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Battle Diffusion Matters: Examining the Impact of Microdynamics of Fighting on Conflict Termination

May 9, 2019

Abstract

How does diffusion of civil war battles influence conflict termination? Recent advances in civil war literature have found that battle dynamics shape conflict termination by affecting the intra-conflict bargaining between disputants. This article extends the theoretical perspective and argues that *how* battles diffuse matters in determining conflict termination. While battlefield dynamics should in principle reveal previously unavailable private information, the relevance of information-revealing effect depends on the diffusion patterns of battles. The persistent, and possibly battle-exacerbated, commitment problem can also alter the prospects of conflict termination. We test the argument by distinguishing two distinct diffusion patterns of battles: distant and proximate. The empirical results reveal that distant diffusion, but not proximate diffusion, of battles makes civil conflicts less likely to terminate. The analysis also demonstrates that incorporating diffusion dynamics significantly improves our ability to predict conflict termination.

Word Count

10,990 (excluding the Online Appendix)

Recent advances in civil war literature have demonstrated a wide cross-national variation in conflict geographies (e.g., Beardsley et al., 2015; Buhaug & Gates, 2002; O’Loughlin & Witmer, 2012; Schutte & Weidmann, 2011). Battle activities in some conflicts gradually diffuse from the originating locations toward geographically contiguous locations, just like a forest fire, while the battle locations in other conflicts spread toward wider areas that have not previously been exposed to violence. Still, in other conflicts, the conflict-affected zones remain relatively contained and stable.

How do such micro-level conflict processes, then, alter the macro-level outcome of conflict termination? Traditionally, much of the conflict termination literature has focused on aggregated and static factors, such as state capacity and the existence of natural resources (e.g., Collier et al., 2004; Fearon, 2004). Consequently, the micro-level, dynamic determinants of civil war termination have been left relatively under-studied in the literature, which could lead us to biased conclusions (Balcells & Kalyvas, 2014, 1391–1392). In recent years, however, a small but growing body of literature has increasingly explored the role of microdynamics, such as battle intensity and locations, in shaping conflict termination (e.g., Greig, 2015; Ruhe, 2015; Wood & Kathman, 2014).

This article joins this emerging debate by exploring not only how the intensity or relative location of the battle matters, but also how different spatio-temporal diffusion patterns of battles impact the likelihood of conflict termination differently. Following previous studies (e.g., Cohen & Tita, 1999; Gould, 1969), we distinguish two distinct diffusion patterns, distant and proximate, of civil war battles and link these microdynamics into conflict termination at the macro level.

We argue that diffusion of battles shapes conflict termination primarily by affecting the underlying bargaining problems. Firstly, battles reveal necessary information for the belligerents’ expectations about victory to converge (Slantchev, 2003b; Smith & Stam,

2004). This is primarily the case for proximate diffusion of battles — resembling conventional warfare tactics with clearer frontlines. In such instances, the information coming from the battlefield should help disputants resolving the bargaining problem and therefore have a positive effect on conflict termination. However, this information becomes uncertain when battles diffuse to geographically distant locations. Such diffusion of battles partly reflects the rebels’ tactical choice to employ guerrilla warfare, or the use terrorism in civil wars. The spread of battles to distant and remote locations means that it is harder to collect reliable information about the opponent (Walter, 2009, 253). In such scenario, as opposed to geographically limited battle zones, the lack of reliable information leaves the information problem unsolved and the conflict is less likely to terminate.

Secondly, the credible commitment problem also needs to be resolved for inefficient fighting to end. The information-revealing effect of battle dynamics, however, does not necessarily resolve the persistent commitment problem. In some situations, the underlying commitment problem can even be exacerbated, rather than resolved, by battlefield dynamics. For example, when battlefield outcomes favor the rebels and the government’s strength fluctuates, costly fighting would make the government’s commitment to war-ending agreements by amplifying its incentives to renege and exploit its recovered bargaining position in the post-conflict environment (cf. Fearon, 2004, 295–296). Successful implementation of guerrilla-like *foco* military strategies by the rebel forces would induce such temporary fluctuations of government capability, spread out battle activities across distant localities, and partly be captured by battle diffusion across remote locations. In such situations, battle diffusion can in turn undermine conflict termination through the commitment channel as well as its limited information-revealing effect.

These two-fold bargaining dynamics, as well as the limited prospects of decisive victories in the face of active fighting, in turn prolong civil conflicts. This effect is, however,

stronger in the cases of distant diffusion of battles whereby such battle dynamics — resembling guerrilla warfare — would not be informative enough to reveal the disputants’ capability and resolve while not solving or even exasperating the credible commitment problem. The impact of proximate diffusion remains indeterminate due to the persistent commitment problem despite the more informative nature of direct confrontations.

Consider, for example, the 17-year civil war in Mozambique. The conflict was characterized by the rebel group RENAMO’s guerrilla tactics across the country. RENAMO’s president Dhlakama famously said that “our aim is not to win the war militarily . . . but to force FRELIMO to accept negotiations” (quoted in Finnegan, 1992, 79). This quote highlights the strategic calculation of the rebels to spread the government’s forces thinly across the Mozambique’s vast territory, which in the end was a significant contributor for the prolonged conflict (Weinstein & Francisco, 2005).

It is precisely the association between these microdynamics of fighting and subsequent course of conflict termination, exemplified by this conflict episode, that we examine systematically in this article. With the help of precisely geocoded and disaggregated monthly data on civil war battles, we find empirical evidence that distant diffusion of battle activities, in particular, make conflict termination less likely. It is also worth noting that incorporating the faster-moving diffusion dynamics of battles in addition to slower-moving structural factors substantially improves our ability to predict conflict termination.

This article contributes to the emerging body of literature about micro-level and dynamic determinants of civil war termination. Balcells & Kalyvas (2014) and Greig et al. (2016) are correct in pointing out that if conflict dynamics, as well as static conditions, matter in altering the chances of conflict termination and outcome, any study on civil war termination remains incomplete without examining how fighting influences the prospects for domestic peace. An empirical investigation on the likely impact of spatio-temporal

dynamics of battles on conflict duration and outcome is therefore a critical step to understand the determinants of civil war termination.

Our study also relates to the broader literature on the relationship between the conflict processes and conflict termination. A notable trend within the recent literature is the renewed call to investigate the question of conflict termination (e.g., Leventoglu & Slantchev, 2007; Powell, 2004, 2012; Reiter, 2009; Slantchev, 2003a,b; Wagner, 2000). Since Fearon (1995), most previous studies have highlighted the question of why costly conflict occurs and have treated war as an outcome to be explained. In contrast, recent literature has increasingly shifted the attention to the puzzle of how and why fighting resolves the bargaining problem that leads to war (Ramsay, 2008, 850–853). Primarily building upon the bargaining model of war, this article offers a theoretical account and empirical tests of the likely impacts of violence diffusion on civil war termination. In so doing, this article specifies the micro-foundations of the relevant theoretical accounts (Kertzer, 2017), and thereby contributes to this ongoing debate on conflict termination.

1 Related Literature and Typology of Diffusion

There has been an increasing scholarly interest in recent years about the microdynamics of violence and dynamic determinants of conflict termination. While informative, previous studies have paid less attention to the possible impacts of spatio-temporal diffusion dynamics of battle activities on conflict termination and duration.

1.1 Conflict Dynamics and Conflict Termination

Previous studies have demonstrated how battle intensity (Ruhe, 2015), battle locations (Greig, 2015; Greig et al., 2016; Ruhe, 2015), civilian victimization (Wood & Kathman,

2014), and acts of terrorism (Fortna, 2015; Thomas, 2014) each influence civil war termination and outcomes. For example, Greig (2015) argues that relative locations and movements of battles toward strategic locations such as capital cities reveal previously unavailable information to warring parties and thereby influence their willingness to participate in war-ending diplomacy. Empirical records show that locations, movement, and dispersion of battles influence the onset and outcomes of peace talks. Ruhe (2015) also emphasizes the role of locations of battles in altering the chances of mediation onset. Battle activities are viewed as costly by warring parties only when they occur at locations with intermediate distances from national capitals and thereby alter the chances of mediation success, or when conflict geography indicates a stalemate rather than either side taking the upper hand over another. Consistent with this expectation, empirical records show that increasing conflict intensity lowers the probability of mediation acceptance when battles take place in the locations close to or far from the capital, while the same escalating battles are followed by an increased probability of mediation acceptance when fighting occurs in intermediate distances to the capital.

Related studies have found that civilian casualties and rebel strategies also invariably affect when and how conflict ends. Rather than direct confrontations between troops, Wood & Kathman (2014) highlight the role of civilian abuse. An intermediate level of civilian victimization improves the bargaining position of the rebels, primarily because continuous civilian victimization imposes costs on the regime and reveals information about the likely costs of the conflict. In contrast, an extremely high level of civilian abuse may hinder the prospects of negotiated settlements because it contributes to a shift in the underlying power balance and thereby exacerbates the credible commitment problem. Consistent with their theoretical claims, Wood & Kathman (2014) find an inverted U-shaped relationship between the intensity of civilian abuse and the chances of negotiated

settlements. Fortna (2015) and Thomas (2014) explore how the use of terrorism rewards rebel groups in achieving their political goals. While these studies disagree on the direction of the causal effect, they generally agree on the correlation between the acts of terrorism and the eventual outcomes of civil conflicts.

Put generally, these studies are a part of the growing civil war literature on the dynamic determinants of conflict process and termination. Similar to these macro-level studies, the micro-level literature demonstrates how faster-moving “process” variables such as civilian attitudes and battle activities, in addition to slower-moving or time-invariant “structural” variables, matter in explaining conflict dynamics (Blair et al., 2017). For example, drawing on the original data from a survey experiment in 204 villages in Afghanistan, Hirose et al. (2017) reveal how insurgents use civilian attitudes toward the counterinsurgents as “cues” to select their targets and tactics. Their empirical results show that incorporating civilian attitudes substantially improves predictive accuracy across multiple insurgent targets and tactics, both in in-sample classification and out-of-sample forecast.

These studies are important in considering how battlefield outcomes shape when and how civil conflicts end. What remains relatively under-investigated in the literature is the possible impact of spatio-temporal violence diffusion on conflict termination, which is the primary interest of this article.

1.2 Typology of Diffusion Dynamics

The typology of micro-level diffusion of violence is a subject of scholarly debate in itself. Recent advances in civil war studies have also yielded valuable insights about how violent activities diffuse in civil conflicts. Beardsley et al. (2015) find that areas exposed to battles tend to relocate when civil wars are fought by rebel groups who lack strong ethnic ties with local population and sufficient military strength, as such rebel groups tend to stay mobile

as a means of survival in the face of government forces. Beardsley & Gleditsch (2015) demonstrate that peacekeeping operations, especially when robust forces are deployed and when rebel groups have strong ethnic ties with local population, tend to contain movement of conflict-affected zones. Schutte & Weidmann (2011) propose a typology of violence diffusion in civil wars and find that the patterns of diffusion in civil wars are expansive in scope (expansion diffusion) rather than changing from one location of a violent event to another (relocation diffusion).

We rely on a similar typology to investigate how different diffusion patterns of violence impact the likelihood of conflict termination differently. Specifically, based on the geographical contiguity between the originating and previously peaceful spatial units, we distinguish two broad categories of diffusion patterns in micro-level battle dynamics: *proximate* diffusion and *distant* diffusion (Cohen & Tita, 1999; Gould, 1969; see also, Baudains et al., 2013; Schutte & Weidmann, 2011; Zhukov, 2012).¹ Proximate diffusion refers to the process in which the status among neighboring spatial units affects the future status of their neighboring units. In the case of civil war violence, this type of diffusion corresponds to the instances where violence spreads to areas contiguous to previously affected areas. In contrast, distant diffusion reflects the spread of phenomena that do not depend on physical contacts between spatial units. The instances of distant diffusion refer to increase of events in physically non-adjacent locations. Since the seminal work by Gould (1969), the distinction between proximate diffusion and distant diffusion processes has been adopted by spatial analysis in crime and civil war studies (Baudains et al., 2013; Cohen & Tita, 1999; LaFree et al., 2012; Schutte & Weidmann, 2011; Zhukov, 2012).

Figure 1 uses a simple grid-cell representation to illustrate these two distinct diffusion patterns. In the current context, proximate diffusion process is exemplified by gradual

¹Proximate diffusion corresponds to “contagious diffusion” while distant diffusion patterns correspond to “hierarchical diffusion” patterns in the typology in Cohen & Tita (1999); Gould (1969).

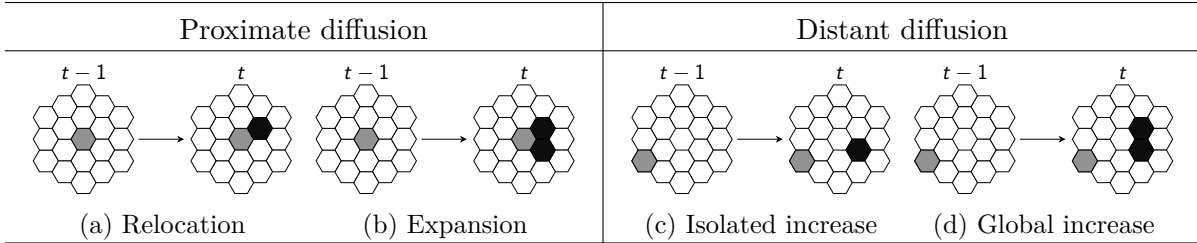


Figure 1: Hexagonal grid representation of diffusion patterns

Note: Examples of (a), (b) proximate diffusion and (c), (d) distant diffusion. Solid cells represent cells with 1+ events at $t - 1$ (gray) and t (black), while cells without events are left blank. Note that distant diffusion does not require the originating locations to continue to experience violence at t .

expansion or movement of front lines, while distant diffusion process is typified by the spread of battles across geographically distant locations within the country. Although it is possible to divide diffusion patterns further into four subcategories, for simplicity, we rely on the dichotomous categorization.

2 Battle Diffusion and Conflict Bargaining

We argue that these diffusion dynamics of battle activities substantially influence the prospects of conflict termination. This section lays out the core argument about the informational value of battle diffusion and logic of the persistent commitment problem, followed by several testable propositions.

2.1 Battle Diffusion, Information, and Credible Commitment

Given that war is *ex post* inefficient as it imposes otherwise unnecessary costs on disputants, there almost always exists a bargaining range that can make both sides better off than dividing the disputed good through fighting (Fearon, 1995; Powell, 2006, 177). In some situations, however, disputants fail to reach efficient pre-war agreements due to asymmetric information about, for example, the distribution of power and likely outcome

of war, combined with incentives to misrepresent private information (information problem, Fearon, 1995). Bargaining may also fail when disputants cannot credibly commit not to renege on an agreement in the absence of a central enforcement mechanism. This may occur due to, for example, a rapid shift in the underlying power balance, resulting in temptations to renege on war-avoiding concessions (commitment problem, Powell, 2006).

Battle diffusion and information problem In the situation where information problem caused the original bargaining failure (Blainey, 1988; Fearon, 1995), the information flows provided by battlefield outcomes such as casualties and location of battles update the belligerents' belief and expectations about the likely outcome of the conflict (Blainey, 1988; Greig, 2015; Powell, 2004; Ramsay, 2008; Ruhe, 2015; Slantchev, 2003b). Fighting itself should in turn produce an agreement to stop inefficient conflicts by revealing previously unavailable information and narrowing the informational asymmetry.

What makes battlefield outcomes a distinguishable source of information is its non-manipulability (Ramsay, 2008; Reiter, 2009). In the battlefields, as Ramsay (2008) argues, “one cannot pretend to be stronger, to have better leadership, or to be better equipped than one actually is” (854). The same cannot be said for diplomacy or negotiation at a table. While warring parties inherently hold strong incentives to manipulate their own image and to misrepresent private information to obtain better deal at the negotiation table, battlefield outcomes reveal true information about the underlying balance of power and willingness to bear costs. Previous studies have highlighted such information-revealing roles of intensity and location of battle activities (Greig, 2015; Ramsay, 2008; Ruhe, 2015; Slantchev, 2003b; Weisiger, 2016).

We argue that while battles are arguably a non-manipulable source of information, the informational values of the costly activities also depend on spatio-temporal diffusion dynamics, as well as the intensity and locations, of battles. For the information coming

from the battlefield to be relevant, it needs to be clear enough to update the disputants' prior beliefs. Belligerents deliberately choose where, when, and how to engage in battles against the opponents, and different diffusion patterns of battles are the outcomes of such tactical considerations of the warring parties. In line with previous studies in civil war strategies, we assume that belligerents broadly employ a mixture of "direct" and "indirect" strategies (Arreguín-Toft, 2001). Battles constitute a classic direct war strategy where the aim of the attacker is to defeat the opponent and capture areas and resources that are of value to the opponent. Indirect strategies on the other hand refer to deployment of, for example, guerrilla tactics that impose costs to the adversary, but are not directly intended to capture territory or resources. Building on the proximate and distant diffusion classification, LaFree et al. (2012) relate observed diffusion patterns of violence to the rebel military strategies. They argue and empirically demonstrate that direct or "control" attacks are more likely to follow proximate diffusion patterns, because such attacks should concentrate around rebel bases. On the other hand, indirect or attrition-based attacks should exhibit patterns of distant diffusion such that attacks in a location are followed by attacks in geographically non-adjacent locations because attacks would target broad areas far beyond the space the rebels hope to eventually govern.

These insights lead us to expect that different diffusion patterns of battles differ in their informational values. Proximate diffusion of battles resembles conventional war strategies and thus efficiently reveals information about the resolve and capabilities to the warring parties. In such instances, the information generated from the battlefield should help in resolving the bargaining problem and therefore have a positive effect on conflict termination. However, the informational value of violent encounters are reduced when battles diffuse to geographically distant locations. The spread of battles to distant and often remote locations means that it is harder to collect reliable information about

the opponent (Walter, 2009, 253). In such situations, as opposed to geographically adjacent battle zones, the lack of reliable information collected from the battlefield remain insufficient to clear the informational asymmetry.

Battle diffusion and commitment problem The flows coming from the battlefield, however, are often not enough to solve the underlying bargaining problem. Even in the situations where costly fighting has effectively reduced uncertainty about the power and intension of disputants, they may find themselves unable to make binding commitments to follow through the war-ending agreement to each other (Reiter, 2009). In this way, regardless of solving the informational problem, civil war termination may be prevented or delayed due to the persistent, and possibly battle-exacerbated, commitment problem.

The credible-commitment accounts of war suggest that large and rapid shifts in underlying power balance cause pre-war negotiations to fail, and inefficient fighting persists until the rate of shift slows (Powell, 2006, 2012; see also, Leventoglu & Slantchev, 2007).² In a situation where a rapid and large shift in the underlying balance of power in its favor is expected, the disputant may be unable to commit itself not to exploit its enhanced bargaining position in the future, as it may also have incentives to renege on prior agreement once the shift takes place. Expecting this scenario, the opposition is likely to be unwilling to accept negotiated settlement over the disputed goods, which in turn results in bargaining breakdown.

A logically equivalent story applies to situations wherein the current balance of capability and bargaining power favors the rebels due to disturbances in the government's capability, caused by a temporary shock (Fearon, 2004). Such temporary fluctuations of government capability can result from both exogenous factors such as a sharp economic

²Another pathway through which a war under commitment problem ends is that third-parties step in to to enforce the war-ending agreements, if they are able to commit to implementation of the agreement and provide credible guarantee on the settlement (Walter, 1997, 2002).

downturn and endogenous factors such as battlefield dynamics. Regardless of the exact causes, in these instances, the temporarily weakened government would commit to concessions to the rebels, reflecting its deteriorating bargaining position. The same government, however, cannot credibly commit to the agreement because, once fighting stops (or is avoided), it would most likely regain its capability. In the post conflict period, the government would have a strong incentive to exploit its enhanced bargaining position and renege on the prior policy concessions. Given that “nothing stops it from overturning or undermining the arrangements” in the absence of enforcement mechanisms, the common knowledge in relation to the temporary shock renders the government’s commitments not to renege incredible (Fearon, 2004, 290, 294).

Another, but related source of the commitment problem in the context of civil war is that rebels are often forced to disarm, demobilize, and disengage their military forces and prepare for peace during or after peace negotiations, which inevitably shifts the underlying power balance in the government’s favor (Walter, 1997, 2002). Armed rebels, however, would have little incentive to accept such conditions. This is primarily because both sides know that in the post-conflict environment where the government no longer faces armed opposition groups, it should have a strong incentive to renege on the war-ending agreements (Walter, 1997; see also, Fearon, 2004; Powell, 2006). Simultaneously, once the rebels “lay down their weapons,” as Walter (1997) argues, “it becomes almost impossible to either enforce future cooperation or survive attack” in the absence of enforcement mechanisms (335–336).

Indeed, the disarmament of rebel forces is often cited by incumbents as the necessary condition for negotiations. For example, former Yemeni Foreign Minister Riad Yassin expressed in 2015 that although Shiite Houthi rebels now controlled the capital and much of the north, that the rebels “must implement the UN resolution and *surrender their*

weapons, and *only then* the dialogue and political process can begin, with the participation of all Yemeni parties” (quoted in Stuster, 2015, emphasis added).

Expecting these probable future pathways, the rebels are likely to be reluctant to negotiate with the government and stop fighting, despite the existence of a mutually-beneficial bargaining range. What exacerbates the underlying bargaining problem is the expectation that the more the government needs to compromise, the stronger its incentive to renege becomes when it finds itself once more in a superior position in the post-conflict environment (Fearon, 2004, 295–296). Consequently, and somewhat paradoxically, the more temporarily powerful the rebels are, the larger the expected size of the post-conflict power shift and the rebels’ fear for the shift become. In extreme cases, this rebels’ fear may make them *more*, rather than *less*, reluctant to accept negotiation when their temporary bargaining position seems to be improving and thus the division of the disputed good currently acceptable to both sides favors them. This combination of warring parties’ incentives leads to the continuation of inefficient conflict.

2.2 Propositions

Based on the combination of information from battles and commitment problem, we generate the following two testable propositions for proximate and distant diffusion battles in relation to conflict termination. Depending on its spatio-temporal dynamics, the costly process of fighting reduces uncertainty about capability and resolves in some situations while not necessarily resolving the credible commitment problem, which in turn generates differing impacts on conflict termination as summarized by Table 1.

First, proximate diffusion dynamics, or battles with clear frontlines, tend to be more informative in revealing each other’s resolve and capability. However, while the uncertainty is likely to have been reduced in the instances of proximate diffusion resembling

conventional battles, the disputants still face the issue of having to solve the credible commitment problem in order for the inefficient fighting to end. In the situations where battle activities diffuse across proximate localities, regardless of whether the rebels or the government is winning on the battlefield, the government would face inherent incentives to renege on war-ending agreements once the conflict is over. More specifically, the government would more likely offer concessions to the rebels if the rebels are winning and the government is losing, however, negotiated settlements are, as discussed above, most often conditional on the rebels disarming and thus the rebels are vulnerable in the post-conflict environment without effective protection. Similarly, if the government is winning on the battlefield, they may also offer some concessions to the rebels, such as granting amnesty, in order to avoid costly fighting in the future. However, the rebels again would be fearful whether such promises would be honored once they have disarmed given the lack of enforcement mechanisms. In other words, the costly fighting with clear frontlines may reveal previously unavailable information, but the same battle dynamics do not necessarily resolve the underlying credible commitment problem.

What these scenarios suggest is that regardless of proximate diffusion reducing the informational deficit in the form of conventional battles, proximate diffusion does not resolve the commitment problem. Given the persistent commitment problem, regardless of the possibility of negotiations taking place, the differing impact of the information and commitment problem on the war ending bargaining, we hypothesize

Hypothesis 1 *Proximate diffusion patterns do not have a clear impact on conflict termination.*

Second, similar to proximate diffusion, distant diffusion of battles may indicate active armed uprising with either rebel or incumbent success in the battlefield. Maneuverability is beneficial for the rebels in their attempts to mobilize citizens and challenge the incum-

Table 1: Battle Diffusion and Conflict Bargaining

| Diffusion pattern | Resolves: | | Total effect |
|---------------------|------------------------|---------------------|--------------------|
| | Asymmetric information | Credible commitment | |
| Proximate diffusion | Yes | No | Indeterminate (H1) |
| Distant diffusion | No | No | Negative (H2) |

bent authority. The desire for geographically dispersed fighting can be traced back to the *foco* guerrilla tactics whereby small fast-moving rebel cadres can spark a mass mobilization against the incumbent (Debray, 1967). At the same time for the government, committing resources to battles across the country is costlier than containing battles in a smaller geographical area. In these instances, the temporarily weakened governments have even greater incentives to offer political concessions to rebels, reflecting the deterioration of their bargaining position in the short term. However, the credible-commitment account of war suggests that the government’s inability to make credible commitments to follow through the war-ending bargain and the rebels’ fear for the post-war power shift impedes the prospects of successful agreements, which in turn prolongs conflicts.

Such patterns were clearly observable in the Angolan civil war. This long and devastating war started in 1975 between the UNITA rebels and the MPLA government, and was widely spread across the vast Angolan territory. While non-military peace brokering attempts took place, they were not successful in bringing the conflict to an end. Eventually in 2002, the UNITA was militarily defeated, but the distant diffusion patterns of this conflict highlight how conflict termination was not successful for 43 years (Ziemke, 2012).

In other instances, distant diffusion of battles implies that the government remains capable of engaging, if not overwhelming, in battles in different geographical locations across the country. Such ability mitigates the likelihood of rebels gaining territorial control or the usefulness of the *foco* strategy. The incumbent forces therefore remain able to prevent the rebels from reaching an outright military victory. Moreover, due to the limited

information-revealing role, it takes longer for guerrilla warfare and battles across distant locations to reveal rebel resolve and strength than conventional or geographically contained, proximate battles. In these situations, continued fighting would not significantly contribute to conflict termination. The combination of the limited information-revealing role, incredible commitment, and military dynamics leads us to expect that:

Hypothesis 2 *Distant diffusion patterns decrease the likelihood of conflict termination.*

3 Data and Empirical Strategy

We examine the relationship between conflict dynamics and conflict termination, drawing on both micro- and macro-level datasets for intra-state armed conflicts in the post-1989 period provided by the Uppsala Conflict Data Program (UCDP). Specifically, we employ the conflict episodes recorded in the UCDP’s Dyadic Armed Conflict Dataset (ACD), which includes rebel-government conflicts that generate at least 25 battle-related casualties in a given calendar year, over some incompatibility classified as control over the central government and/or territorial secession (Gleditsch et al., 2002; Pettersson & Wallensteen, 2015). The coding of specific start and end dates of each conflict is provided by the dyadic version of the UCDP Conflict Termination Dataset, v.2-2015 (CTD, Kreutz, 2010). We choose the civil war (rebel-government) dyad-month as our unit of analysis.

Fine-grained event data are required to specify and compare different types of diffusion patterns of violence. We used the UCDP Georeferenced Events Dataset (GED), version 4.0, which covers incidents of organized violence within civil conflicts in Africa, the Middle East, Asia, and South America during the 1989–2014 period (Sundberg & Melander, 2013). The GED includes data on nearly 110,000 incidents of civil war battles between warring parties (state-based and non-state conflict) as well as their intentional

and direct use of violence against civilians (one-sided violence). Each record in the GED is coded relying on news sources, NGO reports, truth commission reports, historical archives, and other sources of information, and comes with precise geographical locations, dates, and other information including battle deaths and civilian casualties.³

We restrict the temporal scope of the analysis to the 1989–2011 period that is covered by the CTD, GED, and other sources of control variables. Following Ruhe (2015, 249), the GED records are aggregated into the dyad-month level based on the end date variable to ensure that all battle activities have occurred within a month, when an event is attributed to more than two months. Because we are primarily interested in the dynamics of rebel-government confrontations, approximately 10,000 records of non-state violence (battles between non-state actors) have been carefully dropped from the following analysis.⁴ Also note that only those records with relatively high spatial and temporal precision are used.⁵

3.1 Conflict Termination

Our primary dependent variable *Conflict Termination* is coded using the CTD dataset (Kreutz, 2010). The coding procedure described below leaves 7,341 dyad-month observations with 199 unique dyadic conflict episodes (spells). Of the 199 unique conflict episodes, 149 conflicts terminated within the observation period, while the remaining dyadic episodes are coded as ongoing as of December 2011. Dyad-level conflict termination, *Conflict Termination*, is coded using a binary indicator, which takes the value of 1

³An event in the GED is defined as the “incidence of the use of armed force by an organized actor against another organized actor, or against civilians, resulting in at least 1 direct death in either the best, low or high estimate categories at a specific location and for a specific temporal duration” (Sundberg & Melander, 2013, 524).

⁴By “battle” or “violent events,” we refer to instances of direct confrontation between government and rebel troops (“armed conflict events” in the GED), but not violence against civilians (“one-sided violence”) or confrontations between non-state actors (“non-state conflict”).

⁵Specifically, we employ the GED entries with spatial precision scores (“where_prec”) of 1 to 3 (event locations can be located at the second order administrative division or lower level) and temporal precision scores (“date_prec”) of 1 to 4 (event dates can be specified at the month or lower level).

if the conflict has terminated and 0 otherwise. The average duration of dyadic conflict episodes is 59.34 months (4.95 years), and the median duration is 30 months (2.5 years).

3.2 Measuring Diffusion Dynamics

Following the typology above, we distinguished two broad types of diffusion pattern, or proximate diffusion and distant diffusion. We employed the following two-step procedure to detect the instances of these diffusion patterns within empirical records.

Step 1: Generate spatial grids We first constructed a spatial grid with an arbitrary spatial resolution r . The common approach in existing studies is to divide the study region into an artificial spatial grid and then allocate the observed records of violence to each unit at a given time step t (e.g., Baudains et al., 2013; Schutte & Weidmann, 2011). To illustrate the procedure, Figure 2 represents several differently-sized spatial grids overlaid onto Mozambique and the reported geo-coordinates of violent incidents. This geoprocessing operation allows for the diffusion patterns to be interpreted, simply by considering how the presence of violent incidents within individual cells change or remain unchanged over subsequent time periods, or more precisely, counting the number of cases that fulfill the definition of diffusion patterns outlined above.

While existing studies often employ rectangular grids in this procedure, we opted to employ *hexagonal* grids to measure diffusion patterns. The primary reason for the use of hexagonal rather than rectangular grids is that the nearest neighborhood in a hexagonal grid is less ambiguous than in a rectangular grid (Birch et al., 2007). This clarity in neighborhood definition is critical in the current context because our typology of diffusion primarily distinguishes the two diffusion patterns using information about the presence or absence of battle events within its boundary *and* neighborhood.⁶

⁶Moore neighborhood (eight cells surrounding cells are defined as neighbors) and von Neumann neigh-

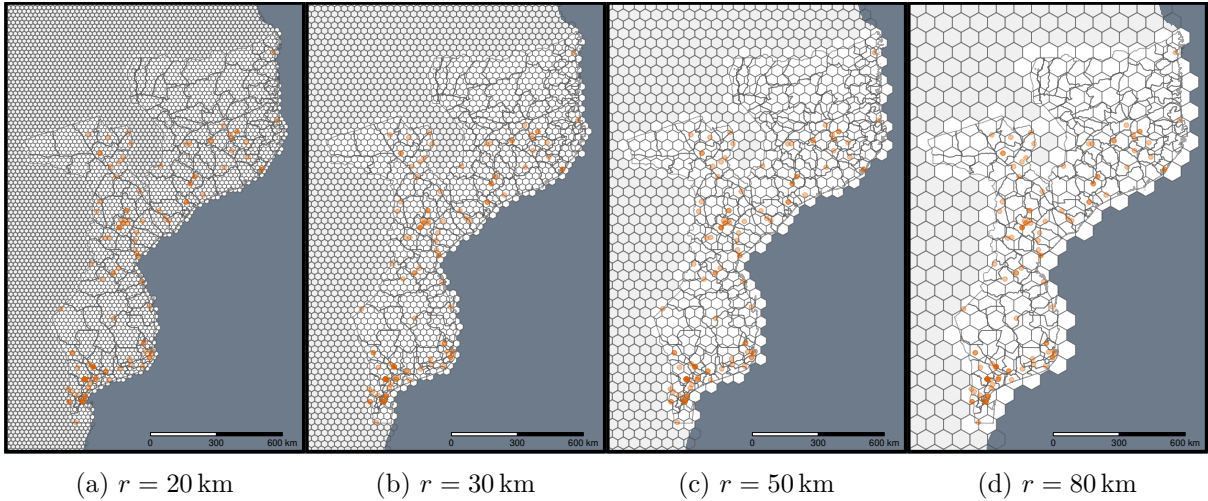


Figure 2: Hexagonal grid generation over Mozambique

Note: (a)–(d) grid cells with $r = 20, 30, 50,$ and 80 km resolution, respectively. White cells represent the cells that fall within the borders of Mozambique, whereas dots indicate the reported locations of battle activities. Segments represent boundaries of second-order administrative divisions.

Another issue is the grid-size selection. Theoretically, we had no prior reasons to select a given grid size over others (Schutte & Weidmann, 2011, 147). Empirically, insights from previous studies suggest that the use of a relatively high resolution is likely to be suitable to capture local-level conflict dynamics. For example, O’Loughlin & Witmer (2012) employ fine-grained spatial grids to explore the diffusion dynamics of insurgent violence in Russia’s North Caucasus and highlight that few significant spatio-temporal patterns are detected when using grid sizes that exceed 50 km. Methodologically and most fundamentally, the selection of grid size, or the choice of areal units, can have a substantial impact on the results of any statistical analysis that draws on discrete spatial units (modifiable areal unit problem, MAUP, Jelinski & Wu, 1996; Openshaw, 1983; Openshaw & Taylor, 1979).⁷

neighborhood (four orthogonal cells are defined as neighbors) are often used to define neighbors in a rectangular grid. In both definitions, the difference between the inter-cell Euclidean distance and the corresponding grid distance increases as the neighborhood order increases (Birch et al., 2007). Appendix C replicates the analysis using a rectangular grid.

⁷More precisely, the effect of the grid-resolution selection on the estimation results is known as the “scale problem,” or a special case of the MAUP in which the variation in results when the same spatial data are aggregated in differently sized spatial units (Jelinski & Wu, 1996).

In order to prevent the MAUP from plaguing the estimates and ensure that our empirical findings were not a result of an arbitrary selection of spatial grid size, we performed the empirical analysis varying the spatial grid specification. Specifically, the following analysis comprises replicated with varying grid size r , ranging from a highest resolution of 10 km to a lowest resolution of 100 km, and a neighborhood order k , ranging from 1 to 3. This approach allows us to address this problem and to investigate whether and how empirical patterns vary over different spatial scales. Similarly, because the definition of two diffusion types essentially depends on the neighborhood structure, an empirical test in which neighborhood order is varied is also critical to examine the robustness of the results in the current context.

Step 2: Identify diffusion patterns Relying on the artificial spatial grids, we interpreted the two diffusion processes of interest in three sub-steps. First, we overlaid the GED points onto the hexagonal grid and specified whether at least one battle event had occurred within each cell in month t . Figure 3(a) represents the spatial distribution of cells at “conflict” (orange) and cells remaining at “peace” (white) in the 50-km resolution grid-cell space over Mozambique for the purpose of illustration.

Second, we constructed an $N \times N$ spatial weight matrix \mathbf{W} to define the adjacency between grid cells, where N denotes the number of grid cells within the topological space. The diagonal elements $w_{ii} = 0$ and non-diagonal elements $w_{ij} \geq 0$ capture the relative connectivity between cells, where $w_{ij} = 1$ if cells i and j are adjacent, and $w_{ij} = 0$ otherwise. Figure 3(b) represents the resultant neighborhood network, with dots denoting the coordinates of individual cells and segments denoting the connectivity. Note that the size of the neighborhood network increases as neighborhood order k increases.

Last, following existing studies (Baudains et al., 2013; Cohen & Tita, 1999), we interpreted the diffusion patterns by considering how the presence (or absence) of violent

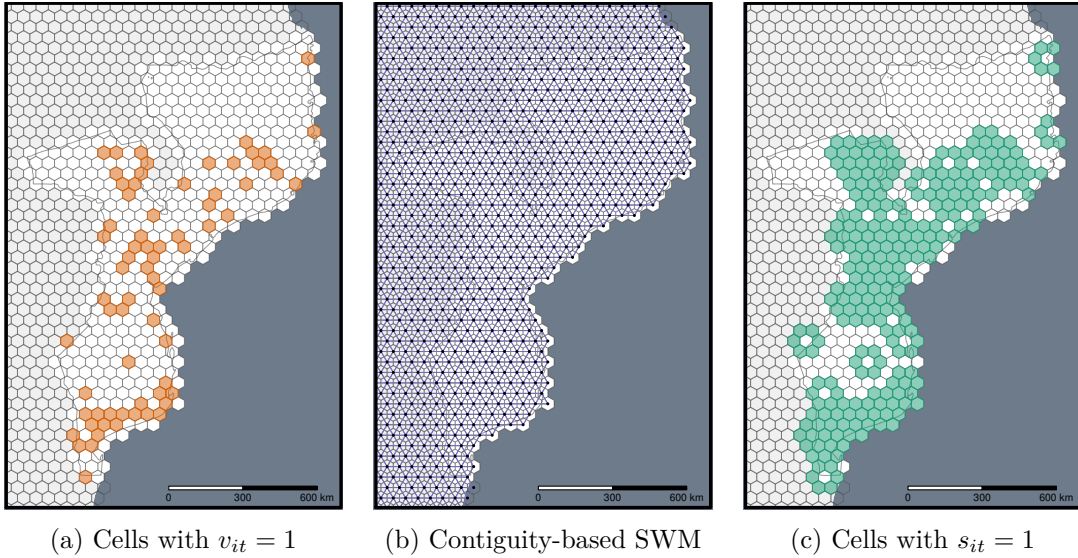


Figure 3: Measuring spatial distribution of violence

Note: (a) distribution of grid cells with 1+ battle events or $v_{it} = 1$ (cells in orange). (b) contiguity-based spatial weight matrix, with segments denoting the connectivity between grid cells. (c) distribution of grid cells with binary spatial lag $s_{it} = 1$ (cells in green).

incidents in a given cell relates to the occurrence of violence in neighbor cells. This procedure was implemented by computing the spatial lag $\sum_{j=1}^n w_{ij}v_{jt}$ for each cell i in a given month t , where n denotes the number of neighboring cells of i , and the binary indicator $v_{it} \in \{0, 1\}$ determines whether at least one battle event has been recorded in cell i at t , with $v_{it} = 1$ denoting the presence of events and $v_{it} = 0$ otherwise. For simplicity, we collapsed the spatial lag into a binary indicator s_{it} , with $s_{it} = 1$ denoting one or more battle events occurring within the neighbors of cell i at t , or $\sum_{j=1}^n w_{ij}v_{jt} \geq 1$, and $s_{it} = 0$ otherwise. Figure 3(c) highlights the distribution of grid cells where one or more battle incidents occurred within their neighbor cells, or $s_{it} = 1$ in green.

Recall that proximate diffusion refers to the spread of phenomena toward adjacent locations, while distant diffusion refers to the diffusion process between non-adjacent locations. Typically, proximate diffusion corresponds to instances in which one or more battle events have occurred in neighboring cells of i at $t - 1$ (i.e., $s_{it-1} = 1$), and one or

more events have occurred in i at t (i.e., $v_{it} = 1$). Another instance of proximate diffusion is a transition where battle events have occurred in i at $t - 1$ (i.e., $v_{it-1} = 1$), and one or more events have occurred in i at t (i.e., $s_{it} = 1$). Similarly, an instance of distant diffusion is defined as the transition where no battle events have been observed in cell i and its neighboring cells of i at $t - 1$ (i.e., $v_{it-1} = 0$ and $s_{it-1} = 0$), while one or more events have occurred in i and/or its neighboring cells at t (i.e., $v_{it} = 1$ and/or $s_{it} = 1$).

Formally, let $(v_i, s_i)_t$ denote the combination of the status of cell i and its neighboring cells at time period t . The instances of proximate diffusion are defined as the transitions between $(v_i, s_i)_{t-1}$, and $(v_i, s_i)_t$, such that $(1, 0)_{t-1} \rightarrow (0, 1)_t$, $(0, 1)_{t-1} \rightarrow (1, 0)_t$, $(0, 1)_{t-1} \rightarrow (1, 1)_t$, or $(1, 0)_{t-1} \rightarrow (1, 1)_t$. As illustrated in Figures 1(a) and (b), violent activities spread toward geographically contiguous areas in these cases. Similarly, an instance of distant diffusion is defined as a process $(0, 0)_{t-1} \rightarrow (0, 1)_t$, $(0, 0)_{t-1} \rightarrow (1, 0)_t$, or $(0, 0)_{t-1} \rightarrow (1, 1)_t$, in which violence diffuses to a wider area that had not experienced violence at $t - 1$ (Figures 1(c) and (d)). To facilitate a better feel for what these diffusion terms indicate, Figure A.2 in the Online Appendix illustrates the observed cases of violence diffusion in the Mozambican civil war.

Diffusion dynamics The resultant terms *Proximate Diffusion* and *Distant Diffusion* count the instances that satisfy the definitions of proximate diffusion and distant diffusion patterns per dyad month, respectively. Note that this three-step procedure was repeated using different grid-cell settings to guard against the MAUP. Note also that conflicts with multiple rebel groups were separated into separate dyads, and instances of diffusion were detected using individual dyads as the unit of analysis. These two diffusion terms were log-transformed due to the notable positive skewness.

Importantly, it is possible that simple changes in the scope of conflict zones, rather than the nature of diffusion, influence the opportunity for peace (Greig, 2015; Greig et al.,

2016). To control for this possibility, we also measured the month-to-month change in the number of grid cells with 1 or more battle events (*Naive Diffusion*). A positive value in this variable indicates the geographic expansion of conflict-affected zones over subsequent months, while a negative value indicates otherwise.

Following previous studies (Greig, 2015; Greig et al., 2016; Ruhe, 2015; Wood & Kathman, 2014), we coded these diffusion terms as a moving average over previous Δt months with $\Delta t = 6$ in the baseline setting.⁸ This reflects the idea that warring actors will update their expectations and beliefs about the course of conflict using the recent history of conflict, but a single monthly record that largely deviates from the recent history is not, on its own, likely to change their overall expectations.⁹

3.3 Control Variables

Structural factors and rebel attributes We included a series of confounding variables that are known to be associated with conflict termination and violence dynamics in our regression models. State-level controls include logged *per capita GDP* as the proxy of state capacity and wealth (Gleditsch, 2002) and *Democracy* (a dummy variable indicating 6+ Polity score) as the proxy of regime type (Marshall, Gurr & Jaggers, 2014). Our models also incorporated the geographical area of the country in logged square kilometers, *Country Size* (Weidmann et al., 2010). A large country size may hinge the government’s power projection and help militarily weak insurgents to survive. It may also constrain the potential of battle activities to spread geographically.

Recent studies suggest that the characteristics of rebel groups shape both battle dy-

⁸We report the robustness check using an alternative temporal window size $\Delta t = 12$ in Appendix D.

⁹For dyad-month observations where no violent incidents are recorded at $t - 1$, these diffusion terms were computed using location information from the previous month with events, as in Greig (2015) and Ruhe (2015). Underlying this imputation rule is the expectation that it is highly unlikely that belligerents will update their belief about the course of conflict in the absence of new information.

namics (Beardsley & Gleditsch, 2015; Beardsley et al., 2015) and the chances of conflict termination (Buhaug et al., 2009; Cunningham, Gleditsch & Salehyan, 2013). Drawing on the Non-State Actor dataset (Cunningham et al., 2013) and the Ethnic Power Relations dataset (Wucherpfennig et al., 2012), we included several binary variables to capture the characteristics of rebel groups. *Territorial Control* takes the value of 1 if rebels exercise a moderate or high level of control over territory. *Rebel Much Weaker* is a dummy variable representing whether rebel forces are extremely disadvantaged relative to government forces.¹⁰ *Ethnic Claim* takes the value of 1 if a rebel group makes an exclusive claim to fight on behalf of a particular ethnic group. Finally, our models incorporated *Multi Dyads*, which takes the value of 1 if the government fights two or more conflicts in a given month, and 0 otherwise.

Conflict dynamics Our regression models incorporated a set of control variables that capture different dimensions of the micro-dynamics of conflicts. First, we included conflict intensity (Greig, 2015; Mason & Fett, 1996; Ramsay, 2008; Ruhe, 2015) and civilian casualties to influence the chances of conflict termination (Wood & Kathman, 2014). We relied on the total number of battle deaths per dyad-month (*Conflict Intensity*) and total troop deaths until month $t - 1$ (*Cumulative Casualty*) to capture the short- and long-term impacts of the attrition that warring parties have suffered. *Rebel OSV* and *Govt OSV* are monthly counts of civilian deaths caused by the intentional and direct use of violence against civilians (one-sided violence) by rebel and government violence, respectively. *Collateral Damage* counts civilian deaths caused indirectly by confrontations between rebel and government troops per dyad month.

¹⁰The multichotomous variable in Cunningham et al. (2013) includes “much weaker,” “weaker,” “parity,” “stronger,” and “much stronger.” The latter four categories are coded 0 in our dataset.

Conflict geography Second, we controlled for a series of variables that represent the local geography. Specifically, the logged mean geodesic distance from reported battle locations to capital cities (*Capital Distance*, Weidmann et al., 2010), average population density in conflict-affected zones (*Local Population*), and shortest geodesic distance between event locations and resource rich areas (*Natural Resource Distance*) were included. The occurrence of battles in territories with strategic, subjective, or objective values may provide belligerents with the incentive to continue fighting as well as the local risks of battle onset (Fearon, 2004; Greig, 2015; Lujala, 2010; Lujala et al., 2007).

We also incorporated accessibility of conflict zones, as difficult terrain may constrain the potential of battle activities to expand and relocate to nearby locations (Beardsley & Gleditsch, 2015; Beardsley et al., 2015; Zhukov, 2012). *Ruggedness* measures mean elevation variance in conflict zones, while *Road Density* indicates kilometers of primary and secondary roads within battle-affected cells per square kilometer (Defense Mapping Agency, 1992).¹¹ We took the moving average with a temporal window Δt and log-transformed the variables, controlling for conflict intensity and geography. Table 2 reports the summary statistics of the variables.

3.4 Spatial-Grid Setups

We employ a grid with $r = 30$ and $k = 1$ as the baseline setting for two reasons. First, this grid setting roughly corresponds to the geocoding accuracy of battle events. Recall that our dataset contains the battle events that can be located at the second or lower level administrative divisions (footnote 5). As the logged mean (median) diagonal distances of second- (city/municipality) and third-level administrative units (town/village) are lo-

¹¹*Ruggedness* is constructed using the grid-based elevation variance data (with a 0.05° (~ 5.56 km) resolution) generated by SpatialGridBuilder (Pickering, 2016) *Natural Resource Distance* measures the average shortest geodesic distance between reported battle locations and locations of lootable diamonds and gemstone deposits, drug cultivation, and hydrocarbon production.

Table 2: Summary statistics

| | Mean | St. Dev. | Min | Median | Max |
|------------------------------|--------|----------|---------|--------|--------|
| Violence diffusion | | | | | |
| Proximate Diffusion | 0.260 | 0.380 | 0.000 | 0.000 | 2.485 |
| Distant Diffusion | 1.645 | 1.024 | 0.000 | 1.638 | 4.570 |
| Naive Diffusion | 0.012 | 0.968 | -20.667 | 0.048 | 10.333 |
| Government attributes | | | | | |
| per capita GDP | 7.601 | 1.054 | 5.217 | 7.520 | 10.163 |
| Democracy | 0.374 | 0.484 | 0 | 0 | 1 |
| Country Size | 13.242 | 1.494 | 9.234 | 13.416 | 16.639 |
| Rebel attributes | | | | | |
| Territorial Control | 0.382 | 0.486 | 0 | 0 | 1 |
| Ethnic Claim | 0.667 | 0.471 | 0 | 1 | 1 |
| Rebel Much Weaker | 0.428 | 0.495 | 0 | 0 | 1 |
| Multi Party | 0.626 | 0.484 | 0 | 1 | 1 |
| Conflict dynamics | | | | | |
| Conflict Intensity | 2.098 | 1.583 | 0.000 | 1.939 | 8.355 |
| Cumulative Casualty | 5.496 | 2.175 | 0.000 | 5.695 | 10.404 |
| Collateral Damage | 0.614 | 0.927 | 0.000 | 0.174 | 6.798 |
| Govt OSV | 0.682 | 1.142 | 0.000 | 0.000 | 10.370 |
| Rebel OSV | 0.830 | 1.206 | 0.000 | 0.000 | 8.786 |
| Conflict geography | | | | | |
| Capital Distance | 5.802 | 1.264 | 0.0003 | 5.959 | 7.689 |
| Local Population | 1.728 | 1.222 | 0.0003 | 1.639 | 6.300 |
| Natural Resource Distance | 5.936 | 0.964 | 2.211 | 6.006 | 7.782 |
| Ruggedness | 1.472 | 0.586 | 0.321 | 1.516 | 3.042 |
| Road Density | 0.013 | 0.008 | 0.000 | 0.012 | 0.063 |

Note: Diffusion terms are constructed with $r = 30$ and $k = 1$.

cated at 3.91 or $e^{3.91} \sim 49.89$ km ($e^{3.89} \sim 49.30$ km) and 12.05 km (10.4 km; Figure A.1 in the Online Appendix), respectively, individual battle events can be located within 30 km grids. Second, the clear correspondence between administrative divisions and artificial grids allows for an intuitive interpretation. Typically, with $r = 30$ and $k = 1$, *Proximate Diffusion* indicates battle diffusion within single municipalities and their neighbors, whereas *Distant Diffusion* captures diffusion beyond such geographical distances.

3.5 Model

Since our dependent variable is a binary indicator and the main independent variables vary over time, we employ a discrete-time event history model with a logit link function (Beck et al., 1998). Following Carter & Signorino (2010), we incorporate a cubic polynomial

of conflict duration to control for duration dependence and model the baseline hazard in the sample dyads.¹² Because observations within the same rebel-government dyads may share unobserved characteristics and thus their standard errors may correlate, we report standard errors that are robust to dyad-level clustering.

4 Empirical Findings

Our primary theoretical interests are on the impacts of distinct diffusion patterns of violence on civil conflict termination. The following section first reports the estimated impact of battle diffusion dynamics on the chances of conflict termination to test our primary propositions against empirical records. It then examines how the predictive performance improves by incorporating the diffusion dynamics.

4.1 Battle Diffusion Shapes When Conflict Ends

Table 3 reports the estimates of discrete-time duration models of conflict termination with different specifications. Model 1 is a minimal model with the diffusion and duration dependence terms, while Model 2 includes all controls. Model 3 utilizes the conditional logit estimator and incorporates dyad fixed effects while dropping the government and rebel attributes with little or no variation across months.¹³

The coefficients on different diffusion patterns vary in size and statistical significance. The coefficient estimates for *Distant Diffusion* are negatively signed and statistically significant across model specifications. The stability of the estimates suggests that the effect of unobservable factors needs to be at least much larger than those of the observed vari-

¹²We include $t/100$ and its square and cube, with t denoting the number of months since conflict onset.

¹³See Beck & Katz (2001) for a discussion on the use of unit fixed effects in a time-series cross-sectional setup with a binomial dependent variable.

Table 3: Discrete-time duration models of conflict termination

| <i>Dependent variable: Conflict Termination</i> | | | |
|---|------------------|------------------|------------------|
| | Model 1 | Model 2 | Model 3 |
| Violence diffusion | | | |
| Proximate Diffusion | -0.139 (0.395) | -0.020 (0.422) | -0.442 (0.531) |
| Distant Diffusion | -0.662** (0.115) | -0.683** (0.136) | -0.670** (0.202) |
| Naive Diffusion | -0.073 (0.086) | -0.058 (0.099) | -0.021 (0.143) |
| Government attributes | | | |
| per capita GDP | | -0.268 (0.213) | |
| Democracy | | 0.091 (0.304) | |
| Country Size | | -0.007 (0.243) | |
| Rebel attributes | | | |
| Territorial Control | | -0.137 (0.214) | |
| Ethnic Claim | | -0.073 (0.195) | |
| Rebel Much Weaker | | -0.128 (0.201) | |
| Multi Party | | 0.048 (0.217) | |
| Conflict dynamics | | | |
| Conflict Intensity | | 0.022 (0.092) | -0.117 (0.122) |
| Cumulative Casualties | | 0.144* (0.057) | 0.790** (0.169) |
| Collateral Damage | | -0.191 (0.171) | -0.172 (0.175) |
| Govt OSV | | -0.102 (0.095) | -0.127 (0.123) |
| Rebel OSV | | -0.197 (0.126) | -0.457* (0.178) |
| Conflict geography | | | |
| Capital Distance | | -0.013 (0.269) | -0.381 (0.596) |
| Local Population | | -0.068 (0.248) | -0.583 (0.415) |
| Natural Resource Distance | | -0.136 (0.170) | 0.388 (0.666) |
| Ruggedness | | -0.070 (0.206) | -0.049 (0.668) |
| Road Density | | 0.047 (0.144) | 0.367 (0.265) |
| Conflict duration polynomials | ✓ | ✓ | ✓ |
| Dyad fixed effects | | | ✓ |
| Observations | 7,341 | 7,341 | 7,341 |
| # conflict episodes (spells) | 199 | 199 | 199 |
| # dyads | 153 | 153 | 153 |
| Log Likelihood | -683.619 | -675.171 | -443.466 |
| AIC | 1,381.238 | 1,398.342 | 918.932 |

Note: * $p < 0.05$; ** $p < 0.01$

Unit of analysis: Conflict dyad-month. Robust standard errors clustered on dyad in parentheses.

Intercepts and conflict duration polynomials are omitted for brevity. Conflict geography

variables, *per capita GDP*, and *Country Size* are standardized using the method in Gelman (2008).

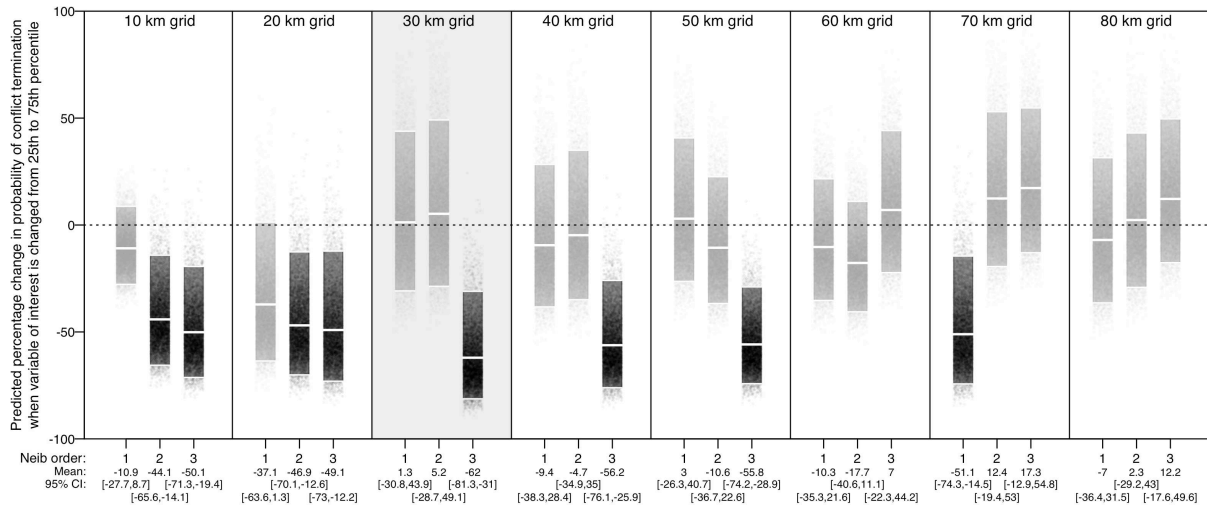
ables to negate the effect of *Distant Diffusion* (Altonji et al., 2005; Bellows & Miguel, 2009).¹⁴ In contrast, the coefficients on *Proximate Diffusion* largely vary across models and fail to retain statistical significance at the conventional 5% level. In addition, the coefficient estimates for *Naive Diffusion* remain small and fail to retain statistical significance across model specifications.

¹⁴A comparison of the coefficients in the models with (Model 2; β_2) and without controls (Model 1; β_1) suggests that the effect of unobserved confounders needs to be $\frac{\beta_2}{\beta_2 - \beta_1} = \frac{-0.683}{-0.683 - (-0.662)} \sim 31.412$ times larger than that of observed controls to fully attenuate away the effect of *Distant Diffusion*.

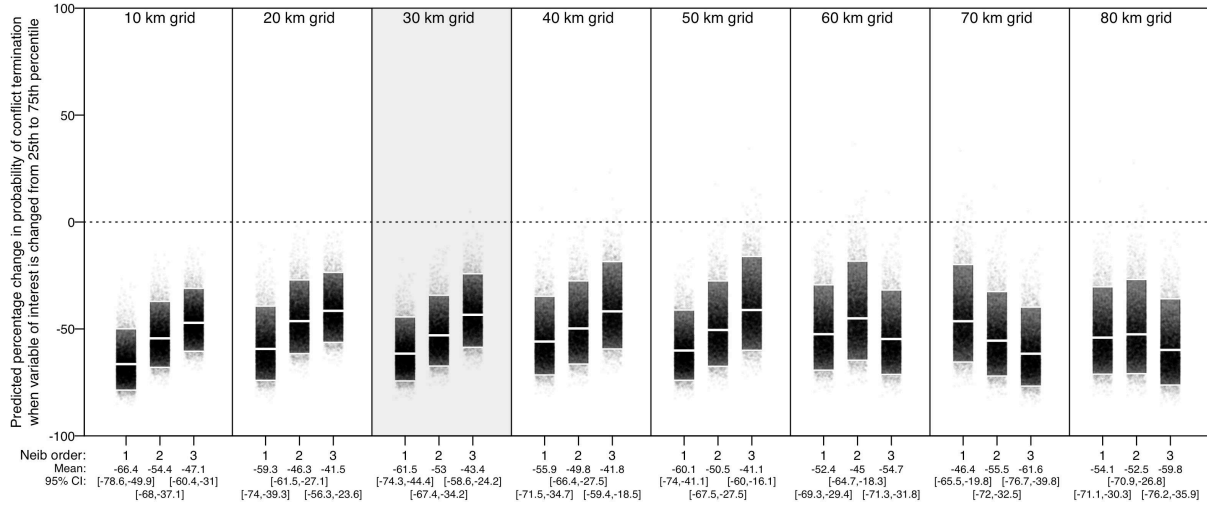
These estimates for the diffusion terms already provide initial support for our argument that micro-level diffusion patterns of violence, as well as the intensity or location of battles, systematically shape the probability of civil conflicts ending. Specifically, the coefficient signs across our model specifications are consistent with Hypothesis 2, which predicts a negative association between distant diffusion and the likelihood of conflict termination. The initial results suggest that *Distant Diffusion*, or the spread of battles across non-adjacent locations, is associated with a lower chance of conflict termination, while *Proximate Diffusion*, or the diffusion of battle activities across adjacent areas, and *Naive Diffusion* may not have a substantial impact on the chances of conflict termination. Combined, these results indicate that it is *how*, rather than *whether*, battles diffuse, that is vital in altering the opportunity for peace.

Nonetheless, unlike in linear models, in non-linear models, raw coefficients alone do not allow for meaningful interpretation of the substantial impacts of the corresponding covariates on the dependent variable. Another methodological issue that requires explicit investigation in empirical analyses employing spatial data is the potential sensitivity of the results to the selection of the choice of grid resolution r and neighborhood order k . Examining the robustness of the results to the definition of spatial units is critical to avoid reporting spurious relationships in any empirical analysis that employs discrete spatial units due to the MAUP.

To account for these two estimation issues, Figure 4 utilizes simulations to assess the impact of diffusion dynamics on the probability of conflict termination across differently-specified spatial grids. Specifically, it plots how a specific level of increase in *Proximate Diffusion* and *Distant Diffusion* (25th to 75th percentile) changes the probability of conflict termination against different grid resolutions and neighborhood orders, holding all other continuous variables constant at their median and binary variables at their



(a) First difference estimates for proximate diffusion



(b) First difference estimates for distant diffusion

Figure 4: Effect of violence diffusion as percentage change in probability of conflict termination across different spatial grid settings

Notes: Each dot indicates a predicted change in probability of conflict termination drawn from a single simulation when *Proximate Diffusion* (*Distant Diffusion*) is changed from the 25th to 75th percentile (first difference estimate), holding all other variables constant at their median (continuous) or mode (binary). Estimates that are statistically significant at 5% level are plotted in black. White stripes indicate the corresponding mean (thick) and 95% confidence intervals (thin) of predicted values. Gray shades indicate the estimation results using the baseline grid size of $r = 30$ km. Uncertainty estimates are obtained by 10,000 simulations.

mode. The left-most row within the gray-shaded area corresponds to the baseline result of Model 3 reported in Table 3, with the grid setting of $r = 30$ and $k = 1$. Statistically significant estimates at the conventional 5% level are plotted in black, whilst insignificant estimates are represented in gray. White stripes represent the mean and 95% confidence intervals of predicted values. Uncertainty estimates for the predicted values are obtained via 10,000 simulations following the recommendation of King et al. (2000).¹⁵

The predicted impact of diffusion dynamics provides compelling empirical support for the theoretical expectation of Hypothesis 2: increasing instances of *Distant Diffusion* substantially lower the chances of conflict termination, and the association remains qualitatively unchanged across different spatial grid resolutions (Figure 4(b)). Specifically, an increase in *Distant Diffusion* from the 25th to 75th percentile is followed by a statistically and substantially significant decrease in the likelihood of conflict termination, with a 61.5% drop in the predicted probability of conflict ending in the baseline spatial grid setting with $r = 30$ km and neighborhood order $k = 1$ (95% Confidence Interval: $-74.3, -44.4$). This association is consistent across different geographic scales, while the predicted impact gradually decreases as grid resolution r and neighborhood order k increase. Indeed, the impact followed by the same amount of change in *Distant Diffusion* decreases to -54.1% (95% CI: $-71.1, -30.3$) with $r = 80$ and $k = 1$, and -43.4% (95% CI: $-58.6, -24.2$) with $r = 30$ and $k = 3$. In contrast, consistent with Hypothesis 1, Figure 4(a) indicates that the association between *Proximate Diffusion* and conflict termination remains sensitive to grid settings at best. The relatively large confidence intervals for the interquartile increase in *Proximate Diffusion* do not allow us to reject the null hypothesis of non-difference from zero in 16 out of 24 grid settings.¹⁶

¹⁵Simulations are based on Model 2 in Table 3. As reported in Appendix B, the nonfinding for *Naive Diffusion* hold across different grid settings.

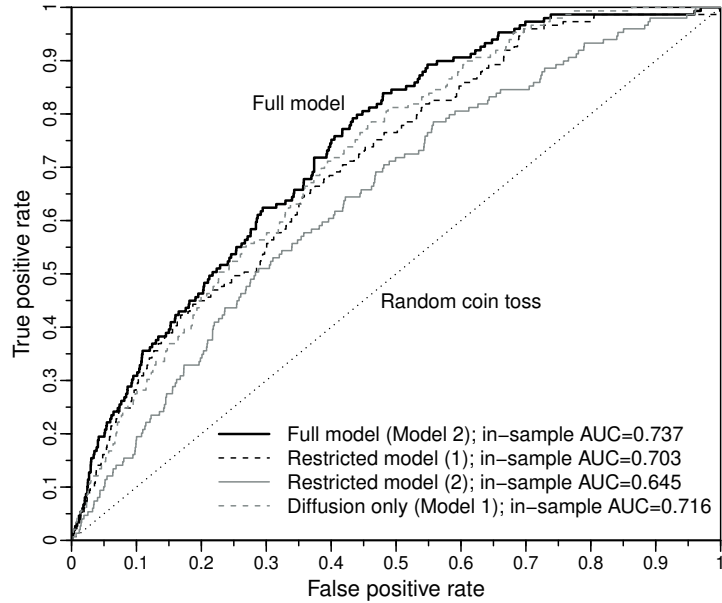
¹⁶Appendix G examines the effect of outliers with a large number of diffusion observations on our estimates by dyad-wise jackknifing. The results suggest that the effect of outliers remains limited.

4.2 Predictive Performance

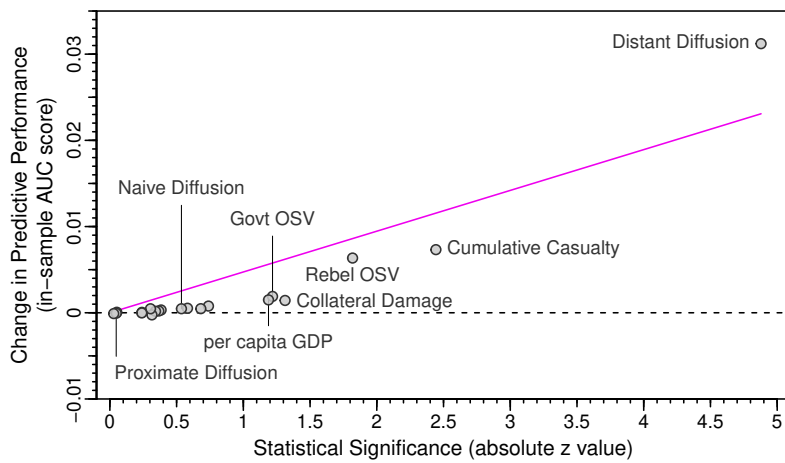
Incorporating diffusion dynamics of battles makes a substantial contribution to our ability to predict conflict termination, both in in-sample and out-of-sample settings. We use the Area Under the Receiver Operator Characteristic (ROC) curve (AUC) score as our primary indicator of predictive accuracy and broadly follow the approach in Ward et al. (2010) to assess each variable’s contribution to the predictive performance. An ROC curve plots true positive rate (TPR) and false positive rate (FPR) as the output of each possible probability threshold for positive prediction (conflict termination). The resultant curve displays the balance between TPR and FPR where a highly predictive model (with high TPR and low FPR) produces a curve up in the top left corner. An AUC score measures the area covered by the corresponding ROC curve and ranges between 0 and 1, where a model with higher classification performance should yield an AUC score closer to 1.

Model 2 in Table 3 (full model) yielded a fairly good predictive performance in both in-sample (AUC = 0.737) and out-of-sample (0.689) settings.¹⁷ Figure 5 graphically summarizes the results of the predictive power analysis. The in-sample ROC curves in Figure 5(a) indicate that the full model (black solid) outperforms the restricted models at almost all possible thresholds. The AUC falls from 0.737 to 0.703 (black dashed) when the three diffusion terms are dropped from the model. Deleting further the four conflict intensity variables is similarly followed by a noticeable decrease in the in-sample AUC score (from 0.703 to 0.649; gray solid). It is also worth noting that the minimal model (Model 1) with *only* the diffusion and duration dependence terms *outperforms* the two restricted models lacking the diffusion terms, which underscores the substantial

¹⁷The out-of-sample AUC score was obtained via a 10-fold cross validation with 10 repetitions. We first randomly divided the dyad-year samples into equally-sized 10 groups. We then used the 9 groups as the training set that was used to estimate parameters and the remaining one as the test set that was used to compute the out-of-sample AUC score. This two-step procedure was repeated for 10 times with different segmentations. The reported AUC score indicates the average of these repetitions.



(a) ROC curves



(b) Contribution to AUC score

Figure 5: In-Sample Predictive Performance

Note: Full model in Panel (a) corresponds to Model 2 in Table 3. Gray dashed segment corresponds to a model with three diffusion terms dropped (Restricted model 1), while the gray solid segment represents a model that excludes the diffusion and conflict intensity terms from the full model (Restricted model 2). The solid segment in Panel (b) represents an OLS estimate without an intercept.

contribution made by *Distant Diffusion*.

To take a closer look at the each predictor's contribution, Figure 5(b) plots the ab-

solute z value of each independent variable on the x -axis against its contribution to the model's in-sample AUC score on the y -axis. We again see the sizable contribution of *Distant Diffusion*, along with the dynamic determinants of conflict termination identified in previous studies such as *Cumulative Casualty*, while the contributions of *Naive Diffusion* and *Proximate Diffusion* remain negligible.

4.3 Disaggregating Conflict Termination

A major sensitivity concern lies in the aggregation of different conflict outcomes into a single binary indicator of conflict termination. Indeed, our theoretical accounts focus on the intra-conflict bargaining rather than military dynamics or decisive victories. Consequently, one may wonder if the empirical pattern holds when focusing on the association between battle diffusion and likelihood of negotiated settlement.

To address this concern, Appendix E reports the regression estimate with conflict outcome as the dependent variable. Reassuringly, the results show that distant diffusion substantially shapes the likelihood of negotiated settlement and suggest that the main results are not a product of the aggregation of different conflict outcomes.

5 Conclusion

Recent advances in civil war studies have increasingly explored the micro-level battle dynamics in civil conflicts. What remains relatively under-studies is how different spatio-temporal diffusion dynamics of violence have differing impacts on the chances of conflict termination. In addition, there has been a limited focus on how these specific micro-level dynamics translate back to the macro-outcomes (Kertzer, 2017). Building on previous studies, this article has proposed and tested preliminary hypotheses that relate the micro-

level diffusion dynamics of battles to the macro-level likelihood of conflict termination. The core argument is that distant diffusion of battles, or the spread of battle activities across distant localities in particular, makes conflict termination less likely due to the limited information-revealing role and the persistent, and possibly battle-exacerbated, credible commitment problem.

The main findings are threefold. First, escalating distant diffusion dynamics, or the spread of battle activities across non-adjacent areas, decrease the chances of conflict termination. Second, although distant diffusion has a robust negative impact on the likelihood of conflict termination, the effect of proximate diffusion, or the spread of violence across geographically contiguous areas, has been found to be sensitive to the selection of spatial units at best. Finally, while not all dimensions of battle dynamics matter in determining subsequent course of civil conflicts, carefully unpacking diffusion dynamics of battles yields a sizable improvement in our ability to predict conflict termination.

These empirical findings speak to scholarly and policy debates of conflict termination. First, this study is an important illustration that merely examining conflict intensity or whether conflict diffusion influences the likelihood of conflict termination does not incorporate the complex dynamics of battle diffusion. By aggregating these distinct patterns, researchers might miss out on important information about belligerents' expectations and behavior during civil conflicts. Just as intensity and geographic locations of battles substantially shape the prospects for peace (e.g., Greig, 2015; Ramsay, 2008; Ruhe, 2015; Weisiger, 2016), distant diffusion of combat activities invariably influences the future course of conflict. Previous studies suggest that the dynamic factors matter in explaining conflict termination. Our analysis has demonstrated how careful disaggregation of spatio-temporal dynamics of battles helps us understand conflict termination.

Second, this article has also yielded implications beyond scholarly debate. The end of

a civil war can depend on “ripe moments” for dialogue between the disputants (Zartman, 2001). These opportunities, however, may often be missed due to the warring parties’ inability to credibly commit to following through with the war-ending agreement in the absence of central enforcement. Previous studies have highlighted the role of a mutually hurting stalemate as a signal of such a moment (Greig, 2015; Ruhe, 2015; Zartman, 2001). This article provides a more nuanced indicator of when such ripe moments might occur and how to design an intervention in order to shorten costly conflicts. The empirical findings suggest that a hurting stalemate may have differing implications for intervention depending on the spatio-temporal dynamics of battles. Escalating battles with proximate diffusion would reduce the informational asymmetry, and helping hands serving as a commitment device would foster conflict termination. Yet the same escalation is not likely to effectively reveal previously unavailable information while not resolving the underlying commitment problem. In such situations, external helping hands would have to address the remaining informational uncertainty while serving as an enforcement device to guarantee the implementation of the war-ending agreement to contribute to conflict termination.

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Online Appendix (Not for Publication)

The following sections provide supplementary figures to the baseline analysis and report a series of additional estimation results to assess the robustness of the empirical findings reported in the main text. Appendix A provides supplementary figures briefly referred in the main text to describe the coding procedure. Appendix B reports the estimation results for *Naive Diffusion* across different spatial grid settings, and Sections C to G each address the major sensitivity concerns of the main empirical results, including the regressions distinguishing the instances of conflict termination with different outcomes. Reassuringly, none of these sensitivity tests yield results that deviate markedly from the main results reported in the main text. These results provide confidence that the specific parameter settings and assumptions are not driving our main empirical findings.

Note that we primarily relied on `sf` and `sp` packages in R (Bivand et al., 2013; Pebesma & Bivand, 2005) and original R implementations in the geoprocessing operations.

Appendix A Supplements to the Main Analysis

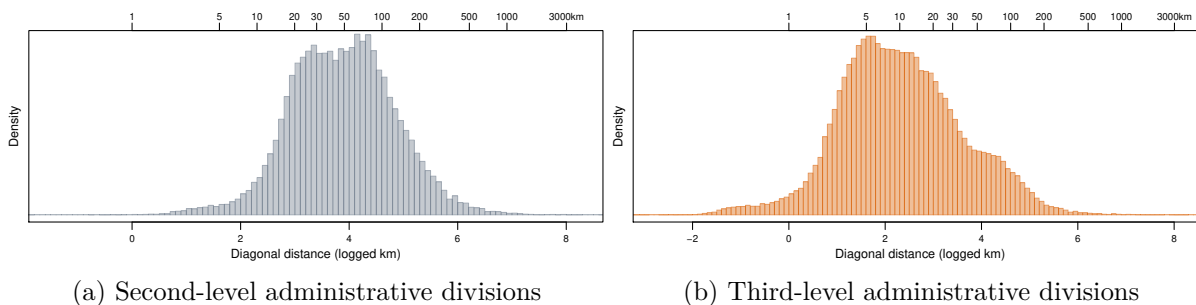


Figure A.1: Distribution of diagonal distance of administrative divisions

Note: Data derived from GADM database of Global Administrative Areas (<http://www.gadm.org/>).

Appendix B Effect of Naive Diffusion

Our empirical analysis suggests that *Naive Diffusion*, or the changes in the scope of conflict zones, are unlikely alter the prospects of conflict termination. Yet, as the MAUP suggests, it is possible that the null findings are specific to the baseline spatial grid setting with resolution $r = 30$ km and neighborhood order $k = 1$.

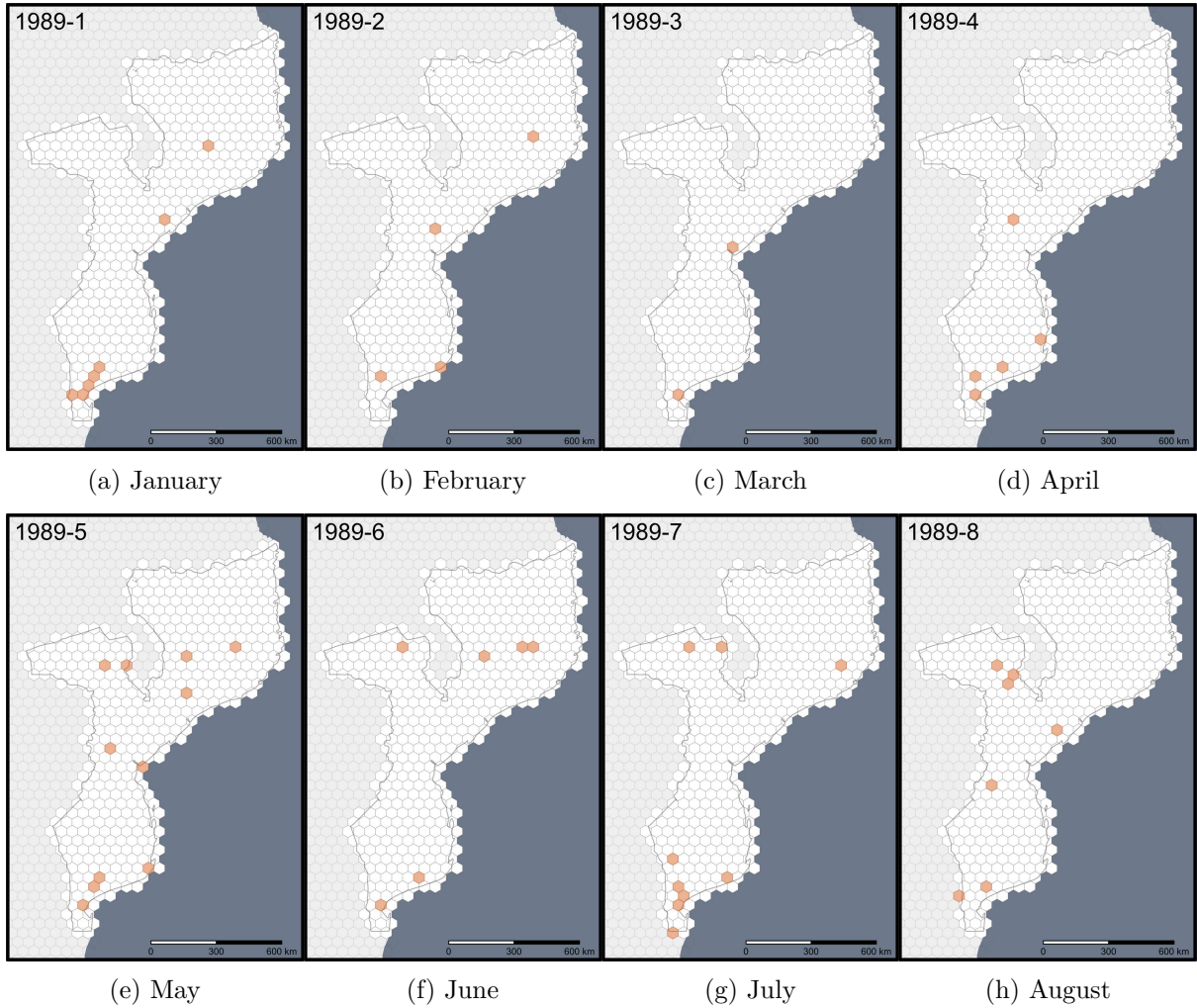


Figure A.2: Evolution of conflict geography in the Mozambican civil war, Jan–Aug, 1989
Note: (a)–(h) distribution of grid cells with 1+ battle events in the Mozambican civil war, 1989 (cells in orange). Spatial grids with resolution $r = 50$ are employed for the visibility purpose.

To address this issue, Figure B.1 replicates the estimates of *Naive Diffusion* on conflict termination and outcomes varying the spatial grid specification. As the results indicate, the effect of *Naive Diffusion* on the chances of conflict termination and rebel- and government-favorable outcomes remain statistically and substantially insignificant across different spatial grid settings, suggesting that the baseline null finding is not likely to be the product of the arbitrary selection of grid resolution and neighborhood order.

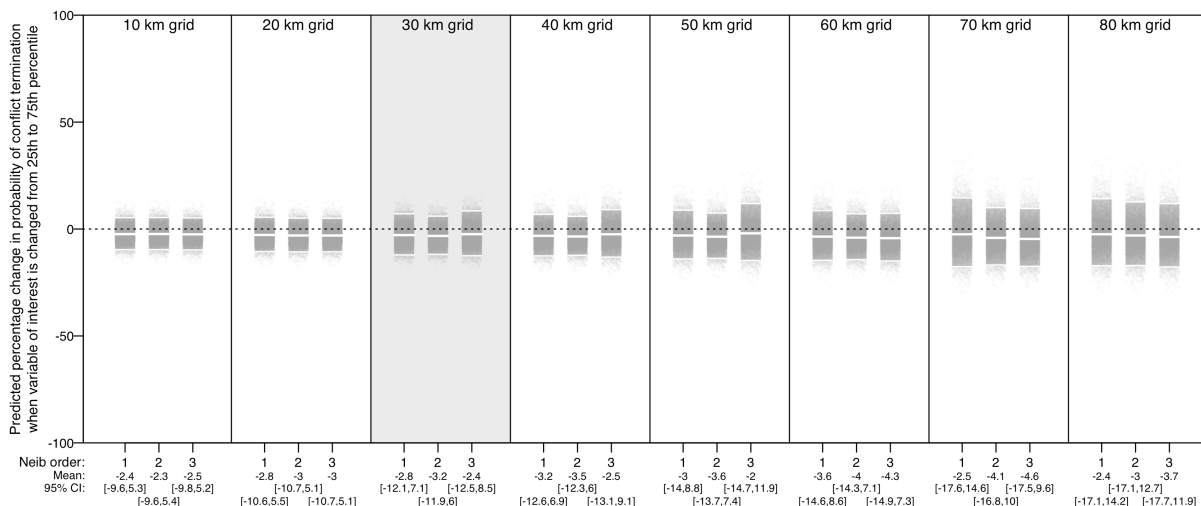


Figure B.1: Effect of *Naive Diffusion* as percentage change in probability of conflict termination across differently specified rectangular grids

Notes: See notes in Figure 4 in the main text.

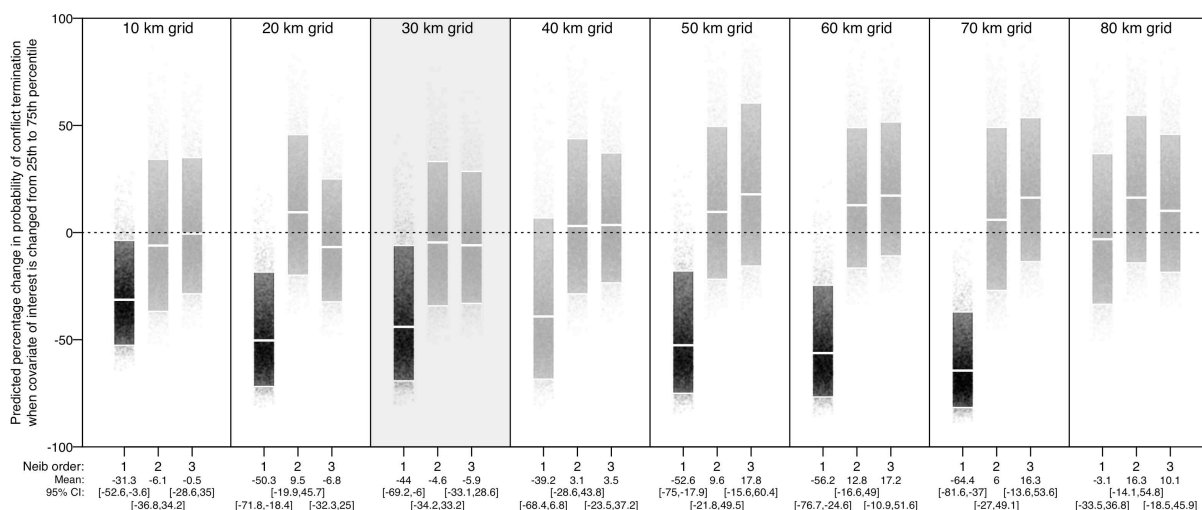
Appendix C Alternative Spatial Grid Specification

The results reported in the main text suggest that the estimations on the effect of diffusion terms can vary, either qualitatively or quantitatively, depending on the selection of grid resolution and neighborhood order. Because the selection of grid *shape* as well as grid resolution and neighborhood size could alter when detecting instances of proximate diffusion and distant diffusion, an explicit statistical examination is needed to ensure that the main findings reported in the main text are not results of arbitrary spatial grid definition.

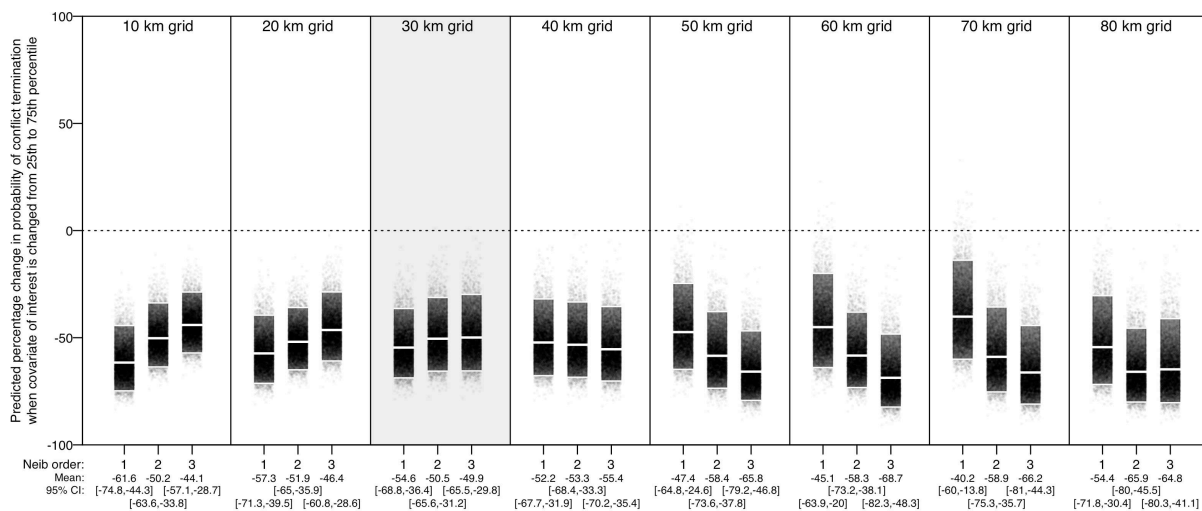
While the analysis in the main text employs a hexagonal grid to detect diffusion patterns, the following robustness check measures battle diffusion using rectangular grids and replicate the main regression models to explore the effect of the selection of spatial units on estimation results. Figure C.1 replicates the estimation reported in Table 3 and Figure 4 in the main text using differently specified rectangular grids.¹

Reassuringly, the estimation results in Figures C.1(a) and C.1(b) do not deviate markedly from the main results: *Distant Diffusion* consistently has a substantial and negative impact on the probability of conflict termination across different grid settings, while the effect of *Proximate Diffusion* remains indeterminate or sensitive to the grid

¹Neighborhood in a rectangular grid is defined as the Moore (Queen) neighborhood, where the neighborhood includes four orthogonal and four diagonal neighbors.



(a) First difference estimates for proximate diffusion



(b) First difference estimates for distant diffusion

Figure C.1: Effect of violence diffusion as percentage change in probability of conflict termination across differently specified rectangular grids

Notes: See notes in Figure 4 in the main text.

specifications. These additional results provide confidence that the specific parameter settings are not driving the main findings.

Appendix D Alternative Temporal-Window Specification

The baseline setting measures the diffusion terms as moving average over previous Δt months with $\Delta t = 6$. As the size of temporal window can affect the detection of the diffusion terms (and the estimates for all other covariates measured as moving-average), Figure D.1 replicates the main regression models with an alternative window size $\Delta t = 12$. These robustness checks do not alter the main findings qualitatively. Although the marginal effect estimates vary depending on the temporal window sizes, the results remain substantially unchanged across different temporal window settings.

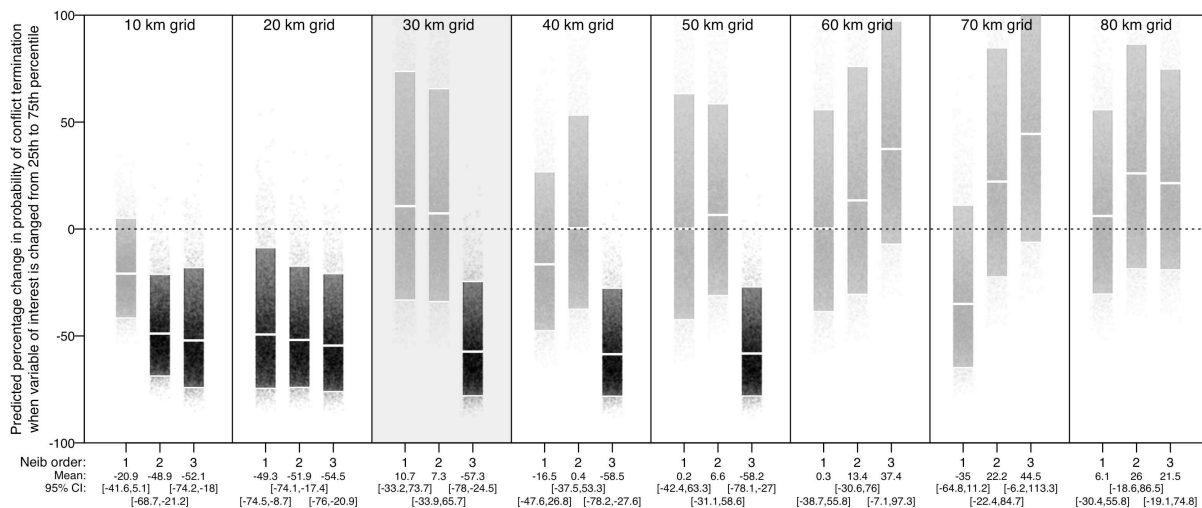
Appendix E Battle Diffusion and Conflict Outcomes

While our empirical analysis primarily focuses on the associations between different diffusion patterns of battle activities and conflict duration, the analysis presented in the following two sections also takes a closer look at *how*, as well as *when*, conflict ends. Specifically, we disaggregate the observations of conflict termination into two broad outcome categories of *Negotiated Settlement* (“ceasefire agreement” and “peace agreement”) and *Military Outcome* (“victory for government side,” “victory for rebel side,” and “low activity” in the original coding of Kreutz, 2010). The analysis allows for exploring how diffusion dynamics shape the course of intra-conflict bargaining, which corresponds to our theoretical accounts. In our dataset, 59 cases of conflict termination are coded as *Negotiated Settlement* and 90 are coded as *Military Outcome*.

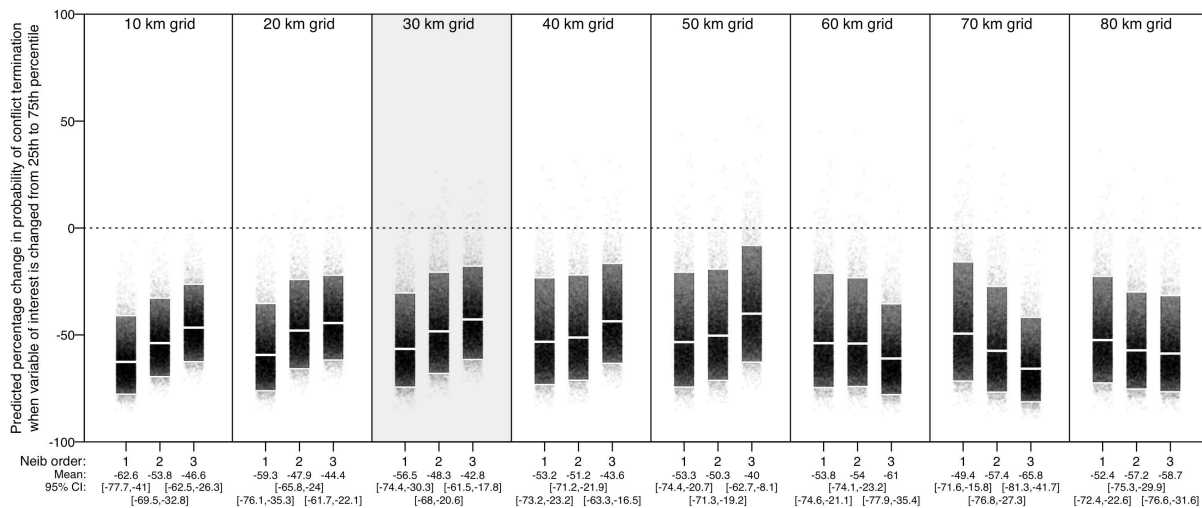
The following analysis utilizes the multinomial logit estimator to examine the determinants of conflict outcomes.² Table E.1 reports the outcome regression estimates, with the model specification following Model 3 in Table 3. Figures E.1 and E.2 simulate and plot the first difference estimates for the impacts of *Distant Diffusion* and *Proximate Diffusion* on the likelihood of each conflict outcome compared to the baseline category of *Continuation*, respectively. Appendix F replicates the estimation with competing-risks regression models.

Two findings emerge and lend additional support for our argument. First, *Distant*

²We replicated the following analysis using multinomial probit estimator and confirm that the alternative estimator yields similar results, suggesting that potential violation of the Independent and Irrelevant Alternatives (IIA) assumption is not likely to alter the main results.



(a) First difference estimates for proximate diffusion



(b) First difference estimates for distant diffusion

Figure D.1: Effect of violence diffusion as percentage change in probability of conflict termination with $\Delta t = 12$

Notes: See notes in Figure 4 in the main text.

Diffusion has a statistically and substantially significant impact on the chances of conflict termination with a negotiated settlement. In the baseline spatial grid setting with $r = 30$ km and $k = 1$, the coefficient estimate of -0.588 translates into a substantial interpretation that an interquartile increase in *Distant Diffusion* results in a 54.4% (95% CI: $-76.8, -18.4$) reduction in the probability of a civil conflict ending in a negotiated settlement. This estimated effect remains qualitatively unchanged across different spatial

Table E.1: Multinomial logit model of conflict outcome

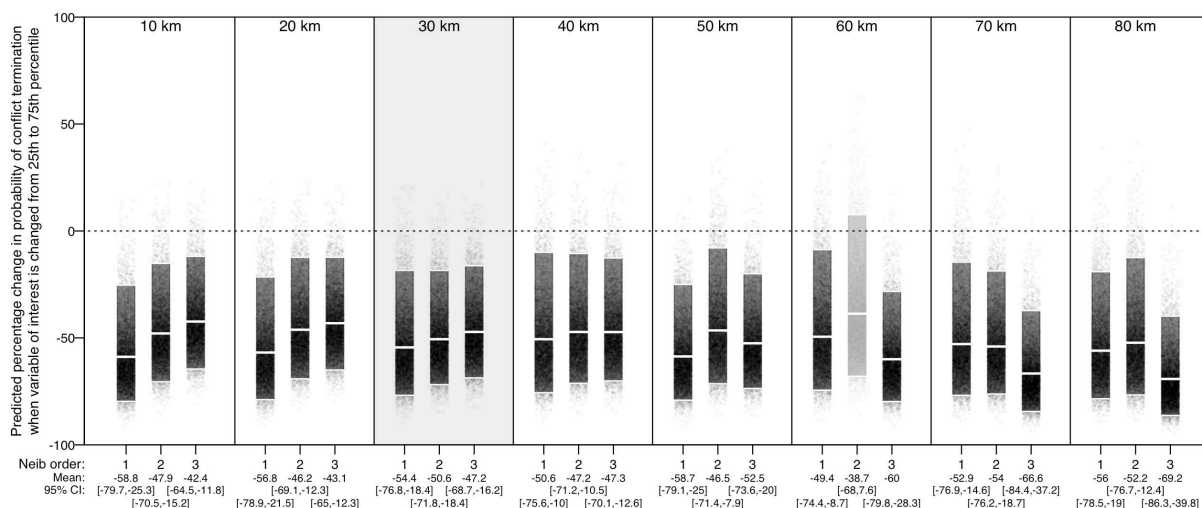
| | <i>Dependent variable: Conflict Outcome</i> | |
|-------------------------------|---|------------------------------|
| | <i>Military Outcome</i> | <i>Negotiated Settlement</i> |
| Violence diffusion | | |
| Proximate Diffusion | 0.200 (0.512) | -0.379 (0.673) |
| Distant Diffusion | -0.788** (0.162) | -0.588** (0.221) |
| Naive Diffusion | 0.164 (0.167) | -0.172 (0.088) |
| Government attributes | | |
| GDP | -0.160 (0.113) | -0.110 (0.215) |
| Democracy | -0.403 (0.351) | 0.845 (0.493) |
| Country Size | -0.093 (0.096) | 0.131 (0.119) |
| Rebel attributes | | |
| Territorial Control | -0.253 (0.274) | 0.004 (0.310) |
| Ethnic Claim | -0.127 (0.245) | 0.092 (0.312) |
| Rebel Much Weaker | 0.627* (0.255) | -1.334** (0.366) |
| Multi Party | 0.232 (0.292) | -0.057 (0.305) |
| Conflict dynamics | | |
| Conflict Intensity | -0.036 (0.123) | 0.091 (0.133) |
| Cumulative Casualties | 0.163* (0.076) | 0.140 (0.079) |
| Collateral Damage | -0.118 (0.196) | -0.280 (0.294) |
| Govt OSV | -0.075 (0.136) | -0.169 (0.116) |
| Rebel OSV | -0.136 (0.184) | -0.272 (0.165) |
| Conflict geography | | |
| Capital Distance | 0.115 (0.125) | -0.251 (0.208) |
| Local Population | 0.177 (0.101) | -0.433 (0.256) |
| Natural Resource Distance | 0.128 (0.124) | -0.346** (0.106) |
| Ruggedness | -0.011 (0.202) | -0.009 (0.296) |
| Road Density | 12.293 (11.772) | -8.891 (14.554) |
| Conflict duration polynomials | ✓ | ✓ |
| Observations | | 7,341 |
| Log Likelihood | | -675.171 |
| AIC | | 1,398.342 |
| Multiclass in-sample AUC | | 0.734 |

Note: * $p < 0.05$; ** $p < 0.01$

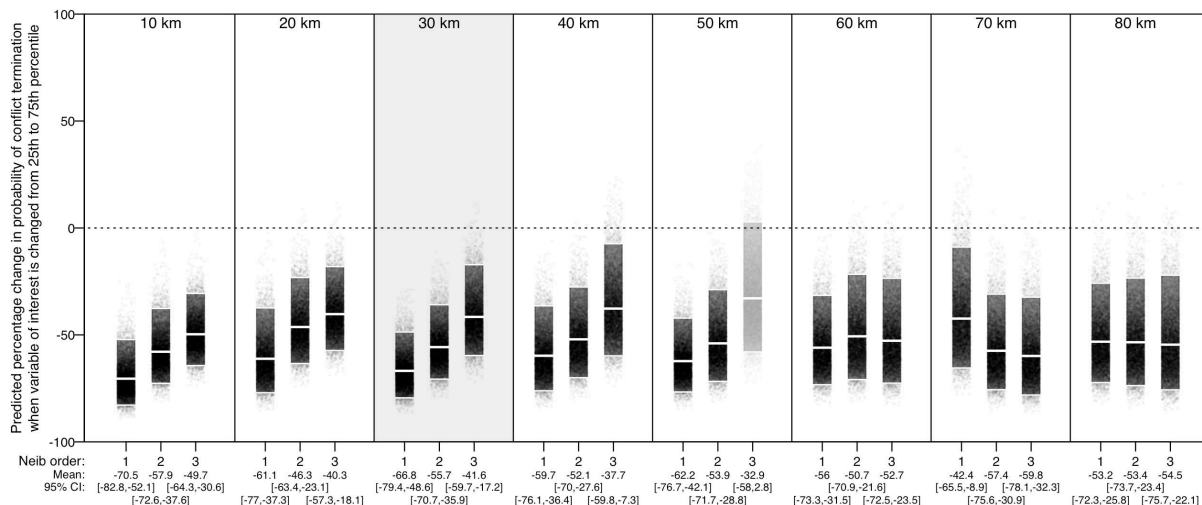
Unit of analysis: Conflict dyad-month. Robust standard errors clustered on dyad in parentheses. Intercepts and conflict duration polynomials are omitted for brevity. The multiclass AUC score is computed using the method in Hand & Till (2001).

grid specifications (Figure E.1(a)). Similarly, increasing instances of *Distant Diffusion* are also followed by a sizable reduction in the probability that a civil conflict terminates without seeing an negotiated settlement (-66.8% , 95% CI: $-79.4, -48.6$, with $r = 30$ km and $k = 1$); and the effect is robust to the selection of spatial grid resolution and neighborhood order (Figure E.1(b)). Second, turning to the estimates for *Proximate Diffusion*, the corresponding coefficient fails to retain a discernible effect at the conventional 5% level in almost all spatial grid settings. The results hold for both negotiated settlements and military outcomes (Figures E.2(a) and E.2(b)).

The estimates underscore the substantial impact of *Distant Diffusion* on conflict termination. Although our argument does not explicitly posit the underlying mechanism



(a) Effect of distant diffusion on probability of negotiated settlements

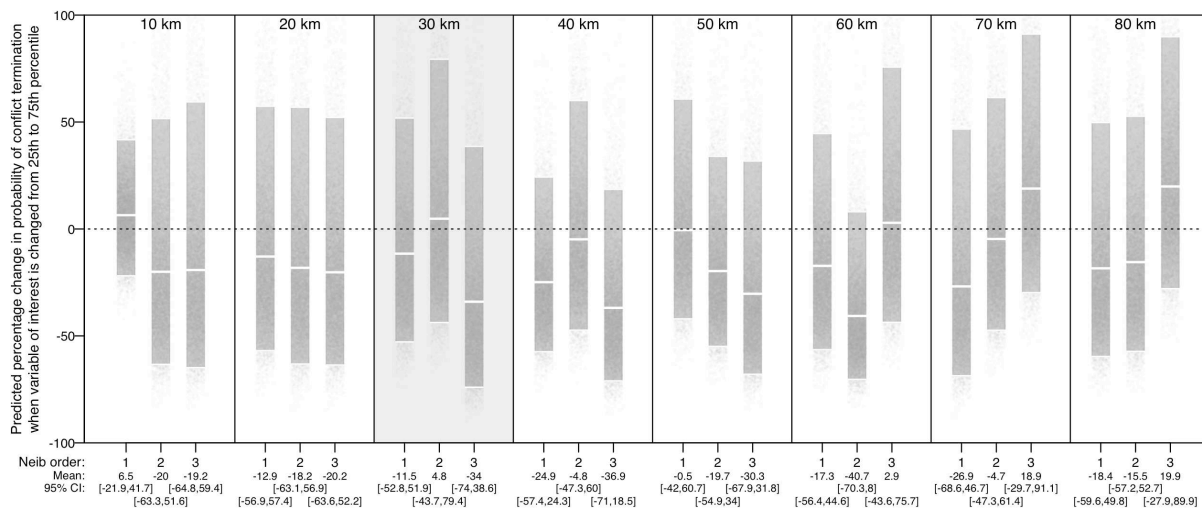


(b) Effect of distant diffusion on probability of military outcomes

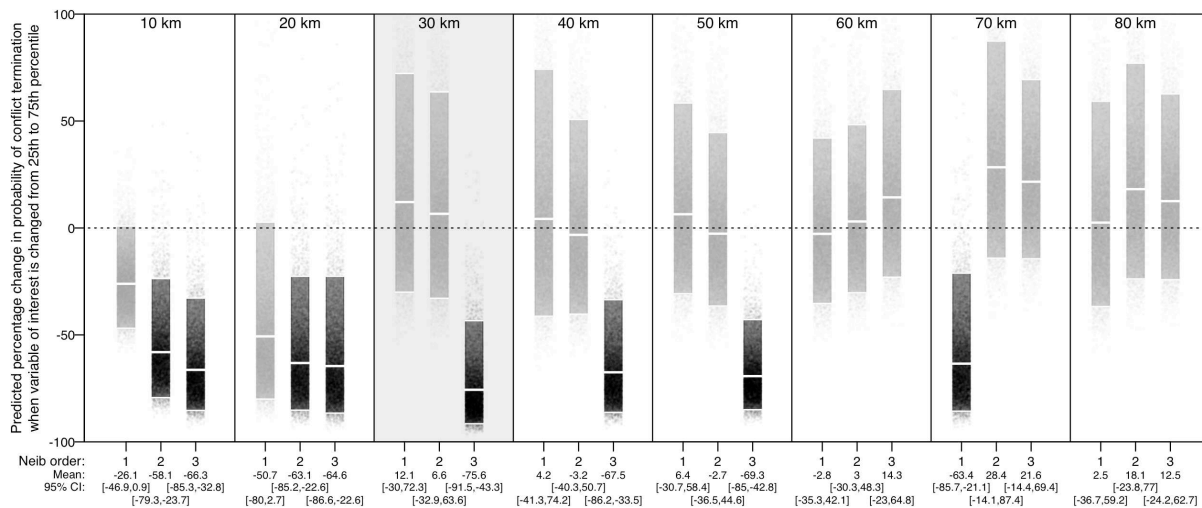
Figure E.1: Effect of distant diffusion as percentage change in probability of government-favorable and rebel-favorable outcome across different spatial grid resolutions

Notes: See notes in Figure 4 in the main text. Simulations are based on regression estimate in Table E.1.

linking battle diffusion and conflict termination with an military outcomes, the sizable negative impact of distant diffusion on conflict termination holds when we focus on negotiated settlements of civil conflict. Combined with the main findings, these results confirm that it is not whether or not battle activities diffuse, but how they diffuse that substantially alters how civil conflict unfolds.



(a) Effect of proximate diffusion on probability of negotiated settlements



(b) Effect of proximate diffusion on probability of military outcome

Figure E.2: Effect of proximate diffusion as percentage change in probability of government-favorable and rebel-favorable outcome across different spatial grid resolutions

Notes: See notes in Figure 4 in the main text. Simulations are based on regression estimate reported in Table E.1.

Appendix F Competing-Risks Regression

While the estimation in the previous section relies on the logit estimator, the following analysis employs the competing-risks Cox regression model as our dataset contains two possible conflict outcomes or competing risks, *Military Outcomes* and *Negotiated Settle-*

ments. `mstate` package in R is used to obtain the estimates (de Wreede et al., 2010, 2011; Putter et al., 2007). The competing-risks estimates are obtained using the model specification in Table E.1 in the main text.³ The spatial grid is specified as in the baseline setting with grid resolution $r = 30$ km and neighborhood order $k = 1$.

Table F.1 reports the cause-specific hazard ratio estimates and corresponding 95% confidence intervals. The cause-specific hazard for cause j refers to the hazard of failing (conflict termination) from cause (outcome type) j in the presence of J competing risks (causes; Putter et al., 2007, 2397). Similar to standard Cox proportional hazard models in the absence of competing risks, cause-specific hazard ratios can be interpreted relative to 1. Cause-specific hazard ratios less than 1 indicate covariates associated with longer duration until conflict termination with a particular outcome, whilst those with cause-specific hazard ratios greater than 1 are associated with shorter duration.⁴ As these estimations show, the main empirical results remain qualitatively unchanged: *Distant Diffusion* has a statistically and substantially negative impact on the chances of both *Military Outcomes* and *Negotiated Settlements*, while the cause-specific hazard ratio estimates for *Proximate Diffusion* and *Naive Diffusion* remain statistically indistinguishable from 1 at the conventional 5% level.⁵

Nonetheless, in the presence of competing risks, the (cause-specific) hazard ratio estimates alone only allow for limited interpretation of the substantial impacts of the corresponding covariates. This is primarily because the effect of a given covariate is modeled for more than one cause of failure (conflict outcomes) in competing-risks Cox regression models. Consequently, the substantial or marginal effect of a change in a given independent variable on cause j depends on its effect on the baseline hazards of all other causes as well as cause j (Beyersmann et al., 2012, 89–121; Putter et al., 2007, 2403–2409). In other words, while a change in a given independent variable can simultaneously affect the baseline cause-specific hazard of more than one cause, the cause-specific hazard ratio

³The multinomial logit model can be regarded as a discrete-time survival model in the presence of competing risks, with t^1, t^2 , and t^3 mimicking the baseline hazard. See Barnett et al. (2009) and Beyersmann et al. (2012, 164–166) for a related discussion.

⁴The key assumption in the competing-risks Cox regression model is the proportional hazard assumption that the effect of a covariate on the baseline cause-specific hazard of cause j is constant over time. Schoenfeld residual-based tests detect no statistically significant violations of the assumption of proportional (cause-specific) hazards at the 5% level.

⁵We also estimated the competing risks model with frailty (random effect) to account for unobserved heterogeneity across rebel-government dyads using `coxme` package in R (Therneau, 2015). The results for the diffusion terms remained qualitatively unchanged.

Table F.1: Competing-risks estimates of conflict outcome

| | <i>Conflict outcome</i> | |
|--------------------------------|--|---|
| | <i>Military Outcome</i> Cause-specific hazard ratio (95% CI) | <i>Negotiated Settlement</i> Cause-specific hazard ratio (95% CI) |
| Violence diffusion | | |
| Proximate Diffusion | 1.250 (0.444, 3.518) | 0.643 (0.174, 2.381) |
| Distant Diffusion | 0.492** (0.354, 0.683) | 0.597* (0.391, 0.912) |
| Naive Diffusion | 1.173 (0.891, 1.545) | 0.907 (0.786, 1.045) |
| Conflict dynamics | | |
| Conflict Intensity | 0.929 (0.712, 1.214) | 1.044 (0.794, 1.372) |
| Cumulative Casualties | 1.180* (1.021, 1.363) | 1.195* (1.007, 1.419) |
| Collateral Damage | 0.921 (0.612, 1.387) | 0.782 (0.445, 1.374) |
| Govt OSV | 0.951 (0.747, 1.211) | 0.820 (0.661, 1.017) |
| Rebel OSV | 0.873 (0.604, 1.261) | 0.774 (0.568, 1.055) |
| Government attributes | | |
| per capita GDP | 0.822 (0.662, 1.021) | 0.887 (0.580, 1.357) |
| Democracy | 0.678 (0.345, 1.335) | 2.193 (0.860, 5.592) |
| Country Size | 0.916 (0.764, 1.098) | 1.167 (0.921, 1.480) |
| Territorial Control | 0.766 (0.458, 1.280) | 0.969 (0.530, 1.771) |
| Rebel attributes | | |
| Ethnic Claim | 0.958 (0.619, 1.483) | 1.175 (0.625, 2.208) |
| Rebel Much Weaker | 1.777* (1.094, 2.887) | 0.310** (0.156, 0.619) |
| Multi Party | 1.198 (0.692, 2.071) | 0.892 (0.468, 1.700) |
| Conflict geography | | |
| Capital Distance | 1.155 (0.906, 1.472) | 0.759 (0.511, 1.126) |
| Local Population | 1.197 (0.990, 1.449) | 0.655 (0.414, 1.036) |
| Natural Resource Distance | 1.128 (0.880, 1.446) | 0.728** (0.590, 0.898) |
| Ruggedness | 0.971 (0.672, 1.403) | 0.942 (0.545, 1.628) |
| Road Density | 1.132 (0.892, 1.438) | 0.927 (0.701, 1.226) |
| Observations (months at risk) | | 7,341 |
| # Spells (conflict dyads) | | 199 |
| # Failures | 90 | 59 |
| Log Likelihood | -346.274 | -206.345 |
| Wald Test (df = 20) | 67.130** | 80.570** |
| LR Test (df = 20) | 54.839** | 53.420** |
| Score (Logrank) Test (df = 20) | 53.184** | 53.829** |

Note: * $p < 0.05$; ** $p < 0.01$

Unit of analysis: conflict dyad-month. 95% confidence intervals computed using robust standard errors clustered on dyad in square brackets.

estimate indicates its effect on the hazard of cause j without taking account for its effect on other causes.⁶

In order to facilitate better understanding of the effects of *Distant Diffusion*, the two panels in Figure F.1 plot the cumulative incidence functions (CIFs) of *Military Outcomes* and *Negotiated Settlements*, for median (dashed) and 99th percentile (solid) values of *Distant Diffusion* holding all other variables constant at their median (continuous) or mode (binary), respectively. Cumulative incidence functions in Figure F.1 represent the proba-

⁶Alternative approaches include regressing directly cumulative incidence functions rather than cause-specific hazards (Fine & Gray, 1999) and reduced rank proportional hazards models (Fiocco et al., 2006).

bility that conflict termination with *Military Outcomes* (left) and *Negotiated Settlements* (right) occur before time (conflict month) t for a given levels of covariates. Because cumulative incidence functions take account for the covariate effects for more than causes, these estimates allow for intuitive interpretation of substantial effect of *Distant Diffusion* on different conflict outcomes.

Figure F.2 plots the stacked transition probabilities to give another graphical representation of the competing-risks regression estimates, with median (left) and 99th percentile (right) values of *Distant Diffusion*. The left panel of Figure F.2 plots the dashed curves in the two panels of Figure F.1 in a single figure, whilst the right panel stacks the probabilities represented by solid curves in Figure F.1. As in Figure F.1, all other variables are held constant at their median or mode. In both panels, the horizontal axis indicates the number of months since the conflict onset, while the distance between two adjacent curves on the vertical axis indicates the estimated probability of being in the corresponding state (*Continuation*, *Military Outcome*, and *Negotiated Settlements*). As noted in the main text, the average duration of dyadic conflict episodes (spells) is 59.34 months (4.95 years), and the median duration is 30 months (2.5 years).

As Figures F.1 and F.2 show, escalating *Distant Diffusion* of battle activities is followed by substantial declines in the probabilities of failure (conflict termination) from *Military Outcomes* and *Negotiated Settlements* and a corresponding increase of probability of conflict continuation. These figures graphically demonstrate the substantial and negative impact of *Distant Diffusion* on conflict termination with different outcomes and provide further empirical support for our argument.

Appendix G Sample Selection and Outliers

The last sensitivity concern is that the sample selection, or the inclusion of outliers with a large number of diffusion observations in a single conflict may have a disproportionate effect on our estimates. To test whether these outliers drive our results, we report a series of subsample coefficient estimation results for the diffusion terms excluding one conflict episode at a time, or groupwise jackknifing of our sample by conflict dyads. As our dataset contains 199 unique dyadic conflict episodes, this dyad-wise jackknife procedure yields 199 distinct subsamples.

Figure G.1 uses a graph to summarize the results of 199 distinct estimations with

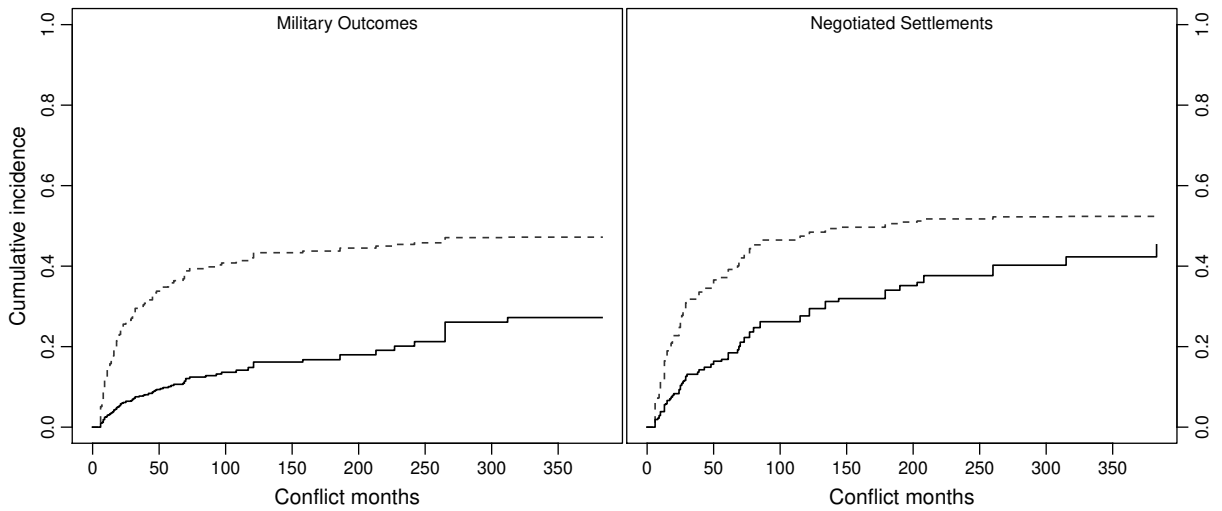


Figure F.1: Cumulative incidence functions for conflict outcomes across different values of *Distant Diffusion*

Notes: Cumulative incidence functions for *Military Outcomes* (left) and *Negotiated Settlements* (right). Solid curves indicate the cumulative incidence functions with *Distant Diffusion* at its 99th percentile value, whilst dashed curves indicate the estimates with *Distant Diffusion* at its median value while holding all other continuous variables constant at their median and binary variables at their mode.

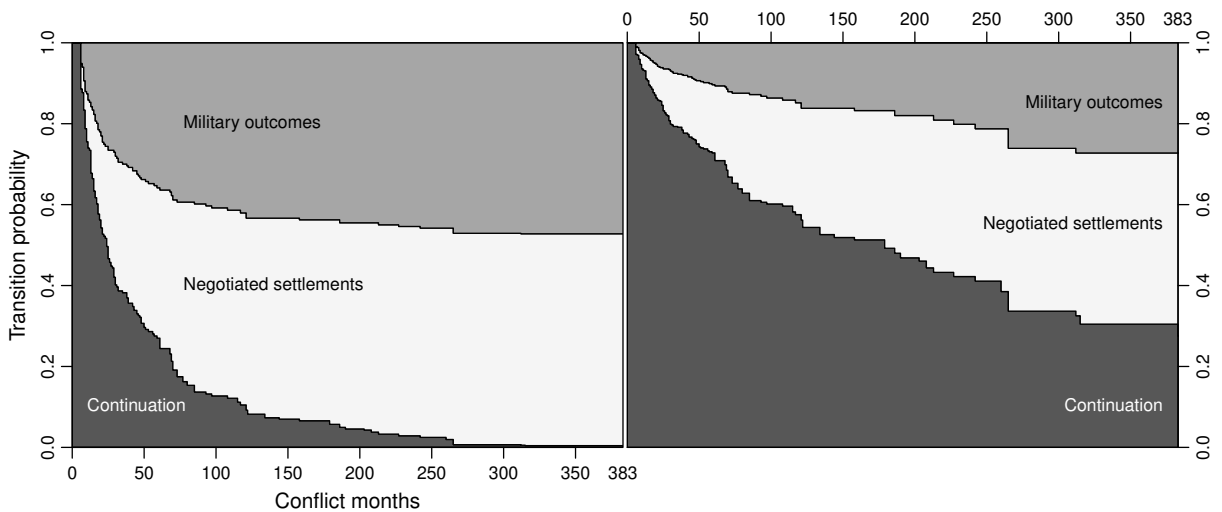
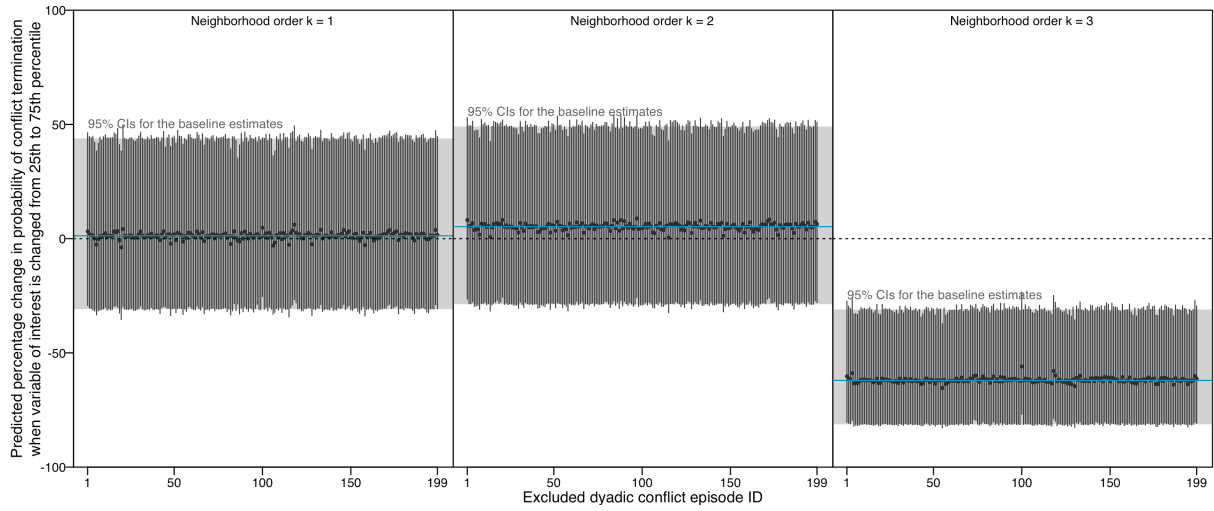
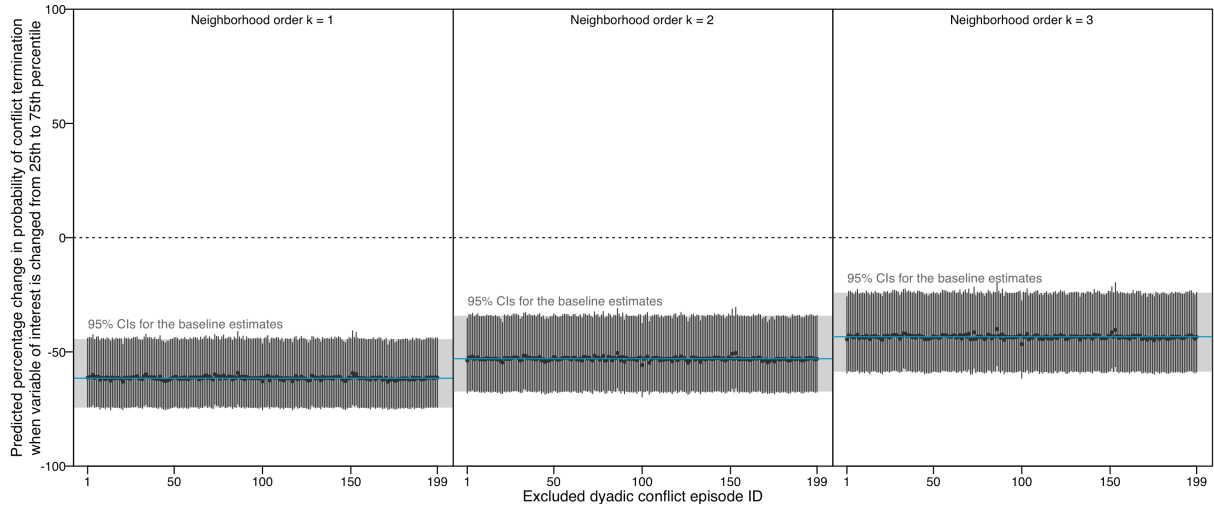


Figure F.2: Stacked transition probabilities of conflict outcomes across different values of *Distant Diffusion*

Notes: The distance between two adjacent curves indicates the estimated probability of being in the corresponding state (*Continuation*, *Negotiated Settlement*, and *Military Outcome*), with median (left) and 99th percentile (right) values of *Distant Diffusion*. All other continuous variables are held constant at their median and binary variables at their mode.



(a) Effect of proximate diffusion on probability of conflict termination



(b) Effect of distant diffusion on probability of conflict termination

Figure G.1: Effect of proximate diffusion as percentage change in probability of conflict termination across subsamples excluding a single dyadic conflict episode

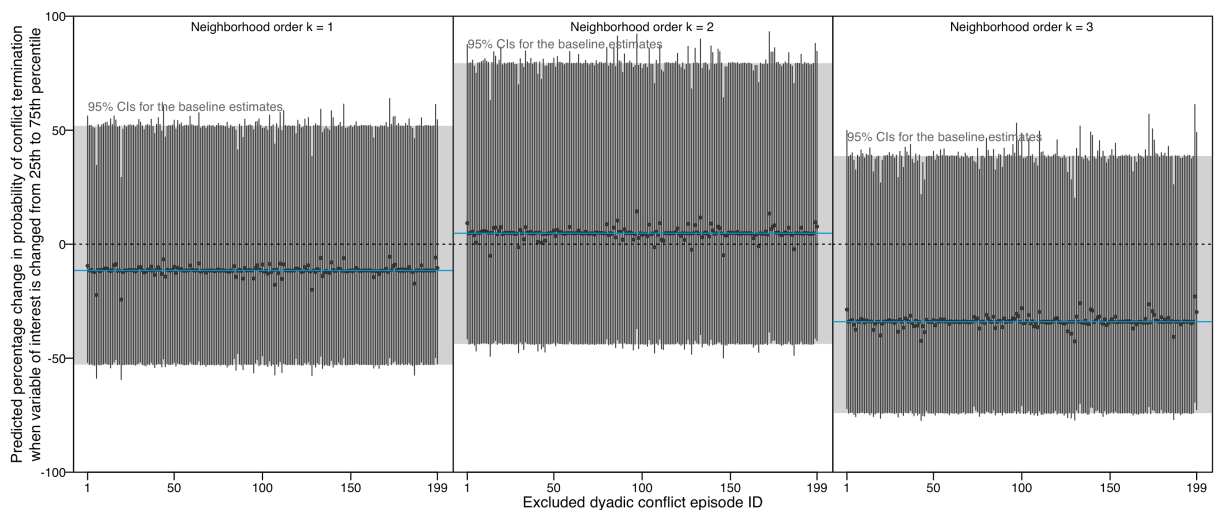
Notes: Each dot indicates a predicted change in probability of conflict termination drawn from a single simulation when *Proximate Diffusion* (*Distant Diffusion*) is changed from the 25th to 75th percentile (first difference estimate), holding all other variables constant at their median (continuous) or mode (binary). Vertical segments indicate the corresponding 95% confidence intervals of predicted values. **Blue solid horizontal segment** indicates the mean estimate for the full sample (baseline) regression, whereas gray shade represents the corresponding 95% confidence intervals. Black horizontal segment running through each panel indicates the zero-reference line. Uncertainty estimates are obtained by 10,000 simulations. Simulations are based on the model specification of Model 3 in Table 3 with grid resolution $r = 30$ km.

a different conflict episode excluded from the sample, with reference to the baseline full sample estimates. Specifically, it plots how a specific amount of increase in *Proximate Diffusion* and *Distant Diffusion* (25th to 75th percentile) changes the probability of conflict termination, holding all other continuous variables constant at their median and binary variables at their mode (first difference estimate). Each dot and vertical segment indicates the median estimates and corresponding 95% confidence intervals for a regression estimate excluding a single episode of dyadic conflict. Three panels represent the estimation results across different neighborhood orders. The grid specification is set as the baseline setting, or $r = 30$ km resolution hexagonal grid with neighborhood order k varying from 1 to 3. Uncertainty estimates for the predicted values are obtained via 10,000 simulations following the recommendation of King et al. (2000).⁷ Blue solid horizontal segment in each panel indicates the mean estimate for the full sample (baseline) regression, whereas gray shade represents the corresponding 95% confidence intervals. Similarly, Figures G.2 and G.3 plot the simulated impact of diffusion terms on *Military Outcomes* and *Negotiated Settlements* across different subsamples, respectively.⁸

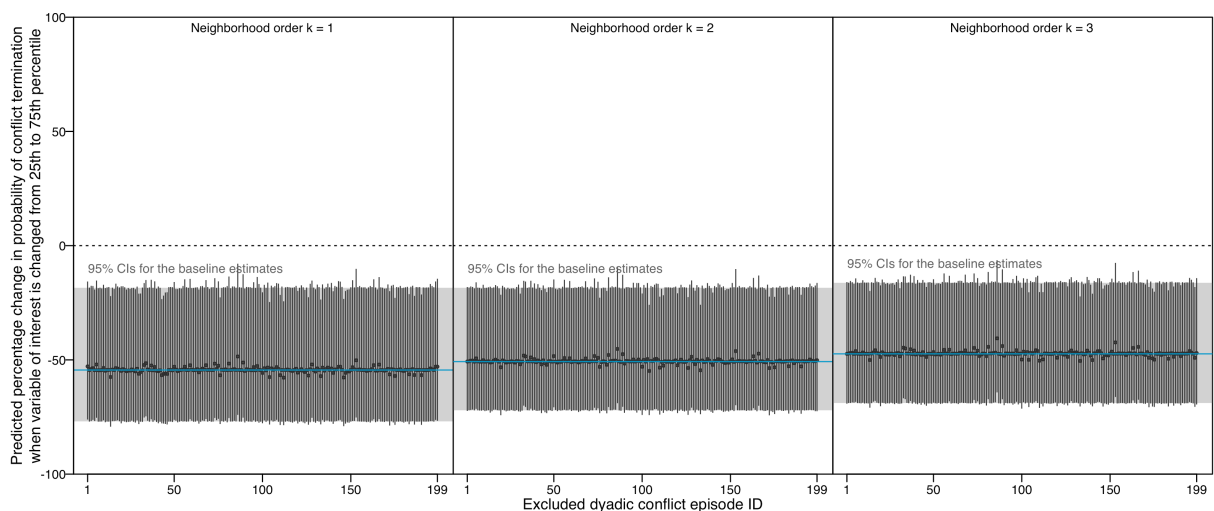
Rather than simply reporting the jackknife estimates, the graphical approach in Figures G.1 to G.3 allows us to easily detect the potential outliers on the estimation results. These three figures indicate heavy overlaps of the confidence intervals in the full sample and individual subsample estimations, suggesting that the main findings are not driven by outliers with an exceptional number of battle diffusion events.

⁷Simulations are based on the model specification of Model 3 in Table 3 in the main text.

⁸Simulations are based on the model specification in Table E.1.



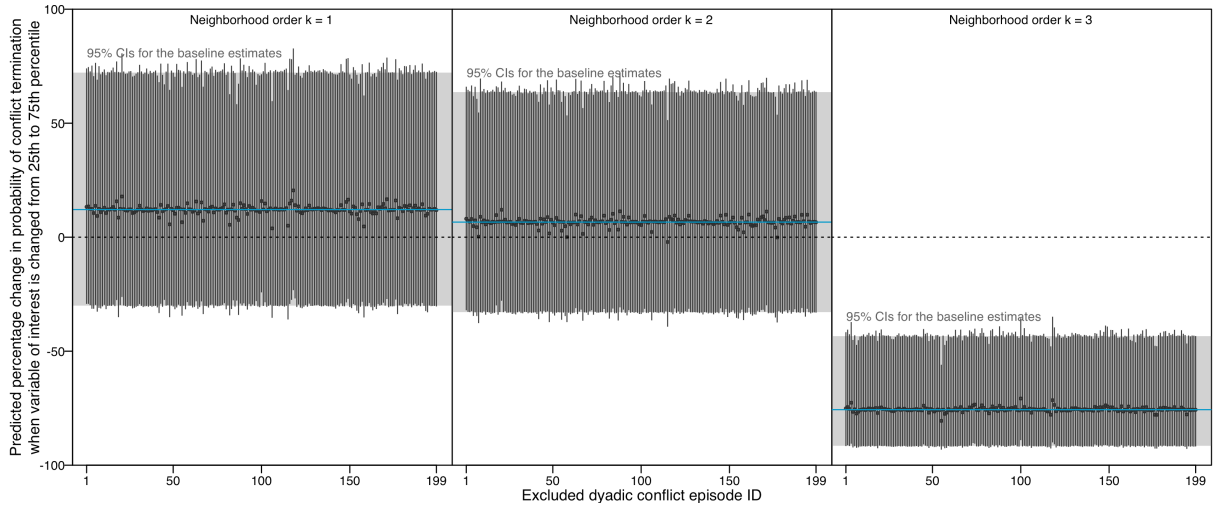
(a) Effect of proximate diffusion on probability of negotiated settlements



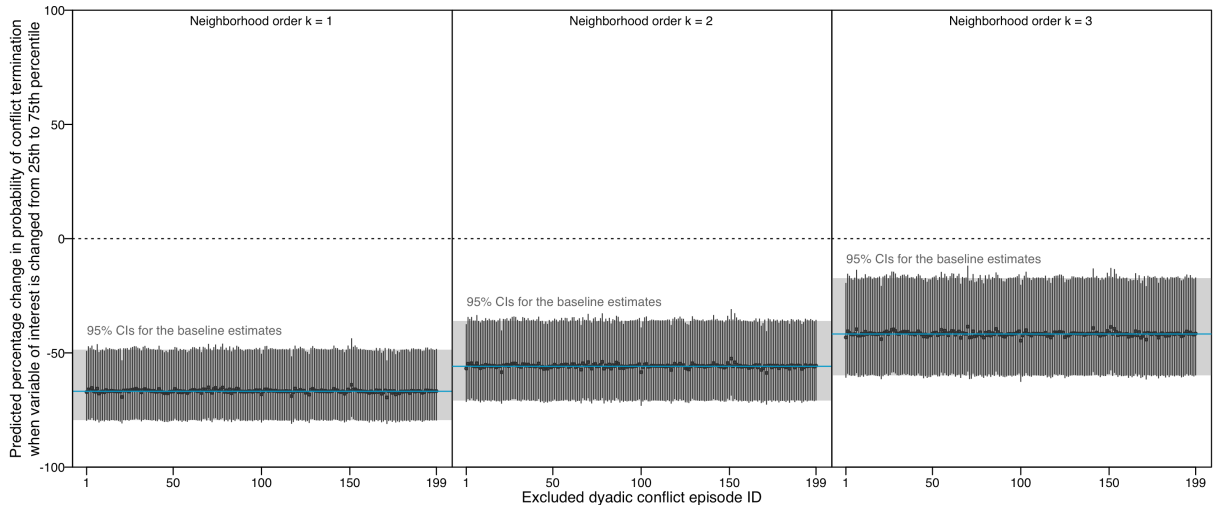
(b) Effect of distant diffusion on probability of negotiated settlements

Figure G.2: Effect of proximate diffusion as percentage change in probability of military outcomes across subsamples excluding a single dyadic conflict episode

Notes: See notes in Figure G.1. Simulations are based on the model specification in Table E.1 with grid resolution $r = 30$ km.



(a) Effect of proximate diffusion on probability of military outcomes



(b) Effect of distant diffusion on probability of military outcomes

Figure G.3: Effect of proximate diffusion as percentage change in probability of negotiated settlements across subsamples excluding a single dyadic conflict episode

Notes: See notes in Figure G.1. Simulations are based on the model specification in Table E.1 with grid resolution $r = 30$ km.

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