DURABILITY ANALYSIS OF CONCRETE BRIDGE DECK EXPOSED TO THE CHLORIDE IONS USING DIRECT OPTIMIZED PROBABILISTIC CALCULATION

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Abstract. *Durability of reinforced concrete structures is a deeply discussed problem recently. Concrete structures in the external environment are very often affected by chloride ions from de-icing salt or sea water. Chloride ions penetrate through the concrete cover layer of the reinforcement and can cause eventually the corrosion of the steel. However, when estimating the durability of the structure, it is not sometimes possible to express the parameters by constant values; therefore, the probabilistic methods come in handy. Then, the variability of inputs and outputs can be expressed by histograms. Two probabilistic approaches were applied in this task – Monte Carlo simulation with Simulation-Based Reliability Assessment method, which is widely used for such type of problems, and the Direct Optimized Probabilistic Calculation, which is still relatively new type of approach. The result is a comparison of mentioned methods in terms of accuracy on the model of one-dimensional chloride penetration with time independent diffusion coefficient by using the Fick's Second Law of Diffusion.*

Keywords

Corrosion initiation, chloride ions, Direct Optimized Probabilistic Calculation, DOProC, durability, Fick's Second Law of Diffusion, Monte Carlo simulation, numerical precision, probabilistic calculation, reinforced concrete, reliability, Simulation-Based Reliability Assessment.

1. Introduction

The reliability and durability of reinforced concrete (*RC*) structures are significantly affected by the harsh environment conditions. Such an example is the de-icing salt, that causes the chloride ingress into concrete structures and hence initiating of the reinforcement corrosion. This led to the uttermost necessity of paying attention to prediction of the durability when designing such kinds of structures [1], [2], [3].

In this task, an ideal non-cracked RC bridge deck made of ordinary Portland cement was analysed. Since the concrete is a heterogeneous material and other input parameters (depth, chloride threshold etc.) can be also described as variable values, histograms are used to characterize their randomness [4], [5], [6]. Result of the probabilistic calculation is usually a qualitative description of the degree of reliability (probability of failure P_f or index of reliability β). The probabilistic approaches (unlike the deterministic ones) can effectively capture the effects of random interactions of interrelated variables [7], [8].

The aim of this project is to determine the probability of corrosion initiation using the Direct Optimized Probabilistic Calculation (*DOProC*), which is based on numerical integration [9]. The outcome is compared to results calculated by Simulation-Based Reliability Assessment (*SBRA*) method with Monte Carlo (*MC*) simulation, which is based on the process of random sampling [10]. To make a comparative assessment of durability analysis, the probability of failure is computed in selected points of service lifetime.

It is also possible to assess the probability of failure based on the design probability of failure P_d, but the value depends on studied problem. The corrosion initiation caused by chloride penetration is considered as an exceedance of a serviceability limit state, so it does not affect the structure in catastrophic consequences. Recommended value of failure probability for SLS is $P_{d,SLS} = 7\times10^{-2}$ [16], for ordinary construction with life-span of 50 years, the recommended failure probability is

$P_{d,SLS}$ = 10^{-2} [11]. According to [27][28] the proposed design value of failure probability is $P_{d,SLS} = 10^{-1}$ in case of depassivation caused by chloride penetration or carbonation and possibility of corrosion.

2. Durability analysis

If the main parameter of the durability of a structure is chloride penetration into the concrete, the service life of that structure can be formulated as:

$$t_{service} = t_{initiation} + t_{propagation}, \qquad (1)$$

where $t_{initiation}$ is time when chloride ions penetrate the cover layer before the corrosion starts and $t_{propagation}$ is time before the corrosion of the reinforcement is on the critical level [12]. The initiation of corrosion depends on many factors – diffusion characteristics, depth of the reinforcement, chloride concentration on the surface of the concrete, temperature, chloride content on the surface of the reinforcement and so on. The propagation phase of the corrosion describes the period at which the reliability of the structure is reduced because of the gradual metamorphosis of the reinforcement due to the corrosive agents.

The chloride penetration through the concrete can be generally modelled as a function of depth and time using Fick's Second Law of Diffusion [2] – eq. 2, because the diffusion is the predominant transport mechanism of chloride into concrete [27].

$$\frac{\partial C(x,t)}{\partial t} = D_c \frac{\partial^2 C(x,t)}{\partial x^2}, \qquad (2)$$

where $C(x,t)$ (% weight of cement) is the chloride ion concentration at a distance x (m) from the surface of the concrete in time t (s); D_c (m^2/s) is effective diffusion coefficient, which characterizes the concrete ability to withstand the penetration of chlorides.

Solution of the differential equation (2) with boundary conditions can be formulated by Crank's solution – eq. (3) [13] – one-dimensional model with time independent diffusion coefficient. However, this model represents the simplification of the reality, for the estimation of the real concentration is necessary to consider also other factors (time dependency of diffusion parameter and surface chloride concentration etc.) [27].

$$C(x,t) = C_0\left\{1 - erf\left(\frac{x}{\sqrt{4 \cdot D_c \cdot t}}\right)\right\}, \qquad (3)$$

where C_0 (% weight of cement) is concentration of the chloride at the surface of the concrete, erf is the error function complement.

Numerical solution can be expressed also as the polynomial solution:

$$C(x,t) = C_0\left\{1 - \frac{2}{\sqrt{\pi}}\sum_{n=0}^{14}\frac{(-1)^n\left(\frac{x}{\sqrt{4 \cdot D_c \cdot t}}\right)^{2n+1}}{n!\,(2n+1)}\right\}, \qquad (4)$$

where n is the number of the polynomial set. According to [14] 14 members of the set are enough for solving this kind of problem.

Values of chloride concentration resulted from the analysis is then compared to the chloride threshold C_{th}, which is a concentration of chloride ions needed to corrosion initiation, to determine whether the corrosion could start or not. It is important to note that C_{th} depends on type and the preparation of reinforcement, on the concrete components and also on the pH of concrete [26]. The reliability function can be expressed by eq. 5 [15]:

$$RF_t = C_{th} - C(x,t), \qquad (5)$$

Probability of failure is the total number of situations of which the concentration of chlorides exceeded the chloride threshold:

$$P_{f,t} = P(RF_t < 0) = P((C_{th} - C(x,t)) < 0). \qquad (6)$$

3. Probabilistic analysis

3.1. Input parameters

Input parameters, that characterize the ideal reinforced concrete bridge deck used in this task, were taken over from a previous research – Table 1, all of them are originally continuous histograms. The distributions of the input parameters are shown in the following Figures 1, 2, 3, 4 and 5.

Tab.1: Input parameters for ideal RC bridge deck.

Parameter	Range	Description
Diffusion coefficient, D_c (m^2/s)	<0; 26> ×10^{-12}	_DiffPA.dis [3], 104 bins
Surface chloride content, C_0 (% wgt/c)	<0.21; 1.63>	c_0V.dis [18], 6 bins
Reinforcement depth, x (m)	<0.04; 0.11>	XDEPTH3.dis [5], 29 bins
Chloride threshold, C_{th} (% wgt/c)	<0.091; 0.505>	thr_b.dis [4], 12 bins

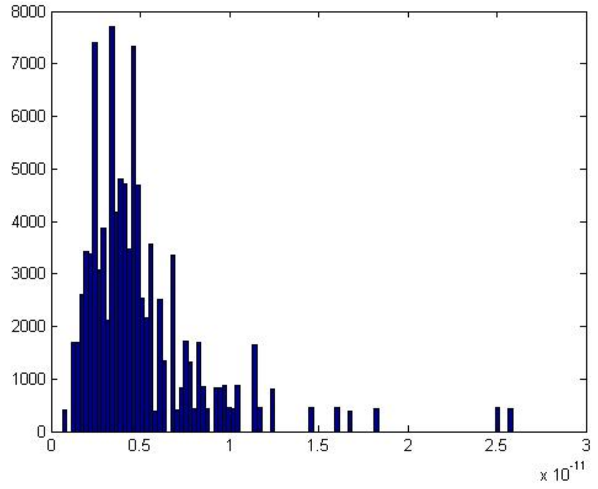


Fig. 1: Histogram of Diffusion coefficient, D_c (m^2/s) [3].

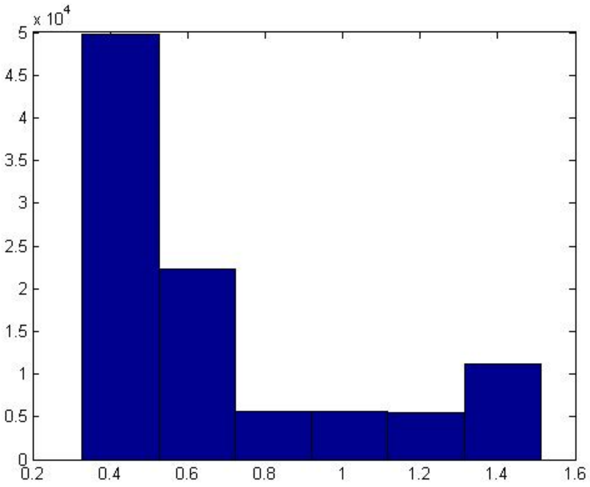


Fig. 2: Histogram of Surface chloride content, C_0 (% wgt/c) [18].

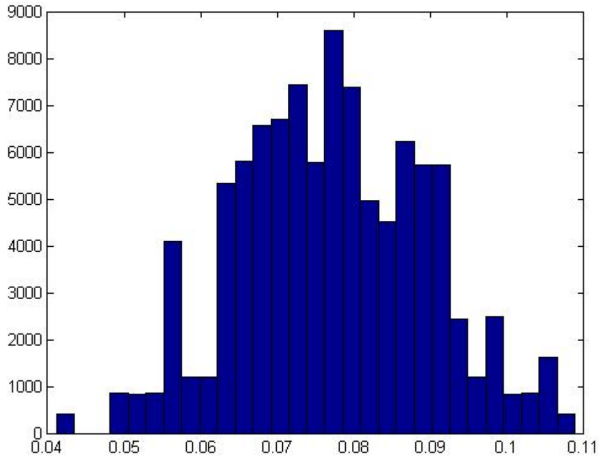


Fig. 3: Histogram of Reinforcement depth, x (m) [5].

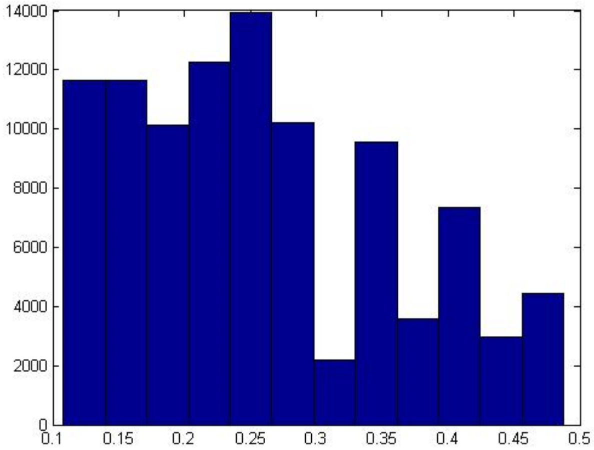


Fig. 4: Histogram of Chloride threshold, C_{th} (% wgt/c) [4].

3.2. Monte Carlo simulation

Monte Carlo method is a technique based on a process of numerical simulation using randomly generated values. The method is easy to use for solution of reliability problems, but in the elementary form can be less effective due to time-consuming computation. Probabilistic analyses are carried out using SBRA method under the scheme of MC simulation to characterize random variables by bounded histograms [17].

The simulation process was repeated 30 times with 10^5 random simulations in order to address the variability of evaluated probability of failure itself. The final probability is expressed as the mean value and expected range (± 3 × standard deviation). Eq. (8) shows that the number of samples is sufficient for required probability, i. e. the range of the inaccuracy of the final failure probability should not be more than 1% in comparison with the exact solution [19].

$$v_{pf} = \frac{1}{\sqrt{N \cdot p_f}} = \frac{1}{\sqrt{10^5 \cdot 10^{-1}}} = 0.01 = 1\% \qquad (8)$$

3.3. Direct Optimized Probabilistic Calculation

Direct Optimized Probabilistic Calculation - DOProC method, is a purely numerical method using no simulation or approximation techniques [20]. The approach is effective for structural assessment of reliability and other probabilistic calculations. Random variables can be entered as empirical or parametrical distribution via histograms. Reliability function can be expressed analytically or numerically. The number of intervals (bins) of each histogram is extremely important for the number of required numerical operations and computing time, but the optimizing tools can be used to reduce the number of numerical operations [21].

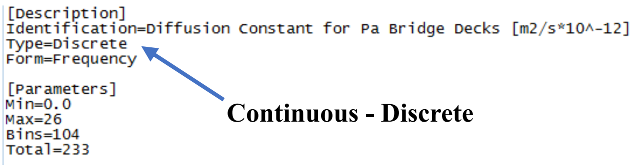
4. Comparison of both methods

There are several distinctions when using the DOProC for calculating of the failure probability in comparison with MC method. These problems must be taken into account before comparison of the resulted failure probability.

4.1. Adjustments before the calculation

4.1.1. Adjustment of the histograms

DOProC uses only discrete or pure discrete histograms [9]. Thus, it is necessary to transform the inputs, which are continuous histograms to discrete histograms. This has been done by rewriting of the type of the histograms in their source code from “continuous” to “discrete” – Fig. 5.



```
[Description]
Identification=Diffusion Constant for Pa Bridge Decks [m2/s*10^-12]
Type=Discrete
Form=Frequency

[Parameters]
Min=0.0
Max=26
Bins=104
Total=233
```

Continuous - Discrete

Fig. 5: A part of the source code of the histogram.

4.1.2. Number of bins

The resulting problem of the discretization of inputs is the appropriate number of intervals (bins) of the histograms. The more bins the histogram has, the more precise but also more computationally difficult the DOProC method is. In this task, calculations were carried out with the original number of bins (as shown in Table 1) and then with approximately 100 bins in every input. The diffusion coefficient has originally about 100 bins in the source code, so for the last series of calculation, bin number of diffusion coefficient was doubled to examine the effect on the results (Table 2). This was done because of the unexpected result in 10th year of service life, as shown below.

Tab.2: Examined numbers of bins in DOProC.

Parameter	Number of bins		
	I.	**II.**	**III.**
Diffusion coefficient, D_c (m^2/s)	104	104	208
Surface chloride content, C_0 (% wgt/c)	6	96	96
Reinforcement depth, x (m)	29	116	116
Chloride threshold, C_{th} (% wgt/c)	12	96	96

4.1.3. Selecting values from intervals

The next problem is that DOProC technique choses mean values of the given interval for calculation [9] but used MC algorithm [22] computes with random values within each bin. Therefore, the MC algorithm was modified in order to select also mean value of the interval.

4.2. Calculation in Probcalc

The DOProC analysis was conducted in a program called ProbCalc [23], specially developed for probabilistic calculations computed by this method.

The first attempt to calculate the probability of failure in ProbCalc has been done by entering the 14 members of the polynomial set to the workspace of the software, because the error function complement is not in the basic program offers. Although it is possible to enter the set to the workspace, the calculation takes a long time and the result is affected by the division with values close to zero (the program refuses to even do the calculation). Because of this problem, it was necessary to program the algorithm of polynomial solution with 14 members in Delphi [24] and use it as a dynamic-linked library. This optimization speeded up the calculation from about an hour to a few seconds, even when the number of bins was increased.

4.3. Results

Fig. 6 shows the comparison of probabilities of failure computed by the aforementioned methods. The dotted lines represent the expected range of results computed by MC simulation with 10^5 steps. Then, the blue, red and green lines represent the probability of failure computed with different number of bins in histograms.

It is worth noticing that probability of failure becomes more accurate with longer time of chloride exposure, as is better shown in Fig. 7. The numerical issues with precision are significant up to 20 years of chloride penetration as can be seen bellow based on the difference between the data points from reference Monte Carlo simulation and DOProC. The improvement of the DOProC precision is related to the number of applied bins. The results with the increased number of bins almost agreed with reference Monte Carlo simulation while the original number of bins agreed at the age of 30 years.

However, an unexpected value appeared in the 10th year of service life - the zero probability of failure. Because of this information, verification is in need. Towards this end, deterministic calculation was done for the minimum and the maximum values of each histogram to get the results as simple numbers. The results computed in Matlab [25] by eq. (3) and eq. (4) were compared. It is observed that for the minimum mean values, the concentrations of chlorides $C(x,t)$ in the 10th year are not the same, as can be seen on Fig. 8. The reason could be the deficient number of the polynomial set. For the maximum mean values the results are identical (Fig. 9).

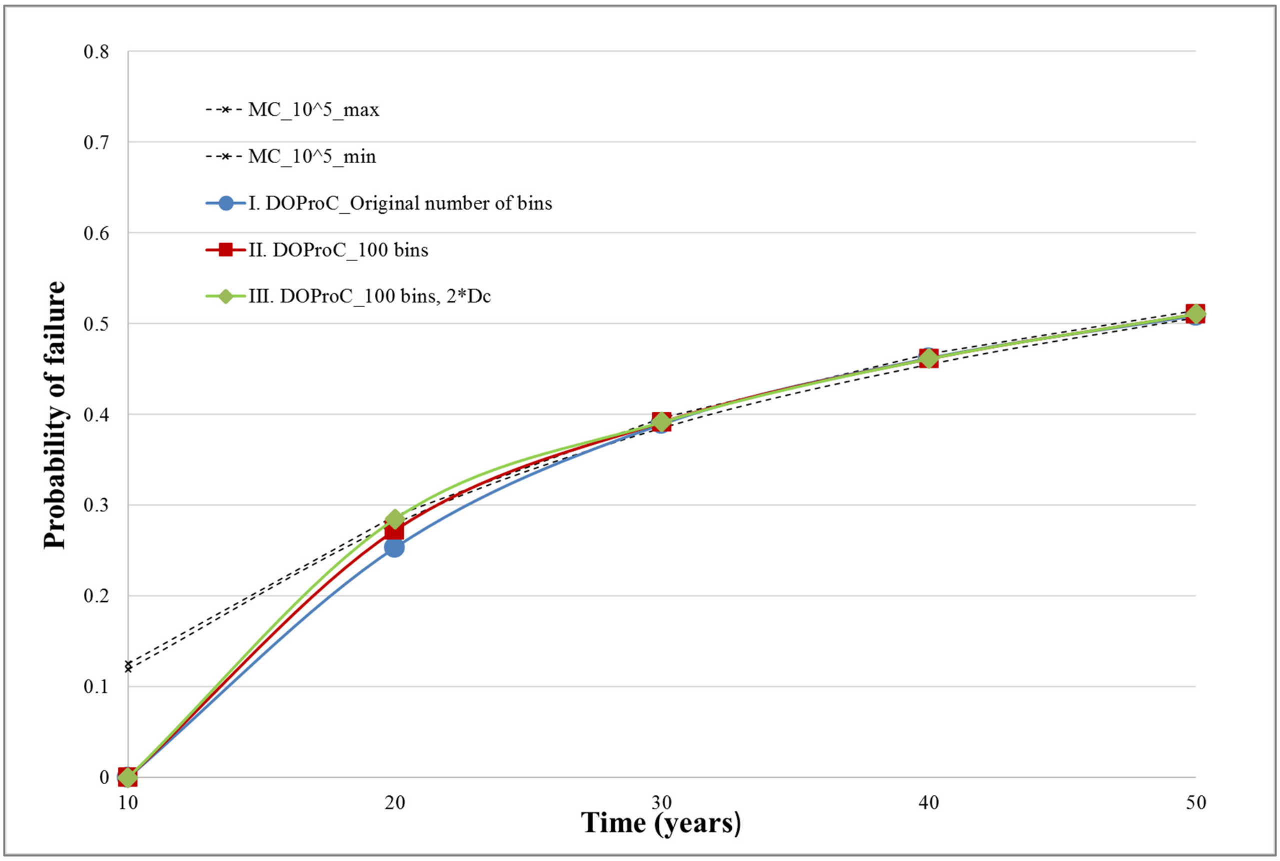


Fig. 6: Comparison of probabilities of failure – MC method and DOProC method

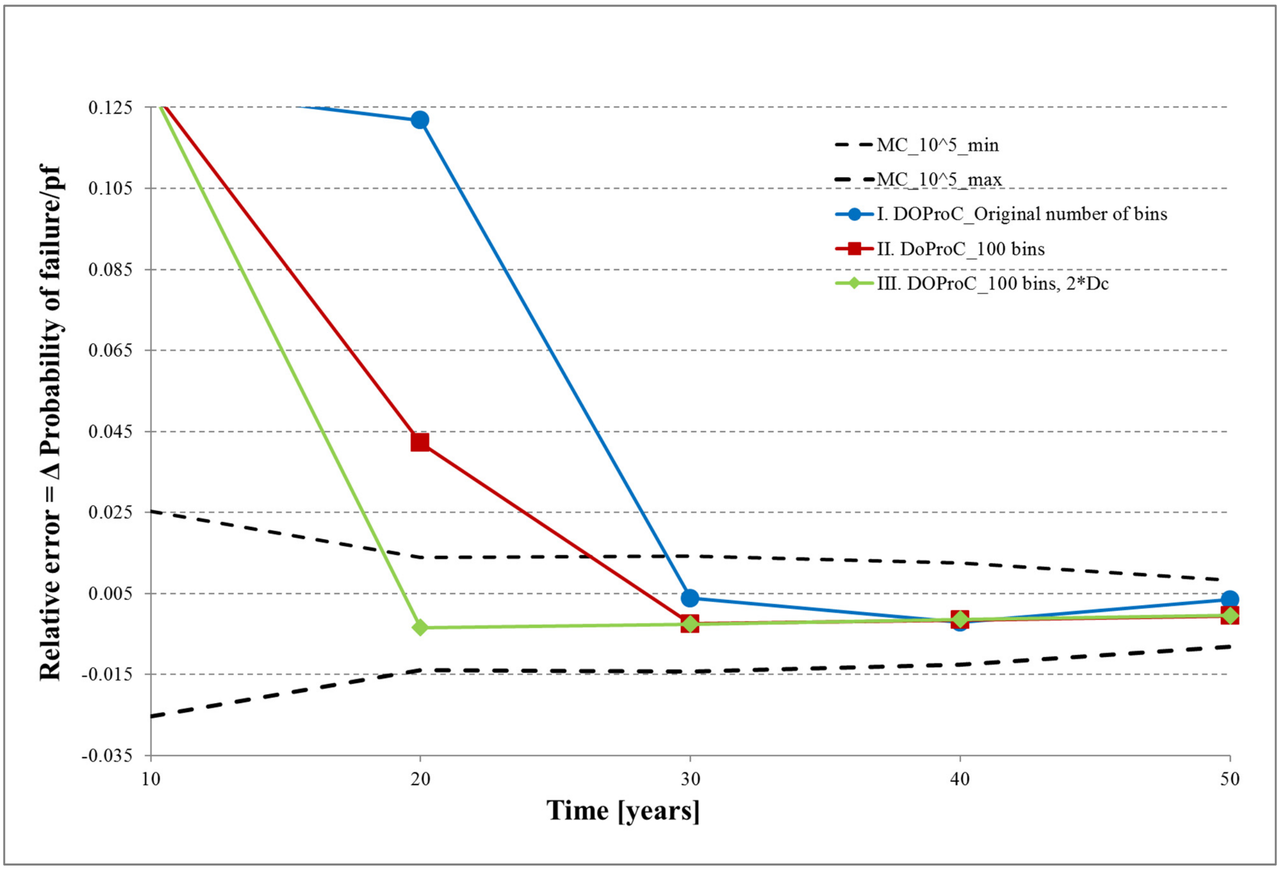
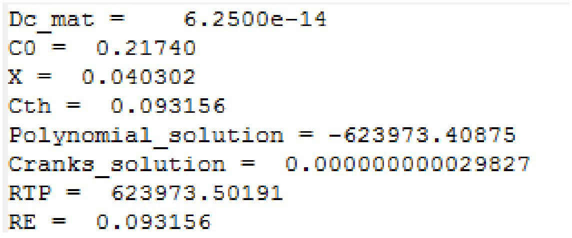
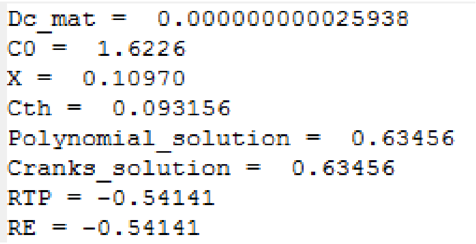


Fig. 7: Relative error depending on the age of the structure



```
Dc_mat =    6.2500e-14
C0 =  0.21740
X =  0.040302
Cth =  0.093156
Polynomial_solution = -623973.40875
Cranks_solution =  0.000000000029827
RTP =  623973.50191
RE =  0.093156
```

Fig. 8: Results of the eq. (3) and eq. (4) computed with the minimum mean value of the first bin of every input.



```
Dc_mat =  0.000000000025938
C0 =  1.6226
X =  0.10970
Cth =  0.093156
Polynomial_solution =  0.63456
Cranks_solution =  0.63456
RTP = -0.54141
RE = -0.54141
```

Fig. 9: Results of the eq. (3) and eq. (4) computed with the maximum mean value of the last bin of every input

It was observed that the numerical problem of minimum values of histogram solved by polynomial set (eq. 4) occurs also in MC method but due to the large number of simulation steps, it does not affect the results in such an extent.

5. Conclusions

Probability-based durability assessment of an ideal non-cracked RC bridge deck was done using DOProC technique and compared to the reference results of MC simulation method. Chloride diffusion was modelled by Fick's Second Law of Diffusion. One-dimensional model with time independent diffusion coefficient was used for calculation of the chloride ions concentration at the reinforcement level. Resulted values of chloride concentration were compared to the chloride threshold to determine if the limiting amount of chloride ions was exceeded. Probability of failure was computed at five different service lifetimes: 10, 20, 30, 40 and 50 years.

As the construction becomes older, the probability of failure computed by DOProC method became closer to the results computed by MC approach. DOProC in comparison with MC simulation shows different results of the chloride concentration $C(x,t)$ in the first two decades of the lifetime of the construction. This can be caused by the insufficient number of bins in the input histograms or number of the polynomial series. By increasing of the number of bins, the probability of failure became more precise in the 20th year, but it has not affected the probability od failure in the first decade of service life.

It is possible that the number of the polynomial set can affect the reliability assessment when using different concrete mixtures with very low diffusion coefficient. This consideration may help address the issues related to the corrosion initiation at early ages of the structure.

It needs to be noted that new update of ProbCalc software with improved precision was prepared at the end of the year 2018, thus, the completion of the effect of the precision update shall be evaluated as well. DOProC method can be successfully applied to a probabilistic calculation, but if a new type of complex task is solved, it is always necessary to compare the results to other method.

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