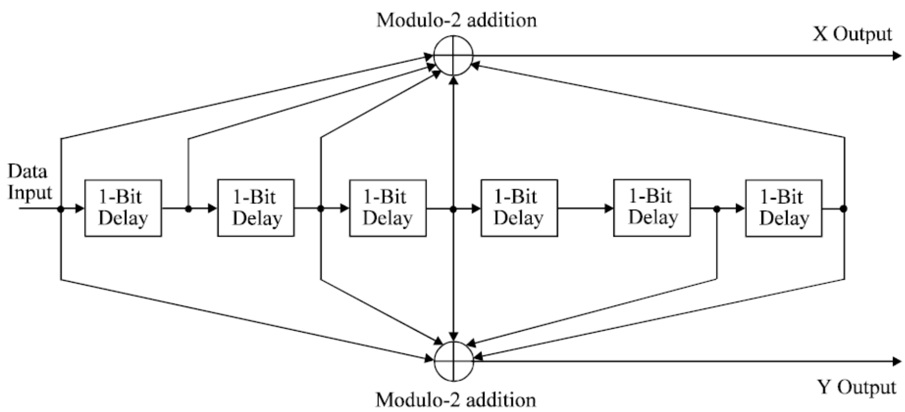

Figure 6.4: Convolutional encoder with code rate of 1/2 [**19**]

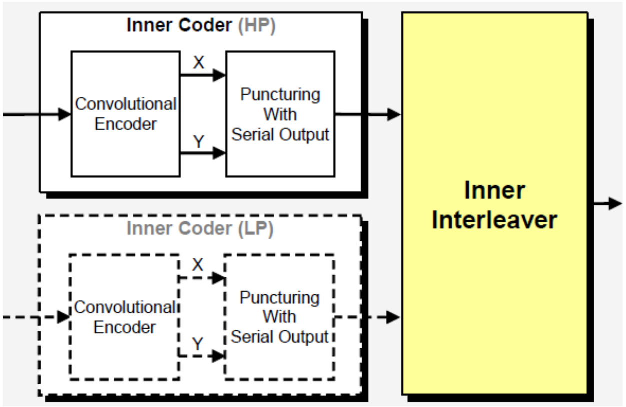

Figure 6.5: Inner coder and inner interleaver HP and LP as in ETSI EN 300 477 [**19**]

1) **Bit-wise interleaver**, whereby the input bit stream is demultiplexed into a number of sub-streams called "v", where v is measured by bits/symbol for each symbol on the constellation map. In 16-QAM, v=4 bit/symbol. This process is different in the hierarchal mode and non-hierarchal mode. In the non-hierarchal mode, each input stream is a demultiplexed v sub-stream [19].

   Whereas, in the hierarchal mode, there are two priority streams, High Priority (HP) and Low Priority (LP). The HP stream is demultiplexed into two sub-streams and the LO as well but with lower priority [19].
2) **Symbol interleaver**, the main function of this process is to map the v bits stream, which is demultiplexed by bit wise interleaver, onto the carriers of the OFDM symbols [19].

The relationship between the bit wise interleaver and the symbol interleaver, for 16-QAM DVB-T system, is depicted in Figure 6.6.

For the hierarchal mode of transmission, interleaving and mapping design is exactly the same as in Figure 6.6. Nevertheless, the second demultiplexer has been removed and the bit interleaver has been fed from just one demultiplexer as there is just one priority stream for

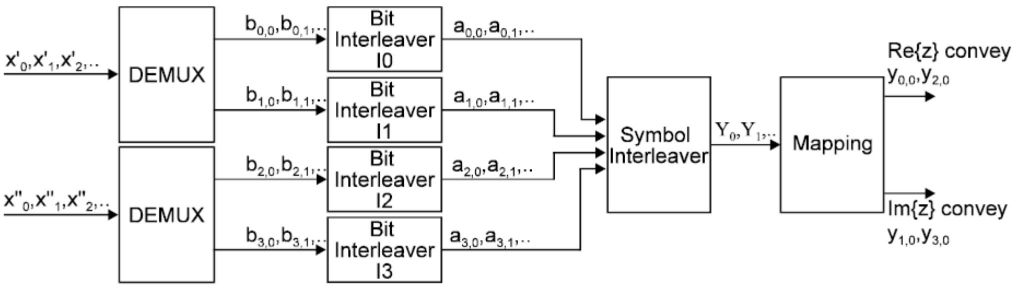

Figure 6.6: Mapping input bits by bit interleaver into symbols by symbol interleaver [**19**]

hierarchal mode. As one level of hierarchal transmission has been used in this research, therefore the chosen code rate of convolutional coder was ½.

6.3.1.4 QUADRATURE AMPLITUDE MODULATOR

Quadrature Amplitude Modulator (QAM) is a modulation technique employed by many of the current high technology communications systems. The reason behind this is that it reduces, and sometimes eliminates, intermodulation interference caused by a continuous carrier near the modulation sidebands. *Quadrature,* in the name, is derived from the fact that QAM can convey a two digital bit stream or two analogue signals by modulating the amplitude of two carrier waves by using amplitude shift keying; and these sinusoids carriers are out of phase by 90° to each other.

It is also called mapping as it maps the information symbols onto a constellation which returns complex values which represent the input information. Each point on the constellation consists of 4 bits, for 16-QAM. Additionally, as the size of M-QAM increases, it can handle higher data rate. The increase of bit rate follows the rule of $2^N = M$, where M is the size of the QAM and N is the bits/symbol, e.g. 2^4= 16-QAM. The QAMs types vary depending on the application it is actually being used for. Generally, it follows the same principle of size to increase the bit rates; however, this comes at a cost of the fact that more the QAM size is used the more it becomes susceptible to errors.

QAM maps the data onto the constellation map by modulating the phase and the magnitude of the information sinusoidal signal periodically. When combining the phase and magnitude of the sinusoidal signal, it produces one symbol on the constellation and vice versa. Each of these symbols represents a digital bit stream with the binary value of the input information signal. As stated earlier, the two sinusoids carriers 90° out of phase are represented as one signal with two components called I and Q, in which $Q = A\sin(\emptyset)$ and $I = A\cos(\emptyset)$; where Q represents the quadrature component and I represents the in-phase component. By subtracting the two components from each other, the result is as follows [85]:

$$I\cos(2\pi f_c t) - Q\sin(2\pi f_c t) = A\cos(2\pi f_c t + \emptyset) \tag{6.1}$$

Where I is the in-phase component and Q is the quadrature component.

f_c is the carrier frequency.

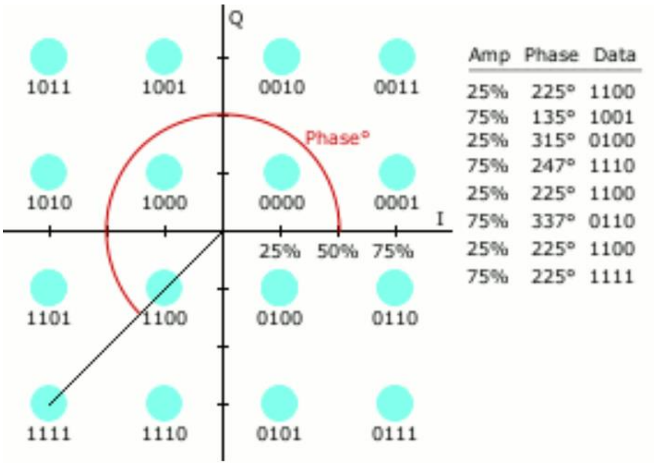

Figure 6.7: Constellation map for 16-QAM with I and Q components position

A is the magnitude of the signal.

$\emptyset$ is the phase shift.

Therefore, by changing the amplitude of the resulting signal, the phase of I and Q components are changed too. For this reason, modulating the carriers digitally is possible through this method by changing the magnitude of the two components. Mapping according to the phase and the magnitude on the constellation is depicted in Figure 6.7.

The QAM used in this research shall be 16-QAM which has a size of 4 bits/symbol and 64-QAM which has a size of 6 bits/symbol. The constellation of data symbols mapping is shown

Table 6.2: 16-QAM Mapping Array Values

Array Index (Input Symbols to 16-QAM modulator)	Array Index Represented in Binary	Array Value -> AB + j CD (Output of 16-QAM modulator)
0	0000	-3 - 3j
1	0001	-3 - 1j
2	0010	-3 + 3j
3	0011	-3 + 1j
4	0100	-1 -3j
5	0101	-1 -1j
6	0110	-1 +3j
7	0111	-1 +1j
8	1000	+3 -3j
9	1001	+3 -1j
10	1010	+3 +3j
11	1011	+3 +1j
12	1100	+1 - 3j
13	1101	+1 - 1j
14	1110	+1 +3j
15	1111	+1 +1j

in Figure 6.7 with the mapping address of each data symbol, 64-QAM follows the same mapping procedure. The array values for the 16-QAM mapper is shown in Table 6.2.

6.3.1.5 THE INVERSE FAST FOURIER TRANSFORM

The IFFT is the inverse process of FFT. This transform is one form of Fourier transform and an effective algorithm to compute DFT. This is the core signal processing technique in any OFDM system and this is what makes OFDM distinctive amongst so many other modulation techniques. IFFT generates an OFDM signal which enables the signal to be transmitted through the channel. The signals, that want to be transformed, must be time limited to avoid aliasing. The reason for this is that the signal needs to be sampled, to ease signal processing, and the sampling of the sinusoid signal in the frequency domain is merely a main pulse, which represents the most valuable information, with unlimited harmony of spikes. The mathematical complexity of this transform, as discussed previously in Chapter 2, Section 2.6.1.1, is $O(Nlog\, N)$.

This core module of OFDM is what this research is mainly focusing on when comparing it with another type of decomposition transform proposed in this research to be an alternative to this core module. Due to the computational complexity and limitations, this research has been conducted to experiment the effectiveness of this technique compared with the brand new proposed technique, as will be described in Section 6.4.

IFFT in OFDM at the transmitter works by converting the input frequency domain signal, from serial to parallel converter, into a time domain signal. Computationally, the input complex values of the data, from QAM, into IFFT is sampled for several sinusoids and then summed up. However, IFFT modulates and multiplexes at one process at the same time. The output of IFFT is further processed by converting from digital to analogue then filtering to have a baseband signal [22].

Mathematically, as it is mentioned earlier in Section 2.6.1.1, that *N* bits are stored and if it is let $a_0, a_1, \dots, a_{N-1}$ represent these *N* bits. Let f_c be lowest frequency of the *N* carriers, and write the k^{th} as:

$$\cos 2\pi\,(f_c + k\Delta f)t, \qquad 0 \leq k \leq N-1 \tag{6.2}$$

Subsequently the entire transmitted signal can be expressed by the following equation, as described by Schwartz [25]:

$$v(t) = Re\left[\sum_{k=0}^{N-1} a_k\, e^{j2\pi(f_c+k\Delta f)t}\right]$$

$$= Re\left[e^{j2\pi f_c t} \sum_{k=0}^{N-1} a_k\, e^{j2\pi k\Delta f t}\right]$$

$$= Re\left[e^{j2\pi f_c t}\, a(t)\right] \tag{6.3}$$

$$\text{Where } a(t) = \sum_{k=0}^{N-1} ak\, e^{j2\pi k\Delta f t}$$

v(t): total transmitted signal.

Re: represent the real part of the function.

ak: *k = 1,…,N*: successive bits stored.

$f_k = f_c + k\Delta f$*, 0 ≤ k ≤ N-1.*

f_k: *k=1, …,N* number of carriers frequencies.

f_c= the lowest subcarrier of *N* parallel subcarriers.

Δf: carrier separation.

N: number of parallel subcarriers.

In this stage, if *a(t)* could be sampled at a rate of $R = \frac{T_s}{N}$ samples/sec. Consequently, *a(t)* will be replaced with the sampled function *a(n)*; and *t* is replaced by nT_s/N for $n = 0, \dots, N-1$. By taking $\Delta f T_s = 1$, the result would be the following [25]:

$$a(n) = \sum_{k=0}^{N-1} a_k\, e^{\frac{j2\pi kn}{N}} \qquad n = 0,1,\dots,N-1 \tag{6.4}$$

Therefore, $a(n)$ is the IDFT form of a_k.

Where: a_k: $k = 1, \ldots, N$: successive bits are stored and N is the number of parallel subcarriers. If we let K < N successive binary digits be stored, generating one of 2^k possible QAM signals, each signal corresponds to a complex number ak.

As is clearly noticeable, Equation 6.4 is in the form of IFFT, and it can be easily evaluated and recovered using FFT [25].

For this research, the length of IFFT used was considered to be proportional to 2^N, where N is the number of filter banks used in the OWDM system. However, the intended IFFT lengths would be 64 associated with 6 levels of filters bank. Then this IFFT lengths increased to up to 2048 to comply with the DVB-T standard's [19] 2k mode. Equivalently, in OWDM system 11 associated levels of filters bank is configured. All available wavelet families and families' members will be examined against OFDM in terms of BER performance and analysed in Chapter 7 and Chapter 8. Systems parameters are also presented in Table 7.1.

6.3.2 THE OFDM CHANNEL

The concept of the channel in communication systems refers to the medium in which the signal will be sent through and has two types::wired or wireless. The interest of this research is the wireless channel, as it considers modulation techniques in the wireless environment. Any channel, in communications, consists of band of frequencies that can be utilised to transmit Electro Magnetic Waves (EM). Additionally, channels have a limitation that must be considered when designing any communication system, these are:

1) Bandwidth, when it is increased, the amount of information that could be sent through increases as well.
2) Transmission power, the signal will become susceptible to noise as the transmission power is decreased. Thus, increasing the power of transmission will reduce the impact of the noise.
3) SNR, the higher the SNR, the lower error that will be introduced onto the signal.

Moreover, the channel's parameters are characterised by the following:

1) Attenuation: This is through several phenomena, e.g. power leakage, wideband noise, introducing ISI by the channel or absorbing interference from other sources measured with (dB/km).

2) Impedance: This is reduced by matching and measured by measuring the load across the input and output ports (Ω).
3) Bandwidth: This is the tube width that limits the data rate of the transmitted signal; it is the band of frequencies (Hz).

In the free space, the signal is propagated through the channel, according to the following formula:

$$v = \frac{c}{\sqrt{\varepsilon_r}}, \qquad \lambda = \frac{v}{f_0} \tag{6.5}$$

There are two mathematical representations for a channel, these are: Linear channel and Non-linear channel. In the linear channel, the recovery of the transmitted signal is possible by following one of the following two formulas:

$$y(t) = x(t - t_d) \tag{6.6}$$

Another formula: $$\frac{y(t)}{k} = x(t - t_d)$$

Where: k: constant, T: time and t_d: $time\ delay$.

In the non-linear channel, the recovery of the transmitted signal is very hard. This channel has the following formula:

$$y(t) = a_1 x(t) + a_2 x(t)^2 + a_3 x(t)^3 + \ \dots + a_n x(t)^n \tag{6.7}$$

In the wireless channel, there are several phenomena that cause attenuation and interference. The distance plays a stronger role in signal attenuation, whereas, the surface curvature and geographical nature of the area plays a stronger role in signal interference and latency. Thus, there are four types of waves that can adapt to these phenomena. These are briefly outlined as follows:

1) Scatter Wave: Where the wave is reflected when it reaches the ionosphere layer.
2) Sky Wave: Where the wave is reflected off the ionosphere layer.
3) Space Wave: This is the line of the sight wave, where the transmitter can see the receiver.
4) Surface Wave: This is where the wave follows the curvature of the earth.

These waves and their behaviour in the wireless channel are illustrated in Figure 6.8.

The transmitted wave can travel through any of the previous phenomena. Each of these journeys is represented mathematically and has been assigned the names of their inventors. These are called fading and they are represented according to the impact of noise and latency. Such fading are as follows:

1) Additive White Gaussian Noise (AWGN): The formula of this noise is just the transmitted signal plus some noise. This kind of noise is statistically independent from the original signal. The AWGN channel is represented in Figure 19 as the line of sight path.
2) The Rayleigh Channel Fading: This can reflect, diffract or bounce against obstacles. This channel is mathematically formulated as in the following equation, as in Schwartz [25]:

$$S_R(t) = \sum_{k=1}^{L} a_k \cos[\omega_c(t - t_0) + \emptyset_k] \tag{6.8}$$

Where S_R: The received signal.

a_k: Amplitude of rays.

ω_c: Random phase.

t_0= d/c, where d ray's traveling distance and c speed of light. t_0 represents the average delay. $\emptyset_k$: Phase of arrived signal.

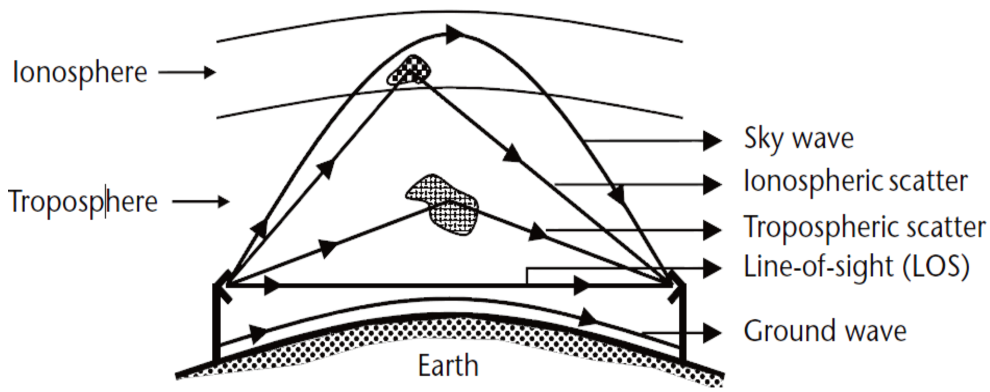

Figure 6.8: Waves propagation through wireless channel [**97**]

The Doppler Effect is the phenomenon that occurs due to the movement of the terminal, either the receiver or the transmitter. This movement introduces a phase shift at the receiver in the case of multipath propagation. This phase shift = $f_k = \frac{v \cos \beta_k}{\lambda}$ in Hz and therefore, $f_c\ becomes\ f_c + f_k$.

With referral to Equation 15, the received signal, due to Rayleigh plus Doppler Effect, will become as follows:

$$S_R(t) = \sum_{k=1}^{L} a_k \cos[\omega_c(t - t_0) + \emptyset_k + \omega_k t] \tag{6.9}$$

Where $\omega_k t = 2\pi f_k t = (2\pi v \cos \beta_k)t/\lambda$ represents the Doppler shift.

3) The Ricean Channel is merely the two previous channels, AWGN (normally line-of-sight) and Rayleigh (multipath distortion), combined together. Due to this fact, the receiver should wait a certain time to receive a reasonable number of reflected, multipath, signals before the commencement of decoding.

The general mathematical expression of this channel is:

$$v(t) = C \cos \omega_c t + \sum_{n-1}^{N} \rho_n \cos\ (\omega_c t + \emptyset_n) \tag{6.10}$$

Where C: the amplitude of AWGN component.

$\emptyset_n$: the phase of *n*th multipath signals.
ρ_n: the amplitude of *n*th multipath signals.
N= 1, 2, …, *N* = index of reflected signal.

In this research, AWGN with SNR was the used channel randomly and manually chosen and tested for SNR between 0 to 30dB. This was taken into consideration during the creation of the code and would be flexible and user friendly for later changes to be easier. The utilised channel was AWGN, where the amount of noise which was added to the signal was reasonably chosen. However, this channel could be changed easily, due to the designed flexibility feature in the code, to any sort of channel and the amount of noise.

The channel capacity which must be known is the transmission bandwidth, prior to the transmission of the signal. This is due to the bandwidth resource which is needed; and to utilise as much of this available resource as possible. The coding rate, QAM size and bandwidth play a vital role in choosing the channel bandwidth. Likewise, the channel bandwidth limits the amount of data that needs to be transmitted and therefore this is taken into consideration when choosing the code rate and QAM size when designing any communication system.

Doppler shift is one of the most sophisticated noise in the channel and often experienced in the mobile channel. This kind of noise is mainly caused by the movement of transmitter or receiver or both at the same time. In the next section, Doppler shift will be discussed in greater details. The channels that are to be used in this research for both systems, OFDM and OWDM, are all the aforementioned channels, AWGN Channel, Ricean Fading Channel, Rayleigh Fading Channel.

6.3.2.1 DOPPLER SHIFT

Doppler shift was first introduced in 1842 with Christian Doppler's hypothesis of how sound wave is affected by movement [86]. Three years later another scientist, Ballot, tested the effect of moving sound sources and confirmed that when the sound source is approaching, the sound pitch is higher than the transmitted frequency and lower when the sound source is moving away. This phenomenon is also seen in our daily life, when a police car or an ambulance is approaching with their sound on, the sound frequency rises as the car approaches and lowers as the car moves away. The increase and decrease in the frequency is directly proportional to the velocity of the source and the receiver or both at the same time. This phenomenon is not exclusively for sound waves, but also applicable on electromagnetic waves. The best way to visualise this is shown in Figure 6.9.

In physics law, the velocity of the signal source and the receiver relative to the medium is much lower than the speed of the waves within the medium. This relationship between the transmitted frequency f_o and the observed frequency f is expressed by [86]:

$$f = \left(\frac{c + v_r}{c + v_s}\right) f_o \tag{6.11}$$

Where c is the velocity of waves, v_r is the velocity of the observer relative to the medium so that it is + if the observer is approaching the source and – if moving away, v_s is the velocity

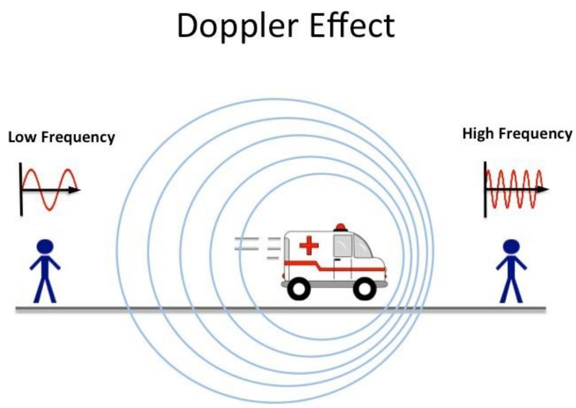

Figure 6.9: Doppler Effect [**149**]

of the source relative to the medium so its sign is + if the source is moving away from the observer and – if the source is approaching the observer. When the velocity of v_s and v_r are small in relation to the velocity of the wave, the relationship between the received frequency f and the transmitted frequency f_o is approximated by:

$$f = \left(1 + \frac{\Delta v}{c}\right) f_o \,, \quad \text{where the change in frequence} \quad \Delta f = \frac{\Delta v}{c}\, f_o \qquad (6.12)$$

Where $\Delta f = f - f_o$ and $\Delta v = v_r - v_s$ is the velocity of the observer relative to the source in which it is + when the source and the receiver approaches each other.

The maximum Doppler shift is calculated by:

$$f' = \frac{vf}{c} \quad , \qquad \text{where v is the mobile speed and } c = 3 * 10^8 \text{ is the speed of light}$$

The used frequency f in this research is 826 MHz as for 2K mode DVB-T, the maximum Doppler shift chosen are 50Hz, 100Hz and 200Hz where the mobile speeds are 65 km/h, 130 km/h and 260 km/h. The rationale behind choosing these specific values of Doppler shifts, supposing that the terminal is traveling with certain speed, is to test the average of the most common speed limits not only in the UK but around the world, where the fast speed is normal in city speed limits, the second is motorways speed limit and the third is for high speed trains.

The channels that are to be used in this research for both systems, OFDM and OWDM, are all aforementioned channels, AWGN Channel, Ricean Fading Channel and Rayleigh Fading

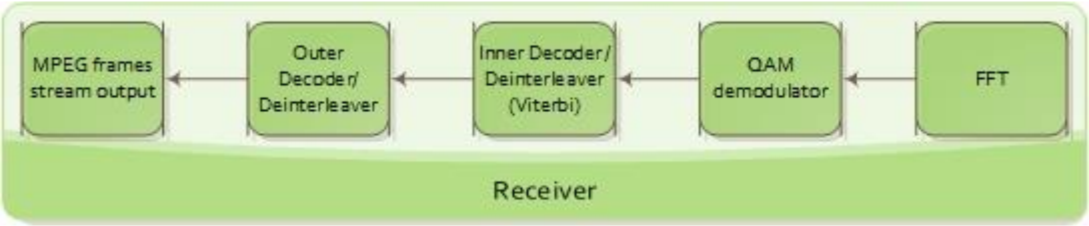

Figure 6.10: OFDM receiver [**19**]

Channel. For further investigation and evaluation, Doppler shift is used with three different frequencies, 50Hz, 100Hz and 200Hz; according to the speed of the mobile, transmitter or receiver or both.

6.3.3 THE OFDM RECEIVER

The receiver is where the EM wave is received and processed to extract the information from this EM wave. The receiver usually is the end user and in the mobile cellular network, can receive signal on the move. Consequently, the received signal will suffer from the Doppler Effect phenomenon, which was mentioned earlier in Section 6.3.2.

Theoretically, receivers in any communication system are almost always conducting the same procedure of processing the signal in reverse order as in the transmitter. However, in an algorithmic sense, the whole algorithm of processing the signal differs in the receiver compared to the transmitter.

In this research, the designed OFDM receiver is depicted in Figure 6.10. In comparison with the transmitter in Figure 6.2 signal processing is in reverse order; as stated earlier.

6.3.3.1 FAST FOURIER TRANSFORM

FFT is the process of converting the received signal from time domain back into the frequency domain. As the signal is converted to time domain, just before transmission, at the receiver; therefore, the first step at the receiver should be bringing back the signal into the frequency domain. Of course, this is facilitated after converting the signal from analogue to digital, filtering the shaping wave and removing the cyclic prefix.

FFT is the inverse process of IFFT, described in Section 6.3.1.5. Literally, FFT is the forward process of computing DFT; however, IFFT is the inverse process. Practically, these processes are converting signals between the time and frequency domain.

As stated earlier, if FFT is applied on a signal in the time domain the result will be the summation of infinite cosine and sin waves with harmonic frequencies starting from zero. The base frequency is represented by$f_0 = 1/T$, where T is duration of *f(t)*. The resulting infinite harmony is called the Fourier series.

The mathematical formula of Fourier series is as follows:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\,x) + \sum_{n=1}^{\infty} b_n \sin(n\,x) \qquad (6.13)$$

Where a_0 is dc value or mean value. $a_n and\ b_n$ are coefficients that need to be computed by FFT to produce the Fourier series.

The coefficients $a_n$and $b_n$can be computed by using the trigonometry's orthogonality feature and multiplying both sides by factors of $\cos(n\,x)$ and $\sin(n\,x)$.

6.3.3.2 QUADRATURE AMPLITUDE DEMODULATOR

The QAM demodulator takes the received signal, after converting to the frequency domain as a QAM 16, for this research, the constellation map. The equaliser is needed for some wireless applications, e.g. mobile wireless, in which the transmitted signals are subject to the Doppler shift. It then traces the symbols' locations, using certain algorithms, to extract the information bits.

However, these recovered bits and symbols, contain noise due to the channel and transmission process, in which the original signal adds up noise. Since the receiver knows the QAM size being used within the system, 16-QAM in this research, it subsequently knows where the data symbols are within the constellation map.

Figure 6.11 shows the difference between transmitted and received data symbols in the QAM constellation map. The transmitted symbols, the left figure, have the original data mapped onto the QAM constellation. The received signal, the right figure, has the original

QAM constellation map, but with noise. The noise around the original data symbols blue aura has the effect on the decision of the QAM demodulator. The greater the noise is, the larger the aura around the original symbol. Whether the received data symbols is received on the right symbols location as the original data is a decision the QAM demodulator must take. Such a decision may cause a loss of some of the original data. Thus, several effective algorithms have been developed over the last two decades to resolve the effect of vital noise and to choose the positioning of the right symbol. One of these algorithms is called the 'hard decision'. This is simply when the QAM demodulator chooses the closest symbol position to the position of the received symbol. Such a decision may damage some the data symbols when the noise around one symbol overlaps with the noise from another symbol.

The Symbol Error Rate (SER) is measured in association with SNR, so that when the SNR value increases; the SER decreases. This is theoretically correct as the noise causes an aura around the symbols on the constellation. As aura noise around the symbol increases, the radius of the aura also increases, and therefore it interferes with the aura of another surrounding symbol. Therefore, the decision will be hard to be taken and it will be taken according to the closest symbol's position.

If the closest symbol's position is in the original position then there is no symbol error; but if the closest symbol's position is the adjacent original symbol, then there will be one symbol error rate counted. The occurrence of the SER increases in the multipath environment higher than the direct path. For this reason a mechanism called the 'soft decision' is developed to reduce the SER. One of these soft decision mechanisms for multipath environment was proposed by Gao and Linfoot [87].

Channel distortion is caused by the multipath environment and rapidly increases in the mobile environment due to the Doppler Effect, which was described earlier in Section 6.3.2. Theoretically, Doppler Effect causes the constellation map to spin around and therefore ISI occurs. In addition, in order to achieve zero ISI on the frequency domain of the received signal, Nyquist Criterion, channel estimation and compensation are required. This compensation reduces the effect of the channel's constructive distortion and can be achieved by adding a channel equaliser in the receiver. However, any equaliser possibly enhances the noise of the channel and therefore, there is a trade-off between minimising the noise and minimising ISI. There are several equalising algorithms which have been developed for this reason, however, the two most popular adaptive equalisers algorithms are as follows:

- Mean Square Error (MSE). This has the advantage of minimising the noise amplification and also minimising the sum of noise and ISI. However, it has the drawback of permitting residual ISI.
- Zero Forcing (ZF). This entirely eliminates ISI. However, it has the drawback of amplifying the noise.

Each of these two algorithms is suitable for a particular M-QAM size when employed with the Linear Equaliser (LE) or Decision Feedback Equaliser (DFE). For instance, MSE/LE and ZF/LE are both suitable for M-QAM with M≤64. Conversely, MSE/DFE is suitable for M-QAM with M>64 [23].

6.3.3.3 INNER DECODER/DEINTERLEAVER

The inner decoder/deinterleaver reflects the same process of the inner coder/interleaver, as in Section 6.3.1.3, but in reverse order. This process, in fact, has two consecutive processes. At the receiver, the first process of these is interleaving, which is responsible for depuncturing. It carries out exactly the opposite process at the receiver and inserts the bits which were removed from the data before transmission. The puncturing rate which was used at the transmitter side must also be used here in order to correctly insert the bits in the right places.

The second process is related to the decoder, which is responsible for decoding the bit stream using a Viterbi decoding algorithm. This decoder is well known as maximum likelihood forward error correction algorithm that is used in digital communications. This algorithm can

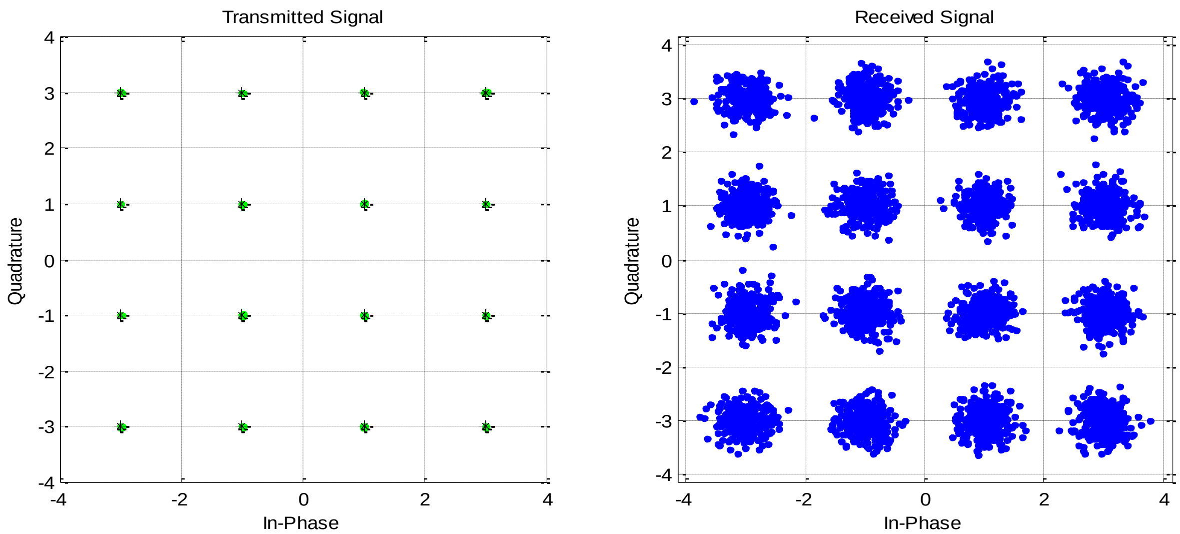

Figure 6.11: QAM constellations for transmitted and received Signal with SNR 20 dB

decode any convolutional encoded binary data that is passed through a noisy channel with an error rate of about10^{-3}.

Assuming that 6 bits of data are transmitted through with ½ code rate; the received data must be decoded at ½ so that for each one bit in the output it is two bits, with a total of 6*½ = 12 bits. These encoded bits possibly may or may not have errors and be unique. However, the data may be all correctly or partially received due to errors. Each bit of the received message has unique mapping to the original bit; and these bits can be traced back using the Trellis diagram. The receiver carries out every possible bit sequence code-words of the message to trace the original message back [88].

When the message was corrupted during transmission and one or more bit was changed and if the bit sequence was not found within the list of the possible code-words for this message, then the decoder would take one of these actions:

1) Compare the received bit sequence to every potential sequence; and compare the hamming distance of each and subsequently, pick the sequence of the smallest hamming distance. This is what is called the "hard decision".
2) Apply a correlation process and subsequently, take the one with the best correlation. This is what is called the "soft decision".

6.3.3.4 OUTER DECODER/DEINTERLEAVER

The purpose of this process is to deinterleave and decode the data stream. As in the previous Section, 6.3.1.2, this process contains two consecutive processes. Firstly, the outer deinterleaver executes, which is the reverse order of the outer interleaving process at the transmitter side in Section 6.3.1.2. This is facilitated by reversing the process of the convolutional interleaving which is based on the Forney algorithm.

Secondly, the outer decoder is invoked, in which the RS coding algorithm is used, as described in Section 6.3.1.2. However, the reverse order of this RS algorithm is used at the receiver. In general, this algorithm can decode the bit stream to recover the transmitted data stream or signal.

6.4 THE OWDM SYSTEM

The OWDM system is the core part of this research. As discussed in the critical review in Chapter 3, the wavelet transform has already been proposed to be used as an alternative to OFDM, however, differently than in this research where added flexibility of sub-bands allocation and all wavelet families and families' members are assessed. The design of this system is generally associated with that of the OFDM system used in the DVB-T standard [19], as depicted in Figure 6.1. Nevertheless, the OWDM system has a core change in the process of subcarrier generation and distribution. The Fourier transform process is used in the OFDM system, was described in Section 6.3.1.5; however, in this system IDWT and FDWT are used in the transmitter and the receiver consecutively.

Asymmetric filters banks or even possibly multibanks are proposed in this section. This has possibly the vital advantage of dynamically or asymmetrically allocating the width of sub-band intervals rather than fixed intervals in the OFDM system. Therefore, this brand new modulation technique is called OWDM for the use of DWT within this system. There is also the advantage that in using this system there will no need to guard the wavelet symbols as opposed to OFDM, where guard interval are necessary between consecutive OFDM symbols. The Wavelet transform and associated wavelet families and families' members were all

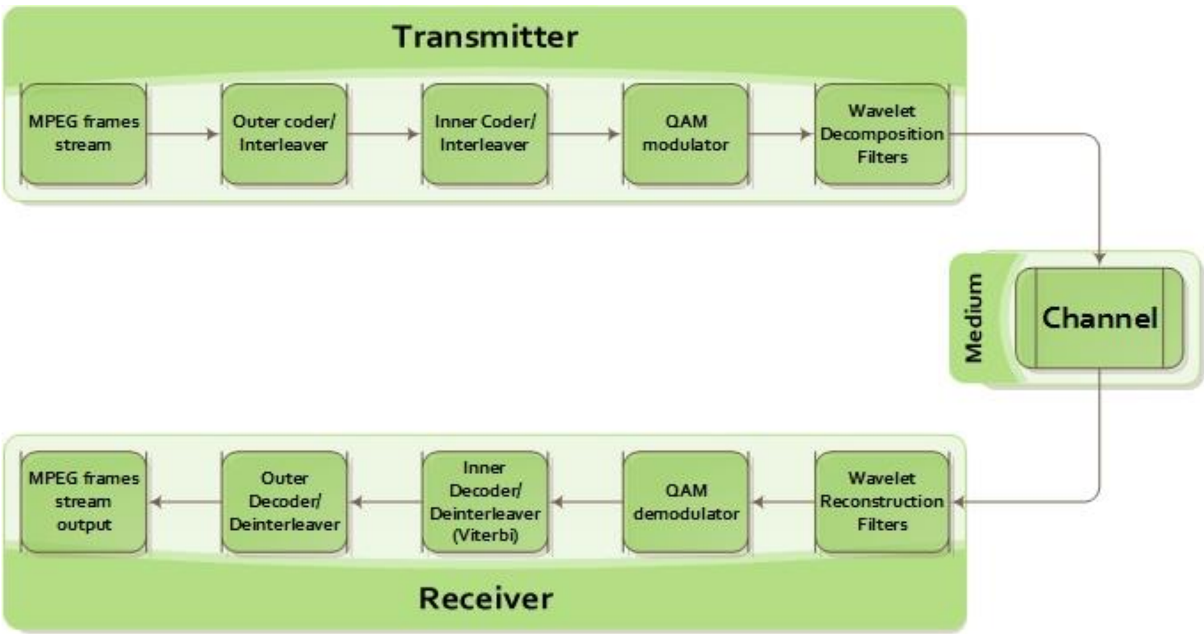

Figure 6.12: The OWDM Transceiver

described and discussed earlier in Chapter 5. The proposed transceiver of the OWDM system has been depicted in Figure 6.12.

The development intended for implementation and testing in this research, which has not been considered in previous works, is the outer coder and channel coding for OWDM system. This has been designed within the OWDM system and was implemented for this research. As discussed earlier, most of these processes are based on the DVB-T standard [19] which utilises the OFDM system, and therefore, the most complicated process in this research is related to the utilisation of the wavelet transform within the system. This is because of the lack of resources associated with using the wavelet transform as a modulation technique for wireless transmission. In the wavelet decomposition in the transmitter and wavelet reconstruction in the receiver is the filters bank that is proposed in this research and is the core scope. The data processing flow procedure for the proposed system is depicted in Figure 6.14**.**

6.4.1 OWDM TRANSMITTER

The transmitter of the OWDM system is mostly the same as the DVB-T OFDM, however, with one vital change in the type of transform that is utilised within OFDM. In the OWDM transmitter, the Fourier transform is replaced with wavelet synthesis (decomposition) filters bank which computes the wavelet transform to convert the signals into the time-frequency domain. The block diagram of this transmitter for this research has been depicted in Figure 6.14 Source signal.

The source signal of the OWDM system is the same as that of the OFDM system, as described in Section 6.3.1.1.

6.4.1.1 OUTER CODER/INTERLEAVER

This coder/interleaver is the same as that in the OFDM which was described in Section 6.3.1.2.

6.4.1.2 INNER CODER/INTERLEAVER

For the inner coder of the OWDM system uses the same coder as in the OFDM system, as described in Section 6.3.1.3.

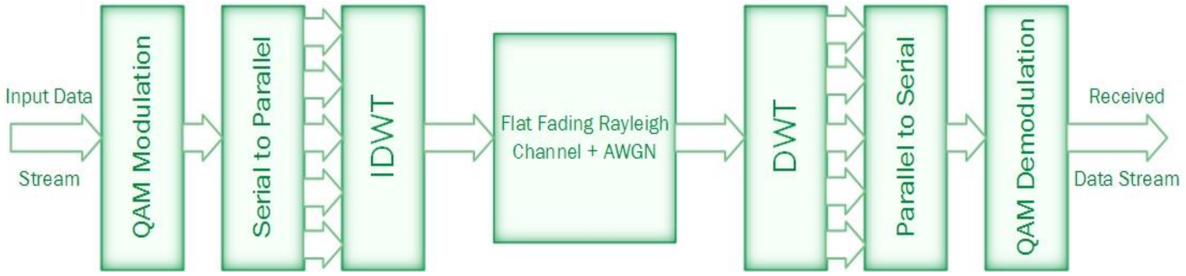

Figure 6.13: System Design Flow Diagram

6.4.1.3 QUADRATURE AMPLITUDE MODULATION

The QAM that the OWDM system uses is exactly the same as that used earlier in the OFDM system, which was described in Section 6.3.1.4.

6.4.1.4 WAVELET DECOMPOSITION FILTERS BANK

This block in the system is the core part of the investigation of this research. As discussed earlier, it replaces IFFT in the transmitter of the OFDM system. It utilises the wavelet transform to transform the signal from the time domain to the time-frequency domain. This powerful feature of the wavelet transform combines the features of analysing the signal in time and frequency domains. The wavelet transform is so far the only process that can do this in signal and image processing science.

The wavelet decomposition filters block theoretically consists of filters banks of LPF and HPF filters. The usability of the filter bank makes the wavelet transform somehow less complex than the Fourier transform in the OFDM. Moreover, the design of the filter bank may vary according to the design and application requirements.

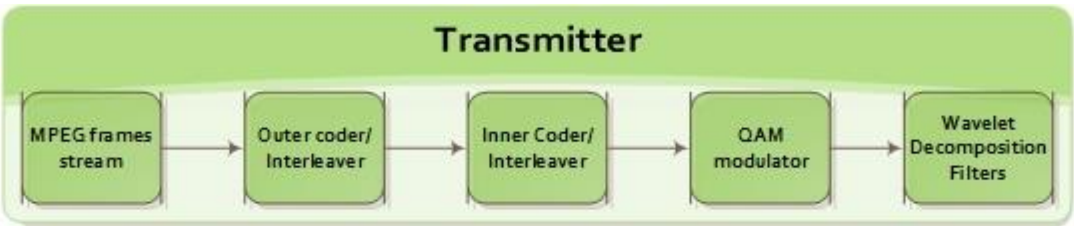

Figure 6.14: The OWDM transmitter

The filters in the filter bank are ordered in levels and in each branch of each level, there are two filters forming a double up. The reason for this is to find the function shift and scaling parameters. The levels of the filters bank correspond to the amount of band separation that the

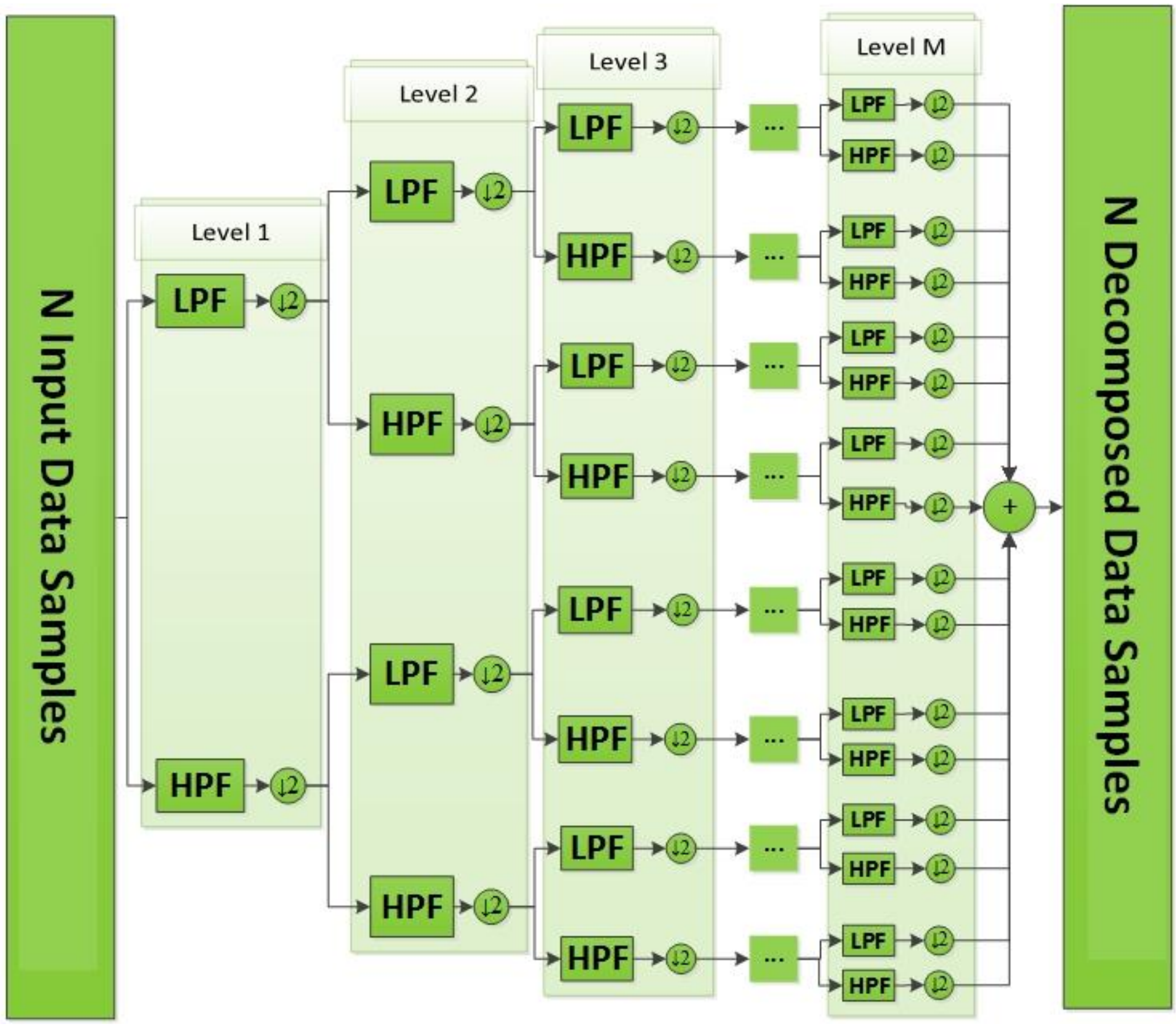

Figure 6.15: M levels of filters banks with fixed sub-band distribution (Reconstruction)

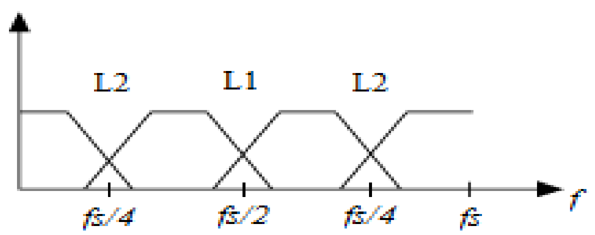

Figure 6.16: Sub-bands symmetric separation with 2-levels

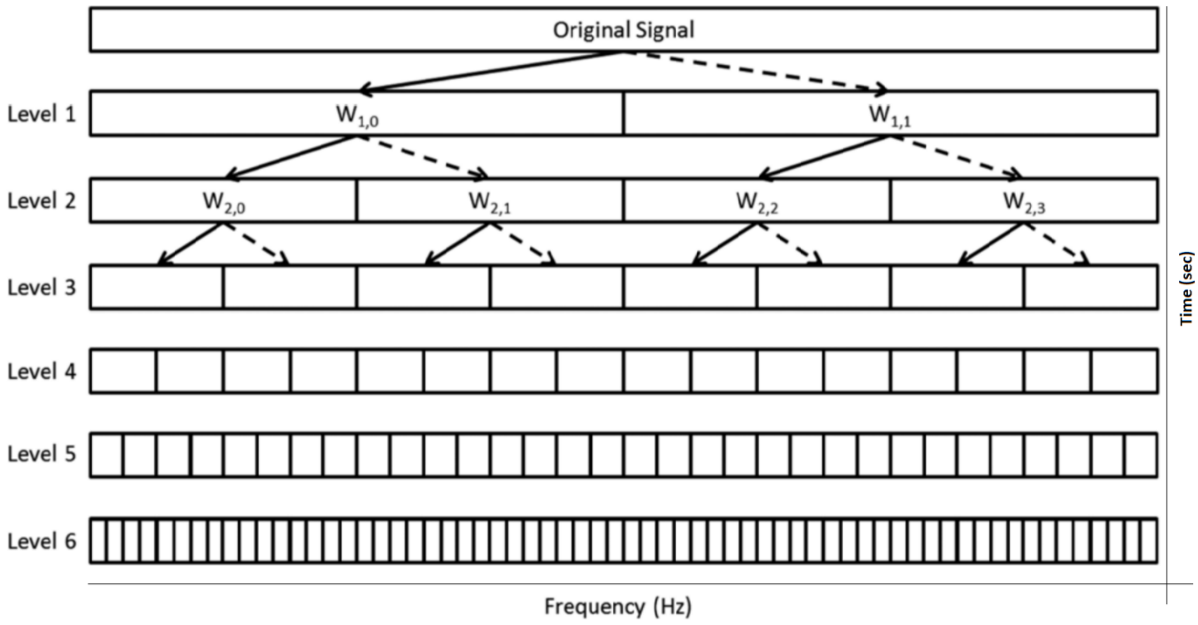

Figure 6.17: Wavelet Packet Spectrum Allocation

synthesis filters can handle. The more levels the synthesis filters has, the more the sub-bands are separated. In this research, the used number of levels ranging from six to eleven as they have not been yet implemented and were chosen in order to carry out more investigation on a higher level. A visualisation of M levels of filters bank has been depicted in Figure 6.15, where the reconstruction filters bank in the receiver is depicted for WPM which was described and discussed earlier in Chapter 5. Figure 6.17 shows the spectrum allocation using WPT, which splits the bandwidth into equally distributed length of sub-bands, with the filters bank depicted in Figure 6.15.

However, the approach in this research is different where one filter path is decomposed, either LPF or HPF depending on the application used, as depicted in Figure 6.17. It is noticeable from Figure 6.15 and Figure 6.18 that on the output of each filter there are ↑2 and ↓2 which correspond to up and down sampling by two. This is used to split up the sampling rate equally between the filters by down sampling the output of each LPF and HPF by the number of next inputs and to keep up with the sampling rate by synthesising the received signal by the same sampling rate at the receiver. As the signal progresses through next level, it is also down sampled by 2 for each filter in each level. Since it has been down sampled twice by 2 then each filter in level two has to further down sample by 2 to obtain a sampling rate of N/4. For generalisation, assume that M level is used and N samples are injected to the synthesis filters,

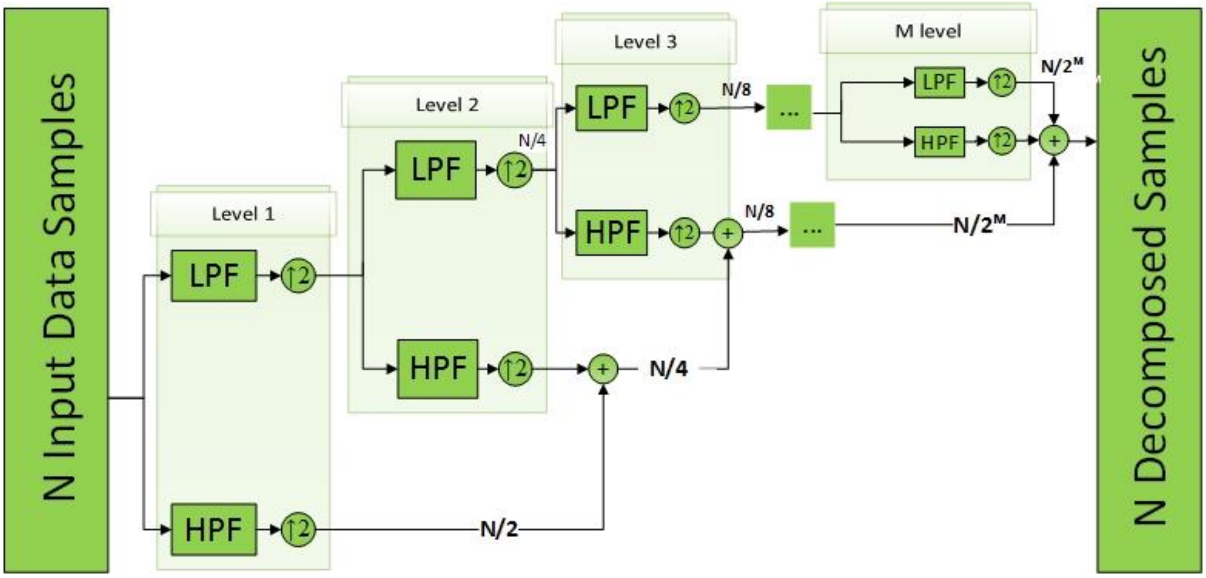

Figure 6.18: Asymmetric Filters Bank Decomposition (Transmitter)

the following condition must be satisfied, $2^M \leq N$ where M is level number and N is samples number, so that Nyquist sampling rate is always satisfied.

Likewise, the output of each filter in level three has sampling rate of N/8 after down sampling and an sampling rate of N/16 for the output of each filters in level four after down sampling. Practically, the output of the decomposition filters must add up the sampling rate of each level to reach the same input sampling rate, which is 1. An illustration has been provided in Figure 6.16 for M levels of levels of filters bank.

According to Chan [89], the continuous wavelet transform of a signal is as follows:

$$CWT(a, \tau) = \frac{1}{\sqrt{a}} \int s(t) \Psi \, \frac{(t-\tau)}{a} \, dt \tag{6.14}$$

Where CWT is Continuous Wavelet Transform $\Psi(t)$ which represents the basic wavelet, sometimes it is called 'mother wavelet'.

$\frac{\Psi \frac{(t-\tau)}{a}}{\sqrt{a}}$ is called the 'child wavelet' [89] and it means either compressing or expanding the mother wavelet which was previously defined. The child wavelets are under families that have different performance. Nevertheless, CWT is unrealistic to be used in calculation in a signal processing device sense. Subsequently, some researchers dedicated their research ideas to finding a method to calculate DWT from CWT to realistically work with the signal processing devices. A researcher named Mallat [53] has come up with an idea to calculate DWT, with inverse reconstruction using FWT, from CWT. FWT is a method to fast compute DWT to enable reliable response during computation. This is done by using a tree of several levels of filters bank, as depicted in Figure 6.15 and in the proposed filters bank in Figure 6.18. A similar but generalised case of filters bank was discussed by Linfoot *et al*. [3] where the higher band was only used for just 2 levels of filters bank. In this proposal, however, several forms of separation is proposed depending on the demand and application being used for. For instance the higher band is used so that the allocation of sub-bands is dynamically distributed in the higher frequencies where HPF is only utilised. Meanwhile, when utilising LPF the lower frequencies will be dynamically allocated. As far as this research concerned the utilisation of lower frequencies is followed where LPF is decomposed as followed in the implementation. Additionally, the higher frequencies are then added to the spectrum to make use of signal analysis in the higher frequencies, however, these frequencies will occupy less bandwidth than the others in the spectrum, illustration of this in depicted in Figure 6.19.

Nevertheless, the opposite is possible by decomposing HPF to have resolution analysis of that band or the LPF for resolution analysis of that band. IDWT is used in the transmitter and it is called “decomposition filters bank” to decompose the signals; where the forward DWT is used in the receiver to recover the decomposed signals and it is called “synthesis or recovery filters bank” to recover the decomposed signals, as will be discussed in the receiver Section, 6.4.3.1. The dynamic sub-band spectrum allocation is depicted in Figure 6.17 for the

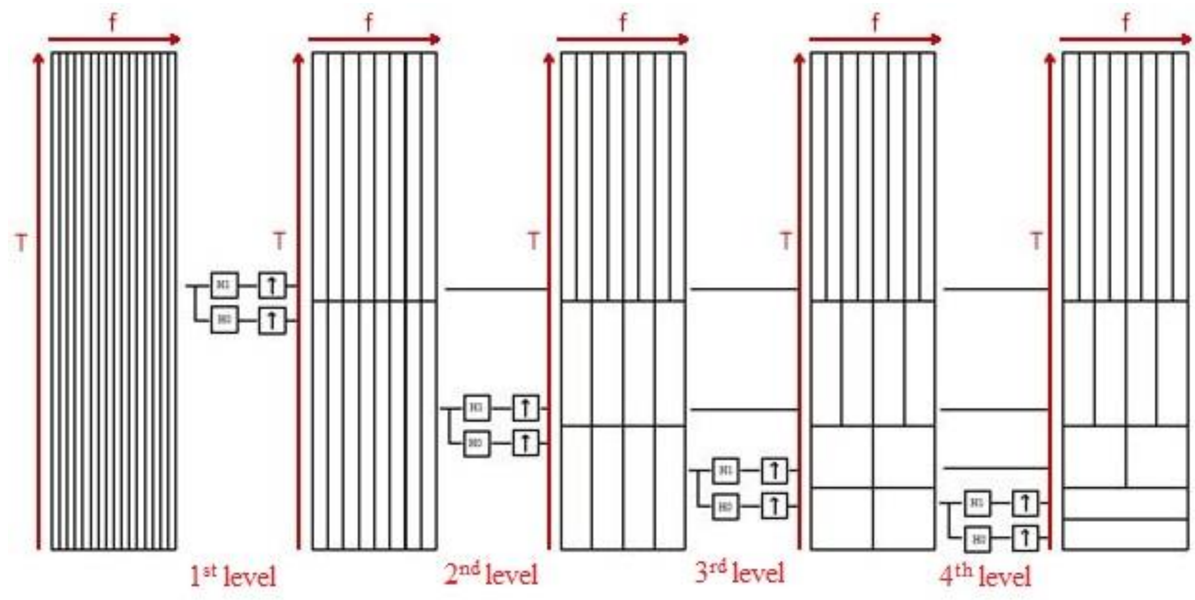

Figure 6.19: Converting 16 sub-bands from frequency domain into time-frequency domain using four levels of filters bank (forward DWT)

first three levels. The more levels the signal travels through, the more separation with a variety of bandwidths is possible, f_s denotes the sampling frequency. As stated previously, in Chapter 5, DWT is time-frequency analysis where the signal is transformed from time to time-frequency domain and this is one of the most important features of DWT, which allows us to better understand the where and when a signal event has occurred. To illustrate how that is done in the context of this research and how the sub-banding is done throughout levels in time-frequency domain, Figure 6.19 shows the forward DWT in the receiver where ↑ is the up sampling by 2 process achieved by zero insertion between every second sample, H1 is LPF and H0 is HPF. The interest of this research is related to investigation of all available compactly supported wavelet families and their associated members, presented in Table 6.3. Since CWT is unrealistic to be calculated in digital signal processing, the fast DWT is used instead.

The DWT is computed for the input signal by convoluting the input signal with the filter and then split into upper and lower halves. In Figure 6.18, if we denote the LPF $h(n)$ and HPF $g(n)$, then to compute DWT for the input signal with $O(N)$ complexity those $h(n)$ and $g(n)$ must have a constant length. Then the the input signal x is convoluted with $h(n)$ and $g(n)$ at a time, $x*h$ and $x*g$, the each of those operations will cost $O(N)$. After that the filters bank splits the signal into further two branches of size $N/2$. However, it splits the upper branch convolved with $h(n)$ recursively, whereas the FFT splits the upper and lower branches recursively. This leads to this recurrence time relation: $T(N) = 2N + T(\frac{N}{2})$ which is in another word $O(N)$ time complexity for the whole operation which also can be show by the geometric series expansion. For instance if we take the simplest wavelet, Haar, it has linear complexity since $h(n)$ and $g(n)$ have a length of 2 constantly, as shown in Equation 6.15 [69]:

$$h(n) = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \quad \text{and} \quad g(n) = \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \tag{6.15}$$

For additional understanding the characteristics of symmetric and dynamic filters banks, Table 6.3, summarises and compares the characteristics of symmetric and dynamic (asymmetric) sub-bands allocation in OWDM. The comparison of both types of decomposition is bought by distilling the information from Fliege [17], Strang and Nguyen [90] and Vaidyanathan [78].

Table 6.3: Summary of symmetric and dynamic sub-bands allocation

Characteristic	M-Level Symmetric	M-Level dynamic (Asymmetric)
Low and High Frequency sub-bands Decomposition	Both the low-frequency and the high-frequency sub-bands in M level are decomposed in the next level.	Each level's low-frequency sub-band is decomposed in the next level, and each level's high-frequency band is an output of the filter bank.
Number of Output Sub-bands	2^n	$n + 1$
Bandwidth and Number of Samples in Output Sub-bands	For an input with bandwidth BW and N samples, all outputs have bandwidth $BW\ /\ 2^n$ and N / 2^n samples.	For an input with bandwidth BW and N samples, y_k has the bandwidth BW_k, and N_k samples, where $BW_k = \begin{cases} \frac{BW}{2^k} & (1 \leq k \leq n) \\ \frac{BW}{2^n} & (k = n + 1) \end{cases}$ $N_k = \begin{cases} \frac{N}{2^k} & (1 \leq k \leq n) \\ \frac{N}{2^n} & (k = n + 1) \end{cases}$ The bandwidth of, and number of samples in each sub-band (except the last one) is half those of the previous sub-band. The last two sub-bands have the same bandwidth and number of samples since they originate from the same level in the filter bank.
Output Sample Period	All output sub-bands have a sample period of $2^n(T_{si})$	Sample period of k^{th}output is: $= \begin{cases} 2^k\ (T_{si}) & (1 \leq k \leq n) \\ 2^n(T_{si}) & (k = n + 1) \end{cases}$ Due to the down sampling by 2, the sample period of each sub-band (except the last one) is twice that of the previous sub-band. The last two sub-bands have the same sample period since they originate from the same level in the filter bank.
Total Number of Output Samples	The total number of samples in all of the output sub-bands is equal to the number of samples in the input (due to down sampling by 2 at each level), so that Nyquist theorem satisfied.	
Wavelet Applications	In wavelet applications, the highpass and lowpass wavelet-based filters are designed so that the aliasing introduced by the down sampling is cancelled in reconstruction stage.	

The orthogonality of OWDM comes from the fact that the scaling function is orthogonal to the wavelet or shifting function. Unlike the usage of both paths through LPF and HPF which would fix the width of the sub-bands so that the sub-bands allocation is equally distributed, as depicted in Figure 6.15 and Figure 6.16, the allocation of the dynamic width of sub-bands is therefore flexible due to the fact that one path, through first LPF, is only used to transform signals. Additionally, in order to clarify this dynamic sub-bands allocation; assume that one has 16 sub-bands with four levels of filters bank; if these sub-bands were passed through first LPF path with four levels of filters bank decomposition, the result would be allocating these 16 sub-bands dynamically with different widths, as depicted in Figure 6.17 and Figure 6.18. This method of allocation is subject to a must have condition to satisfy Nyquist theorem. This condition is defined as $2^M \leq N$ where M is level number and N is number of samples. If otherwise, the sampling theorem will be broken and start to sample to the half which will cause aliasing between adjacent samples.

Since DWT is not fast enough to be compute in real time systems, as discussed previously in Chapter 5, Section 5.4, FDWT was built by Mallat [53] to compute DWT faster. Since this section is about the transmitter of OWDM, IDWT will be presented here whereas the forward DWT will be discussed later in the receiver Section, 6.4.3.1. Following on the four equations of scaling and wavelet functions in Chapter 5, within the practical wavelet transform Section 5.6, given that the coefficients sequence s^M for some of $M < J$ and the differences sequences $d^k, k = M, \dots, J-1$ the IDWT can be recursively computed by:

$$s_n^{(k+1)} = \sum_{k=-N} a_k \, s_{2n-k}^{(k)} + \sum_{k=-N}^{N} b_k \, d_{2n-k}^{(k)} \qquad (6.16\ a)$$

Or it can also be written as:

$$s^{k+1}(z) = a(z).(\uparrow 2)\Big(s^k(z)\Big) + b(z).(\uparrow 2)\Big(d^k(z)\Big) \qquad (6.16\ b)$$

Where $\uparrow 2$ is the upsampling operator, which is achieved by filling the coefficients sequence by zero in every second element of the resulted sequence, so that every second element is zero or can be represented as:

$$(\uparrow 2)\big(c(z)\big) = \sum_{n\in\mathbb{Z}} c_n z^{-2n} \qquad (6.17)$$

This is a linear operator and can be reversed by the down sampling operator $(\downarrow 2)$, as will be explained later in the receiver Section 6.4.3.1 Equations 6.18, 6.19 and 6.24. Down sampling in the forward DWT is achieved by discarding any alternative samples among the samples.

In the next section, the used channels are presented and then later in the receiver Section 6.4.3.1, forward DWT will be discussed and describe how it connects to IDWT, described this section.

Table 6.4: Wavelet families and members

Wavelet families	**Wavelets Family's Member**																	
Daubechies	db1(haar)	db2	db4	db6	db10	db14	db18	db20	db23	db25	db27	db29	db32	db35	db38	db40	db43	db45
Symlets	sym1	sym2	sym4	sym6	sym10	sym14	sym18	sym20	sym23	sym25	sym27	sym29	sym32	sym35	sym38	sym40	sym43	sym45
Coiflets	coiflet1	coiflet2	coiflet3	coiflet4	coiflet5													
Dmeyer	Dmey																	
Biorthogonal	bior1.1	bior1.3	bio1.5	bior2.2	bior5.5	bior2.6	bior2.8	bior3.1	bior3.3	bior3.5	bior3.7	bior3.9	bior4.4	bior5.5	bior6.8			
Reverse Biorthogonal	rbio1.1	rbio1.3	rbio1.5	rbio2.2	rbio2.4	rbio2.6	rbio2.8	rbio3.1	rbio3.3	rbio3.5	rbio3.7	rbio3.9	rbio4.4	rbio5.5	rbio6.8			

Each of these children will be implemented in two various SNR; they intended to be from 0dB to 40dB. For this research, the level of filters bank is directly proportional to the length of IFFT was used in OFDM system, the levels number of filters banks in OWDM follows the logarithm to the base of 2, $log_2(N)$ where N is IFFT length. However, the intended levels number is 11 levels with IFFT length of 2048, as per DVB-T standard specifications. The best BER performance of those families' members presented in Table 6.3 will be picked for further comparison analysis. All these analysis and comparisons are demonstrated and analysed in Chapter 7 and Chapter 8.

6.4.2 THE OWDM CHANNEL

The channels used here in OWDM system are a replica of those channels that are used in the OFDM system which was described earlier in Section 6.4.1.4. The used frequency f in this research is 826 MHz as for 2K mode DVB-T, the maximum Doppler shift chosen are 50Hz, 100Hz and 200Hz where the mobile speeds are 65 km/h, 130 km/h and 260 km/h. The rationale behind choosing these specific values of Doppler shifts, supposing that the terminal is traveling with certain speed, is to test the average of the most common speed limits not only in the UK but around the world where the fist speed is normal in-city speed limit, the second is motorways speed limit and the third is for high speed trains.

The channels that are to be used in this research for both systems, OFDM and OWDM, are all aforementioned channels, AWGN channel, Ricean fading channel and Rayleigh fading channel. For further investigation and evaluation Doppler shift is used with three various frequencies, 50Hz, 100Hz and 200Hz; according to the speed of the mobile, transmitter or receiver or both.

6.4.3 OWDM RECEIVER

The receiver here in this system is mostly the same as the OFDM system used in the DVB-T standard [19], however, with the proposal of using wavelet reconstruction (recovery) filters bank instead of FFT in the OFDM. The receiver of OWDM designed to be implemented for this research is depicted in Figure 6.20.

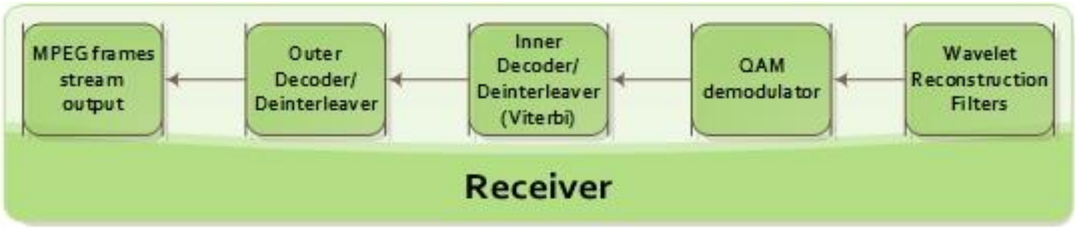

Figure 6.20: The OWDM receiver

6.4.3.1 WAVELET RECONSTRUCTION FILTERS

Wavelet reconstruction filters at the receiver are practically the inverse process of wavelet decomposition (synthesis) filters at the transmitter, which was described earlier in Section 6.4.1.4. Technically and for compatibility, reconstruction filters level must be the same number of levels of filters bank used in the decomposition at the transmitter. It must also use the same wavelet family and the same family's member that was used in the decomposition at the transmitter. The reason behind this is to match the analysis of the signal with the transmitter and also to insure that the original sub-bands are fully compatible for perfect reconstruction with the same quality of separation at the transmitter. The wavelet reconstruction filters bank at the receiver uses the forward DWT to recover the decomposed samples, as represented in Figure 6.21. It reconstructs the signal by extracting the samples from the sub-bands followed by up sampling by the same sampling rate of the relevant level before it is injected into the next level of filters, as depicted in Figure 6.21, where ↓2 is down sampling by 2 achieved by discarding the alternate samples or removing the inserted zeros. Figure 6.22 shows how the filters banks detect information sub-bands, where H1 is LPF and H0 is HPF.

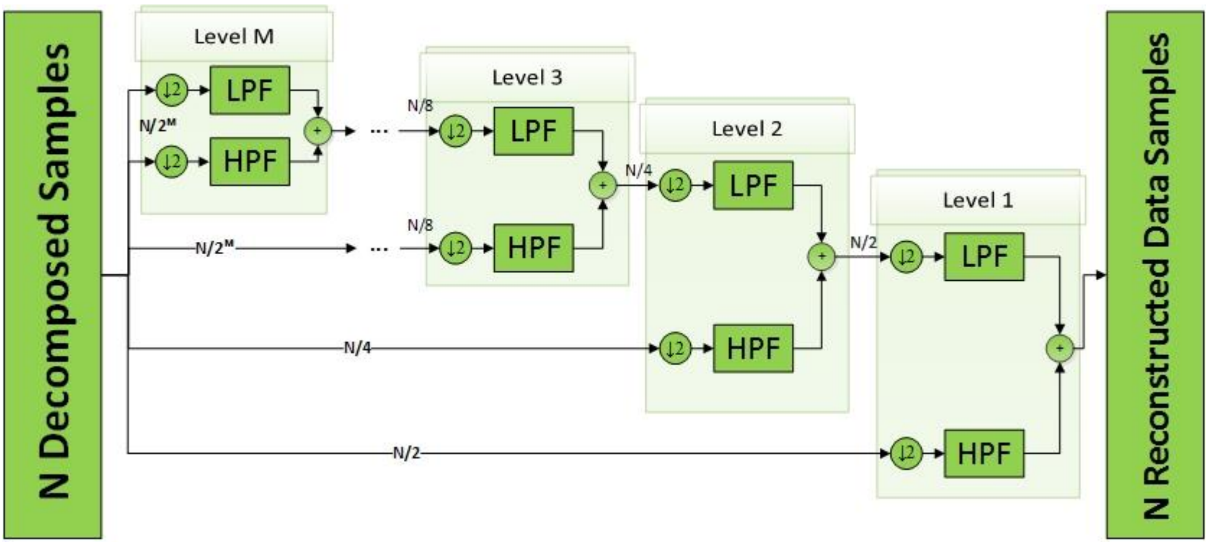

Figure 6.21: Reconstruction Filters Bank (Receiver) Forward DWT

As mentioned earlier, the sub-bands separation take place at the transmitter with different levels as depicted in earlier figure, Figure 6.18 and Figure 6.19. However, reconstruction filters bank at the receiver start to reconstruct the decomposed data from sub-bands to reconstruct the original data. This is by taking the received signal through the same levels of filters bank, used in the decomposition at transmitter, starting at the highest level through to the lowest until it reaches the first level where the data signal is recovered and passed to the demapper. This process is depicted in Figure 6.21 for the receiver and Figure 6.18 for the transmitter.

The type of transform used in the reconstruction filters bank is the forward DWT, the IDWT was described earlier in Section 6.4.1.4. The computation of the forward fast DWT is achieved by starting with the coefficients sequence s^J and going down through from $k = J - 1$ to $M < J$ [53]:

$$s_n^k = \frac{1}{2}\sum_{m=-N}^{N} a_m\, s_{2n+m}^{k+1} \qquad \text{or} \qquad s^k(z) = (\downarrow 2)\Big(a^*(z).\, s^{k+1}(z)\Big) \tag{6.18}$$

And also:

$$d_n^k = \frac{1}{2}\sum_{m=-N}^{N} b_m\, s_{2n+m}^{k+1} \qquad \text{or} \qquad d^k(z) = (\downarrow 2)\Big(b^*(z).\, s^{k+1}(z)\Big) \tag{6.19}$$

For $k = J - 1, J - 2, J - 3, \ldots, M$ and all $n \in \mathbb{Z}$ in Z-transform notation must implies:

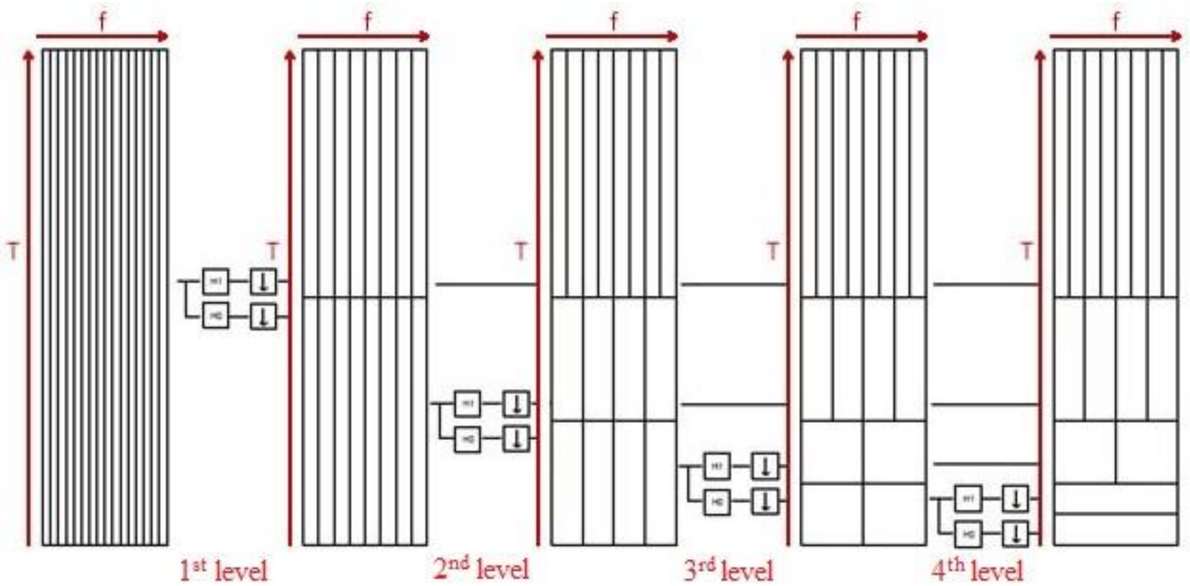

Figure 6.22: Synthesising 16 sub-bands from time-frequency domain into frequency domain using 4 levels of filters bank

- The starred $a^*(z)$ is the adjoint filter and has a time reversed adjoint coefficients: $a^*(z) = \sum_{n=-N}^{N} a_{-n}^*$; i.e. a real number has itself as adjoint, a conjugate of the complex number is its adjoint, a real matrix has its transpose as adjoint.
- Polynomial multiplication, which is the same as the convolution of the coefficients sequences.
- The down sampling operator $(\downarrow 2)$ decrease the infinite sequence, by its Z-transform, to a sequence of coefficients of:

$$(\downarrow 2)\big(c(z)\big) = \sum_{k\in\mathbb{Z}} c_{2k} z^{-k} \tag{6.20}$$

It follows that

$$P_k[f](x) = \sum_{n\in\mathbb{Z}} s_n^k \, \varphi(2^2 x - n) \tag{6.21}$$

is the orthogonal omission of the signal f for a minimum the first $P_k[f](x)$ approximation onto the subspace V_k and with sampling rate of 2^k per interval. The difference with the first approximation can be found and t is expressed by:

$$P_j[f](x) = P_k[f](x) + D_k[f](x) + \cdots + D_{J_1}[f](x) \tag{6.22}$$

where the details signals are obtained from the details coefficients and can be computed by:

$$D_k[f](x) = \sum_{n\in\mathbb{Z}} d_n^k \, \psi(2^k x - n) \tag{6.23}$$

where ψ is the mother wavelet of the wavelet transform, depending on the wavelet family chosen. From Equations 6.18 and 6.19, one can obtain the IDWT as described earlier in the wavelet decomposition section 6.4.1.4 Equations 6.16 a and 6.16 b.

In this design DWT of the signals will be computing by passing the signals through the designed filters bank, IDWT in the transmitter and DWT in the receiver. The data samples are passed through LPF with an impulse response g resulting in the convolution of the two:

$$y[n] = (x * g)[n] = \sum_{k=-\infty}^{\infty} x[k] g[n-k] \tag{6.24}$$

HPF h simultaneously decomposes the data samples resulting in the details coefficients. Therefore, now the approximation coefficients are known from LPF output and the details coefficients from the HPF output. To compute both LPF and HPF we apply Equation 6.24 to both:

$$y_{low}[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-k] \qquad \text{and} \qquad y_{High}[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \tag{6.25}$$

Now the data samples are decomposed through the first level and hence the time resolution is halved since each halve filter's output characterises the data. Nevertheless, the frequency resolution is doubled since each filter has half the frequency band, as depicted in Figure 6.23. The input signal into the filters bank must be a multiple of 2^n where *n* is the number of levels, for instance 32 samples signal with frequency ranging from 0 to f_n and 3 levels are used to decompose this signal and the result will produce 4 scales as shown in Table 6.4 associated with Figure 6.23. With the down sampling operator ↓ the summation in the last equation can be more concisely written as:

$$y_{low} = (x * g) \downarrow 2 \qquad \text{and} \qquad y_{High} = (x * h) \downarrow 2 \tag{6.26}$$

However, * is a convolution and hence convolution computation with subsequent down sampling will cost time for real time transmission system where no latency is tolerated. Therefore, for optimisation purposes the lifting scheme is used where those two computations are interleaved, this was described earlier in Chapter 5, Section 5.4.3. One condition that must

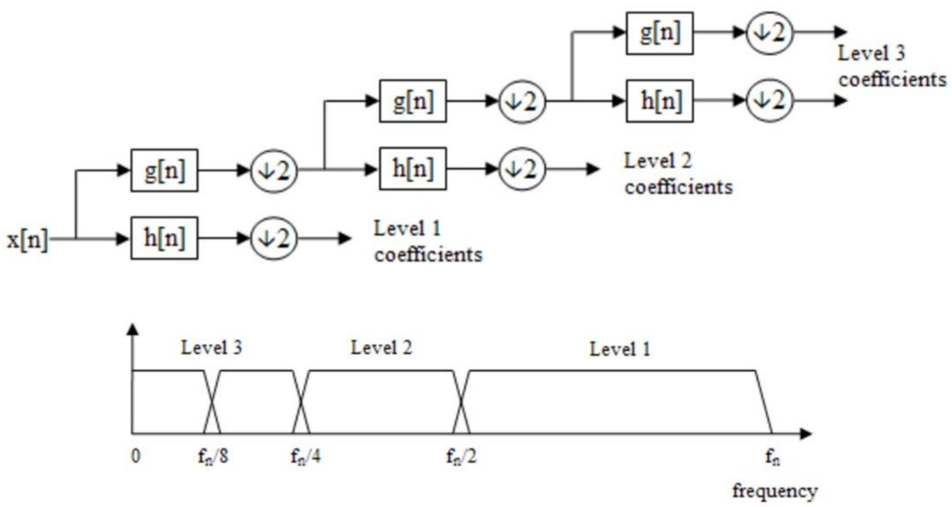

Figure 6.23: Doubling the Frequency Resolution with 3 levels of Filters Bank

be satisfied in order to fully reconstruct the signal is that the LPF and HPF in the filters banks must be a quadrature mirror filters. The quadrature mirror filter denote that the magnitude response of one is the mirror image of the other around $\frac{\pi}{2}$, i.e. a filter $H_1(z)$ is a quadrature mirror filter of $H_0(z)$ if $H_1(z) = H_0(-z)$; the filter response are symmetric about $\Omega = \frac{\pi}{2}$ => $\left|H_1(e^{j\Omega})\right| = \left|H_0(e^{j(\pi-\Omega)})\right|$, further about that can be found in [91].

Table 6.5: Number of Samples per Frequency Band for 3 Decomposition levels

Number of Levels	Frequency Bands	Number of Samples
3	0 to $f_n/8$	4
	$f_n/8$ to $f_n/4$	4
2	$f_n/4$ to $f_n/2$	8
1	$f_n/2$ to $f_n/$	16

From this stage of processing onward, in the receiver, through the demapper, decoder, forward error detection and correction and deinterleaver are all exactly as per the DVB-T standard specification, all described earlier in OFDM receiver Section 6.3.3. Therefore, for redundancy evasion, these will not be-described again in this section.

6.5 SUMMARY

In summary to the design of the proposed system, DVB-T standard [19] is used to validate the overall design and simulations results. Since wavelet transform has many families and even multiple associated members, those families that satisfy orthogonality, compactly supported, biorthogonality and discrete conditions are being used in this research. All their associated members are also being examined and compared to the baseline system, OFDM. Asymmetrical filters bank for dynamic bandwidth allocation, as designed in this chapter, are to be simulated and results will be presented in Chapter 7. In the next chapter, all results of designed systems and subsystems are presented, analysed and discussed.

CHAPTER 7: SYSTEM SIMULATION AND RESULTS

Objectives:

- To describe the system's parameters.
- To describe the implementation of baseline system.
- To describe the implementation of the proposed system, Orthogonal Wavelet Division Multiplex.
- To list the results of experiments for both systems over Additive While Gaussian Noise Channel.
- To list the results of experiments for both systems over Multipath Ricean Fading Channel.
- To list the results of experiments for both systems over Multipath Rayleigh Fading Channel.
- To demonstrate a technical observations discussion.

7.1 INTRODUCTION

In order to justify and evaluate the analysis of different wavelet families and families' members' performance, a baseline standard model has been adopted as a baseline for the analysis process to validate the proposed systems' results and analysis. In this case DVB-T (ETSI EN 300 744) standard [19] is being used. The designed systems both, OFDM and OWDM, which were presented in the previous chapter, are implemented in this chapter. In this chapter, the simulation parameters and results will be presented as well as a discussion and analysis of these results. During the critical review, there were almost no mention of time processing relative to the data set size and the type and width of the wavelet member used, therefore, to cover this gap of time complexity, four data sets are processed while conducting the simulation phase; all 100, 1k, 10k and 100k frames. This chapter is directly connected to the next chapter where an overall comparison and analysis of the results will be provided.

7.2 OFDM IMPLEMENTATION AND VALIDATION

The DVB-T standard [19], which utilises OFDM system as its multiplexer, in 2K mode, shall be the baseline for the comparison with the proposed modulation scheme, OWDM. This standard is the most recent practically implemented transmission system adopted by several broadcasters across the globe. In the UK, British Broadcasting Corporation (BBC), ITV and Digital 3&4 utilised this standard alongside the second generation of the same standard which they switched over to during 2012 and 2013. The implementation of this system has replicated the DVB-T standard [19] design and parameters from MPEG-2 frame generation in the transmitter through error detection and correction, encoding, symbol mapping, multiplexing and transmitting through channel to MPEG-2 receiving end in the receiver. The OFDM system has many parameters involved in operating the system and these depend on the application that OFDM is being used for and the quality and capacity of the system being manipulated by changing some of these parameters. The flow of the research has run smoothly and therefore, the implementation of both systems has been on the MATLAB command as .m code, and would therefore not be a Simulink block.

As mentioned earlier in Chapter 6, the considered baseline system is OFDM as in DVB-T represented by the EN ETSI 300 744 standard [19]. The overall design of this system was described earlier in Chapter 6, Section 6.2. The parameters of the implemented system are provided in Table 7.1 as follows:

Table 7.1: OFDM system parameters

Parameters Names	**Value/type**
Number of encoded frames	from 100 to 100k frames
Size of each encoded frame	96 bits/frame
Total number of encoded bits stream	from 96 to 9600 Kbits
Convolutional coding rate	½ and ¾
Polynomials generator	171 and 133 octal notation
Modulation scheme	16-QAM and 64-QAM
IFFT Length	64(test) and 2048 (real)
Number of active carriers	Varies, 64, 1705 carriers
Number of pilot tones	128 pilots. 1 every 13 bits
Cyclic prefix	25% (512 bits)
Channel type	AWGN, Ricean and Rayleigh
SNR	0 dB to 40 dB

For design choices and running time simulation the testing procedure will be carried out on a range of data sizes to confirm the hypothesis with realistic data rates. The data rate for this research shall be ranging from 1000 frames to 100k frames with 96 bits/frame, which give a total of up to 9600 Kbits stream, which is equivalent to 6 minutes HD video. These values were chosen to put a certain number of bits into each subcarrier (or sub-band). SNR is chosen to be from 0 to 40dB to check the effect of SNR versus BER for wider range of SNR values as per DVB-T standard specification. In addition, at 16 or more of SNR in DVB-T systems BER is negligible. As demonstrated by Linfoot et al. [3] and as per the standard specification [19], the ideal SNR where the signal can possibly be fully reconstructed in 16-QAM without noise in DVB-T systems with AWGN channel is around 40dB.

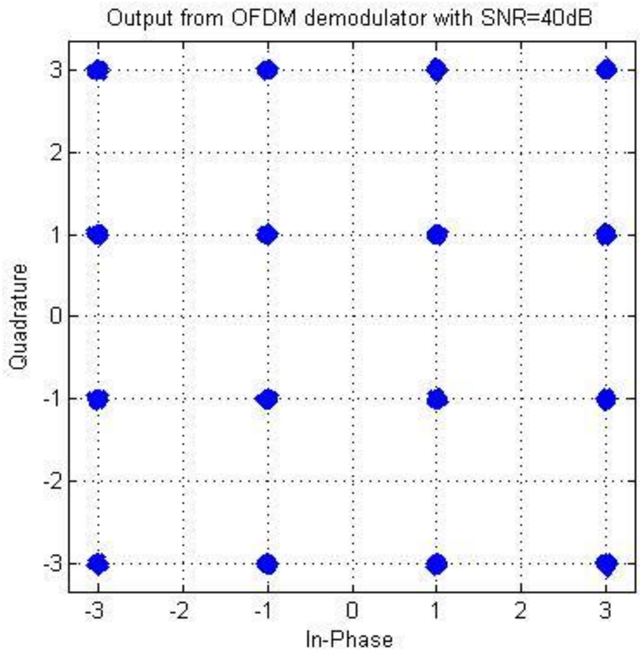

Figure 7.1: OFDM 16-QAM Demodulator output with 40 dB SNR

To validate this against the simulation and for testing and troubleshooting purposes, the following early stage validation against the standard was carried out for both OFDM and OWDM. The 16-QAM, as shown in Figure 7.1, is the result of this simulation when transmitting 1000 frames over 64 IFFT, which shows that at SNR 40dB the 16-QAM has

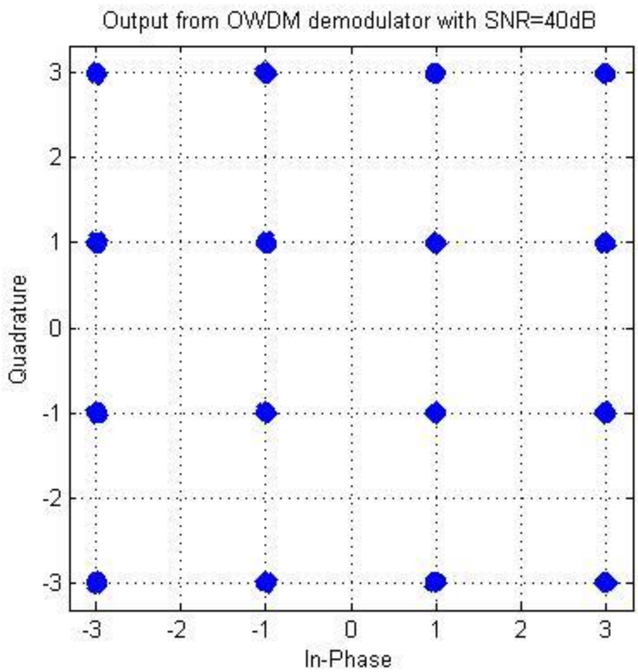

Figure 7.2: OWDM db6 16-QAM Demodulator output with 40dB SNR

minimum distortion. The resulting BER vs SNR of the same maximum SNR is shown in Figure 7.3. For dynamic OWDM with db6 and 6 levels of filters bank (equivalent to 2^6=64 IFFT) and for the same maximum SNR, the 16-QAM has the same distortion as OFDM QAM and the BER vs SNR is similar to that of OFDM at the maximum SNR 18dB, the BER is 10^{-5}, as shown in the same Figure 7.3, with OFDM. Figure 7.2 shows the received signal at the 16-QAM demodulator output which is comparable with that of OFDM in Figure 7.1. In both systems, there is no abnormality in performance which indicates that both systems are realistic and are normal.

This is the validation step of the simulated OFDM system against DVB-T standard. Since OFDM is the baseline comparison of this research, the results and discussion will be mingled with OWDM system. However, the simulation of all the parameters including data rate, QAM type and channels model will be presented in separate sections, thereby the results and analysis of each system is provided. Following that, OWDM with the same parameters and channels, which were used in OFDM system, for all different wavelet families and families' members analysis will also be presented at separate subsections in OWDM section. In the next section the OFDM system as used in BBC, ITV and Channel 3&4 and the equivalent OWDM are

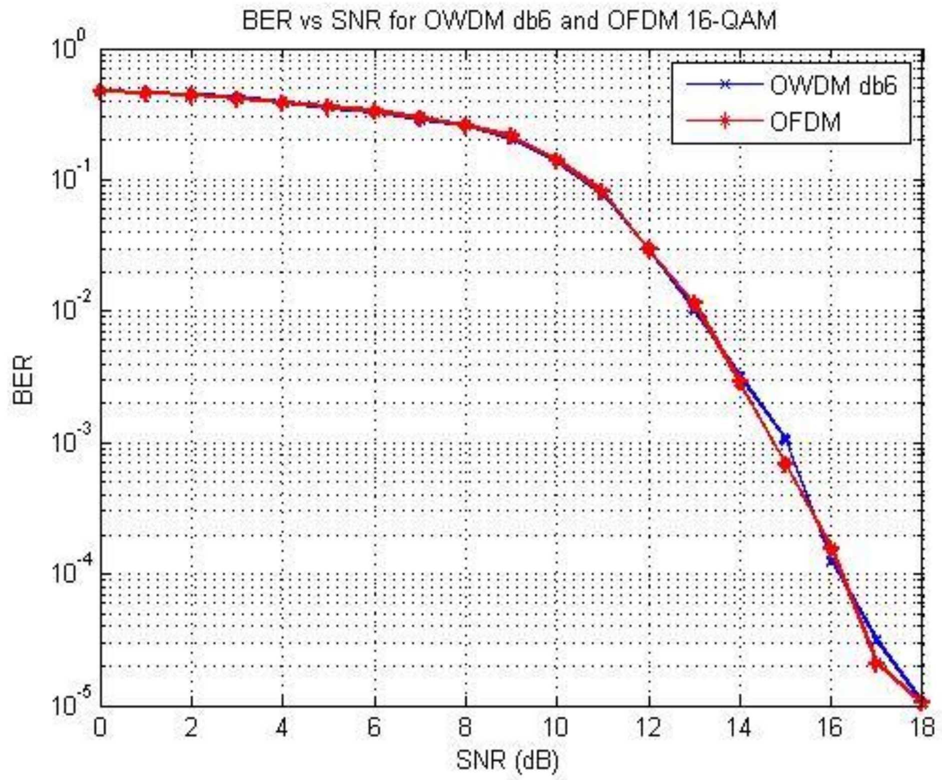

Figure 7.3: BER vs SNR for OFDM

presented with comparison of OWDM and over different channels as described in the design chapter, Chapter 6, in Section 6.2.

7.3 OWDM IMPLEMENTATION AND COMPARISON WITH OFDM

The OFDM system is the baseline for the comparison with the proposed modulation technique, OWDM. The implementation of this system has taken the DVB-T standard [2] as a baseline design. The OFDM system has many parameters involved in operating the system and these depend on the application that OFDM is being used for and the quality and capacity of the system being manipulated by changing some of these parameters. The number of filter bank levels, in OWDM, is directly proportional to the length of IFFT, in OFDM, it follows: $log_2(N)$ where N is IFFT length. There are two configurations of OWDM: Using wavelets and using wavelet packets. In this research, the first will be used since wavelet packets use a symmetric synthesis, meaning that each sub-band generated by the OWDM modulator will have identical bandwidth in the same way that OFDM does, whereas the proposed technique is to dynamically allocate bandwidth and this is achieved by varying the width of sub-bands, which is achieved by asymmetrical filter banks. All parameters of both systems are equivalent, however, the filters bank level, in OWDM, is proportional to the length of IFFT, in OFDM, by following the statement earlier. Therefore, it is possible to perform a “like-for-like” comparison.

In the experiments detailed in this research, the simulation is based on the DVB-T specification model and includes ½ and ¾ encoding rate, 16-QAM and 64-QAM and RS(204,188,16) outer encoder. The simulation will use the 2K mode DVB-T for OFDM and 11 levels of decomposition for OWDM, both giving sub-bands widths of 4464Hz. The simulation will run over a range of data size from 100 frames to 100k frames, which is capable of transmitting 6 minutes HD video, through AWGN, Ricean and Rayleigh fading channels, with SNR powers ranging from 0dB to 40dB. The wavelet families that are being investigated in this research are: Haar (db1), Daubechies (dbs), Symlets (syms), Coiflets (coifs), Biorthogonals (biors), Reverse Biorthogonals (rbios) and Discrete Meyer (dmey). The full list of investigated wavelet families and their associated members were presented earlier in Chapter 6, Section 6.2 Table 6.3. The full system parameters used in the implementation of proposed system are presented in Table 7.2.

However, SNR might be increased in some cases where it is needed to adapt the channel noise and test signal delivery. The usage of 11 levels is to match OWDM system with OFDM where IFFT length is 2048, following the logarithmic relationship between M levels and IFFT length.

The next subsection will present the performance analysis of randomly chosen wavelet families and families' members and compared not only among each other but also against OFDM performance. These results were partially published in four published papers and articles in leading conferences and journals in the field, thereafter, more families and associated members were also included in the investigation when it was submitted to a journal and being reviewed, as the turnaround is about 9 months. Due to the vast amount of results and for better visualisation of results and also for space management, multiple curves of results are represented in the same figure with 8 curves per figure. Each of those curves is a separate experiment that was carried out individually. Additionally, since the BER vs SNR can vary slightly, therefore, for maximising accuracy of results, each of those experiments was run on average 45 times and the average result was saved for later use.

Table 7.2: OWDM system parameters

Parameters Names	**Value/type**
Number of encoded frames	from 100 to 100k frames
Size of each encoded frame	96 bits/frame
Total number of encoded bits stream	from 96 to 9600 Kbits
Convolutional coding rate	½ and ¾
Polynomials generator	171 and 133 octal notation
Modulation scheme	16-QAM and 64-QAM
Wavelet Families	Daubechies, Symlet, Coiflet, biorthognal, reverse biorthogonal and discrete Meyer
Number of filters bank levels	6 to 11 levels
Channel type	AWGN, Ricean and Rayleigh
SNR	0 dB to 40 dB

7.3.1 OWDM OVER AWGN FADING CHANNEL

In this section and its associated subsections, all simulations of OWDM proposed system and also OFDM system are presented using the parameters stated in Table 6.1and Table 6.2 over AWGN channel, which represents the direct line of sight between the transmitter and the receiver. The symbol mappers that are used are 16-QAM and 64-QAM. The number of encoded frames will be ranging from 100 frames to 100k frames to test further data rates and also test the time complexity that each set of families and their associated members need to respond. Apart from that, we are also testing the memory occupancy during processing for those families and their members.

The following subsections will be categorised by the wavelet families where all associated members are compared against each other and also against OFDM at the same graph, for better visualisation of results. Discussion and analysis will also be presented at each subsection and more importantly, at the conclusion of this section.

7.3.1.1 HAAR AND DAUBECHIES WAVELETS FAMILY ANALYSIS

This subsection presents all the results of proposed OWDM system for asymmetrical filter banks with dynamic bandwidth allocation. The used channel is AWGN, 16-QAM is used at the beginning and then 64-QAM is also included. The convolutional coding rate used is ½ code rate which was changed to ¾ later but will be stated when any parameter is changed. Due to the vast number of Daubechies wavelet members, 45 members, and for better visualisation of results, they will be divided into several graphs with 7 members in each graph. Those tested Daubechies members are described in Table 6.3 in Chapter 6, Section 6.4.1.4. For redundancy avoidance and due to the very close similarity between consecutive members and due to the large number of Daubechies members, a step of 3 vanishing moments were skipped and the next was chosen, i.e. db20, db23, db27 are chosen so on so forth; however, the first 6 members were all tested to see any changes in the behaviour pattern. Haar wavelet is the db1 so it will be called Haar, but will be included here rather than as a standalone subsection.

To start with, for less data size of 100 frames, the jumps in the members is higher since the effect in performance will be close to random, due to the short data sequence that is processed. In Figure 7.4, the difference in performance for Haar, db2, db4, db10, db21, db30, db40 and db45 is measured for 100 frames over AWGN using 16-QAM and ½ code rate. It can be seen that between the performances of members of the two groups, the best are Haar, db10, db21, db30 and db40; they performed very similarly and achieved less BER than the rest. Whereas, db2 and db4 performed similarly but both are the worst among; however, db45 performed in the middle of both groups where it achieved better BER than the second group but worse than the first.

Nevertheless, since this performance is not realistic, especially those are with higher vanishing moments, i.e. db40 and db45 varies in performance between them but similar to those of a lower vanishing moments, the performance is random at this stage due to several factors. Firstly, the used data size is not long enough to test the system's susceptibility to errors and errors occur randomly within transmitted bits. When 100 frames were used, the real transmitted bits were 9600 bits, where errors that occurred could have more impact on the received signals during transmission than longer data streams. Therefore, to avoid this random behaviour, the data size is increased to a factor of x10 for the next experiment where the same parameters are used, however, more importantly the transmitted frames are 1000 rather than 100, which makes the transmitted data stream 96 kbits. Apart from that, the step between each

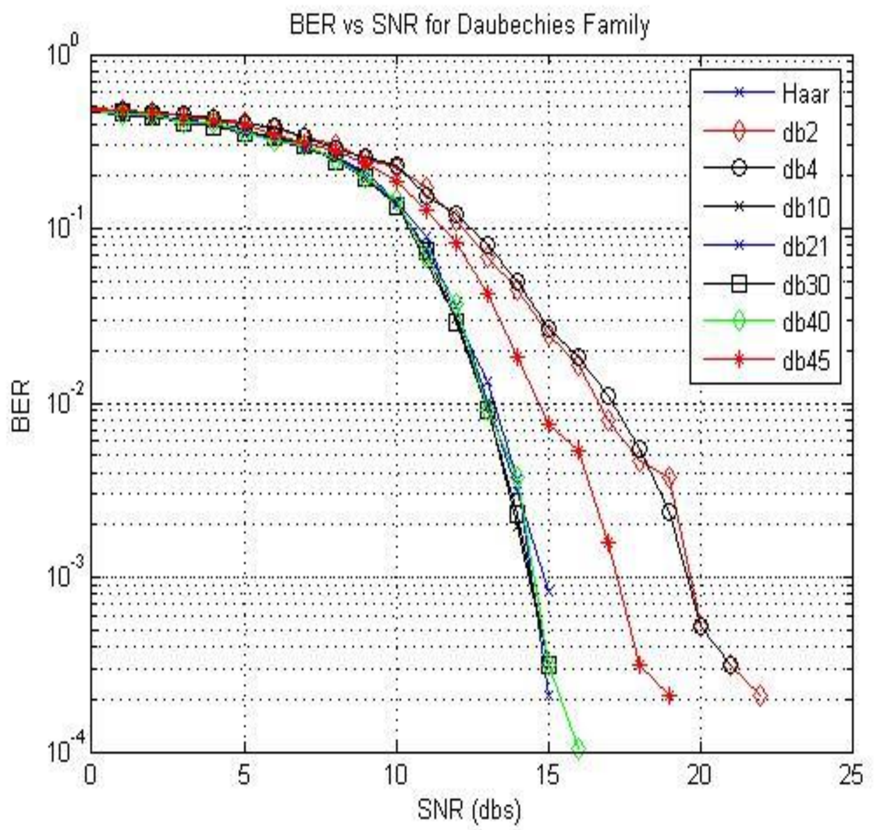

Figure 7.4: Daubechies members' performance over AWGN, 16-QAM and ½ code rat

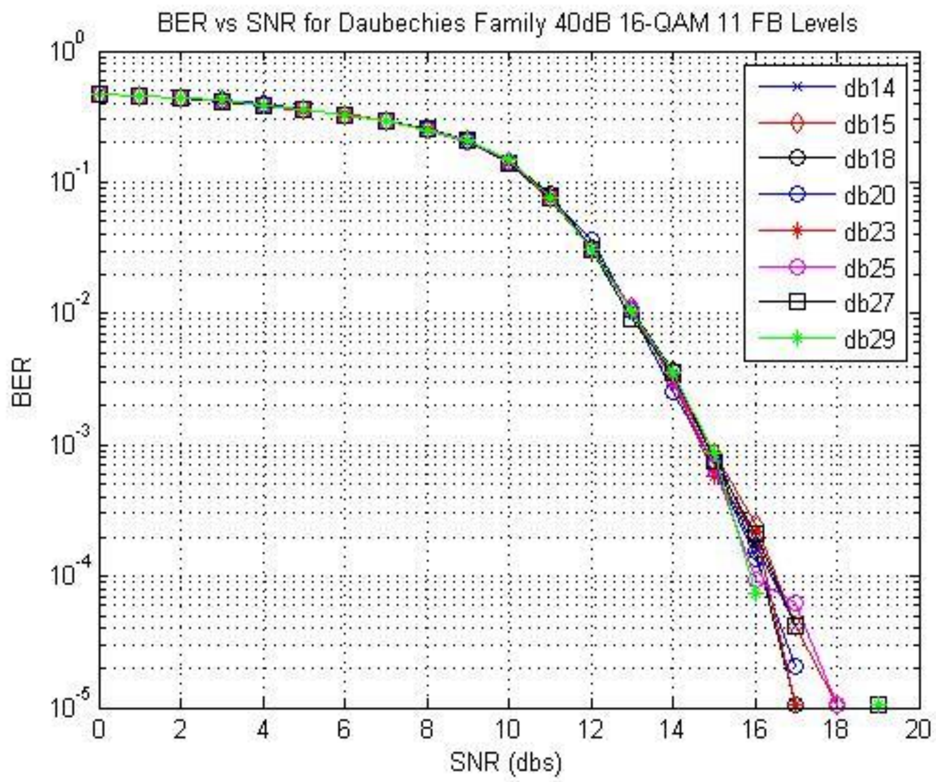

Figure 7.6: db14 to db29 BER vs SNR Performance with 40dB 16-QAM and 11 Levels

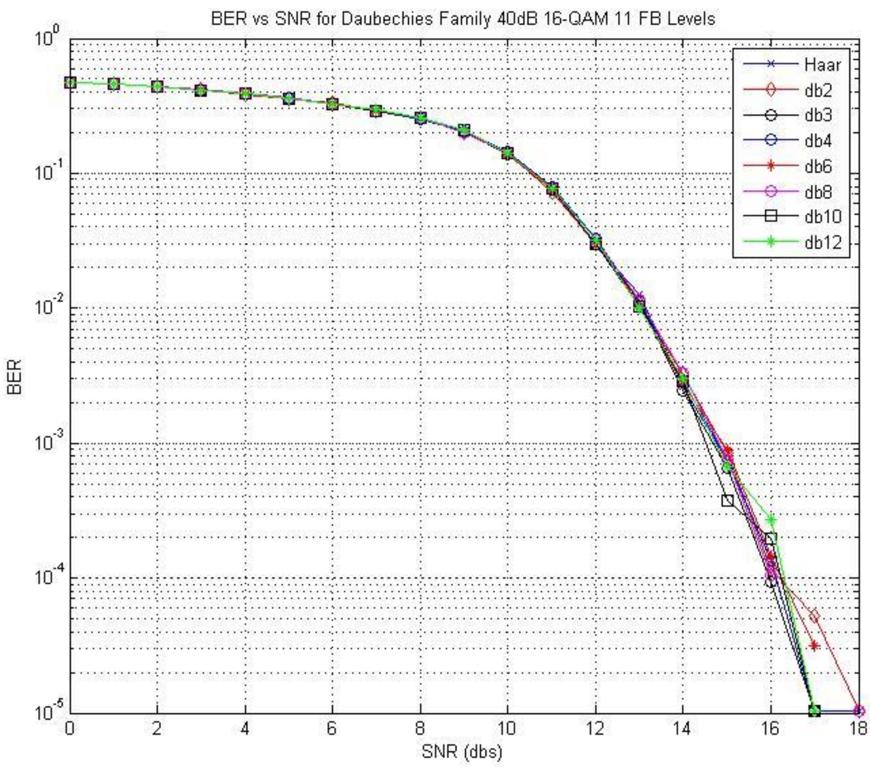

Figure 7.5: db1 to db12 BER vs SNR Performance with 40dB 16-QAM and 11 Levels

consecutive member is reduced from 10 with the same factor, so that instead of skipping 10 members now just 1 member is skipped at the first 20 members, thereafter 2 members are skipped for the rest. This is for managing the large set of data analysis without compromising the desired differences analysis between consecutive members. For better visualisation of results among these large number of members, each 7 members are grouped and presented in separate graph rather than individually and due to the large number of members, each 8 members are presented in separate graph rather than congesting them in one graph.

Further analysis of why the performance of different families and their associated members varies depends on the number of vanishing moments and family type will be discussed in the critical analysis and overall comparison section later in this chapter, Section 7.4.

The Figures, Figure 7.5, Figure 7.6 and Figure 7.7 shows the performance of Daubechies' members whereas Figure 7.7 also show the performance of OFDM as among the best within Daubechies members. From these three figures, it is noticed that the performance with longer data stream came to a better rhythm of data flow, compared to the random rhythm when using

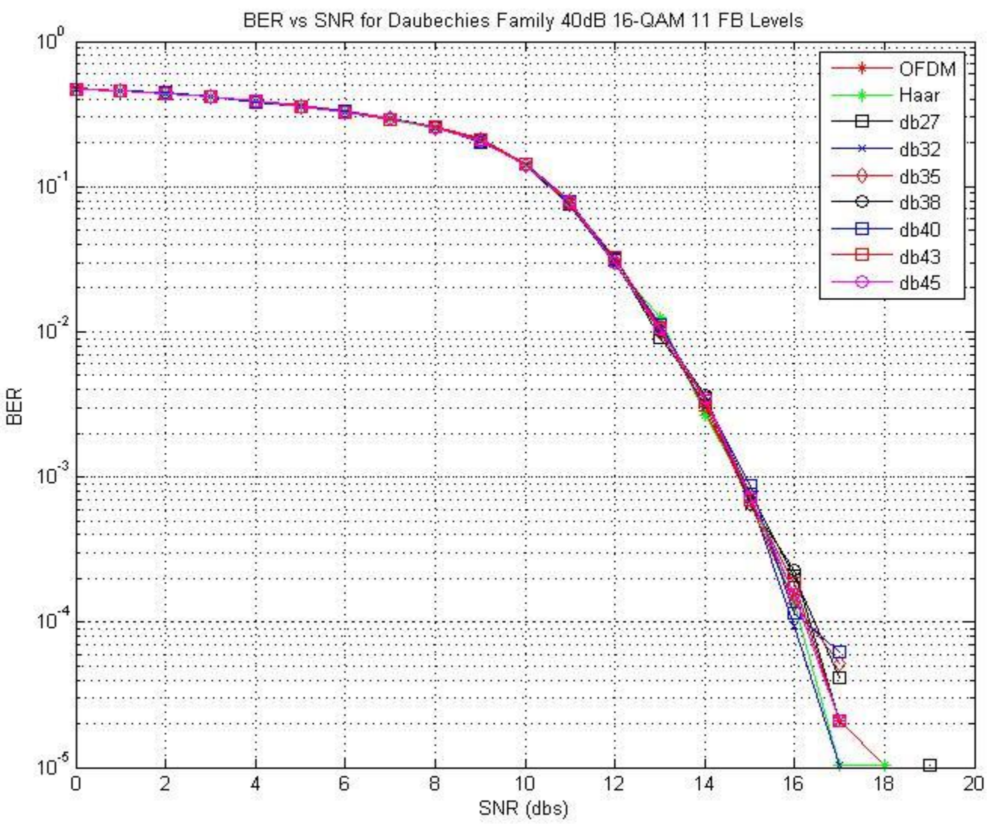

Figure 7.7: OFDM vs Haar and db27 to db45 BER Performance

unrealistically short data stream, as shown previously in Figure 7.4. Although most Daubechies members have performed similarly regardless of the used length of data streams, some have slightly performed differently with longer data stream, e.g. db2 and db4. Nevertheless, it is well know that when measuring error rates in data transmission and since the channel is AWGN, where a random white noise is added to the transmitted signal, the burst of noise will affect the signal with short data streams considerably more when using longer data stream where the receiver can maximise the chance of determining and correcting some occurred errors using error detection and correction algorithm.

It is clearly seen from Figure 7.5, Figure 7.6 and Figure 7.7 how closely all the members are performing for a large data set. It can also be seen that most members compete with OFDM performance with some even achieve less errors than OFDM, i.e. Haar and db32. However, each of those curves is a standalone experiment of that particular member and thus the exhibited response time of each of those members was observed to indicate the time complexity, including OFDM. The PCs processor, RAM and other relevant specifications are identical, so that no compromise in processors power or memory occurred. The most important observation of all, regarding time processing, is that as the number of vanishing moments is increased, the processing time is increased too. This is true since the number of filter coefficients is doubled with each single vanishing moment and hence the number of multiplications is doubled too. More on this time complexity will be discussed later in this chapter, in the critical analysis Section 7.4 and Chapter 8, and will also be revisited in the next chapter.

Therefore, to further validate this processing time complexity and also the performance exhibited by the members with longer data stream, the data stream will be increased to a factor of x10 where it becomes 10k frames = 960 kbits, to further confirm the behaviour of the family's members. The same members over the same system's parameters including OFDM are examined and results are shown in the following figures.

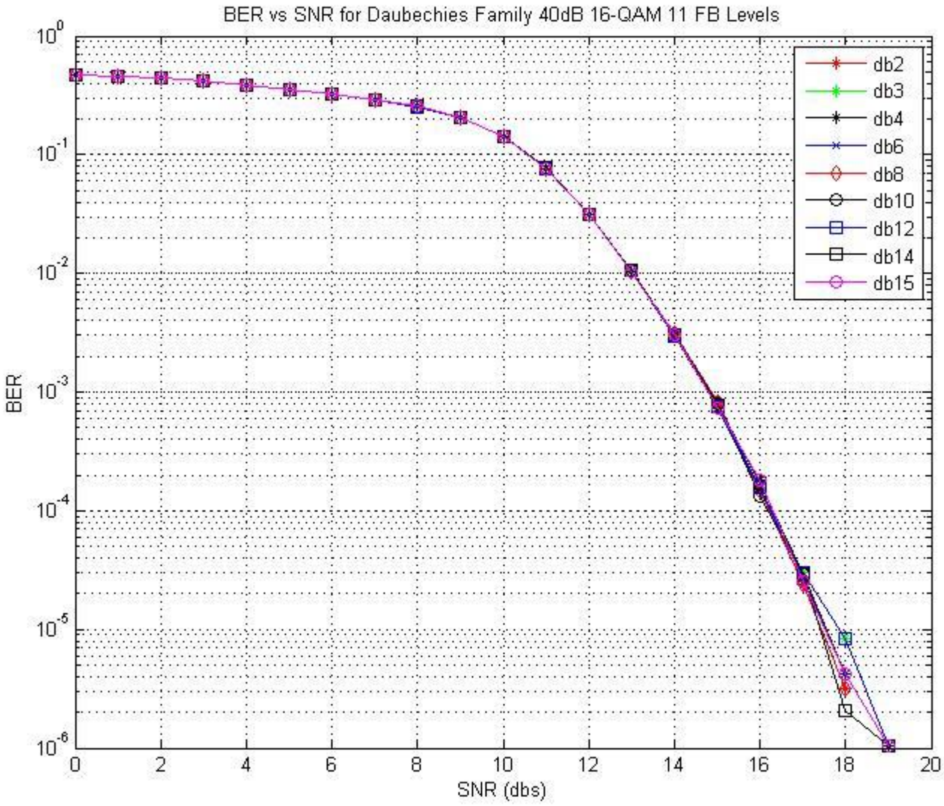

Figure 7.8: db2 to db15 BER Performance for 10 kbits

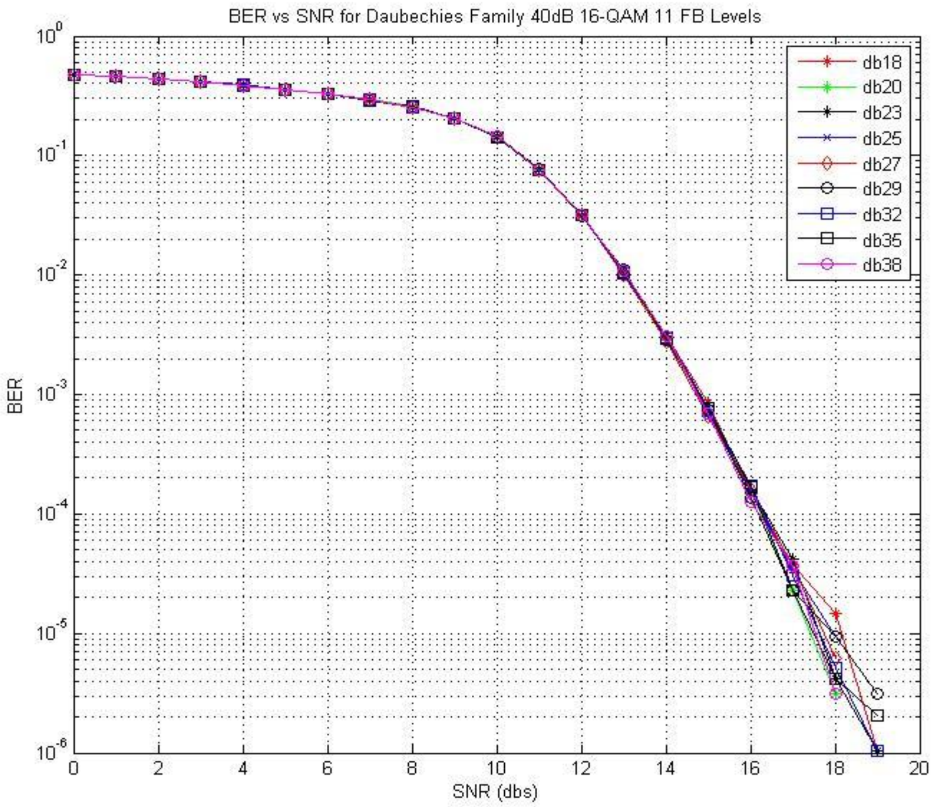

Figure 7.9: db18 to db38 BER Performance for 10 kbits

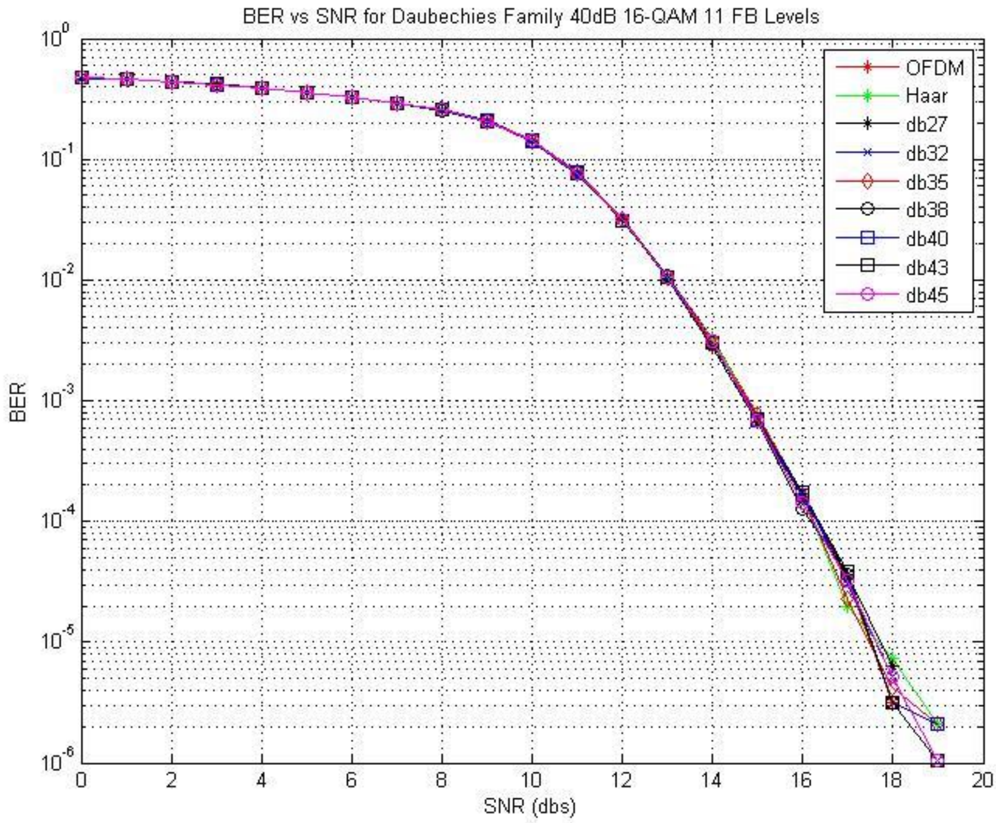

Figure 7.10: OFDM, Haar and db27-db45 BER Performance

It can be noticed from Figure 7.8, Figure 7.9 and Figure 7.10 that the behaviour of Daubechies members are now more stable and they are acting normally with longer data stream, in this case 960 kbits. This is because with the longer bits stream, error detection and correction can detect and correct corrupted bits and pilots can comprehend what the channel behaviour looks like. We can also see that members were behaving better than OFDM, Haar and db32 continue to perform as they did with 1 kbits. However, some other members also performed better than OFDM, such as db43 and db45, who achieved less BER at higher data rate with 5.3 errors each 90 kbits, compared to OFDM with 5.9 errors every 90 kbits. The processing time, however, is the cost that has to be paid, especially when processing high data rate with higher orders of members, which confirms the time complexity when 1 kbits were transmitted, as described earlier.

Therefore, for last test of the behaviour and time complexity, the data size was increased further to a factor of x10 where 100k frames = 100 kbits are transmitted, which is capable of transmitting 6 minutes of HD video. The following figures show the performance of OFDM, Haar and db2 to db45 for transmission rate of 100 kbits.

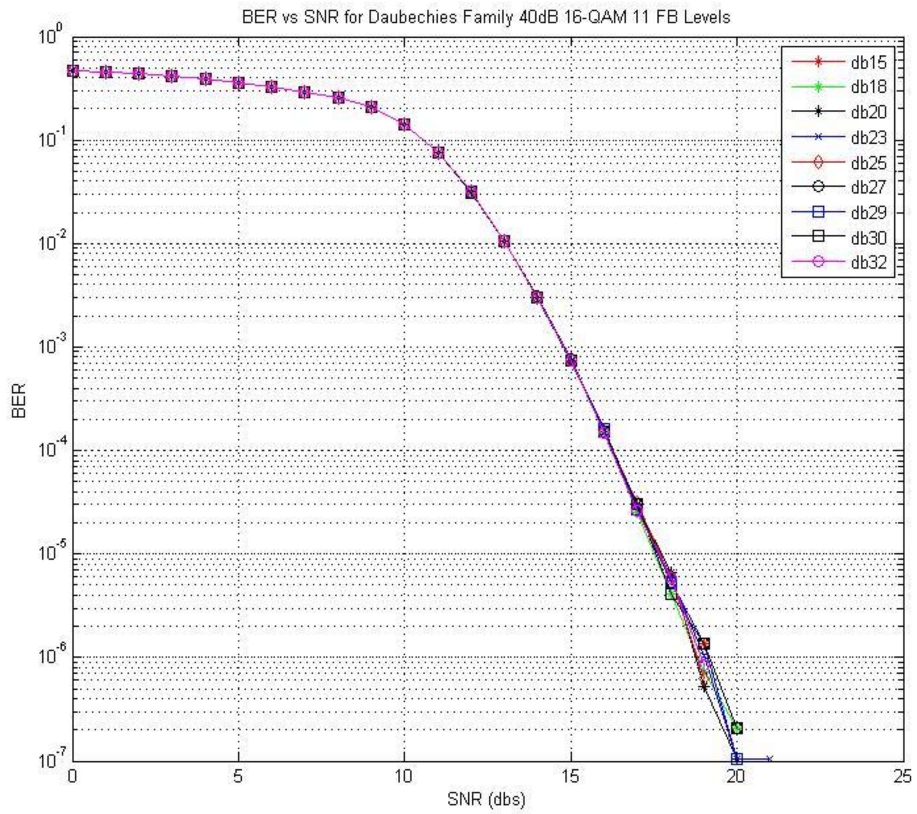

Figure 7.12: db15 to db32 BER Performance

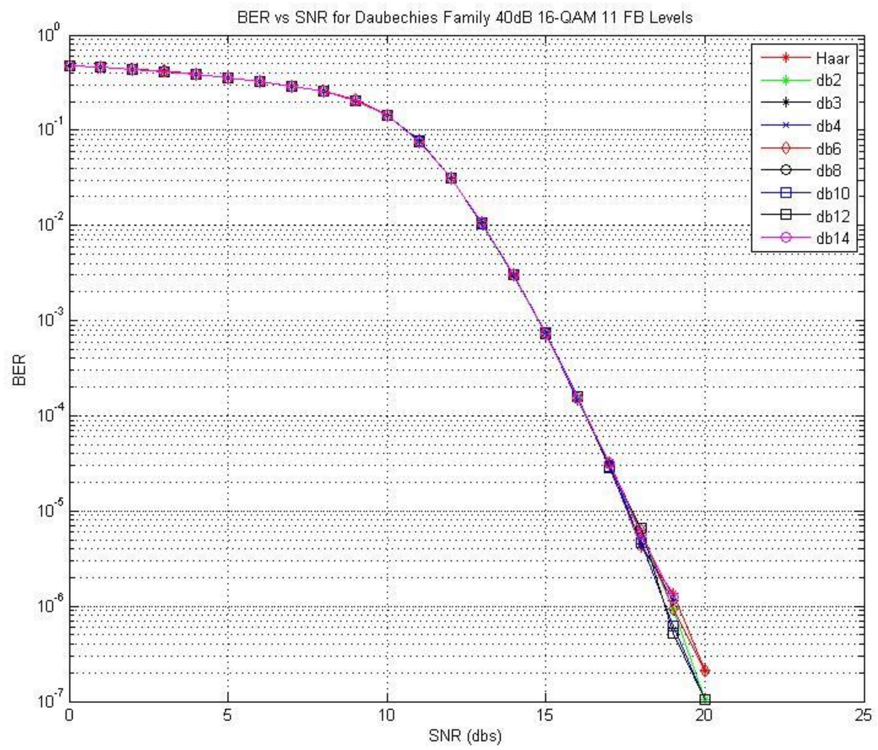

Figure 7.11: Haar and db2 to db14 BER Performance

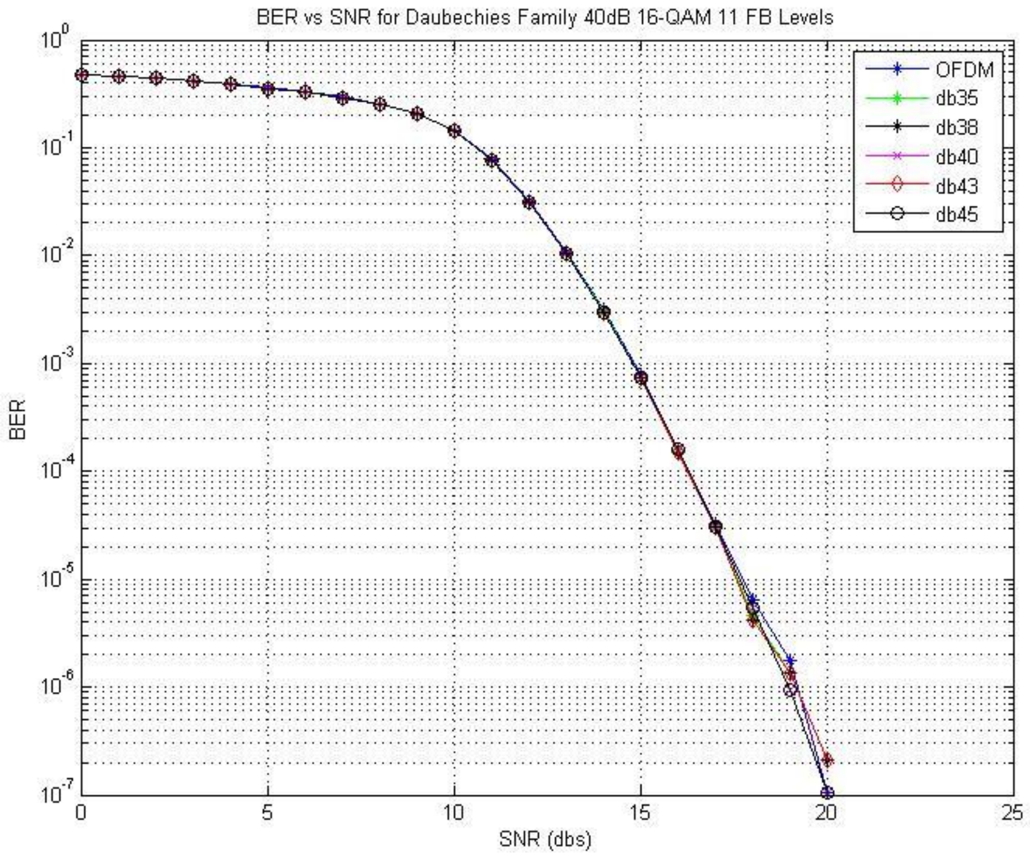

Figure 7.13: OFDM, Haar and db35 to db45 BER Performance

It can be noticed form these three Figure 7.11, Figure 7.12 and Figure 7.13 that the performances of the members are still very close and since this huge data rate is transmitted, hence the performance is more likely to continue to be the same for those members alongside OFDM. In Figure 7.11, although all members are visibly similarly performing at high data rate, however, the performance of those members are very close starting from zero until BER reaches $10^{-5.4}$ where differences becomes clearly visible (after magnifying), in which db10 and db12 distinctively performed with less BER achieved at 19dB SNR. These differences in performance though come at a cost of both memory size needed to complete the computation and computational time. As observed previously at 10 kbits data size, the higher the order of the filters (member, coefficients), the higher the time consumption and also the higher the memory space needed.

In Figure 7.12 and Figure 7.13, however, the same observation was observed in which the higher the order of the family, the longer time it take to process, especially for db43 and db45 where it took tremendously long time (about two weeks) in a normal MATLAB code, although that could be optimized in other platforms. However, this is a trade-off between better system performance at less cost, since there are no need for guard intervals and also dynamic bandwidth allocation, alongwith the complexity of response time and memory size. As we can see in Figure 7.13, the best performing member, among OFDM and other members, is the one that has the highest order (db45) where it achieved less error at 19dB SNR with 10^{-6} and again at 20dB SNR where it achieved better BER than not just OFDM but all other members.

These presented results will be discussed in comparison with other families and their associated members for the same system parameters and over the same conditions later in this chapter. Also all results will be critically analysed and evaluated in the next chapter, Chapter 8.

7.3.1.2 COIFLET WAVLET FAMILY ANALYSIS

As described earlier in Chapter 5, Section 5.4.3, Coiflet wavelet is a derivative of Daubechies wavelet; however, it is a nearly symmetric family. It has just 5 members which is the minimum number of family members among other wavelet families. The naming pattern follows (coif*N*) to have a number of vanishing moments of 3*N* with *N* up to 5, so that coif5 has 3*5=15 vanishing moments, and hence, it has 30 filter taps [84]. In this section, Coiflet wavelet

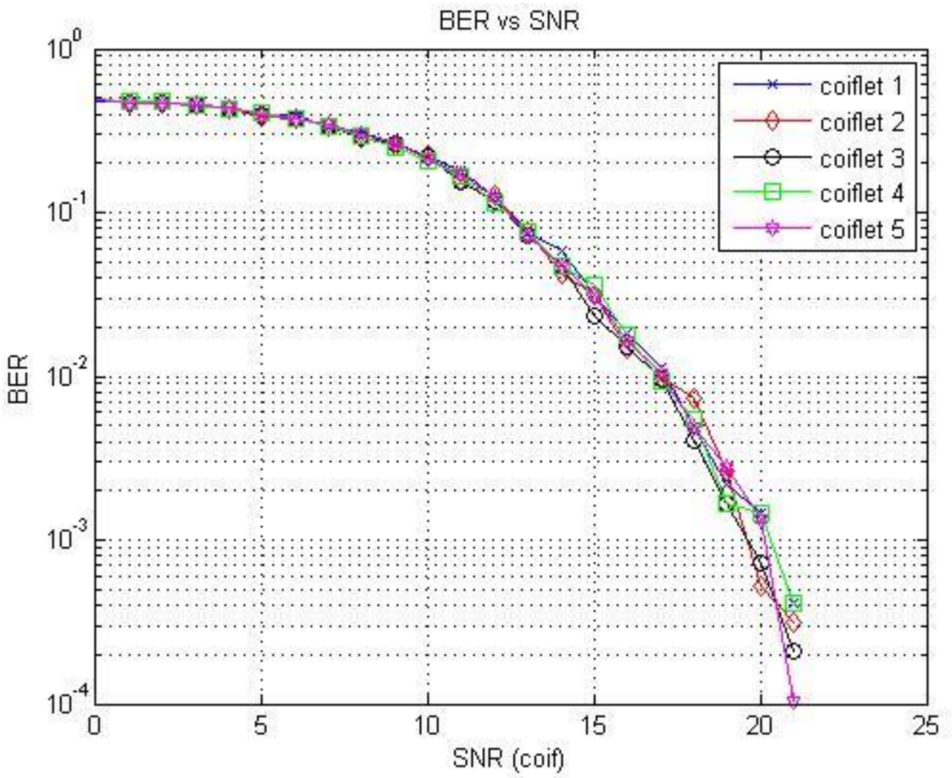

Figure 7.14: Coiflets Members over AWGN with 100 frames

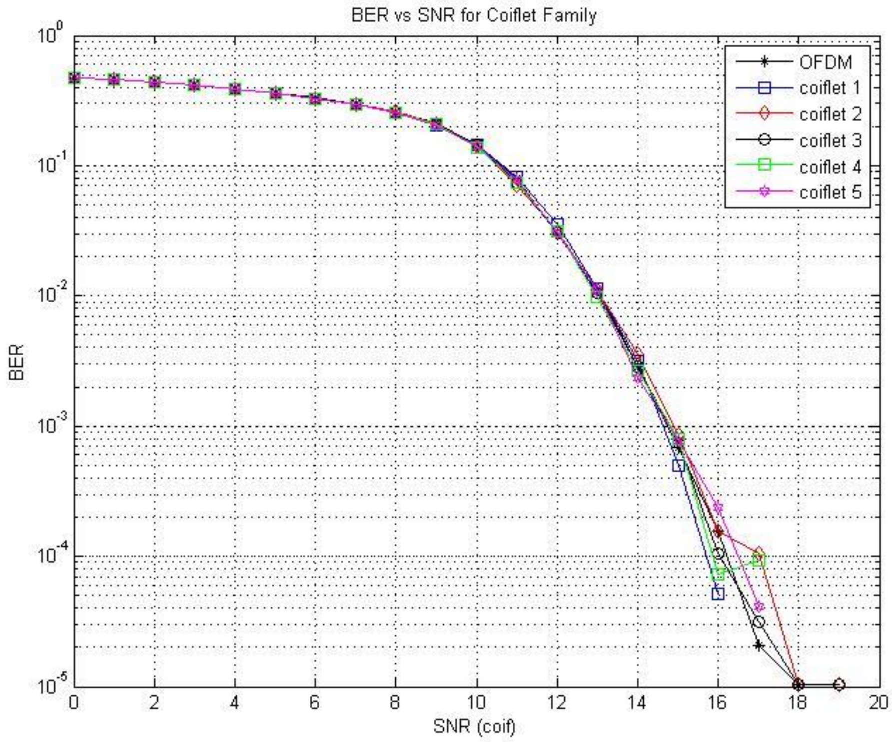

Figure 7.15: All Coiflet Members over AWGN 1k frames

family members are tested with the same parameters that were described earlier in Section 7.3 and over AWGN fading channel, ½ code rate, 16-QAM and 6 levels of asymmetrical filter banks that matches 64 IFFT length in OFDM system.

The same procedure of data size increase by a factor of x10, that was followed in Daubechies family investigation, will also be followed here not only to maximise the accuracy of the investigation but also to measure the time complexity associated with each member of this family. Increasing the data dramatically from a simulation test data size (100 frames) to a real world scenarios (100k frames) would realistically indicate the time complexity that a member of family would need to take to respond. Apart from that, we are also measuring the consumption of memory size that would be occupied during processing. The next three figures show the BER vs SNR performance of all the 5 coiflets members over AWGN with the parameters indicated in the beginning of this section, 7.3. From the four previous figures, Figure 7.14, Figure 7.15, Figure 7.16 and Figure 7.17, the performance of coiflets members remains stable and progressive independent of the data size that is being processed.

Figure 7.17 shows the resulting BER vs SNR performance for 960 kbits processed data stream, which shows that the overall performance of all members are very similar to that of

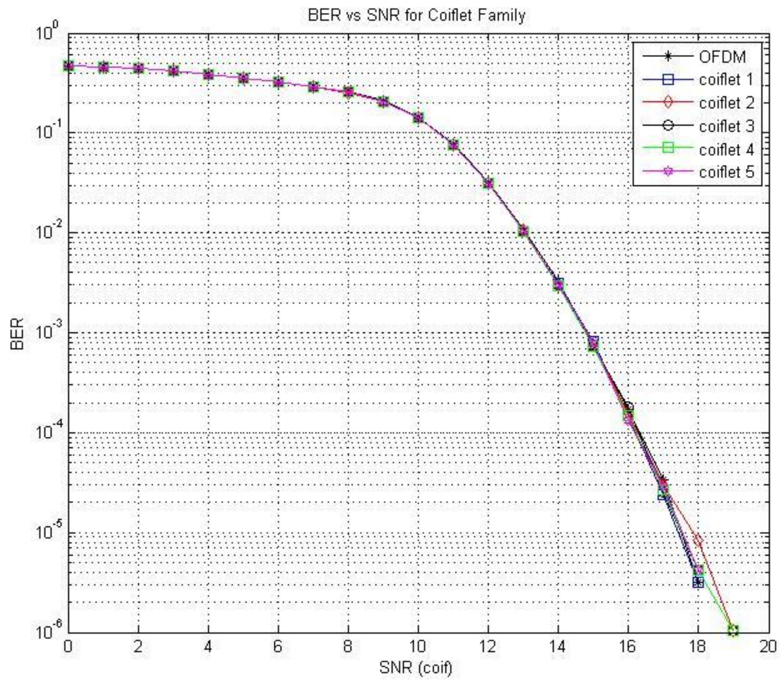

Figure 7.16: Coiflets Memebrs Performance over AWGN with 10k frames

OFDM. However, the time complexities for each of those members differ due to the fact that each member has different filters taps, which is directly proportional to the number of multiplications and summations during the computation of functions during processing. More on these analyses will be discussed in the results analysis chapter, Chapter 8.

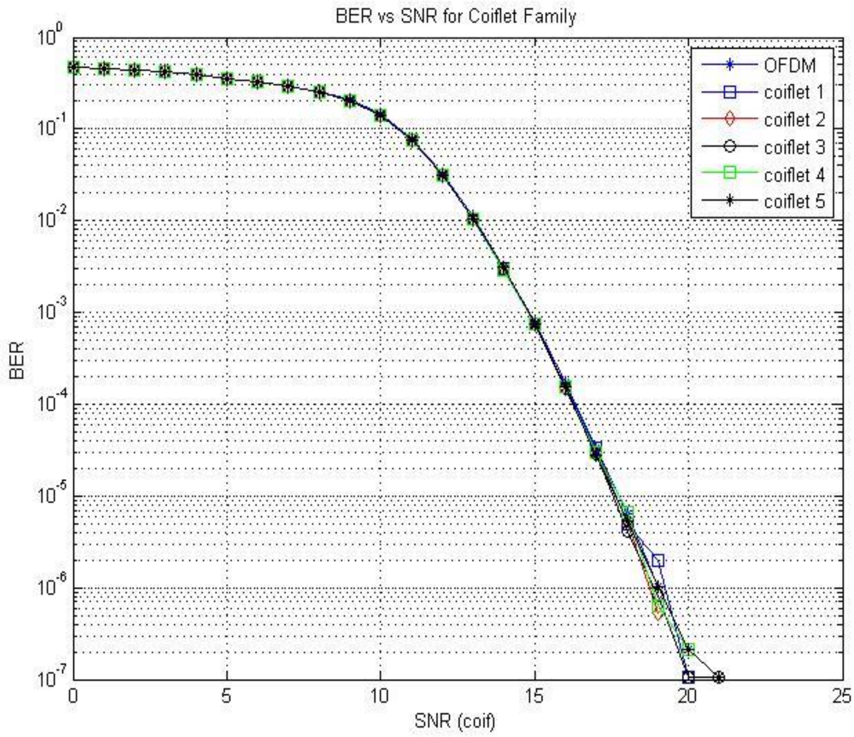

Figure 7.17: Coiflets Memebrs Performance over AWGN with 100k frames

7.3.1.3 BIORTHOGONAL WAVELET FAMILY ANALYSIS

The Biorthogonal family was described earlier in Chapter 5, Section 5.4.4. In this section, the implemented biorthogonal members will be presented. The system parameters used for this simulation were presented earlier in this chapter in The next subsection will present the performance analysis of randomly chosen wavelet families and families' members and compared not only among each other but also against OFDM performance. These results were partially published in four published papers and articles in leading conferences and journals in the field, thereafter, more families and associated members were also included in the investigation when it was submitted to a journal and being reviewed, as the turnaround is about 9 months. Due to the vast amount of results and for better visualisation of results and also for space management, multiple curves of results are represented in the same figure with 8 curves per figure. Each of those curves is a separate experiment that was carried out individually. Additionally, since the BER vs SNR can vary slightly, therefore, for maximising accuracy of results, each of those experiments was run on average 45 times and the average result was saved for later use.

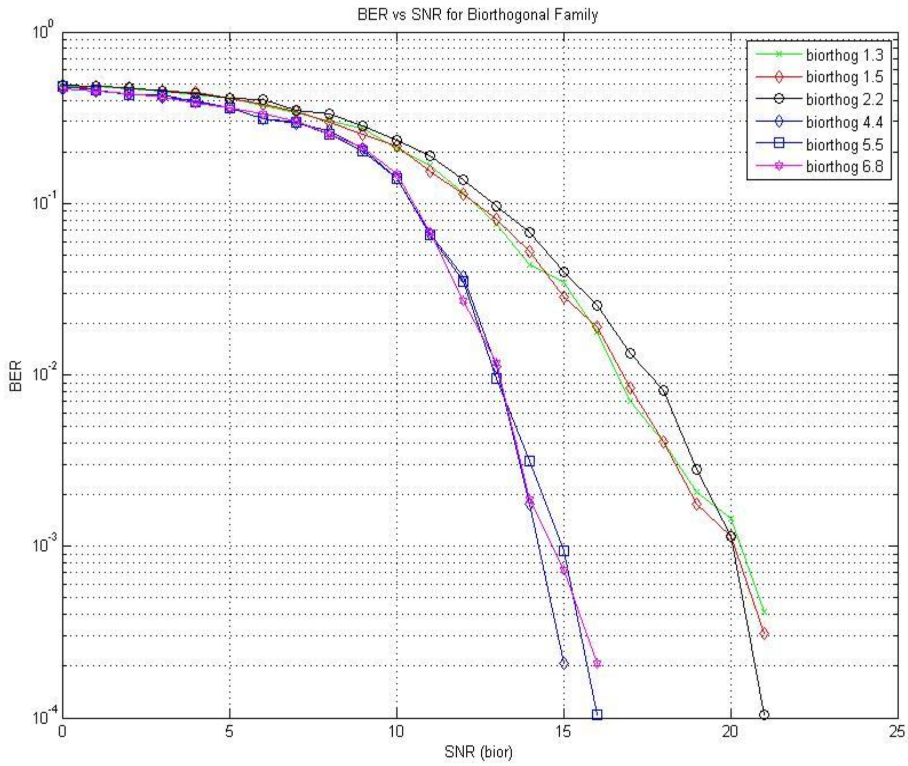

Figure 7.18: Biorthogonal BER vs Performance (100 frames)

Table 7.2, in Section 7.3, shows the system parameters. The procedure followed in justifying the results for short average and long data streams are same as previous sections, where the simulation of each member will start with short data stream to measure the effect of computation time and memory size needed for computation of individual members starting from 100 frames. Thereafter, the data stream is increased by a factor of x10 three times and up to 100k frames, equivalent to 6 minutes HD video.

In Figure 7.18, the data rate used is as small as 100 frames. It can be seen that the performance of members vary where those with higher vanishing moments, i.e. bior4.4, bior5.5 and bior6.8 performed better than the others, where they achieved considerably less BER. However, since the data rate is very small and was intended just to test the response time with less bit stream, the data rate was increased by a factor of x10 for which the results are shown in Figure 7.19 and Figure 7.20. It can be seen from those two figures that behaviour of those three aforementioned members have slightly changed due to the significant increase in data length. Nevertheless, all other members are behaving similarly no matter what the length of data is. The processing time, though, increased by a factor of x10 since the data has been increased and the response time has also noticeably gone up by the same factor as data size,

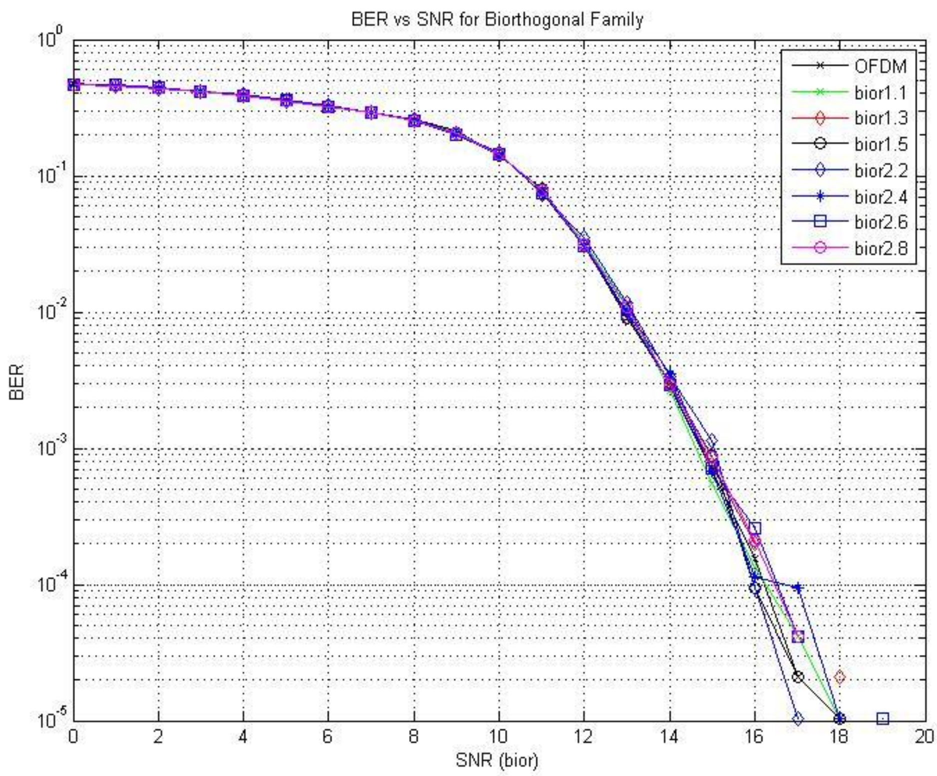

Figure 7.19: Bior1.1 to bior2.8 BER vs SNR Performance (1k frames)

except for bior6.8 where the time response has gone up by close to twice that of the other. This is due to not only the considerable number of vanishing moments (filter taps) but also the different number of filters taps in the decomposition than in the reconstruction.

The same response was noticed when increasing the data rate twice to a further x10 factor, 1k frames. The results are shown in Figure 7.19 and Figure 7.20, where the performance is still similar for all members, with minor differences at a higher level of SNR. We can also notice that the members with symmetrical decomposition and reconstruction, i.e. bior 1.1 and bior 2.2, have achieved less errors at higher SNR compared to not just the other members but also OFDM. To further validate the simulation and for maximising the measuring accuracy of BER, the data size increased to a further factor of x10 giving 10k frames and then again to 100k frames. The results of all these simulations are shown in Figure 7.22-Figure 7.23 in the next few pages. The times response measured for those members are observed to have performed better than what the previous two experiments had measured. The response time increased proportionally to the increase in data size for all members except for those members with large discrepancy in the number vanishing moments in the decomposition and reconstruction filters.

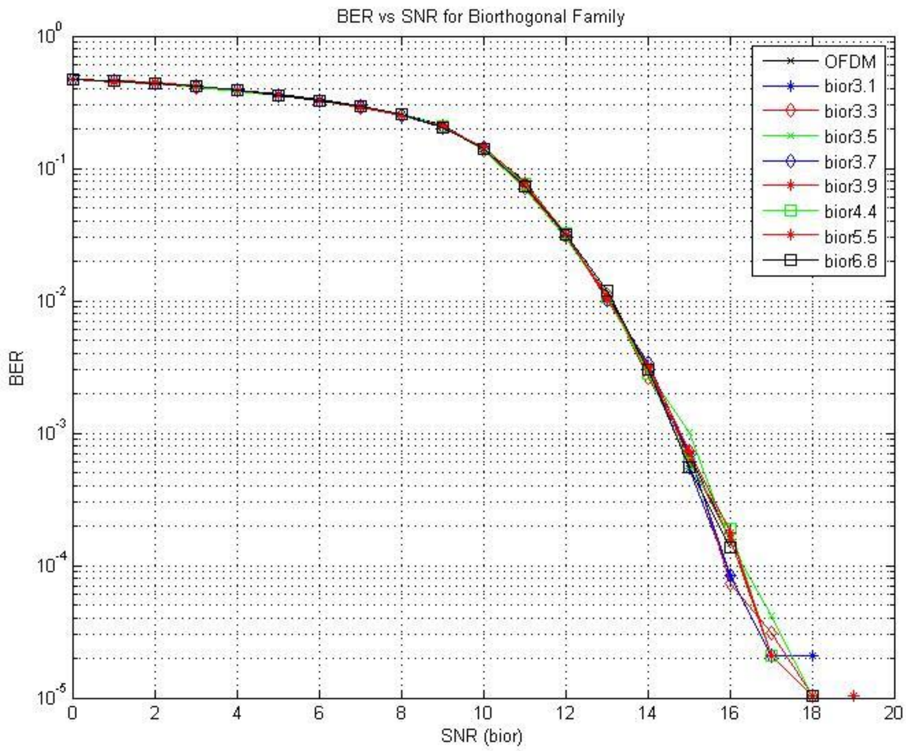

Figure 7.20: Bior3.1 to bior6.8 and OFDM BER vs SNR Performance (1k frames)

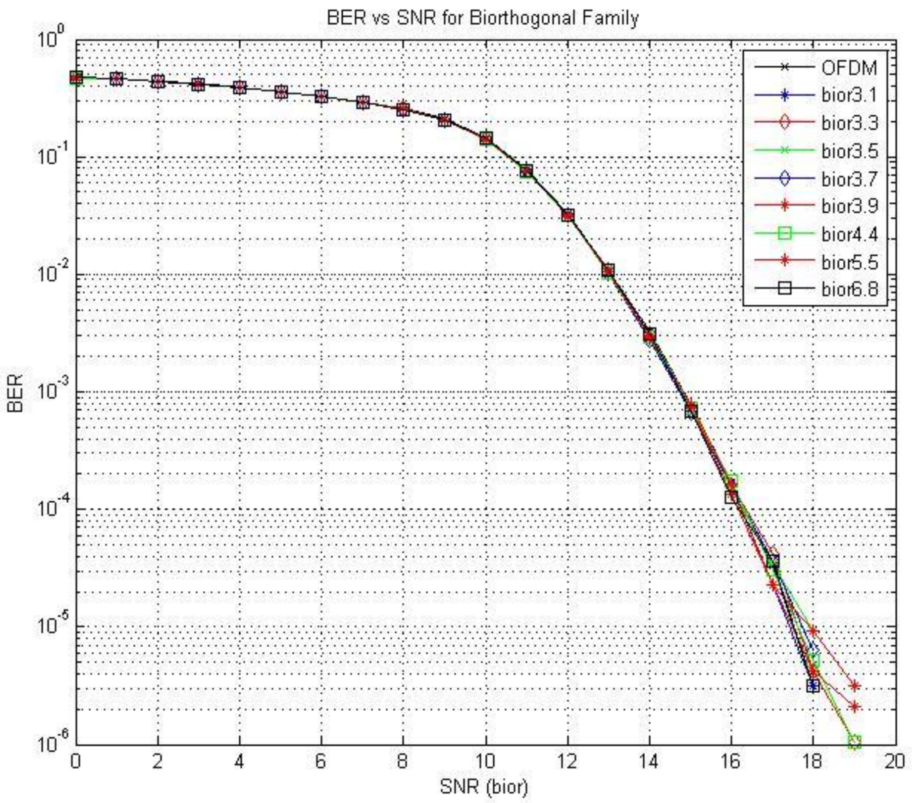

Figure 7.21: Bior3.1 to bior6.8 and OFDM BER vs SNR Performance (10 frames)

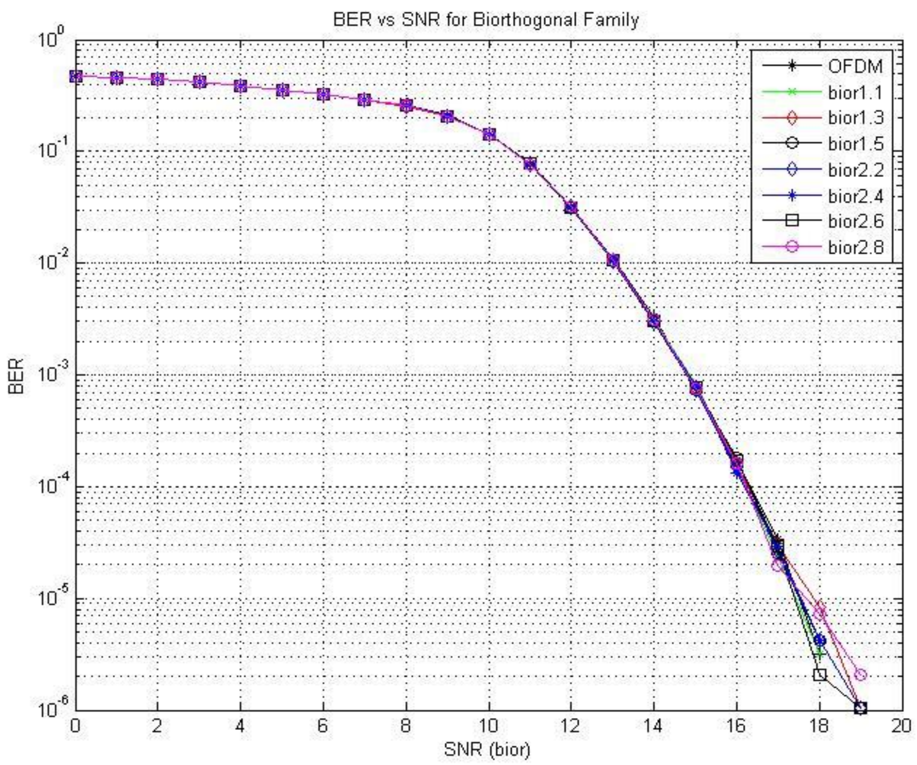

Figure 7.22: Bior1.1 to bior2.8 and OFDM BER vs SNR Performance (10 frames)

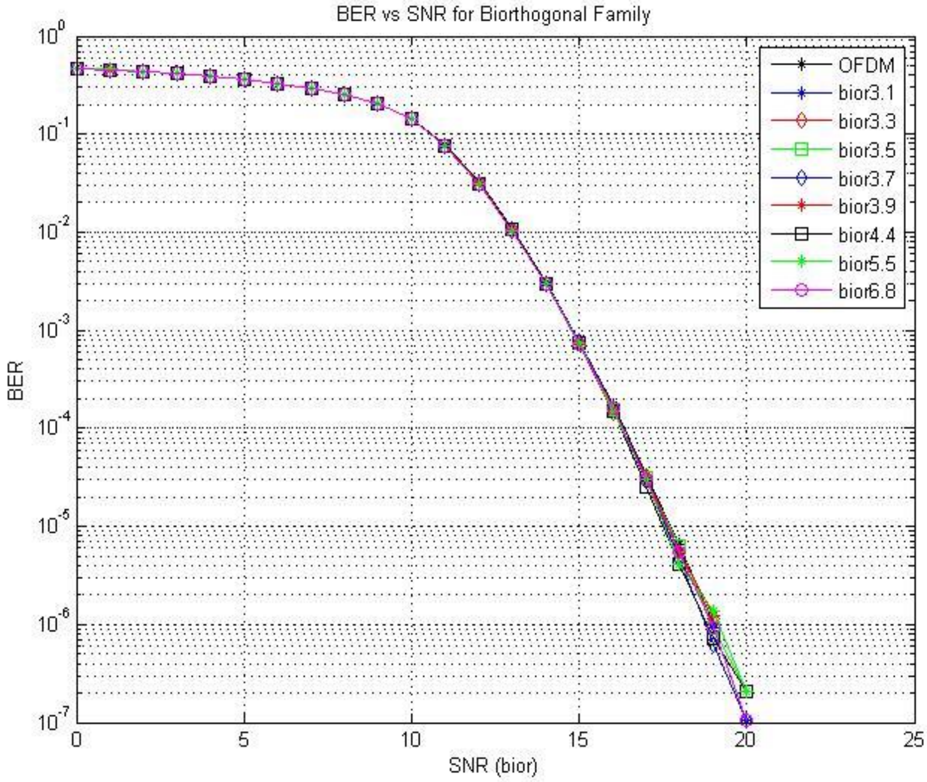

Figure 7.23: Bior3.1 to bior6.8 and OFDM BER vs SNR Performance (100k frames

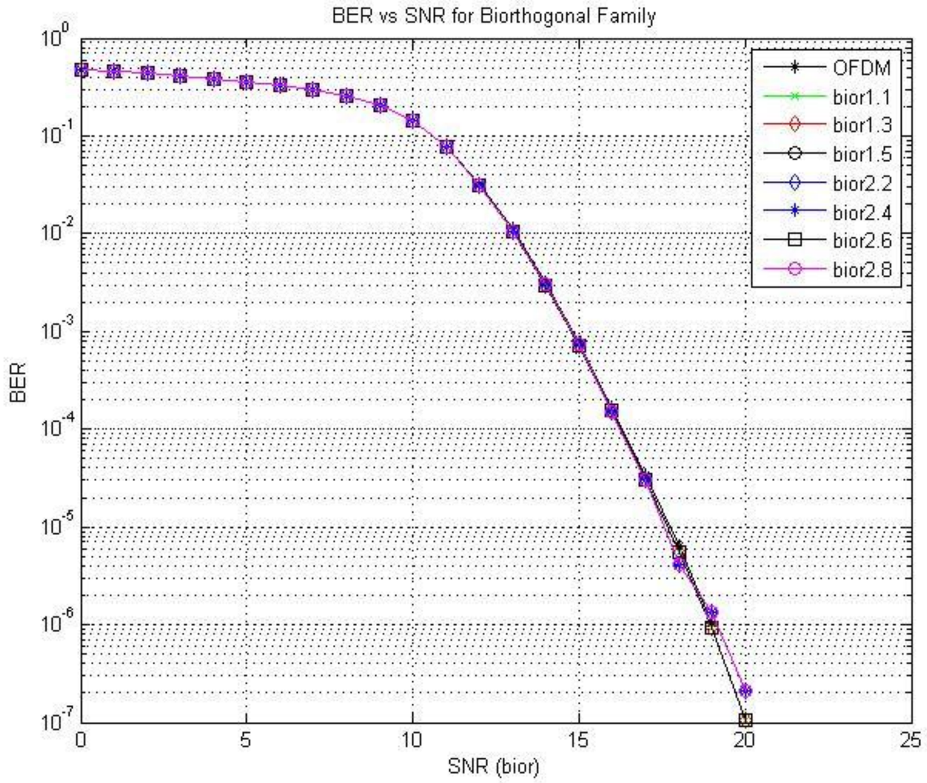

Figure 7.24: Bior1.1 to bior2.8 and OFDM BER vs SNR Performance (100k frames)

Further analysis of why the performance of different families and their associated members varies depends on the number of vanishing moments and family type will be discussed in the results and critical analysis chapter, Chapter 8.

7.3.1.4 REVERSE BIORTHOGONAL WAVELET FAMILY ANALYSIS

Reverse Biorthogonal family and its associated members are technically the same as Biorthogonal family; however, the reconstruction filters are reversed to be the decomposition filters instead and vice versa. Thus the naming pattern here is reversed, i.e. instead of bior*Nr.Nd,* here it is rbio*Nd.Nr*, where *Nd* is the decomposition filters order and *Nr* is the reconstruction filters order. The simulation experiments here have followed the same authentication procedure, as we start with data rate as low as 100 frames then increase it three times each time by a factor of x10 to better understand the effect in performance due to longer data streams and also to measure the time response, in order to indicate the time complexity and compare it later with those of other families' members and also with OFDM. Likewise in this family, the experiment started with low data rate of just 100 frames, thereafter the data rate increased dramatically to a factor of x10 three times and up to 100k frames.

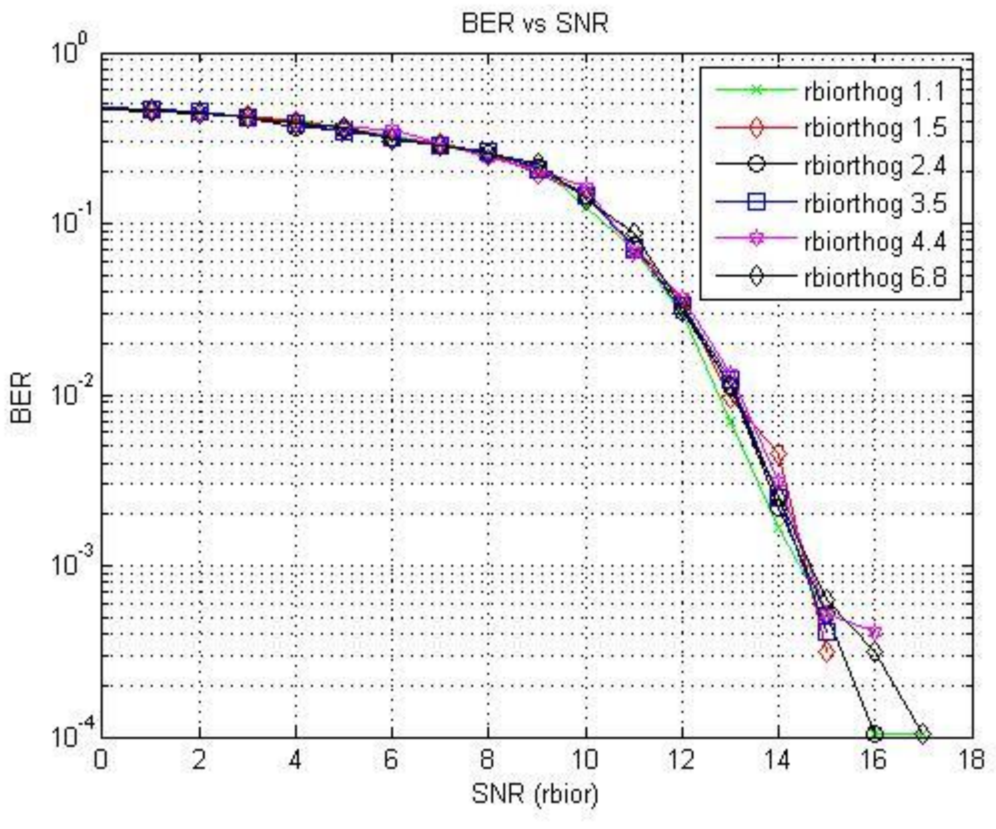

Figure 7.25: Rbiorthogonals BER vs SNR Performance (100 frames)

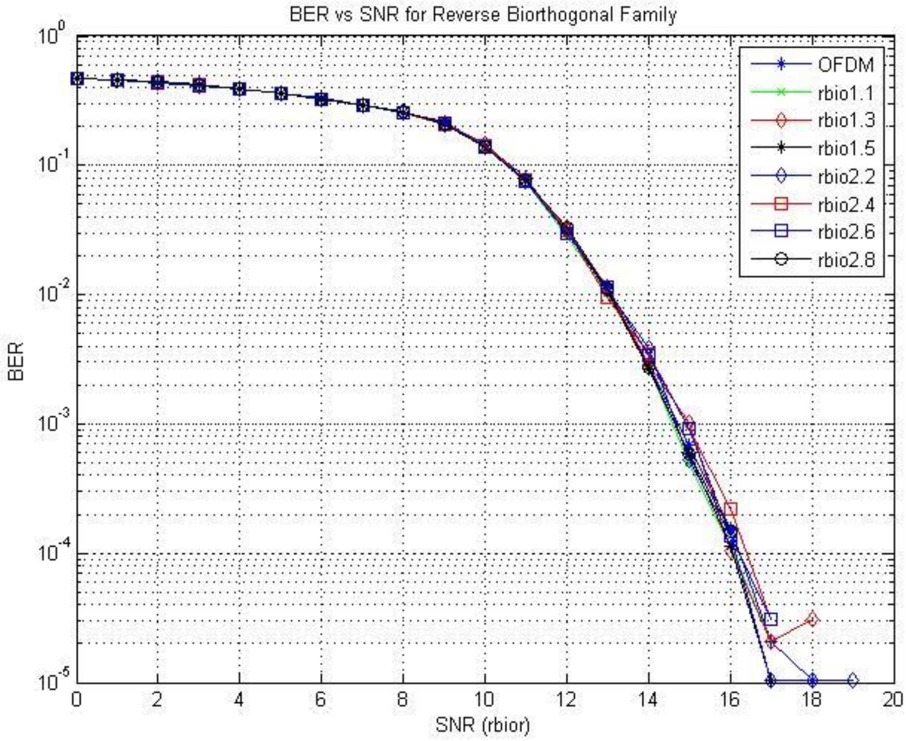

Figure 7.26: Rbio1.1 to rbio2.8 and OFDM BER vs SNR Performance (1k frames)

Figure 7.25 shows the performance of selected Reverse Biorthogonals members where we can see the similarity in performance with some achieving less errors at lower SNR but higher at higher SNR and the others vice versa. The next step is to increase the data rate by a factor of x10 to 1k frames not just for selected members but for all members of this family.

From Figure 7.26 and Figure 7.27, we can see that the performance of all the members in Figure 7.25 stayed stable and achieved similar errors rates. Compared to OFDM, most members performed very similarly to OFDM; however, at higher SNR, rbio3.5 and rbio6.8 were found to achieve lesser errors than OFDM and also the other members. Further increase of data rate by a further factor of x10 is performed and the results are shown in Figure 7.28 and Figure 7.30. From those two figures, we can clearly see that all the members continue to perform as they did with lower data rate, so it is evident that there will not be irregularity in performance even for higher data rates. To compute the response times for these members, a further increase in the data rate is performed by a further x10 factor to reach 100k frames = 9600 kbits, equivalent to 6 minutes HD video. All the results are shown in Figure 7.29 and Figure 7.31. Further discussion and critical analysis of time and computational complexity of not just this family's members, but also all other families' members discussed previously, will be presented later in this chapter in Section 7.4 and will also be revisited in the next chapter, Chapter 8.

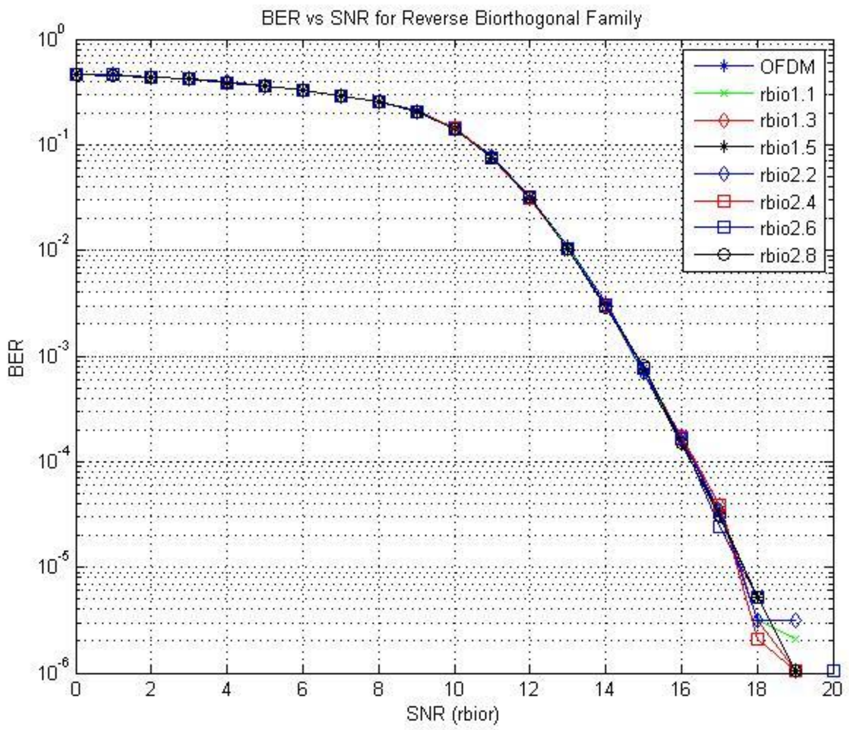

Figure 7.28: Rbio1.1 to rbio2.8 and OFDM BER vs SNR Performance (10k frames)

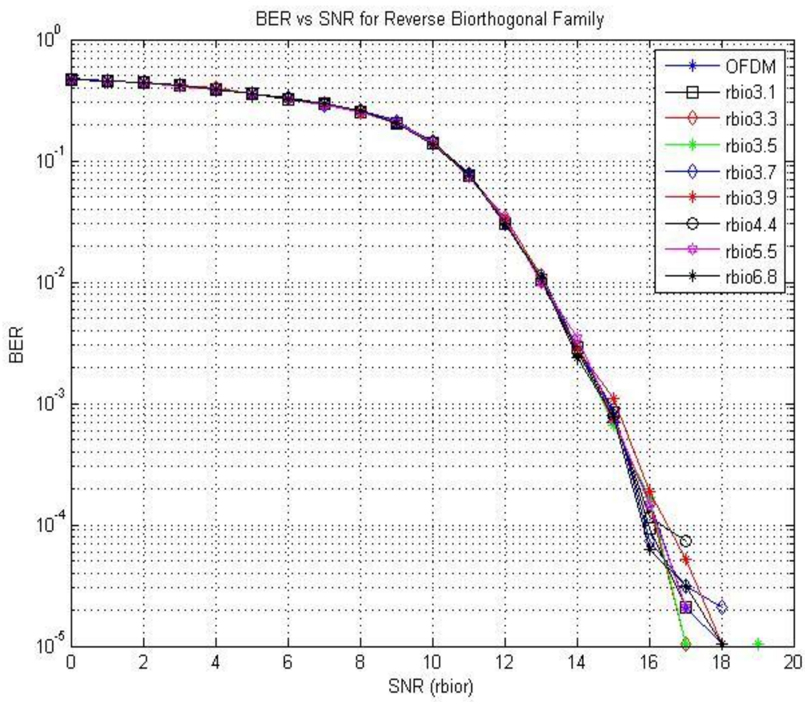

Figure 7.27: Rbio3.1 to rbio6.8 and OFDM BER vs SNR Performance (1k frames)

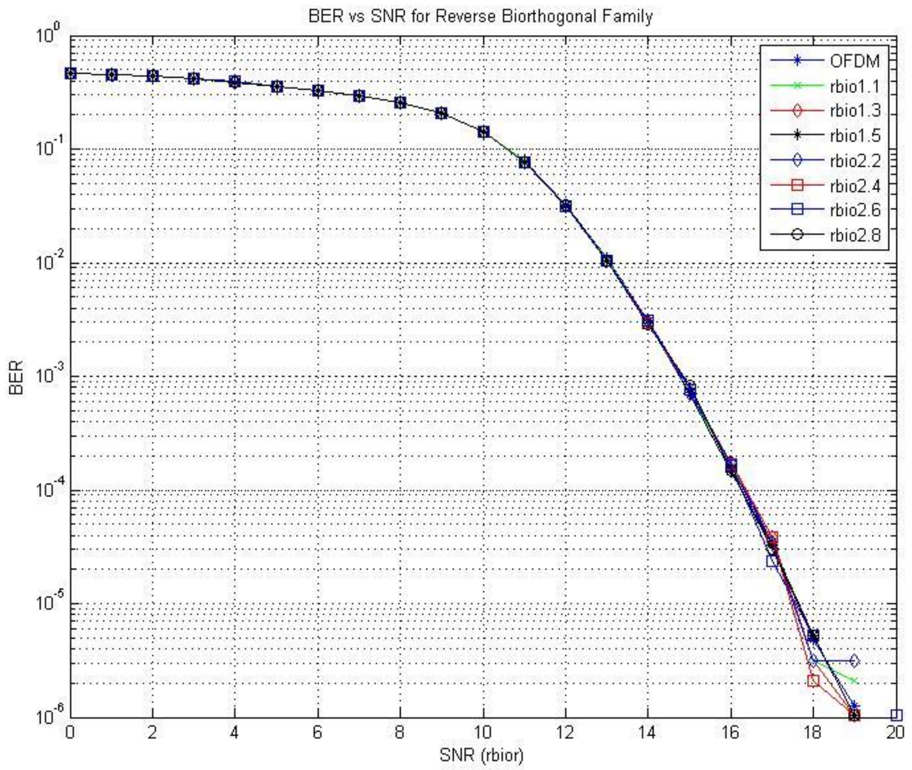

Figure 7.29: Rbio1.1 to rbio6.8 and OFDM BER vs SNR Performance (100k frames)

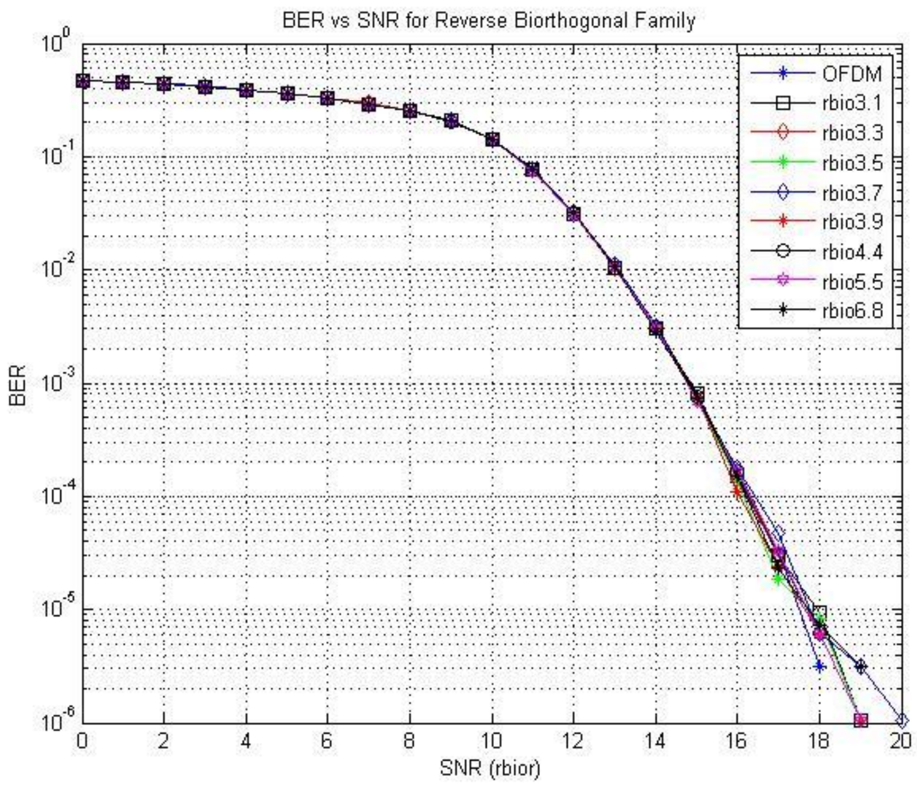

Figure 7.30: Rbio1.1 to rbio2.8 and OFDM BER vs SNR Performance (10k frames)

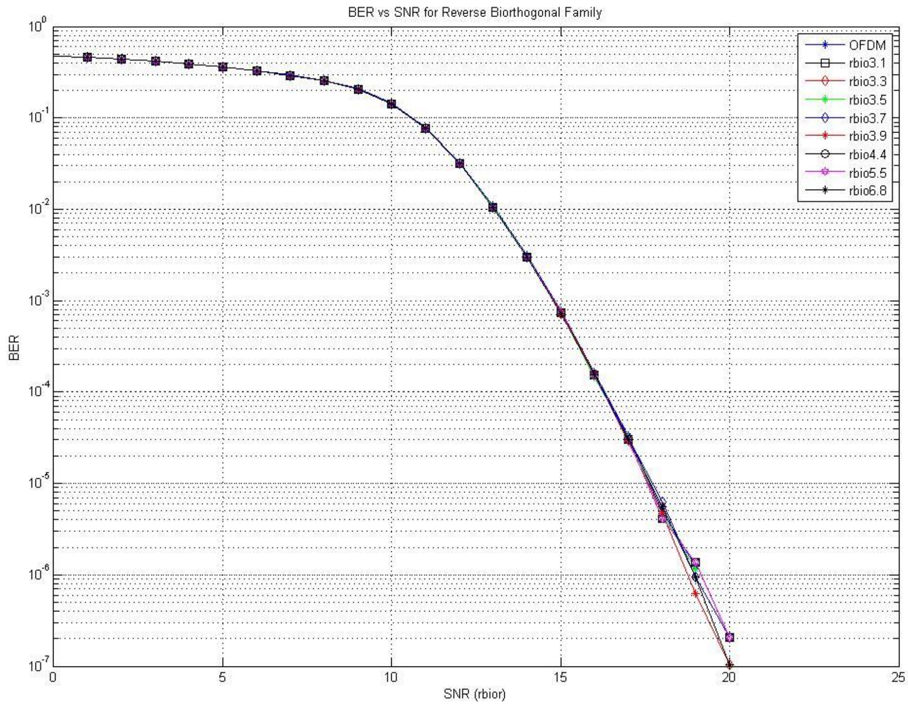

Figure 7.31: Rbio3.1 to rbio6.8 and OFDM BER vs SNR Performance (100k frames)

7.3.1.5 DISCRETE MEYER WAVELET ANALYSIS

As discussed previously, Discrete Meyers (dmey) is the only family that is orthogonal, biorthogonal and symmetrical. In this section, the results of dmey are compared to OFDM and also to Haar, as they are both orthogonal and symmetrical. However, before presenting the results, we will illustrate the symmetry feature of dmey, as seen in Figure 7.32 for all four filters of decomposition and reconstruction. Since the dmey family has just one member, the results for 1k, 10k and 100k frames will be shown on the same figure. Figure 7.34 shows the results of dmey and OFDM for 1k, 10k and 100k frames. From Figure 7.34, the performance of dmey is very competitive compared to OFDM for all tested data rates. However, the time response for dmey is considerably longer than OFDM during the processing of each of those data rates, 1k, 10k and 100k frames. The longer the data rate being processed, the longer the computation time required. The measured response time for dmey is a factor of around x1.8 of the times consumed by OFDM to process the same data rate. This is clearly evident from Figure 7.32 which shows the many filter taps that dmey have, so that the computational complexity increase due to the multiple multiplications and additions that have to be carried out throughout the processing.

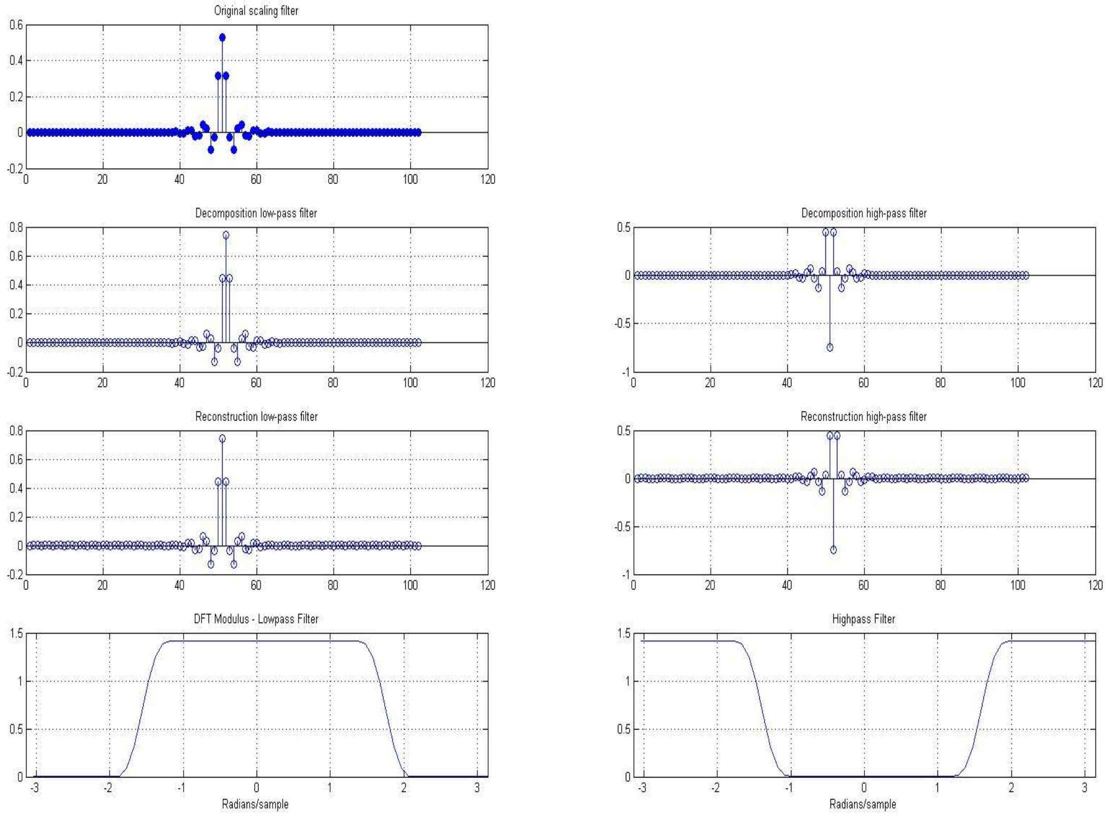

Figure 7.32: Dmey Decomposition and Reconstruction Filters Symmetry

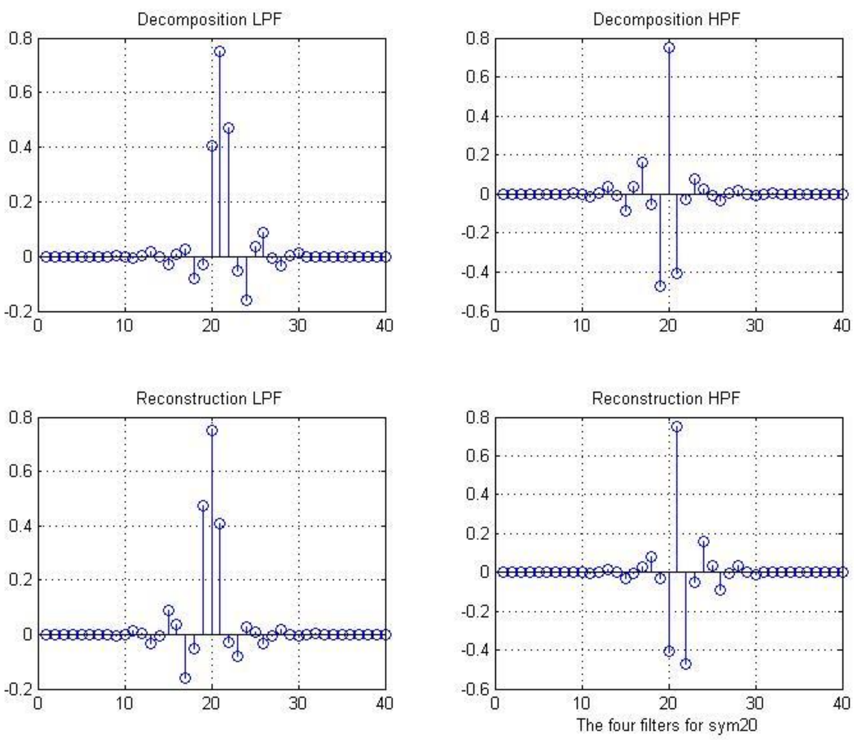

Figure 7.33: The four filters of Sym20

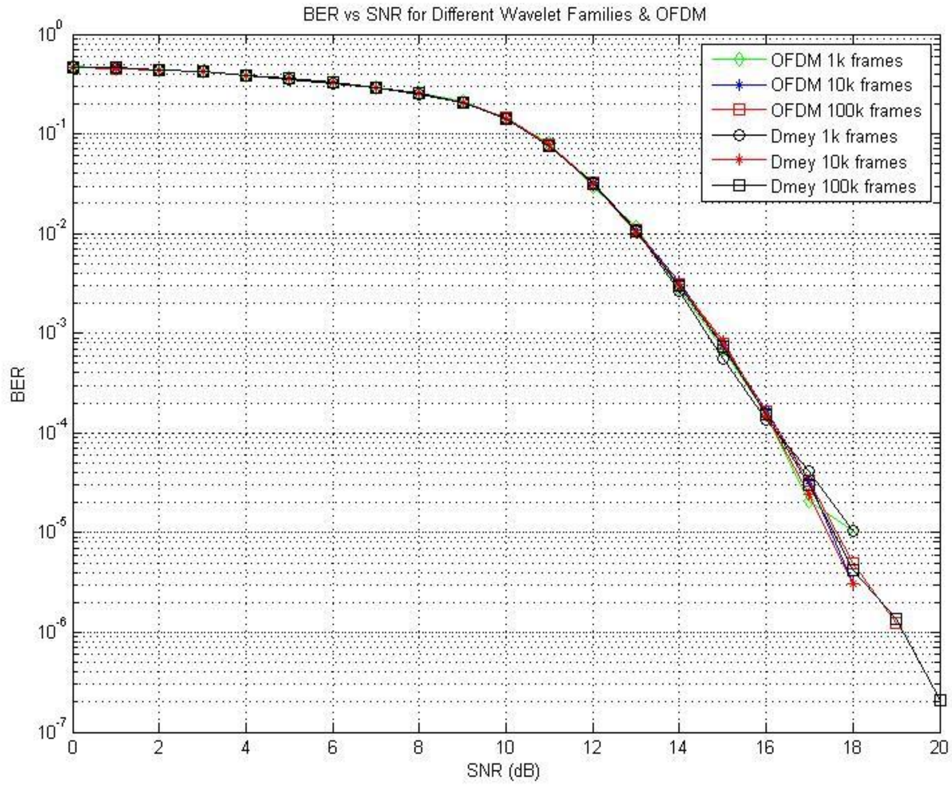

Figure 7.34: Dmey vs OFDM BER vs SNR Performance for 1k, 10k and 100k frames

7.3.1.6 SYMLET WAVELT FAMILY ANALYSIS

As discussed in Chapter 5, Section 5.4.3, the symlet family is a derivative of Daubechies family but with improved symmetry and it follows the same naming pattern and numbers of vanishing moments. Before presenting the simulation results for this family, let us see the filters taps and how it is closer to symmetry than the original Daubechies family. Figure 7.33 shows filters taps of sym20. The first data rate used to examine the members' behaviour was just 100 frames, where results are shown in Figure 7.35. The Symlet family is very close to Daubechies in performance with 100k frames but with very minor differences in some members in lower data rates, especially at 1k frames and 10k frames. Therefore, the data rates examined here are for 1k frames and 10k frames where the results are presented in Figure 7.36 to Figure 7.40.

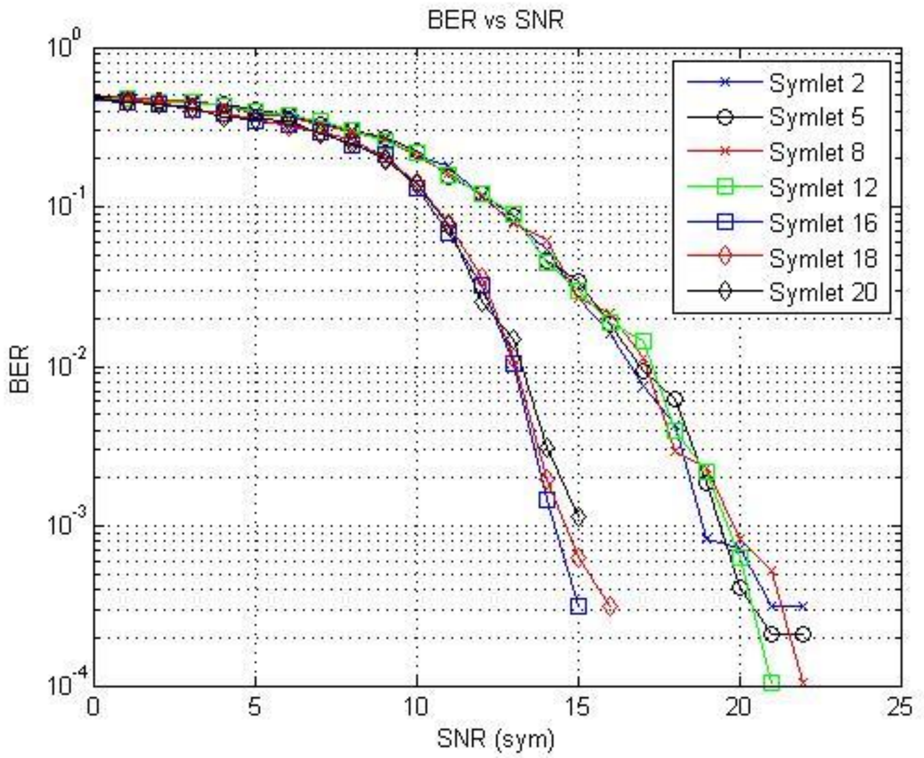

Figure 7.35: Selected Symlets members BER vs SNR Performance (100 frames)

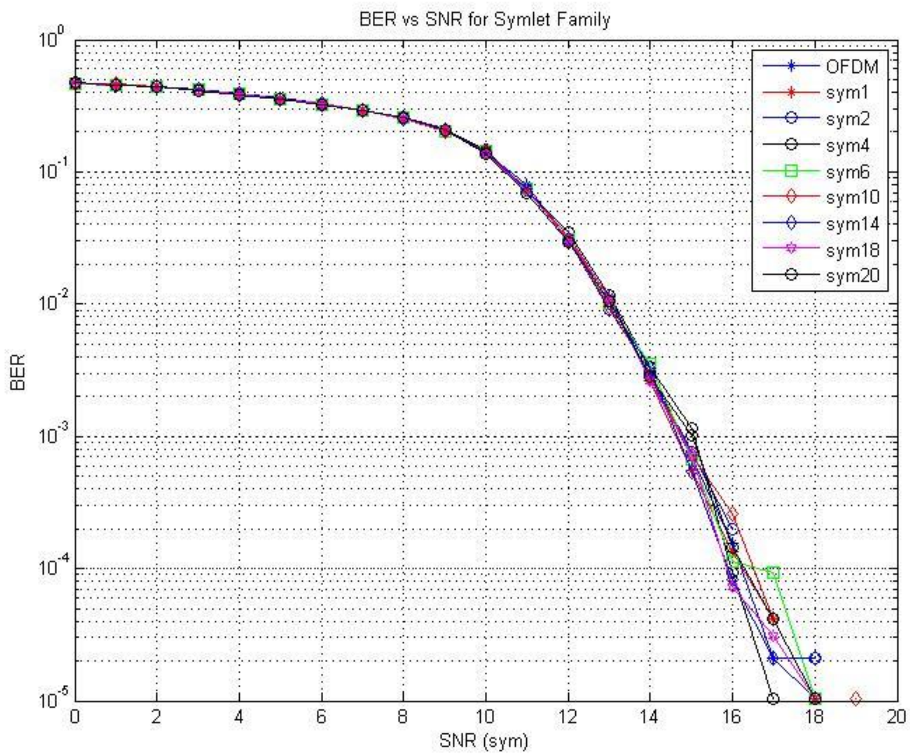

Figure 7.36: Sym1-sym20 and OFDM BER vs SNR Performance (1k frames)

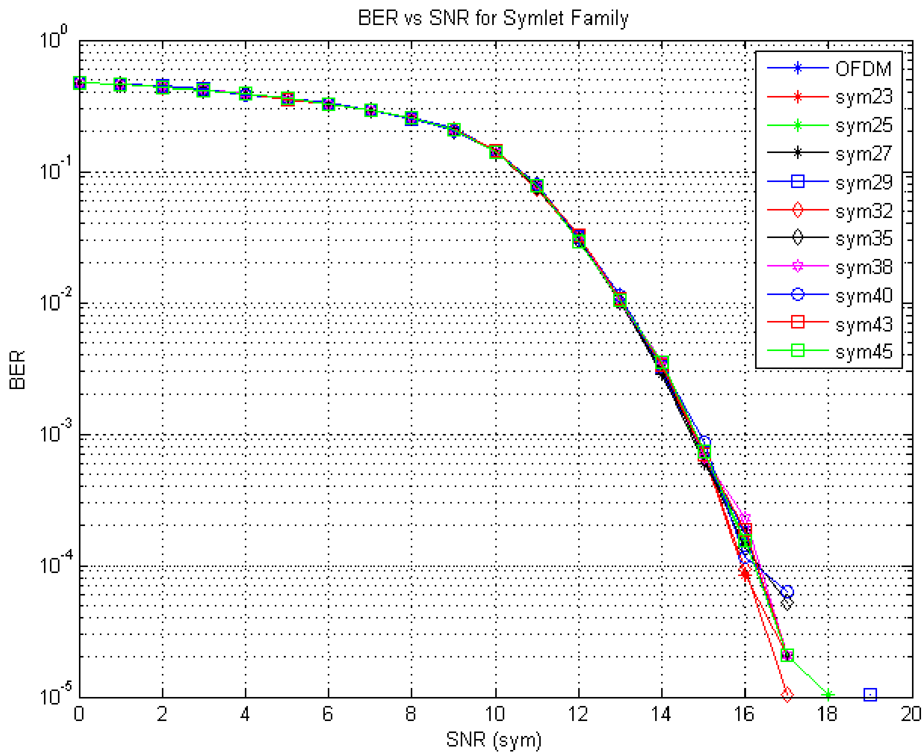

Figure 7.37: Sym23-sym45 and OFDM BER vs SNR Performance (1k frames)

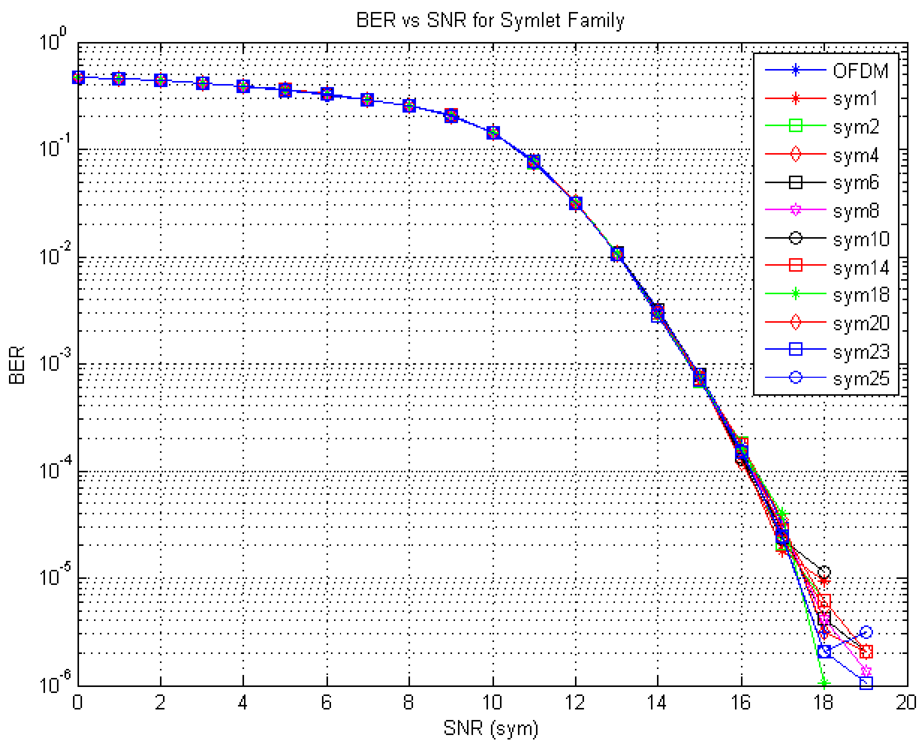

Figure 7.38: Sym27-sym45 and OFDM BER vs SNR Performance (10k frames)

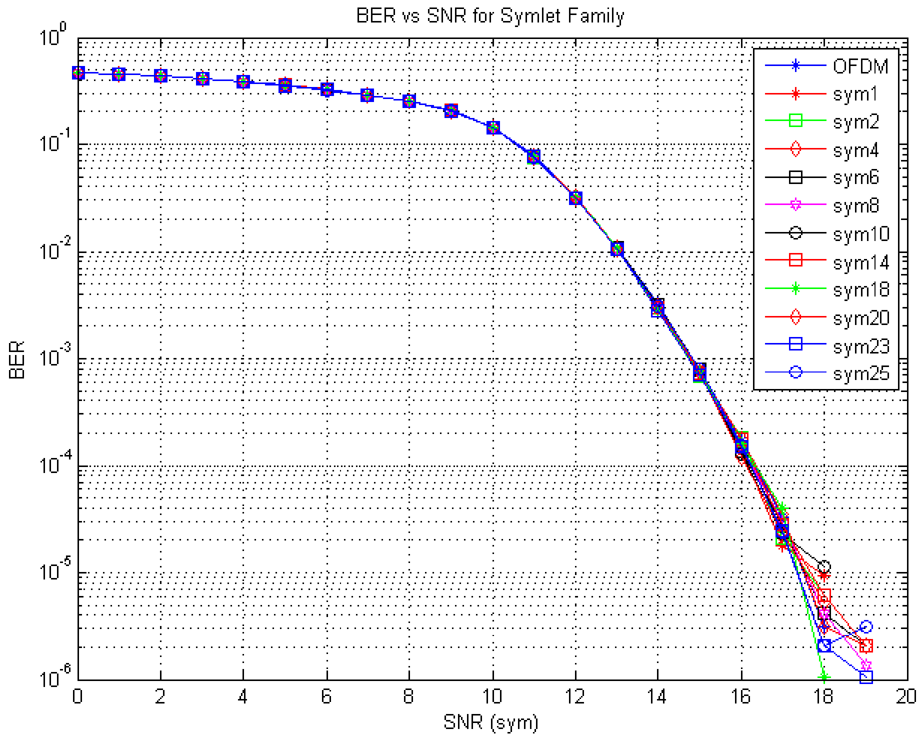

Figure 7.39: Sym1-sym25 and OFDM BER vs SNR Performance (10k frames)

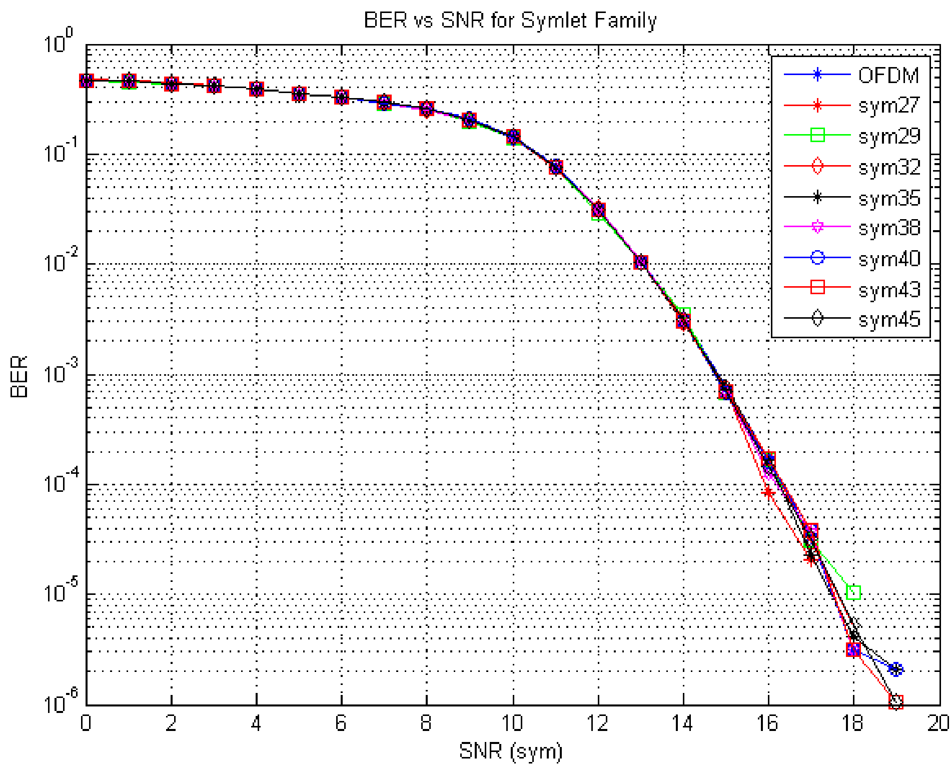

Figure 7.40: Sym1-sym25 and OFDM BER vs SNR Performance (10k frames)

7.3.1.7 PERFORMANCE OF 16-QAM VS 64-QAM OVER AWGN CHANNEL FOR OWDM AND OFDM

In this section, the performance of two of the most commonly used data mappers are compared head to head over AWGN channel. The same system parameters as prescribed by DVB-T standard [19] are used for both systems, however, the filter bank level used here is 11 levels, with only Haar wavelet. Haar is the simplest wavelet family, having only single vanishing moment, just two coefficients and with the added advantage that they are symmetrical, which makes them rapidly responsive. Figure 7.42 shows the performance of 64-QAM OFDM and OWDM, whereas Figure 7.41 shows the performance of 16-QAM for both systems.

From Figure 7.42 with 64QAM, it is clearly visible that the performance of BER vs SNR for OWDM is competitive and very similar to that of OFDM across almost all SNR regions. However, at SNR 24 dB, OFDM achieved less error rates as opposed to the higher SNR of 26 dB, where OWDM achieved less error rates than that of OFDM at the same SNR value. In Figure 7.41 with 16-QAM, the performance of OWDM is still very similar to that of OFDM, with better performance of OWDM than OFDM at higher SNR values. In that figure, OWDM achieved lower error rates at SNR of 16, 17 and 19 dBs, whereas OFDM achieved lower error rate at only 18dB.

It is concluded from the above that OWDM with Haar wavelet very likely produces a very strong candidate to replace IFFT in OFDM systems. They are similar in performance, which was slightly even better for OWDM, with the added advantage for OWDM that it does not need guard intervals, which saves up to 25% of the bandwidth, giving a very promising potential that one day we could see OWDM standardised for industrial utilisation. Apart from that, there is also the advantage that by using the asymmetrical filter banks, dynamic sub-bands allocation is possible and hence, will result in more bandwidth saving. More about this will be included in the results analysis and conclusion chapters, Chapter 8 and Chapter 9.

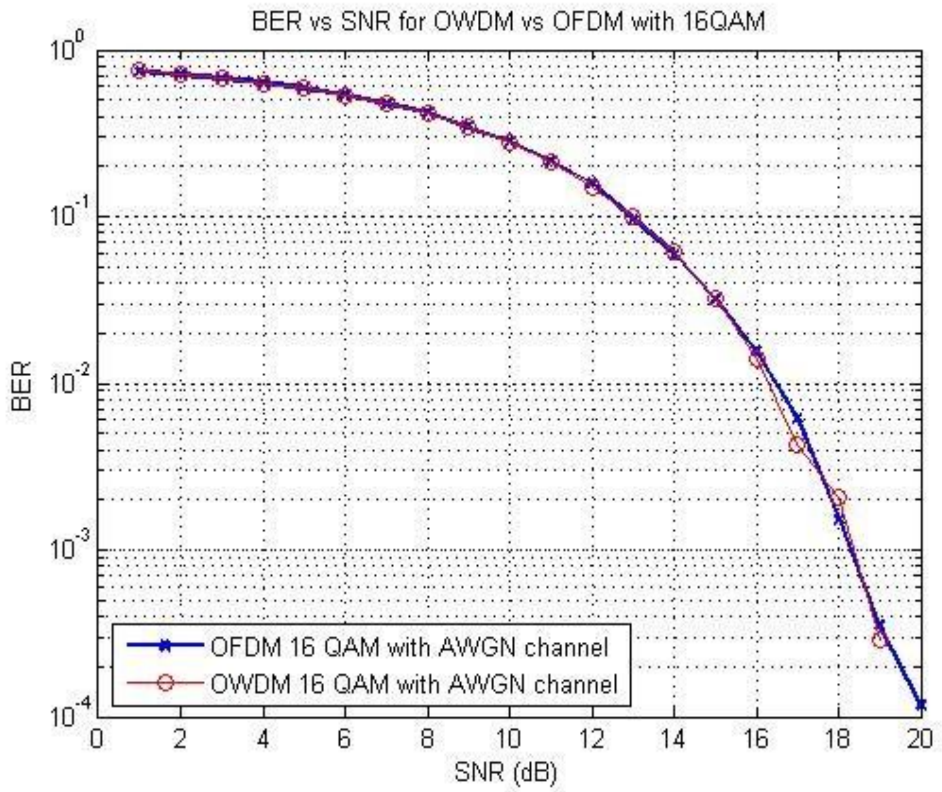

Figure 7.41: 16-QAM OFDM vs OWDM over AWGN

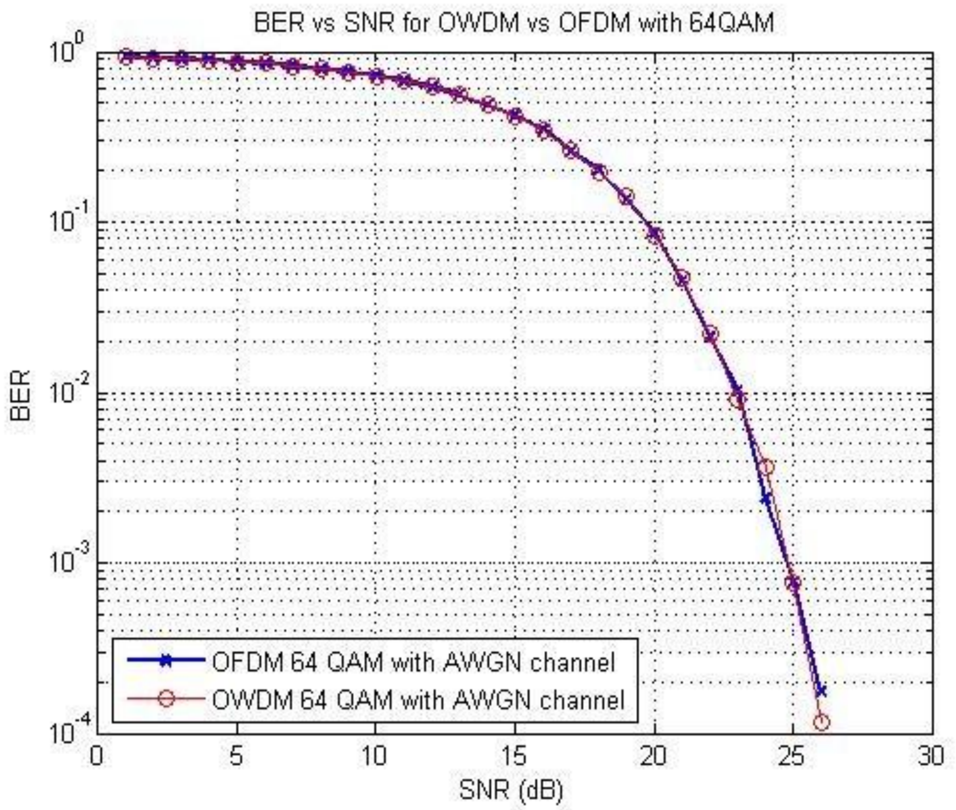

Figure 7.42: 64QAM OFDM vs OWDM over AWGN

7.3.2 OWDM OVER MULTIPATH CAHNNEL (RICEAN FADING)

In this section OWDM and OFDM are simulated over multipath fading channel using Ricean model which is effectively the reflected path where the transmitter cannot see the received. The signals takes different multipath to reach the receiver, where it arrive at different time through different paths. This model always suffer from multipath scattering effects and time dispersion. In this section both systems OFDM and OWDM are implemented using a slightly different parameters than previously mentioned to test various conditions and real world scenarios phenomena.

7.3.2.1 BBC, ITV AND CHANNEL 3&4 MULTIPLEXERS

In this subsection one of the most used multiplexers, parameters, of DVB-T industrial standard are simulated and tested to further validate the proposed system in this research. The results of OFDM will be presented here alongside OWDM, however, the discussion here will be just about OFDM, whereas in the OWDM section the discussion will be about the performance of OWDM for the same set of parameters. It will then both jointly discussed and analysed in the comparison section before the end of this chapter and will also be revisited in the next chapter, Chapter 8. The set of parameters used in this specific simulation is summarised in Figure 7.3. Due to the nature of multipath channel latency, the simplest wavelet family, Haar, is selected which has a single vanishing moment with 2 symmetrical filter's taps.

Table 7.3: Simulation Parameters for BBC, ITV and Channel 3&4

Parameter	Value
OFDM Size	2K
OFDM Active Subcarriers	1705
OWDM Decomposition levels	11
Multiplex 1 Parameters	16-QAM, 3/4 rate
Multiplex 2 Parameters	64-QAM, 2/3 rate
RS Encoding	RS(204,188,16)
SNR range	0-30dB
Number of frames	1000
Number of multipath in Ricean	2

In this work, two UK multiplexer values are used as test scenarios:

- The BBC owned multiplex (Multiplex 1).
- The ITV and Digital 3&4 owned multiplex (Multiplex 2).

These multiplexes have parameters of 16-QAM, 3/4 rate and 64-QAM, 2/3 rate respectively. These two multiplex parameter sets were used in a commercial system until the 2012 UK digital switchover. MUX 1 is the pre-switchover multiplexing for British Broadcasting Corporation (BBC) and British Telecommunications Company (Arqiva). Whereas, MUX 2 is the pre-switchover multiplexing for ITV corporation and Digital 3&4 (Channel 4 consortium).

The experiment involved increasing the SNR of the channel, comparing to a measure of BER taken at the output of the RS decoder. In order to ensure compliance with the standard, the SNR which gave the Quasi Error Free (QEF) state (a BER 10-11 at the output of the RS Decoder) was evaluated and compared to the values provided in the DVB-T standards (which gives all SNRs at QEF state for all combinations of QAM and Inner coding rate).

Once the simulation was compliance validated according to the standards, the IFFT/FFT blocks were replaced and the experiment repeated using the system discussed earlier in Chapter 6, Section 6.4.1.4 in which the IFFT and FFT are replaced with the Wavelet Synthesis Filter and Decomposition filters respectively.

Each system, OFDM and OWDM, was repeated with 4 scenarios:

1. Multiplex 1 (MUX 1) – AWGN Channel.
2. Multiplex 2 (MUX 2) – AWGN Channel.
3. Multiplex 1 (MUX 1) – Ricean Channel.
4. Multiplex 2 (MUX 2) – Ricean Channel.

Figure 7.43 shows the SNR-BER curves for OFDM under AWGN while Figure 7.45 shows the SNR-BER curves for OWDM under the same conditions. From these two plots, it can be seen that there is a similarity between the two curves. From Figure 7.43, it can be seen that the SNR which gives a BER of 10^{-4} is 14.7dB and 20.9dB for MUX 1 and MUX 2 respectively. Comparing these results to the values for OWDM in Figure 7.45 of 15dB and 20.8dB respectively shows that both systems offer a similar performance under AWGN.

Figure 7.44 and Figure 7.46 show the SNR-BER curves for OFDM and OWDM in a Ricean fading channel respectively. In OFDM, a BER of 10^{-4} is achieved with an SNR of 19dB and 24dB for MUX 1 and MUX 2 respectively. Comparing this to OWDM as shown in Figure 7.45, the same BER is achieved through an SNR of 24dB and 26dB respectively. It can, therefore, be seen by comparing these results, OWDM has a small advantage over OFDM in terms of BER vs SNR. However, taking into account the advantage of guard interval elimination this small difference in BER comes with value of more bandwidth saving in OWDM adding to that the dynamic allocation of bandwidth adds in to the saving.

In conclusion to this part of the research, it has provided a comparison between OFDM and OWDM as applied to two pre-2012 UK digital television multiplexes. The purpose of this research part is to demonstrate that OWDM has the potential of acting as a successor for OFDM because of some of the potential advantages it has over OFDM. The computational complexity of wavelet decomposition filters in OWDM is less than Fourier Transform in OFDM. There is also a potential exploitation of guard interval in OWDM instead of wasting it in OFDM.

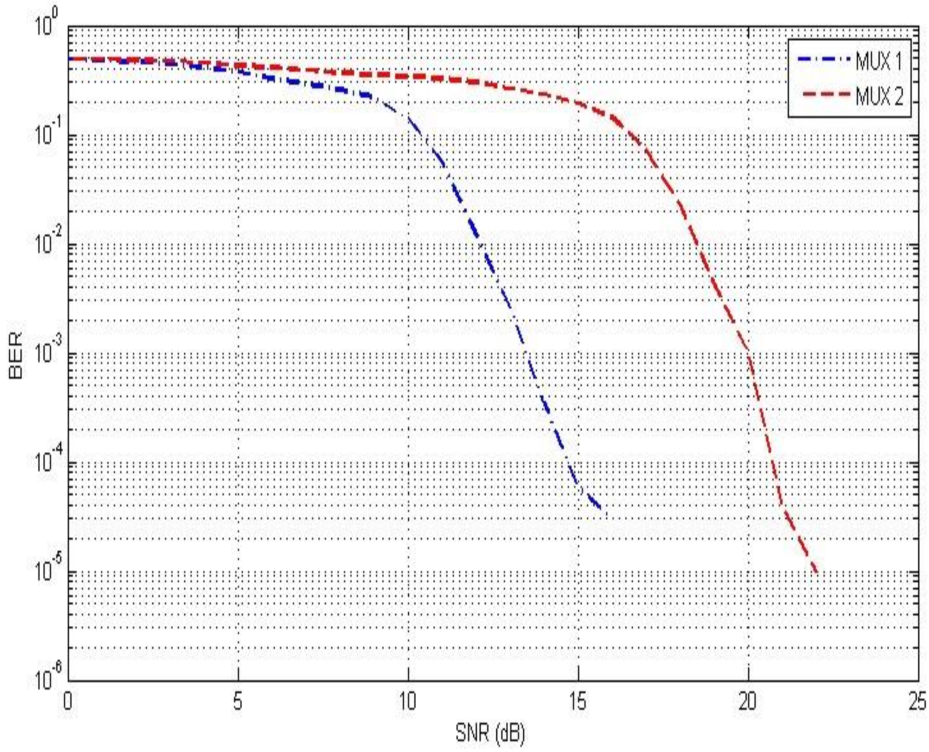

Figure 7.43: OFDM MUX 1 (BBC) and MUX 2 (ITV and Digital 3&4) Performance

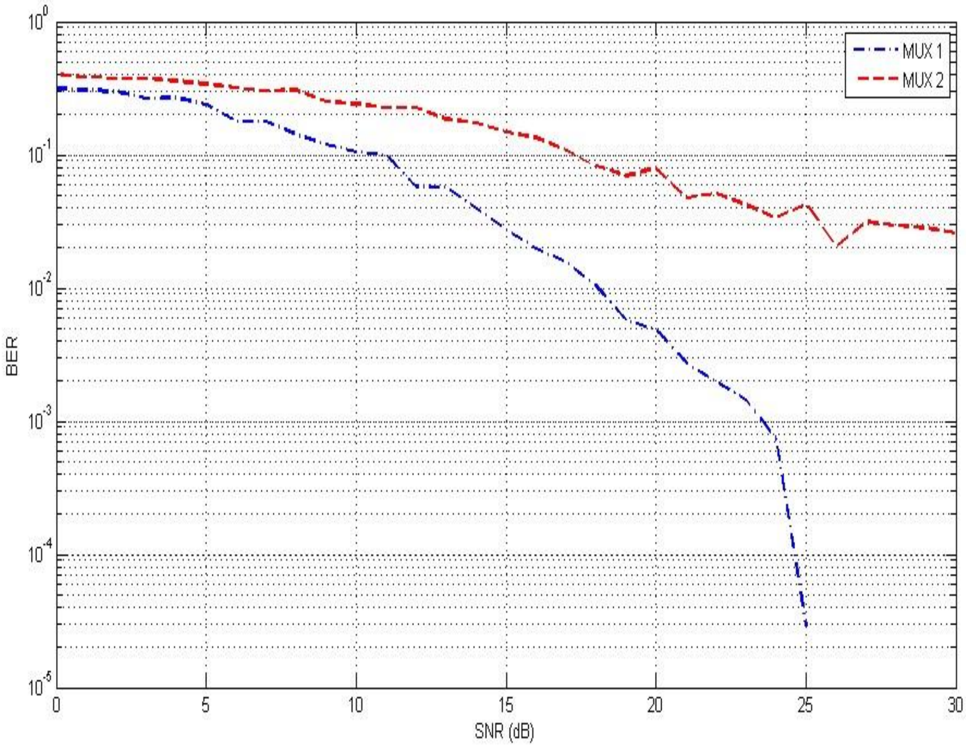

Figure 7.44: OWDM MUX 1 and MUX 2 over Ricean Fading Channel

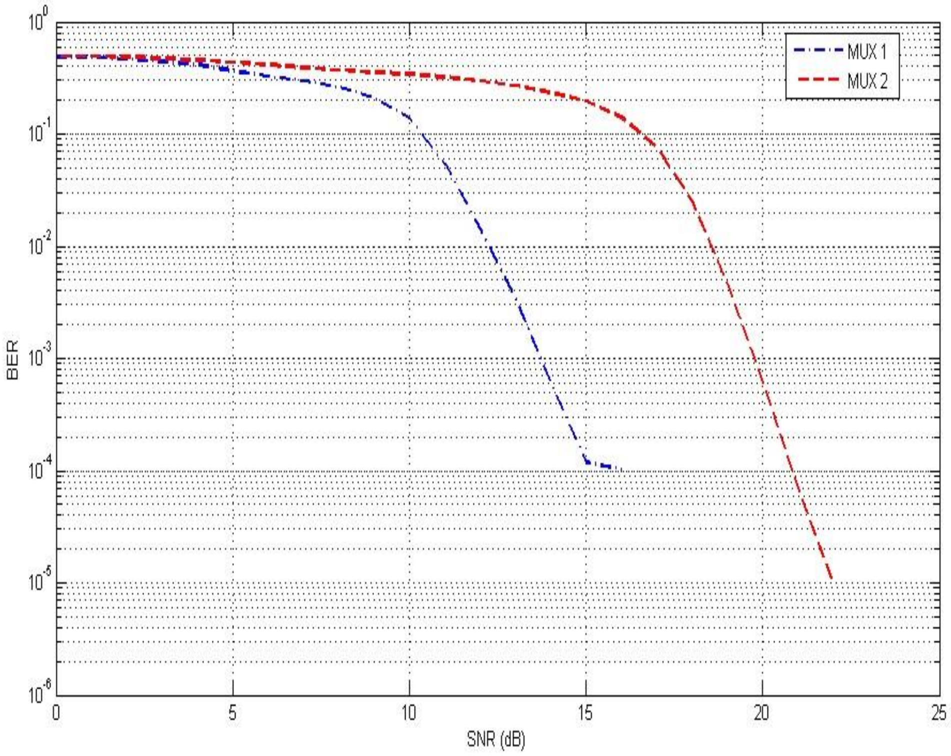

Figure 7.45: OWDM MUX 1 (BBC) and MUX 2 (ITV and Digital 3&4) Performance

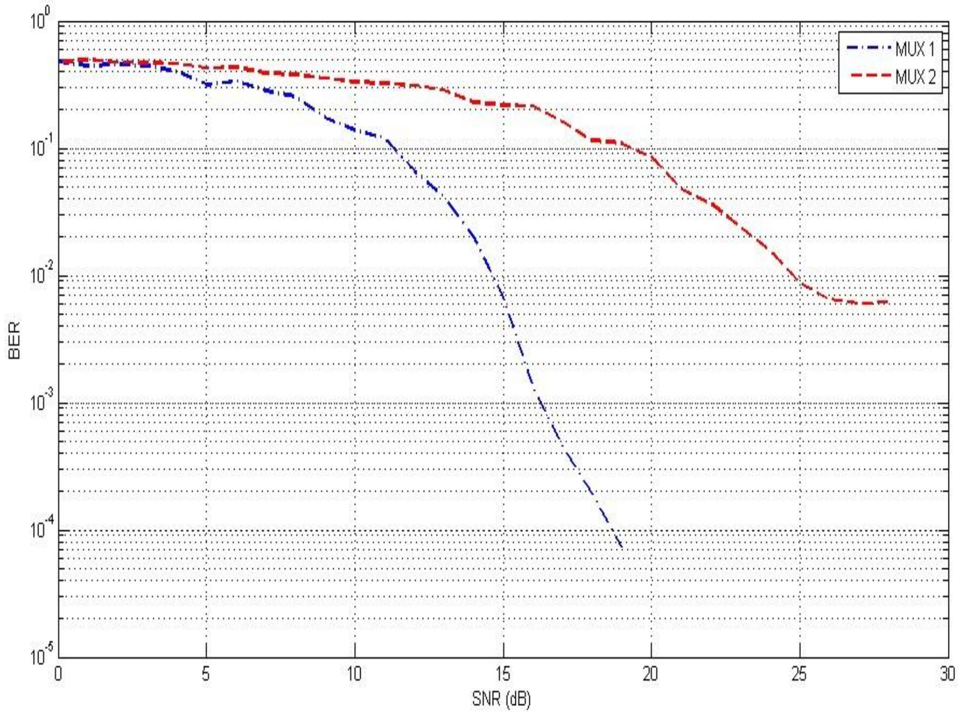

Figure 7.46: OFDM MUX 1 and MUX 2 over Ricean Fading Channel

Two scenarios have been presented in this paper which demonstrates that under AWGN, the performance of OWDM is matched to that of OFDM. Under a Ricean fading channel, however, the results have predicted a small decrease in performance when comparing OWDM to OFDM, as in Figure 7.45 and Figure 7.44. It is suspected that the cause for this is due to the guard interval insertion under OFDM which is used to eliminate multipath [22] but does not exist within the current OWDM model. Further equalization will be required to get a similar response.

7.3.3 OWDM OVER MULTIPATH CAHNNEL (RAYLEIGH FADING)

For more practicality of this research, the simulation is carried out for dynamic bandwidth allocation for different wavelet families and their associated members over a real world scenario wireless channel model. Rayleigh fading model is the most well-known channel model which has different channel variables and parameters depending on the application it is used for. In this research DVB-T Rayleigh model with 2 paths is taken into consideration with adaptive equalizer, Least Mean Squares (LMS), is used for both OFDM and OWDM systems. In the next subsections, wavelet families and their associated members are presented with some discussion, however, more analysis and discussion is presented in section 7.4 and also revisited in the next chapter, Chapter 9, with further analysis and justifications. All experiments are done for mobile wireless channel with different mobile terminals speeds, 65 km/h=50Hz, 130 km/h=100Hz and 260 km/h=200Hz as stated in the system design earlier in Chapter 6, Section 6.2. Those chosen speed simulate three most used forms of transports which are:

- Buses which travels inside cities usually with maximum 65 km/h (about 40 miles/h).
- Cars which travels at a national speed limit of 130 km/h (about 80 miles/h).
- High speed trains at an average speed of 260 km/h (about 161 miles/h).

Moreover, one set of data stream will be used over three different Doppler shifts. This chosen data set is based on the average data used in the previous experiments. Therefore, the data length to be used here is 10k frames = 960 kbits. This process is carried out on each family's member individually and then grouped in one figure to minimise the vast quantity of results and used space. In the next subsections, wavelet families are presented in separate subsection where all results of their associated members are presented and discussed. A further technical observation, discussion and result analysis will be presented in Section 7.4 and Chapter 8 respectively.

7.3.3.1 DAUBECHIES FAMILY

As said earlier that Daubechies 1 (db1) is Haar wavelet which was discussed earlier is Chapter 5, Section 5.4.2. There were three different experiments carried out for each member in this family. These experiments included processing 10k frames = 960 kbits over Rayleigh channel at Doppler shifts mentioned earlier, 50Hz, 100Hz and 200Hz for terminals speeds of 65 km/h, 130 km/h and 260 km/h consecutively. Results for 50Hz are shown in Figure 7.47 and Figure 7.48; results for 100Hz are shown in Figure 7.49 and Figure 7.50; and results for 200Hz are shown in Figure 7.51 and Figure 7.52 below.

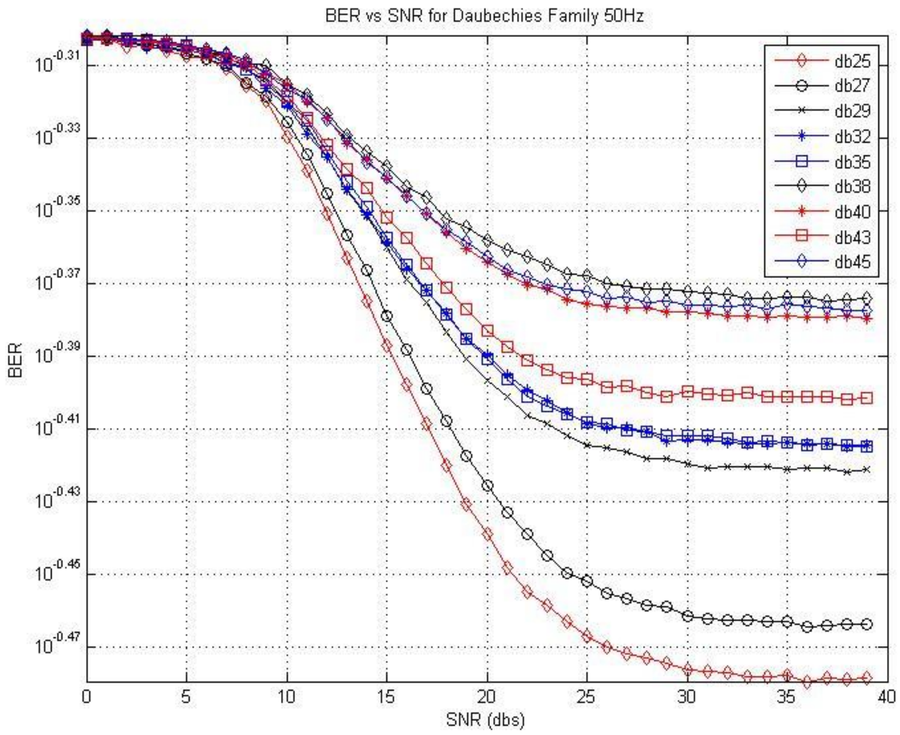

Figure 7.47: Daubechies 25 to db45 BER vs SNR over Rayleigh 50Hz

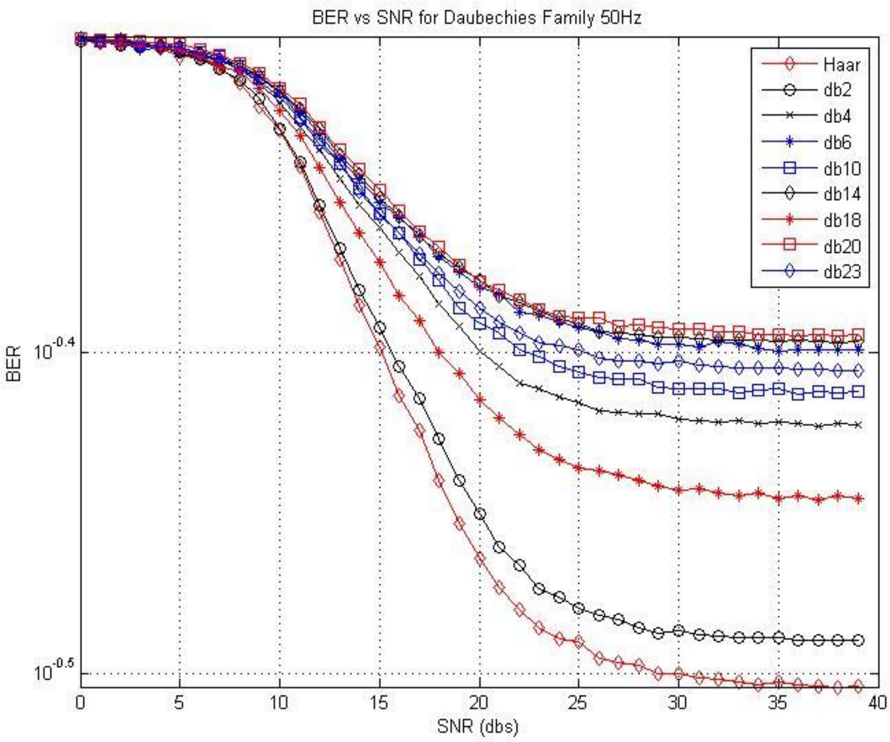

Figure 7.48: Daubechies 1 (Haar) to db23 over Rayleigh 50Hz

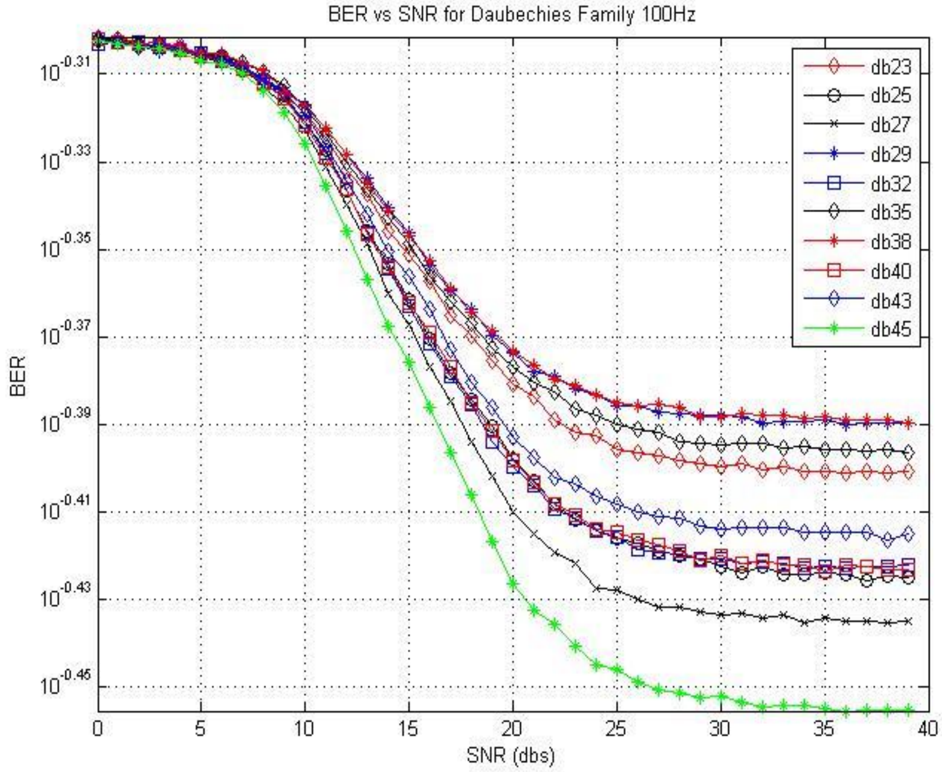

Figure 7.49: Daubechies 23 to db45 BER vs SNR over Rayleigh 100Hz

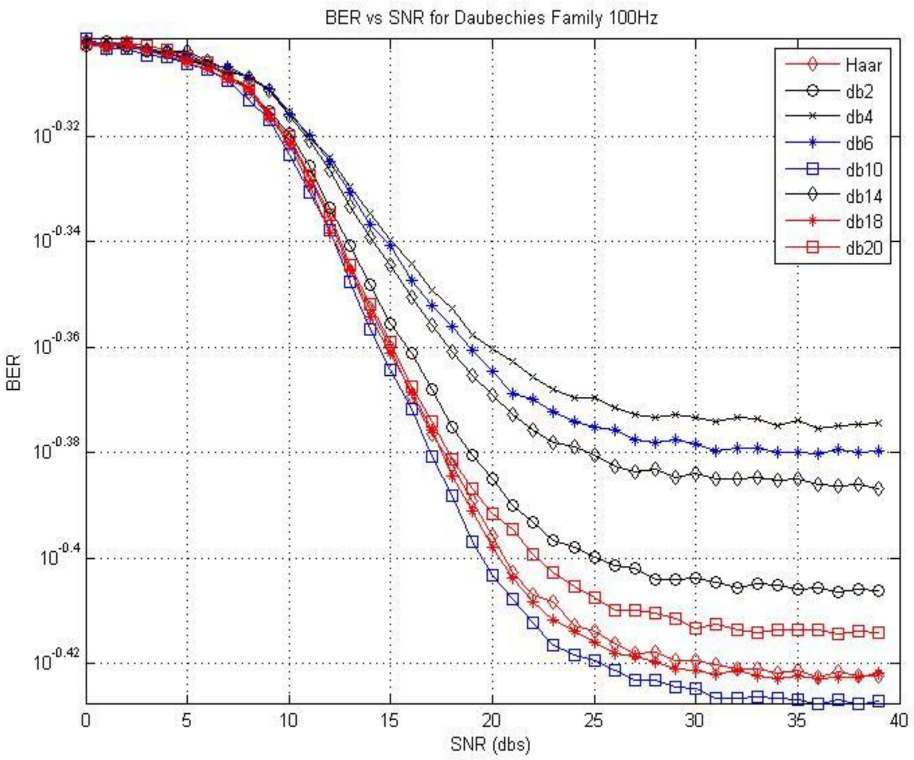

Figure 7.50: Daubechies 1 (Haar) to db20 BER vs SNR over Rayleigh 100Hz

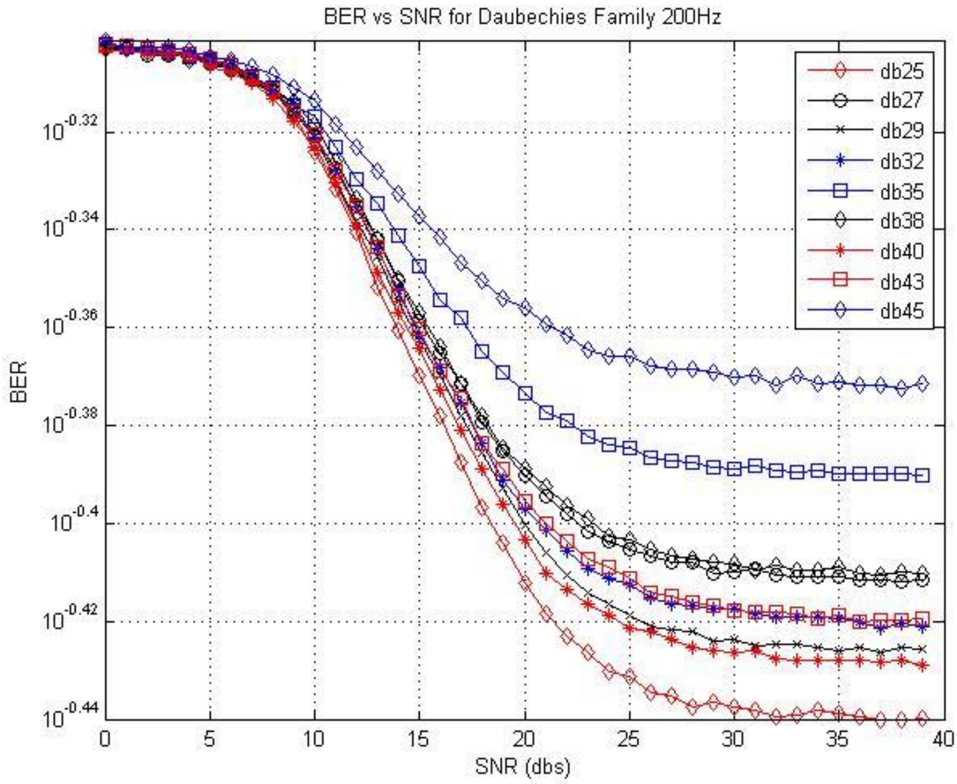

Figure 7.51: Daubechies 25 to db45 BER vs SNR over Rayleigh 200Hz

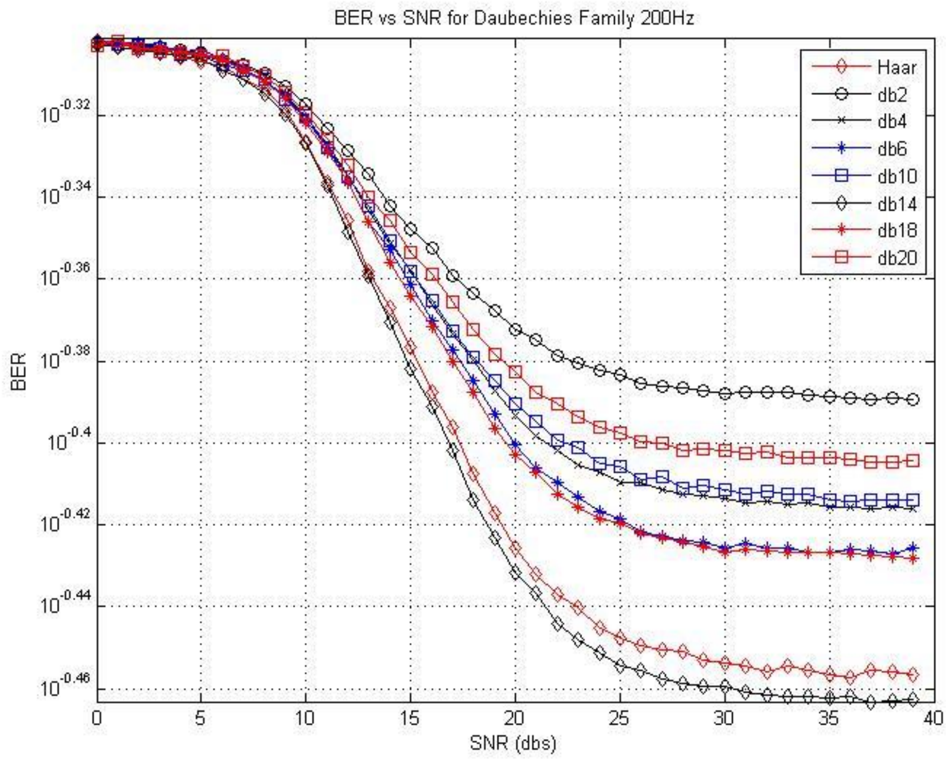

Figure 7.52: Daubechies 1 to db20 BER vs SNR over Rayleigh 200Hz

7.3.3.2 SYMLETS FAMILY

Symlets family was explained earlier in Chapter 5, Section 5.4.3. The filters' coefficients of this family is near symmetry and is a derivative of Daubechies family [69]. Likewise in the previous experiment, the data rate that will be processed in this experiment is 10k frames = 960 kbits and over the Rayleigh fading channel with three different Doppler effects, 50Hz, 100Hz and 200Hz. Each of those frequencies is experimented on each family member, nevertheless, for better visualisation of results and for space management the results shall be grouped in one figure at approximately 8 curves per figure. Figure 7.53 and Figure 7.54 show the results for symlet's members over 50Hz; Figure 7.55 to Figure 7.56 show the results over 100Hz and Figure 7.57 to Figure 7.58 show the results over 200Hz.

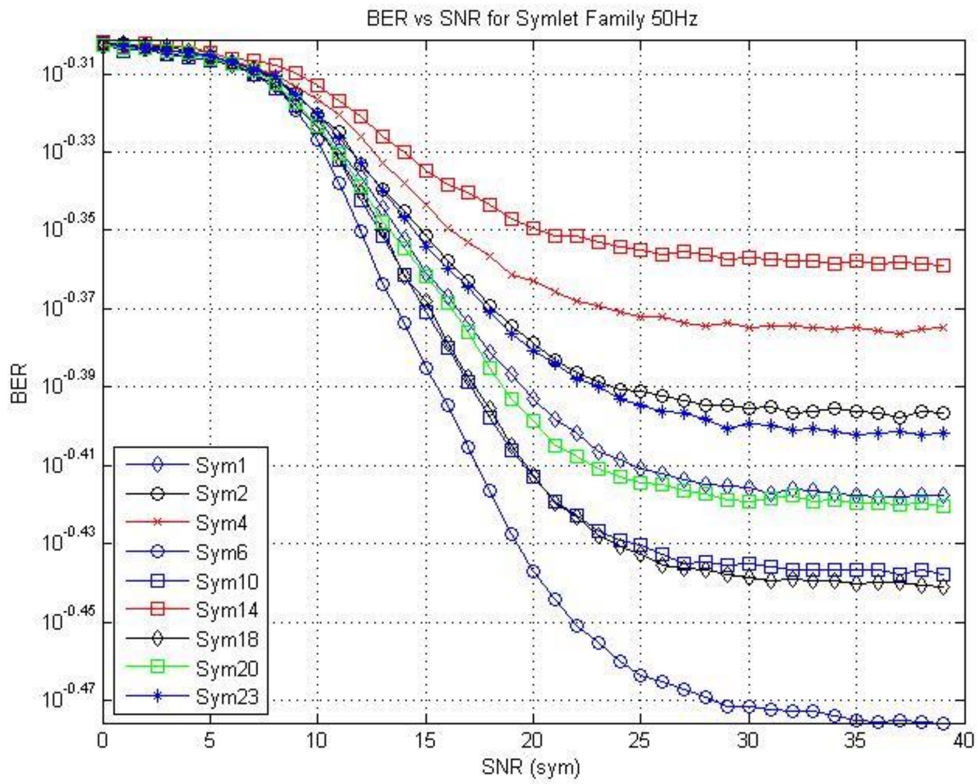

Figure 7.53: Symlets 1 to sym23 BER vs SNR 10k over Rayleigh 50Hz

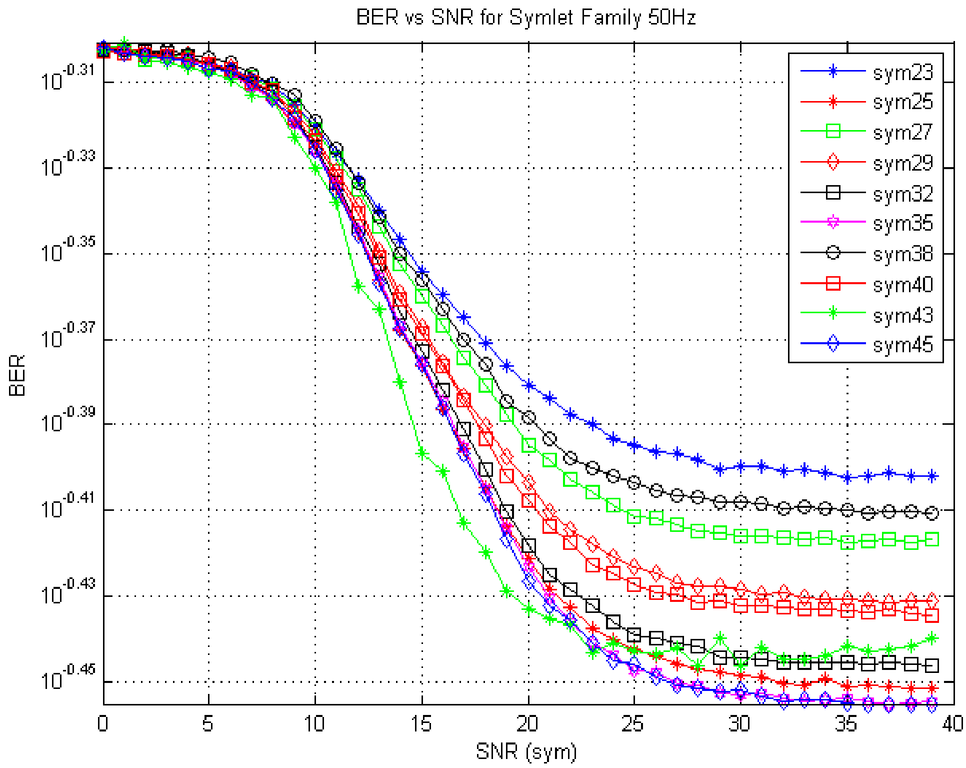

Figure 7.54: Symlets 23 to sym45 BER vs SNR 10k over Rayleigh 50Hz

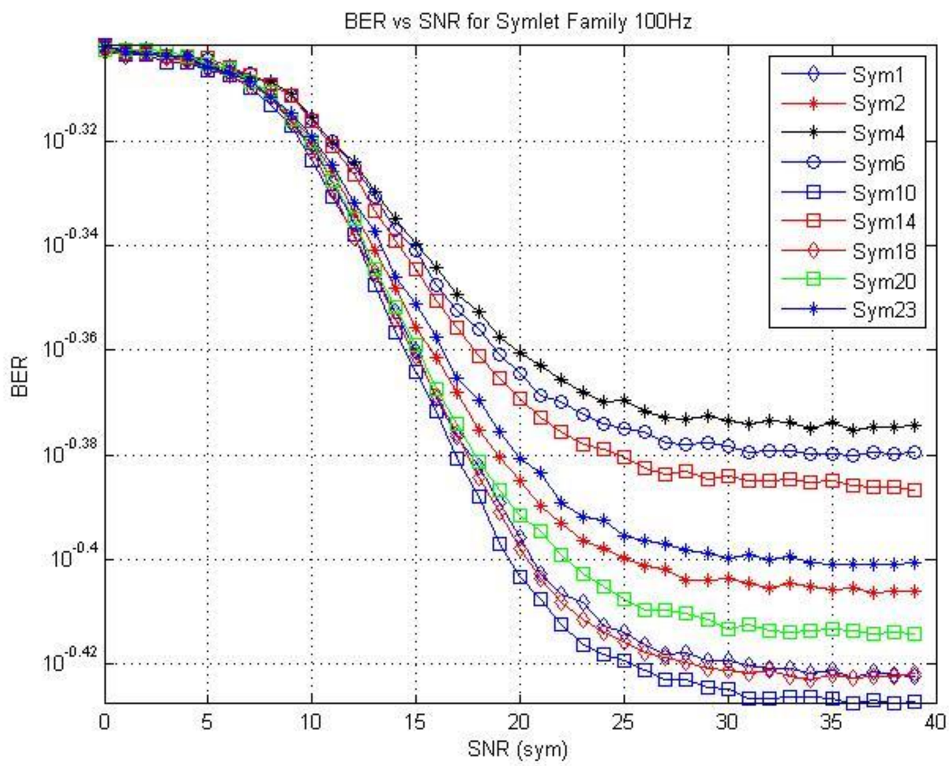

Figure 7.55: Symlets 1 to sym23 BER vs SNR 10k over Rayleigh 100Hz

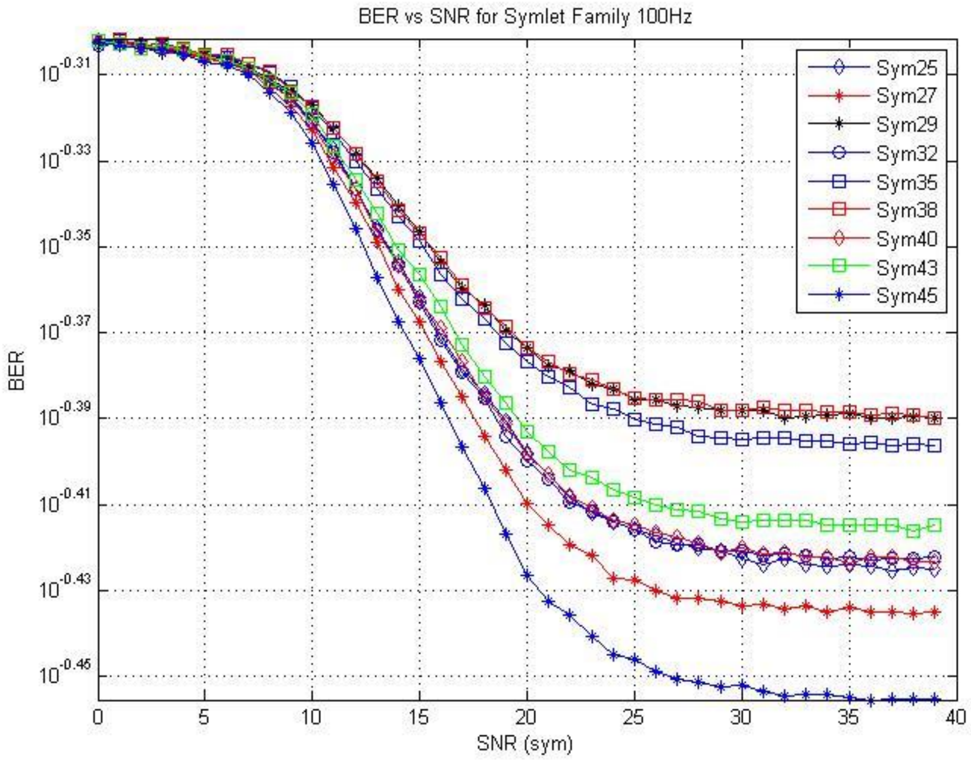

Figure 7.56: Symlets 25 to sym45 BER vs SNR 10k over Rayleigh 100Hz

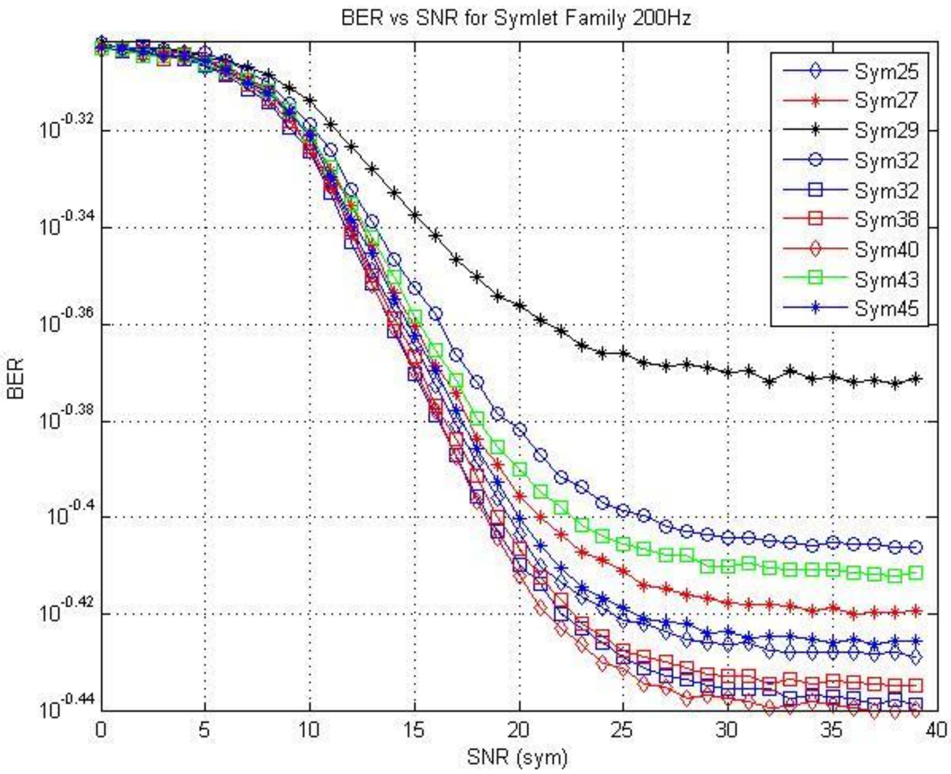

Figure 7.57: Symlets 25 to sym45 BER vs SNR 10k over Rayleigh 200Hz

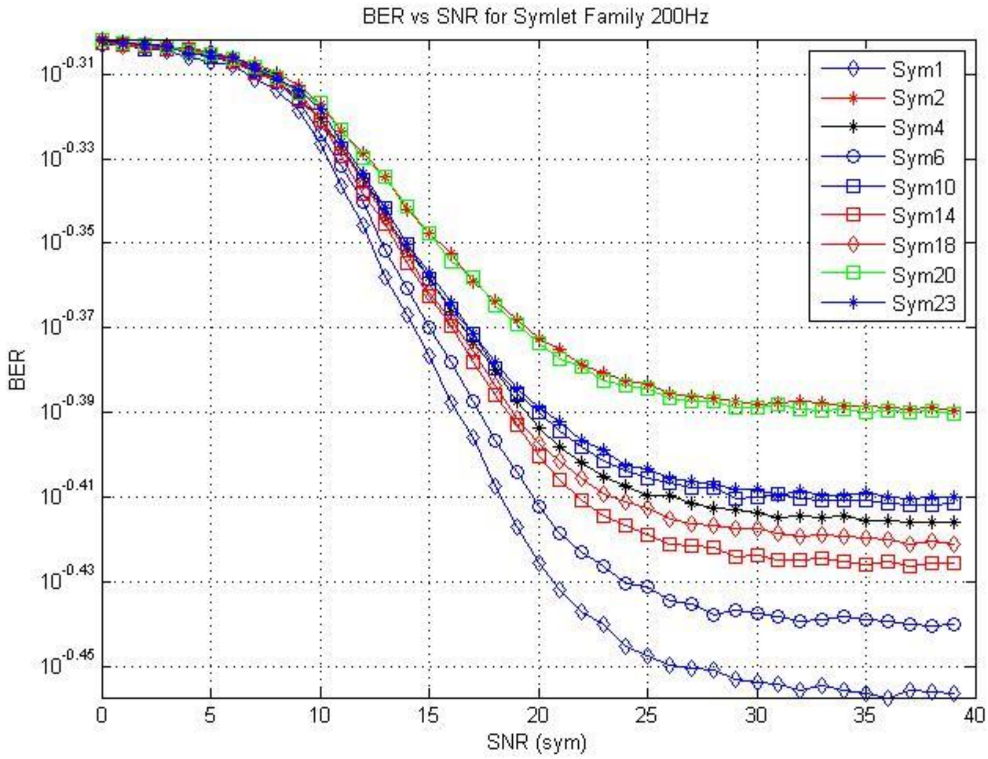

Figure 7.58: Symlets 1 to sym23 BER vs SNR 10k over Rayleigh 200Hz

7.3.3.3 COIFLETS FAMILY

Coiflets family were described mathematically in Chapter 5. Herein, the proposed system will be examined for all coiflets family's members, coif1, coif2, coif3, coif4 and coif5. The number of vanishing moments for each of those members is *N*, however, unlike other families the number of coefficients for this family is 3*N* i.e. for coif4 4*3=12 coefficients or filter taps. The data rate examined is 10k frames which is equivalent to 960 kbits. The three Doppler frequencies examined are 50Hz, 100Hz and 200Hz to replicate the speeds of 65 km/h, 130 km/h and 260 km/h. Figure 7.58 to Figure 7.60 show the results for 50Hz, 100Hz and 200Hz consecutively.

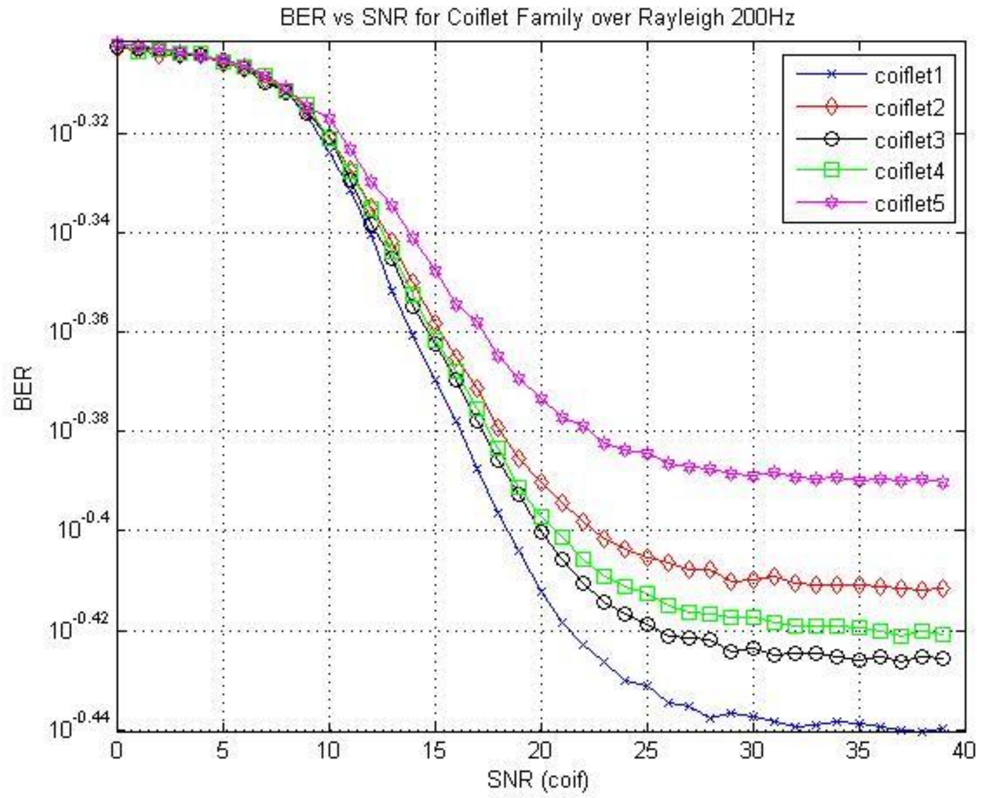

Figure 7.59: Coifets 1 to coif5 BER vs SNR 10k over Rayleigh 200Hz

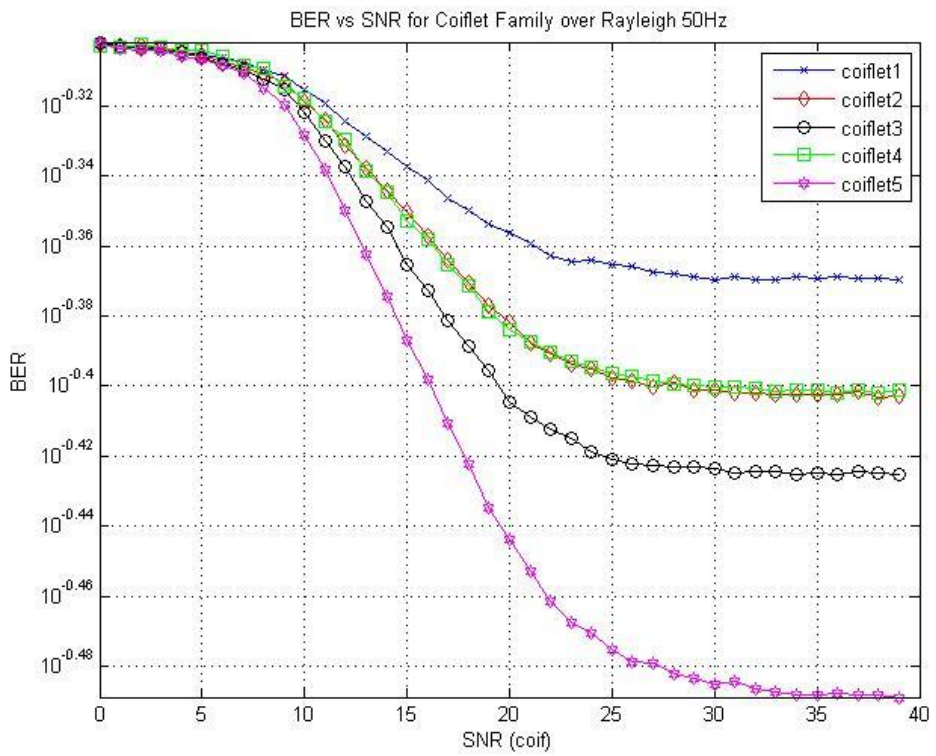

Figure 7.60: Coiflets 1 to coif5 BER vs SNR 10k over Rayleigh 50Hz

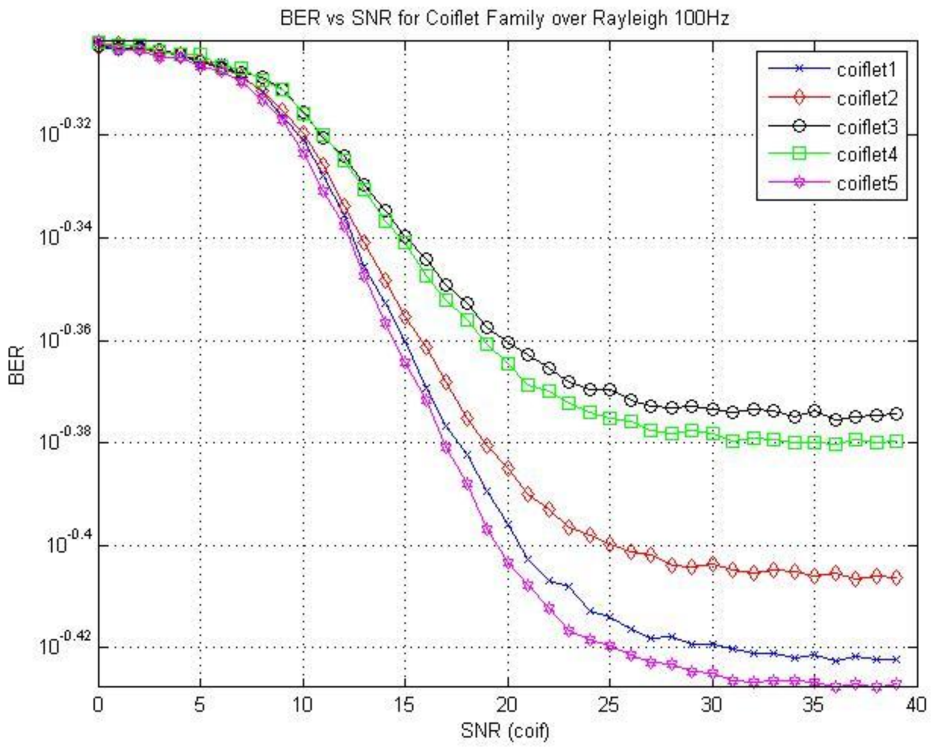

Figure 7.61: Coiflets 1 to coif5 BER vs SNR 10k over Rayleigh 100Hz

7.3.3.4 BIORTHOGONAL FAMILY

As with previous families, Biorthogonal were described mathematically in Chapter 5, Section 5.4.4. This wavelet family is distinctive among others where it is possible to reconstruct the signal with different filters vanishing moments than the one used in decomposition. This family also is biorthogonal and thus it follows slightly different naming pattern than others. The naming is bior*Nr.Nd* where *Nr* is the reconstruction filters bank and *Nd* is the decomposition filters bank order. Figure 7.61 to Figure 7.64 show the performance of all members of this family over 50Hz, 100Hz and 200Hz consecutively.

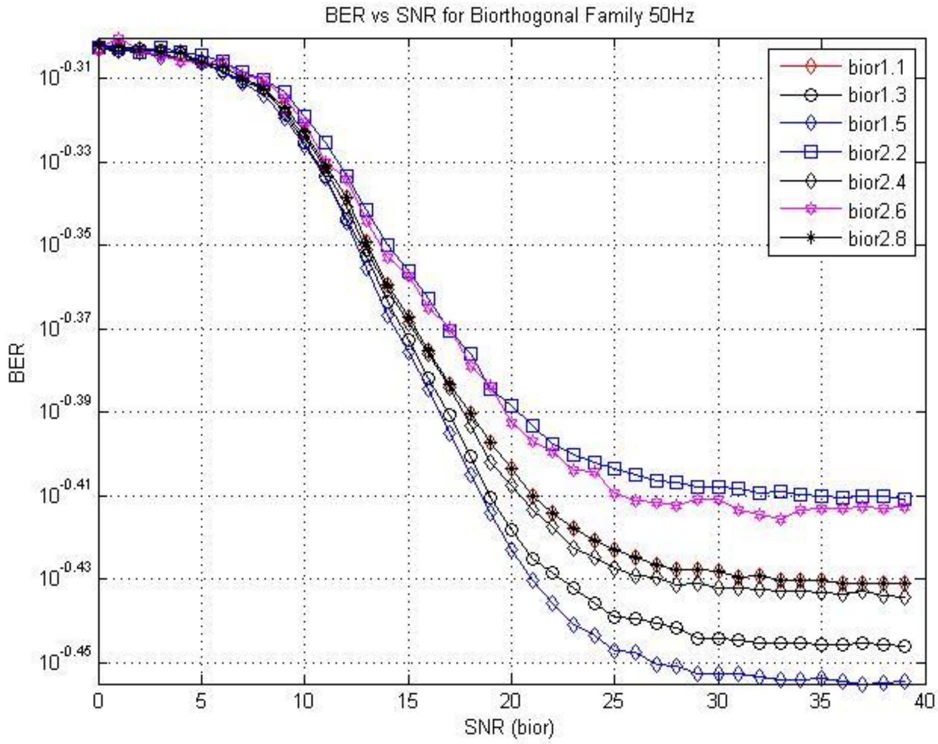

Figure 7.62: Biorthogonal 1.1 to bior2.8 10k over Rayleigh 50Hz

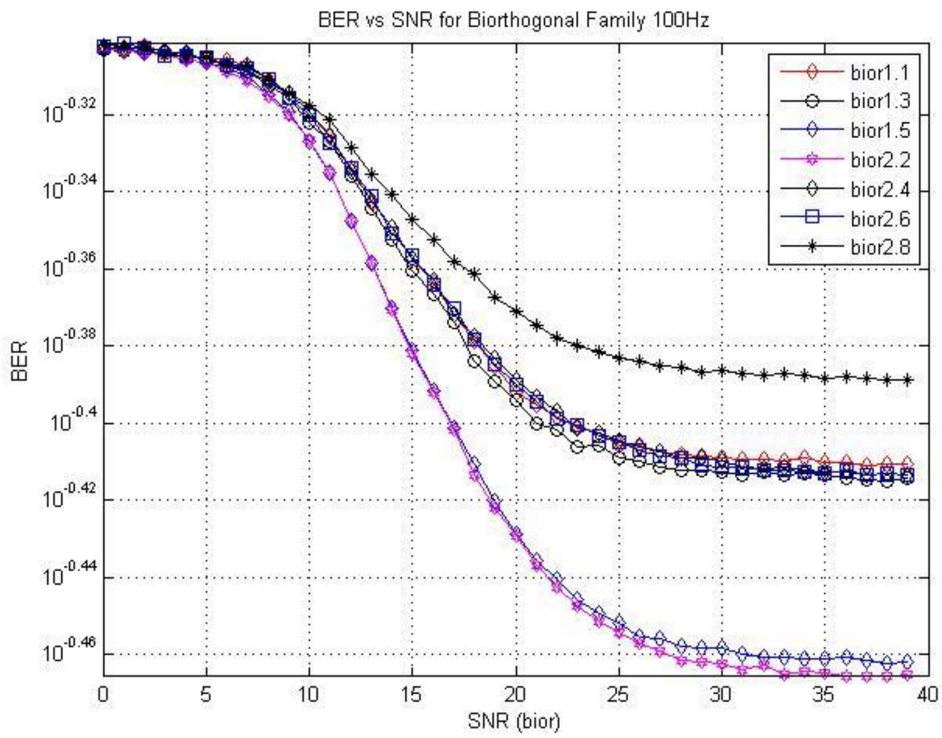

Figure 7.63: Biorthogonal 1.1 to bior2.8 10k over Rayleigh 100Hz

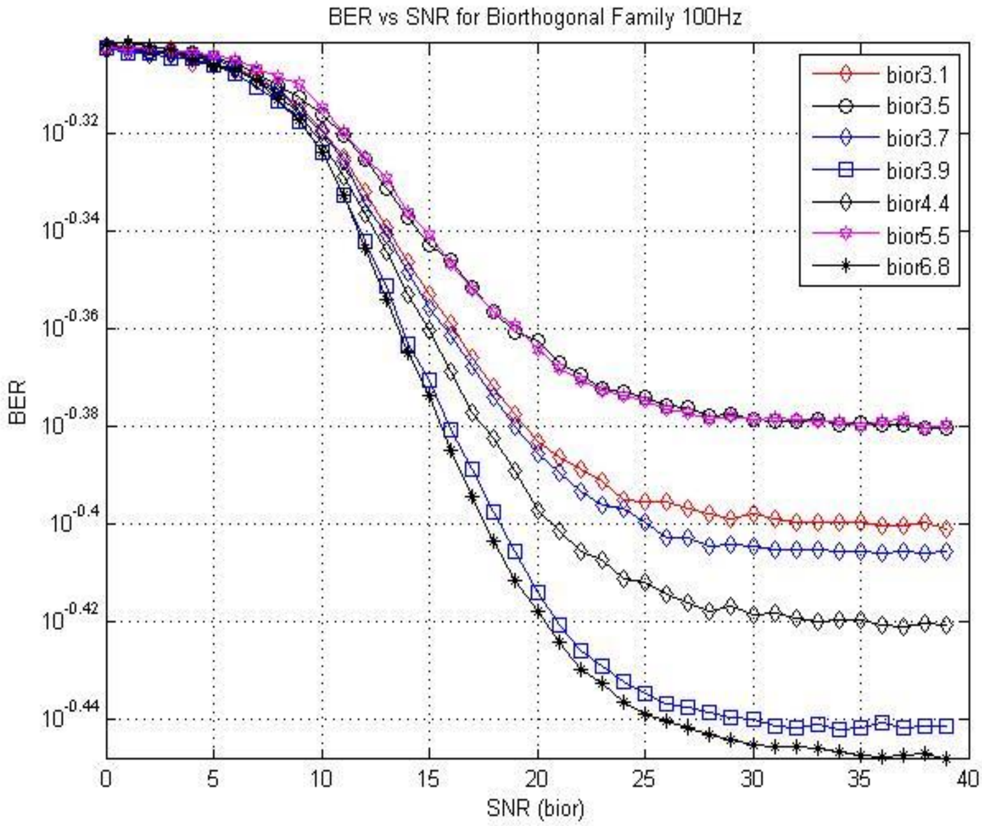

Figure 7.64: Biorthogonal 3.1 to bior6.8 10k over Rayleigh 100Hz

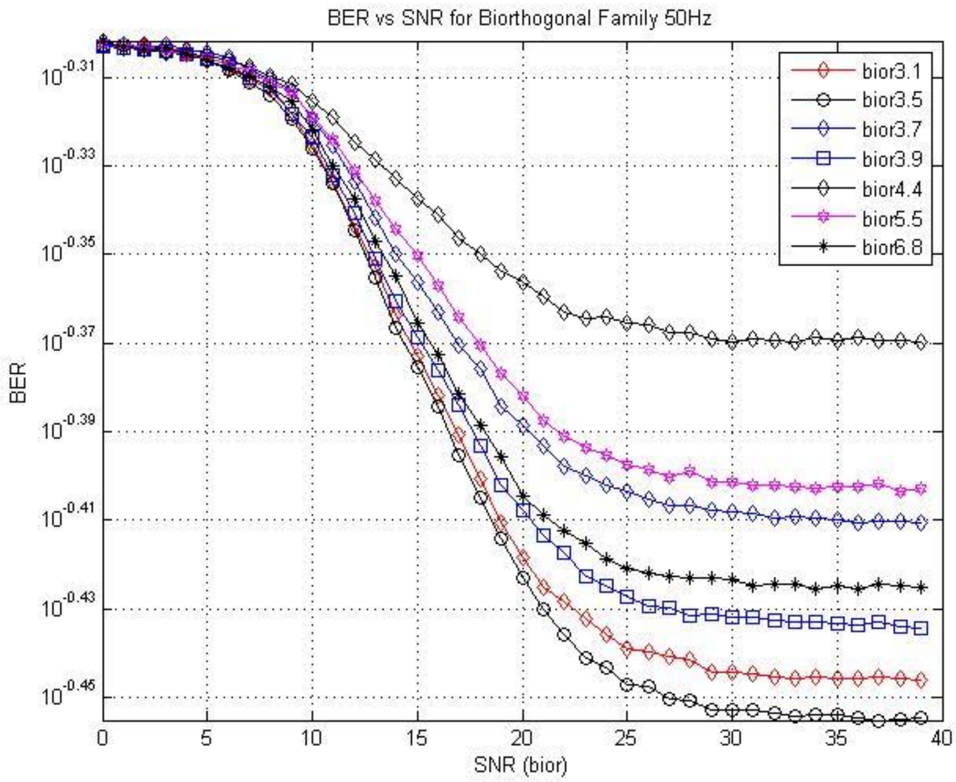

Figure 7.65: Biorthogonal 3.1 to bior6.8 10k over Rayleigh 50Hz

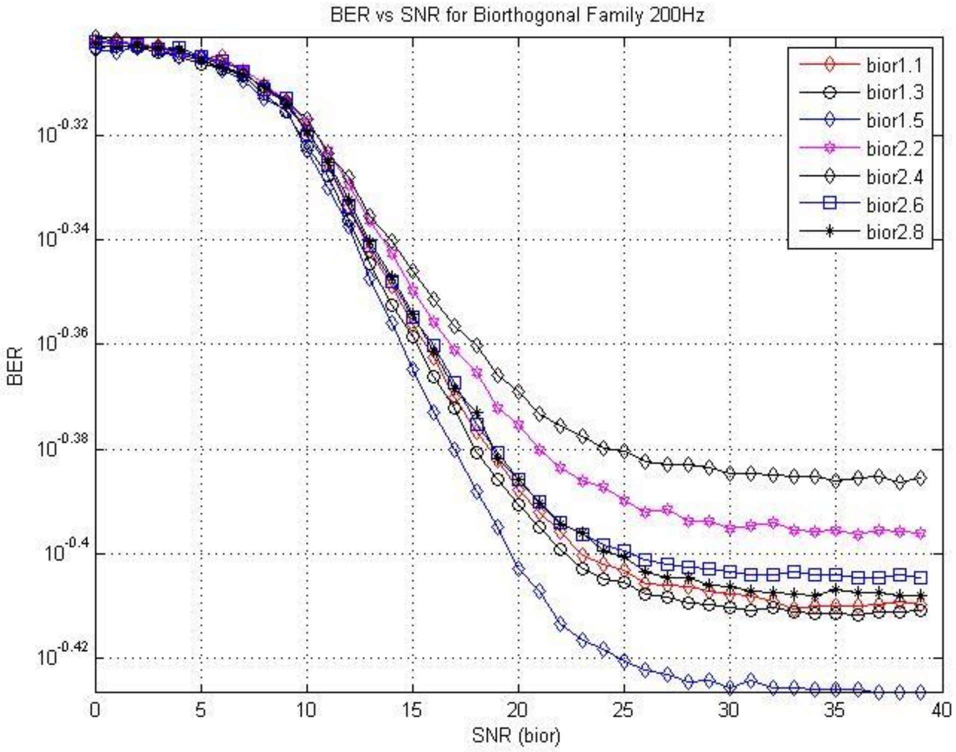

Figure 7.66: Biorthogonal 1.1 to bior2.8 10k over Rayleigh 200Hz

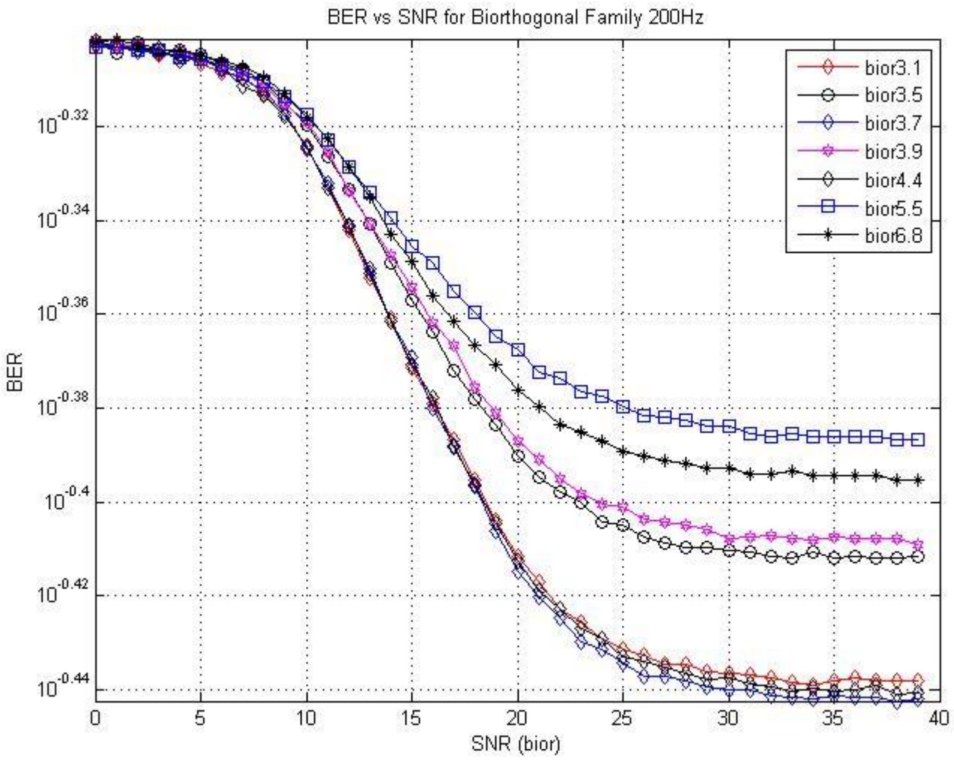

Figure 7.67: Biorthogonal 3.1 to bior6.8 10k over Rayleigh 200Hz

7.3.3.5 REVERSE BIORTHOGONALS FAMILY

This family is the reverse of the biorthogonal family, in which the decomposition and reconstruction filters bank are reversed. This becomes clearer from the naming pattern that this family follows which is rbio*Nd*.*Nr* where *Nd* is the decomposition filters bank and *Nr* is the reconstruction filters bank. Herein, the results of this family's members of them all are presented for all three Doppler frequencies, 50Hz, 100Hz and 200Hz. Figure 7.66 to Figure 7.72 show the results for all members over Rayleigh channel with Doppler frequencies 50Hz, 100Hz and 200Hz consecutively.

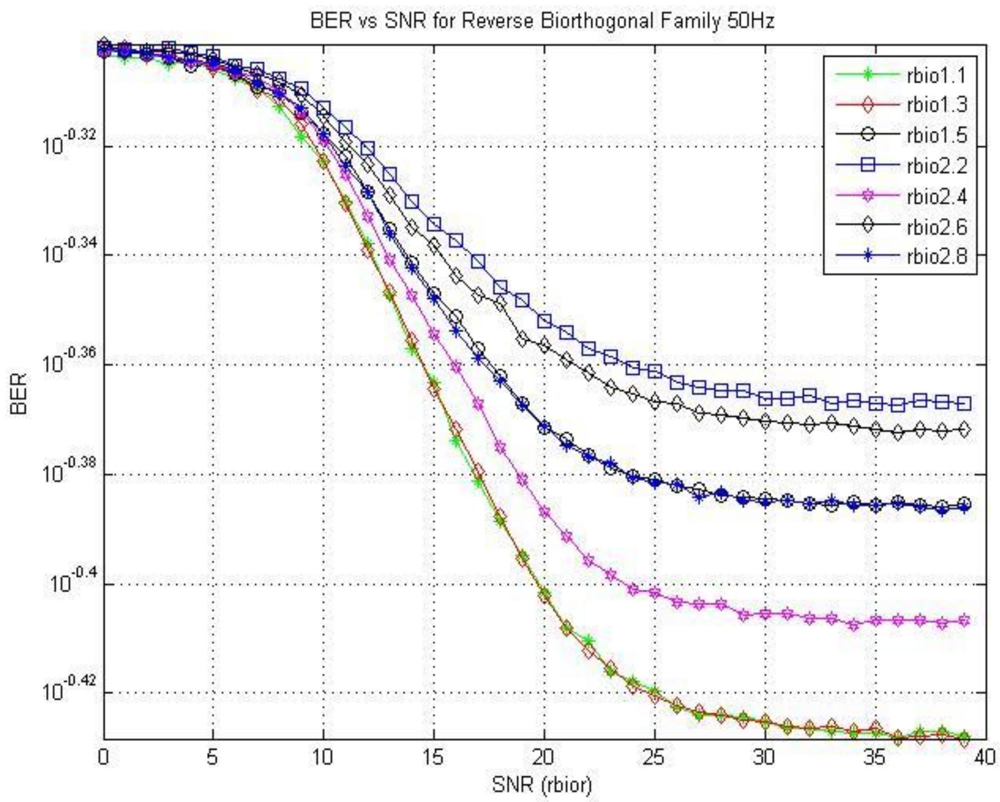

Figure 7.68: Reverse biorthogonal 1.1 to rbio2.8 BER vs SNR over Rayleigh 50Hz

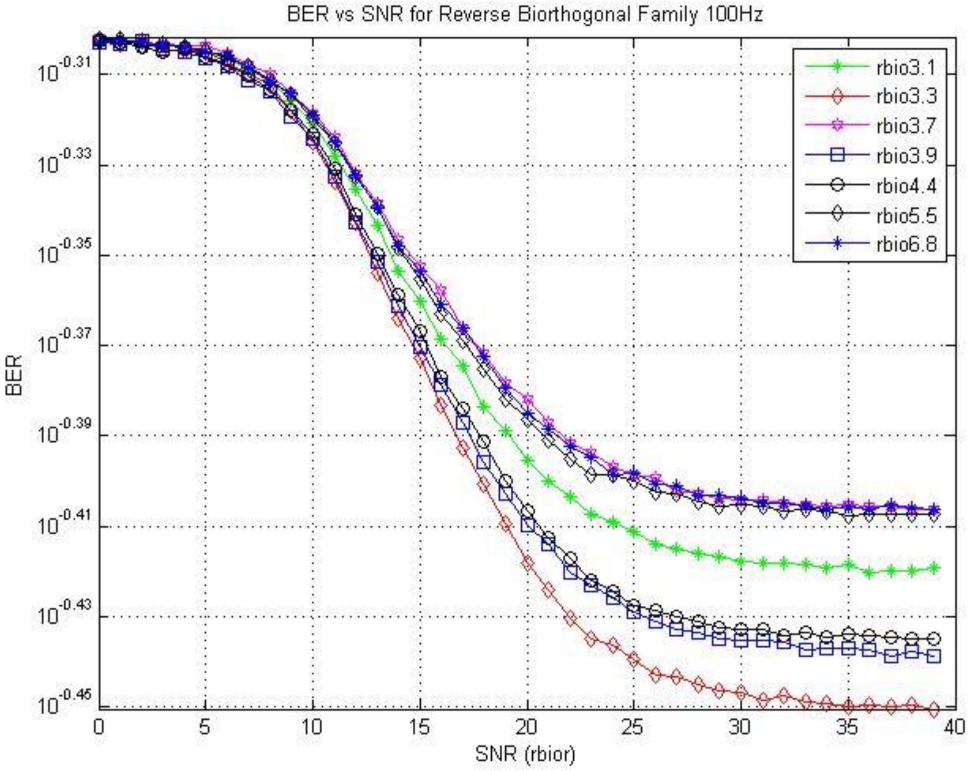

Figure 7.69: Reverse biorthogonal 3.1 to rbio6.8 BER vs SNR over Rayleigh 100Hz

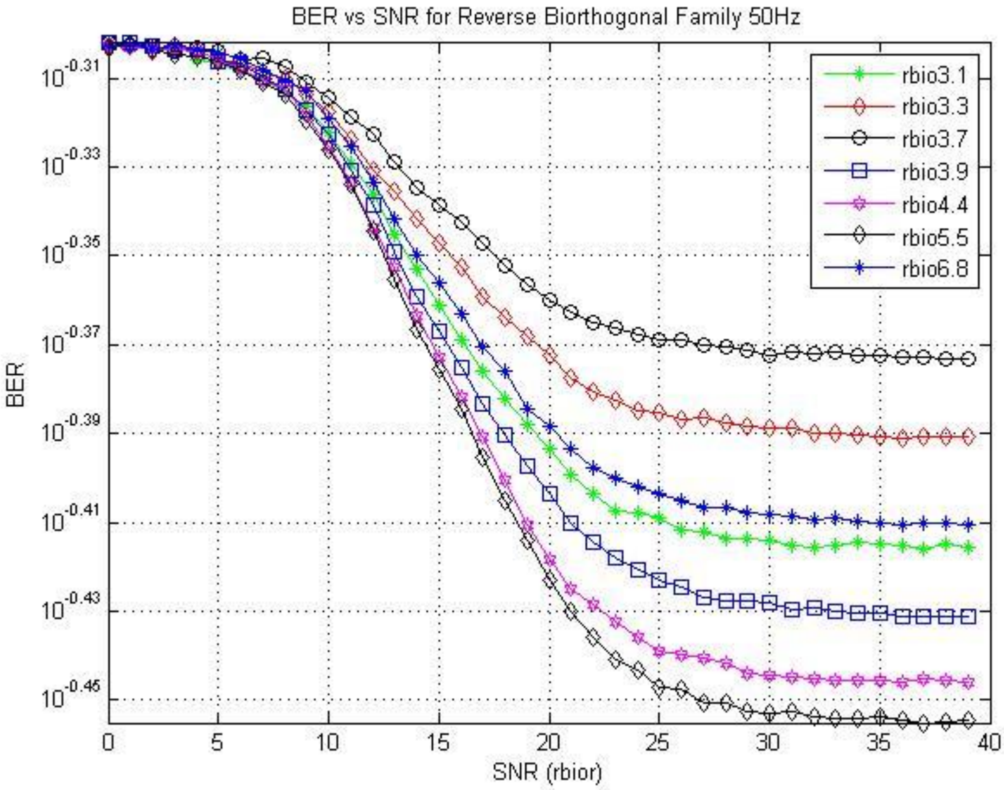

Figure 7.70: Reverse biorthogonal 3.1 to rbio6.8 BER vs SNR over Rayleigh 50Hz

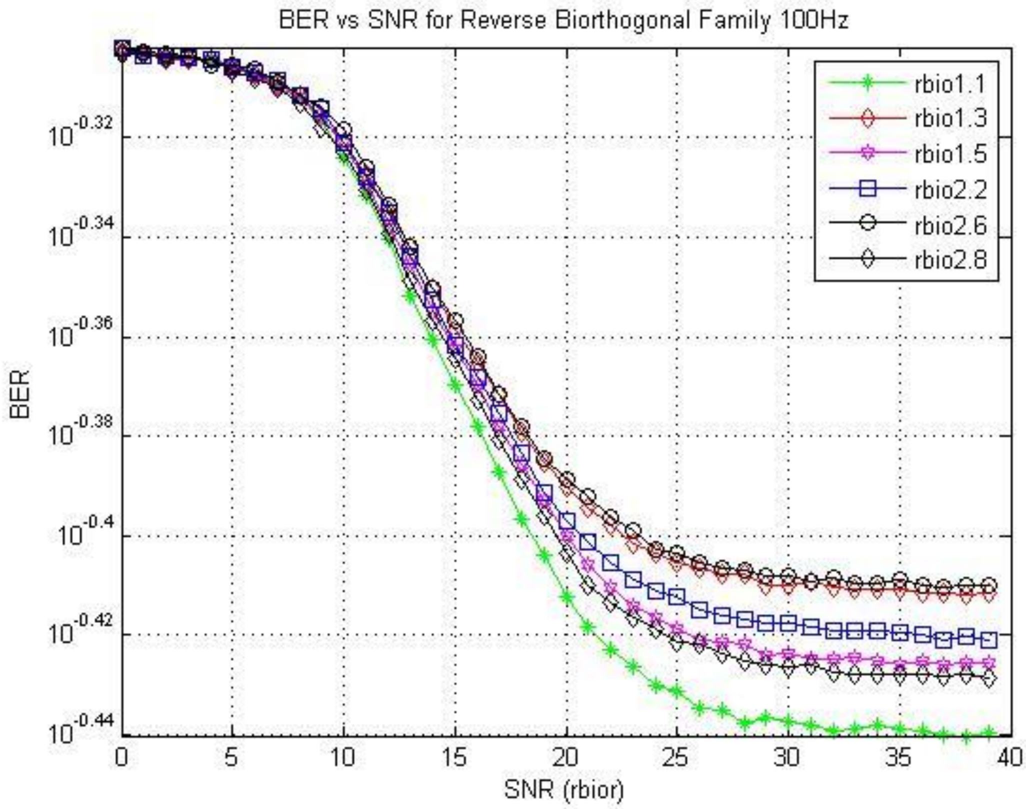

Figure 7.71: Reverse biorthogonal 3.1 to rbio6.8 BER vs SNR over Rayleigh100Hz

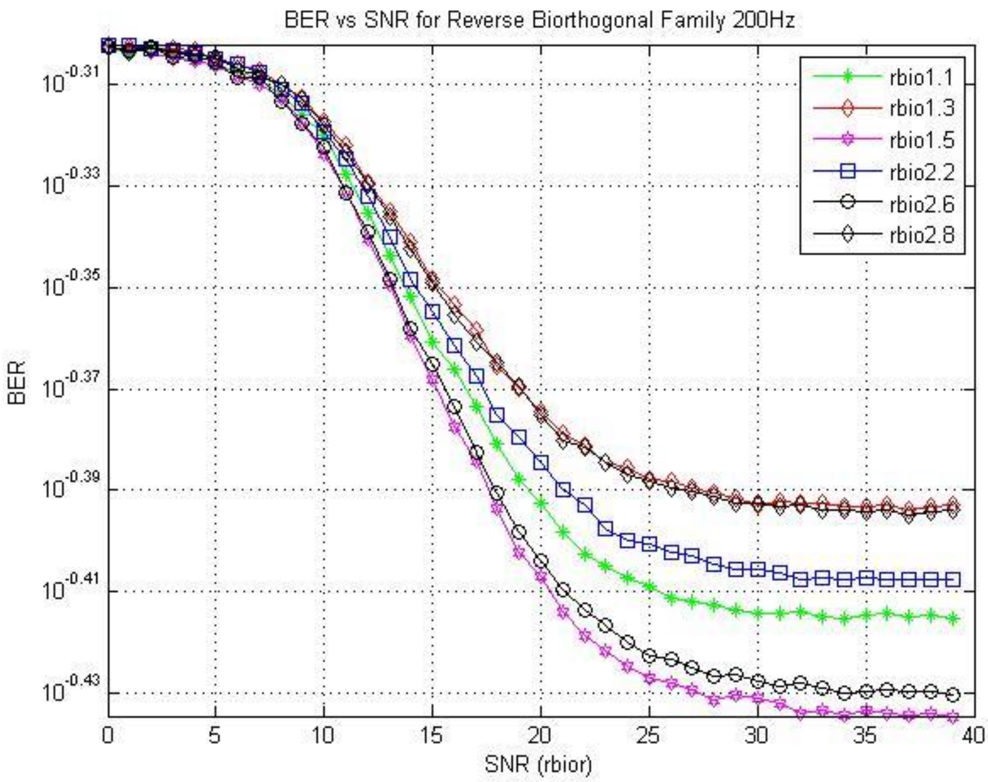

Figure 7.72: Reverse biorthogonal 1.1 to rbio2.8 BER vs SNR over Rayleigh 200Hz

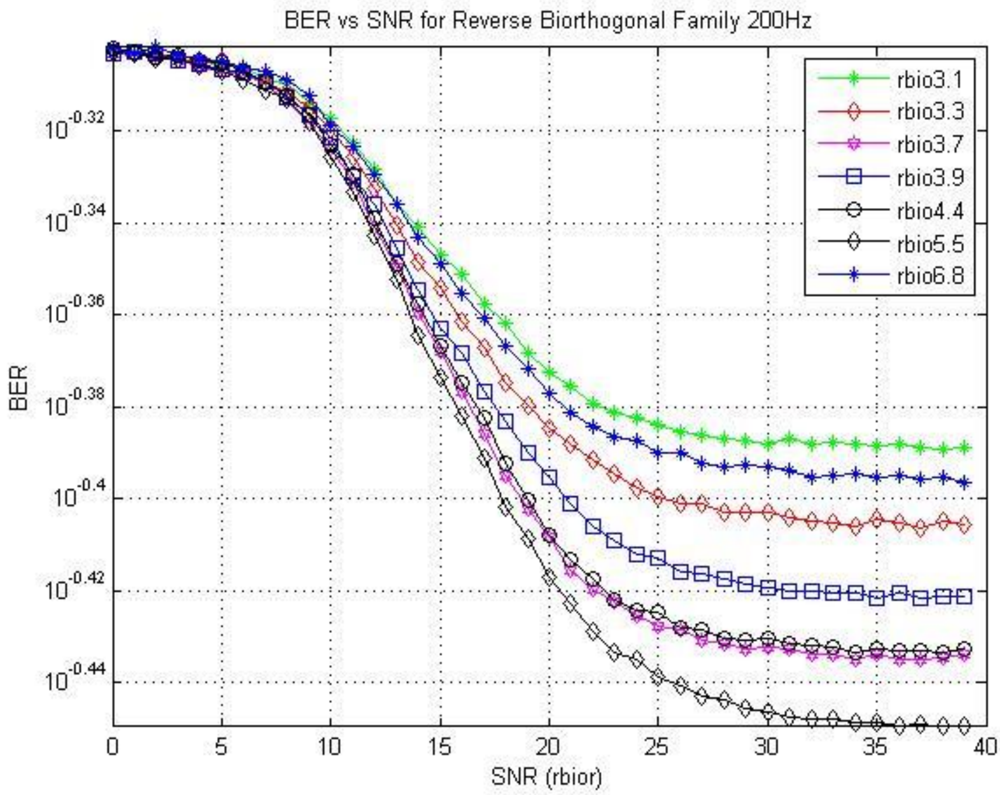

Figure 7.73: Reverse biorthogonal 3.1 to rbio6.8 BER vs SNR over Rayleigh 200Hz

7.3.3.6 DISCRETE MEYER

Discrete Meyer (dmey) has been described previously in Chapter 5 and in this chapter in Section 7.3.1.5. This family is the only family that has a single member as well as being orthogonal, biorthogonal and symmetrical at the same time. This makes it rapidly responsive during computation, however, the huge number of coefficients, 102 filter taps, might be the reason behind the disinclined of researches around it. Nevertheless, it has been included in this research to examine the time response of this family and measure the trade-off between filter symmetry and the large number of vanishing moments. The data length used herein is 10k frames at 96 its per frame over three different Doppler frequencies, 50Hz, 100Hz and 200Hz. Figure 7.72 show dmey and also Haar for those specified Doppler frequencies in the same figure.

7.4 TECHNICAL DISCUSSION AND OBSERVATIONS

The first main focus of comparison is to be made on comparing against Additive White Gaussian Noise (AWGN), the overall throughput based on 11 levels of filters bank, as equivalent to 2048 subcarriers in OFDM, in OWDM. Apart from these, also all available wavelet families and families' members are compared against each other and also against OFDM for 16-QAM and also 64-QAM. Generally speaking, in OFDM, when SNR is 16dB, BER is almost zero. The same observation was noticed in OWDM, especially with Haar wavelet, in which case, there was almost no BER at this value of SNR. Consequently, it was derived from this that the chosen OFDM system's parameters matched the number of filters bank levels in OWDM because of the similar behavior and the received transmitted frames.

In OWDM over the same channel, wavelet families' members are very similarly behaving, where it was found that (db45 over 11 levels) is found to be the best for performance in terms of BER versus SNR with the same family for the lower data rate, 100 frames. However, for higher data rates of 1k and higher, this performance degrades slightly, but not severely with medium size of data rates, 10k frames; but, it recovers well with very competitive performance with rapid data rates of 100k frames, where it achieves less error rate than all other members and also OFDM, as shown in Figure 7.13.

If the linear complexity of OWDM is taken into consideration along with the unnecessary guard intervals, OWDM shows high possibility of producing a strong candidate as an alternative to OFDM. Figure 7.21 and Figure 7.22 show the similarity of performance of OWDM (db45/11-levels) to OFDM (2048 length of IFFT).

For using the symmetric allocation of sub-bands, one can argue whether by decomposing this way it does what IFFT in OFDM system does. The answer is yes, however, with the interesting advantage that guard intervals are no further necessary when using the banks of filters. An additional advantage is that the linear complexity of OWDM is less than the logarithmic complexity of IFFT in OFDM.

The first noticeable observation during results analysis was that in OFDM, as shown in Figure 7.10, when SNR was at 16dB, the BER reached the minimum value and there was almost no error at this value of SNR. The same observation was noticed in OWDM as well, as seen in Figure 7.10, where there was almost no BER at this value of SNR. Consequently, it was derived from this that the chosen OFDM system's parameters matched the number of filters bank levels in OWDM because of the same behaviour as OWDM (db45 with 11 levels), as shown in Figure 7.13.

Another observation is that when running the db45 wavelet family, the simulation takes more time to produce the results than any other wavelet family and even the OFDM system. Despite being this time consuming, db45 is found to be the best performer in terms of BER versus SNR in the tested wavelets families. This time consumption is not related to the chosen SNR, but it is more likely to be in the algorithm for calculating the coefficients of db45 itself, which is a built-in MATLAB function. Therefore, this particular observation has not been looked at further, since it is a minor thing, but might be looked at in-depth in future works to investigate the real causes behind this observation.

7.5 SUMMARY

In summary to this chapter, the designed systems that were proposed in Chapter 6, DVB-T OFDM and OWDM, have been simulated and results were presented in this chapter. The proposed systems were first simulated over AWGN channel, as prescribed by DVB-T standard [19]. Thereafter, for wavelet families and their associated members, described in Chapter 6 Table 6.3, results over AWGN for different data lengths - 100 frames, 1k frames, 10k frames and 100k frames, have been presented. Technical observations from these results were discussed. A comparison between the nominated best performing wavelet families and their associated members with OFDM are presented with a discussion of the technical observation.

Following that, a Ricean Fading Channel has also been modelled according to DVB-T standard [19] for both systems. The parameters of both systems were a replica for BBC, ITV and Channel 3&4 multiplexing values to study the effect on a real world scenario. The results of those two systems, over AWGN and again over Ricean channels, are then discussed and analysed. Although OFDM showed less error rates than OWDM over Ricean channel, however, further channel equalisation for OWDM system would more likely achieve better results.

Finally, the simulation results for the wavelet families and their associated members over multipath fading channels are presented. Doppler Effect frequencies were chosen for three most common maximum speeds, 65 km/h, 130 km/h and 260 km/h which correspond to in city buses, national motorway speed for cars and high speed trains, respectively. The channel equaliser used in OWDM is the same one used for OFDM, which does not seem to be as effective as with OFDM. Therefore, there are a need for further research on finding the most effective adaptive equaliser or filter to mitigate Doppler effect on OWDM signals.

The examined wavelet families for all aforementioned experiments are: Haar, Daubechies, Symlets, Coiflets, Biorthogonals, Reverse Biorthogonals and Discrete Meyers. For those with family members, all their associated members have also been examined and assessed against each other. In the next chapter, further analysis of the results in term of BER vs SNR performance, time, space and computational complexities is provided.

CHAPTER 8: RESULTS ANALYSIS

Objectives:

- To demonstrate a critical comparison between OFDM and OWDM systems.
- To critically compare OFDM and OWDM over AWGN.
- To critically compare OFDM and OWDM over Multipath Ricean Fading Channel.
- To critically compare OFDM and OWDM over Multipath Rayleigh Fading Channel.
- To validate the results of different wavelet families and their children against the standard.
- To evaluate the results and articulate recommendations.

8.1 INTRODUCTION

In the previous chapter we examined the proposed system for all wavelet families and their associated children with comparison against the-state-of-the-art OFDM as like-for-like comparison with accordance to DVB-T standard [19]. Not only that but also we examined them over different fading channels and different data mapping modulations and various data lengths. The results over AWGN fading channel, however, were at a lower data rate the children of one family behaves differently with some children outperformed OFDM for the same data length. This is not the case for longer data rates where the data stream extended to a factor of x10 twice to make the system have a longer data rate to understand the behaviour of the channel and subsequently a better error detection and correction. This rapid increase of data stream simulates what real world scenario might look like. The results of children of one family over longer data streams are explicitly very similar to each other and also to that of OFDM.

Thereafter, and to validate the comparison on more reliable and industrial standards systems, two of the most used multiplexers in the not only the UK but across the world were built to validate the proposed system against. Those multiplexers' values are used by BBC, ITV and Channel 3&4, Section 7.3.2, in the UK and other TV broadcasters use similar multiplexers across the world. The examined wavelet family in this comparison was Haar wavelet due to its simplicity and reliability in time and space complexities. The results, however, were very competitive; while OFDM outperformed OWDM slightly a further equalisation of OWDM could be used.

To expand on that, Rayleigh fading channel were then simulated for all aforementioned wavelet families and their associated children. The purpose of expanding the investigation to this level is to further investigate more reliable mobile environment channels and also to investigate whether the type of equalisers used in OFDM systems are suitable and compatible for the use with the proposed system, OWDM. In terms of data length reliability, two set of data lengths were examined, 1k frames and 10k frames. Results, however, in this part shown that there are different performance among one family and across families. This is due to the various length of filters used in each individual child and also the type of channel used.

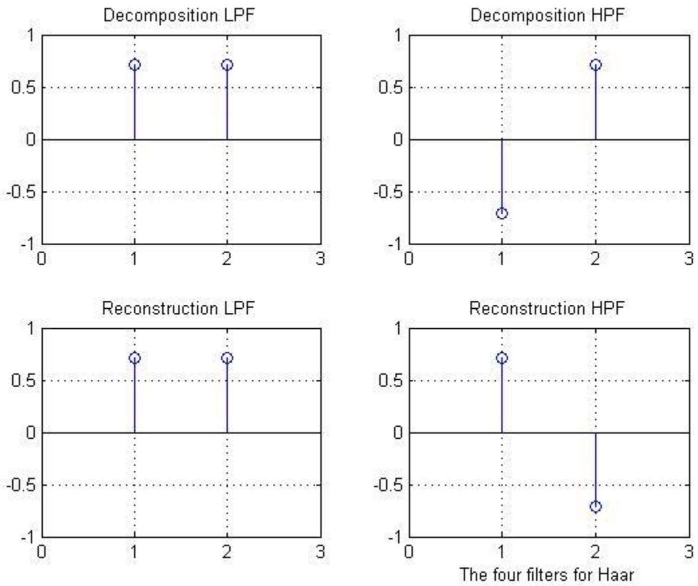

Figure 8.1: Haar Decomposition and Reconstruction Filters' Taps

In this chapter a further analysis and discussion of the performance of each wavelet family and their associated children is presented. Further validation of results and further type of examination are also presented and discussed in this chapter. The type and length of filters used for each of those family and children will also be included in the discussion. A summary and conclusion of findings will be presented at the end of this chapter and revisited in the conclusion chapter, Chapter 9. Furthermore, all type of complexities including time, computational cost and space will also be discussed and concluded in this chapter.

8.2 CRITICAL COMPARISON OF OWDM AND OFDM OVER AWGN

In this section a collection of wavelet families and selection of best performed families' children are presented and discussed. The wavelet families and their associated members are, thereafter, discussed further in their time, computation and memory size complexities. Consequently, all those families and associated members will then be compared to IFFT in OFDM system for the same categories of complexities. The channel type used for this investigation were AWGN which BER vs SNR throughput results were presented in Chapter 7, Section 7.3.1. All examined wavelet families' children including OFDM reconstructed the transmitted frames efficiently, as we will see some of them for example but not for limitation. Recapping the computation of BER ration, it is the number of bit errors divided by the total number of transmitted bits for a given period of time interval that want to be analysed.

The Haar wavelet, which is also db1, is the simplest wavelet, as discussed earlier in Section 5.4.2, due to the fact that it has only single vanishing moment as shown in Figure 8.2. Further to the results presented earlier, in Section 7.3.1.1, the transmitted message were measured which successfully been fully recovered after error detection and correction block, as shown in Figure 8.2. As we also saw earlier in Figure 7.4, Figure 7.5, Figure 7.7 and Figure 7.10 has slightly outperformed the rest of Daubechies' children and also OFDM. However, this slightly better performance is actually very significant when combined with the save of bandwidth of up to 25% due to the elimination of guard interval and also the dynamic allocation of bandwidth. The symmetry and simplicity of Haar filter, having only single vanishing moment which means just two filters taps, is one of the main reasons why this type of family is in fact outperforming other families' children. Figure 8.1 show the decomposition and reconstruction of LPF and HPF of Haar, where we can clearly see how many filters' taps how symmetrical they are.

The symmetrical distribution and the very low number of filters' taps implies that during operation the complexities of time response, computational and space are very low and proportional to the number of multiplications and additions during process. The time response

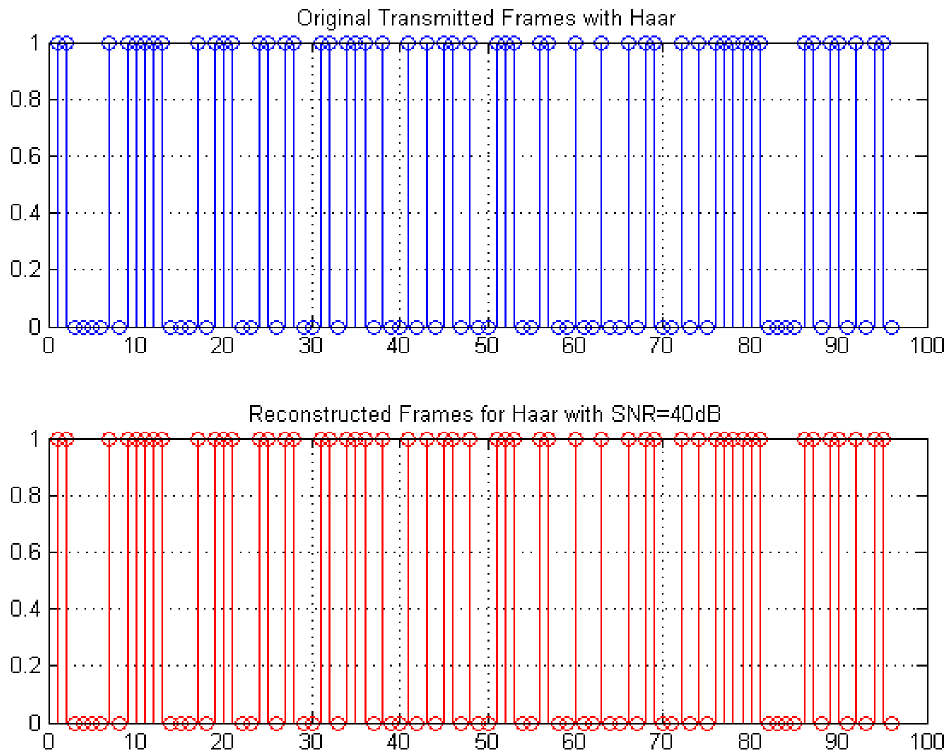

Figure 8.2: Transmitted and Reconstructed Frames using Haar Wavelet

was confirmed when timer set in parallel simulation run of OFDM and OWDM for Haar, both systems were fast, however, OWDM responded within 3.281 seconds whereas OFDM responded within 3.320 seconds with 4 milliseconds. This conclude that Haar wavelet is a very promising family that can potentially replace FFT in the-state-of-the-art OFDM systems.

The second competitor to OFDM and also among other Daubechies' children were db32, db40 and db45. Those three Daubechies delivered a good BER vs SNR performance compared to other children of the same family and also to OFDM as we saw earlier in Chapter 7, Section 7.3 in Figure 7.4, Figure 7.7 and Figure 7.13. Nonetheless, those children have considerably higher vanishing moments, filters length, than Haar as shown in Figure 8.3 for bd45. The asymmetrical distribution of filters' taps, which can be seen from the seen in the same figure, add up significantly to the computational time complexity, as seen in earlier Chapter 5, Sections 5.5 and 5.6. Consequently, higher number of multiplications and additions throughout the data processing and subsequently more computations, longer time response and more memory space required to store computed coefficients which results in added cost. In contrast with FFT in OFDM, which has $O(NlogN)$ complexity, but in hardware operation this

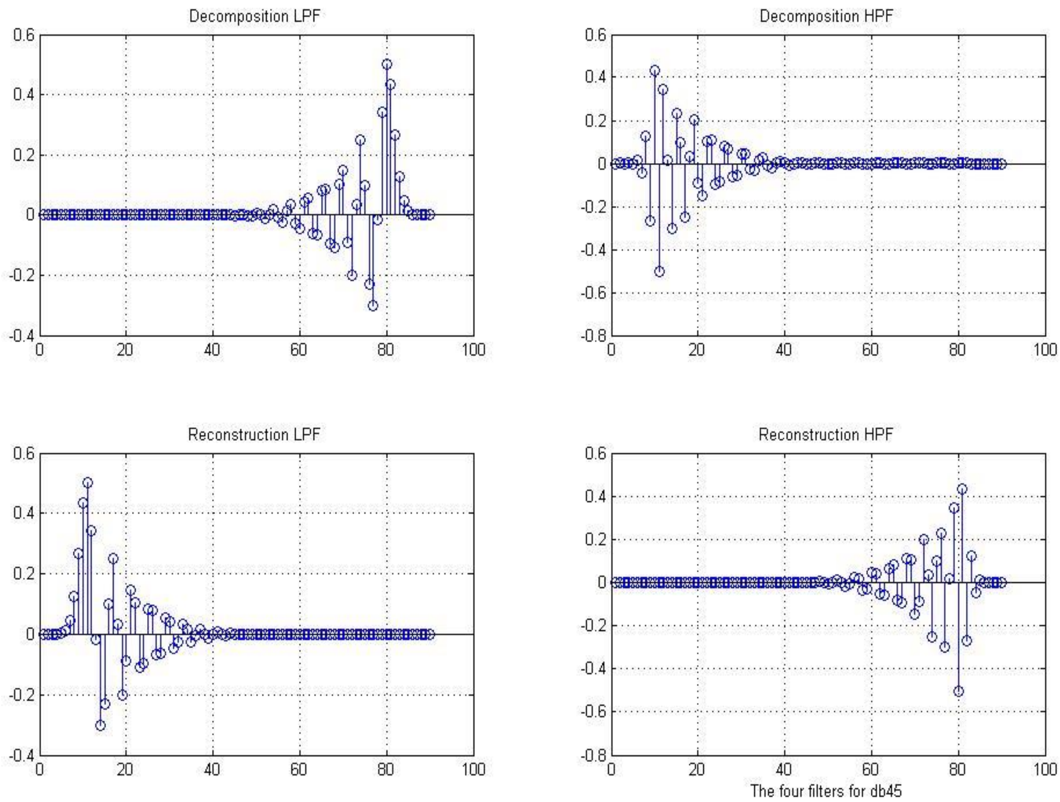

Figure 8.3: db45 Decomposition and Reconstruction Filters' Taps

FFT operation can be done through well-established data structure algorithms including heapsort, quicksort and mergesort which does not require as large memory space as that of $O(N)$ complexity of Daubechies with high vanishing moments would need. Therefore, Haar wavelet is nominated to be compared with other families' members in the next stage of comparison. While all these three Daubechies' children outperformed others and signals were successfully reconstructed, the transmitted and the reconstructed frames using db45, which outperformed them all, is shown in Figure 8.4.

Moreover, in Daubechies' children the more vanishing moments there are, the better performance it has, nevertheless, this comes on the cost of more time consumption, more memory space required and more computation of coefficients. This is due to the increasing number of coefficients computations which is directly proportional to the number of filters' taps, which implies more multiplications and additions are required during computation, as explained in Chapter 5, Sections 5.5 and 5.6. All experiments were carried out consistently, to maximise accuracy, on Intel Core i5-3470 CPU at 3.20GH, 6M cache memory, and 3.88GB active RAM over 64-bit windows 7 operation system. While OFDM responded within 3.320 seconds to process 100 frames, OWDM with db15 and db32 needed 5.929 and 6.314 seconds

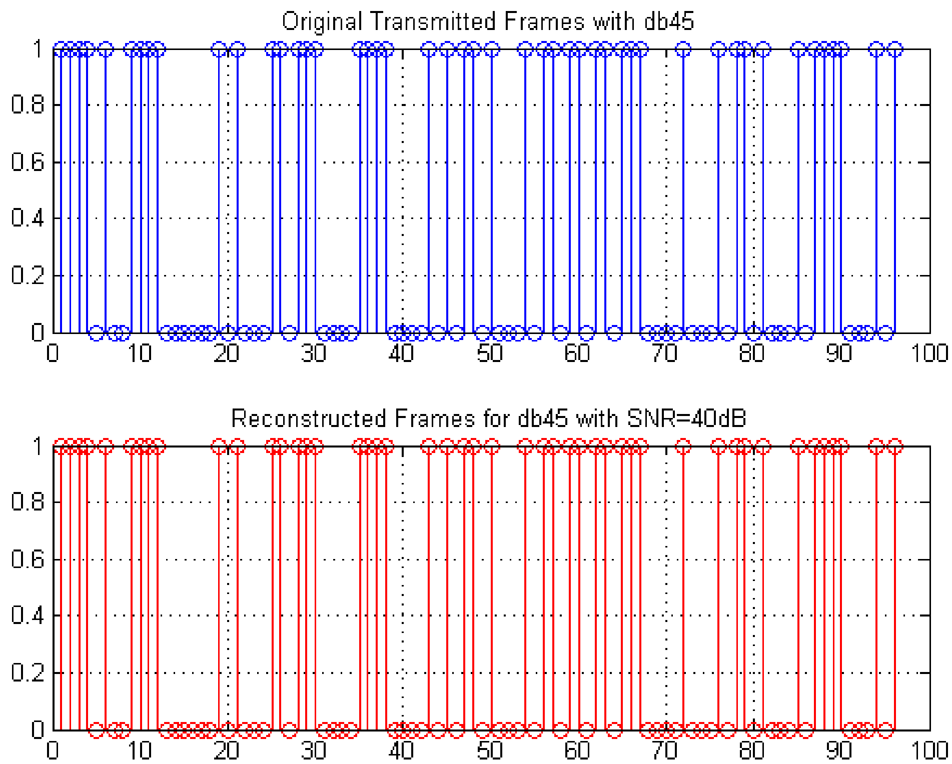

Figure 8.4: Transmitted and Reconstructed Frames using db45

respectively to fully process the same length of data. Whereas for db40 it took 6.649 seconds for the full process and db45 consumed staggering 7.854 seconds which is more than double the time that took OFDM to fully respond.

Although this is the case, all these three Daubechies' children achieved less errors rate than OFDM. Due to the restricted access to the memory architecture within the campus there could not be anyway of precisely, in exact numbers, measure the memory consumption of each wavelet family. However, during running the simulation on MATLAB there were an indications that local memory is building up upon the stored values. For more accurate observation, the calculated coefficients are stored in matrices during the process where these matrices are observed to fills up with computed values. To the best of the author's knowledge those children with higher filter orders were observed to occupy more memory than of those with lower orders and also than OFDM.

The next analysed family were Coiflets, where the performance of all 5 children of this family were competitively similar to not only each other but also to OFDM. However, there were slightly difference between those with higher order children, for instance coif5, which

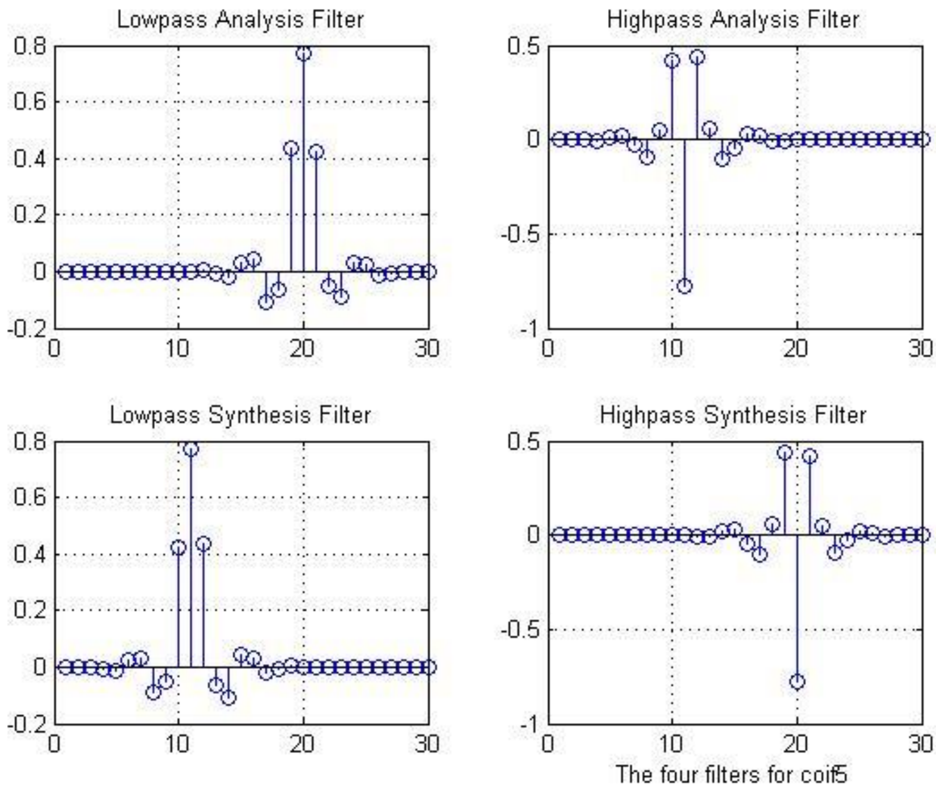

Figure 8.5: Coiflets 5 Decomposition and Reconstruction Filters' Taps

has 30 filters' taps, outperformed other children and also OFDM. This is due to the high number of coefficients and thus increasing precision of calculations. Although this comes at the cost of memory size and time consumption which are both not tolerated in real time systems, however, some other high precision applications could use those higher order children. During running the simulation, with 100 frames, an accurate automated stopwatch were used to measure the time consumption of coif1, coif3 and coif5 and compared to OFDM. It was found that coif1, which has 6 filters' taps, responded within 4.012 seconds whereas OFDM responded quicker with 3.320seconds. On the other hand, however, coif3 and coif5 took longer with 5.166 and 5.684 seconds respectively.

In contrast with higher order Daubechies, i.e. db45, it was noticed that the time response for coifl5 is slightly less than db15, 5.684 and 5.929 respectively, despite that both have exactly the same filters length, 30 each. It is probable that this difference is due to the near symmetry of coiflets filters, as shown in Figure 8.5 with coif5, as opposed to the asymmetry of Daubechies filters, as shown in Figure 8.3 with db45. A further probable justification of these differences in time response is that if we measure the impulse response of coif5 and db15, which can be seen in Figure 8.6 and Figure 8.7 respectively, we indeed can see that the smoothness of coif5 is more than db15. Not only that but also the very close symmetry of the filters of coif5 is obviously for both the scaling and the wavelet functions as opposed to the far

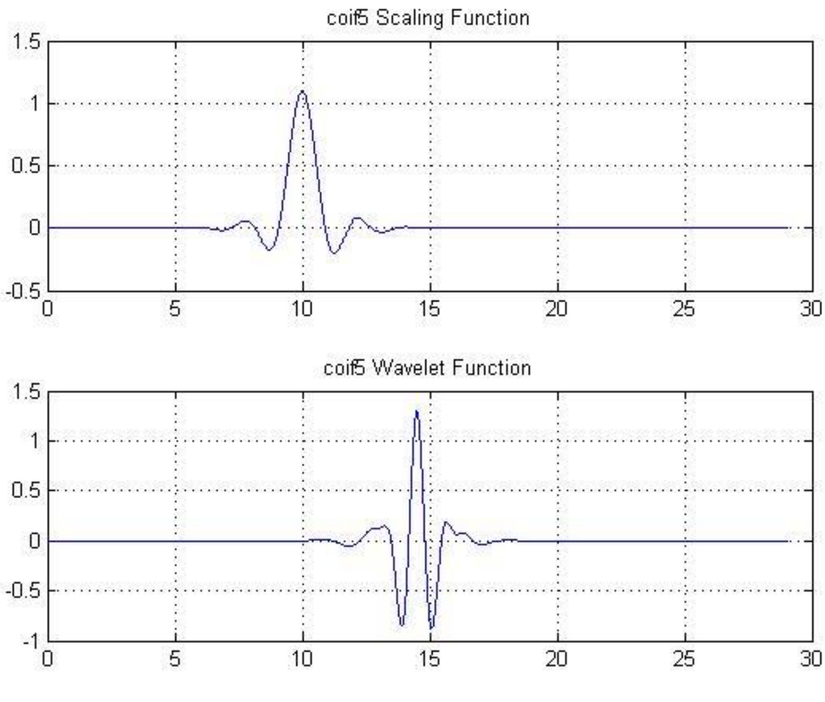

Figure 8.6: db15 Scaling and Wavelet Functions Impulse Response

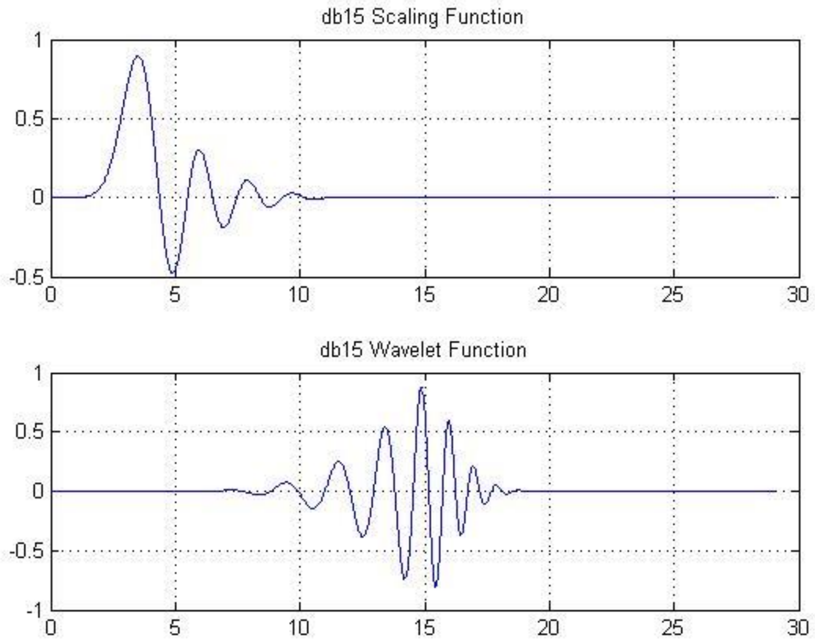

Figure 8.7: db15 Scaling and Wavelet Functions Impulse Response

from symmetry of db15. This directly contributes to the ease of computations of coefficients during data process. All these children including OFDM have indeed delivered and successfully reconstructed the frames as shown in Figure 8.4.

Therefore, we can conclude that the coiflets can be used in other applications than real time systems where latency and memory space are tolerated. Coif1, however, performed closer than OFDM but with very slight latency that can be tolerated in some real time systems for the sake of bandwidth saving and the added dynamic flexibility. The memory space consumption for the same child is not significant and can also be tolerated in some real time systems for the trade-off between the added bandwidth saving, the less computational complexity and flexibility of bandwidth allocation. Other children of this family are not as feasible as coif1 to implement in real time systems, however, it can be used in other application for added accuracy, i.e. image watermarking, compression or biomedical imaging.

Thereafter, biorthogonals and reverse biorthogonals were examined through four different data lengths. Those two families are the same, however, the only difference between them is that the reverse biorthogonal is substituting the decomposition filters bank with the reconstruction filters bank, as explained earlier in Chapter 5, Section 5.4.4. It was found that in the lower data rate, the biorthogonals children behaves differently than reverse biothogonals, thus, the data rate increased drastically to a reliable level of 100k frames. These increases in data rate were done at different stages while observing the behaviour of all children and also OFDM against them all. It was found that bior2.2 and bior5.5 both achieved slightly lower errors rate than OFDM. To our surprise the same observation were found for the same children in the reverse biorthogonals, rbio2.2 and rbio5.5. To show the similarity of filters for both families for the same children, bior5.5 and rbio5.5 are shown in Figure 8.8 and Figure 8.9 respectively. It is probable that the performance of those particular filters is due the same number of vanishing moments used in the decomposition and reconstruction filters bank.

On the other hand, however, those children with asymmetrical vanishing moments in both of these families, for example but not for limitation bior 3.9, rbio3.9, bio6.8 and rbio6.8, are observed to consume more time and also more memory during process than other children of the symmetrical vanishing moments in both sides. In contrast with OFDM, bior1.1 which has the fastest response time and less memory consumption performed slightly worse than OFDM. This particular child could be used in real time transmission systems but with sacrificing some of errors rate efficiency towards saving some bandwidth and flexibility of sub-bandwidth allocation. The good thing though is that this sacrifice can be compensated by increasing the signal power to achieve higher signal to noise ratio or perhaps invest deploying more efficient error detection and correction algorithms. The other symmetrical children of higher order are

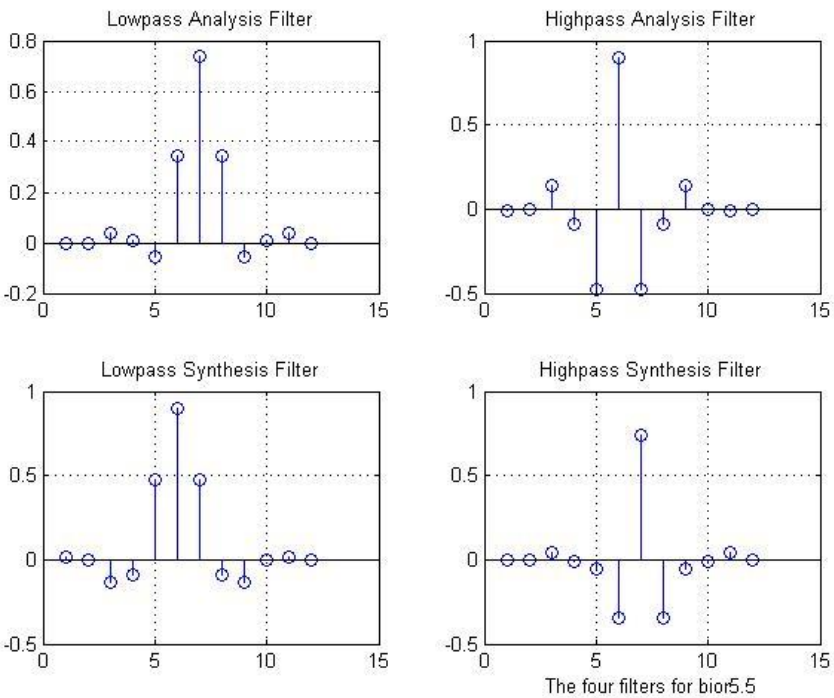

Figure 8.8: bior5.5 Decomposition and Reconstruction Filters' Taps

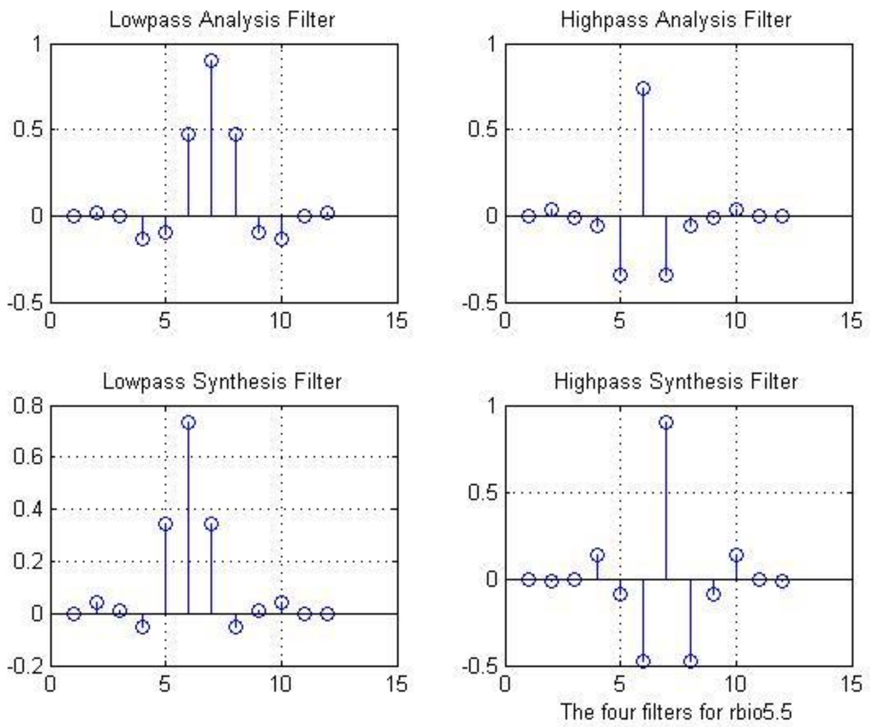

Figure 8.9: rbio5.5 Decomposition and Reconstruction Filters' Taps

more costly to implement due to the increased requirement of memory size. Having said that, it could still possibly be used for some real time transmission systems with the desired efficiency by deploying the appropriate wavelet child. The trade-off between the increased efficiency and flexibility and the more investment increasing memory sizes and sometimes the power of processors depending on the range of the required efficiency and the wavelet child length.

Thereafter next, discrete Meyer were examined through 1k, 10k and 100k frames and then compared directly to OFDM with the same length of data. Although discrete Meyer performed competitively against OFDM through all three examined data lengths, the implementation of this particular type of family is considerably more costly than not just OFDM but also other families and children. This is due to the significant number of filters coefficients, staggering 102 taps as shown previously in Chapter 7, Figure 7.32, which implies more multiplications and additions are required and hence more memory size. While the dmey filters are symmetrical, which in turn ease the computation, the staggering number of filters' coefficients, which is the longest compactly supported wavelet, make it unfeasible to implement. This assumption is confirmed when dmey were run in parallel to OFDM and both systems have an automated stopwatch where demy responded at a considerable 11.143 seconds where OFDM responded within just 3.320 seconds. The memory consumption were also measured by automated export of computed values to a matrices during operation. It was found that dmey consume enormously vast size of memory compared to OFDM and also those wavelet children with lower filters order. Therefore, dmey is not feasible to deploy in real time transmission systems due to the latency and large memory consumption issues.

The last examined family was Symlets were most children of this family were examined and contrasted with OFDM, as in section 7.3.1.6. As seen previously, in Sections 5.4.3 and 7.3.1.6, that symlets family is similar to Daubechies family and was originally developed by Daubechies [69], however, the differences is only the improved symmetry of the original Daubechies as shown in Figure 7.74 for db45 and Figure 8.10 for sym45 respectively. The overall comparison among the children of this family was as expected followed the Daubechies' children characteristics since this family is a derivative of Daubechies but with improved symmetry of filters' taps. Since it is not completely symmetrical, which could improve time response, but closer to symmetry than Daubechies then it was expected that their

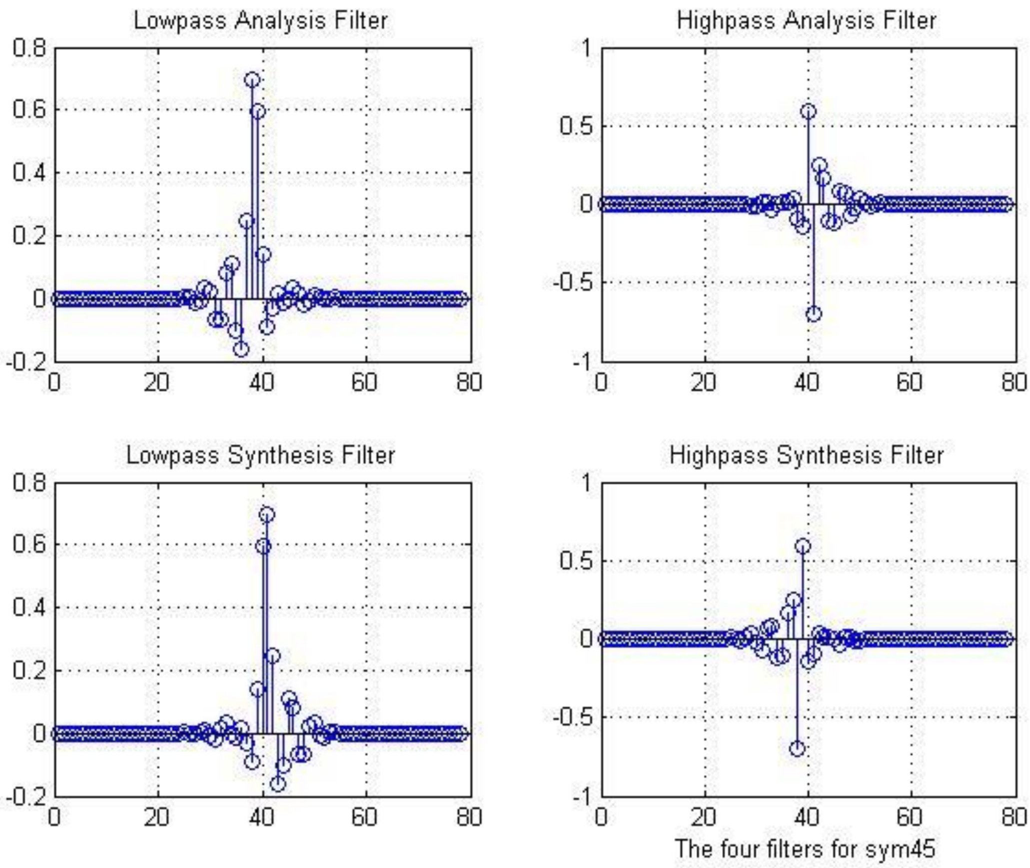

Figure 8.10: sym45 Decomposition and Reconstruction Filters' Taps

performance will be similar which was confirmed indeed by experiments and examining the twin children of both families. The hypothesis were that the lower the vanishing moments the lower time response and memory space requirement for the system to respond. The higher the vanishing moment then the opposite occurs.

Last but not least, in this phase, two type of data mapping were further examined to validate the usability of OWDM on different mappers. 16-QAM and 64-QAM were examined for both OFDM and OWDM with Haar wavelet since Haar proved feasibility and usability. It was found that OWDM is very competitively performing in contrast with OFDM. Over higher SNR the OWDM achieved slightly lower errors rate as opposed to OFDM which achieved slightly higher errors. In comparison with the performance over 16-QAM OWDM performed similarly to its performance over 64-QAM where it achieved lower error rate in the higher SNR. It is concluded form this that the OWDM using Haar wavelet is very likely stands as a robust candidate to replace FFT in OFDM systems. The similarity in performance, even slightly better for OWDM, with the advantage that OWDM does not need guard interval, which saves up to 25% of the bandwidth, gives a very promising potential that one day we could see OWDM standardised for industrial utilisation. Not only that but also the advantage that by using the asymmetrical filters bank dynamic sub-bands allocation is possible and hence more bandwidth saving which adds up to this potential.

8.3 CRITICAL COMPARISON OF OWDM OVER MULTIPATH FADING CHANNELS

Since AWGN is a simulation of just the line of sight propagation between the transmitter and the receiver, however, in the real world scenario this type of propagations are very rare more specifically inside cities and built up areas. Therefore, to enhance the investigation more and replicate a real world scenarios, multipath channel are included in this investigations. Ricean channel, as described in Chapter 7, Section 7.3.2, were considered with a parameters that has been used by three of the industrial leading broadcasters including BBC, ITV and Channel 3&4 whose multiplexers' parameters are 16-QAM, 3/4 rate and 64-QAM, 2/3 rate respectively. The experiment involved increasing the SNR of the channel from 0-30, comparing to a measure of BER taken at the output of the RS decoder. In order to ensure compliance with the standard, the SNR which gave the Quasi Error Free (QEF) state, a BER 10^{-11} at the output of the RS Decoder, was evaluated and compared to the values provided in

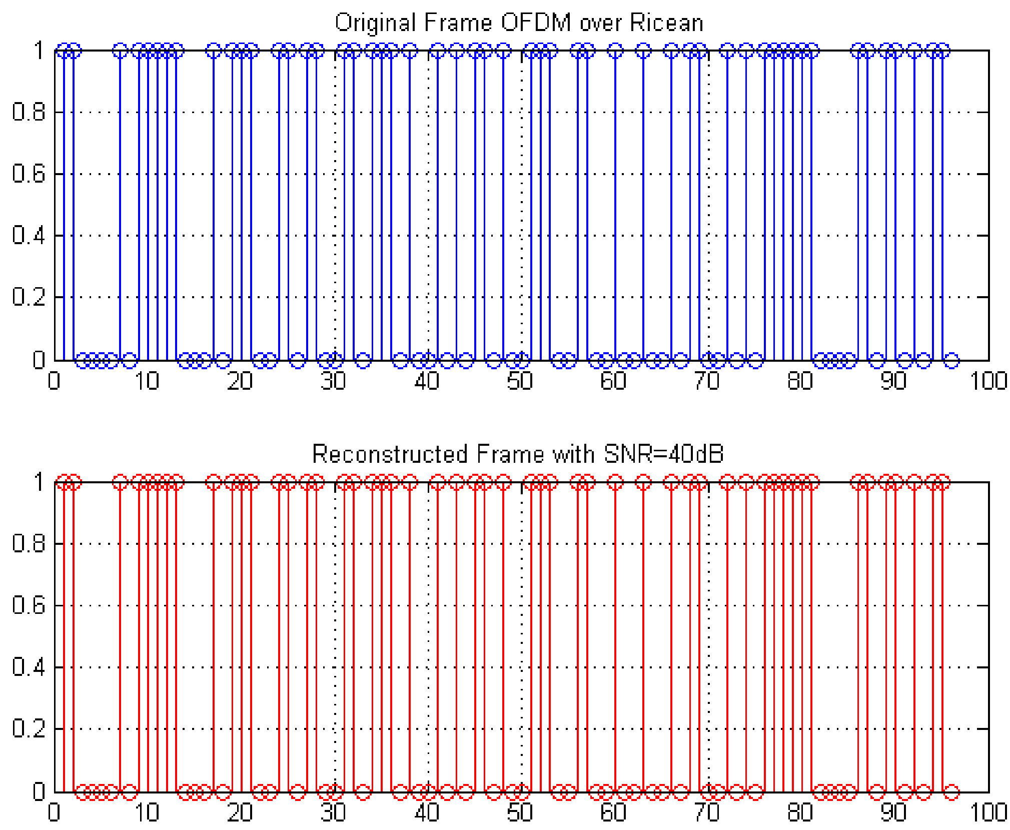

Figure 8.11: OFDM recovered frames over Ricean

the DVB-T standards, which gives all SNRs at QEF state for all combinations of QAM and Inner coding rate.

Once the simulation was compliance validated according to the standard, the IFFT/FFT blocks were replaced with the designed algorithm for computing wavelet transform using filters bank, explained in Chapter 6, Section 6.4. The equivalence of IFFT and FFT in the designed system are the wavelet decomposition filters and wavelet reconstruction filters respectively. The transmitted frames for both multiplexers and over Ricean channel were successfully recovered, as shown in Figure 8.11 for OFDM and Figure 8.12 for OWDM.

This part of the research has provided a comparison between OFDM and OWDM as applied to two pre-2012 UK digital television multiplexes. The purpose of this paper is to demonstrate that OWDM has the potential of acting as a successor for OFDM because of some of the potential advantages it has over OFDM. The computational complexity of wavelet decomposition filters in OWDM is less than Fourier Transform in OFDM. There is also a potential exploitation of guard interval in OWDM instead of wasting it in OFDM.

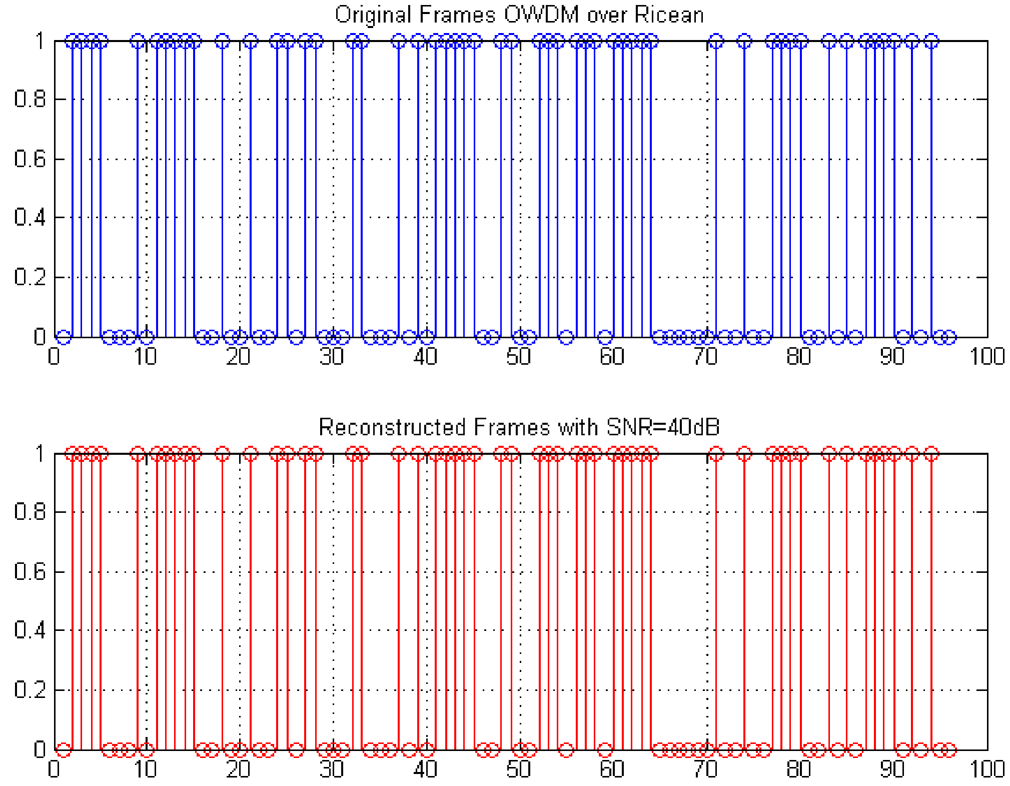

Figure 8.12: OWDM recovered frames over Ricean

Two scenarios have been presented in this part of the research which demonstrates that under AWGN, the performance of OWDM is matched to that of OFDM. Under a Ricean fading channel, however, the results have predicted a small decrease in performance when comparing OWDM to OFDM, as in Figure 7.46 and Figure 7.44. It is suspected that the cause for this is due to the guard interval insertion under OFDM which is used to eliminate multipath [22] but does not exist within the current OWDM model. Further equalization will be required to get a similar response.

Thereafter, another type of complicated multipath model is used, Rayleigh. This time though Doppler shift is present to enhance the investigation more, as described in Chapter 6, Section 6.3.2.1. For more practicality of this research, the simulation is carried out for dynamic bandwidth allocation for different wavelet families and their associated members over a real world scenario wireless channel model. Rayleigh fading model is the most well-known channel model which has different channel variables and parameters depending on the application it is used for. In this research DVB-T Rayleigh model with 2 paths is taken into consideration with adaptive equalizer, Least Mean Squares (LMS), is used for both OFDM and OWDM systems. All available wavelet families and their associated children were presented and discussed with more analysis and discussion presented in section 7.4. All experiments were done for mobile wireless channel with different mobile terminals speeds to imitate real world scenarios, 65 km/h=50Hz, 130 km/h=100Hz and 260 km/h=200Hz as stated in the system design earlier in Chapter 6, Section 6.2. Those chosen speed simulate three most used forms of transports which are:

- Buses which travels inside cities usually with maximum 65 km/h (about 40 miles/h).
- Cars which travels at a national speed limit of 130 km/h (about 80 miles/h).
- High speed trains at an average speed of 260 km/h (about 161 miles/h).

Moreover, one set of data stream used over three different Doppler shifts. This chosen data set is based on the average data used in the previous experiments. Therefore, the data length to be used here is 10k frames = 960 kbits. This process is carried out on each family's member individually and then grouped in one figure to minimise the vast quantity of results and used space. In the next subsections, wavelet families are presented in separate subsection where all results of their associated members are presented and discussed. A further technical observation, discussion and result analysis will be presented in Section 7.4 and Chapter 8 respectively.

8.4 CRITICAL ANALYSIS OF OVERALL COMPARISON OF OWDM VERSUS OFDM

The principle concept behind this research is to examine the asymmetrical filters bank for all available wavelet families with varying depths and to compare their performance with OFDM performance as in DVB-T standard. In the design all parameters for both systems were equivalent. The behaviour of OWDM system is more likely to be the excellent alternative candidate for FFT in OFDM as the system behaves in the same way as OFDM. This was addressed previously by Linfoot *et al.* [3] who examined two levels of dynamic (asymmetric) allocation of sub-bands for only one wavelet family. The difference in this investigative research is that the levels used are six and eleven and the sub-band allocation used are the two types symmetrical and asymmetric respectively. Symmetrical filters bank is where both branches are sub-banded, as explained in Chapter 5, whereas asymmetrical is where one branch is sub-banded. The symmetric allocation allocates a fixed width of sub-bands. This allocation

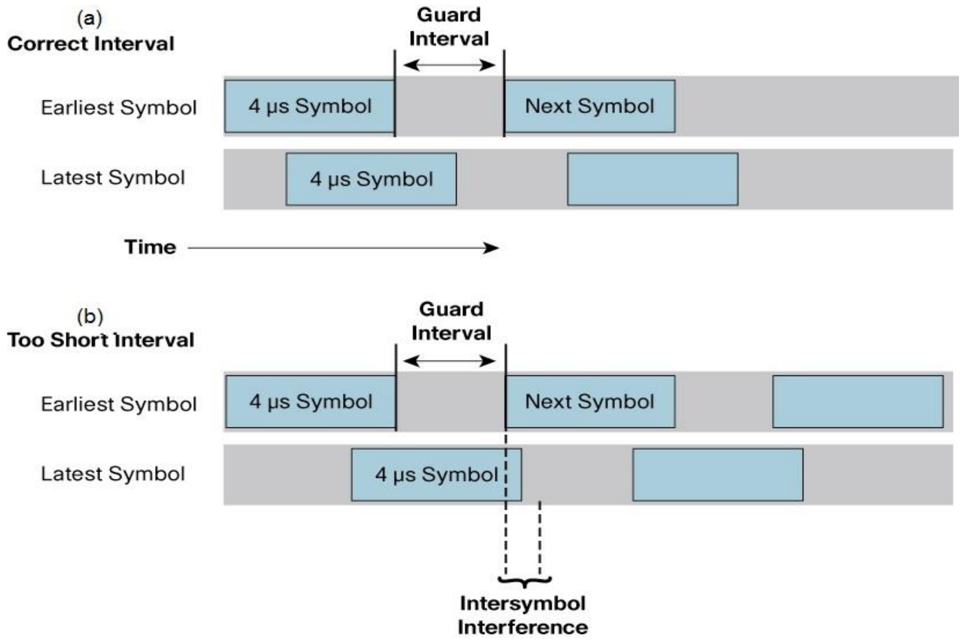

Figure 8.13: Guard intervals needed between successive symbols in OFDM system

method is referred to by some as WPM or WP-OFDM. However, the asymmetrical filters bank allocates dynamic sub-bands with varying lengths and can be used for more flexibility of sub-bandwidth allocations.

The outcome of the simulation model, in general, is countless numbers of results which required further filtering and the best were chosen for further examination. Each curve in those presented figures was a standalone experiment. However, the backbone focus of results involved the comparison between the performances of OFDM with parameters as described earlier, in Chapter 7, in Table 7.1, and the OWDM system as described in Chapter 7, Table 7.2. Since the BER vs SNR can vary slightly, therefore, for maximising accuracy of results, each

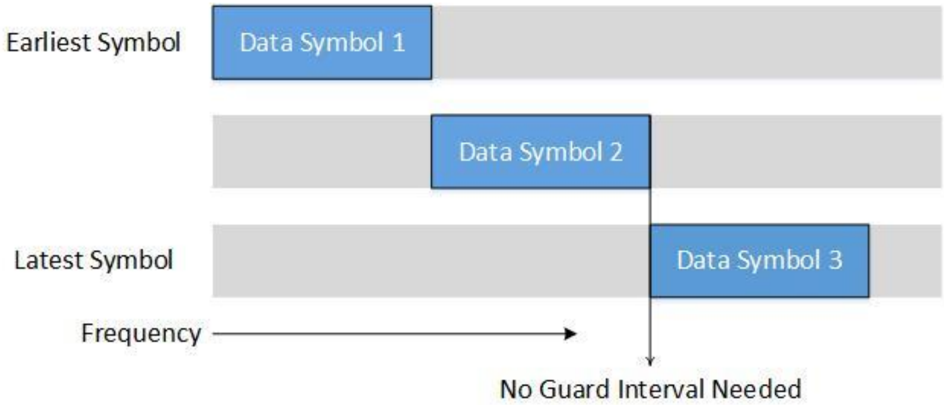

Figure 8.14: OWDM does not need guard intervals between successive symbols

of those experiments was run on average 45 times and the average result was saved for later use. A vast amount of results and curves were produced throughout those experiments and thus grouped in multiple figures, as seen in the previous chapter. These results were partially published in four papers and articles in leading conferences and journals in the field, further article is being processed at ISI journal as the turnaround is about 9 months. However, the most interesting results among wavelet families were that of the Daubechies family including Haar, db1. The comparison associated with the different children of the Daubechies filters were shown in Chapter 7, in Figure 7.4 to Figure 7.13.

In conclusion, of the overall comparison, if the linear complexity of OWDM is taken into consideration along with unnecessary guard intervals, OWDM shows the high possibility of producing a strong candidate as an alternative to OFDM. Figure 8.14 and Figure 8.13 show an illustration of guard interval in OFDM and OWDM. The similarity of performance in most of children with OFDM performance also show the possibility of replacing FFT with FWT. The most suitable, feasible and usable wavelet family is found to be the Haar for several reasons:

1. The simplicity of computations which is just $O(N)$ as opposed to $O(NlogN)$ in FFT OFDM.
2. The low memory space and time consumption, since there is just two filters' taps associated with Haar and hence memory occupation is not as much as if higher filter order are used. Additionally, the symmetry of those two filters' taps make the computation simultaneous.

Further tests are carried out on the QAM demodulator for the same parameters, and with SNR ranging from 0 to 40dB for both systems over AWGN channel. Since Haar wavelet found to be most feasible and suitable family, hence it is taken to a further level of comparison directly with OFDM over AWGN for 10k, 100k and 100k frames, as shown in Figure 8.15. In regard to other wavelet families' comparison, Daubichies 43 and 45, Coflet 4 and 5 and discrete Meyer outperformed OFDM, however, since they are unfeasible and unsuitable to implement due to the large number of multiplications and additions which result in large memory consumption and longer time response which both are not tolerated in real time transmission systems. Having said that, they are still usable for other applications where higher resolutions and precision are required in which memory and time response can be tolerated for the desired higher resolution analysis.

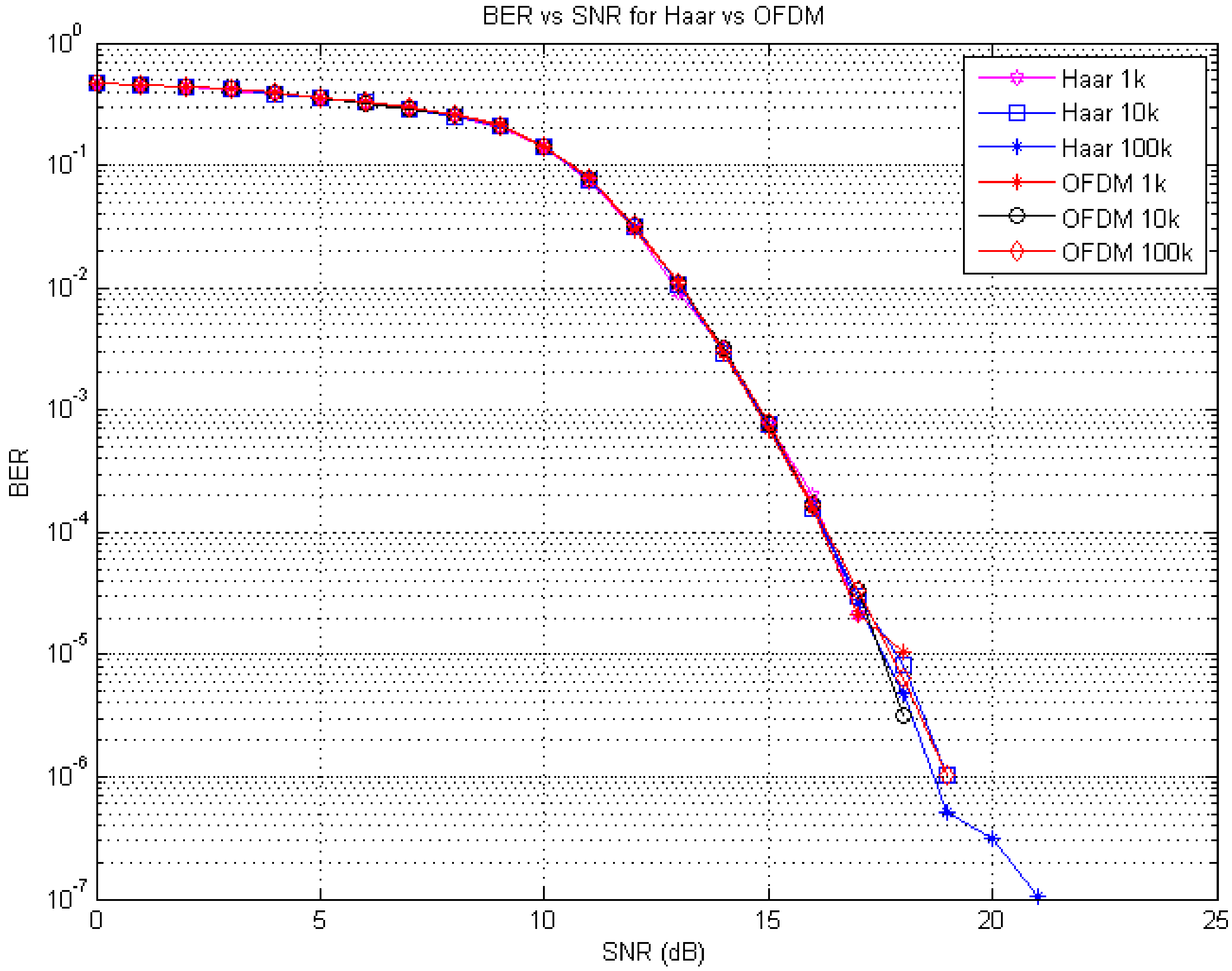

Figure 8.15: Haar versus OFDM performance for 1k, 10k and 100k frames

As stated earlier in Chapter 7, the computation of DWT for Haar is implemented by convoluting the input signal with the filter then split the signal into two halves, upper and lower. The DWT is computed for the input signal by convoluting the input signal with the filter and then split into upper and lower halves. In the filters banks, explained in Chapter 6, Figure 6.18, by denoting the LPF to be $h(n)$ and HPF $g(n)$, then to compute DWT for the input signal with $O(N)$ complexity those $h(n)$ and $g(n)$ must have a constant length. Then the the input signal x is convoluted with $h(n)$ and $g(n)$ at a time, $x*h$ and $x*g$, the each of those operations will cost $O(N)$. After that the filters bank splits the signal into further two branches of size $N/2$. However, it splits the upper branch convolved with $h(n)$ recursively, whereas the FFT splits the upper and lower branches recursively. This leads to this recurrence time relation: $T(N) = 2N + T(\frac{N}{2})$ which is in another word $O(N)$ time complexity for the whole operation which also can be show by the geometric series expansion. For instance if we take the simplest wavelet, Haar, it has linear complexity since $h(n)$ and $g(n)$ have constantly length of 2 [69]:

$$h(n) = \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \quad \text{and} \quad g(n) \left[\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right] \tag{8.1}$$

Since there are a vast range of wavelet families that often confused with each other, who are not all compatible to be used in the context scope of this research, it worth recapping the differences between them. CWT is the continuous form of wavelet transform where it performs convolution at every position and thus costly to implement. Conversely, DWT is used instead where it performs convolution at an approximation discrete portions and thus is more practical to implement. Nevertheless, there are many wavelet families that are probable to construct DWT from. Although they are all discrete, however, those which can be utilised within the proposed system in this research were discussed and presented in Chapter 5 Table 5.3 and Chapter 6 Table 6.4. Those families which referred to as “compactly supported” must satisfy the following: discrete analysis, orthogonality, fast algorithm (FWT), exact reconstruction and FIR filters and in some biorthogonality can also be used. The properties of all compactly supported wavelet families were presented, earlier in Table 5.3 in Chapter 5.

The overall results showed that OWDM is the potential replacement of OFDM since there was significant conservation of bandwidth through the elimination of guard interval which could be up to 25%. In term of wavelet families and their children, the best found candidate is the Haar wavelet, or db1, due to many advantages, these are:

1. It has a single vanishing moment this corresponds to just two filter' taps. This means lower memory requirements and faster time response.
2. Those two filter's taps are symmetrical which also could save more computational time.

Other wavelet families' children, however, outperformed OFDM, nevertheless, they have a larger number of vanishing moments where it could not be used in real time systems due to some latency associated with their use depends on how many vanishing moments there are. This particular has slightly outperformed OFDM over AWGN. However, over the two examined multipath channels, Ricean and Rayleigh, the performance were similar. Unsurprisingly, this family has also outperformed OFDM over both 16-QAM and 64-QAM.

To our surprise, over Ricean and Rayleigh channels wavelet families including Haar performed worse than OFDM. It is suspected that this unexpected performance because of the LSE adaptive equaliser used in both system does not equalise OWDM signal as it does well for OFDM which impacts the signal in which considerable number of errors occur. This is an interesting other research question, however, due to time limitations this question could not be look at deeply on time. Therefore, this could be added to the future work with some other interesting research questions arose during conducting this research.

8.5 RESULTS EVALUATION

The two systems, OFDM and OWDM, were equivalent for like-for-like comparison with full system being considered from frame generation up to frames recovery. As we saw earlier in Chapter 5, Chapter 6 and Chapter 7, that all results have been validated against DVB-T standard [19] which is the state-of-the-art broadcasting standard. The experiment involved increasing the SNR of the channel from 0-30, comparing to a measure of BER taken at the output of the RS decoder. In order to ensure compliance with the standard, the SNR which gave the Quasi Error Free (QEF) state (a BER 10-11 at the output of the RS Decoder) was evaluated and compared to the values provided in the DVB-T standards (which gives all SNRs at QEF state for all combinations of QAM and Inner coding rate).

OFDM cannot operate without guard interval, often called "cyclic prefix" to eliminate the ISI between successive symbols in frequency selective multipath channels. Technically the cyclic prefix is the repetition of the last null part of OFDM symbol add to the front or back of each symbol. The receiver is configured to default discard these cyclic prefix, hence it is a waste of some bandwidth. These cyclic prefix are inserted at a certain rate, in DVB-T case this ranges from 1/32; 1/16; 1/8; 1/4 which can be up to 25% from the used bandwidth. Each guard interval must be longer than the channel's impulse response. It also guard the orthogonality of OFDM sub-carriers by keeping symbols periodic over the extended symbol duration and thus mitigating ICI. However, there are two main disadvantages of using cyclic prefix:

1. Transmitting partially unused bandwidth which results in added cost of more bandwidth requirement.
2. The reduction of SNR since the efficiency will be reduced by repetition of symbols, this loss of SNR can be measured by:

$$SNR_{loss} = -10log_{10}\left(1 - \frac{T_{cp}}{T}\right) \tag{8.2}$$

Where T_{cp} is the cyclic prefix length and $T = T_{cp} + T_s$ which is the length of transmitted symbol.

Since all this was proven to be unnecessary when using OWDM, as discussed in Chapter 3 and Chapter 5, while maintaining the orthogonality, hence there will be a save of bandwidth of up to 25% as opposed to OFDM. Additionally, wavelet transform analyses signals time-frequency domain where better localisation of signal in time and frequency simultaneously is achieved. Hence better multiresolution analysis is achieved which result in less errors and more robustness against noise.

Talk about the linear complexity of wavelet transform and STFT and DFT and how the data growth for each and what does it do for the storage and look up data. Also explain again what was explained in the Time-Frequency Analysis Chapter 5. Talk about different wavelet families and families' member's performance and time consumption during simulation and how the large set of coefficients could affect proportionally the requirements for larger memories and thus more associated cost. Talk also about the trade-off between simplicity of computation and the cost for data storage associated with wavelet families and members.

As the number of vanishing moments increases, the number of coefficients increases too and hence the number of computations and the need for more memory for data storage will increase too which eventually will increase the cost of systems implementation. Not even mentioning the real time transmission systems which cannot tolerate any kind of latency in the system due to the high and very fast and high rate of data transmission.

8.6 SUMMARY

In summary to the results analysis chapter, there were different performance among the children of the same family where some behaves better and some worse than OFDM. Those children with low vanishing moments are more feasible to use with real time in real time transmission systems where no tolerance to time latency. The time latency, space complexity and computational cost are included in this investigation. Since the time and space consumptions were noticed in the lower data rate with higher order wavelet children, the data rate were increased to a factor of x10 three times up to 100k frames, which is capable of transmitting 6 minutes HD video. During the process of these three stages data increase, all wavelet families and their associated children were examined and observed very carefully for the three factors of investigations. It is concluded that wavelet transform using FWT is less complex than FFT in OFDM with computational complexity of just $O(N)$ as opposed to the complexity of FFT of $O(NlogN)$. However, although FFT during operation does not consume more space in memory where the FWT does use more and this depends on the type of wavelet family and children used. The higher number of vanishing moments, filters' taps, for the wavelet used the larger memory is required to store the computed coefficients during operation. This also increase the computational time for the system to respond. The time and space complexities are both directly proportional to the data size that are being processed.

On the other hand, however, the FFT does not consume such time and space during processing and there are other operations that can also be performed identical to FFT operation, i.e. heapsort, quicksort and mergsort, which are all well-established. They store, locate and recall data from memory quicker than FWT. Nonetheless, those wavelet children with very low vanishing moments, filters' taps, found to perform competitively similar to, and even some outperformed, FFT in OFDM. For instance, Haar wavelet has a single vanishing moment, having only two simple symmetrical filters' taps, has slightly outperformed FFT in OFDM achieving less errors rate. Not only that but it also responded within significantly similar time to that of FFT consumed. Surprisingly, the memory consumption during operations is also dramatically similar that FFT occupied. Adding to that, the saving in bandwidth and also the flexibility of sub-bandwidth allocation is one of the main demands that industry and vendors need to cut some budget without sacrificing efficiency. Therefore, Haar wavelet found to be the best candidate among all other wavelet families and families' children for the use in the proposed system in this research.

Those wavelet families' children with higher vanishing moments which consume more memory size and time, further optimisation could possibility increase response time and reduce memory consumption. Further research on the computational cost of wavelet families and their associated children especially those with vast number of filters' taps could be facilitated to enhance space and time computational operation. It is apparent that the linear complexity of wavelet is less than the logarithmic complexity of FFT, however, the FFT is more efficient in memory and time operational cost but not with the computational complexity. Having said that, those wavelet's children with lower filters' taps are competitively less computational complexity, memory and time cost as well, and the most efficient one of those all is the Haar wavelet.

FWT is the fast algorithm that can compute DWT in a faster way than DWT itself and this was to adapt the rapid increase in demand to not only higher data rate communications systems but also faster and reliable processing time that real time system demands, less stacking memory usage and less cost involvement. The less computational cost of FWT of only $O(N)$ than the FFT of $O(NlogN)$ make hardware more efficiently running in term of computational complexity and cost. Nevertheless, this is not the case for all wavelet families and their members where the high number of filters taps induces higher number of multiplications and additions operations which in turn adds to the space complexity and thus larger memory is needed which means more cost. While FWT is less complex than FFT, however, FFT is less complex in space requirement than FWT. However, this is not always the case for all wavelet families and members, where Haar, with just two filters taps, can operate within the same space required for FFT operations. In the context of this research, these type of complexities will be analysed during the analysis of results and a conclusion will be drawn and contrasted with what we discovered in this chapter.

CHAPTER 9: CONCLUSIONS AND FUTURE WORK

Objectives:

- To summarise the research carried out through this thesis.
- To revisit and answer the research questions.
- To list the main contribution of this research.
- To articulate recommendations.
- To draw the future works.

9.1 CONCLUSION OF THE RESEARCH

Wavelet transform naturally analyses signals in time-frequency domain which gives more freedom to localise frequent behaviour of signals in frequency domain. Although STFT does analyse signals in time-frequency domain, it is restricted due to the window size that determines the length of the transform as well as the sweeping over the entire signal, which implies more processing time and computation. Therefore, wavelet transform is considered the current state-of-the-art transform. Utilising this transform in communications systems and more specifically for transmission is one of the vast number of open research areas that are yet to be explored. This research has investigated three main questions, which are; the newly proposed filters bank that is flexible for sub-bandwidth allocations, examining all available wavelet families and their associated children for their suitability and viability to be used in this specific concept and assessing the computation, time and space complexities for those wavelet families and their children.

The research specifications, including aims and objectives that were aimed to be met at the beginning of this research, were discussed through the nine presented chapters. The main motivation of this research is to examine the flexibility of OWDM over different channel environments and also to examine all available wavelet families and their associated children. Another motivation is to assess the suitability and feasibility of implementation of OWDM as real time transmission system and assess the computational cost and time for all those families and their children. The scope and contribution to knowledge, which include the benefit from carrying out this research, were addressed throughout this thesis as follows:

- The computational cost, time and space complexities of both DWT and FFT were presented in Chapter 5 and Chapter 8.
- The performance of different wavelet and their children were presented in Chapter 7 and Chapter 8.
- The comparison of BER vs SNR and system throughput over different channel environments were addressed in Chapter 6, Chapter 7 and Chapter 8.

In the second chapter, a survey of communications and telecommunications systems were conducted to have an overview of the history and the current state-of-the-art technologies. Not only the techniques used among those systems were explored, but also their strengths and weaknesses were reviewed. There have been a series of unstoppable developments of modulation schemes within telecommunications systems that addressed and solved whatever weaknesses or issues a predecessor systems suffered since the first built communications system till the current technology. It was concluded from, Chapter 2, that there are further evidences that wavelet transform can be utilised as transmission scheme and potentially as an alternative to OFDM, scientists and researchers are actively looking at this hot topic during the conduct of this thesis, as can be seen from several researches published during the life of this research and also beyond as discussed previously in Chapter 2 and Chapter 3. DWT as a modulation technique for signal transmission was reviewed, critically discussed and compared to in this research. In DWP modulation cases, it was found that DWT offers better BER vs SNR and better system throughput than FFT in OFDM. However, different wavelet families and the families' members have varying performances. No comprehensive research thus far investigated all wavelet families and families' members neither in DWP modulation nor in the proposed system in this research.

In Chapter 3, an in depth critical review of modulation schemes was presented, focusing in the time-frequency analysis and their advantages and disadvantages. The derived conclusion of that chapter was, some researchers proposed what is called "DWPT" [36] others called it wavelet based FFT [39] [42], wavelet packet based OFDM "WP-OFDM" [60] [61], DWT-OFDM [4], or even Wavelet Packet Multicarrier Modulation "WPMCM" [41]. Nevertheless, whatever terminology is used, the main principle is that the filters banks that are used to synthesis and reconstruct signals are symmetrical and similar to that used by Jain and Myers [36] in their patent. However, in this research the approach is different, where, filters banks are asymmetrically oriented for a more flexible system, flexible bandwidth allocation is used, no guard intervals are used, different channel conditions are investigated and full system is investigated from energy dispersal, Forward Error Correction (FEC), convolutional coding and interleaving to symbol mapping and channel coding. Furthermore, the synthesis filters banks in the transmitter are differently oriented and distributed than what those described researches were considering. The synthesis filters in the transmitter starts with singly level of LPF and HPF, then increases in pairs as the number of levels increases. In the receiver, however, the opposite operation is carried out where the signal to be reconstructed is entered from the highest level and goes through to the lowest level before it is entered into the demodulation (demapping) process.

Some researchers have also investigated multiwavelets based OFDM where they built based on multiwavelet rather than the conventional wavelet. The difference between multiwavelet and wavelet transforms is that in multiwavelet, there are multiple associated wavelets and scaling functions instead of a single wavelet and single scaling function as used in conventional wavelet transform. Although the increase in the number of wavelets and scaling functions can achieve a higher approximation power over the conventional wavelet transform [62], however, this comes at a significant cost of considerably higher memory usage and added computational time complexity and latency, which cannot be tolerated by any real time transmission systems where real time transmission is required. Nonetheless, such types of schemes can be used in image processing and watermarking, although certainly not for transmission systems.

Nonetheless, there are still a considerable number of research questions thus far yet to be answered concerning this particular field of research before OWDM or Asymmetric OWDM could be used in practice. These include, the wavelet families' performance, families' members' performance or the so called "children", which type of wavelets family and family's members are best performing in this modulation scheme and how much difference will it make for different communications applications. Most importantly, what if there is a comprehensive investigation of all wavelet families and families' children that can support orthogonality and can perform outstandingly against OFDM. Furthermore, can there be a system that can add more flexibility to the bandwidth allocation, system efficiency and cost effectiveness? Therefore, this research has been widening the investigation to include not only all wavelet families but also all families' members, which was discussed in the critical analysis chapter. More importantly, this research is also proposing a novel filters banks orientation in order to make the system more effective and dynamic for bandwidth allocation and hence, more cost effective and bandwidth efficient. Those questions were answered in Chapter 5, Chapter 6, Chapter 7 and Chapter 8 respectively.

From the personal perspective of the author, this research has not previously been implemented in exactly the same investigative manner as in this approach. Nevertheless, the wavelet transform has existed previously, although it was used for other applications, e.g. data compression and image compression. It has recently been proposed to be used as a modulation technique for Wireless Communications but no real work has been facilitated on this so far. Thus the author believes that this research is unique and therefore, worthy of implementation and further investigation. To the best of author's knowledge, there has been no consideration of investigating wavelet based OFDM in a fully standardised system that comply with current industrial standards. No error detection and correction, encoding or interleaving were included; instead, most of these researches examined the signal from mapper onward, which is much easier to simulate but not practical. Additionally, there are no previous comprehensive investigation studies carried out on a complete system for all available compactly supported wavelet families and their associated members. Moreover, most of the critically surveyed researches did not consider reliable data size in their investigation, while some other researches did not declare the data size they investigated which indicate either unreliability of the used data size or that it is not known to them. Also, the system examined was a replica of Jain and Meyers patent [36] which uses symmetrical filter banks. Therefore, this research shall consider the full DVB-T system and comply with the DVB-T standard [19] and propose a dynamic bandwidth allocation scheme using asymmetrical filter banks. Moreover, not only all available wavelet families but also all their associated members shall be investigated. The data size examined in this research shall be ranging from 1000 frames with 96 bits per frame to 100k frames, which is equivalent to 6 minutes long HD video.

STFT is primarily DFFT operation, however, with a time limited window that restricts the transform and that window sweeps over the signal to compute STFT for the entire signal. Since this operation consists of multiple DFT operations, proportional to the window size, it adds to the computational complexity of the system and certainly to the latency, which is often not tolerable in real time transmission systems. However, this trades off with the advantage of that it analyses the signals in time-frequency domain which gives much better approximation and resolution of the signal compared to the conventional DFT, which transform the signal from time to frequency domain. Furthermore, STFT is restricted by the window size which cannot be changed once it has been set and this is the main reason which drove researchers and scientists to further develop transforms that are more flexible and adaptable for various applications.

The driving force ended up with the discovery of wavelet transform in both CWT and DWT. Since CWT is significantly costly due to the huge memory needed to store infinite number of coefficients, DWT is used instead. The primary advantage of DWT over STFT is that it performs multiresolution analysis with more flexibility and varying depths, thanks to the wavelet and scaling functions. Apart from that, it also has the possibility of analysing and approximating signals using multiple wavelet and scaling functions, as used in multiwavelets. However, since it is not yet feasible to use multiwavelet for transmission schemes due to its added complexity of computation and consequent memory size and latency occurrence, the conventional wavelet family is considered in this research.

The comparison between the complexities of DWT and STFT and how it operates in hardware context, CPU and memory consumptions and of course more importantly, the cost associated with data buffering and storage were answered in Chapter 5. Data structure of both complexities, $O(N)$ and $O(NlogN)$ are reviewed and compared against each other. It is found that fast wavelet transform has a complexity of $O(N)$ which allows a linear operations. On the other hand, however, DFT found to have a complexity of $O(NlogN)$ which is more complex than wavelet, however, in hardware data structures, FFT operation can be performed by other operations including heapsort, quicksort or mergesort, which are well-established. The practicality of using DWT in real time transmission systems is evaluated. In short, the Big-O notation represents the complexity factor of operations needed to complete a task. A rule of thumb that there is always a trade-off between time complexity and space complexity. Very often, algorithms with $O(N \log N)$ time complexity would have $O(\log N)$ space complexity as we saw with the quicksort algorithm, which presents an issue with processing extremely large datasets, which is always the case for a real time transmission systems where a recursive function runs on the system, often with limited stack memory.

FWT is the fast algorithm that can compute DWT more rapidly than DWT itself and this was to adapt to the rapid increase in demand to not only higher data rate communications systems but also faster and more reliable processing time that real time systems demand, less stack memory usage and less cost involvement. The lower computational cost of FWT of only $O(N)$, rather than the FFT of $O(NlogN)$, makes hardware run more efficiently in terms of computational complexity and cost. Nevertheless, this is not the case for all wavelet families and their members, where the high number of filters taps induces higher number of multiplications and additions operations, which in turn adds to the space complexity and thus larger memory is needed, which means more cost. While FWT is less complex than FFT, FFT is less complex in space requirement than FWT. However, this is not always the case for all wavelet families and members, where Haar, with just two filters taps, can operate within the same space required for FFT operations. In the context of this research, these types of complexities has been analysed during the analysis of results, in Chapter 5, and a conclusion has been drawn and contrasted with what we discovered in the results of experiments, Chapter 7.

Systems evaluations were made according to DVB-T for OFDM and OWDM, where they both have been compiled to, as in Chapter 6, and all results were presented in Chapter 7. The proposed system was first simulated over AWGN channel as prescribed by DVB-T standard [19]. Thereafter, wavelet families and their associated members, described in Chapter 6, Table 6.3 , were considered and results over AWGN for different data length, 100 frames, 1k frames, 10k frames and 100k frames were presented in Chapter 7. The main finding from this part of the research is that wavelet families, as well the children of the same family, differ in their performance. The Haar wavelet was found to be not only the best performing family but also it has less computational time and memory consumption, thanks to its single vanishing moment and symmetry.

Following that, to validate the comparison on more reliable and industrial standards systems, two of the most used multiplexers, in not only the UK but across the world, were built to validate the proposed system against. As described in Section 7.3.2, those multiplexers' values are used by BBC, ITV and Channel 3&4 in the UK and other TV broadcasters use similar multiplexers across the world. The examined wavelet family in this comparison was the Haar wavelet, due to its simplicity and reliability in time and space complexities. The results, however, were very competitive; while OFDM outperformed OWDM slightly, further equalisation of OWDM could be used. To comprehend performance over multipath channels, Rayleigh fading channels were then modelled for all aforementioned wavelet families and their associated children. In terms of data length reliability, two set of data lengths were examined, 1k frames and 10k frames. Doppler Effect frequencies were chosen for three most common maximum speeds, 65 km/h, 130 km/h and 260 km/h, which correspond to in-city buses, national motorway speed for cars and high speed trains, respectively. All the results for all wavelet families and their associated children are presented in Chapter 7. The channel equaliser used in OWDM is the same one used for OFDM, which does not seem to be as effective as with OFDM. Thus, the conclusion of this part of the research is that the channel estimation and the adaptive equaliser for OFDM is probably not compatible with OWDM systems. Therefore, further researches could be facilitated for this standing research question.

It is concluded from the observations that OWDM with Haar wavelet very likely produces a very strong candidate to replace IFFT in OFDM systems. The similarity in performance, even slightly better for OWDM, with the advantage that OWDM does not need guard intervals, which saves up to 25% of the bandwidth, gives a very promising potential that one day we could see OWDM standardised for industrial utilisation. Apart from that, there is an additional advantage that by using the asymmetrical filter banks, dynamic sub-bands allocation is possible, which results in more bandwidth saving. The detailed discussion about this has been included in the results analysis, Chapter 8.

On the other hand, however, other wavelet families or their children with higher vanishing moments can be used in applications other than real time systems where latency and memory space are tolerable, e.g. image watermarking, compression or biomedical imaging. The results, in Chapter 7, have shown that the performance of those families and children with higher vanishing moments is competitively comparable to OFDM, however, OFDM outperforms those in term of processing time reliability and also memory space consumption. This is due to the higher number of multiplications and additions required and asymmetrical filters' taps, which lead to more time latency and higher memory occupation.

Therefore, the conclusion of the main findings in this research are that there are trade-offs between not only one element, but multiple elements and one cannot justify the viability of DWT just because it is less computationally complex than FFT, which does not by any means prove viability. Hardware restricts efficiency but not the standards or theory, since it is true that there are many other crucial elements of the comparison that, to the best of the author's knowledge, have not yet been researched. Those include the computational cost, memory cost and time latency during operations and the reliability of the system. While the computational cost of FFT is less than DWT, Haar wavelet is the simplest wavelet and is proven to compete with FFT in terms of this specific complexity. The memory consumption of this wavelet is very low due to having a single vanishing moment with symmetrical filter's taps, which simplifies the number of multiplications and additions carried out during operation and hence, it can be real time capable and involve low cost of memory. Thus, there is a potential for Haar to be seen implemented in OWDM systems, which have many advantages, these are summed up as:

- Elimination of guard intervals, which results in saving up to 25% of the bandwidth.

- Dynamic sub-bandwidth allocation, which results in more bandwidth savings and control.

- Less system complexity, which implies lower costs with even higher efficiency.

Having said that, those wavelet families' children with higher number of vanishing moments and/or has a symmetrical filter's taps could be used in real time transmission systems but with sacrificing some of errors rate efficiency towards saving some bandwidth and flexibility of sub-bandwidth allocation. The good aspect, though, is that this sacrifice could be compensated by increasing the signal power to achieve higher signal to noise ratio or perhaps by investing more in deploying more efficient error detection and correction algorithms. Those asymmetrical children of higher order are more costly to implement due to the increased requirement of memory size. Having said that, it could still possibly be used for some real time transmission systems with the desired efficiency, by deploying the appropriate wavelet child with the appropriate filters' length. The trade-off between the increased efficiency and flexibility and the added cost of increasing memory sizes and sometimes the power of processors, in OFDM and OWDM, with different wavelet families' children, primarily depends on the range of the required efficiency and the wavelet child's filters length.

9.2 RESEARCH QUESTIONS REVISIT AND EVALUATION

The research questions of this thesis were presented in Chapter 1, Section 1.2. All these 8 questions were answered throughout this research. The questions, their brief answers and the location of their detailed answers in this thesis, are as follow:

1. Is the current IFFT modulation technique used in OFDM system is sufficiently using the bandwidth?
    - The answer was no, because it wastes some unused bandwidth to guard OFDM symbols from interfering with each other, the length of these guards vary and can reach up to 25% of the available bandwidth. This was answered in details in Chapter 2.

2. Would using wavelet transform instead of FFT in OFDM system be more sufficient and would it conserve some bandwidth?

- The answer was yes, because it does not need guard intervals and is less computationally complex. This was answered in Chapter 3 and Chapter 5.

3. Has this modulation scheme been previously proposed?
    - The answer was yes, it was proposed with wavelet packet modulation. This was illustrated and discussed in Chapter 3.

4. Has all available wavelet families and families' members been investigated in term of their computation, time and space complexities?
    - The answer was no, there were some discrete studies that partially discuss some families with wavelet packet modulation but not asymmetrical wavelet. Additionally, to the best of the author's knowledge, there are no comprehensive studies that analyse computational complexity, operational time, operational memory and cost effectiveness of employing wavelet in transmission systems, compared to OFDM. The detailed answers were presented in Chapter 3.

5. Which wavelet family and family's member outperform FFT with lower computation, time and space complexities?
    - The answer of this question was discussed in Chapter 5, Chapter 7 and Chapter 8.

6. Can the current wavelet modulation be further developed to contribute to a cost effective system?
    - The answer was yes, by using asymmetrical filters bank for added flexibility of sub-bandwidth allocations. This was discussed in Chapter 6.

7. Can this modulation be practically implemented and how much could be the complexity?
    - The answer was yes, and this was explained in Chapter 5 and Chapter 6.

8. Which wavelet family and family's member could outperform others families and their associated members and also IFFT?
    - The Haar wavelet proved viable, as explained in Chapter 7 and Chapter 8.

The indication of success as prescribed in Chapter 1, Section 1.5, has been addressed throughout this thesis. The summary of locations of those answers is described in the answers for researches questions above.

9.3 FUTURE WORK

Since wavelet transform has been just recently proposed and has not yet been standardised, there are still many yet to be answered research questions. The first priority for future research will be given to the investigation of multipath channel estimation and equaliser for OWDM systems. Multipath frequency selective fading is one of these issue that even OFDM suffers from, which causes phase off-set. Due to the many advantages that OWDM offers over OFDM, it is the author's main interest to investigate further the estimation and equalisation in frequency selective fading channels. There is also the issue of signal power which needs to be researched when frequency selective fading channel is used, to what level should the SNR be? Also the length of filters bank is another question, what is the maximum sub-banding that filters bank can reach under frequency selective channels, before ICI occur?

While conducting the critical review, there were many potentials for further researches that can be facilitated. The flexibility of wavelet offers many characteristics that can facilitated in many other aspects. Security is one, where sub-banding could possibly be implemented in a way that distribute one signal over many sub-bands, such that the receiver can be configured in an identical reversed way to the transmitter for signal reconstruction, which cannot be duplicated by any other receiver.

Peak to Average Power Ration (PAPR) is one of the problem that current state-of-the-art modulators suffer from, how would it behave if wavelet transform been used? Potentially intelligent system in cognitive radio systems could utilise wavelet filter banks. Spectrum sensing, MIMO OFDM, multiple access OWDM (OWDMA possibly) and LTE-OWDM - all these are some of the spectrum of wavelet applications in wireless communications systems. Fifth generation (5G) has started rolling out in part of Europe; however, 6G could possibly oversee OWDM or use wavelet transform as part of the standard.

Multi-filters banks which could probably increase data rate and data transmission speed rate is another possible avenue of research. Multi-wavelet with asymmetric filter banks can also be research for added flexibility, however, with computational optimisation of the multiple wavelet and scaling functions, if possible. What if 2-D wavelet transform is used for signal transmission, could this be implemented?

REFERENCES

[1] Abbas Hasan Kattoush, Waleed Ameen Mahmoud, Atif Mashagbah, and Ahed Ghodayyah, "Multiwavelet Computed Radon-Based Ofdm Trasceiver Designed and Symulation under Different Channel Conditions," *Journal of Information and Computing Science*, vol. 5, no. 2, pp. 133-145, May 2010.

[2] H. Nikookar and M. K. Lakshmanan, "COMPARISON OF SENSITIVITY OF OFDM AND WAVELET PACKET MODULATION TO TIME SYNCHRONIZATION ERROR," in *The 19 Annual IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Cannes, 2008, pp. 1-6.

[3] Scott L. Linfoot, Mohammad K. Ibrahim, and Marwan M. Al-kaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[4] Khaizuran Abdullah and Zahir M. Hussain, "Studies on DWT-OFDM and FFT-OFDM Systems," in *INTERNATIONAL CONFERENCE ON COMMUNICATION, COMPUTER AND POWER*, Muscat, 2009, pp. 382-386.

[5] Abbas Hasan Kattoush, Waleed A. Mahmoud, and S. Nihad, "The performance of multiwavelets based OFDM system under different channel conditions," *ELSEVIER Digital Signal Processing*, vol. 20, no. 2, pp. 472-482, March 2010.

[6] Homayoun Nikookar, *Wavelet Radio: Adaptive and Reconfigurable Wireless Systems Based on Wavelets*. New York, United States of America: Cambridge University Press, 2013.

[7] The Cisco® Visual Networking Index (VNI), "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2015–2020," Cisco Incorporate, White Paper March 2016.

[8] Juha Heiskala and John Terry, *OFDMA Wireless LANs: A Theoretical and Practical Guide*. Indianapolis, United States of America: SAMS Publishing, 2004.

[9] Akansu A. N. and Agirman Tosun H., "Generalization Discrete Fourier Transform with Nonlinear Phase," *IEEE Transactions on Signal Processing*, vol. 58, no. 9, pp. 4547-4556, September 2010.

[10] K. Toh, *Ad Hoc Mobile Wireless Network: Protocols and Systems*. Englewood Cliffs, New Jersey, United State of America: Prentice Hall, 2001.

[11] Edward A. Lee and David G. Messerschmitt, *Digital Communications*, 2nd ed.: Kluwer Academic Publishers, 1988.

[12] Aashirwad Viswanathan Anand, "A BRIEF STUDY OF DISCRETE AND FAST FOURIER," *School of Mathematics, University of Chicago*, vol. 41, no. 6, December 2010.

[13] Oran E. Brigham, *The Fast Fourier Transform*. New Jersey, United States of America: Prentice Hall Inc., 1974.

[14] James W. Cooley and John W. Tukey, "An Algorithm for the Machine Calculation of Complex Fourier Series," *Mathematics of Computation*, vol. 19, no. 90, pp. 297-301, April 1965.

[15] Robert W. Chang and Richard A. Gibby, "Theoretical Study of Performance of an Orthogonal Multiplexing Data Transmission Scheme," *IEEE Transactions on Communications*, vol. 16, no. 4, pp. 529-540, 1968.

[16] Randy C. (2011, July) Discrete Nonlinear Fourier Transform Characteristics.

[17] N. J. Fliege, *Multirate Digital Signal Processing: Multirate Systems, Filter Banks, Wavelets*. West Sussex, England: John Wiley & Sons, 1994.

[18] Marvasti Farokh A., *Nonuniform sampling: Theory and Practice*. New York, United States of America: Kluwer Academic / Plenum Publishers, 2001.

[19] European Telecommunications Standards Institute, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series) ETSI EN 300 744, 2009.

[20] "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers," , 2006.

[21] IEEE 802.16 standard for Lacal and Metropolitan area networks, "Part 16, Air Interface for Boadband Wireless Access Systems, Amendment 3: Advanced Air Interface," , 2011.

[22] Prasad Ramjee, *OFDM for Wireless Communications Systems*. London, United Kingdom: Artech House Inc., 2004.

[23] L. J. Cimini, "Analysis and Simulation of a Digital Mobile Channel using Orthogonal Frequency Division Multiplexing," *IEEE Transactions on Communications*, vol. 33, no. 7, pp. 665-675, July 1985.

[24] Robert W. Chang, "Orthogonal Frequency Division Multiplexing," Patent 3,488,445 filed 1966, January 6, 1970.

[25] Mischa Schwartz, *Mobile Wireless Communications*. New York, United States of America: Cambridge University Press, 2005.

[26] W. C. Y. Lee, *Mobile Communications Design Fundamentals*. New York, United States of America: John Wiley and Sons Inc., 1992.

[27] Nima Soltani, "Comparison of Single Carrier FDMA vs. OFDMA as 3GPP Long-Term Evolution Uplink," EE359 Project, Project 4th December 2009.

[28] Yupeng Jia, Dongfeng Yuan, Haixia Zhang and Xinying Gao, "Application of Different Basis and Neural Network Turbo Decoding in Nueral Networks," *Second International Symposium on Neural Networks*, vol. Part III, pp. 303-309, May 2005.

[29] T. Cover and A. E. Gamal, "Capacity Theorems for the Relay Channel," *IEEE Transactions on Information Theory*, vol. 25, no. 5, pp. 572-584, September 1979.

[30] Sendonaris E. Erkip and B. Aazhang, "User cooperation diversity-Part I: System Description.," *IEEE Transactions on Communications*, vol. 51, no. 11, pp. 1927-1938, November 2003.

[31] G. Kramer, M. Gastpar and P. Gupta, "Cooperative Strategies and Capacity Theorems for Relay Networks," *IEEE Transactions on Iformation Theory*, vol. 51, no. 9, pp. 3037-3063, September 2005.

[32] S. A. Grandhi, R. Vijayan, D. J. Goodman and J. Zander, "Centralized Power Control in Cellular Radio Systems," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 466-468, November 1993.

[33] G. J. Foschini and Z. Miljanic, "A Simple Distributed Autonomous Power Control Algorithm and its Convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641-646, November 1993.

[34] Mitra, "An Asynchronous Distributed Algorithm for Power Control in Cellular Radio Systems," in *4th WINLAB Workshop*, 1993, pp. 249-259.

[35] T. S. Rappaport, *Wireless Communications: Priciples and Practice*, 2nd ed. Upper Saddle River, New Jersey, United States of America: Prentice Hall, 2002.

[36] Vijay K. Jain and Brent A. Myers, "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," Patent 7058004, June 06, 2006.

[37] Joint Photographic Experts Group Committee, "Joint Photographic Experts Group Standard and Coding System (JPEG2000)," , 2000.

[38] K. Max Wong, Jiangfeng Wu, Timothy N. Davidson, Qu Jin, and P. C. Ching, "Performance of Wavelet Packet-Division Multiplexing in Impulsive and Gaussian Noise," *IEEE TRANSACTIONS ON COMMUNICATIONS*, vol. 48, no. 7, pp. 1083-1086, July 2000.

[39] Rana Shahid Manzoor, Regina Gani, Varun Jeoti, Nidal Kamel, and Muhammad Asif, "Implementation of FFT Using Discrete Wavelet Packet Transform (DWPT) and Its Application to SNR Estimation in OFDM Systems," in *International Symposium on Information Technology*, Kuala Lumpur, August 2008, pp. 1-6.

[40] Lukasz Zbydniewski and Tomasz Zieliński, "OFDM VS Wavelet-OFDM and Circular Wavelet-OFDM in High Speed Communication over Power Lines," in *IEEE Signal Processing Algorithms, Architectures, Arrangements, and Applications Conference Proceedings (SPA), 2009*, Poznan, Poland, September 2009, pp. 67-81.

[41] M. K. Lakshmanan, D. Karamehmedovi´c, and H. Nikookar, "An Investigation on the Sensitivity of Wavelet Packet Modulation to Time Synchronization Error," *Wireless Presonal Communications*, vol. 58, no. 3, pp. 483-502, June 2011.

[42] A. Bajpai, M.K. Lakshmanan, and H. Nikookar, "Channel Equalization in Wavelet Packet Modulation by Minimization of Peak Distortion," in *International Symposium on Personal, Indoor and Mobile Radio Communications*, Toronto, Canada, 2011, pp. 152-156.

[43] John Proakis and Massoud Salehi, *Digital Communications*, 5th ed. New York, United States of America: McGrow-Hill Higher Education, 2008.

[44] Rama Kanti and Manish Rai, "Comparative Analysis of Daubechies wavelets in OWDM with OFDM for DVB-T," *International Journal of Scientific and Engineering Research*, vol. 4, no. 3, March 2013.

[45] Jianguo Liu and Xia Hua, "A Novel Approach to Compute Discrete W Transform," *Circuits, Systems and Signal Processing*, vol. 33, no. 9, pp. 2867-2880, September 2014.

[46] Bhawana Mohanty and Srinivas Ramavath, "A Companding Technique for PAPR Reduction in DWT OFDM Systems," in *IEEE 8th International Conference on Advanced Communication Control and Computing Technologies*, Haryana, 2014, pp. 837-841.

[47] B. Torun, M. K. Lakshmanan, and H. Nikookar, "On the Analysis of Peak-to-Average Power Ratio of Wavelet Packet Modulation," in *European Wireless Technology Conference. EuWIT 2009.*, Rome, 2009, pp. 1-4.

[48] Esteban Gutiérrez, José A López-Salcedo, and Gonzalo Seco-Granados, "Systematic Design of Transmitter and Receiver Architectures for Flexible Filter Bank Multi-Carrier Signals," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. 12, December 2014.

[49] Manuvinakurike Narasimhasastry Suma, Somenahalli Venkatarangachar Narasimhan, and Buddhi Kanmani, "Orthogonal Frequency Division Multiplexing Peak-to Average Power Ratio Reduction By Best Tree Selection Using Coded Discrete Cosine Harmonic Wavelet Packet Transform," *IET Communications*, vol. 8, no. 11, pp. 1875-1882, November 2014.

[50] M. N. SUMA, S. V. NARASIMHAN, and B. KANMANI, "Analytic discrete cosine harmonic wavelet transform based OFDM system," *SADHANA Academy Processings in Engineering Sciences*, vol. 40, no. 1, pp. 173-181, February 2015.

[51] Peijia Zheng and Jiwu Huang, "Implementation of the discrete wavelet transform and multiresolution analysis in the encrypted domain," in *19th ACM international conference on Multimedia*, Scottsdale, Arizona, 2011, pp. 413-422.

[52] R. L. Rivest, L. Adleman, and M. L. Dertouzos, "On Data Banks and Privacy Homomorphisms," *Foundations of Secure Computation*, pp. 169-178, 1978.

[53] Stephane G. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Transactions on Pattern and Machine Intelligence*, vol. 11, no. 7, pp. 674 - 693, July 1989.

[54] Michael B. Martin and Amy E. Bell, "New Image Compression Techniques Using Multiwavelets and Multiwavelet Packets," *IEEE TRANSACTIONS ON IMAGE PROCESSING*, vol. 10, no. 4, pp. 500-510, April 2001.

[55] Abbas Hasan Kattoush, Waleed Ameen Mahmoud Al-Jawher, and Osama Q. Al-Thahab, "A Radon-Multiwavelet Based OFDM System Design and Simulation Under Different Channel Conditions," *Wireless Personal Communications*, vol. 71, no. 2, pp. 857-871, July 2013.

[56] Dinesh Divakaran, "OWSS And MIMO-STC-OFDM: Signaling Systems for the Next Generation of High," Ph.D Thesis, November 2008.

[57] "Long Term Evolution (4G LTE) Standard," , December 2008.

[58] Chinmaya Mahapatra, Victor CM Leung, and Thanos Stouraitis, "An orthogonal wavelet division multiple-access processor architecture for LTE-advanced wireless/radio-over-fiber systems over heterogeneous networks," *EURASIP Journal on Advances in Signal Processing*, vol. 2014, no. May, May 2014.

[59] M. N. Suma, S. V. Narasimihan, and B. Kanmani, "A New OFDM System Based on Discrete Cosine HarmonicWavelet Transform (DCHWT) for PSK and QAM," *Wireless Personal Communications*, vol. 80, no. 4, pp. 1733-1746, February 2015.

[60] Sashuang Wang, Jufeng Dai, Chunping Hou, and Xueqing Liu, "Progressive Image Transmission Over Wavelet Packet Based OFDM," in *IEEE 2006 Canadian Conference on Electrical and Computer Engineering CCECE '06*, Ottawa, 2006, pp. 950-953.

[61] Yupeng Jia, Dongfeng Yuan, Haixia Zhang, and Xinying Gao, "Application of Different Basis and Nueral Network Turbo Decoding Algorithm in Multicarrier Modulation System over Time-Variant Channels," in *Advances in Nueral Networks*, Chongqing, China, 2005, pp. 303-308.

[62] Tai-Chiu Hsung, Yu-Hing Shum, and Daniel Pak-Kong Lun, "ORTHOGONAL SYMMETRIC PREFILTER BANKS FOR DISCRETE MULTIWAVELET TRANSFORMS," in *International Symposium on Intelligent Signal Processing and Communication Systems. ISPACS 2005. Proceedings of December 2005*, Hong Kong, December 2005, pp. 677-680.

[63] Dennis Gabor, "Theory of Communication," *Institute of Electrical Engineering*, vol. 93, pp. 429-457, 1946.

[64] Beigi Homayoon, *Fundamentals of Speaker Recognition*. United States of America: Springer US, 2011.

[65] Mark H. Holmes, *Introduction to Perturbation Methods*. New York, United States of America: Springer-Verlag, 2013.

[66] Austin Mohr, "Quantum Computing in Complexity Theory and Theory of Computation," Southern Illinois University at Carbondale, Carbondale, Technical Paper 2014.

[67] James W. Cooley, Peter A. W. Lewis, and Peter D. Welch, "Historical notes on the fast Fourier transform," *IEEE Proceedings*, vol. 55, no. 10, pp. 1675-1677, October 1967.

[68] S. V. Narasimhan, Nandini Basumallick, and S. Veena, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[69] Ingrid Daubechies, *Ten Lectures on Wavelets*, 7th ed. Philadelphia, Pennsylvania, United Satates of America: Society for Industrial and Applied Mathematics, 2002.

[70] Stephane Mallat, *A Wavelet Tour of Signal Processing: The Sparse Way*, 3rd ed. New York, United States of America: ELSEVIER, December 2008.

[71] Alfred Mertins, *Signal Analysis: Wavelets, Filter Banks, Time-Frequency Transforms and Applications*. New Jersey, United Staes of America: John Wiley and Sons Ltd, 1999.

[72] John J. Benedetto, Christopher Heil, and David F. Walnut, "Gabor systems and the Balian-Low Theorem," in *Gabor Analysis and Algorithms*, Hans G. Feichtinger and Thomas Strohmer, Eds. New York, USA: Springer Science+Business Media, 1998, ch. 2, pp. 85-122.

[73] Steven G. Johnson and Matteo Frigo, "A modified split-radix FFT with fewer arithmetic operations," *IEEE Transactions on Signal Processing*, vol. 5, no. 1, pp. 111-119, 2007.

[74] Matteo Frigo and Steven G. Johnson, "The Design and Implementation of FFTW3," *IEEE Proc.*, vol. 39, no. 2, pp. 216-231, 2005.

[75] S. V. Narasimhan, Nandini Basumallick and S. Veena, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[76] Ingrid Daubechies and Wim Sweldens, "Factoring Wavelet Transforms into Lifting Steps," *Journal of Fourier Analysis and Applications*, vol. 4, no. 3, pp. 247-269, May 1998.

[77] Mark J. T. Smith and Thomas P. Barnwell, "Exact Reconstruction for Tree-structured Subband Coders," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 34, no. 3, pp. 434-441, June 1986.

[78] P. P. Vaidyanathan, *Multirate Systems And Filter Banks*. Englewood, New Jersey, United States of America: Prentice Hall, 1993.

[79] A. Haar, "Die Minkowskische Geometrie und die Annäherung an stetige Funktionen," Goettinger DigitalisierungsZentrum (GDZ), Goettingen, Germany, Technical Report Math Ann. 78, 294-311, 1918.

[80] Abdullah S. Almuttiri and Scott L. Linfoot, "An Advanced Modulation Scheme For Mobile Broadcasting," in *IEEE 17th Mediterranean Electrotechnical Conference*, 13-16 Arpil 2014, Beirut, Lebanon, 2014, pp. 33-38.

[81] A. Haar, "Zur Theorie der orthogonalen Funktionensysteme," Goettinger DigitalisierungsZentrum (GDZ), Goettingen, Germany, Thesis Math. Ann. 69, 331-371, 1910.

[82] A Cohen, I Daubechies, and J C Feauveau, "Biorthogonal basis of compactly supported wavelets," *Communication on Pure and Applied Mathematics*, vol. 45, pp. 485 - 560, 1992.

[83] Hassan F. S., "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, March 2008.

[84] G. BEYLKIN, R. COIFMAN, and V. ROKHLIN, "Fast Wavelet Transform and Numerical Algorithms I," *Communications on Pure and Applied Mathematics*, vol. 44, no. 2, pp. 141-183, March 1991.

[85] D Patranabis, "Symbols and Codes," in *Telemetry Principles*. New Delhi, India: Tata McGraw-Hill, 1999, ch. 3, pp. 69 - 115.

[86] Alec Eden, *The Search for Christian Doppler*. Salzburg, Austria: Springer-Verlag Wien GmbH, 1992.

[87] Lin-Peng Gao and Linfoot S.L., "A Novel Soft Decision QAM Demodulator for DVB-T for multipath system," in *Proceedings of The Ninth International Symposium on Consumer Electronics 2005 (ISCE 2005)*, Macau, 2005, pp. 348-352.

[88] S. B. Wicker, *Error Control System for Digital Communication and Storage*. Englewood Cliffs, New Jersey, United States of America: Prentice Hall, 1995.

[89] Y. T. Chan, *Wavelet Basics*, 3rd ed., Jonathan Allen, Ed. Norwell, Massachusetts, United States of America: Kluwer Academic Publishers Group, 1996.

[90] Gilbert Strang and Truong Nguyen, *Wavelets and Filter Banks*, 2nd ed. Cambridge, England: Wellesley-Cambridge Press, 1997.

[91] Fredric J. Harris, *Multirate Signal Processing for Communication Systems*. New Jersey, United States of America: Prentice Hall, 2004.

[92] J. Zander, "Performance of Optimum Transmitter Power in Cellular Radio Systems," *IEEE Transactions on Vehiclar Technology*, vol. 41, no. 1, pp. 57-62, February 1992.

[93] M J Shensa, "The discrete wavelet transform: wedding the a trous and Mallat algorithms," *IEEE Transactions of Signal Processing*, vol. 40, no. 10, pp. 2464 - 2482, October 1992.

[94] David K Ruch and Patrick J Van Fleet, *Wavelet Theory: An Elementary Approach with Applications*. New Jersey, USA: John Wiley & Sons, November 2009.

[95] Awatif Rouijel, Benayad Nsiri, and Mohammed ET Tolba, "Performance of Iterative Multiuser Detection in OWDM-IDMA System," in *5th International Symposium on Communications and Mobile Network*, Rabat, October 2010, pp. 1-4.

[96] Yan-Lei Li and Yue Wang Ran Tao, "Short-Time Fractional Fourier Transform and Its Applications," *IEEE Transactions On Signal Processing*, vol. 58, no. 5, pp. 2568-2580, May 2010.

[97] Dr. Ifiok Otung, *Communication Engineering Principles*. Basingstoke, United Kingdom: Palgrave Macmillan, February 2001.

[98] Mohamed H. M. Nerma, Nidal S. Kamel, and Varun Jeoti, "An OFDM System Based on Dual Tree Complex Wavelet," *Signal Processing: An International Journal (SPIJ)*, vol. 3, no. 2, pp. 14-21, May 2009.

[99] Michel Misiti, Yves Misiti, Georges Oppenheim, and Jean-Michel Poggi, "Wavelet Toolbox User's Guide," The MathWorks, Inc., Boston, Manual R2015b, September 2015.

[100] Y Meyer, *Ondelettes et opérateurs (English Translation Wavelets and operators)*, Hermann ed., Tome 1, Ed. Cambridge, United Kingdom: Cambridge University Press, 1990.

[101] Murroni Maurizio and Popescu Vlad, "Cognitive Radio Communications for Vehicular Technology – Wavelet Applications," in *Vehicular Technologies: Increasing Connectivity*, Miguel Almeida, Ed.: InTech, 2011, ch. 13, pp. 223-239.

[102] S. L. Linfoot, "Wavelet Families for Orthogonal Wavelet Division Multiplex," *IET Electronics Letters*, vol. 44, no. 18, pp. 1101-1102, September 2008.

[103] S. L. Linfoot, "A Study of Different Wavelets in Orthogonal Wavelet Division Multiplex for DVB-T," *IEEE Consumer Electronics*, vol. 54, no. 3, pp. 1042 - 1047, August 2008.

[104] Abbas Kattoush, "A Novel Radon-Wavelet-Based Multi-Carrier Code Division Multiple Access Transceiver Design and Simulation under Different Channel Conditions," *The International Arab Journal of Information Technology*, vol. 9, no. 3, pp. 225-234, May 2012.

[105] Haleh Hosseini, Sharifah Kamilah, Syed Yusof, Norsheila Fisal, and Ali Farzamnia, "Wavelet Packet-Based Transceiver for Cognitive UWB," *CANADIAN JOURNAL OF ELECTRICAL AND COMPUTER ENGINEERING*, vol. 37, no. 2, pp. 59-64, May 2014.

[106] Fadel S. Hassen, "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, pp. 131-147, March 2008.

[107] Mahmudul Hasan, "Performance Comparison of Wavelet and FFT Based Multiuser MIMO OFDM over Wireless Rayleigh Fading Channel," *International Journal of Energy, Information and Communications*, vol. 3, no. 4, November 2012.

[108] Jaideva C. Goswami and Andrew K. Chan, *Fundamentals of Wavelets: Theory, Algorithms, and Applications*, 2nd ed., Kai Chang, Ed. New Jersey, United States of America: John Wiley & Sons, 2011.

[109] LKCE Ghaziabad and NITTTR Chandigarh, "Ber Performance Analysis of Daubechies Wavelet In Dvb-T For Rayleigh Channel," *International Journal of Engineering Research and Technology*, vol. 1, no. 10, December 2012.

[110] D. Lee Fugal, *Conceptual Wavelets In Digital Signal Processing*. San Diego, United States of America: McGrow Hill, 2009.

[111] Gerard J. Foschini and Zoran. Miljanic, "A Simple Distributed Autonomous Power Control Algorithm and its Convergence," *IEEE Transactions on Vehicular Technology*, vol. 42, no. 4, pp. 641-646, November 1993.

[112] Karthik Divakaran, Naveen Manikandan, and Shri Hari, "Wavelet Based Spectrum Sensing Techniques For Cognitive Radio - A Survey," *International Journal of Computer Science and Information Technology*, vol. 3, no. 2, pp. 123-137, April 2011.

[113] R. V. Dalal, *Orthogonal Wavelet Division Multiplexing (OWDM) for Broadband Wireless Communications*. South Florida: M.S thesis, University of South Florida, 1999.

[114] A Cohen, "Ondelettes, analyses multirésolution et traitement numérique du signal," University of Paris IX, Dauphine, Paris, PhD Thesis 1992.

[115] John R. Barry, Edward A. Lee, and David G. Messerschmitt, *Digital Communications*, 3rd ed.: Springer, 2003.

[116] Mostafa Alimosaymer and Reza Mohseni, "Systematic Approach in Designing Wavelet Packet Modulation-Orthogonal Frequency Division Multiplexing Radar Signal by Applying the Criterion of Least-Squares," *IET Signal Processing*, vol. 8, no. 5, pp. 475-482, October 2014.

[117] Yahya Ali Lafta Al-hussiany, "Novel Dwt -Dapsk Based Transceivers For Dvb- T Transmission And Next Generation Mobile Networks," Ph.D Thesis, December 2012.

[118] Anders E. Zonst, *Understanding Fast Fourier Transform*. Titusville, Florida, United States of America: Citrus Press, 2008.

[119] Jonathan Y. Stein, "The Fast Fourier Transform," in *Digital Signal Processing: A Computer Science Perspective*. New Jersey, United States of America: John Wiley & Sons, 2000, ch. 14, pp. 531-568.

[120] J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*. New York, United States of America: Springer-Verlag Inc., 1993.

[121] Abdullah S. Almuttiri, "A Transmission Scheme for Communications Systems Using Wavelet Transform and Comparison with Fourier Transform," *European Journal of Scientific Research*, vol. 137, no. 3, March 2016.

[122] Abdullah S. Almuttiri and Scott L. Linfoot, "Orthogonal Wavelet Division Multiplex as a Modulation Scheme for Digital Television," in *2013 IEEE International Conference on Consumer Electronics - Berlin (ICCE-Berlin)*, 8-11 September 2013, Berlin, Germany, 2013, pp. 136-139.

[123] Gerhard Doblinger, *MATLAB-Programmierung in der digitalen Signalverarbeitung*. Germany: Schlembach Fachverlag, 2001.

[124] Manju Mathew, A. B. Premkumar, and C. T. Lau, "Scalar Wavelet Packet Based Multi-User Cognitive Radio Transceiver," *International Journal of Engineering and Technology*, vol. 11, no. 5, pp. 70-77, August 2012.

[125] "IEEE Standard for Information technology-- Local and metropolitan area networks-- Specific requirements-- Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 6: Wireless Access in Vehicular," , July 2010.

[126] M. Mathew, A. B. Premkumar, and A. S. Madhukumar, "A study on the performance of Multifilter Bank," in *IEEE International Conference on Communication Systems (ICCS)*, Singapore, 2012, pp. 433-437.

[127] Yong Soo Cho, Jaekwon Kim, Won Young Yang, and Chung G. Kang, *MIMO OFDM wireless communications with MATLAB*. London, United Kingdom: John Wiley & Sons (IEEE Press), October 2010.

[128] Tao Xu, G. Leus, and U. Mitra, "Orthogonal Wavelet Division Multiplexing for Wideband Time-Varying Channels," in *International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Prague, May 2011, pp. 3556-3559.

[129] Ling Wang, Jianming Lu, Yeqiu Li, and Takashi Yahagi, "Multiwavelet-Domain Filtering for Degraded Images with Gaussian Noise," in *IEEE International Conference on Industrial Technology, 2005. ICIT 2005*, Hong Kong, 2005, pp. 28-32.

[130] Chinmaya Mahapatra, Ashwin Ramakrishnan, Thanos Stouraitis, and Victor C. M. Leung, "A Novel Implementation of Sequential Output based Parallel Processing - Orthogonal Wavelet Division Multiplexing for DAS on SDR Platform," in *19th IEEE International Conference on Electronics, Circuits and Systems (ICECS), 2012*, Seville, December 2012, pp. 320-323.

[131] Antony Jamin and Petri Mahonen, "Wavelet Packet Modulation for Wireless Communications," *WIRELESS COMMUNICATIONS and MOBILE COMPUTING JOURNAL*, vol. 5, no. 2, pp. 1-18, March 2005.

[132] Wim Sweldens, "The Lifting Scheme: A New Philosophy in Biorthogonal Wavelet Constructions," in *Proceeding of the SPIE Conference on Wavelet Application in Signal and Image Processing III*, vol. 2569, Bellingham, 1995, pp. 68-79.

[133] N Akansu and MJ Medley, "Wavelet and Subband Transforms: Fundamentals and Communication Application," *IEEE Communication Magazine*, vol. 35, pp. 104-115, 1997.

[134] Yves Meyer, "Principe D'Incertitude, Bases Hilbertien nes et Algebres," *Séminaire Bourbaki*, vol. 28, pp. 209-223, 1986.

[135] Donderdag, "LTE Frequency Utilisation," National Instruments, Technical Report June 2011.

[136] James W. Cooley, Peter A. W. Lewis, and Peter D. Welch, "The Fast Fourier Transform and its Applications," *IEEE Transactions on Education*, vol. 12, no. 1, pp. 27-34, March 1969.

[137] Charles Van Loan, *Computaional Frameworks for the Fast Fourier Transform*. Philadelphia, United States of America: Society for Industrial and Applied Mathematics, 1992.

[138] Jörg Brothers, *Einführung in die analytische Zahlentheorie (Introduction to analytic number theory)*. Berlin Heidelberg, Germany: Springer-Verlag, 1995.

[139] Remazi Arpaci-Dusseau and Andrea Arpaci-Dusseau, "Log-structured File Systems," in *Operating Systems: Three Easy Pieces: Three Easy Pieces*, 090th ed.: Arpaci-Dusseau Books, LLC, 31 March 2015, ch. 43.

[140] P. P. Vaidyanathan, "Quadrature Mirror Filter Banks, M-Band Extensions and Perfect-Reconstruction Techniques," *IEEE Acoustics, Speech and Signal Processing Magazine*, pp. 4-20, July 1987.

[141] Ali N. Akanu and Richard A. Haddad, *Multiresolution Signal Decomposition*, 2nd ed. New Jersey, United States of America: Academic Press, A Harcourt Science and Technology Company, 2001.

[142] Fritz Keinert, *Wavelets and Multiwavelets*. New York, United States of America: CHAPMAN & HALL/CRC, 2004.

[143] A. Jensen and A. la Cour-Harbo, *Ripples in Mathematics: The Discrete Wavelet Transform*. Heidelberg, Germany: Springer-Verlag, 2001.

[144] Franz Hlawatsch and Francois Auger, Eds., *Time-Frequency Analysis: Concepts and Methods*. New Jersey, United States of America: John Wiley & Sons, Inc., 2008.

[145] Michael Weeks, *Digital Signal Processing: Using MATLAB and Wavelets® and Wavelets*. Massachusetts, United States of America: INFINITY SCIENCE PRESS LLC, 2007.

[146] Cooervo. (2016, January) Algorithms and Data Structures. [Online]. http://cooervo.github.io/Algorithms-DataStructures-BigONotation/index.html

[147] Stefano Galli and Oleg Logvinov, "Recent Developments in the Standardization of Power Line Communications within the IEEE," *IEEE Communications Magazine*, vol. 46, no. 7, pp. 64-71, July 2008.

[148] Joe Rosen and Lisa Quinn Gothard, *Encyclopedia of Physical Science*. New York, United States of America: Facts on File, May 2010.

[149] Princeton Journal of Science and Technology. (2016, February) Innovation, Princeton Journal of Science and Technology. [Online]. http://princetoninnovation.org/magazine/2015/12/09/redshift-universes-doppler-effect/

[150] Anthony Teolis, *Computational Signal Processing with Wavelets (Applied and Numerical Harmonic Analysis)*. Boston, United States of America: BirkhAuser Boston, 1998.

[151] Yves Nievergelt, *Wavelets Made Easy*, 2nd ed. Boston, United States of America: Birkhauser, 2000.

[152] Yves Meyer, *Ondelettes et Opérateurs*. Paris, France: Hermann, 1990.

[153] Yves Meyer and R. R. Coifman, *Ondelettes et Opérateurs, tome 3 : Opérateurs Multilinéaires*. Paris, France: Hermann, 1991.

[154] Srinadh Godavarthy, "Generating Spline Wavelets," in *Proceedings of the 36th Annual Southeast Regional*, New York, 1998.

[155] Scott L. Linfoot, Mohammad K. Ibrahim and Marwan M. Al-Akaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[156] Nandini Basumallick and S. Veena S. V. Narasimhan, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[157] Jain V. K. and Myera B. A., "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

[158] Hassan F. S., "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, March 2008.

[159] A. Lee and D. G. Messerschmitt, *Digital Communications*, 2nd ed.: Kluwer Academic Publishers, 1988.

[160] European Telecommunications Standards Institute, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series) V.2 ETSI EN 300 744, 2011.

[161] European Telecommunications Standards Institute, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.

[162] "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers," , December 2006.

APPENDIX A: COEFFICIENTS TABLES

Table A.1: Daubechies coefficients for Daubechies 4, 5, 6, 7, 8 and 9

	Daubechies 4	Daubechies 5	Daubechies 6	Daubechies 7	Daubechies 8	Daubechies 9
h0	0.230 377 813 309	0.160 102 397 974	0.111 540 743 350	0.077 852 054 085	0.054 415 842 243	0.038 077 947 364
h1	0.714 846 570 553	0.603 829 269 797	0.494 623 890 398	0.396 539 319 482	0.312 871 590 914	0.243 834 674 613
h2	0.630 880 767 940	0.724 308 528 438	0.751 133 908 021	0.729 132 090 846	0.675 630 736 297	0.604 823 123 690
h3	−0.027 983 769 417	0.138 428 145 901	0.315 250 351 709	0.469 782 287 405	0.585 354 683 654	0.657 288 078 051
h4	−0.187 034 811 719	−0.242 294 887 066	−0.226 264 693 965	−0.143 906 003 929	−0.015 829 105 256	0.133 197 385 825
h5	0.030 841 381 836	−0.032 244 869 585	−0.129 766 867 567	−0.224 036 184 994	−0.284 015 542 962	−0.293 273 783 279
h6	0.032 883 011 667	0.077 571 493 840	0.097 501 605 587	0.071 309 219 267	0.000 472 484 574	−0.096 840 783 223
h7	−0.010 597 401 785	−0.006 241 490 213	0.027 522 865 530	0.080 612 609 151	0.128 747 426 620	0.148 540 749 338
h8	-	−0.012 580 751 999	−0.031 582 039 317	−0.038 029 936 935	−0.017 369 301 002	0.030 725 681 479
h9	-	0.003 335 725 285	0.000 553 842 201	−0.016 574 541 631	−0.044 088 253 931	−0.067 632 829 061
h10	-	-	0.004 777 257 511	0.012 550 998 556	0.013 981 027 917	0.000 250 947 115

h11	-	-	−0.001 077 301 085	0.000 429 577 973	0.008 746 094 047	0.022 361 662 124
h12	-	-	-	−0.001 801 640 704	−0.004 870 352 993	−0.004 723 204 758
h13	-	-	-	0.000 353 713 800	−0.000 391 740 373	−0.004 281 503 682
h14	-	-	-	-	0.000 675 449 406	0.001 847 646 883
h15	-	-	-	-	−0.000 117 476 784	0.000 230 385 764
h16	-	-	-	-	-	−0.000 251 963 189
h17	-	-	-	-	-	0.000 039 347 320

Table A.2: Symlets wavelet coefficients for sym 2, 3, 4, 5, 6, 7 and 8

	Symlet 2	Symlet 3	Symlet 4	Symlet 5	Symlet 6	Symlet 7	Symlet 8
h0	0.482 962 913 145	0.332 670 552 951	0.032 223 100 604	0.019 538 882 735	−0.007 800 708 325	0.010 268 176 709	0.001 889 950 333
h1	0.836 516 303 737	0.806 891 509 313	−0.012 603 967 262	−0.021 101 834 025	0.001 767 711 864	0.004 010 244 872	−0.000 302 920 515
h2	0.224 143 868 042	0.459 877 502 119	−0.099 219 543 577	−0.175 328 089 908	0.044 724 901 771	−0.107 808 237 704	−0.014 952 258 337
h3	−0.129 409 522 551	−0.135 011 020 010	0.297 857 795 606	0.016 602 105 765	−0.021 060 292 512	−0.140 047 240 443	0.003 808 752 014
h4	-	−0.085 441 273 882	0.803 738 751 807	0.633 978 963 458	−0.072 637 522 786	0.288 629 631 752	0.049 137 179 674
h5	-	0.035 226 291 882	0.497 618 667 633	0.723 407 690 402	0.337 929 421 728	0.767 764 317 003	−0.027 219 029 917
h6	-	-	−0.029 635 527 646	0.199 397 533 977	0.787 641 141 030	0.536 101 917 092	−0.051 945 838 108
h7	-	-	−0.075 765 714 789	−0.039 134 249 302	0.491 055 941 927	0.017 441 255 087	0.364 441 894 835
h8	-	-	-	0.029 519 490 926	−0.048 311 742 586	−0.049 552 834 937	0.777 185 751 701
h9	-	-	-	0.027 333 068 345	−0.117 990 111 148	0.067 892 693 501	0.481 359 651 258
h10	-	-	-	-	0.003 490 712 084	0.030 515 513 166	−0.061 273 359 068
h11	-	-	-	-	0.015 404 109 327	−0.012 636 303 403	−0.143 294 238 351
h12	-	-	-	-	-	−0.001 047 384 889	0.007 607 487 325
h13	-	-	-	-	-	0.002 681 814 568	0.031 695 087 811

h14	-	-	-	-	-	-	−0.000 542 132 332
h15	-	-	-	-	-	-	−0.003 382 415 951

Table A.3: Coiflet scaling filters coefficients

	Coiflet 1	Coiflet 2	Coiflet 3	Coiflet 4	Coiflet 5
h0	−0.072 732 619 513	0.016 387 336 464	−0.003 793 512 864	0.000 892 313 669	−0.000 212 080 840
h1	0.337 897 662 458	−0.041 464 936 782	0.007 782 596 427	−0.001 629 492 013	0.000 358 589 688
h2	0.852 572 020 212	−0.067 372 554 722	0.023 452 696 142	−0.007 346 166 328	0.002 178 236 358
h3	0.384 864 846 864	0.386 110 066 823	−0.065 771 911 282	0.016 068 943 965	−0.004 159 358 782
h4	−0.072 732 619 513	0.812 723 635 450	−0.061 123 390 003	0.026 682 300 156	−0.010 131 117 521
h5	−0.015 655 728 135	0.417 005 184 424	0.405 176 902 410	−0.081 266 699 681	0.023 408 156 788
h6	-	−0.076 488 599 079	0.793 777 222 626	−0.056 077 313 317	0.028 168 028 974
h7	-	−0.059 434 418 647	0.428 483 476 378	0.415 308 407 030	−0.091 920 010 569
h8	-	0.023 680 171 946	−0.071 799 821 619	0.782 238 930 921	−0.052 043 163 181
h9	-	0.005 611 434 819	−0.082 301 927 107	0.434 386 056 491	0.421 566 206 733
h10	-	−0.001 823 208 871	0.034 555 027 573	−0.066 627 474 263	0.774 289 603 730
h11	-	−0.000 720 549 445	0.015 880 544 864	−0.096 220 442 034	0.437 991 626 216
h12	-	-	−0.009 007 976 137	0.039 334 427 123	−0.062 035 963 969

h13	-	-	−0.002 574 517 689	0.025 082 261 845	−0.105 574 208 714
h14	-	-	0.001 117 518 771	−0.015 211 731 528	0.041 289 208 754
h15	-	-	0.000 466 216 960	−0.005 658 286 687	0.032 683 574 270
h16	-	-	−0.000 070 983 303	0.003 751 436 157	−0.019 761 778 945
h17	-	-	−0.000 034 599 773	0.001 266 561 929	−0.009 164 231 163
h18	-	-	-	−0.000 589 020 756	0.006 764 185 449
h19	-	-	-	−0.000 259 974 552	0.002 433 373 213
h20	-	-	-	0.000 062 339 034	−0.001 662 863 702
h21	-	-	-	0.000 031 229 876	−0.000 638 131 343
h22	-	-	-	−0.000 003 259 680	0.000 302 259 582
h23	-	-	-	−0.000 001 784 985	0.000 140 541 150
h24	-	-	-	-	−0.000 041 340 432
h25	-	-	-	-	−0.000 021 315 027
h26	-	-	-	-	0.000 003 734 655
h27	-	-	-	-	0.000 002 063 762
h28	-	-	-	-	−0.000 000 167 443

h29	-	-	-	-	−0.000 000 095 177

Table A.4: Biorthogonal Scaling Filters Coefficients

Biorthogonal 1.3				Biorthogonal 1.5				Biorthogonal 2.2				Biorthogonal 2.4				Biorthogonal 2.6				Biorthogonal 3.3			
h_0	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-3}$	$-\frac{1}{16}\sqrt{2}$	h_0	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-5}$	$\frac{3}{256}\sqrt{2}$	h_{-1}	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_{-2}$	$-\frac{1}{8}\sqrt{2}$	h_{-1}	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_{-4}$	$\frac{3}{128}\sqrt{2}$	h_{-1}	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_{-6}$	$-\frac{5}{1024}\sqrt{2}$	h_{-1}	$\frac{1}{8}\sqrt{2}$	$\tilde{h}_{-4}$	$\frac{3}{64}\sqrt{2}$
h_1	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-2}$	$\frac{1}{16}\sqrt{2}$	h_1	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-4}$	$-\frac{3}{256}\sqrt{2}$	h_0	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-1}$	$\frac{1}{4}\sqrt{2}$	h_0	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-3}$	$-\frac{6}{128}\sqrt{2}$	h_0	$\frac{1}{2}\sqrt{2}$	$\tilde{h}_{-5}$	$\frac{10}{1024}\sqrt{2}$	h_0	$\frac{3}{8}\sqrt{2}$	$\tilde{h}_{-3}$	$-\frac{9}{64}\sqrt{2}$
		$\tilde{h}_{-1}$	$\frac{1}{2}\sqrt{2}$			$\tilde{h}_{-3}$	$-\frac{22}{256}\sqrt{2}$	h_1	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_0$	$\frac{3}{4}\sqrt{2}$	h_1	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_{-2}$	$-\frac{16}{128}\sqrt{2}$	h_1	$\frac{1}{4}\sqrt{2}$	$\tilde{h}_{-4}$	$\frac{34}{1024}\sqrt{2}$	h_1	$\frac{3}{8}\sqrt{2}$	$\tilde{h}_{-2}$	$-\frac{7}{64}\sqrt{2}$
		$\tilde{h}_0$	$\frac{1}{2}\sqrt{2}$			$\tilde{h}_{-2}$	$\frac{22}{256}\sqrt{2}$			$\tilde{h}_1$	$\frac{1}{4}\sqrt{2}$			$\tilde{h}_{-1}$	$\frac{38}{128}\sqrt{2}$			$\tilde{h}_{-3}$	$-\frac{78}{1024}\sqrt{2}$	h_2	$\frac{1}{8}\sqrt{2}$	$\tilde{h}_{-1}$	$\frac{45}{64}\sqrt{2}$
		$\tilde{h}_1$	$\frac{1}{16}\sqrt{2}$			$\tilde{h}_{-1}$	$\frac{1}{2}\sqrt{2}$			$\tilde{h}_2$	$-\frac{1}{8}\sqrt{2}$			$\tilde{h}_0$	$\frac{90}{128}\sqrt{2}$			$\tilde{h}_{-2}$	$-\frac{123}{1024}\sqrt{2}$			$\tilde{h}_0$	$\frac{45}{64}\sqrt{2}$
		$\tilde{h}_2$	$-\frac{1}{16}\sqrt{2}$			$\tilde{h}_0$	$\frac{1}{2}\sqrt{2}$							$\tilde{h}_1$	$\frac{38}{128}\sqrt{2}$			$\tilde{h}_{-1}$	$\frac{324}{1024}\sqrt{2}$			$\tilde{h}_1$	$-\frac{7}{64}\sqrt{2}$
						$\tilde{h}_1$	$\frac{22}{256}\sqrt{2}$							$\tilde{h}_2$	$-\frac{16}{128}\sqrt{2}$			$\tilde{h}_0$	$\frac{700}{1024}\sqrt{2}$			$\tilde{h}_2$	$-\frac{9}{64}\sqrt{2}$
						$\tilde{h}_2$	$-\frac{22}{256}\sqrt{2}$							$\tilde{h}_3$	$-\frac{6}{128}\sqrt{2}$			$\tilde{h}_1$	$\frac{324}{1024}\sqrt{2}$			$\tilde{h}_3$	$\frac{3}{64}\sqrt{2}$

						$\tilde{h}_3$	$-\frac{3}{256}\sqrt{2}$							$\tilde{h}_4$	$\frac{3}{128}\sqrt{2}$			$\tilde{h}_2$	$-\frac{123}{1024}\sqrt{2}$				
						$\tilde{h}_4$	$\frac{3}{256}\sqrt{2}$											$\tilde{h}_3$	$-\frac{78}{1024}\sqrt{2}$				
																		$\tilde{h}_4$	$\frac{34}{1024}\sqrt{2}$				
																		$\tilde{h}_5$	$\frac{10}{1024}\sqrt{2}$				
																		$\tilde{h}_6$	$-\frac{5}{1024}\sqrt{2}$				

APPENDIX B: JOURNAL ARTICLES

The following article is currently under review in the International Journal of Wavelets, Multiresolution and Information Processing (IJWMIP), ISI impact factor 0.742.

International Journal of Wavelets, Multiresolution and Information Processing

COMPARISON OF COMPACTLY SUPPORTED WAVELET FAMILIES AS A MODULATION SCHEME IN WIRELESS TRANSMISSION

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Abstract – Recently, issues surrounding wireless communications have risen to prominence because of the increase in the popularity of wireless applications. Bandwidth problems, and the difficulty of modulating signals across carriers, represent significant challenges.

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Every modulation scheme used to date has had limitations, and the use of the Discrete Fourier Transform in OFDM (Orthogonal

Frequency Division Multiplex) is no exception. This paper investigates a new modulation scheme called OWDM "Orthogonal Wavelet Division Multiplex", which utilizes the wavelet transform as a substitute to Fourier transforms in OFDM.

This paper comprehensively investigates the new modulation scheme, and proposes multi-level dynamic sub-banding as a tool to adapt variable signal bandwidths. Furthermore, all compactly supported wavelet families and members of those families are investigated and evaluated against each other and compared with OFDM.

The evaluation reveals which wavelet families perform more effectively than OFDM, and for each wavelet family identifies which family members perform the best. Based on these results, it is concluded that the wavelet modulation scheme has some advantages over OFDM, such as lower complexity and bandwidth conservation, due to the elimination of guard intervals and dynamic bandwidth allocation, which result in better cost effectiveness.

Keywords: OFDM, OWDM, Wavelet, Orthogonal Wavelet Division Multiplex.

1. INTRODUCTION

Wireless communications systems underwent rapid development at the beginning of the 1960s. Conserving bandwidth resources and transmitting as much information as possible down a single channel was the main objective leading to the discovery and introduction of Orthogonal Frequency Division Multiplexing (OFDM). This modulation scheme resolved a critical problem affecting wireless communication systems at that time, namely multipath propagation.

OFDM utilizes the Discrete Fourier Transform (DFT) within the communication system. It implements the Inverse Fast Fourier Transform

(IFFT) at the transmitter and the Fast Fourier Transform (FFT) at the receiver. Issues arose with the modulation scheme as wireless communication systems became increasingly advanced and more complex. Cyclic prefixes and the insertion of guard intervals between OFDM symbols were introduced in an attempt to resolve these problems. However, these adaptations have proven insufficient to keep up pace with the rapid advancement of wireless communication systems and modulation schemes.

Over the last three decades, the wavelet transform has become increasingly popular in many application areas, including in signal and image processing and in communications. The wavelet transform is the direct result of the efforts to improve on the key shortcomings of Fourier transforms. Unlike Fourier transforms which are only localised in frequency, wavelets are localised in both time and frequency offering true multiresolution signal analysis. Many wavelet families and wavelet children have been developed over time, offering quite a wide choice of wavelets for different applications.

One of the more recent application areas for wavelets is in wireless communication systems. Recently, Linfoot *et al.* [1] proposed using Orthogonal Frequency Division Multiplexing (OWDM) as an alternative to OFDM. This proposal considered a DVB-T system and simulations were carried using a standard Moving Picture Expert Group (MPEG) frame stream. Additionally, the system was investigated without using error detection and correction coding, and without interleaving and channel coding, which are all part of the DVB-T system.

The aim of the research described in this paper was to further investigate and assess the benefits of OWDM using the wavelet transform. The simulation results obtained for different wavelet families are compared with the results obtained for OFDM. The complexity of each modulation scheme was also considered as part of this evaluation. Different wavelet families and their members were investigated at different levels of decomposition. This paper presents a technical discussion, evaluation of the transforms, results validation, and a conclusion summarizing the overall results and recommendations.

1. ORTHOGONAL FREQUENCY DIVISION MULTIPLEX

1.1 History of OFDM

OFDM is a modulation scheme that squeezes the signal through a channel with as little Intersymbol Interference (ISI) as possible. This technology was introduced in 1968 after Chang and Gibby [2] rendered it practical for use as a transmission scheme. The aim of this paper is to find a modulation scheme that is more effective and robust than OFDM, and which can satisfy the rising demand to receive signals on the move.

OFDM has become one of the most popular wireless modulation and transmission schemes over the last two decades. It has been adopted for several world class wireless standards; for example, Digital Video

Broadcasting Terrestrial (DVB-T) [3]; Digital Audio Broadcasting (DAB); IEEE 802.16a Metropolitan Area Network (MAN) and IEEE 802.11a Local Area Network (LAN) standards [4].

Characteristically, channels in the mobile environment receive several signals for the same user at the receiver at different times. This is termed multipath fading, and occurs because of a combination of obstacles and diverse geographical features in the wireless environment. Therefore, a complex equalizer is needed at the receiver to resolve the multipath issue. The activity at the receiver takes longer as the data rate increases. According to Ramjee [5], OFDM offers an option to resolve the multipath issue without the requirement of a complex equalizer at the receiver side.

However, a key problem is that OFDM suffers from ISI. It mitigates this by dividing the bandwidth into N non-overlapping narrow subcarriers and modulates the signals over the subcarriers. These N subcarriers are frequency multiplexed and sent in parallel to produce the OFDM symbol, as will be shown later in section 2. By adding the guard interval to the beginning or the end of each OFDM information symbol, the effect of spreading the delay over the spectrum would subsequently appear as a multiplication in the Fourier domain. Thus, the receiver and the transmitter must be synchronized perfectly. In addition, guard intervals also prevent ISI [6]. The N subcarriers that carry the information signals are orthogonal to one another preventing interference between subcarriers.

1.2 Technical Background

The orthogonality principle states that when subcarriers are orthogonal to one another, the peak of one carrier coincides with the trough of the next. This also applies to consecutive carriers, where the duration between two successive carriers is 1/Ts. Organizing subcarriers in this way guarantees no interference or collision between them.

According to Ramjee [5], the OFDM transmission process is sufficient and effectively conserves bandwidth usage by 50%. The step response of OFDM subcarriers in the frequency domain is a sinc function $\sin\frac{x}{x}$. Consequently, overlap occurs in the frequency domain. This overlap does not cause interference, because the subcarriers are orthogonal to each other [2].

The receiver in OFDM has to use the FFT to return the time waveform to a frequency form. Consequently, the FFT picks up some discrete frequency samples at the carrier peaks. All other subcarriers that pass through zero disregard interference between subcarriers.

FFT is sensitive to the changes in any of the following:

- The integer number of cycles during the symbol period for any symbol.
- The integer number of cycles separating the subcarriers.

OFDM generates subcarriers from the incoming binary stream, supposing R bps. It requires a bandwidth of BT = R(1+r), where r is the roll-off factor for

Nyquist shaping. Subsequently N bits are stored during the Ts = N/R (OFDM symbol interval). Each of these N bits can then be applied individually to modulate a carrier. All the modulated carriers are transmitted simultaneously over the long Ts interval [7].

The following function summarizes the generalization of OFDM:

$$a(n) = \sum_{k=0}^{N-1} a_k \, e^{\frac{j2\pi kn}{N}} \qquad n = 0,1, \dots, N-1 \qquad (1)$$

where: $a(n)$ is the IDFT of ak. ak: k = 1,…,N: represents successive bits stored.

K < N describes successive binary digits stored, generating one of 2^k possible QAM signals. Each signal corresponds to a complex number, ak.

Equation 1 is a form of the Inverse Discrete Fourier Transform (IFFT) [7], and can be evaluated easily using FFT.

In DVB-T, which is used as an example in this paper, the OFDM modulator buffers *M* symbols from QPSK or the QAM modulator, where *M* is the number of subcarriers. Using DVB-T with *M= 1705* QAM symbols [3] will result in 2048-IFFT points in total after zero padding.

2. ORTHOGONAL WAVELET DIVISION MULTIPLEX

The wavelet transform originates from the research carried out by Daubechies and Cohen [8] in the 1990s. The wavelet transform converts the signal from the time domain into the time-frequency domain. Unlike the Fourier transform, wherein the sinusoidal wave repeats itself for infinity, the wavelet transform occurs only within a finite domain, and zeroes are used elsewhere. Hence, it is possible for the wavelet transform to model changes in frequency by altering the time domain scale. Additionally, changes in time can be modelled by altering the position of the wavelets; allowing the frequency and location of the frequency to be modeled. Consequently, this transform refers to the time-frequency domain [9].

The wavelet transform takes two forms: the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The CWT is complex and costly because it performs convolutions at each position due to the infinite scales making its computation very expensive. Whereas, the DWT resolves both complexity and cost issues by discretely storing the signal data which most needs to be processed. In addition, the Nyquist sampling theorem indicates that the highest frequency that can be modelled with a discrete data signal is half of the sampling frequency [9].

The principal benefit of the wavelet transform is that it analyses the signal in the time-frequency domain, which has the advantage of extracting all the key features of the input signal.

In this paper, OWDM is investigated as an alternative to OFDM. Unlike the

OFDM which uses the FFT, OWDM utilizes the discrete wavelet transform. The DWT is utilized to separate sub-bands of bandwidths symmetrically. This task is usually referred to as Wavelet Packets Modulation (WPM) and has previously been proposed and studied by several authors. However, here the analysis involves utilizing DWT to allocate sub-bands with different widths dynamically. The use of DWT offers a lower computational complexity than in OFDM.

Due to the sampling theorem and calculation of scale convolutions at the wavelet transform, the first path that the wavelet picks up at a single sample point is unnecessary. Conversely, the most important scale deemed necessary for consideration is the smallest scale at which the wavelet picks up two sample points. Therefore, it is suggested that this process can be achieved by a combination of orthogonal filters; i.e. the Lowpass Filter (LPF) and the Highpass Filter (HPF). The LPF consists of the results of the scaling function, whereas HPF consists of the results of the wavelet function. Figure 1 shows the proposed system block diagram [10].

Calculation of DWT and IDWT can be achieved using filter banks comprised of LPF and HPF. These filters are finite impulse response (FIR) filters with variable lengths and different coefficients, dependent on the wavelet family and the family member being used. As shown in figure 2, every LPF is subdivided into a further level of HPF and LPF. Each LPF or HPF is treated as an individual signal, and can be traced along the subdivision (sub-bands). These signals do not interfere with each other because of the scaling feature.

The dilation equation (2), also called the refinement equation, acts as a starting point from which to determine the scaling function φ and the wavelet function ψ:

$$\frac{1}{2}\phi\left(\frac{x}{2}\right) = \sum_{n\in Z} w_n \phi(x-n) \qquad (2)$$

The DWT and IDWT filters are closely related to the sequence $(w_n) \in Z$

Whenever φ is compactly supported, the sequence w_n is finite, and could act as a filter. W (normalized) is the scaling filter, and it is a FIR filter with a $2N$ length and it has a coefficients' sum of 1 of a low-pass filter with norm of $\frac{1}{\sqrt{2}}$.

The number of filter bank levels used depends on the application and bandwidth availability. The number of levels can be counted using the single split of LPF and HPF. The OWDM communication system is comprised of two main sections: the synthesis section and a channel interface. The synthesis section consists of a filter bank with a number of inputs and an output that supplies the OWDM signal [11].

Each input receives a symbol associated with the super-symbol, which is selected from the modulation scheme. The synthesis section generates an OWDM signal composed of weighted OWDM pulses. Each weighted pulse represents a symbol in the super-symbol. The OWDM pulses are also included in an OWDM Spread Spectrum [11], known as OWSS. OWSS allows the operation of wireless channels with the use of an equalizer.

As explained by Hassan [12], the filters mentioned above are typically single lowpass filters (ho(i)), such as:

$$h(i) = -1I\ ho(L-1-i)$$

$$(i) = h(L-1-i)$$

$$(i) = -(-1)iho(i) \qquad (3)$$

where L is the length of the filter and 0≤ i ≤L-1.

OWDM is based on the concept of orthogonal multi-pulse signalling [11].

Lee and Messerschmitt [13], considered that pulses, e.g. $\varphi m(t)$, m = 0, 1, . . .

,

$M - 1$, form an orthonormal set over a set period of time, and that each pulse is orthogonal to itself as shifted by the non-zero integer multiples of a certain interval T. Each basis pulse (t), can serve to create a 'virtual' channel, over which the symbol am is carried. The vector of symbols $A = [a_0, a_1, \dots, a_{M-1}]T$ is called a super-symbol, and the interval $T = MT_s$ is the super-symbol/block interval, where T_s is the basic symbol interval.

Subsequently the signal transmitted by the baseband becomes:

$$\mathrm{S(t)} = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m} \ \phi_{m}(t - iT) = \sum_{i=-\infty}^{\infty} A_i^T \ \phi\,(t - iT) \qquad (4)$$

At the receiver, symbol block timing extraction is performed and the received signal is correlated with $(t - iT)$ to detect the nth super-symbol at time iT and time $iT + \tau$, where τ denotes the optimum timing phase [14]. Since the complementary metal oxide semiconductor and very large scale integration implementation of signal processing schemes are often less complicated and more economical in the discrete time domain, a discrete time formulation for orthogonal multi-pulse signaling will be used henceforth. The discrete time orthogonal multi-pulses are $\varphi m(n)$, $m = 0, 1, \dots, M-1$; thus, the corresponding transmitted orthogonal multi-pulse signal becomes:

$$s(n) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m\,(n - iM) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(n - iM) \qquad (5)$$

However, for convenience and simplicity, the variable t will be used to denote both the continuous time variable and the discrete time sample index. In addition, M and T will be used interchangeably to denote block length.

Orthogonal multi-pulse signaling is associated with the following distinct advantages:

- Possibility of dynamic separation of sub-bands;
- Potentially less sensitive to multipath fading;
- No requirement for guard intervals [1];
- Potential to reach channel capacity and counter selective fading; and ▪ Potential for multiplexing at the physical layer.

For a more detailed review of OWDM pulses, refer to Dalal [15]. The OWDM system uses wavelet pulses as basis pulses in orthogonal multi-pulse signaling.

The orthogonality of OWDM arises because the scaling function is orthogonal to the wavelet, or shifting, function. Unlike WPM, in which subbands are fixed, OWDM dynamically allocates various widths of sub-bands because only one path, through first level LPF, is used. Additionally to clarify the dynamic sub-bands allocation; we can assume that one has 16 sub-bands with four levels of filters banks. If those sub-bands were passed along the first LPF path with four levels of filters bank decomposition, the results would allocate these 16 sub-bands dynamically with different widths, as depicted in

Figure 2.

3. DESIGN OF PROPOSED SYSTEM

The OFDM system is used as the standard for comparison with the proposed

OWDM modulation scheme. The implementation of this system takes the DVB-T standard [3] as a baseline. With the OFDM system multiple parameters are involved in operating the system' these depend on the application OFDM is being used for. The quality and capacity of the system can be manipulated by changing some of these parameters. The number of filter bank levels, in OWDM, is directly proportional to the length of IFFT in OFDM: $log_2(N)$ where N is IFFT length.

As a prerequisite, this method of allocation requires the user to satisfy the Nyquist theorem. This condition is defined as $2^M \leq N$, where M is level number and N is number of samples. If the condition is not satisfied then the sampling theorem will not be satisfied resulting in signal distortion. The condition can be satisfied by passing the received signal through the same level of filter banks as those used in the transmitter (figure 3). This is done by starting at the highest level and working down to the lower level until the first level where the original data samples can be recovered is reached, as depicted in figure 4.

The running time chosen for the simulation of the OFDM system will be 100 frames at 96 bits/frame. These values were chosen to identify a certain number of bits per each subcarrier (sub-band). SNR is selected in the range from 0 to 20dB to measure the effect of SNR versus BER for a wider range of SNR values; the results of the tested simulation BER did not exceed 16 or 17 dB.

The parameters for the OWDM system are 100 frames to be coded thus: 96 bits/frame, ¾ convolutional coding, 171 and 133 polynomial generator, 16-QAM modulator, 11 levels of filters banks ($log_2(2048) = 11$), and noise level from 0 to 30dB.

The primary focus of this paper is to examine the performance of all compactly supported wavelet families, and the performance of all wavelet families' members, by comparing them against each other and with the OFDM system. Therefore, the investigation is carried out to test all compactly supported Wavelet families and families' members, including Haar, Daubechies, Symlet, Coiflet, Biorthogonal, ReverseBiorthogonal and Meyer.

4. RESULTS ANALYSIS

The design implementation was flexible, using M level filter banks. In

OWDM, 11 levels of filter banks were chosen in association with a 2048 IFFT length in the OFDM system which follows $\log_2 N$ where N is IFFT length. Those parameters were replicated based on the DVB-T standard [3] which the proposed system was compared with; however, if desired, other standards could also be used. It was observed that the BER performance for an IFFT length of 2048 with 11 level filter banks was better than for 512 IFFT length with 9 levels of filter banks. Therefore, the results presented related to 2048 IFFT in OFDM and 11 levels of filter banks in OWDM.

The main comparisons made were:

1. Comparison of performance in the Additive White Gaussian Noise (AWGN) channel.

2. The overall throughput based on M with different levels of filter banks in OWDM;

3. Comparison of different Wavelet families against OFDM and against each other; and

4. Comparison of different Wavelet family members against OFDM and against each other.

The following wavelet families were proven by [10] to be orthogonal, compactly supported and suitable for multi-resolution analysis: Haar,

Daubechies, Symlet and Coiflet. Biorthogonal, ReverseBiorthogonal and

Discrete Meyer are sometimes compactly supported orthogonals, but not generally. The name of any given family refers to the family and its family members; for example db*N* denotes a Daubechies wavelet where *N* describes a number of vanishing moments in both scaling and wavelet functions and every vanishing moment has two filter taps or coefficients. However, for some families the number of vanishing moments differs between the synthesis wavelet and the analysis wavelet. An example of this is the Biorthogonal and ReverseBiorthogonal families; for example bior3.5 has three vanishing moments in the synthesis and five in the analysis wavelet. Figure 6 shows the scaling and wavelet functions and impulse response for the db4 as an example. As discussed by Daubechies [10], db4 has the following filter coefficients: 0.1629, 0.5055, 0.4461, -0.0198, -0.1323, 0.0218, 0.0233 and -0.0075. Figure 7 shows the original scaling function, and the decomposition and reconstruction filters for db4 with a DFT modulus for each.

The first observation to report was that in OFDM, as shown in Figure 8 (blue diamond curve), last error occurs when SNR is 16dB and the total number of errors is 17 bits in error out of a 10k bit stream. Similarly, the Haar wavelet (green squared curve), had the same number of errors as OFDM, with some lower error rates at 10dB and 12dB. However, 15dB SNR returned a slightly better performance than OFDM. This is because the Haar wavelet is the simplest wavelet, as it has just a single vanishing moment. This means it has just two filter taps, and so every two taps represents a single vanishing moment. Consequently, it was found that the chosen IFFT length (OFDM system) matched the chosen number of filter bank levels in OWDM, because of the similar behaviour of both systems, meaning both systems are interchangeable.

When examining the results of the wavelet families' members' performance, there is a clear variation in performance between families and also between family members within a wavelet family. In figure 8, the chosen wavelet families, as based on their performance against their members, are compared together with and against OFDM. Overall performance can be divided into three different behaviours. OFDM, Haar, ReverseBiorthogonal 2.4, Symlet 20 and Discrete Meter behave similarly, with all having a probability of 4 bits in error out of each

10k of transmitted bits, with a 16dB SNR. This is with the exception of Haar, which showed a slightly better performance of 3.7 errors for the same transmitted bit stream with a 15dB SNR.

If the linear complexity of OWDM is taken into consideration along with unnecessary guard intervals, OWDM shows a higher probability of producing a strong candidate as an alternative to the logarithmic complexity of IFFT in

OFDM. A further investigation into all compactly supported orthogonal wavelet and wavelet families' members db*N*, sym*N*, coifN, bior*Nr.Nd* and rbio*Nd.Nr* is shown in figures 9, 10, 11, 12 and 13 respectively, and confirms this hypothesis.

Duabechies family members' extremal phase (db*N*) wavelets are in the order *N* for up to 45 vanishing moments, 90 filter taps. Although the number of filter coefficients for db45 is very large, and hence increases computational complexity compared to that of the lower orders of vanishing moments, this complexity remains linear and below the logarithmic complexity of IFFT in OFDM. Figure 14 shows the 90 filter taps for the LPF and HPF decomposition and reconstruction filters for the db45 family. The reconstruction filter is the quadrature mirror filter of the decomposition filter has no symmetry. The Haar wavelet, also known as as db1, is the only Daubechies wavelet that is symmetrical [10].

The Symlet family (sym*N*), where *N* is the number of vanishing moments, is referred to as Daubechies' least asymmetric wavelets. Symlet filters are closer to symmetry than the Daubechies extremal phase filters. Similar to the Daubechies extremal phase, Symlets have an order *N* of up to 45 vanishing moments [10]. Figure 10 shows the performances of the various Symlet families' members. To further analyze the performance we looked at how the filters for this family decompose signals, Figure 15 shows the decomposition for Symlet20 and the reconstruction filters for both LPF and HPF, which clearly show the near symmetry in the filter taps distribution. The differences between this family's children's performance are illustrated in Figure 10, which shows a clear tipping point in performance occurring between the symlet12, which has 12 vanishing moments, and symlet16, which has 16 vanishing moments. Meanwhile, the symlet2 to symlet12 group of wavelets show very similar performance. Similarly, simulations show that the symlet16 to symlet20 wavelet children group behaves very similarly performance wise. It is clear that as the number of vanishing moments increases above the tipping point, the performance increases substantially.

The Coiflet family has the least number of members among all other wavelet families, with only 5 members. Unlike wavelet families that were previously discussed, the Coiflet wavelets (coif*N*) have a number of vanishing moments of 3*N* with *N* up to 5, so that coif5 has 3*5=15 vanishing moments, and hence it has 30 filter taps [16]. The performance of this family is shown in Figure 11, which reveals that all members of this family perform very similarly. This is because the numbers of vanishing moments are very close to each other. Figure 16 shows the near symmetry of the filter taps for coif5 in the decomposition and reconstruction filters.

Unlike the previously explored families, the Biorthogonal (biorNr,Nd) and Reverse Biorthogonal (rbioNd,Nr) wavelets can have a different number of vanishing moments and filter taps in the analysis and respectively synthesis filters, this can be know from the name, i.e. biorNr,Nd where Nr is the number of vanishing moments for reconstruction filters and Nd is the number of vanishing moments for decomposition filters. These can be determined by their naming structure, i.e. bior3.5, has 3 vanishing moments in the reconstruction filters and 5 vanishing moments in the decomposition filters.

Although the Haar wavelet is the only orthogonal family with a linear phase, Biorthogonal wavelets with a linear phase can be designed [8]. A larger number of vanishing moments can be used in analysis (decomposition) filters for sparse representation and smoother wavelets for reconstruction. The ReverseBiorthogonal family, denoted as rbioNd.Nr, is obtained by reversing the Biorthogonal wavelet pairs. Like in the case of the Biorthogonal family, Nd represents the number of vanishing moments for the decomposition filters, while Nr is the number of vanishing moments for the reconstruction filters.

Figures 12 and 13 show the performance of the Biorthogonal and ReverseBiorthogonal family members respectively. In Figure 12, the performance of Biorthogonal family's children is clearly divided into two groups; one for those children with higher vanishing moments in the reconstruction filters (order 4, 5 and 6) and the other for those with lower vanishing moments (1 and 2). Results show that the first group that has a higher number of vanishing moments in the reconstruction filters, comprised of bior4.4, bior5.5 and bior6.8 children, substantially outperforms the second group which contains the children with lower vanishing moments, bior1.3, bior1.5 and bior2.2. Reverse Biorthogonal wavelets on the other hand, perform very similarly to each other with some minor variation occurs between rbior1.1, rbior2.4 and rbior6.8. While Biorthogonal wavelets make use of the smoother filters for reconstruction, this is no longer the case for the Reverse Biorthogonal wavelets due to the reversed filter pairs. Their reconstruction filters are less smooth. Furthermore, as a result of the filter pair reversal, the vanishing orders of the reconstruction filters are also different between the two families. The end result is a different behavior of the Reverse Biorthogonal wavelets compared to their Biorthogonal counterparts, as illustrated in figures 12 and 13.

Note that in order to keep figures legible and the length of this paper manageable, the results included in this paper do not show all wavelet children for all wavelet families. Only the most representative wavelet children are presented. The other wavelet children show a similar behavior to the one described for their respective family.

5. CONCLUSION

This paper examined and compared different wavelet families and wavelet children with varying decomposition levels and further compared their performance against the OFDM system. In the system design, all parameters for both systems were equivalent and OFDM system was validated against the standard [3]. The behavior and performance of the OWDM system suggests that OWDM is a viable alternative to OFDM, and even though performance wise the OWDM results are very similar to OFDM, these results are obtained with reduced computational complexity compared to OFDM. OWDM complexity was previously addressed by Linfoot *et al.* [1] who examined two levels of dynamic (asymmetric) allocation of sub-bands, considering data entered directly to QAM mapping.

The difference in this paper is that eleven levels were used, the sub-band allocation was asymmetric, and the LPF branch was utilized. The asymmetric allocation results in sub-bands of various widths, which enables flexibility of bandwidth allocation. With regard to comparing wavelet families, the db40 wavelet family promised to provide the most efficient performance amongst the other wavelet families followed by a similarly performing db10, db21 and db30. The Haar wavelet offered the closest performance to OFDM although with less acceptable SNR. This is the simplest wavelet ever built, it is symmetrical and exhibits the fastest response time due to the single vanishing moment and use of only two filter taps (two coefficients).

The differences in wavelet performance reflected the differences in the properties of all wavelet families and family members. The properties that differed between families and members within families, which can explain the behaviour shown in the simulations, can be summarized as follows:

- The symmetry and asymmetry of the wavelet; Haar, Biorthogonal, Reverse Biorthogonal and Discrete Meyer families are symmetrical whereas the reset of families are asymmetrical.
- The phase linearity of the perfect reconstruction filters;
- The number of vanishing moments: wavelets with more vanishing moments, facilitate sparser representation of a greater class of signals;
- Wavelet regularity: the smoother the wavelets the sharper the frequency resolution they provide; and
- Existence of a scaling function, φ.

Based on the OWDM simulation results, the best performing wavelets and wavelet children are found to be those who have a higher vanishing moment but at the same time are symmetrical or near symmetrical. A good example to illustrate this is db45 vs db40. Db45 is far from symmetry but db40 is closer to symmetry, and although their number of vanishing moments is similar, the results are significantly different. Haar, DMeyer, Biorthogonal and ReverseBiorthogonal are all symmetrical wavelet families.

Finally, OWDM offers the dual advantage of eliminating the guard interval, which is mandatory in OFDM, and also having the added advantage of a lower (linear) computational complexity compared to that of the IFFT in

OFDM, which make OWDM a viable alternative to OFDM.

REFERENCES

[1] Scott L. Linfoot, Mohammad K. Ibrahim and Marwan M. Al-Akaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[2] R.W Chang and R.A Gibby, "Theoretical Study of Performance of an Orthogonal Multiplexing Data Transmission Scheme," *IEEE Transactions on Communications*, vol. 16, no. 4, pp. 529-540, 1968.

[3] ETSI EN 300 744 V1.6.1, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.

[4] IEEE 802.16 standard for Lacal and Metropolitan area networks, "Part 16, Air Interface for Boadband Wireless Access Systems, Amendment 3: Advanced Air Interface," , 2011.

[5] Prasad Ramjee, *OFDM for Wireless Communications Systems*. London, United Kingdom: Artech House Inc., 2004.

[6] L. J. Cimini, "Analysis and Simulation of a Digital Mobile Channel using Orthogonal Frequency Division Multiplexing," *IEEE Transactions on Communications*, vol. 33, no. 7, pp. 665-675, July 1985.

[7] Mischa Schwartz, *Mobile Wireless Communications*. New York, United States of America: Cambridge University Press, 2005.

[8] A Cohen, I Daubechies, and J C Feauveau, "Biorthogonal basis of compactly supported wavelets," *Communication on Pure and Applied Mathematics*, vol. 45, pp. 485 - 560, 1992.

[9] Nandini Basumallick and S. Veena S. V. Narasimhan, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[10] Ingrid Daubechies, *Ten Lectures on Wavelets*, 7th ed. Philadelphia, Pennsylvania, United Satates of America: Society for Industrial and Applied Mathematics, 2002.

[11] Jain V. K. and Myera B. A., "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

[12] Hassan F. S., "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, March 2008.

[13] Edward A. Lee and David G. Messerschmitt, *Digital Communications*, 2nd ed.: Kluwer Academic Publishers, 1988.

[14] John R. Barry, Edward A. Lee, and David G. Messerschmitt, *Digital Communications*, 3rd ed.: Springer, 2003.

[15] R. V. Dalal, *Orthogonal Wavelet Division Multiplexing (OWDM) for Broadband Wireless Communications*. South Florida, United State of America: M.S thesis, University of South Florida, 1999.

[16] G BEYLKIN, R COIFMAN, and V ROKHLIN, "Fast Wavelet Transforms and Numerical Algorithms I," *Communications on Pure and Applied Mathematics*, vol. 44, no. 2, pp. 141 - 183, March 1991.

[17] David K Ruch and Patrick J Van Fleet, *Wavelet Theory: An Elementary Approach with Applications*. New Jersey, USA: John Wiley & Sons, November 2009.

[18] M J Shensa, "The discrete wavelet transform: wedding the a trous and Mallat algorithms," *IEEE Transactions of Signal Processing*, vol. 40, no. 10, pp. 2464 - 2482, October 1992.

[19] Y Meyer, *Ondelettes et opérateurs (English Translation Wavelets and operators)*, Hermann ed., Tome 1, Ed. Cambridge, United Kingdom: Cambridge University Press, 1990.

[20] A Cohen, "Ondelettes, analyses multirésolution et traitement numérique du signal," University of Paris IX, Dauphine, Paris, PhD Thesis 1992.

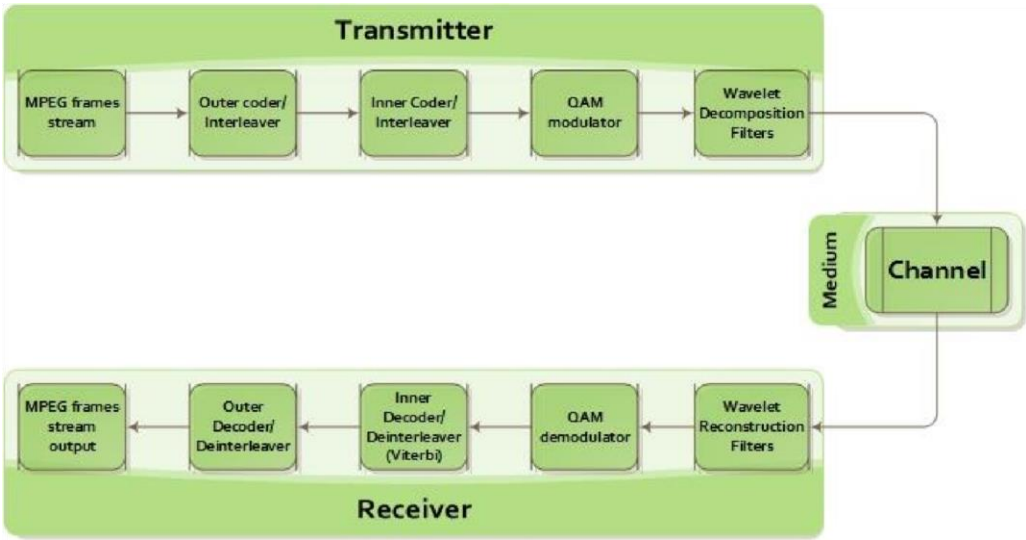

Figure 1: OWDM system block diagram

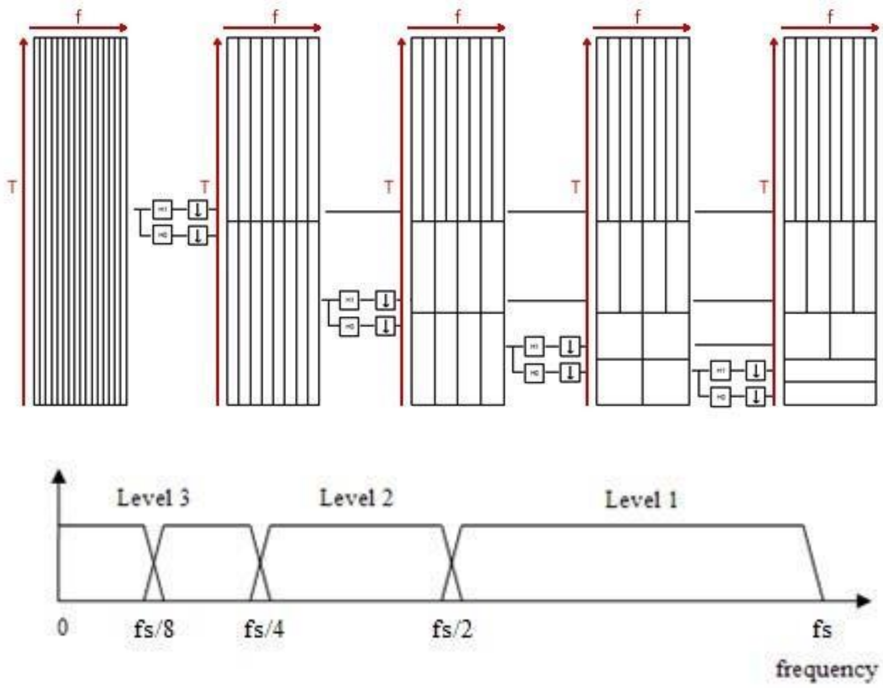

Figure 2: Sub-bands Domain allocation in Time-Frequency Domain

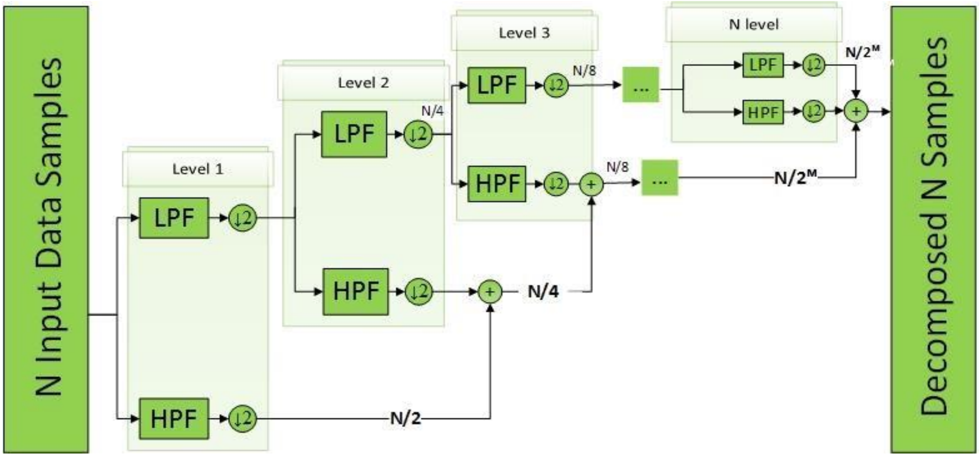

Figure 3: OWDM: M levels of Synthesis Filters Bank

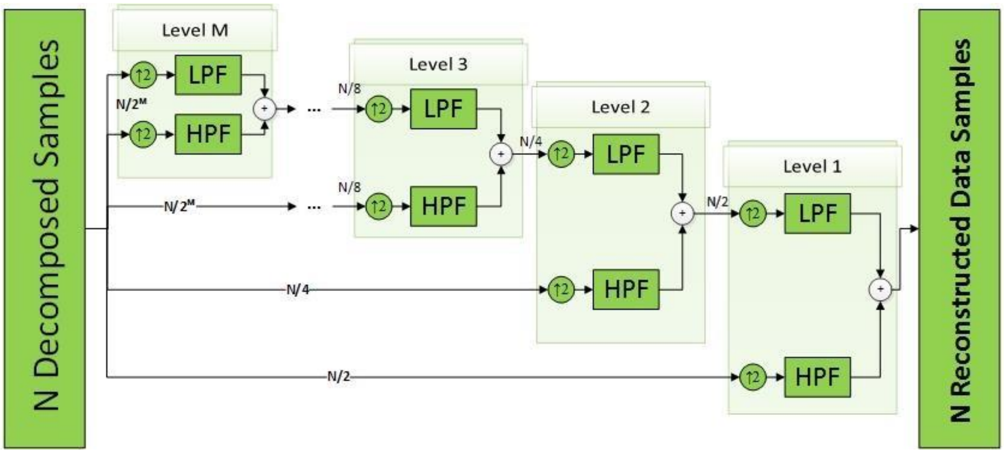

Figure 4: OWDM: M levels of Decomposition Filters Bank

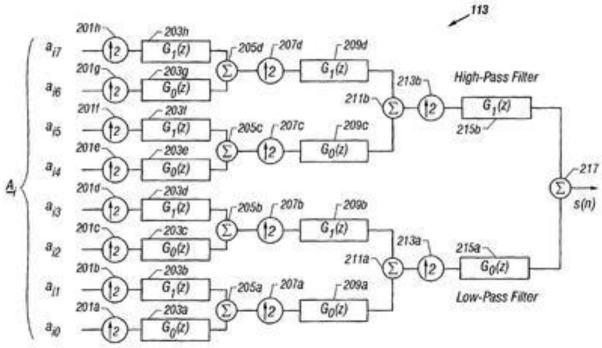

Figure 5: Wavelet Symmetric Synthesis (Wavelet Packets) [9]

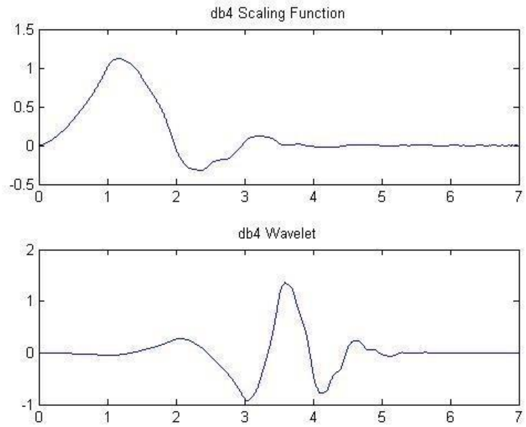

Figure 6: db4 Wavelet and Scaling Functions Impulse Response

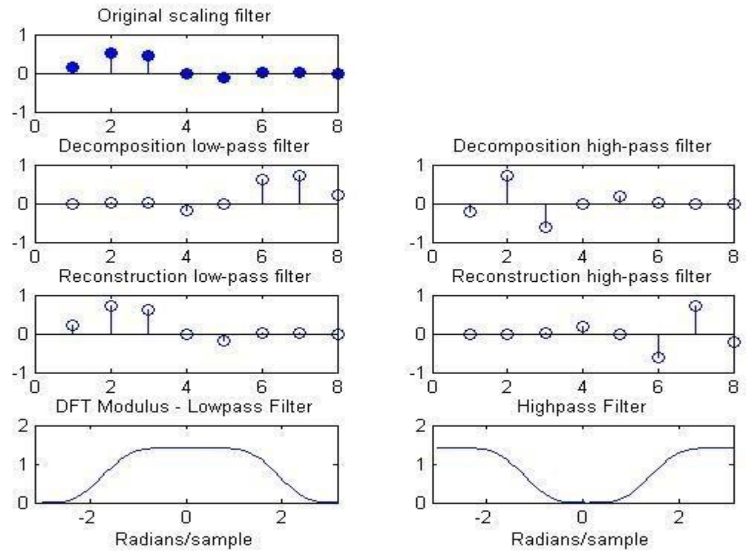

Figure 7: db4 Decomposition and Reconstruction of LPF and HPF with DFT Modulus

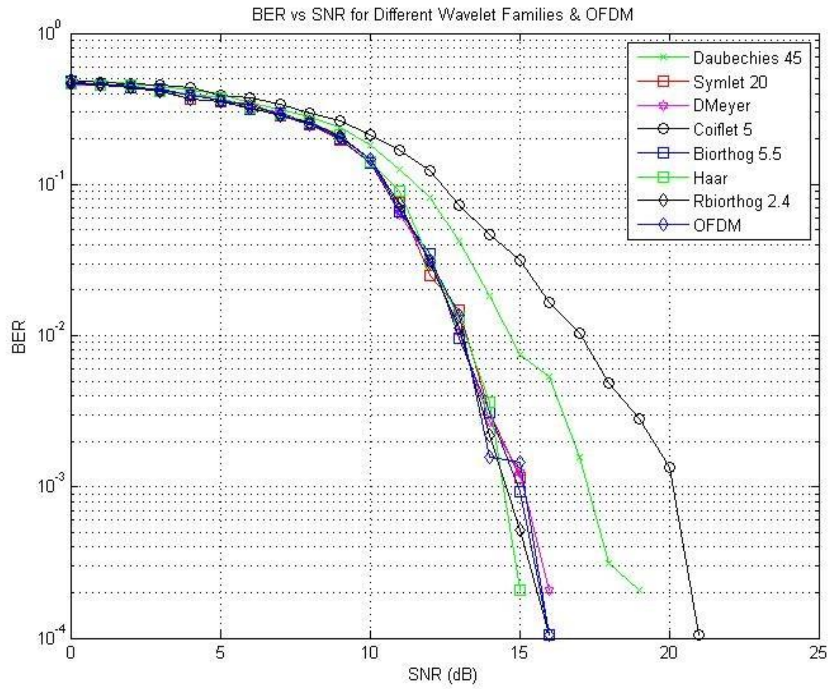

Figure 8: All Wavelet Families Compared to OFDM Performance

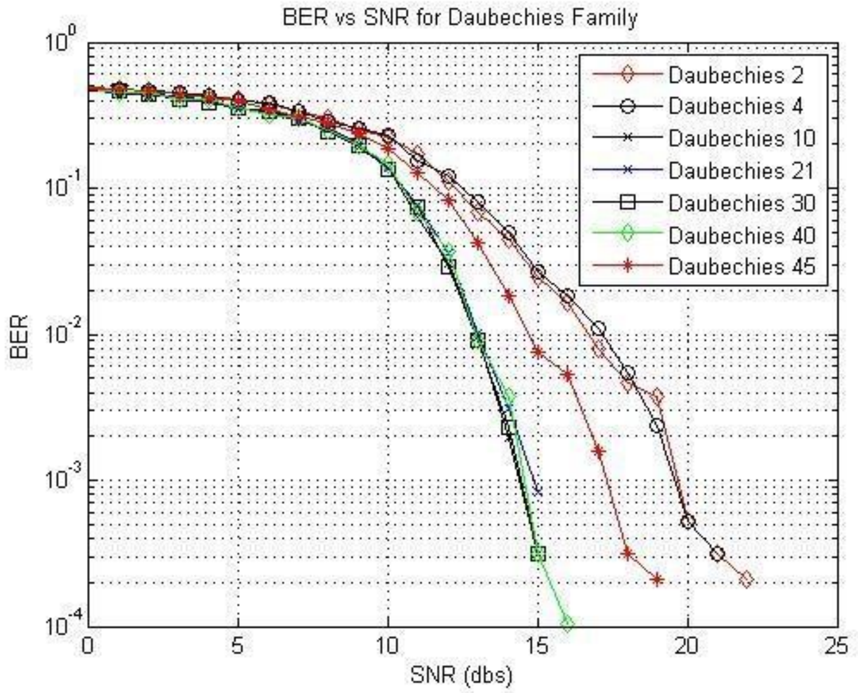

Figure 9: Daubechies Family's Members Performance

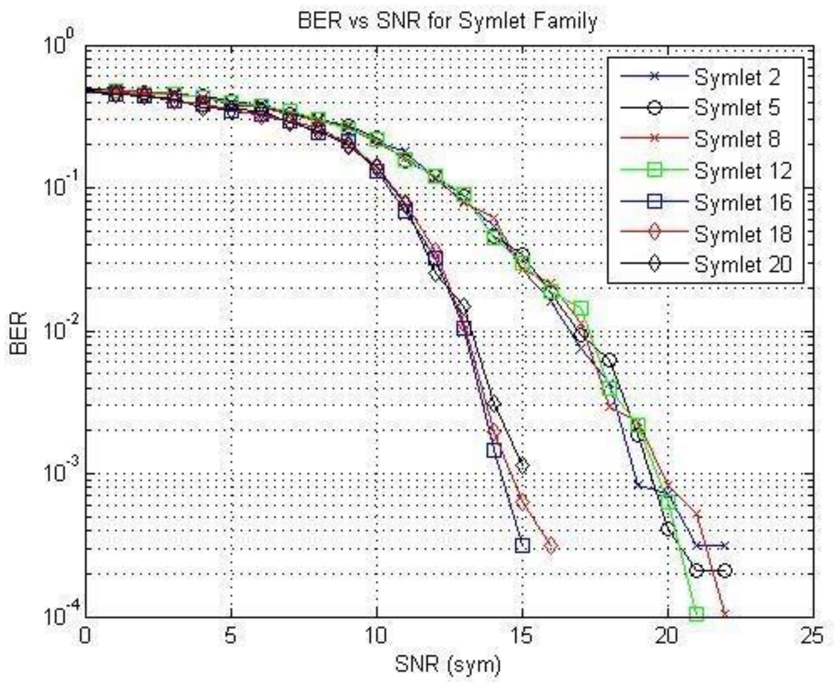

Figure 10: Symlet Family's Members Performance

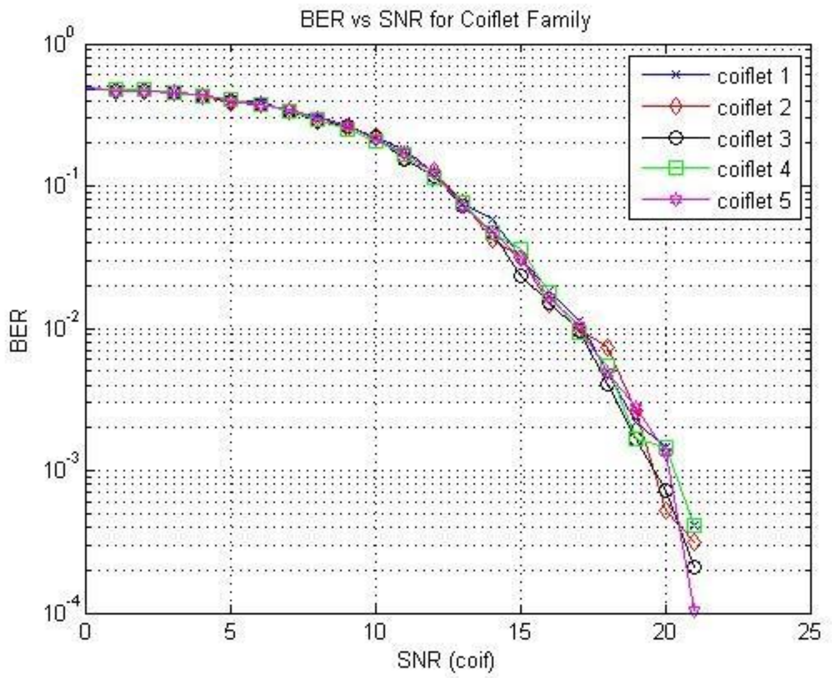

Figure 11: Coiflet Family's Members Performance

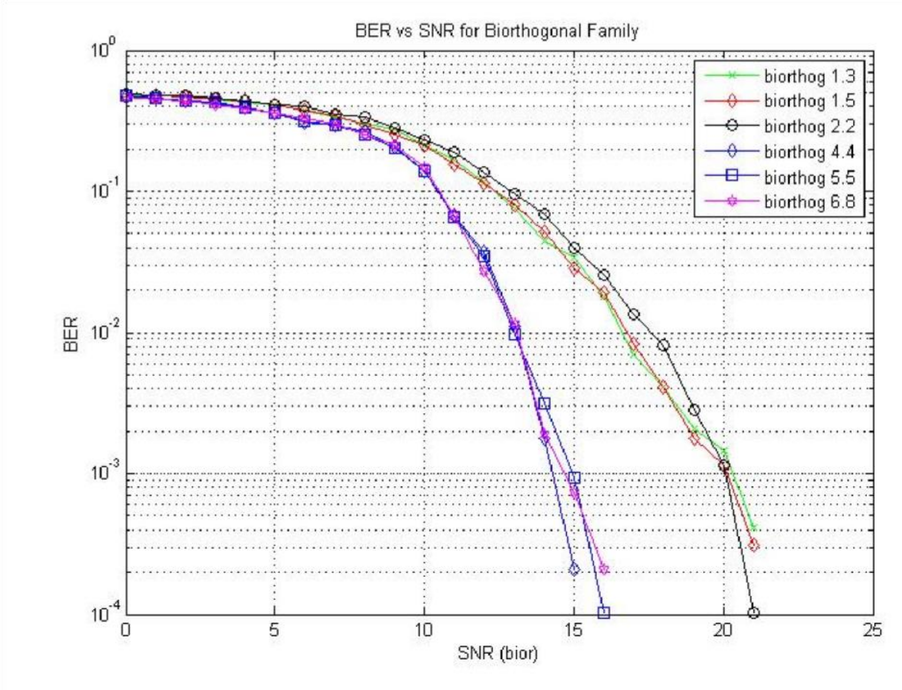

Figure 12: Biorthogonal Wavelet Family's Members Performance

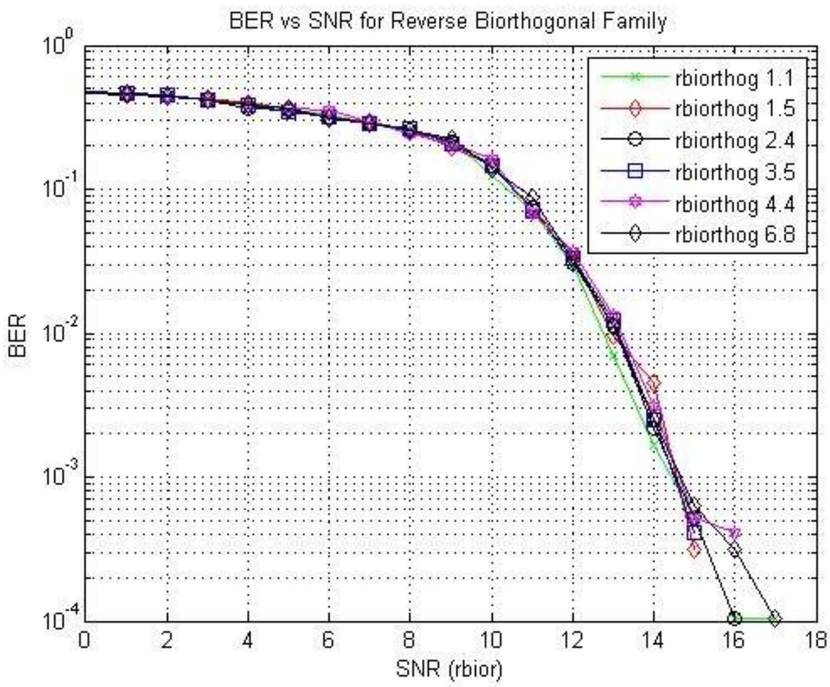

Figure 13: Reverse Biorthogonal Family's Members Performance

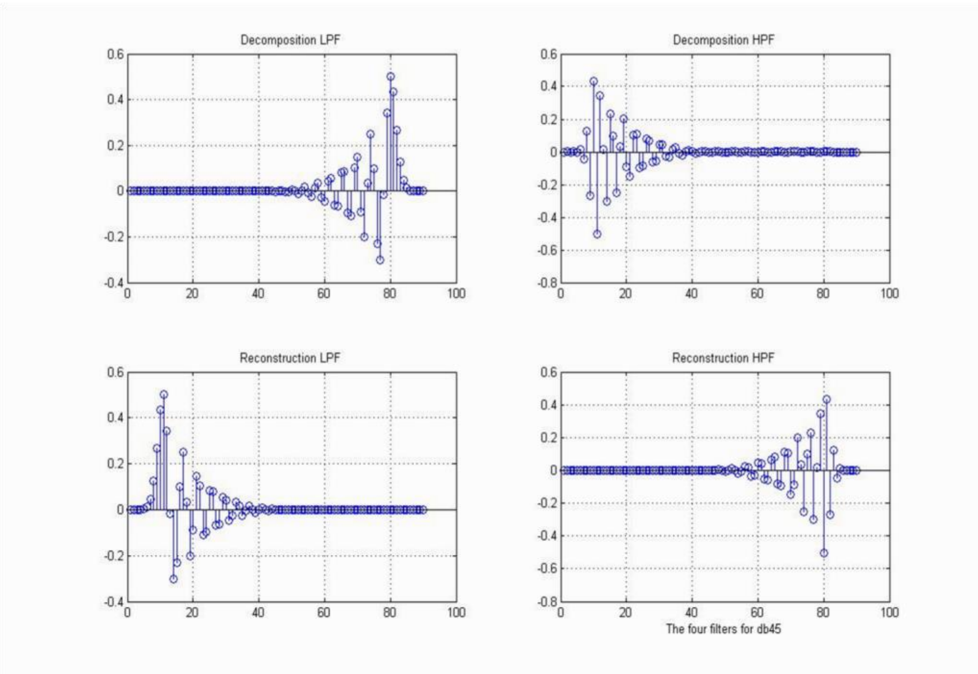

Figure 14: db45 Family Filters' Impulse Response

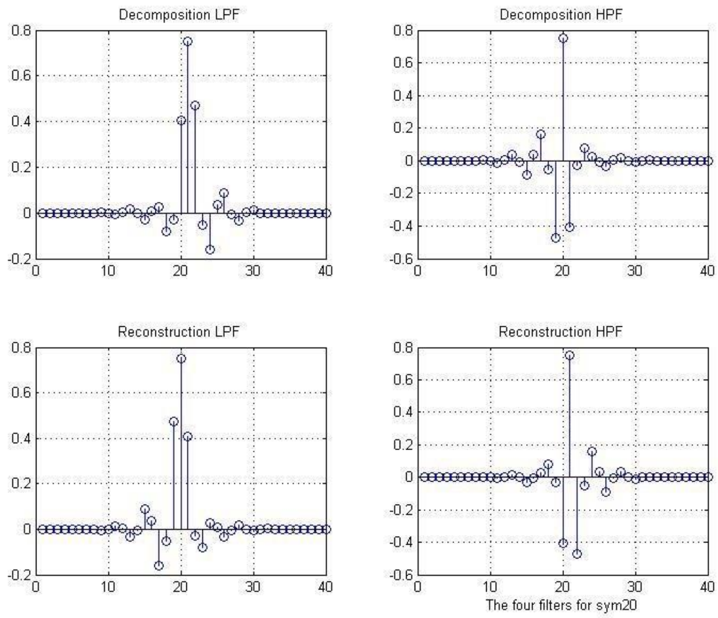

Figure 15: Symlet20 Filters' Impulse Response

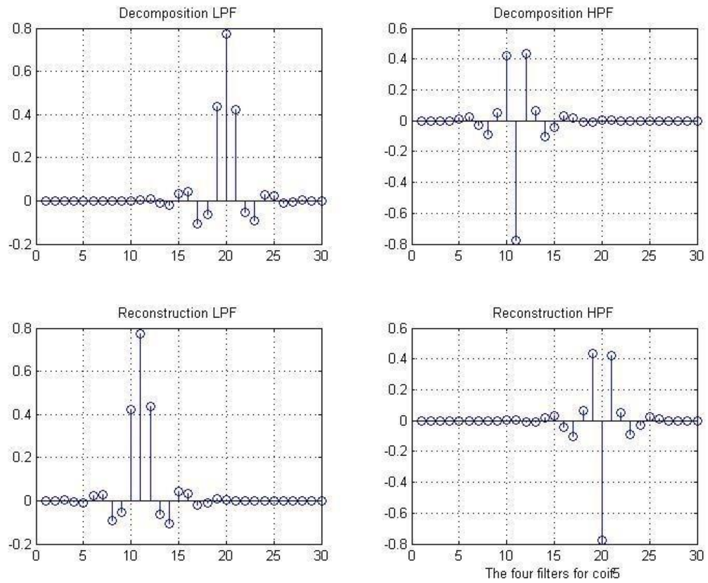

Figure 16: Coiflet 5 Filters' Impulse Response

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A Transmission Scheme for Communications Systems Using Wavelet Transform and Comparison with Fourier Transform

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Abstract

Over the last three decades the demand for wireless communications has increased dramatically. Consequently, the need for adapting such a demand has also increased putting pressure on scientist and researchers to develop more advanced techniques that are capable of handle such a demand. However, the bandwidth limitation increased the pressure on developers as it is the focus point and band limited of any communication system. During the 1970s scientists came up with Orthogonal Frequency Division Multiplex (OFDM) technology with the use of Fast Fourier Transform (FFT) which was remarkable discovery at that time as it saves up to 50% of bandwidth. Since then OFDM is the adapted technology in most of communication standards including DVB-Ts, LAN, MAN, Wi-Fi, WiMAX and VANET. Short-Time Fourier Transform (STFT) was developed due to the need for analyzing the signal in time-frequency domain in order to visualize the features of any signal. However, due to the fact that this technique is time limited and localized due to the usage of fixed time sliding window. The discovery of Wavelet transform is a result of the need for further development from the limitation of Fourier Transform (FT) and STFT. It is being used now in several applications across signal and image processing, i.e. image and video compression, watermarking and medical applications. This paper suggests the use of Discrete Wavelet Transform (DWT) to modulate signals for transmission. Orthogonal Wavelet Division Multiplex (OWDM) is proposed and DVB-T standard is used for baseline comparison with OFDM system. It is shown that OWDM modulation technique could save up to 25% of the bandwidth over conventional OFDM system. Investigating this brand new technology could open a new window for new modulation technique with lower complexity, bandwidth conservation, flexibility and cost effectiveness.

Keywords: OFDM, Discrete Wavelet Transform, Short-time Fourier Transform, STFT, OWDM, Wavelets, DWT, Orthogonal Wavelet Division Multiplex.

Introduction

Wireless Communication Systems have been dramatically developing since the start of 1960s. During 1960s researchers and scientists developed a brand new modulation technique to adapt the rapid users increase and also the need for faster speed of transmission. The cost of conventional communication systems was also one of the main drivers for further development and therefore they need to remarkably increase the data rate with keeping the speed as high as possible. In wireless communication systems transmitted signals are exposed to environmental factors which lead to signal distortion, interference and sometimes elimination. As a result of their efforts Orthogonal Frequency Division Multiplex (OFDM) was invented and brought to life. This modulation technique mitigated most of the vital issues that communication systems are exposed to and achieved, at that time, the required speed, high data rate and bandwidth resource saving.

OFDM system uses Discrete Fourier Transform (DFT) at the heart of its signal processing system. Inverse Fast Fourier Transform (IFFT) is utilized in the transmitter and Fast Fourier Transform (FFT) in the receiver. Over the last two decades issues started to arise within this modulation technique due to the fast rapid increase of users and therefore the complexity of the system. Cyclic Prefix (CP) insertion between OFDM symbols was introduced to OFDM systems to mitigate the Inter-Symbol Interference (ISI) of multiple OFDM symbols. Multiple Input Multiple Output OFDM (MIMO-OFDM) was also another form of improvement with the use of adaptive linear channel equalizer, which has made OFDM system more complex but more efficient and robust in minimizing channel effectiveness. Nevertheless, adding more components and advancing OFDM system has not been sufficient to keep up-to-date with the rapid advancing modulation techniques of the communication systems.

In the 1980s scientists came up with a brand new signal processing technique, called Wavelet, to compress data, images and videos. The main driver behind this discovery is the need for further development from Fourier transform. Wavelet Transform (WT), likewise Fourier transform, has continues and discrete form of algorithm. Wavelet transform is capable of transforming the signals from time domain into time-frequency domain, unlike the Fourier transform which transform the signals from time domain to frequency domain. Subsequently, scientists and researchers have introduced greater developments on wavelet transform and its features. Currently, there are several wavelet families which are used within wavelet transform which vary in performance according to their parameters, i.e. Haar, Daubechies, Symlets and Coiflets. Over the last decade utilizing the wavelet transform in several different applications, other than main use in image, date and video compression, has become the hot research focus for researchers.

Introducing the wavelet transform in communication systems can be possibly achieved in various communications applications. Recently, have proposed the usage of Discrete Wavelet Transform

(DWT) to modulate signal which they named it Orthogonal Wavelet Division Multiplex (OWDM) and it could be an alternative modulation technique for OFDM system. DVB-T standard was utilized as a guideline simulation with input of Moving Picture Expert Group (MPEG) frames stream as an input. Nevertheless, such a proposal has brought up the idea of using wavelets to transmit signals, however, the testing scenario was not sufficient and practical. In their simulation test there were some omissions:

- A pure Additive White Gaussian Channel (AWGN) was used.
- No error detection and correction coding was used.
- No data interleaving was used.
- No channel coding was used in the test scenario.

This paper is a further investigation of this brand new modulation technique, OWDM, utilizing different wavelets in a comprehensive and complete model of a well known broadcasting system. Simulation baseline is derived from DVB-T standard ETSI EN 300 744 [2], theoretical and practical results are compared against this standard. The comparison taken into account are complexity, viability, cost effectiveness and bandwidth resource saving. Simulation results of different wavelets families are compared with each other and also against OFDM system in terms of Bits Error Rate (BER) vs Signal to Noise Ratio (SNR) performance. Validation and a conclusion of the overall results are presented.

Orthogonal Frequency Division Multiplex

Historical Background

OFDM system was brought to live in 1968 when Chang and Gibby [3] made it practically possible to transmit signals using OFDM modulation technique. During the last four decades OFDM system witnessed a series of improvements and became the primary drive force of communication systems for various communications applications. With its vast spread popularity it has been adopted by several worldwide industrial standards, e.g. DVB-T standard [2], Digital Audio Broadcasting (DAB) standard [4], IEEE 802.16a Metropolitan Area Network (MAN) standard [5] and the IEEE 802.11p Local Area Network (LAN) standard [6].

Unlike single carrier system where a one signal representing each bit uses the entire available spectrum, OFDM works by splitting the available spectrum into a stream of many parallel narrow sub bands and transmits the stream of data signals through these sub bands, this is the reason of calling it multicarrier system. Therefore, OFDM created a saving of 50% of bandwidth compared to the coexisted conventional modulation systems at that time as illustrated in figure 1. OFDM utilizes DFT in the heart of its structure in the form of FFT and IFFT. Since the discovery of OFDM, in late 1960s, development on OFDM system is carried out throughout the years to produce more robust system and reduce the impact of severe multipath channels.

The mobile channel environment is characterized by several factors which cause the receiver to receive multiple signals arriving at different time intervals for the same user. This is due to the nature of the channel, which is characterized by the geographical nature of the area, causing refraction, diffraction and reflection of signals. This phenomenon is called multipath channel and has been the research focus for the last two decades. Channel estimation and equalization have been introduced and placed at the receiver side to minimize the impact of multipath channel on transmitted signals. ISI also impact OFDM signals, however, it mitigates this by dividing the bandwidth into N non-overlapping narrow subcarriers and signals are modulated over these subcarriers. These subcarriers are frequency multiplexed to be propagated in parallel and consequently produces the OFDM symbols, as shown in figure 2.

However, adding a guard interval to the end or the beginning of each OFDM symbol is essential to prevent ISI through the channel, as shown in figure 3. This guard interval must be equal to or higher than the length of the multipath channel spread, otherwise ISI will occur. Subsequently, the effect of the delay spread would therefore appear as a multiplication in the frequency domain. Thus, the receiver and the transmitter must be 100% synchronized. Due to the fact that subcarriers, that carry the information symbols, are orthogonal to each other due to the orthogonality feature of OFDM, therefore no interference is possible between subcarriers. OFDM system was brought to live in 1968 when Chang and Gibby [3] made it practically possible to transmit signals using OFDM modulation technique. During the last four decades OFDM system witnessed a series of improvements and became the primary drive force of communication systems for various communications applications. With its vast spread popularity it has been adopted by several worldwide industrial standards, e.g. DVB-T standard [2], Digital Audio Broadcasting (DAB) standard [4], IEEE 802.16a Metropolitan Area Network (MAN) standard [5] and the IEEE 802.11p Local Area Network (LAN) standard [6].

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Technical Background

Orthogonality of subcarriers in OFDM systems achieved through the fact that the subcarriers have a step response in the frequency domain of a sinc function $(\sin x)/x$ and, thus, they are orthogonal to each other so there is no interference occurs between the subcarriers. Subsequently, overlap between subcarriers in the frequency domain occurs; however, this overlapping does not cause any interference due to the orthogonality of subcarriers [3]. The duration between two successive subcarriers is 1/Ts, as shown in figure 2.

In the transmitter OFDM uses IFFT to transform signals from time to frequency domain, therefore, FFT has to be used in the receiver to convert the signals vice versa. Consequently, FFT picks up some discrete frequency samples according to the peaks of the carriers. All other carriers which pass through zero disregards interference between the subcarriers. According to Change and Gibby [3] FFT is very sensitive to any change of the following factors:

1. Integer number of cyclic during any symbols period.
2. The integer number of cyclic that separate the subcarriers between each other.

OFDM generates the subcarriers by taking the incoming binary stream, suppose R bps. It requires a bandwidth of BT = R(1+r) where r is the roll-off factor for Nyquist shaping. Subsequently N bits are stored during an interval of Ts = N/R (OFDM symbol interval). Each of these N bits is used to

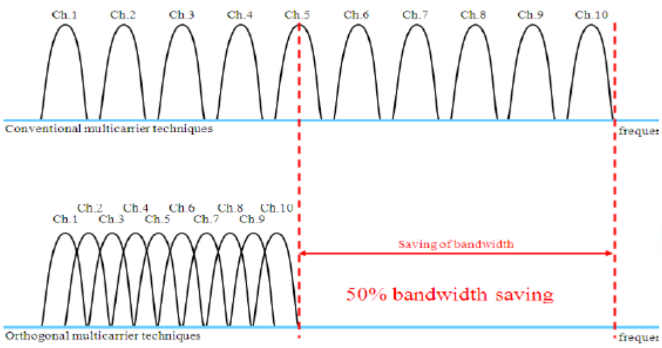


Fig. 1: OFDM bandwidth savings compared to conventional modulation techniques [16]

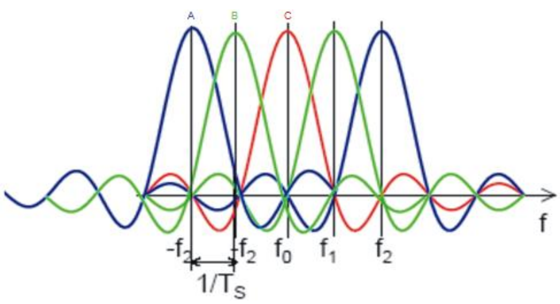


Fig. 2: Orthogonal OFDM Symbols [16]

individually modulate a carrier. All these modulated carriers are transmitted simultaneously over the long interval of Ts[7].

The generalization of the OFDM is summarized in the following function:

$$a(n) = \sum_{k=0}^{N-1} a_k \, e^{\frac{j2\pi kn}{N}} \qquad n = 0,1,\dots,N-1 \qquad (1)$$

Where: $a(n)$ is the IDFT of ak. ak: k = 1,…,N: successive bits stored.

If we let K < N successive binary digits be stored, generating one of 2^k possible QAM signals. Each signal corresponds to a complex number ak. As it is noticed that equation 1 is the form of Inverse Discrete Fourier Transform (IFFT) [7], and can be easily evaluated using FFT.

As in DVB-T, which is used as a platform application in this paper, the OFDM modulator buffers *M* symbols, from QPSK or QAM modulator, where *M* is number of subcarriers. Using DVB-T *M= 1705* QAM symbols [2] and zero padding up to 2048 IFFT points [2].

Short-Time Fourier Transform

Short-Time Fourier Transform (STFT), or sometimes it is called Short-Term Fourier Transform, was first introduced in 1946 when Dennis Gabor [8] utilized Fourier transform to analyze signals in time-frequency domain. He suggested that a fixed time limited window can be used to analyze a portion of a sinusoidal signal at a time by applying Fourier transform at that specific time. This window then can slid over the signal to compute the transform for the whole signal. This maps the signal into two domains at once, time-frequency domain. This transform however, is different than conventional Fourier transform due to the fact that the conventional Fourier transform transforms the signal from time to frequency domain; nevertheless, the STFT transforms the signal from time to time-frequency domain [8].

Figure 4 illustrates how STFT window is sliding over the signal and representation in time-frequency domain. This analysis can solve some of the limitation of conventional Fourier transform, as

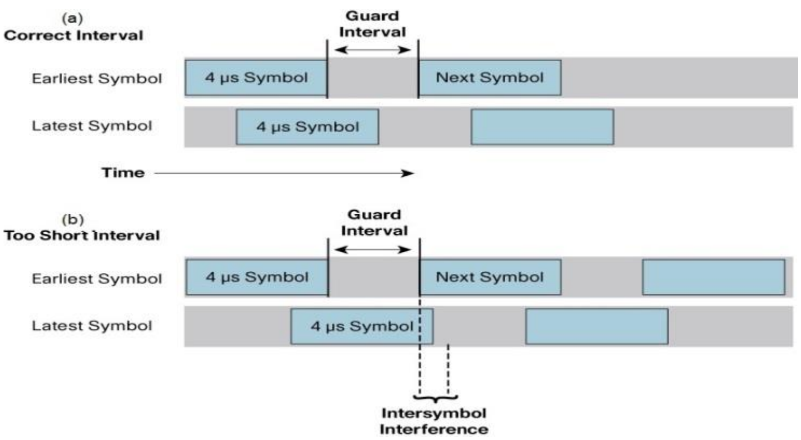


Fig. 3: OFDM Guard Interval between symbols

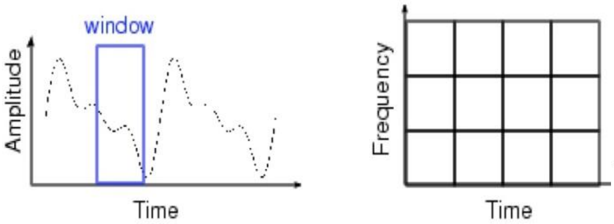


Fig. 4: STFT Window Sliding over Signal and Time-

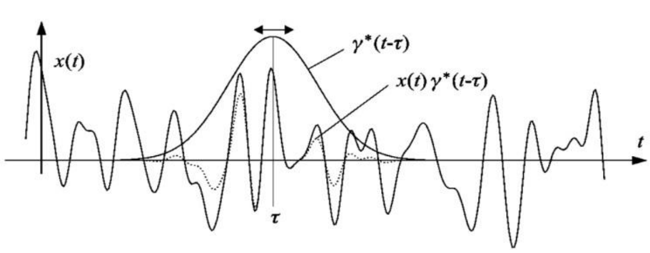


Fig. 5: Windowing a Signal, STFT

it can provide the information of the frequency and at which time that a specific signal event occurred. However, there is still even limitation when using STFT which is the size of the sliding window which determines the precision of the obtained information [8]. Additionally, the size of the sliding window cannot be changed once it has been chosen and it will remain the same for all frequencies of the signal. Complex signals require a much more flexible approach to analyze signal in time-frequency domain.

STFT can be mathematically obtained by multiplying the signal $\boldsymbol{x(t)}$ with the analysis window of $\boldsymbol{\gamma^*(t-\tau)}$, then compute the conventional Fourier transform for this particular period. The transform of the windowed signal is:

$$\mathcal{F}_x^{\gamma} \int_{-\infty}^{\infty} x(t)\, \gamma^*(t-\tau)\ e^{-j\omega t}\, dt. \qquad (2)$$

What $\gamma^*(t-\tau)$ does is that it ignores any part of $x(t)$ that falls outside of the window. Then Fourier transform is implemented over this particular part of the signal inside the local window. Figure 5 illustrate the process of windowing a signal [9]. This is, however, is for continues time signals which is most suits audio signals analysis.

From equation 2, $x(t)$ can be reconstructed by the following equation [9]:

$$x(t) = \sum_{m=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \mathcal{F}_x^{\gamma}(mT, k\omega_{\Delta})\, g(t-mT)\, e^{jk\omega_{\Delta}t}. \quad (3)$$

Where $F_x^{\gamma}(mT, k\omega_{\Delta})$, m , k ∈ Z of the STFT are the coefficients of the series expansion of the signal $x(t)$. It is observed that the functions set utilized to reconstruct the signal are built from the modulated and shifted versions of the same $g(t)$ [9]. Moreover, perfect reconstructions can be also obtained, for more details and derivations please refer to [9].

In Discrete time signals, however, the critical sampling for $T\omega_{\Delta} = 2\pi$ and equal analysis and synthesis, it is not possible to obtain good time and frequency resolutions. Whenever the time window $\gamma(t) = g(t)$ allows perfect reconstruction, then one of Δ_t or Δ_ω will be infinite. This notion of relationship is widely known as *Balian-Low theorem* [10]. Balian-Low theorem shows that it is not impossible to construct an orthonormal STFT where the window is differentiable and compactly supported [10].

Therefore, when computing STFT mathematically for any given discrete time signal $x(n)$, we substitute the integration in equation 2, STFT for continues time signal equation, with summation [10]:

$$\mathcal{F}_x^\gamma\left(m, e^{j\omega}\right) = \sum_n x(n)\, \gamma^*(n - mN)\, W_M^{kn} \qquad (4)$$

It is assumed that the sampling rate of the signal is higher than the rate of calculating the spectrum by a factor of $N \in \mathbb{N}$. Where γ^* and g denote the analysis and synthesis windows respectively. The frequency ω is normalized to the sampling frequency. Due to the limitation of STFT a further development from Fourier type transform was needed and as a result a wavelet transform was invented as will be discussed in the next section.

Proposed System Orthogonal Wavelet Division Multiplex

The wavelet transform was firstly used in the 1910s when Haar [11] constructed an equation that is capable of extracting a given signal's feature in time-frequency domain. Developments continued when Dennis Gabor [8] when he constructed similar wavelet to Haar's wavelet. Comnitues wavelet transform was also discovered by Zweig in 1975. Further discoveries of wavelets construction was carried out in the late 20th century by several scientists including Goupillaud, Morlet and Daubechies.

Wavelet transform has two kinds of transforms: the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The CWT is dramatically costly because it performs convolutions at all positions. Conversely, the DWT resolves this cost issue due to the fact that the signal data which mostly needs to be processed is discretely stored. In addition, Nyquist sampling theorem indicates that with the discrete data signal the highest frequency which can be modeled is half of the sampling frequency [12].

DWT is widely used in multimedia communications from image and video compression, image watermarking to data analysis, however, recently it was proposed, by Jain and Myers [13], that wavelet transform could be used for data signals transmission. Due to its recent discovery, there is a lack of research into using wavelet transform for signal transmission. In 2007 Linfoot et al. [1] proposed OWDM system which could be an alternative to OFDM with more advantages over OFDM. It is utilized to separate the bandwidth into sub-bands and dynamically allocate these sub-bands with variables widths. It was also proposed by Linfoot et al. [1] that it offers a lower computational complexity and more flexible than that of OFDM.

Unlike OFDM which utilizes Fourier transform, OWDM utilizes wavelet transform. Wavelet transform converts the signal from time domain into time-frequency domain, unlike Fourier transform which transfer the signal form time domain into frequency domain. Another good feature of wavelet transform is that it is located within a finite domain and zeros anywhere else, whereas, in Fourier transform the sinusoidal wave repeats itself infinitely. Hence it is possible in the wavelet transform to model changes in frequency by changing the scale of the time domain.

Due to the sampling theorem and scale convolutions calculations in the wavelet transform, the first path in which the wavelet picks up at a single sample is unnecessary. Conversely, the smallest scale whereby the wavelet picks up two sample points is deemed to be more important. Subsequently, it is suggested that this process can be achieved by a combination of orthogonal filters of Lowpass Filter (LPF) and Highpass Filter (HPF) forming the filters bank. LPF consists of the resulted scaling function, whereas HPF consists of the resulted wavelet function.

There is some miss-understanding between Wavelet Packet Modulation (WPM) and Asymmetric wavelet, the second; however, is the interest of this research. WPM spread the signal over fixed allocated bands, this process has some advantage over OFDM, which is the elimination of guard interval, but it is still fixed sub-band allocation which is similarly like what OFDM does. WPM is usually referred to as symmetric filter banks, or wavelet, and it has been proposed and researched over the last decade.

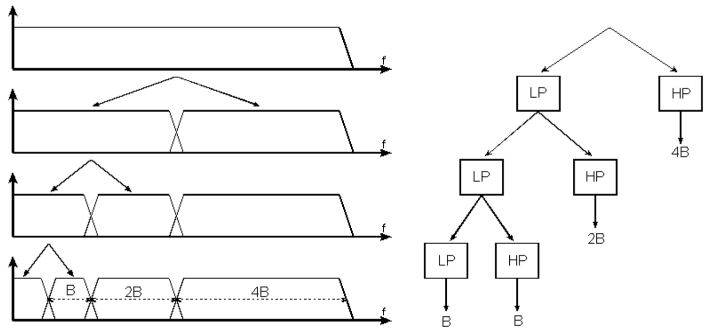
Nevertheless, the proposal of this paper is to investigate the performance of asymmetric wavelet filter banks to be used as modulation scheme for signal transmission. This technique will illuminate the guard interval which take zero data spectrum and conserve this wasted spectrum, figure 6 illustrates how data symbols are transmitted in sequence without guard interval, compared to OFDM in figure 3 (OFDM section). The proposal is to decompose the first Lowpass branch then after decompose both branches just similar to WPM. The first Highpass, however, is just down sampled by 2 and not decomposed, as shown in figure 7. This scheme of decomposition will allocate signals dynamically and therefore conserve the bandwidth and give more flexibility to providers and end users, figure 7 illustrates how this decomposition allocates sub-bands.

Wavelet has a wide range of families and families' members, so called children, which vary mainly in coefficients and number of vanishing moments or filter taps. Almuttiri and Linfoot [14] have investigated two wavelet families with three members of both, Daubechies and Symlet. In this paper, however, a wider investigation of the effect of not only the simplest wavelet family but also the most complex wavelet family will be explored. The main criteria of each chosen wavelet family and family's member are that is orthogonal and compactly supported [15].

In the experiments detailed in this paper, the simulation is based on the Digital Video Broadcasting Terrestrial (DVB-T) specification model, which uses OFDM system as its modulation scheme. This investigation considered the whole system from MPEG stream generation including ½ rate encoding, 16 RS(204,188,16) outer encoder, 16-QAM, 1/25 guard interval for OFDM but not for OWDM and 2048 IFFT length. The simulation adapted the 2K mode for OFDM and 11 levels of decomposition for OWDM both giving subband widths of 4464Hz. These two systems are like-for-like comparable due to the fact that number of levels in OWDM follows $\log_2 N$ where N is IFFT length in OFDM system.

The complexity of DWT is $O(N)$ which is clearly less than FFT in OFDM which has a well-known complexity of $O(N \log_2 N)$. As discussed by Hassan [6], the filters discussed above are mainly single lowpass filters (ho(i)), such as:

$$hi(i) = -1I\, ho(L-1-i)$$
$$go(i) = ho(L-1-i)$$
$$gi(i) = -(-1)iho(i) \qquad (5)$$



Where L is the length of the filter and 0≤ i ≤L-1.

OWDM is based on the concepts of orthogonal multipulse signaling [9]. Lee and Messerschmitt [7], considered that the pulses $\varphi m(t)$, $m = 0, 1, \ldots, M-1$, form an orthonormal set over a certain period of time and that each pulse is orthogonal to itself shifted by the non-zero integer multiples of a certain interval T. Each basis pulse $\varphi m(t)$, can then serve to create a 'virtual' channel over which the symbol am is carried. The vector of symbols $A = [a_0, a_1, \ldots, a_{M-1}]T$ is called a super-symbol, and the interval $T = MT_s$ as the super-symbol/block interval, where T_s is the basic symbol interval. Subsequently the base band transmitted signal becomes:

$$\mathrm{S(t)} = \sum_{\mathrm{i}=-\infty}^{\infty} \sum_{\mathrm{m}=0}^{\mathrm{M}-1} \mathrm{a_{i,m}}\, \varphi_{\mathrm{m}}(\mathrm{t}-\mathrm{iT}) = \sum_{\mathrm{i}=-\infty}^{\infty} \mathrm{A_i^T}\ \varphi(\mathrm{t}-\mathrm{iT}) \quad (6)$$

At the receiver, symbol and block timing extraction is performed; and the received signal is correlated with $\varphi(t - iT)$ to detect the nth super symbol at time iT and time $iT + \tau$, where τ denotes the optimum timing phase [7]. Since complementary metal oxide semiconductor and very large scale integration implementation of signal processing techniques are often less complicated and economical in the discrete time domain, discrete time formulation of orthogonal multipulse signaling will be used from now on. The discrete time orthogonal multipulse's are $\varphi m(n)$, $m = 0, 1, \ldots, M-1$; and the corresponding transmitted orthogonal multipulse signal becomes:

$$s(n) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m\,(n - iM) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(n - iM) \quad (7)$$

However for convenience and simplicity, the variable t will be used to denote both the continuous time variable as well as the discrete time sample index. In addition, M and T will be used interchangeably to denote the block length. Orthogonal multipulse signaling has the following distinct advantages:

- Possible dynamic separation of sub-bands.
- Potentially less sensitive to multipath fading.
- No need for guard intervals [14].
- Potential to reach channel capacity and counter selective fading.

- Potential for multiplexing at the physical layer.

System Design and Simulation

The simulation platform considered for this proposed system is Matlab using various multiple toolboxes. DWT has a wide range of wavelet families each differ in, number of filter's coefficients, vanishing moments or filter's taps and subsequently performance. Each of those families has a subdivision of wavelet members or sometimes it is called "wavelet children". All compactly supported wavelets have scaling function and wavelet function computed by wavelet analysis and synthesis filter banks, or decomposition and reconstruction respectively. For instance, if we take Daubechies 4 (db4) it the name of wavelet family is Daubechies and the wavelet children is 4, now this number refers to the number of vanishing moments. From that we can know the number of coefficients or number of filter taps, these follows the rule that each vanishing moment represent two filter taps or coefficients [15]. Nevertheless, those number of filter's vanishing moments can be differ in some wavelet families from analysis filters and synthesis filters. A popular example of these families is Biorthogonal (bior) and Reverse Biorthogonal (rbio); for example bior2.8 has two vanishing moments in the analysis, or decomposition, filter and eight in the synthesis, or reconstruction, filters.

Moreover, although all compactly supported wavelets share similar properties, however, there are still some differences between those families. Properties of all compactly supported wavelet families' important properties are summarized in table 1 [15].

Table 1: Wavelet Families' Properties [16]

Property \ Wavelet Family	Haar	dbN	symN	coifN	biorNr.Nd	rbiorNr.Nd	dmey
Arbitrary regularity		✓	✓	✓	✓	✓	
Compactly supported orthogonal	✓	✓	✓	✓			
Compactly supported biothogonal					✓	✓	
Symmetry	✓				✓	✓	✓
Asymmetry		✓					
Close to symmetry			✓	✓			
Arbitrary number of vanishing moments		✓	✓	✓	✓	✓	
Vanishing moments for ϕ				✓			
Existence of ϕ	✓	✓	✓	✓	✓	✓	
Orthogonal analysis	✓	✓	✓	✓	✓	✓	
Biorthogonal analysis	✓	✓	✓	✓	✓	✓	
Exact reconstruction	✓	✓	✓	✓	✓	✓	✓
FIR filters	✓	✓	✓	✓	✓	✓	✓
Continuous transform	✓	✓	✓	✓	✓	✓	
Discrete transform	✓	✓	✓	✓	✓	✓	✓
Fast algorithm	✓	✓	✓	✓	✓	✓	✓
Explicit expression	✓				For Spline	For Spline	
FIR-based approximation							✓

DVB-T standard [2], which uses OFDM as the signal framing and multiplexing scheme, is used as the baseline system to simulate and compare the proposed system against. The DVB-T 2k mode is used and the parameters of the system are shown in table 2. Number of levels in OWDM is directly proportional to the number of IFFT in OFDM and follows $log_2(N)$ where N is IFFT length or for simplicity 2M where M is IFFT length. The experiment involved increasing the Signal to Noise Ratio (SNR) of the channel, comparing to a measure of the Bit Error Rate (BER) taken at the output of the

Table 2: Full System Parameters for OFDM and OWDM

Parameter	Value
OFDM Size	2K
Number of IFFT	1705
OWDM composition levels	11 levels
Multiplex 1 parameters	16-QAM, 3/4 rate
Multiplex 2 parameters	64-QAM, 2/3 rate
RS Encoding	RS(204,188,16)
SNR range	0-30dB
Number of frames	100
Number of bits/frame	96

RS decoder.

In order to ensure compliance with the standard, the SNR which gave the Quasi Error Free (QEF) state (a BER 10-11 at the output of the RS Decoder) was evaluated and compared to the values provided in the DVB-T standards (which gives all SNRs at QEF state for all combinations of QAM and Inner coding rate). Once the simulation was compliance validated according to the standards, the IFFT/FFT blocks were replaced and the experiment repeated using the system depicted in figure 4 in which the IFFT and FFT are replaced with the Wavelet Synthesis Filter and Decomposition filters respectively.

Results and Analysis

Two QAM modulations were considered to widen the comparison, 16-QAM and 64QAM. Figure 8 shows the result for OFDM and OWDM with 16-QAM over AWGN channel with the parameters as in table 2. The red circled is OWDM whereas the blue star is OFDM curve. It is clear that OFDM and OWDM performing similarly with a slight better performance of OWDM when using Haar wavelet. Figure 9 shows the performance of OWDM and OFDM for 64QAM over the same conditions as table 2. From these two plots, it can be seen that there is a significant similarity between the two curves. From figure 8, it can be seen that the SNR which gives a BER of 10^{-4} is 20 dB and 19 dB for 16-QAM for OFDM and OWDM respectively. Figure 9 shows a closer performance of OWDM to OFDM with 64QAM and under the same conditions as in table 2. It is clearly seen that both systems competing with each other and both achieved BER of 10^{-4} at a similar SNR of 27 dB.

Consequently, it was derived from this that the chosen OFDM system's parameters matched the number of filters bank levels in OWDM because of the similar behavior. In the wavelet modulation the overall throughput based on not only the M different levels of filters banks, in this paper 11 levels, but

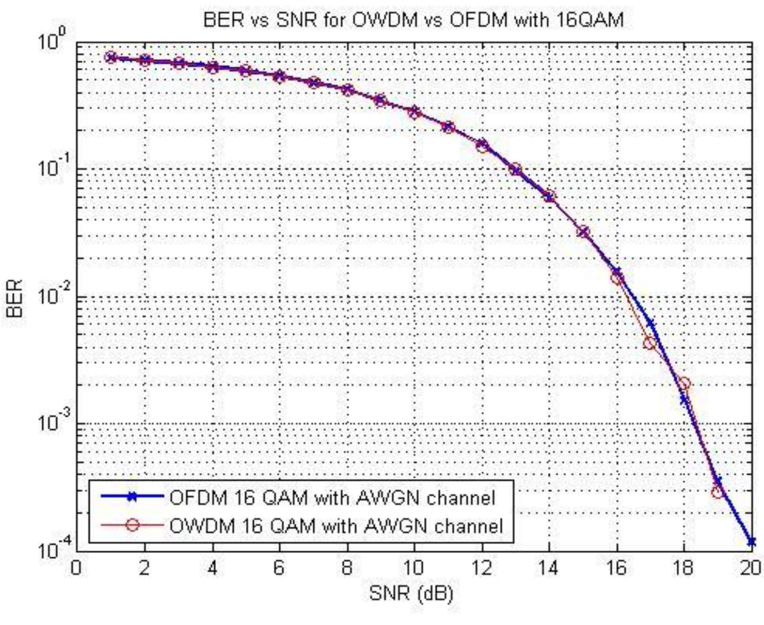


Fig. 8: OWDM vs OFDM with 16-QAM over AWGN channel

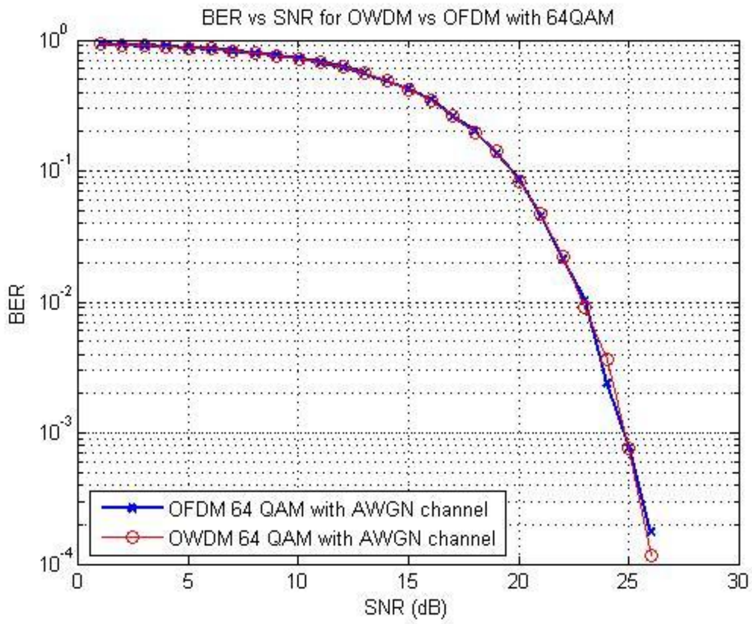


Fig. 9: OWDM vs OFDM with 64QAM over AWGN channel

also the wavelet family, in this paper Haar family.

Haar wavelet is the smiplist wavelet family with a single vanishing moment and two coefficients or filter taps, Figure 10 shows decomposition and reconstruction LPF and HPF filters of Haar family. If the linear complexity of OWDM is taken into consideration along with unnecessary guard intervals, OWDM shows the high possibility of producing a strong candidate as an alternative to OFDM. Figures 8 and 9 show the similarity of performance of OWDM (Haar/11-levels) to OFDM (2048 length of IFFT). For using the symmetric allocation of sub-bands, one can argue that by decomposing this way it does what IFFT in OFDM system does. The answer is yes, however, with interesting advantage that the guard interval is not further necessary when using the banks of filters. Additional advantage is that the linear complexity of OWDM less than the logarithmic complexity of IFFT in OFDM. When decomposing asymmetrically, the advantages are even more interesting as the flexibility of allocating the spectrum to different users are possible and subsequently conserving the wasted unoccupied bandwidth compared to fixed spectrum allocation.

Conclusion

This paper has provided a comparison between Orthogonal Frequency Division Multiplex and Orthogonal Wavelet Division Multiplex with two different QAM sizes and in accordance to Digital Video Broadcasting Terrestrial standard [2]. The purpose of this paper is to demonstrate that OWDM has the potential of acting as a successor for OFDM because of some of the potential advantages it has over OFDM. The computational complexity of wavelet decomposition filters in OWDM is less than Fourier Transform in OFDM. There is also a potential exploitation of guard interval in OWDM instead of wasting it in OFDM. Additionally, dynamic allocation of bandwidth is potential with the use of asymmetric filter banks. Two scenarios have been presented in this paper which demonstrates that under AWGN, the performance of OWDM is matched to that of OFDM.

The principle concept behind this paper was to examine OWDM with varying depths of levels and to compare their performance with OFDM system. In the design all parameters for both systems were equivalent and evaluated against the DVB-T standard [2]. The behavior and performance of OWDM system is more likely to be a sufficient alternative for OFDM as the system behaves in the same way as OFDM, however, with less complexity. OWDM complexity was addressed previously by Linfoot et al. [1] who examined two levels of dynamic (asymmetric) allocation of sub-bands considering data entered directly to QAM mapping without forward error correction, encoding or interleaving. The difference in this investigative paper is that the levels used are eleven and the sub-band allocation is symmetric, where both LPF and HPF are utilized. The symmetric allocation allocates a fixed width of sub-bands.

References

[1] Scott L. Linfoot, Mohammad K. Ibrahim, and Marwan M. Al-Akaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[2] ETSI EN 300 744, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series) ETSI EN 300 744, 2009.

[3] Robert W. Chang and Richard A. Gibby, "Theoretical Study of Performance of an Orthogonal Multiplexing Data Transmission Scheme," *IEEE Transactions on Communications*, vol. 16, no. 4, pp. 529-540, 1968.

[4] "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers," , 2006.

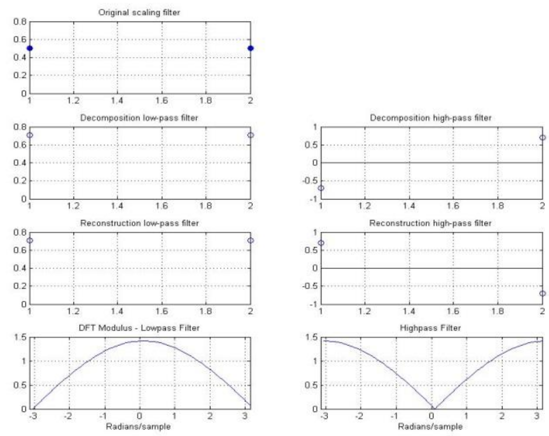


Fig. 10: Decomposition and Reconstruction LPF and HPF filters of Haar Wavelet family

[5] IEEE 802.16 standard for Lacal and Metropolitan area networks, "Part 16, Air Interface for Boadband Wireless Access Systems, Amendment 3: Advanced Air Interface," , 2011.

[6] "802.11p-2010 - IEEE Standard for Information technology-- Local and metropolitan area networks-- Specific requirements-- Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications Amendment 6: Wireless Access in Vehicular," , July 2010.

[7] Mischa Schwartz, *Mobile Wireless Communications*. New York, United States of America: Cambridge University Press, 2005.

[8] Dennis Gabor, "Theory of Communication," *Institute of Electrical Engineering*, vol. 93, pp. 429-457, 1946.

[9] Alfred Mertins, *Signal Analysis: Wavelets, Filter Banks, Time-Frequency Transforms and Applications*. New Jersey, United Staes of America: John Wiley and Sons Ltd, 1999.

[10] John J. Benedetto, Christopher Heil, and David F. Walnut, "Gabor systems and the Balian-Low Theorem," in *Gabor Analysis and Algorithms*, Hans G. Feichtinger and Thomas Strohmer, Eds. New York, USA: Springer Science+Business Media, 1998, ch. 2, pp. 85-122.

[11] A. Haar, "Die Minkowskische Geometrie und die Annäherung an stetige Funktionen," Goettinger DigitalisierungsZentrum (GDZ), Goettingen, Germany, Technical Report Math Ann. 78, 294-311, 1918.

[12] S. V. Narasimhan, Nandini Basumallick, and S. Veena, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[13] Vijay K. Jain and Brent A. Myers, "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

[14] Abdullah S. Almuttiri and Scott L. Linfoot, "Orthogonal Wavelet Division Multiplex as a Modulation Scheme for Digital Television," in *2013 IEEE International Conference on Consumer Electronics (ICCE-Berlin)*, Berlin, Germany, 2013, pp. 136-139.

[15] Ingrid Daubechies, *Ten Lectures on Wavelets*, 7th ed. Philadelphia, Pennsylvania, United Satates of America: Society for Industrial and Applied Mathematics, 2002.

[16] Klaus Witrisal, Orthogonal Frequency Division Multiplexing (OFDM): Concept and System-Modeling, December 15, 2010.

[17] S. V. Narasimhan, Nandini Basumallick and S. Veena, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[18] A. Haar, "Zur Theorie der orthogonalen Funktionensysteme," Goettinger DigitalisierungsZentrum (GDZ), Goettingen, Germany, Thesis Math. Ann. 69, 331-371, 1910.

[19] Scott L. Linfoot, Mohammad K. Ibrahim and Marwan M. Al-Akaidi, "Orthogonal Wavelet

Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[20] ETSI EN 300 744 V1.6.1, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.

[21] Prasad Ramjee, *OFDM for Wireless Communications Systems*. London, United Kingdom: Artech House Inc., 2004.

[22] Nandini Basumallick and S. Veena S. V. Narasimhan, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[23] Jain V. K. and Myera B. A., "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

[24] Hassan F. S., "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, March 2008.

[25] A. Lee and D. G. Messerschmitt, *Digital Communications*, 2nd ed.: Kluwer Academic Publishers, 1988.

[26] Michel Misiti, Yves Misiti, Georges Oppenheim, and Jean-Michel Poggi, "Wavelet Toolbox User's Guide," The MathWorks, Inc., Boston, Manual R2015b, September 2015.

APPENDIX C: CONFERENCE PAPERS

The following paper has been published in the IEEE 2013 International Conference on Consumer Electronics, September 2013 Berlin, Germany. And is indexed in ieeeexplore library.

Orthogonal Wavelet Division Multiplex as a Modulation Scheme for Digital Television

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***Abstract*—Issues surrounding wireless communications have arisen as a result of an increase in popularity and usability. The complexity of modulating signals over carriers is considered to be a significant challenge and Orthogonal Frequency Division Multiplex has proven to be a popular technology. The discovery of the wavelet transform, while mainly used in image processing applications, has led to a recently proposed, innovative new modulation technique called Orthogonal Wavelet Division Multiplex. This paper looks at the application of different families of wavelet as applied to digital terrestrial television.**

***Index Terms*— OFDM, OWDM, Wavelet, Orthogonal Wavelet Division Multiplex.**

Introduction

Over the last three decades, the discovery of the wavelet transform has been introduced as an application for image compression. It is used as an algorithm to analyze the data by transforming the signals from time to time-frequency domains as opposed to just the frequency domain as in the Fourier transform. Presently, there are several types of wavelet which are used in wavelet transform and vary in performance.

Utilizing the wavelet transform in wireless communication systems can be achieved through a recently proposed application by described by Linfoot et al., [1] and uses Orthogonal Wavelet Division Multiplexing (OWDM) as an alternative modulation technique for Orthogonal Frequency Division Multiplex (OFDM). In their proposal, a Digital Video Broadcast – Terrestrial (DVB-T) system was considered as a platform for the testing simulation. However, the system only considered a signal without error detection and correction coding, interleaving and channel coding.

This paper is a further extension of research associated with this brand new proposed modulation technique, OWDM, as an alternative to OFDM for DVB-T. A comprehensive experimental implementation of different wavelet families with different levels of filter banks will be conducted in the full paper with a technical discussion, evaluation, results validation and a conclusion of the overall results.

Orthogonal Frequency Division Multiplex

Over the last two decades OFDM has been developing and is becoming a more popular wireless modulation and transmission technique. It is been adopted in several world class wireless standards such as DVB-T [2]; Digital Audio Broadcasting (DAB); IEEE 802.16a Metropolitan Area Network (MAN) and the IEEE 802.11a Local Area Network (LAN) standards.

The channel in the mobile environment is characterized by several signals for the same user arriving at the receiver during different times known as multipath fading. This is due to certain obstacles and the geographical nature of the area in the wireless environment. A complex equalizer at the receiver is needed to resolve the multipath issue; but can consume more time as the data rate increases. According to Ramjee [3], OFDM has been presented to resolve the multipath issue without the need for a complex equalizer at the receiver side through the inclusion of a guard interval – a cyclic extension of the transmitted signal.

OFDM can suffer from Inter Symbol Interference (ISI). However, it mitigates this by dividing the bandwidth into *N* non-overlapping narrow subcarriers and it modulates the signals over these subcarriers. Subsequently these *N*

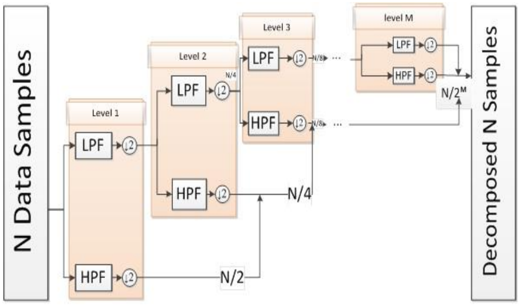

Figure 1: OWDM M levels of filter banks decomposition in the transmitter

subcarriers are frequency multiplexed to be sent in parallel to produce the OFDM symbol.

FFT is sensitive to the changes of any of the following:

- Integer number of cycles during any symbol's period.
- The integer number of cycles that separate the subcarriers.

The generalization of the OFDM is summarized in the following function:

$$a(n) = \sum_{k=0}^{N-1} a_k \, e^{\frac{j2\pi kn}{N}} \qquad n = 0,1,\ldots,N-1 \tag{1}$$

Where: $a(n)$ is the IDFT of ak.
ak: k = 1,…,N: successive bits stored.
If we let K < N successive binary digits be stored, generating one of 2^k possible QAM signals. Each signal corresponds to a complex number ak.

It is noticed that equation 1 is the form of Inverse Discrete Fourier Transform (IFFT) [4], and can be easily evaluated using FFT.
As in DVB-T, which is used as an example in this paper, the OFDM modulator buffers M symbols, from QPSK or QAM modulator, where M is number of subcarriers. Using DVB-T $M= 1705$ QAM symbols [2] and zero padding 2048- IFFT points.

Orthogonal Wavelet Division Multiplex

OWDM is investigated to be an alternative to OFDM. It is utilized to separate the sub-bands of bandwidth and dynamically allocate these sub-bands with different widths. It has been proposed that it offers a lower computational complexity to that of OFDM; and is more flexible [1]. This is unlike OFDM which uses FFT and OWDM which utilises the wavelet transform.

The wavelet transform was discovered due to the need for further development from the Fourier transform. The wavelet transform converts the signal from the time domain into the time-frequency domain. Unlike the Fourier transform where the sinusoidal wave repeats itself for infinity, the wavelet transform is located only within finite domain and zeroes anywhere else. Hence it is possible in the wavelet transform to model changes in frequency by changing the scale of the time domain. Additionally, modeling the changes in time could be achieved by shifting the position of the wavelets; therefore the frequency and location of the frequency could be modeled. Consequently, this transform is called the time-frequency domain [5].

The wavelet transform has two types of transform: the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The CWT is rapidly costly because it performs convolutions at every position. Conversely, the DWT resolves the cost problem due to the fact that the signal data which mostly needs to be processed is discretely stored. In addition, Nyquist sampling theorem indicates that with the discrete data signal the highest frequency which can be modeled is half of the sampling frequency [5].

Due to the sampling theorem and calculation of scale convolutions at the wavelet transform, the first path which the wavelet picks up at a single sample point is unnecessary. Conversely, the more important scale deemed necessary for consideration is the smallest scale whereby the wavelet picks up two sample points. Therefore, it is suggested that this

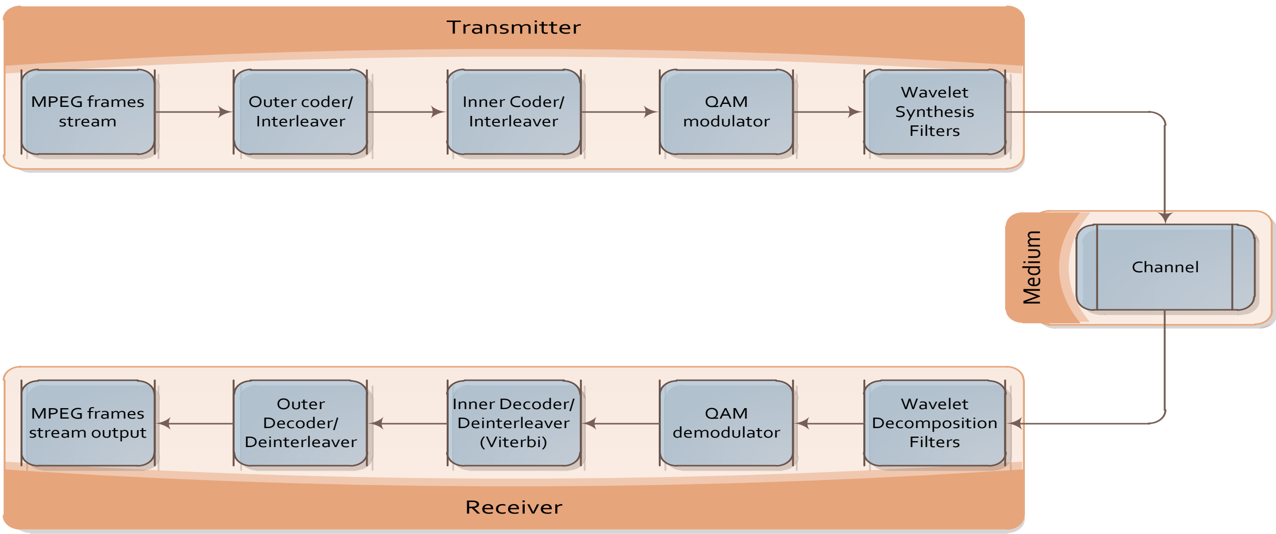

Figure 2: OWDM system block diagram

process can be achieved by a combination of orthogonal filters of Lowpass Filter (LPF) and Highpass Filter (HPF) forming the filters bank. LPF consists of the resulted scaling function, whereas HPF consists of the resulted wavelet function.

As shown in figure1, it is considered that every LPF is subdivided into HPF and LPF. Each LPF or HPF is treated as an individual signal and can be traced along the subdivision (sub-bands). These signals will not interfere with each other due to the scaling feature.

The number of filter bank levels depends on the application that it is used for and the bandwidth availability. The number of levels can be counted as per single level of filters.

However, the OWDM communication system is made up of two main sections: the synthesis section and a channel interface. The synthesis section consists of a filter bank with a number of inputs; with an output that supplies an OWDM signal [5].

Each input receives a symbol associated with the super-symbol, which are selected from a modulation scheme. The synthesis section generates an OWDM signal made up of weighted OWDM pulses; each weighted pulse representing a symbol of the super-symbol. The OWDM pulses are also used with an OWDM Spread Spectrum, known as OWSS. OWSS allows operation for wireless channels with the use of an equalizer.

As discussed by Hassan [6], the filters discussed above are mainly single lowpass filters (ho(i)), such as:

$$hi(i) = -1I\ ho(L-1-i)$$
$$go(i) = ho(L-1-i)$$
$$gi(i) = -(-1)iho(i) \qquad (2)$$

Where L is the length of the filter and $0 \le i \le$L-1.

OWDM is based on the concepts of orthogonal multipulse signaling [9]. Lee and Messerschmitt [7], considered that the pulses $\varphi m(t)$, $m = 0, 1, \ldots, M-1$, form an orthonormal set over a certain period of time and that each pulse is orthogonal to itself shifted by the non-zero integer multiples of a certain interval T. Each basis pulse $\varphi m(t)$, can then serve to create a 'virtual' channel over which the symbol am is carried. The vector of symbols $A = [a_0, a_1, \ldots, a_{M-1}]T$ is called a supersymbol, and the interval $T = MT_s$ as the supersymbol/block interval, where T_s is the basic symbol interval. Subsequently the base band transmitted signal becomes:

$$\mathrm{S(t)} = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m(t-iT) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(t-iT) \qquad (3)$$

At the receiver, symbol and block timing extraction is performed; and the received signal is correlated with $\varphi(t - iT)$ to detect the nth super symbol at time iT and time $iT + \tau$, where τ denotes the optimum timing phase [7]. Since complementary metal oxide semiconductor and very large scale integration implementation of signal processing techniques are often less complicated and economical in the discrete time domain, discrete time formulation of orthogonal multipulse signaling will be used from now on. The discrete time orthogonal multipulse's are $\varphi m(n)$, $m = 0, 1, \ldots, M-1$; and the corresponding transmitted orthogonal multipulse signal becomes:

$$s(n) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m\,(n-iM) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(n-iM) \qquad (4)$$

However for convenience and simplicity, the variable t will be used to denote both the continuous time variable as well as the discrete time sample index. In addition, M and T will be used interchangeably to denote the block length. Orthogonal multipulse signaling has the following distinct advantages:

- Possible dynamic separation of sub-bands.
- Potentially less sensitive to multipath fading.
- No need for guard intervals [1].
- Potential to reach channel capacity and counter selective fading.
- Potential for multiplexing at the physical layer.

Design of proposed system

The OFDM system is the baseline for the comparison with the proposed modulation technique, OWDM. The implementation of this system has taken the DVB-T standard [2] as a baseline design. The OFDM system has many parameters involved in operating the system and these depend on the application OFDM is being used for and the quality and capacity of the system being manipulated by changing some of these parameters. The number of filter bank levels, in OWDM, is directly proportional to the length of IFFT, in OFDM, it follows: $log_2(N)$ where N is IFFT length.

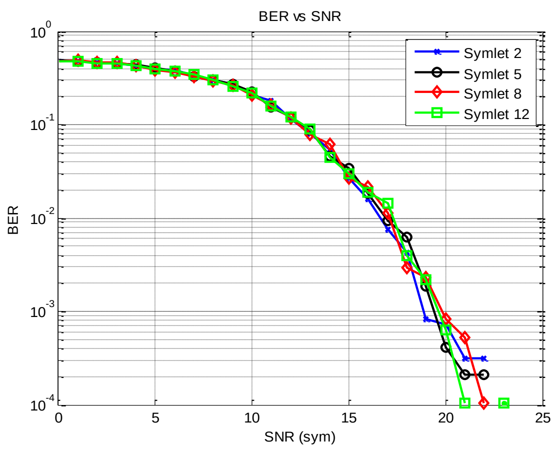

Figure 4: SNR vs BER for different Symlet children

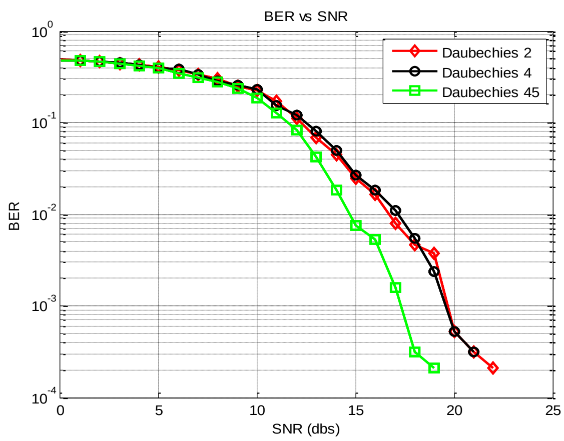

Figure 5: SNR vs BER for different Daubechies children

There are two configurations of OWDM: Using wavelets and using wavelet packets. In this paper, the latter will be used because wavelet packets use a symmetric synthesis meaning that each subband generated by the OWDM modulator will have identical bandwidth in the same way that OFDM does. Therefore, it is possible to perform a “like-for-like” comparison.

In the experiments detailed in this paper, the simulation is based on the DVB-T specification model and includes ½ rate encoding, 16-QAM and RS(204,188,16) outer encoder.

The simulation will use the 2K mode for OFDM and 10 levels of decomposition for OWDM both giving subband widths of 4464Hz.

The simulation will run over 100 frames through an Additive White Gaussian Noise (AWGN) channel of Signal to Noise Ratio (SNR) powers from 0dB to 25dB.

The wavelet families tested in this paper are: Haar (Daubechies 2), Daubechies (4 and 45) and Symlets (2, 5, 8 and 12).

Results and Analysis

The main comparison focus was to be made on comparing against *Additive White Gaussian Noise* (AWGN), the overall throughput based on M different levels of filters bank in OWDM and on different Wavelet children against each other. In OFDM, when SNR is 16dB, the BER is almost zero. The same observation was noticed in OWDM whereby there was almost no BER at this value of SNR. Consequently, it was derived from this that the chosen OFDM system’s parameters matched the number of filters bank levels in OWDM because of the same behavior as OWDM (db45 and 11 levels). Daubechies (db45) is found to be the best performance in terms of BER versus SNR in the tested wavelets families.

The first noticeable observation during results analysis was that in OFDM, as shown in figure 3, when SNR 16dB then the BER reach the minimum value and almost there was no error. The same observation was noticed in OWDM, as in figure 5 (Daubechies 24), whereby there was almost no BER at this value of SNR. Consequently, it was derived from this that the chosen OFDM system’s parameters matched the number of filters bank levels in OWDM because of the same behavior as OWDM (db45 and 11 levels). Daubechies (db45) is found to be the best performance in terms of BER versus SNR in the tested wavelets families.

If the linear complexity of OWDM is taken into consideration along with unnecessary guard intervals, OWDM shows the high possibility of producing a strong candidate as an alternative to OFDM. figures 4 and 5 show the similarity of performance of OWDM (db45/11-levels) to OFDM (2048 length of IFFT).

For using the symmetric allocation of sub-bands, one can argue that by decomposing this way it does what IFFT in OFDM system does. The answer is yes, however, with interesting advantage that the guard interval in not further necessary when using the banks of filters. Additional advantage is that the linear complexity of OWDM less than the logarithmic complexity of IFFT in OFDM.

Conclusion

The principle concept behind this paper was to examine the different wavelet families with varying depths and to compare their performance with OFDM system. In the design all parameters for both systems were equivalent. The behavior and performance of OWDM system is more likely to be a sufficient alternative for OFDM as the system behaves in the same way as OFDM, however, with les complexity. OWDM complexity was addressed previously by Linfoot et al. [1] who examined two levels of dynamic (asymmetric) allocation of sub-bands considering data entered directly to QAM mapping.

The difference in this investigative paper is that the levels used are six and the sub-band allocation is symmetric, where both LPF and HPF are utilized. The symmetric allocation allocates a fixed width of sub-bands. In regard to wavelet families comparison, Daubichies 45 wavelet family looks promising to providing most efficient performance amongst other wavelet families.

References

[1] Scott L. Linfoot, Mohammad K. Ibrahim and Marwan M. Al-Akaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," *IEEE Transactions on Consumer Electronics*, vol. 53, no. 2, pp. 278 - 284, May 2007.

[2] ETSI EN 300 744 V1.6.1, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.

[3] Prasad Ramjee, *OFDM for Wireless Communications Systems*. London, United Kingdom: Artech House Inc., 2004.

[4] Mischa Schwartz, *Mobile Wireless Communications*. New York, United States of America: Cambridge University Press, 2005.

[5] Nandini Basumallick and S. Veena S. V. Narasimhan, *Introduction to Wavelet Transform: A Signal Processing Approach*. Delhi, India: Alpha Science International Ltd, 2011.

[6] Hassan F. S., "The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel," *Journal of Engineering and Development*, vol. 12, no. 1, March 2008.

[7] A. Lee and D. G. Messerschmitt, *Digital Communications*, 2nd ed.: Kluwer Academic Publishers, 1988.

[8] Jain V. K. and Myera B. A., "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

Abdullah S. Almuttiri (Student M'10) was born in 1982 in Riyadh, Saudi Arabia. He received his BEng degree in Electronics and Communications Engineering from Amman University, Jordan in 2008. In 2008 he was granted a scholarship to pursue his master and doctoral studies under His Majesty King Abdullah Bin Abdulaziz Al-Saud Scholarships Program. He also completed his MSc in Communications Engineering in De Montfort University, UK in 2011. He is currently a PhD researcher in the Centre for Electronics and Communications Engineering in the Faculty of Technology at De Montfort University, UK. His research interests are OFDM systems Wavelet Packet and Wavelet Transform as modulation technique for Wireless Multimedia Communications and Networks. He is a Student Member of IEEE since 2010.

Scott L. Linfoot (M'03 – SM'10) was born in Chatham, UK in 1976. He received his B.Eng degree in Electronic Engineering from The University of Reading, UK in 1998. He also holds a PhD in multipath equalization in DVB-T also from The University of Reading obtained in 2003. His research interests are broadcast and communication systems. Dr Linfoot is the CTO of MediaTag Ltd – a company focusing on asset management for the media industry. Dr. Linfoot was the Technical Chair if the 8th International Symposium on Consumer Electronics held in Reading in 2004; and is on the Technical Programme Committee for ICCE and organizing committee for the IEEE International Conference on Consumer Electronics (ICCE) in Las Vegas and Berlin. He is a Past Treasurer and Secretary for the IEEE Consumer Electronics Society and is a current standing member of the IEEE CE Society Administrative Committee.

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An Advanced Modulation Scheme For Mobile Broadcasting

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***Abstract* — Issues surrounding wireless communications have arisen as a result of an increase in popularity and usability. The complexity of modulating signals onto carriers while optimizing bandwidth usage has always been considered to be one of the most significant challenges.**

The discovery of the wavelet transform has opened up a number of potential applications from image compression to watermarking and encryption. In the past, work has been done to investigate the potential of using wavelet transforms within the communication space. This paper will further investigate a recently proposed, innovative, modulation technique, Orthogonal Wavelet Division Multiplex, which utilizes the wavelet transform opening a new avenue for an alternative modulation scheme with some interesting potential characteristics.

Index Terms* — *Orthogonal Frequency Division Multiplex, Orthogonal Wavelet Division Multiplex, Wavelet.

Introduction

This Wireless communications systems started to rapidly develop at the beginning of the 1960s, whereby a revolutionary wireless modulation technique was brought to life by several researchers [1], [2]. Saving the bandwidth resource and squeezing as much information down a channel as possible was the main force behind discovering and introducing this modulation technique, Orthogonal Frequency Division Multiplexing (OFDM), and it was revealed at that time, not only to provide a modulation scheme that would allow data rates approaching the Shannon Rate but also to resolve one of the important channel conditions imposed on a signal, multipath propagation.

In 1994, Daubechies [3] developed a transform that has been applied to a number of applications, including data compression and, more recently, for image and video compression.

In addition, it was suggested [4] that wavelet decomposition could be applied in the communications domain as an alternative to OFDM.

The reason for this work is that it has been hinted [5] that there are several potential advantages that wavelet decomposition could have over OFDM such as dynamic separation of the sub-bands, physical layer multiplexing leading to multiple access and dynamic sub-band allocation.

This work will present a series of simulations based on the pre-2011 system parameters of the United Kingdom Multiplex 1 and 2 in order to demonstrate the potential of this new modulation scheme.

While this paper does indeed focus on the Digital Video Broadcast – Terrestrial (DVB-T) [6] specification, it should be noted that the core of this work is to demonstrate the capabilities of the modulation scheme and its application can be extended beyond DVB-T into other systems that employ OFDM as the modulation scheme.

Background

In 2006, there was a proposal introduced by Jain [4] to use Orthogonal Wavelet Division Multiplexing (OWDM) as an alternative modulation technique to OFDM. Linfoot *et al.*, [5] expanded on this theoretical work by considering OWDM as a modulation scheme as applied to the United Kingdom's Digital Video Broadcast – Terrestrial (DVB-T).

In the system presented by Linfoot, *et al*, in order to simplify the proposal, there were three omissions from the work:

1. Only the Quadrature Amplitude Modulation (QAM) along with the OWDM and OFDM components of the

DVB-T system were included, with no consideration of error correction, interleaving or channel coding.

2. Only Additive White Gaussian Noise channel is considered.
3. Asymmetric wavelet filtering was used as opposed to Symmetric filtering. In order to compare to OFDM, symmetric filtering should be used.

In order to demonstrate that OWDM has the potential of replacing OFDM in a typical communications system, further investigations are needed which address these three omissions.

This paper presents is a further extension of the research of Linfoot, et al and compares OFDM with OWDM using a fully compliant DVB-T simulation model as opposed to the incomplete model previously used [5]. The major difference is that the work presented here considers a full EN 300 744 [6] compliant simulation model is used.

In addition, this paper will not only review the AWGN channel but will also consider the effects of a mobile channel by applying Ricean Fading to the signal.

Orthogonal Frequency Division Multiplex

OFDM is a modulation technique that optimizes the channel bandwidth while minimizing the Inter-Symbol Interference (ISI). This technology has existed since 1966 when Chang [1] proposed a solution that made it practical for use as a transmission technique in order to find a more effective and robust modulation technique which could satisfy the increasing demands for receiving signals in a communications channel.

The channel in the mobile environment is characterized by several signals for the same user arriving at the receiver during different times known as multipath fading. This is due to certain obstacles and the geographical nature of the area in the wireless environment. A complex equalizer at the receiver is needed to resolve the multipath issue; but can consume more time as the data rate increases.

OFDM utilizes the Discrete Fourier Transform (DFT) within the communication system. It uses the Inverse Fast Fourier Transform (IFFT) at the transmitter and the Fast Fourier Transform (FFT) at the receiver.

In order to eliminate the effects of multipath, a cyclic prefix is added to the transmitted symbol and providing that any echoes arrive within the guard interval, they can be converted into constructive interference.

According to Ramjee [7], OFDM has been presented to resolve the multipath issue without the need for a complex equalizer at the receiver side.

OFDM suffers from ISI but is able to mitigate this by dividing the bandwidth into N non-overlapping narrow subcarriers and it modulates the signals over these subcarriers. Subsequently these N subcarriers are frequency multiplexed to to produce the OFDM symbol. By adding the guard interval to the beginning or the end of each OFDM information symbol, the effect of the delay spread would subsequently appear as a multiplication in the Fourier domain; and thus the receiver and the transmitter must be perfectly synchronized. In addition, guard intervals also prevent the ISI [7]. The N subcarriers that carry the information signals have a step response of a sinc function (sin(x)/x) and are, thus, orthogonal to each other so there is no interference between the subcarriers.

Over the last fifty years, a great deal of research has been done into many different aspects of OFDM because it has become an increasingly popular wireless modulation and transmission technique. It is been adopted in several globally adopted wireless standards such as Digital Video Broadcasting Terrestrial (DVB-T) [6]; Digital Audio Broadcasting (DAB) [8]; IEEE 802.16a Metropolitan Area Network (MAN) [9] and the IEEE 802.11a Local Area Network (LAN) standards [10].

Orthogonal Wavelet Division Multiplex

Wavelet Decomposition

Because of its recent discovery, to date, there has only been a small amount of research into OWDM as used in a communications channel [4], [5], [11].

The wavelet transform converts a signal from the time

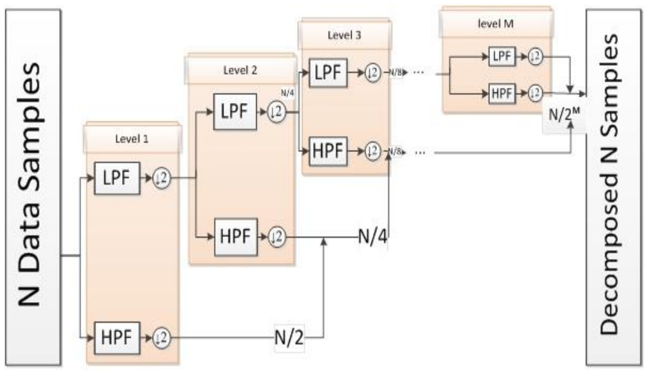

Fig. 1. OWDM - M levels of decomposition at the transmitter

domain into the time-frequency domain. Unlike the Fourier transform, where the sinusoidal wave repeats itself to infinity, the wavelet transform is located only within finite

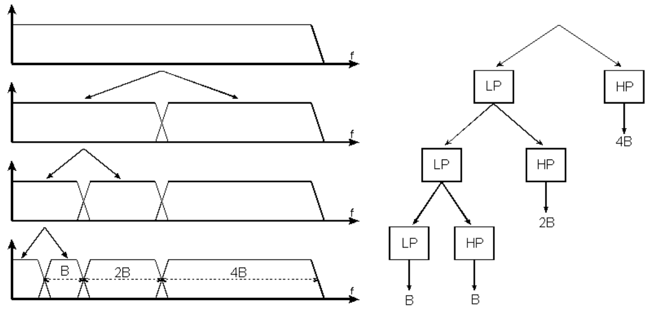

Fig. 2. Splitting the frequency using Wavelet Transform

domain and zeroes anywhere else. Hence it is possible in the wavelet transform to model changes in frequency by changing the scale of the time domain. Additionally, modeling the changes in time could be achieved by shifting the position of the wavelets; therefore the frequency and location of the frequency could be modeled. Consequently, this transform is called the time-frequency domain [4].

The wavelet transform has two kinds of transforms: the Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT). The CWT is rapidly costly because it performs convolutions at every position. Conversely, the DWT resolves the cost problem due to the fact that the signal data which mostly needs to be processed is discretely stored. In addition, Nyquist sampling theorem indicates that with the discrete data signal the highest frequency which can be modeled is half of the sampling frequency [2].

Due to the sampling theorem and calculation of scale convolutions at the wavelet transform, the first path which the wavelet picks up at a single sample point is unnecessary. Conversely, the more important scale deemed necessary for consideration is the smallest scale whereby the wavelet picks up two sample points. Therefore, it has been suggested [4] that this process can be achieved by a combination of orthogonal Lowpass Filter (LPF) and Highpass Filter (HPF) forming the filter bank where the LPF consists of the resulted scaling function and the HPF consists of the resulted wavelet function as depicted in figure 1.

As with OFDM, it is important that each sub-band remains orthogonal with respect to each other. In OWDM, this is achieved through the fact that the scaling function is orthogonal to the wavelet, or shifting function.

Orthogonal Wavelet Division Multiplex

OWDM is based on the concept of orthogonal multipulse signaling. Lee and Messerschmitt [12], considered that the pulses, $\varphi m(t)$, where $m = 0, 1, \ldots, M-1$, form an orthonormal set over a certain period of time and that each pulse is orthogonal to itself shifted by the non-zero integer multiples of a certain interval, T. Each basic pulse, $\varphi m(t)$, can then serve to create a 'virtual' channel over which the symbol a_m is carried.

The vector of symbols, $A=[a_0,a_1,\ldots,a_{M-1}]T$ is called a super-symbol and the interval $T = MT_s$ as the super-symbol/block interval, where T_s is the basic symbol interval. Subsequently the base band transmitted signal becomes

$$S(t) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m(t - iT) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(t - iT) \qquad (1)$$

At the receiver, symbol and block timing extraction is performed; and the received signal is correlated with $\varphi(t - iT)$ to detect the nth super symbol at time iT and time $iT + \tau$, where τ denotes the optimum timing phase [12].

In reality, discrete domain implementations are used as opposed to continuous time versions. In the discrete time domain, time orthogonal multipulses are $\varphi m(n)$ where $m=0,1,\ldots,M-1$. As a result, the corresponding transmitted orthogonal multipulse signal becomes

$$S(n) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{i,m}\, \varphi_m\,(n - iM) = \sum_{i=-\infty}^{\infty} A_i^T\, \varphi(n - iM) \qquad (2)$$

However, for convenience and simplicity, t will be used to denote both the continuous time variable as well as the discrete time sample index. In addition, M and T will be used interchangeably to denote the block length.

In practice, the OWDM communication system is made up of two main sections: the synthesis section and a channel interface. The synthesis section consists of a filter bank with a number of inputs; with an output that supplies an OWDM signal [4].

Each input receives a symbol associated with the super-symbol, which are selected from a modulation scheme. The synthesis section generates an OWDM signal made up of weighted OWDM pulses; each weighted pulse representing a symbol of the super-symbol. The OWDM pulses are also used with an OWDM Spread Spectrum, known as OWSS. OWSS allows operation for wireless channels with the use of an equalizer.

Asymmetric decomposition

When looking at wavelet decomposition, there are two approaches – asymmetric and symmetric. A block diagram of the asymmetric decomposition is depicted in figure 1. From figure 1, it can be seen that that every LPF output is subdivided into additional HPF and LPF components. Each LPF or HPF component is treated as an individual signal and can be traced along the subdivision (sub-bands). What is interesting here is that due to the scaling feature of the wavelet filter banks, these components signals will not interfere with each other even though it appears that aliasing may occur.

With asymmetric decomposition, the bandwidth given to each sub-band subsequent level of decomposition is halved.

Symmetric decomposition

In addition to asymmetric decomposition, it is possible to not only further decompose the LPF component, it is equally possible to decompose each HPF component in the same way as shown in figure 3. The difference between symmetric and asymmetric decomposition is that in symmetric decomposition, each sub-band has the same bandwidth as all other sub-bands in that level. If sufficient levels of decomposition are employed, the resulting waveform will appear similar to that of OFDM.

Simulation

In this work, two pre-2011 UK multiplex values are used as test scenarios: the BBC owned multiplex (Multiplex 1) and the Digital 3&4 owned multiplex (Multiplex 2). These multiplexes have parameters of 16-QAM, 3/4 rate and 64-QAM, 2/3 rate respectively.

These two multiplex parameter sets were used in a commercial system until the 2012 UK digital switchover. MUX 1 is the pre-switchover multiplexing for British Broadcasting Corporation (BBC) and British Telecommunications Company (Arqiva). Whereas, MUX 2 is the pre-switchover multiplexing for ITV corporation and Digital 3&4 (Channel 4 consortium).

TABLE I. SIMULATION PARAMETERS

Parameter	Value
OFDM Size	2K
OFDM useful symbols	1705
OWDM composition levels	11
Multiplex 1 parameters	16-QAM, 3/4 rate
Multiplex 2 parameters	64-QAM, 2/3 rate
RS Encoding	RS(204,188,16)
SNR range	0-30dB
Number of frames	10
Number of multipaths in Ricean	1

The full parameters of the simulation can be seen in Table 1. The experiment involved increasing the Signal to Noise Ratio (SNR) of the channel, comparing to a measure of the Bit Error Rate (BER) taken at the output of the RS decoder.

In order to ensure compliance with the standard, the SNR which gave the Quasi Error Free (QEF) state (a BER 10-11 at the output of the RS Decoder) was evaluated and compared to the values provided in the DVB-T standards (which gives all SNRs at QEF state for all combinations of QAM and Inner coding rate).

Once the simulation was compliance validated according to the standards, the IFFT/FFT blocks were replaced and the experiment repeated using the system depicted in figure 4 in which the IFFT and FFT are replaced with the Wavelet Synthesis Filter and Decomposition filters respectively.

Each system, OFDM and OWDM, was repeated with 4 scenarios:

1. Multiplex 1(MUX 1) – AWGN Channel
2. Multiplex 2(MUX 2) – AWGN Channel
3. Multiplex 1 (MUX 1) – Ricean Channel
4. Multiplex 2 (MUX 2) – Ricean Channel

Results

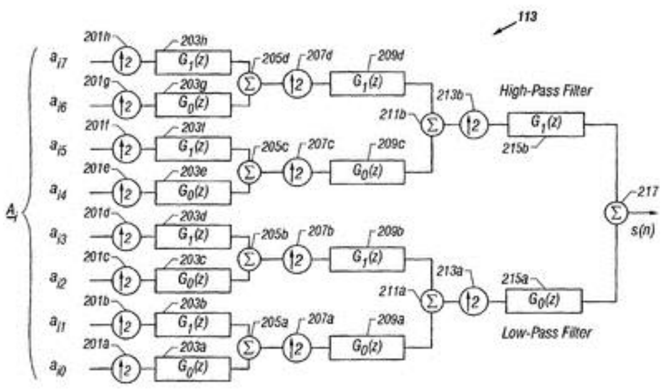

Fig. 3. Wavelet Symmetric synthesis [4]

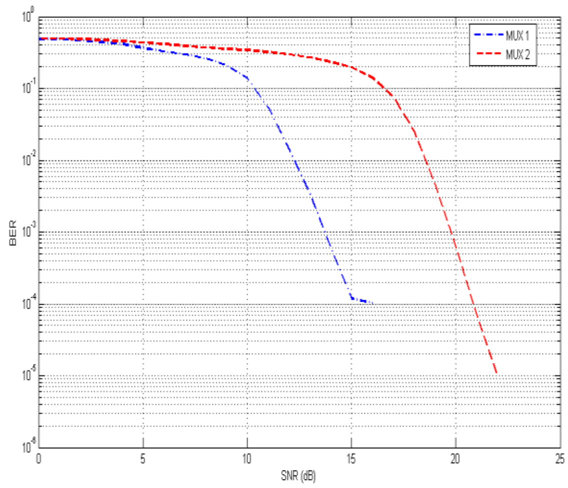

Fig. 6. SNR vs BER for OWDM with AWGN Channel

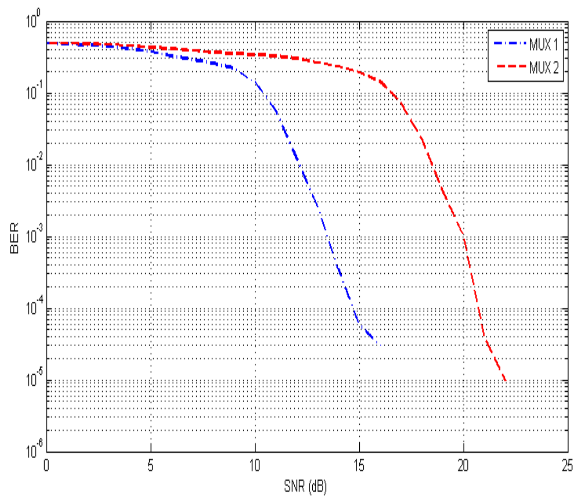

Fig. 5. SNR vs BER for OFDM with AWGN Channel

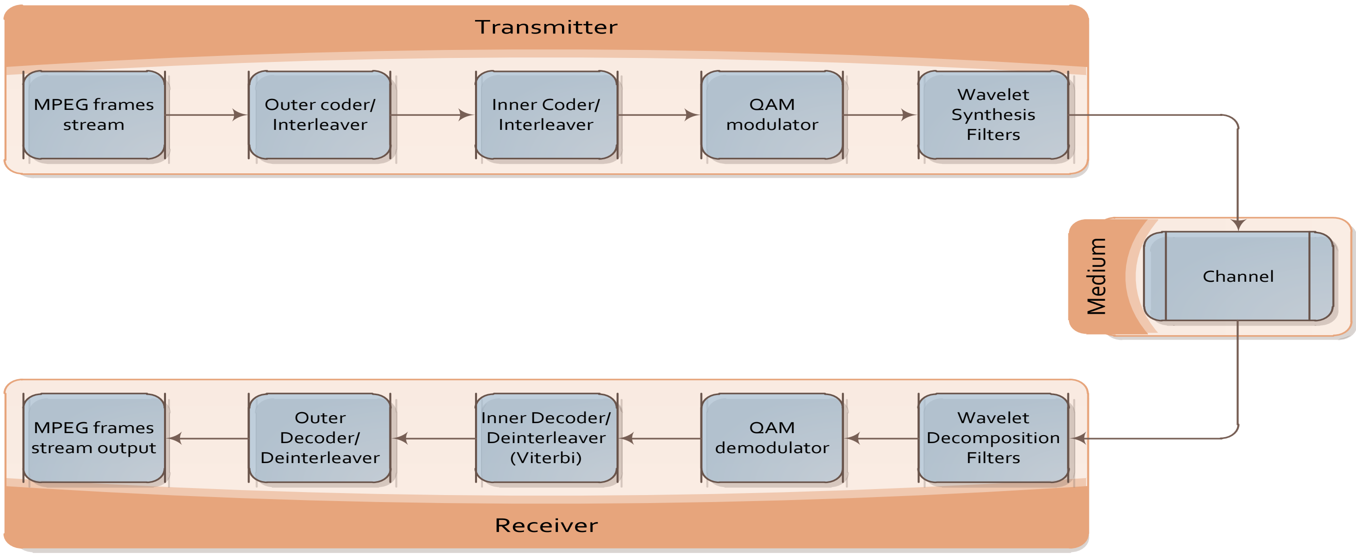

Fig. 4. OWDM system block diagram

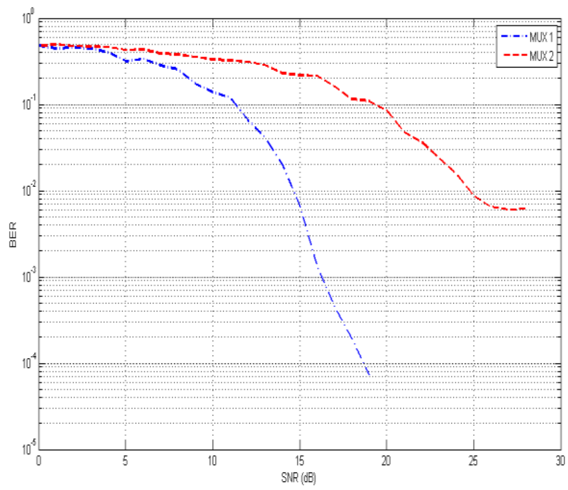

Fig. 7. SNR vs BER for OFDM with Ricean Channel

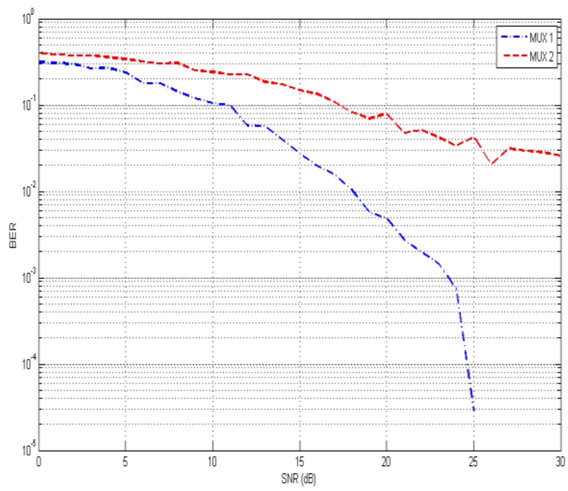

Fig. 8. SNR vs BER for OWDM with Ricean Channel

Figure 5 shows the SNR-BER curves for OFDM under AWGN while figure 6 shows the SNR-BER curves for OWDM under the same conditions. From these two plots, it can be seen that there is a similarity between the two curves. From figure 5, it can be seen that the SNR which gives a BER of 10^{-4} is 14.7dB and 20.9dB for MUX 1 and MUX 2 respectively. Comparing these results to the values for OWDM in figure 6 of 15dB and 20.8dB respectively shows that both systems offer a similar performance under AWGN.

Figures 7 and 8 show the SNR-BER curves for OFDM and OWDM in a Ricean fading channel respectively. In OFDM, a BER of 10^{-4} is achieved with an SNR of 19dB and 24dB for MUX 1 and MUX 2 respectively. Comparing to OWDM as shown in figure 8, the same BER is achieved through an SNR of 24dB and 26dB respectively. It can, therefore, be seen by comparing these results, OWDM has a small advantage over OFDM in terms of SNR.

Conclusion

This paper has provided a comparison between Orthogonal Frequency Division Multiplex and Orthogonal Wavelet Division Multiplex as applied to two pre-2011 UK digital television multiplexes.

The purpose of this paper is to demonstrate that OWDM has the potential of acting as a successor for OFDM because of some of the potential advantages it has over OFDM. The computational complexity of wavelet decomposition filters in OWDM is less than Fourier Transform in OFDM. There is also a potential exploitation of guard interval in OWDM instead of wasting it in OFDM.

Two scenarios have been presented in this paper which demonstrates that under AWGN, the performance of OWDM is matched to that of OFDM.

Under a Ricean fading channel, however, the results have predicted a small decrease in performance when comparing

OWDM to OFDM. It is suspected that the cause for this is due to the guard interval insertion under OFDM which is used to eliminate multipath [7] but does not exist within the current OWDM model. Further equalization will be required to get a similar response.

References

[1] R. Chang, "Synthesis of a Band-Limited Orthogonal Signals for Multichannel Data Transmission", the Bell Technical Journal, Vol. 45, 1966, pp 1775-1996.

[2] Weinstein S. and Ebert P. "Data transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform". IEEE Transactions on Communications, COM-19, 5:628-634, October, 1971.

[3] I. Daubechies, "Ten Lectures on Wavelets", Society for Industrial and Applied Mathematics, 3rd Edition, 1994, ISBN 0-89871-274-2.

[4] Jain V. K. and Myera B. A., "Communication System using Orthogonal Wavelet Division Multiplexing (OWDM and OWDM-spread spectrum (OWSS)) Signalling," 7058004, June 06, 2006.

[5] Scott L. Linfoot, Mohammad K. Ibrahim and Marwan M. Al-Akaidi, "Orthogonal Wavelet Division Multiplex: an Alternative to OFDM," IEEE Transactions on Consumer Electronics, vol. 53, vol. 2, May 2007, pp. 278 - 284.

[6] ETSI EN 300 744 V1.6.1, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.

[7] Prasad Ramjee, OFDM for Wireless Communications Systems. London, United Kingdom: Artech House Inc., 2004.

[8] DAB standard ETSI EN 300 401: "Radio Broadcasting Systems; Digital Audio Broadcasting (DAB) to mobile, portable and fixed receivers", 2006.

[9] IEEE 802.16 standard for Lacal and Metropolitan area networks, "Part 16, Air Interface for Boadband Wireless Access Systems, Amendment 3: Advanced Air Interface", 2011.

[10] IEEE 802.11 standard for Local Area Network "Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications", 2012.

[11] Linfoot S. L., "A Study of Different Wavelets in Orthogonal Wavelet Division Multiplex for DVB-T," IEEE Consumer Electronics, vol. 54, no. 3, pp. 1042 - 1047, August 2008.

[12] A. Lee and D. G. Messerschmitt, Digital Communications, 2nd ed.: Kluwer Academic Publishers, 1988.

BIOGRAPHIES

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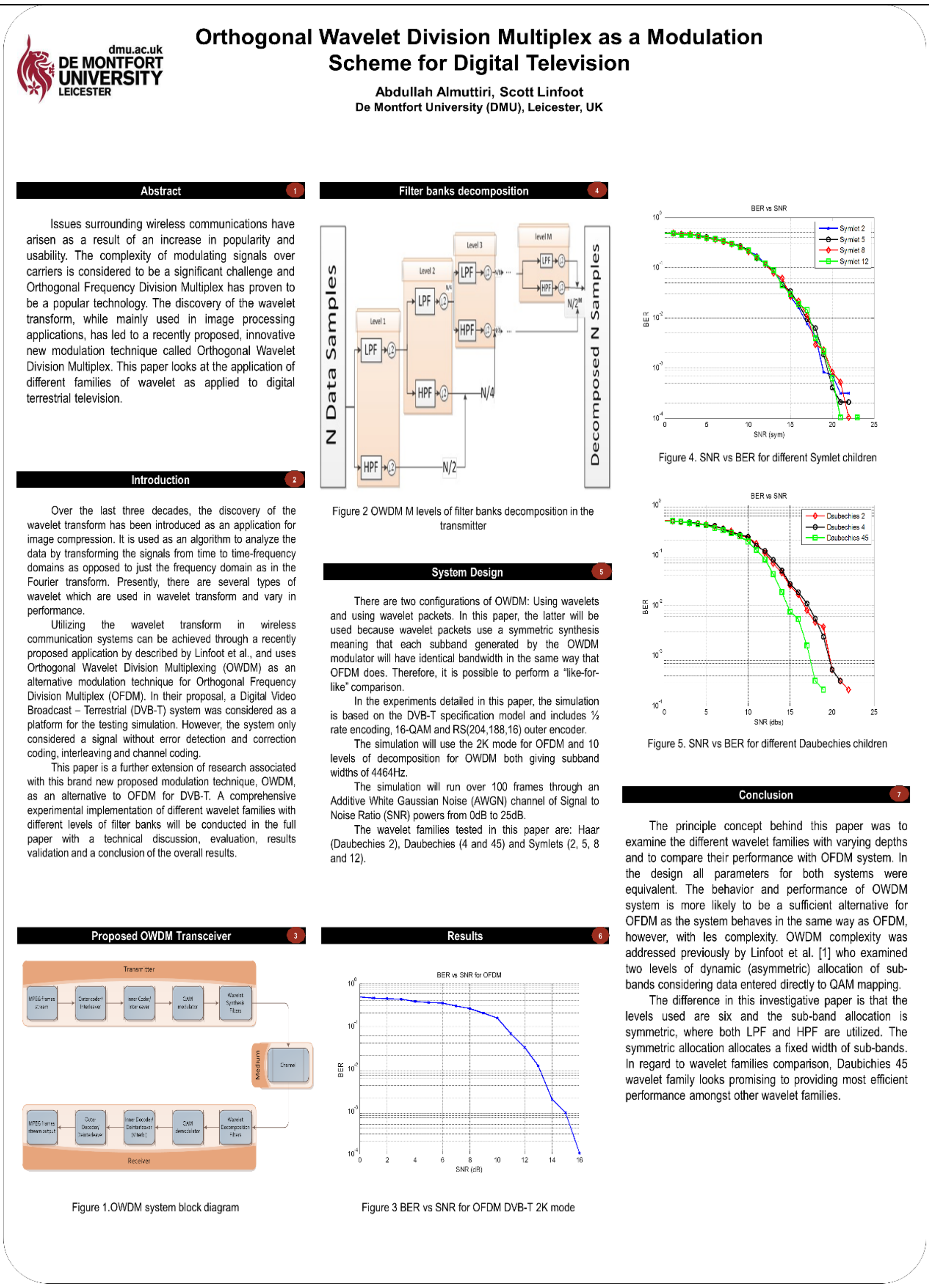
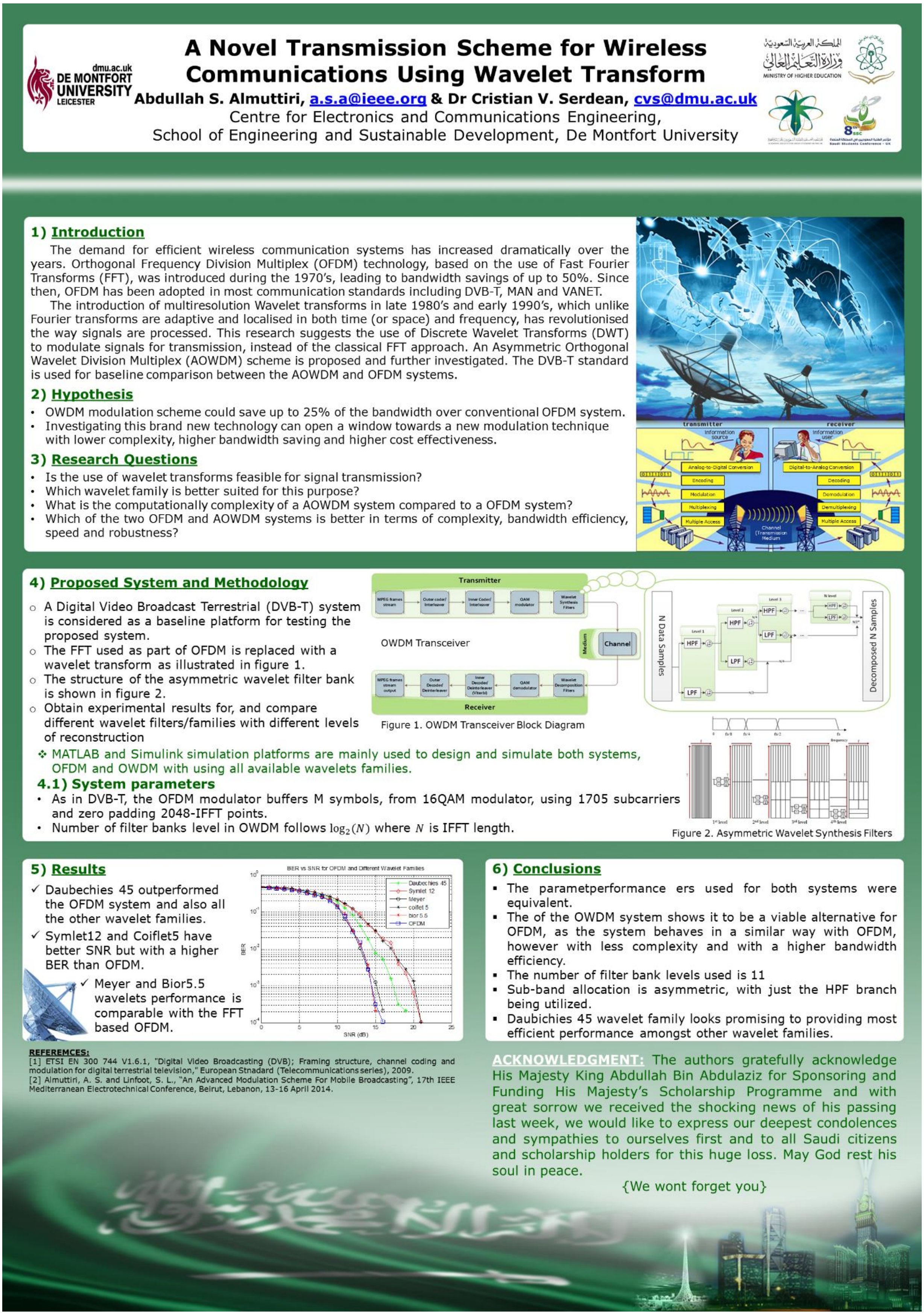
APPENDIX D: CONFERENCE POSTERS

The following Poster was abstracted and published as full paper in ieeeexplore library and presented in IEEE International Conference in Consumer Electronics 2013 (ICCE-Berlin 2013), Berlin, Germany.



The following Poster was abstracted and published in the Imperial College Press and was presented in the 8th Saudi Students Conference in the UK, February 2015, London.



A Novel Transmission Scheme for Wireless Communications Using Wavelet Transform

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1) Introduction

The demand for efficient wireless communication systems has increased dramatically over the years. Orthogonal Frequency Division Multiplex (OFDM) technology, based on the use of Fast Fourier Transforms (FFT), was introduced during the 1970's, leading to bandwidth savings of up to 50%. Since then, OFDM has been adopted in most communication standards including DVB-T, MAN and VANET.

The introduction of multiresolution Wavelet transforms in late 1980's and early 1990's, which unlike Fourier transforms are adaptive and localised in both time (or space) and frequency, has revolutionised the way signals are processed. This research suggests the use of Discrete Wavelet Transforms (DWT) to modulate signals for transmission, instead of the classical FFT approach. An Asymmetric Orthogonal Wavelet Division Multiplex (AOWDM) scheme is proposed and further investigated. The DVB-T standard is used for baseline comparison between the AOWDM and OFDM systems.

2) Hypothesis

- OWDM modulation scheme could save up to 25% of the bandwidth over conventional OFDM system.
- Investigating this brand new technology can open a window towards a new modulation technique with lower complexity, higher bandwidth saving and higher cost effectiveness.

3) Research Questions

- Is the use of wavelet transforms feasible for signal transmission?
- Which wavelet family is better suited for this purpose?
- What is the computationally complexity of a AOWDM system compared to a OFDM system?
- Which of the two OFDM and AOWDM systems is better in terms of complexity, bandwidth efficiency, speed and robustness?

4) Proposed System and Methodology

- A Digital Video Broadcast Terrestrial (DVB-T) system is considered as a baseline platform for testing the proposed system.
- The FFT used as part of OFDM is replaced with a wavelet transform as illustrated in figure 1.
- The structure of the asymmetric wavelet filter bank is shown in figure 2.
- Obtain experimental results for, and compare different wavelet filters/families with different levels of reconstruction

❖ MATLAB and Simulink simulation platforms are mainly used to design and simulate both systems, OFDM and OWDM with using all available wavelets families.

OWDM Transceiver

Figure 1. OWDM Transceiver Block Diagram

Figure 2. Asymmetric Wavelet Synthesis Filters

4.1) System parameters

- As in DVB-T, the OFDM modulator buffers M symbols, from 16QAM modulator, using 1705 subcarriers and zero padding 2048-IFFT points.
- Number of filter banks level in OWDM follows $\log_2(N)$ where N is IFFT length.

5) Results

- ✓ Daubechies 45 outperformed the OFDM system and also all the other wavelet families.
- ✓ Symlet12 and Coiflet5 have better SNR but with a higher BER than OFDM.
- ✓ Meyer and Bior5.5 wavelets performance is comparable with the FFT based OFDM.

BER vs SNR for OFDM and Different Wavelet Families

6) Conclusions

- The parametperformance ers used for both systems were equivalent.
- The of the OWDM system shows it to be a viable alternative for OFDM, as the system behaves in a similar way with OFDM, however with less complexity and with a higher bandwidth efficiency.
- The number of filter bank levels used is 11
- Sub-band allocation is asymmetric, with just the HPF branch being utilized.
- Daubichies 45 wavelet family looks promising to providing most efficient performance amongst other wavelet families.

REFEREMCES:
[1] ETSI EN 300 744 V1.6.1, "Digital Video Broadcasting (DVB); Framing structure, channel coding and modulation for digital terrestrial television," European Stnadard (Telecommunications series), 2009.
[2] Almuttiri, A. S. and Linfoot, S. L., "An Advanced Modulation Scheme For Mobile Broadcasting", 17th IEEE Mediterranean Electrotechnical Conference, Beirut, Lebanon, 13-16 April 2014.

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{We wont forget you}

APPENDIX E: PROJECT MANAGEMENT

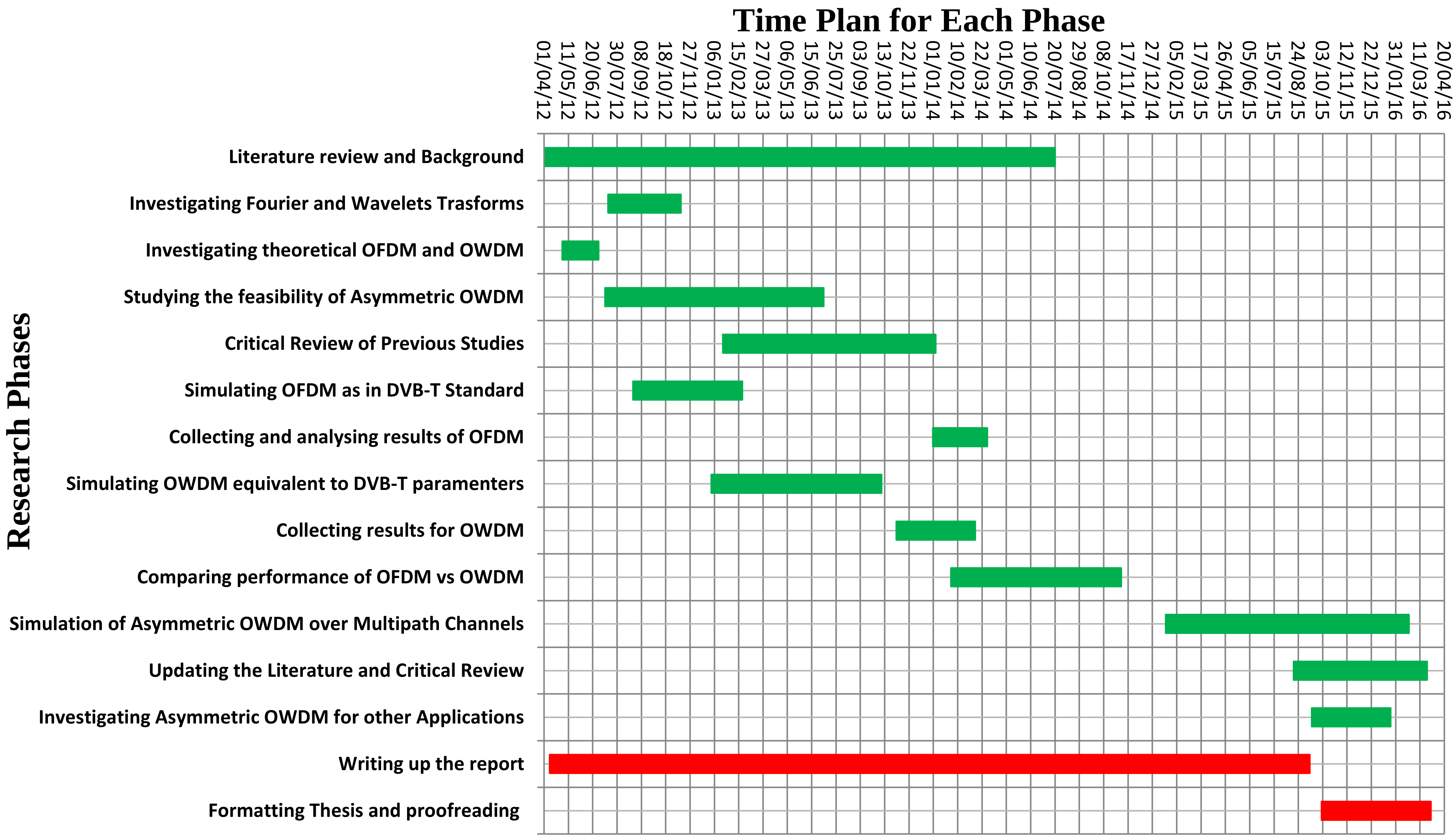

Figure E.1: Gantt chart for the research phases

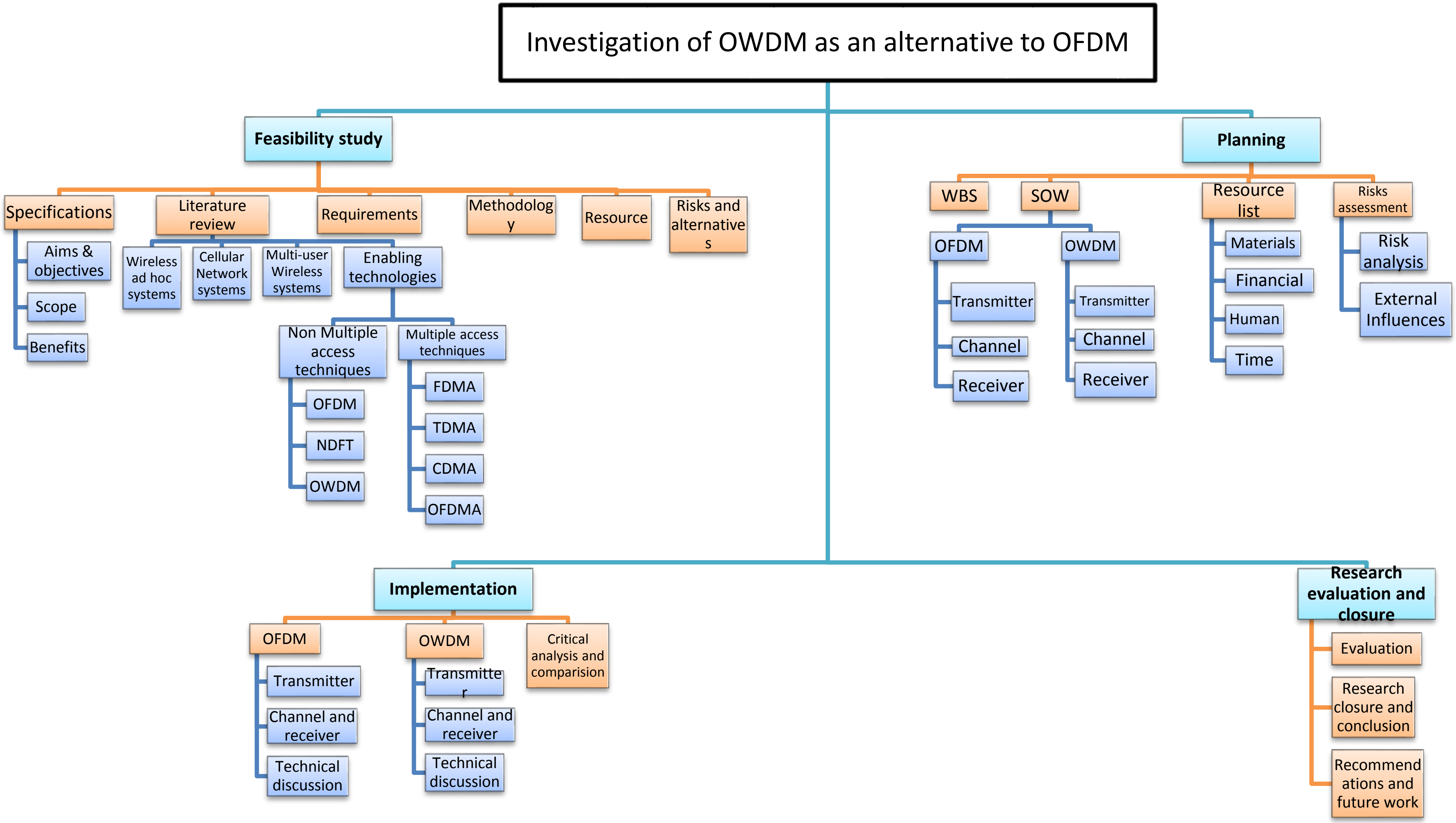

Figure E.2: Work Breakdown Structure (WBS) of the research

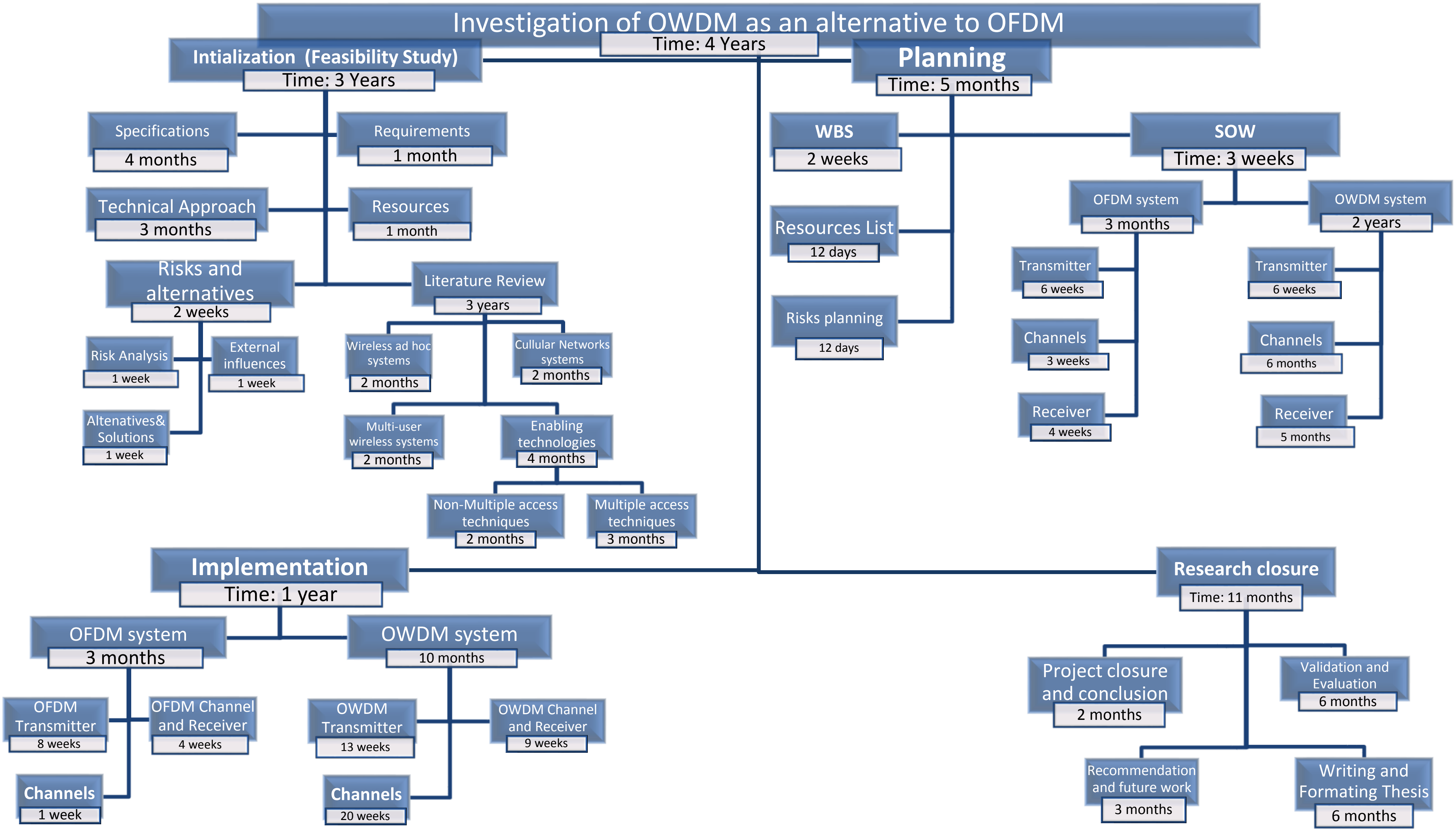

Figure E.3: Detailed Time Plan for Individual Tasks

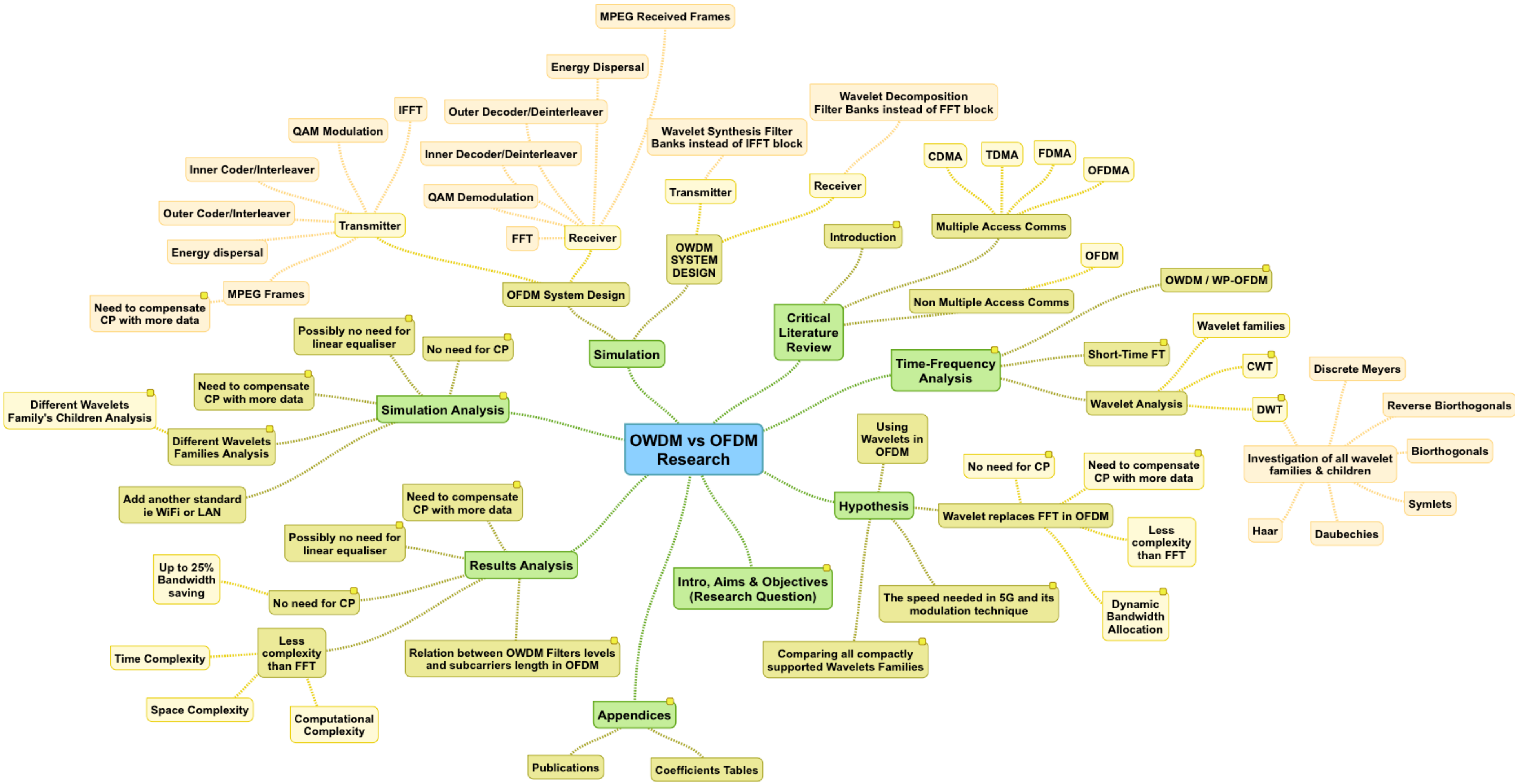

Figure E.4: Research Mind Map