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1 Article

# 2 **Delay-differential-equation modeling of mode-** 3 **locked vertical-external-cavity surface-emitting lasers** 4 **in different cavity configurations.**

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12 **Abstract:** A simple, versatile model for the dynamics of electrically and optically pumped vertical-  
13 external-cavity surface-emitting lasers mode locked by semiconductor saturable absorber mirror is  
14 presented. The difference between the laser operation in the linear and folded cavity, as well as  
15 the potential for colliding pulse operation, are studied.

16 **Keywords:** vertical-external-cavity surface-emitting lasers; mode locking; SESAM; frequency combs

17

## 18 **1. Introduction**

19 Vertical External-Cavity Surface-Emitting Lasers (VECSELs) first developed in 1997 [1] have the  
20 advantage of the mature VCSEL technology for the semiconductor gain chip consisting of an epitaxial  
21 Distributed Bragg Reflector (DBR) and several Quantum Well (QW) or Quantum Dot layers. This  
22 allows wavelength flexibility, output power scalability and mass production. The external cavity  
23 makes possible lasing to occur in a single transverse and longitudinal mode by implementing,  
24 respectively, spherical mirrors and Fabry-Perot filters in the external cavity [1-5]. On the contrary,  
25 multi-transverse mode lasing is typical for VCSELs with output power in excess of several mW due  
26 to spatial hole burning [6, 7]. Spectral coverage of VECSELs extends from 390 nm [8] to 5  $\mu\text{m}$  [9] and  
27 even to 244 nm [10] by an intracavity fourth harmonic generation. By efficient thermal management  
28 the output power is scaled by simply scaling the area of the optically pumped spot reaching 100W in  
29 CW operation [11]. VECSELs are very well-suited for mode-locked operation by utilizing  
30 Semiconductor Saturable-Absorber Mirror (SESAM) either in the external cavity [12] or integrated in  
31 the gain-chip [13]. Tremendous progress has been achieved since the first demonstration of an  
32 SESAM mode-locked VECSEL in 2000 [14]: the pulse durations in fundamentally mode locked  
33 operation has been decreased to 107 fs [15] and even to 60fs in a burst operation [16], the average  
34 output power has been increased to 6.4W [17] and the peak power increased to 4.35 kW [18]. While  
35 the shortest pulses have been achieved with optically pumped active layers, electrically pumped  
36 structures are promising for a number of applications. A versatile, but fairly complex theoretical  
37 model has been presented [19, 20] for mode locking in electrically pumped VECSELs, but only for  
38 the simplest linear geometry and using a semi-microscopic model for the optical properties of  
39 Quantum-Well active layers. Later, a delay-differential model of a similar type but with a simpler  
40 gain model was used for inclusion of complex transverse/lateral effects in a MIXCEL structure, in

41 which the active layer and the saturable absorber are hosted within the same chip [21], with a  
 42 possibility to substantially enhance computational efficiency by narrowing the temporal window  
 43 when simulating the pronounced ML regime [22]. In recent papers [23,24], a version of a delay-  
 44 differential model was developed taking into account the folded, as well as simple linear, cavity  
 45 geometries. These papers centred on lasers designed for ultrashort pulse generation and were mainly  
 46 dedicated to the issue of multiple pulse (pulse molecule) generation observed under some operating  
 47 conditions in such lasers; therefore the gain chip and saturable absorber were treated essentially as  
 48 short travelling-wave amplifiers, ignoring their reflective nature and resonator properties (indeed,  
 49 for femtosecond pulse generation it is beneficial to suppress the resonator properties of the chips by  
 50 applying antireflection coatings, to minimise any narrowing of the emission spectra).

51 An alternative to the delay-differential equation modelling is an iterative pulse shaping  
 52 approach [25,26,27], where gain, saturable absorption, and dispersion in each round trip are, as in  
 53 classic mode locking theories, represented by pulse shaping operators in time or frequency domain,  
 54 as appropriate. Such an approach is by necessity somewhat artificial as it separates dispersion from  
 55 gain and absorption and also effectively assumes a unilateral ring cavity. Still it has proven very  
 56 useful for many cases of practical significance, particularly when the main limitation for the pulse  
 57 duration is the group velocity dispersion of the cavity (soliton mode locking regime). With simulation  
 58 parameters deduced from measurements for a known laser, such a model is capable of providing  
 59 very good agreement with experiment in both picosecond [26] and femtosecond [27] regimes;  
 60 however it may be not the most appropriate for describing regimes and designs where more than one  
 61 pulse can exist in the cavity, and for including the cavity parameters at the design stage.

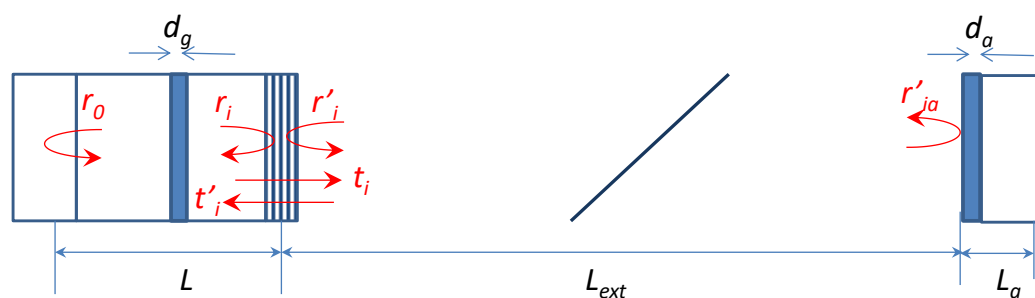
62 Here we continue the work started in the earlier conference papers [28] and present a model  
 63 based on an approach similar to that of [19, 20] and so treating the cavity properties of the gain and  
 64 absorber chip consistently, but using a simpler, generic active layer model that can be used for both  
 65 linear and folded-cavity geometry. The model can be, and is in this study, applied to study different  
 66 regimes of laser operation, including the possibility of colliding pulse and multiple colliding pulse  
 67 mode locking, and also can be used for inclusion of polarization effects in future.

68 The paper is organized as follows. In section 2 we present the derivation of the model and its  
 69 application to a straightforward linear cavity. Sections 2 and 3 deal with different versions of a folded  
 70 cavity, with either gain or the absorber forming the central chip. Finally, in section 4 a brief  
 71 discussion and summary are presented.

## 72 2. Vertical External-Cavity Surface-Emitting Laser: time-delay model.

### 73 2.1. Derivation of a simple equation for the active cavity dynamics.

74 A schematic of the mode-locked VECSEL consisting of vertical-cavity amplifier chip (left side)  
 75 and a SESAM chip (right side) is shown in Fig. 1.



76 **Figure 1.** Mode-locked VECSEL consisting of vertical-cavity amplifier chip with two DBRs with  
 77 reflectivities  $r_0$  and  $r_1$  and active region with thickness  $d_g$  and a SESAM chip with a single DBR and  
 78 active region thickness  $d_a$ .  $L$  and  $L_a$  are their effective lengths and  $L_{ext}$  is the length of the external  
 79 cavity.

80 The derivation of the model is a somewhat simplified version of that of [19] and is shown here for  
 81 completeness. As in [19], we start with the frequency domain approach and then convert it into time  
 82 domain. The equation for the reflected "field" (more accurately, wave amplitude) leaving the gain chip  
 83 reads

$$E_r^{(g)} = r_g E_{inc} = \left( \frac{r_i^2 r_o \tilde{G} e^{-j2kL}}{1 - r_i r_o \tilde{G} e^{-j2kL}} + r_i' \right) E_{inc} = E_{rc} + r_i' E_{inc}, \quad (1)$$

84 In this equation:

85  $E_{inc}$  is the complex amplitude of the incident field.

86  $E_{rc}$  is the complex amplitude of the field exiting the active resonator chip into the passive compound  
 87 cavity.

88  $r_i$  and  $r_o$  are the (wavelength-dependent) reflectances of the mirrors of the resonator facing inside  
 89 (i.e. the incident light) and outside the cavity  $r_i'$  is the reflectance seen by the light incident on the  
 90 mirror from the external cavity side, which has the same amplitude as  $r_i$  but different phase, as usual.  
 91  $L$  is the geometric cavity length.

92  $k$  is the complex wave vector. We can define a reference frequency and the corresponding wave  
 93 vector  $k_{ref} = \frac{n}{c} \omega_{ref}$  - for convenience it is easiest to assume that  $\omega_{ref}$  is the frequency of one of the cold  
 94 cavity modes. Then,  $k = k_{ref} + \frac{n}{c} \Delta\omega - \frac{j}{2} \alpha_{int}$ ,  $\alpha_{int}$  being the internal loss in the passive part of the  
 95 resonator.

96  $\tilde{G}$  is the single-pass dimensionless complex gain by all the QWs in the active layer of the resonator.  
 97 Assigning the active layer a thickness  $L_a$  and introducing the equivalent distributed complex gain  
 98  $\tilde{g} = g' + jg''$  we can write  $\tilde{G} = e^{\xi_g \Gamma_{\perp} \tilde{g} d_g} = e^{\Gamma \tilde{g} L}$ , where the total confinement factor, including the  
 99 enhancement, or relative confinement, factor due to the standing wave profile  $\xi_g$  is

$$\Gamma = \xi_g \Gamma_{\perp} d_g / L \quad (2)$$

100 This formalism is most natural in the case of a relatively thick, distributed, gain region, in which case  
 101 the standing wave factor is  $\xi_g=1$ . In the case of one or several QWs, when  $d_g \ll L$ , the notion of  $g$  is  
 102 somewhat artificial, but can be introduced heuristically alongside a  $\xi_g$  value of  $1 < \xi_g < 2$  (see below).  
 103 Using lumped gain per well, as in [19], is more rigorous (e.g. it gives  $\xi_g > 1$  self-consistently), but also  
 104 more complex, particularly in the case of multiple Quantum Well (QW) or Quantum Dot (QD) active  
 105 layers separated by a substantial distance (e.g. located in different wave antinodes). The present  
 106 formalism, in principle, applies to an arbitrary active layer thickness and location, though in this  
 107 paper we shall concentrate on the most usual one using a thin active layer in a single resonant  
 108 location.

109 The usual differential equation for an injected laser (i.e. a vertical cavity amplifier with two strongly  
 110 reflecting mirrors, operated above or near lasing threshold) is obtained by taking the absolute value

111 of the denominator in eqn. (1) to be small (which means operating above or near threshold, and  
 112 simultaneously with a small frequency detuning from the cold cavity mode frequency). In the more  
 113 general case of a resonator with arbitrary reflectances (in electrically pumped VECSELS, reflectances  
 114 of 70-90% can be used [19, 20]), we cannot assume the absolute value of the denominator in eqn. (1)

115 to be small but can assume small frequency detuning  $\left|2\frac{nL}{c}\tilde{G}_{net}\Delta\omega\right|\ll 1$ . Then,

$$1 - r_i r_o \tilde{G} e^{-j2kL} \approx 1 - \tilde{G}_{net} + jT_{rt} \tilde{G}_{net} \Delta\omega, \quad (3)$$

116 where

$$\tilde{G}_{net} = \exp(\tilde{g}_{net} L) = r_i r_o \exp[(\Gamma \tilde{g} - \alpha_{int}) L] = \mathcal{G}_c \tilde{G} \quad (4)$$

117 is the complex net roundtrip gain, with

$$\mathcal{G}_c = r_i r_o \exp(-\alpha_{int} L) \quad (5)$$

118 the (real) cavity attenuation factor. We have also introduced the round trip time of the cavity which,

119 as usual in the theory of vertical cavity structures, is evaluated as  $T_{rt} = 2\frac{n_g L_{eff}}{c}$ , where  $n_g = n + \omega \frac{dn}{d\omega}$

120 is the group refractive index, and  $L_{eff} = L + \frac{c}{n_g} \frac{d}{d\omega} (|\arg r_o + \arg r_i|)$  is the effective cavity length.

121 Then, the resonator equation becomes

$$(1 - \tilde{G}_{net} + jT_{rt} \tilde{G}_{net} \Delta\omega) E_{rc} = t_i t_o r_o \tilde{G} e^{-j2kL} E_{inc}. \quad (6)$$

122 The active layer in a VECSEL is always thin so  $|\tilde{G}|$  is never high above one (indeed, the measured  
 123 chip reflectance has been reported [29,30] as  $R_g = |r_g|^2$  for  $r_i \approx 0.9$  and  $R_g \approx 1.55$  for  $r_i \approx 0.96$  (notations as in  
 124 Eq. 1)), from which the value of  $|\tilde{G}|-1$  can be estimated to be of the order of  $10^{-2}$  at most, meaning it  
 125 is safe to approximate in Eq. (6)

$$\tilde{G} \approx 1 + \delta\tilde{G}, \quad \delta\tilde{G} = \Gamma \tilde{g} L$$

126 Strictly speaking, the expression (6) includes the dispersion of both the VECSEL active subcavity and  
 127 the complex gain  $\tilde{G} = \tilde{G}(\Delta\omega)$ . If (as is usually the case) the operating wavelength is near the gain peak,  
 128 we can use a Lorentzian gain spectrum approximation with a width  $\Delta\omega_g$ .

129 Then, assuming as usual  $\Delta\omega \ll \Delta\omega_g$ , the usual substitution  $j\Delta\omega \rightarrow \frac{d}{dt}$  gives a single differential  
 130 equation for the determination of the field reflected from the cavity

$$\left( T_{rt} \mathcal{G}_c + \frac{f_g \delta\tilde{G}}{\Delta\omega_g} \right) \frac{dE_{rc}}{dt} = [\delta\tilde{G} - (1 - \mathcal{G}_c)] E_{rc} + t_i t_o r_o e^{-\alpha_{int} L} E_{inc} \quad (7)$$

131 where  $\delta\tilde{G} = \Gamma \tilde{g} L$  is evaluated at the reference frequency  $\omega_{ref}$  and  $f_g = \frac{1}{1 + j(\omega_{ref} - \omega_p) / \Delta\omega_g}$

132 represents the carrier density dependent detuning between  $\omega_{ref}$  and the spectral peak  $\omega_p$  of gain.  
 133 In a QW active material, the carrier density dependence of the detuning is weaker than in bulk  
 134 material so we can take  $f_g = 1 = \text{const}$ . The equation combines cavity selectivity and gain dispersion

135 and thus can, in general, describe the performance of a gain chip with arbitrarily small or large  
 136 reflectances. In electrically pumped VECSEL designs, however, typically  $|r_i| > \sim 0.5$  [19,30], so, for  
 137 realistic semiconductor active media ( $\Delta\omega_g \gg \sim 10^{13} \text{s}^{-1}$ ) we can comfortably assume that the spectral  
 138 selectivity of the cavity dominates over the gain dispersion:

$$T_{rt} \mathcal{G}_c \gg \frac{\Gamma g L}{\Delta\omega_g}$$

139 Thus in the first approximation, we can neglect gain dispersion and write the equation in the simple  
 140 form

$$T_{rt} \mathcal{G}_c \frac{dE_{rc}}{dt} = \left[ \delta\tilde{G} - (1 - \mathcal{G}_c) \right] E_{rc} + t_i t_i' r_o' e^{-\alpha_{im} L} E_{inc} \quad (8)$$

141 Equations (7) and (8) are the main result of this section.

142

143 *2.2. The full mode-locked laser model: the linear cavity.*

144 To consider the full cavity, we need to recall that with a single incident and single reflected beam in  
 145 the linear cavity,

$$E_r^{(g)} = E_{rc} + r_i' E_{inc} \quad (9)$$

146 and note that

$$E_{inc} = \frac{\sqrt{\gamma}}{w'} E_r^{(a)} (t - T_{ext}) \quad (10)$$

147 where  $T_{ext} = L_{ext}/c$  is half of the round-trip time of the external cavity, and  $E_{ra}$  is the field reflected  
 148 from the absorber chip,  $\gamma$  is the dimensionless attenuation between the chips (including the out-  
 149 coupling, if it is located between the chips, and the attenuation in the contact layer of an electrically  
 150 pumped VCSEL), defined for the intensity as usual, hence square root in the equation for the field  
 151 amplitude. Technically speaking, the value needs to be complex, with the phase dependent on the  
 152 sub-wavelength variation in the length  $L_{ext}$  of the internal cavity, but in the linear cavity this can be  
 153 set to zero with no loss of generality. The dimensionless parameter  $w' > 1$  is the ratio of the beam  
 154 cross-section diameters in the gain (broader) and absorber (tighter to ensure saturation).

155 The absorber chip itself in the linear cavity can be described as in [19] by equations symmetric to  
 156 those of the gain chip, with the incident field given by

$$E_{inc}^{(a)} = \sqrt{\gamma} w' E_r^{(g)} (t - T_{ext}) \quad (11)$$

$$E_r^{(a)} = r_{ia}' E_{inc}^{(a)} + E_{rc}^{(a)} \quad (12)$$

157 Here, the field reflected from the inside of the chip is, assuming a (detuned) Lorentzian absorption  
 158 spectrum, evaluated using an equation formally identical to (7):

$$\left( T_{rt}^{(a)} \mathcal{G}_{ca} - \frac{f_a \delta\tilde{A}}{\Delta\omega_a} \right) \frac{dE_{rc}^{(a)}}{dt} = - \left[ \delta\tilde{A} + 1 - \mathcal{G}_{ca} \right] E_{rc}^{(a)} + t_{id}' t_{ia}' r_{oa}' e^{-\alpha_{im,a} L_a} E_{inc}^{(a)} \quad (13)$$

159 As in the case of the gain chip,  $\mathcal{G}_{ca} = r_{ia} r_{oa} \exp(-\alpha_{int} L)$ ,  $r_{oa}$  and  $r_{ia}$  being the outer and inner  
 160 reflectances of the absorber cavity,  $T_r^{(a)}$  is the SA cavity round-trip time, and the absorption in the  
 161 cavity, evaluated at  $\omega_{ref}$ , at is quantified as

$$\delta\tilde{A} = \Gamma_a \tilde{\alpha} L_a. \quad (14)$$

162 The detuning factor  $f_a = \frac{1}{1 + j(\omega_{ref} - \omega_{pa}) / \Delta\omega_a}$  needs to take into account that the SA is typically  
 163 operating at the spectral slope of the absorber  $(\omega_{ref} - \omega_{pa}) / \Delta\omega_a \sim 1$ .

164 The complex gain and absorption in the case of bulk or QW active layers (QD layers may need more  
 165 complex analysis) can be parametrized using the phenomenological concepts of gain and absorption  
 166 compression coefficients  $\varepsilon_g$ ,  $\varepsilon_a$ , and linewidth enhancement factors for the carrier-dependent  
 167 gain and absorption  $\alpha_H, \alpha_{Ha}$  and the nonlinearities  $\alpha_\varepsilon, \alpha_{\varepsilon a}$ :

$$\tilde{g} = g(N_g) \left( \frac{1}{(1 + \varepsilon_g S_g)} + j(\alpha_H - \alpha_\varepsilon \varepsilon_g S_g) \right) \quad (15)$$

$$\tilde{\alpha} = \alpha(N_a) \left( \frac{1}{(1 + \varepsilon_a S_a)} + j(\alpha_{Ha} - \alpha_{\varepsilon a} \varepsilon_a S_a) \right) \quad (16)$$

168 where  $S_{g,a}$  are the effective photon densities in the gain and absorption chip active layers. To write  
 169 out the rate equations for carrier densities in both chips, we need the explicit expressions for these  
 170 quantities, which depend on the geometry. Since  $|\delta\tilde{G}| \ll 1$ ,  $|\delta\tilde{A}| \ll 1$ , we can estimate the field in  
 171 the active layer of both chips in the passive resonator approximation. In the case of active layers in  
 172 the form of (single or multiple) thin (QW or QD) layers very near the field antinodes, which we shall  
 173 follow in the analysis below, we can write

$$S_g = \left| \overline{E}_{int}^{(g)} \right|^2, \quad S_a = \left| \overline{E}_{int}^{(a)} \right|^2, \quad (17)$$

174 where the fields inside the gain and SA active layers are evaluated assuming a resonant QW position  
 175 as

$$E_{int}^{(g)} \approx \frac{1 + r_o}{t_i r_0} E_{rc}^{(g)}; \quad E_{int}^{(a)} \approx \frac{1 + r_{oa}}{t_{ia} r_{oa}} E_{rc}^{(a)} \quad (18)$$

176 From these expressions, the enhancement factors can be evaluated as

$$\xi^{(g)} \approx \frac{|1 + r_o|^2}{1 + |r_0|^2}; \quad \xi^{(a)} \approx \frac{|1 + r_{oa}|^2}{1 + |r_{0a}|^2} \quad (19)$$

177 which in the case of  $|r_o| \sim 1$  gives  $\xi^{(g)} \approx 2$ , the well-known result for the wave enhancement factor in an  
 178 isolated VCSEL cavity with the QW active layer at the resonant position.

179 In the case of distributed (multilayer of bulk) gain or absorbing layers occupying a substantial fraction  
180 of the cavity, the enhancement factors  $\zeta$  could be taken as  $\zeta \approx 1$ , with the intensities calculated as

$$S_g \approx \left(1 + |r_o|^2\right) \left|E_{rc}^{(g)} / t_i\right|^2, \quad S_a \approx \left(1 + |r_{oa}|^2\right) \left|E_{rc}^{(a)} / t_{ia}\right|^2 \quad \text{but that case is not considered here.}$$

181 The effective photon densities  $S_{g,a}$  then are used in the carrier rate equations:

$$\frac{dN_g}{dt} = \frac{j}{ed_g} - \frac{N_g}{\tau_g(N_g)} - \frac{\nu_g g(N_g)}{1 + \varepsilon_g S_g} S_g \quad (20)$$

$$\frac{dN_a}{dt} = -\frac{N_a}{\tau_a(V_a)} + \frac{\nu_g \alpha_p(N_a)}{1 + \varepsilon_a S_a} S_a, \quad (21)$$

182 where as usual  $\nu_g = c/n_g$ ,  $\tau_a(V_a)$  is the absorber recovery time,  $V_a$  being the (reverse) bias applied  
183 to the chip, if any. For the carrier dependences of gain and absorption, in this paper we use the  
184 standard phenomenological expressions

$$g_p(N_g) = G_0 \ln \frac{N_g + N_s}{N_r + N_s} \quad (22)$$

$$\alpha_p(N_a) = \alpha_0 - \sigma N_a \quad (23)$$

185 The dependences of gain and absorption bandwidths (BW) on respective carrier densities are estimated  
186 in the first approximation as linear, i.e.

$$\Delta\omega_g(N_g) = \frac{d\Delta\omega_g}{dN_g}(N_g - N_{g0}),$$

$$\Delta\omega_a(N_a) = \Delta\omega_{a0} + \frac{d\Delta\omega_a}{dN_a} N_a.$$

### 187 2.3. The main “observable” parameters.

188 It is useful to establish the relations between parameters used in the model and the measurable  
189 values typically quoted in experiment, such as the threshold of laser operation, the saturation fluence  
190 of the absorber, and the modulation contrast of the SA chip.

191 The threshold condition of the compound cavity is given by a transcendental equation which in our  
192 notations is written as

$$\left( \frac{t_i^2 r_o \tilde{G}_{th} e^{-j2kL}}{1 - \tilde{G}_{th} r_i r_o e^{-j2kL}} + r_i \right) \left( \frac{t_{ia}^2 r_{oa} \tilde{A}_0 e^{-j2kL_a}}{1 - \tilde{A}_0 r_{ia} r_{oa} e^{-j2kL_a}} + r_{ia} \right) \gamma e^{-j2k_0 L_{inrcavity}} = 1 \quad (24)$$

193 where  $\tilde{G}_{th} = 1 + \delta\tilde{G}_{th}$  and  $\tilde{A}_0 = 1 - \delta\tilde{A}_0$  are (complex) threshold gain and unsaturated absorption,  
194 and  $k_0 = k_{ref} + \frac{\Delta\omega}{c}$  the wave vector in vacuum at the resonant (modal) frequency. The threshold  
195 condition is taken as the lowest gain of all the multiple solutions of the transcendental equation (24),  
196 which correspond to modes of the compound cavity. The numerical solution of essentially the same  
197 problem, though in different notations, illustrates [19] that, since the cavity length  $L_{inrcavity} \gg L, L_a$ , the  
198 modes are spaced closely enough for there always to be a few near the resonance of both chip  
199 resonators, essentially allowing us to count  $e^{-j2k_0 L_{inrcavity}} \approx 1$ . In that case, assuming that the gain chip  
200 and SESAM are resonant cavities, we can establish an analytical estimate for the threshold in the form



$$g_{th} = \frac{1}{\Gamma L} \ln G_{th} \approx \frac{1}{\Gamma L} (G_{th} - 1) \approx \frac{1}{\Gamma L} \left( \frac{(1 + r_i r_{oa} \gamma)(1 + \alpha_{int} L)}{(r_{oa} \gamma + r_i) r_o} - 1 \right) \quad (25)$$

201 where

$$r_{oa} = \left| \frac{t_{ia}^2 \mathcal{G}_{ca} (1 - \Gamma_a a_0 f_{la} L_a)}{r_i (1 - \mathcal{G}_{ca} (1 - \Gamma_a a_0 f_{la} L_a))} + r_{ia} \right| \quad (26)$$

202 is the unsaturated SA chip reflectance.

203 The saturation fluence of the absorber in the model we use (Eq.(23)) is

$$F_{sat} \approx \frac{\hbar \omega}{\sigma} \left| \frac{1 - |\tilde{A}_0 r_{ia} r_{oa}|}{t_{ia} (1 + r_{oa})} \right|^2 \quad (27)$$

204 The reflectance contrast is estimated most easily neglecting the self-phase modulation in the SA (since  
205 the Henry factor in the absorber is usually believed to be smaller than in the amplifier) and assuming  
206 small detuning from resonance, in which case

$$\Delta R \approx 2 |r_{sA}| (|r_{sA}| - |r_{oa}|) \approx 2 |r_{sA}| \left| \frac{t_{ia}^2 \mathcal{G}_{ca}}{r_i (1 - \mathcal{G}_{ca})^2} \Gamma_a a_0 L_a \right| \quad (28)$$

207 where

$$r_{sA} = \frac{t_{ia}^2 \mathcal{G}_{ca}}{r_i (1 - \mathcal{G}_{ca})} + r_{ia} \quad (29)$$

208 is the amplitude reflectance of a fully saturated absorber.

#### 209 2.4. Numerical Results.

210 The gain-chip and the saturable absorber parameters used in this section, unless specified  
211 otherwise, are listed in tables I and II, respectively.

212  
213

Table I. Gain-chip parameters

Parameter	Nota-Tion	Value	Units
back mirror amplitude reflectivity	$r_o$	0.9995	
front mirror amplitude reflectivity	$r_i$	0.7	
internal losses	$\alpha_i$	0.001	$\mu\text{m}^{-1}$
effective length	$L_a$	1.5	$\mu\text{m}$
QWs cumulative thickness	$d_g$	0.024	$\mu\text{m}$
radius of the active region	$\rho_g$	3	$\mu\text{m}$
group refractive index	$n_{rg}$	3.5	
confinement factor	$\Gamma_g$	0.06	
linewidth enhancement factor	$\alpha_g$	3	
carriers lifetime	$\tau_g$	1	ns
gain compression factor	$\varepsilon_g$	$0.5 \cdot 10^{-5}$	$\mu\text{m}^{-3}$

parameter carrier density	$N_s$	$-0.4 \cdot 10^6$	$\mu\text{m}^{-3}$
transparency carrier density	$N_{tr}$	$1.6 \cdot 10^6$	$\mu\text{m}^{-3}$
gain coefficient	$G_0$	0.18	$\mu\text{m}^{-1}$
Coefficients of gain BW dependence	$d\Delta\omega_g/dN_g$	0.012	$\mu\text{m}^3/\text{n}$
	$dN_g$		s
	$N_{g0}$	$7.5 \cdot 10^5$	$\mu\text{m}^{-3}$

214

215

Table II. Saturable absorber parameters

Parameter	Notation	Value	Units
back mirror amplitude reflectivity	$r_{oa}$	0.97	
front mirror amplitude reflectivity	$r_{ia}$	0.565	
internal losses	$\alpha_{ia}$	0.001	$\mu\text{m}^{-1}$
effective length	$L_a$	1.5	$\mu\text{m}$
group refractive index	$n_{ra}$	3.5	
confinement factor	$\Gamma_a$	0.06	
linewidth enhancement factor	$\alpha_a$	3	
carriers lifetime	$\tau_a$	0.03	ns
compression factor	$\varepsilon_a$	$1.5 \cdot 10^{-5}$	$\mu\text{m}^{-3}$
absorber saturation cross-section	$\sigma$	$2 \cdot 10^{-7}$	$\mu\text{m}^{-3}$
absorption coefficient	$\alpha_0$	0.5	$\mu\text{m}^{-1}$
Coefficients of absorption bandwidth dependence	$d\Delta\omega_a/dN_a$	0.048	$\mu\text{m}^3/\text{n}$
	$dN_a$		s
	$\Delta\omega\alpha_0$	15199	1/ns

216 The values of the external cavity parameters are: the time of flight between the gain and the absorber  
 217 cavities  $\tau=0.02$  ns, the transmission coefficient  $\gamma=1$  and the ratio of the beam cross-section diameters  
 218 onto the gain and the absorber chips  $\omega' = 3$ .

219

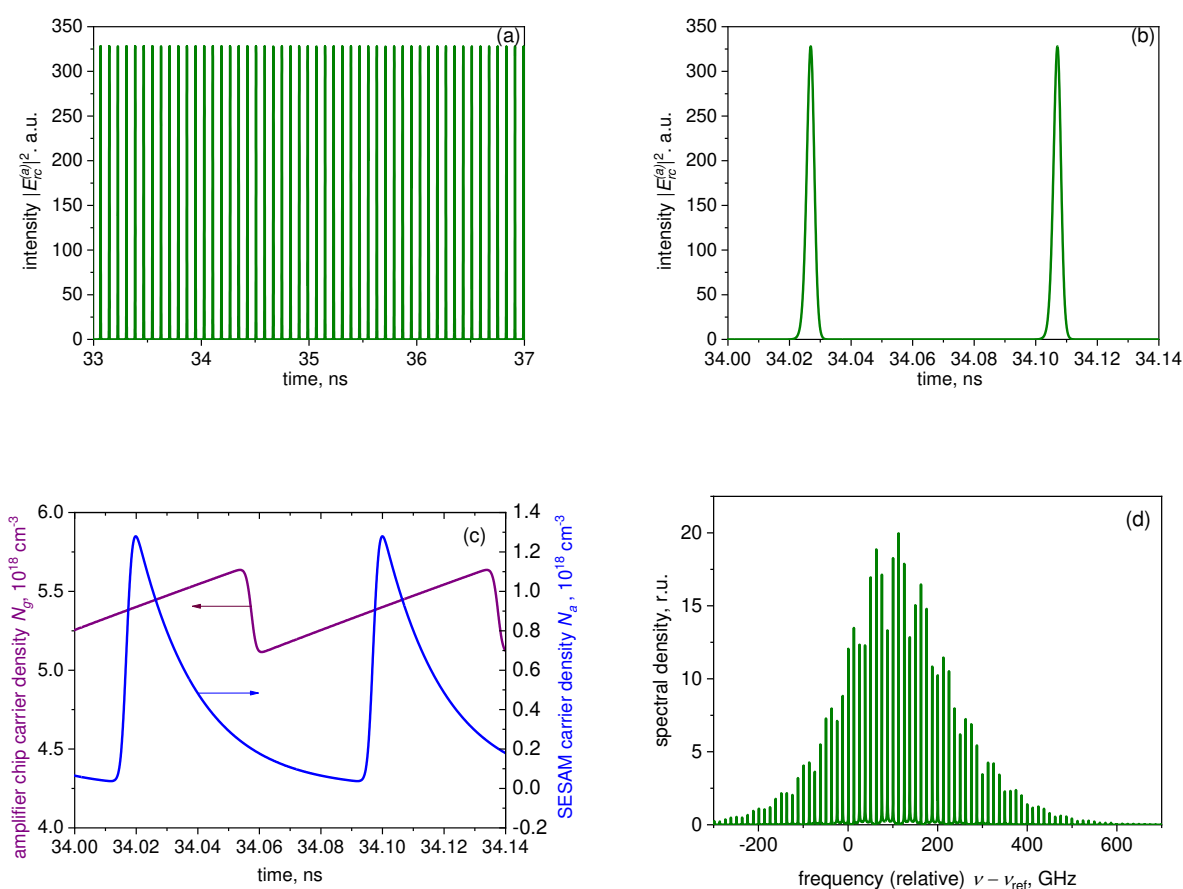
220 Figures 2-4 present an example of simulated mode-locked operation of a VECSEL – SESAM

221 configuration. Long- and short-term time traces of photon density  $|E_{rc}^{(a)}|^2$  are shown in Fig. 2 (a)

222 and (b), respectively. Fig. 2(c) shows the transient of carrier densities  $N_a$  and  $N_g$  corresponding to the

223 photon density transient of Fig. 2(b). The optical spectrum of the time trace of Fig. 2 (a) is shown in

224 Fig.2 (c).



225

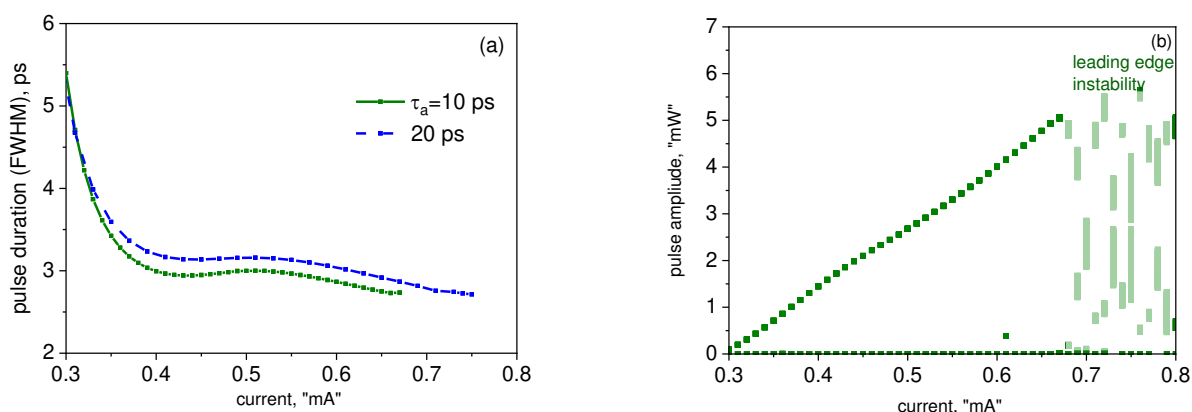
226

227 **Figure 2.** Time traces of the long-term (a) and short-term (b) evolution of the photon density  
 228 reflected from the SESAM chip. (c) the corresponding evolution of the carrier densities; (d) the  
 229 spectrum of (a). Gain chip current  $i_c=0.6$  mA (stable mode locking).

230 As the modification of the pulse by a single round-trip is only moderate in the ML VECSEL, the pulse  
 231 shape is fairly symmetric; however the up-chirp usual in passively mode locked semiconductor lasers  
 232 is still present, if relatively modest, with the time-bandwidth product of  $\Delta\nu\Delta\tau\approx 0.6$  (the pulse duration  
 233  $\Delta\tau$  and the spectral width  $\Delta\nu$  being evaluated at half maximum) at the current shown. The chirp also  
 234 manifests itself in the asymmetry and some envelope modulation of the spectrum.

235

236 The evolution of the pulse duration and amplitude with current is illustrated in Fig. 3 a and b  
 237 respectively. the pulse duration is in the picosecond range and, as in [19], decreases overall with  
 238 current. As normal in mode-locked semiconductor lasers (see [32,33] and references therein), the  
 239 pulses become longer with an increase in the absorber relaxation time.



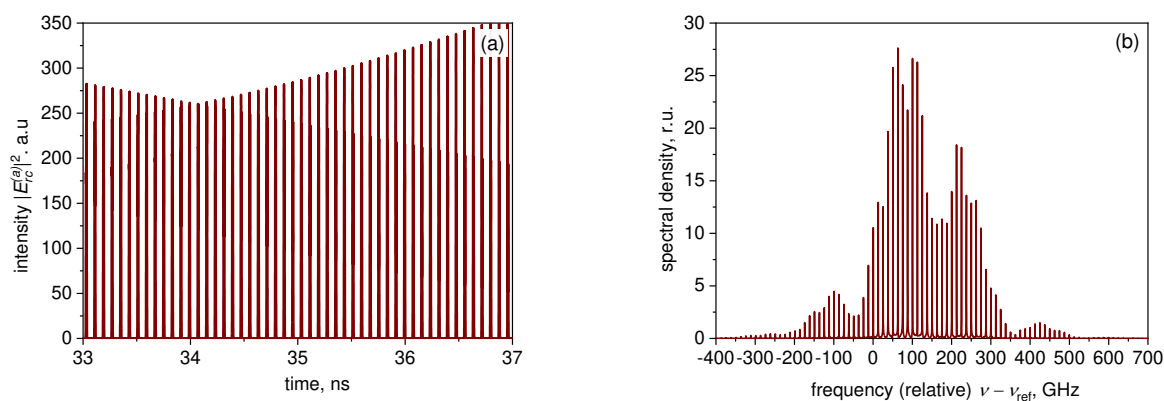
240

241 **Figure 3.** Current dependences of the pulse duration (a) and amplitude (b). In (b), the absorber recovery time  
 242 is 10 ps.

243

244 As in [19], no trace of the “trailing edge” self-pulsing instability was observed in our simulations; this  
 245 can be attributed to both the relatively low repetition frequency and the weak pulse modification per  
 246 pulse. However, at high currents, the irregular “leading edge” instability, in the form of two or, at  
 247 higher currents, several non-periodically competing pulse trails (Fig. 4a) is present; its onset has been  
 248 chosen as the upper extent of curves in Figure 3a.

249



250

251 **Figure 2.** Time traces of the long-term evolution of the photon density reflected from the SESAM  
 252 chip (a) and the corresponding spectrum (b). Gain chip current  $i_c=1.2$  mA (unstable operation)

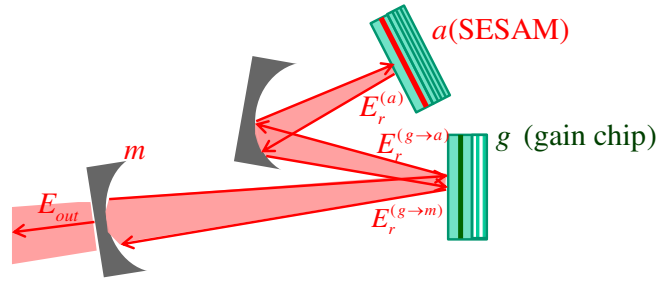
253 As in edge-emitting ML lasers, the spectral signature of this unstable regime is the spectral shape  
 254 (Fig. 4b) less regular and with more envelope modulation than the spectrum of stable ML (Fig. 2b).

### 255 3. VECSEL – SESAM in a folded geometry.

#### 256 3.1. Formulation of delay-differential model for the folded cavity

257 In this section, we consider the case of geometry alternative to the linear one treated in [19] and the  
 258 previous section: the folded geometry. In this case, the three “reference points” of the cavity (Fig. 5)  
 259 are the output mirror ( $m$ ), the gain chip ( $g$ ) and the SESAM ( $a$ ).

260



**Figure 5.** Mode-locked VECSEL –SESAM in folded geometry with an additional mirror.

The purpose of the intermediate fully reflecting mirror is essentially for establishing the correct value of the width ratio  $w'$ .

In the folded cavity designs realised so far [21] it is the gain chip that is located in the middle of the cavity (Fig. 5). In this case, the equation for the SESAM chip remains the same as Eq. (13),

$$T_n^{(a)} \mathcal{G}_{ca} \frac{dE_{rc}^{(a)}}{dt} = -[\delta\tilde{A} + 1 - \mathcal{G}_{ca}] E_{rc}^{(a)} + t_{ia}' t_{ia}' r_{oa} e^{-\alpha_{int,a} L_a} E_{inc}^{(a)}, \text{ and we still have}$$

$$E_{inc}^{(a)} = \sqrt{\gamma w'} E_r^{(g \rightarrow a)} (t - T_{a-g}) \quad (30)$$

where  $T_{a-g} = L_{a-g}/c$  is the flight time between the gain and absorber chips, and  $E_r^{(g \rightarrow a)}$  is the field reflected from the absorber chip in the direction of the gain chip.

For the gain section, the equation is functionally different, taking into account reflections in two directions. For the field leaving the cavity towards the output mirror, we would have

$$T_n \mathcal{G}_c \frac{dE_{rc}^{(g \rightarrow m)}}{dt} = [\delta\tilde{G} + \mathcal{G}_c - 1] E_{rc}^{(g \rightarrow m)} + t_i t_i' r_o e^{-\alpha_{int} L} E_{inc}^{(a \rightarrow g)} \quad (31)$$

The total field measured near the gain chip and propagating towards the output mirror then is

$$E_r^{(g \rightarrow m)} = r_{ia}' E_{inc}^{(a \rightarrow g)} + E_{rc}^{(g \rightarrow m)} \quad (32)$$

For the field leaving the cavity towards the SESAM, we have

$$T_n \mathcal{G}_c \frac{dE_{rc}^{(g \rightarrow a)}}{dt} = [\delta\tilde{G} + \mathcal{G}_c - 1] E_{rc}^{(g \rightarrow a)} + t_i t_i' r_o e^{-\alpha_{int} L} E_{inc}^{(m \rightarrow g)} \quad (33)$$

The total field travelling from the VECSEL gain chip towards the SESAM then is

$$E_r^{(g \rightarrow a)} = r_i' E_{inc}^{(m \rightarrow g)} + E_{rc}^{(g \rightarrow a)} \quad (34)$$

Finally, the field returning to the gain chip from the mirror is

$$E_{inc}^{(m \rightarrow g)} = r_m \gamma_m e^{j\Delta\phi_{g-m}} E_r^{(g \rightarrow m)} (t - 2T_{g-m}) \quad (35)$$

with  $T_{g-m}$  the flight time between the gain chip and the mirror, and the factor  $e^{j\Delta\phi_{g-m}}$  taking into account wavelength-scale cavity length variation. The field returning to the gain section from the SESAM will be the same as in the linear cavity:

$$E_{inc}^{(a \rightarrow g)} = \frac{\sqrt{\gamma_g}}{w'} E_r^{(a)}(t - T_{g-a}) \quad (36)$$

282 The output field at the time  $t$  then is given by

$$E_{out} = t_m \sqrt{\gamma_m} E_r^{(g \rightarrow m)}(t - T_{g-m}), \quad t_m = \sqrt{1 - r_m^2} \quad (37)$$

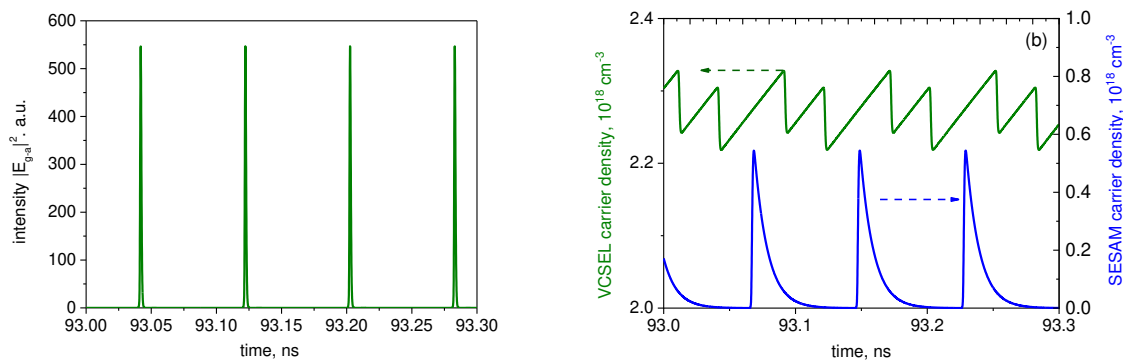
283 For the carriers in the VECSEL gain cavity we have the rate equation identical to (20), but the intensity  
 284 in the cavity is now due to propagation in both directions. In the plane wave approximation and, as in  
 285 [23,24], assuming in this study incoherent addition of the counter propagating signals (applicable given  
 286 a wide enough aperture) the intensity within a thin resonantly positioned gain layer is

$$S_g \approx \left| \frac{1 + r_o}{t_i} \right|^2 \left( |E_{rc}^{(g \rightarrow a)}|^2 + |E_{rc}^{(g \rightarrow m)}|^2 \right) \quad (38)$$

287 3.2. Results of the simulations for the folded cavity.

288

289 Figure 6 illustrates the dynamics of the photon (a) and carrier (b) densities in a short folded cavity,  
 290 with  $T_{g-a} + T_{m-g} = 40$  ps, corresponding to the repetition rate of  $\approx 12.5$  GHz (in the example shown,  
 291  $T_{g-a} = 25$  ps,  $T_{m-g} = 15$  ps). As in the case of the linear cavity, there is only one pulse in the cavity  
 292 per round trip; however the pulse is amplified in the gain chip twice per round-trip, which thus has  
 293 substantially shorter time to recover than in a linear cavity with the same overall length.



294 Figure 6. dynamics of the photon (a) and carrier (b) densities in a short folded cavity

296

297 Figure 7 shows the pulse duration and amplitude for the case of a short folded cavity as functions of  
 298 the gain chip current. As in the linear cavity, and as is typical in all mode-locked semiconductor  
 299 lasers, the pulse duration is somewhat longer for longer absorber recovery time. The current  
 300 dependence of the pulse duration in this case is non-monotonic, decreasing with current at lower  
 301 currents, as predicted also by [19] as well as by early generic theories of ML lasers with weak pulse  
 302 modification per pulse (see e.g. [31]), but increasing at higher currents, when pulse modification per  
 303 pulse is more significant, in common with most edge-emitting ML lasers [32,33]. As in Fig.3, the  
 304 upper limit of the curves is set by the onset of leading-edge chaotic instability.

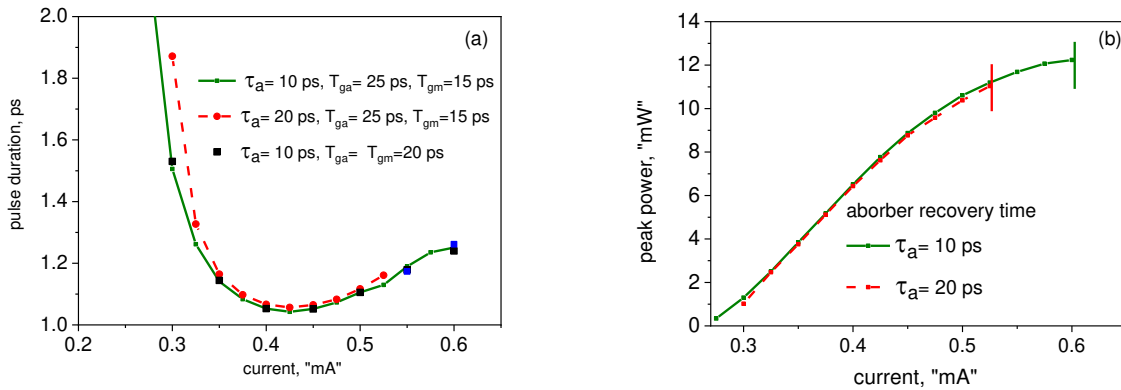


Figure 7. Current dependences of the pulse duration (a) and amplitude (b) in a short folded cavity.

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For the relatively high repetition rate shown in Figures 6-7, the pulse parameters show almost no dependence on the relative length of the two branches of the cavity, so long as  $T_{g-a} + T_{m-g}$  is kept constant (note the rectangular dots in Figure 7a). This is understandable, because given  $T_{g-a}, T_{m-g} \ll \tau_g$ , the recovery of the population inversion in gain chip (strictly speaking, exponential) is virtually linear and so the total depletion of the gain chip by both pulses does not depend on the relative magnitudes of the flight times  $T_{g-a}, T_{m-g}$ . This dependence becomes more pronounced in longer cavities, when the flight times approach  $\tau_g$  by order of magnitude. This is illustrated in Fig. 8, calculated for  $T_{g-a} + T_{m-g} = 200$  ps, or the repetition rate of  $\approx 2.5$  GHz. As seen in the figure, there is an optimal relation of the cavity branch lengths in this case yielding the shortest ML pulse width, which, at least for the values of reflectances studied, corresponds to the gain chip near the middle of the cavity.

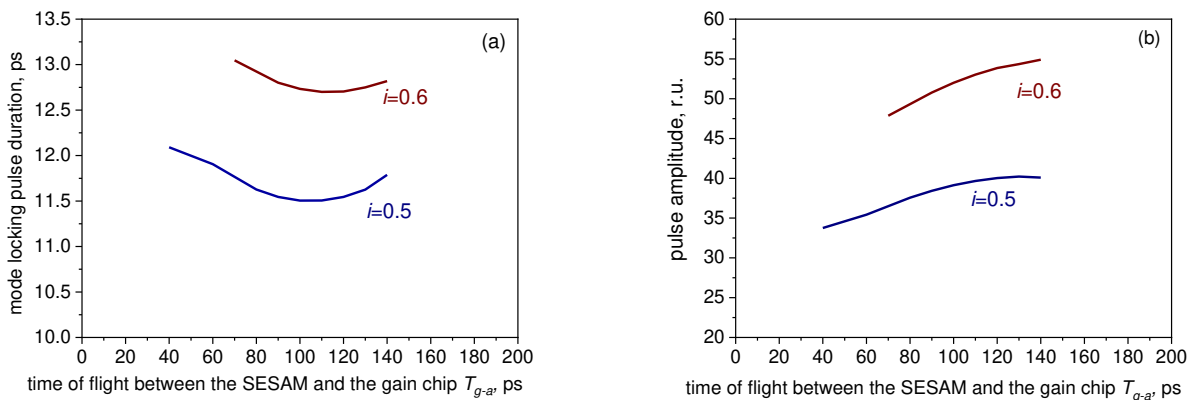


Figure 8. Dependences of the pulse duration (a) and amplitude (b) in a long folded cavity ( $T_{g-a} + T_{m-g} = 200$  ps) for two values of current

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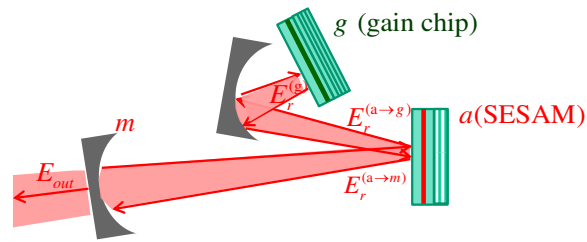
At relatively high currents, close to the onset of the leading edge instability, the geometry affects the stability limits: with the lengths of the branches strongly unbalanced, the gain chip current limit of stability is lower (the extent of the curves in Fig. 8 corresponds to stable single-pulse emission limit).

## 329 4. Colliding-pulse mode locking configuration.

330

### 331 4.1. Time-delayed model

332 We consider last an alternative, and so far hypothetical, case of the central chip being the SESAM, with  
 333 the gain chip and the mirror  $m$  terminating the cavity, which is more difficult to realise (and has not,  
 334 to the best of our knowledge, been realised in this form so far) but offers greater functionality,  
 335 potentially offering Colliding Pulse Mode locking (CPM) option. The equation system for this case  
 336 is obtained from one for the folded cavity with the *gain chip* in the middle by simple permutation of  
 337 the symbols  $g$  and  $a$  in the notations.



338

339 **Figure 9.** Colliding-pulse Mode-locking folded geometry.

340

341 Indeed, in a folded cavity with the SESAM in the “middle”, the equation for the gain chip remains the  
 342 same as eqn. (8)

$$T_{rt} \mathcal{G}_c \frac{dE_{rc}}{dt} = [\delta \tilde{G} - (1 - \mathcal{G}_c)] E_{rc} + t_i t'_i r'_o e^{-\alpha_{im} L} E_{inc}$$

343 and we still have

$$E_{inc} = \frac{\sqrt{\gamma}}{w'} E_r^{(a \rightarrow g)}(t - T_{a-g}) \quad (39)$$

344 where  $T_{a-g} = L_{a-g}/c$  is the flight time between the gain and absorber chips, and  $E_r^{(a \rightarrow g)}$  is the field  
 345 reflected from the absorber chip in the direction of the gain chip.

346 For the SESAM, the equation is functionally different, taking into account reflections in two  
 347 directions. For the field leaving the cavity towards the output mirror, we would have

348

$$T_{rt}^{(a)} \mathcal{G}_{ca} \frac{dE_{rc}^{(a \rightarrow m)}}{dt} = -[\delta \tilde{A} + 1 - \mathcal{G}_{ca}] E_{rc}^{(a \rightarrow m)} + t_{ia} t'_{ia} r_{oa} e^{-\alpha_{im} L} E_{inc}^{(g \rightarrow a)}. \quad (40)$$

349 The total field towards the output mirror then is

$$E_r^{(a \rightarrow m)} = r'_{ia} E_{inc}^{(g \rightarrow a)} + E_{rc}^{(a \rightarrow m)} \quad (41)$$

350 For the field leaving the cavity towards the gain (VECSEL) chip, we have

$$T_{rt}^{(a)} \mathcal{G}_{ca} \frac{dE_{rc}^{(a \rightarrow g)}}{dt} = -[\mathcal{G}_{ca} \delta \tilde{A} + 1 - \mathcal{G}_{ca}] E_{rc}^{(a \rightarrow g)} + t_{ia} t'_{ia} r_{oa} e^{-\alpha_{im} L} E_{inc}^{(m \rightarrow a)}. \quad (42)$$

351 The total field travelling from the SESAM towards the VECSEL chip then is

$$E_r^{(a \rightarrow g)} = r'_{ia} E_{inc}^{(m \rightarrow a)} + E_{rc}^{(a \rightarrow g)}. \quad (43)$$

352 Finally, the field returning to the SESAM from the mirror is



$$E_{inc}^{(m \rightarrow a)} = r_m \gamma_m e^{j\Delta\phi_{a-m}} E_r^{(a \rightarrow m)}(t - 2T_{a-m}) \quad (44)$$

353 whereas the field returning to the SESAM from the gain section will be the same as in a linear cavity:

$$E_{inc}^{(g \rightarrow a)} = \sqrt{\gamma_g} w' E_r^{(g)}(t - T_{g-a}). \quad (45)$$

354 The output field at the time  $t$  then is given by

$$E_{out} = t_m \sqrt{\gamma_m} E_r^{(a \rightarrow m)}(t - T_{a-m}), \quad t_m = \sqrt{1 - r_m^2} \quad (46)$$

355 Inside the SESAM cavity we still have the same rate equation but the fields exist due to propagation in  
356 both directions. In the plane wave approximation and with a thin resonantly positioned absorber,

$$S_a \approx \left| \frac{1 + r_{oa}}{t_{ia}} \right|^2 \left( |E_{rc}^{(a \rightarrow g)}|^2 + |E_{rc}^{(a \rightarrow m)}|^2 \right) \quad (47)$$

357 The absorber saturation fluence in the folded cavity, with either absorber position, would be the same  
358 as in the linear geometry (though effectively in the colliding pulse design it will become twice smaller  
359 with 2 pulses arriving simultaneously), and the threshold condition in the CPM cavity becomes

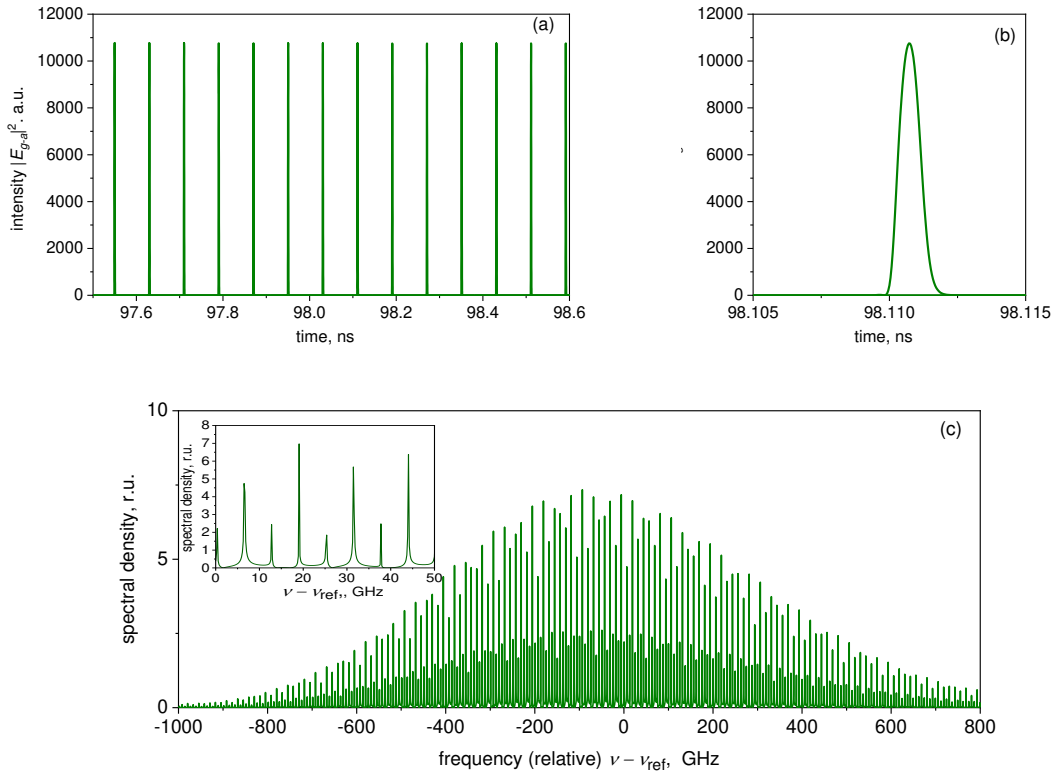
$$g_{th} \approx \frac{1}{\Gamma L} (G_{th} - 1) \approx \frac{1}{\Gamma L} \left( \frac{(1 + r_i r_m r_{oa}^2 \gamma_m)(1 + \alpha_{int} L)}{(r_m r_{oa}^2 \gamma_m + r_i) r_o} - 1 \right). \quad (48)$$

360 **4.2. Calculations and results.**

361 Fig. 10 presents the schematic of evolution of the output photon density in a CPM configuration (a,  
362 b) and the corresponding spectrum (c), for one operating current in a short ( $T_{g-a} = T_{m-g} = 40$  ps)  
363 cavity. The time-domain pulse trail (Fig. 10a) shows complete repetition frequency doubling  
364 compared to a linear cavity (two pulses per round trip; note the identical amplitudes of adjacent  
365 pulses and the repetition period same as in Figure 2 despite a twice longer cavity), as expected for  
366 well developed CPM operation. However, in the spectrum, the doubling of frequency interval  
367 between modes is not complete; intermediate modes corresponding to the round-trip of the entire  
368 cavity are somewhat suppressed but still present (see inset to Fig. 10c). Similar performance was  
369 simulated for edge emitting mode locked lasers under certain conditions [34].

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**Figure 10** Mode-locked VECSEL-SESAM-output mirror structure time traces of  $|E_{g-a}|^2$ : (a) pulse trail and (b) single pulse. (c) shows the spectrum of the time trace of (a), with a fragment in an inset. Injection current is  $j=1.1$  mA; absorber recovery time  $\tau_a=10$  ps.

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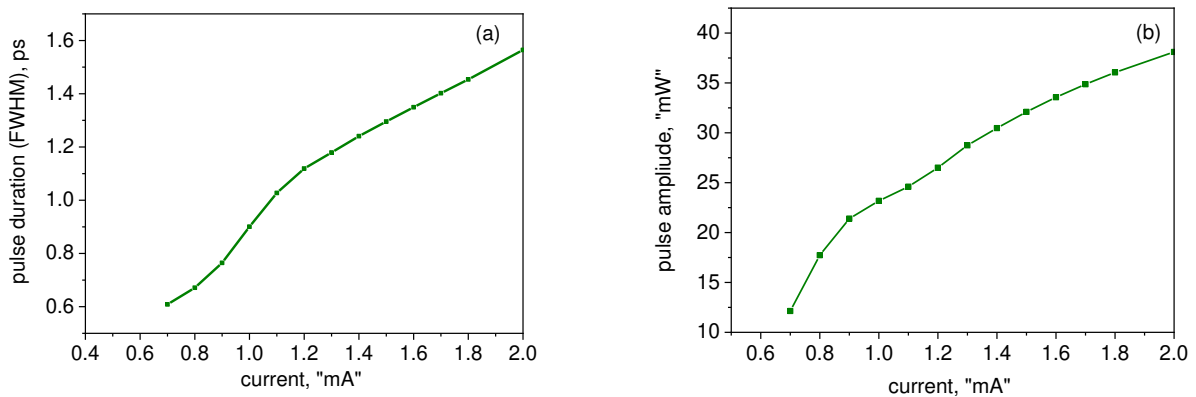
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Figure 11 shows the evolution of the CPM pulse duration and amplitude with current. In this geometry, the simulated pulsewidth increases with current through the current range studied; as mentioned above this is typical for edge emitting lasers and can be associated with relatively strong modification of the pulse per round trip. In edge emitters, such a situation is associated with an asymmetric shape with the longer trailing edge, which indeed is observed also in our simulations (Figure 10b).



383

384

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**Figure 7.** Current dependences of the pulse duration (a) and amplitude (b) in the CPM design.  $\tau_a=10$  ps

386 The upper extent of the curve, as in Figures 3 and 7, is the onset of the leading edge instability. The  
387 stability range for this design, with two pulses saturating the absorber simultaneously, is substantially  
388 higher than in the case of a simple linear cavity with the same repetition rate, and the pulses, shorter,  
389 by virtue of more efficient absorber saturation, which is one of advantages of CPM [32]. The cavity  
390 thus looks suitable, in principle, for picosecond pulse generation. In the case of the femtosecond  
391 regime using a low-dispersion optically pumped active chip, the more complex, but also more reliable  
392 ring CPM geometry used in recent studies [35] may be preferable (the folded cavity studied here  
393 would need micrometer-scale balancing of subcavity lengths). More detailed investigation is  
394 reserved for future work.

## 395 5. Conclusions

396 We have presented a simple, versatile model for the dynamics of electrically and optically  
397 pumped vertical-external-cavity surface-emitting lasers mode locked by semiconductor saturable  
398 absorber mirror. Time delays in the external cavities formed by the VECSEL gain chip and the  
399 saturable absorber mirror and output mirror are accounted for. Analytical expressions for the  
400 experimentally accessible characteristics of the system are provided, namely, the threshold gain and  
401 saturation fluence and reflection contrast of the absorber. For realistic parameters of the  
402 semiconductor cavities, the model predicts fundamental mode locking with ps pulse duration. The  
403 dependences of the pulse width and pulse amplitude, as well as the frequency chirp are investigated  
404 as a function of injection current. The model is easily generalized for different VECSEL and SA  
405 configurations and examples for the case of folded geometry with central chip being either the gain  
406 section or the SESAM are presented. Future work can concentrate on perfecting the model for the  
407 folded cavities, including account of partly coherent addition of signals as in [35] and possibly the  
408 lateral effects, as well as polarization properties and more diverse geometries.  
409

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411 the simulations. Both authors participated in the writing, review and editing the manuscript.

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415 **Conflicts of Interest:** The authors declare no conflict of interest.

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