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Performance Analysis of a Polling Model with BMAP and Across-Queue State-Dependent Service Discipline

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ABSTRACT As various video services become popular, video streaming will dominate the mobile data traffic. The H.264 standard has been widely used for video compression. As the successor to H.264, H.265 can compress video streaming better, hence it is gradually gaining market share. However, in the short term H.264 will not be completely replaced, and will co-exist with H.265. Using H.264 and H.265 standards, three types of frames are generated, and among different types of frames exist dependencies. Since the radio resources are limited, using dependencies and quantities of frames in buffers, an appropriate time division transmission policy can be applied to transmit different types of frames sequentially, in order to avoid the occurrence of video carton or decoding failure. Polling models with batch Markovian arrival process (BMAP) and across-queue state-dependent service discipline are considered to be effective means in the design and optimization of appropriate time division transmission policies. However, the BMAP and across-queue state-dependent service discipline of the polling models lead to the large state space and several coupled state transition processes, which complicate the performance analysis. There have been very few researches in this regard. In this paper, a polling model of this type is analyzed. By constructing a supplementary embedded Markov chain and applying the matrix-analytic method based on the semi-regenerative process, the expressions of important performance measures including the joint queue length distribution, the customer blocking probability and the customer mean waiting time are obtained. The analysis will provide inspiration for analyzing the polling models with BMAP and across-queue state-dependent service discipline, to guide the design and optimization of time division transmission policies for transmitting the video compressed by H.264 and H.265.

INDEX TERMS Across-queue state-dependent, Batch Markovian arrival process, H.264/H.265, polling model, video streaming.

I. INTRODUCTION

WITH the fast development in the fields of mobile communications and Internet of Things (IoT) technologies, the number of network access devices is increasing dramatically. Ericsson's mobility report [1] reveals that, by the end of 2024 there will be 8.9 billion mobile subscriptions, excluding the cellular IoT connections and fixed wireless access subscriptions. Moreover, the video services are becoming common, such as video part of most online content (news, ads, social media, etc.), video anywhere and anytime, emerging immersive media formats and applications (360-degree video, AR, VR), etc. In the future, the video streaming will dominate the mobile data traffic. Ericsson's mobility report [1] reveals that, in 2024 video will account for around 74% of mobile data traffic. Thus, it is worth improving the quality of video services in mobile networks with limited

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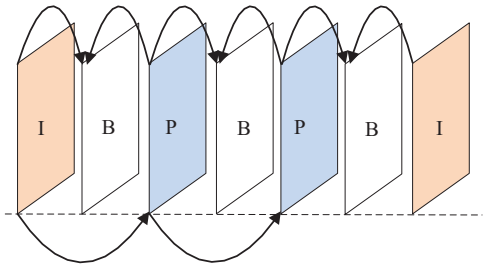

FIGURE 1. Diagram of the relationship among I-frame, P-frame and B-frame.

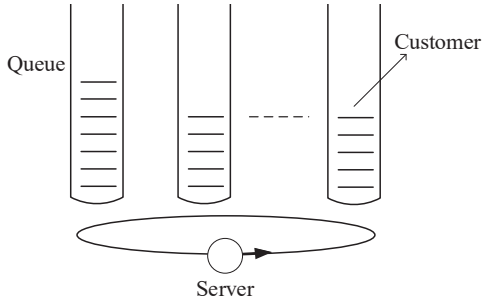

FIGURE 2. Structure of the classical polling model.

radio resources.

The H.264 video compression standard was developed in 2003, and it is suitable for encoding High Definition (HD) video (1920×1080 resolution or higher) [2]. Up till now, H.264 has been widely used for video compression. In 2013, the video compression standard H.265 was developed as the successor to H.264. H.265 has better compression for video streaming, and it is suitable for high-resolution compression such as 4K (4096×2160 resolution) and 8K (7680×4320 resolution) [3]. Compared with H.264, H.265 can save up to 50% of bandwidth and storage for the same video quality. Nevertheless, the H.265 standard needs more resources to decode or encode. In view of the cost and user's necessary needs, H.264 will not be completely replaced in the short term, and it will co-exist with H.265. Using H.264 and H.265 standards, three types of frames are generated in video compression, as I-frame (intra frame), P-frame (predictive frame) and B-frame (bi-predictive frame). As shown in Fig. 1, I-frames are complete pictures and don't require other types of frames to decode; P-frames require I-frames and other P-frames to decode; B-frames require I-frames and P-frames to decode. If one I-frame is lost, some P-frames and B-frames may not be decoded accurately; if one P-frame is lost, some B-frames and other P-frames may not be decoded accurately.

In wireless networks, some devices and data centers are responsible for transmitting the collected or stored video data to the destination. For example, the cameras transmit the collected video data to the processing center through wireless networks, and the cloudlet/fog is deployed on the edge of wireless networks to transmit the stored video to the requesting user. The video data can be compressed by H.264 and H.265 in the upper layer. Due to the dependencies among the generated frames and the limited radio resources, different types of frames can be assigned different transmission priorities, as I-frame $>$ P-frame $>$ B-frame, and they are arranged in different buffers of the data link layer. An appropriate time division transmission policy is implemented in the scheduler to transmit different types of frames orderly.

Polling models are considered to be effective means in the design and optimization of time division transmission policies. The classical polling model consists of one server and several queues (see Fig. 2), and the server renders services to customers in different queues, according to the given polling and service discipline. For the key concepts and performance measures, the correspondence between the polling model and the time division transmission policy is shown in Table 1. In this paper, we study the polling model and its performance analysis method which can be used in the design and optimization of time division transmission policies for transmitting the video compressed by H.264 and H.265.

A. RELATED WORKS

Up to now, various polling models have been analyzed. These polling models can be divided into two categories, i.e. ones with intra-queue autonomous service discipline and ones with across-queue state-dependent service discipline, according to whether the service discipline attached to one queue depends on the states of other queues.

1) Polling models with intra-queue autonomous service discipline

In this type of polling models, the service discipline attached to each queue is independent of the states of other queues. For each queue, the common used service disciplines are exhaustive, gated, number-limited, time-limited and their variations. Some examples are listed in the following.

In [4], Sikha and Manivasakan analyzed the polling model which consists of one server and two finite-buffer queues. The service disciplines attached to the two queues both are number-limited, and the limited number for the first queue is more than the one for the second queue. Customers arrive at the first queue according to the Poisson process, and the second queue is assumed to be saturated. The steady state queue length distribution of the first queue at server departure epochs was analyzed, and its calculation formula was obtained. In [5], Winands et al. analyzed the polling model which consists of one server and two infinite-buffer queues. The server alternately attends the two queues, and the service disciplines attached to the first and second queue are respectively exhaustive and number-limited. The customer arrival processes of the two queues are independent Poisson processes. The probability generating functions of the joint and marginal queue length distributions both at service completion epochs and at arbitrary instants were obtained, and they are the basis of calculating the mean queue lengths.

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TABLE 1. The correspondence between the polling model and the time division transmission policy

Key concepts and performance measures of the polling model	Corresponding elements of the time division transmission policy
Customer	Frame
Queue	Buffer
Customer arrival process	The process under which the corresponding frames enter into the buffer
Server	Transmission unit
Polling and service discipline	Transmission discipline
Queue length	The number of frames in the corresponding buffer
Customer waiting time	The time from the instant when the corresponding frame enters into the buffer to the instant when the frame transmission is finished
Customer blocking probability	Frame blocking probability due to the buffer overflow

In [6], Boxma et al. analyzed the polling model which consists of one server and multiple infinite-buffer queues. The server attends these queues periodically according to a general service order table, and the queues with higher priority are attended frequently. Each queue is attached to one of the three service disciplines, including exhaustive, gated and number-limited (1-limited). For each queue, the numbers of arrivals in every time slots are independent and identically distributed random variables. The pseudoconservation law for this model was derived, and it can be used to obtain approximations for individual mean waiting times. In [7], van Wijk et al. analyzed the polling model which also consists of one server and multiple infinite-buffer queues. The server attends these queues in a cyclic manner, and each queue is served according to the multigated service discipline. The customer arrival processes of the queues are independent Poisson processes. The mean visit time of each queue, the pseudoconservation law, the distribution of waiting times and the mean waiting times were derived.

In [8], Saffer and Telek presented a unified analysis method for the cyclic polling model which consists one server and multiple infinite-buffer queues. The server is entitled to serve the queues in a cyclic manner, and the service disciplines attached to the queues have the following properties: memoryless property, work-conservation property, non-preemptive service property, determination property. The most commonly known disciplines, such as exhaustive, gated, binomial exhaustive, binomial-gated, non-exhaustive, semi-exhaustive, limited-N and non-preemptive limited-T, all satisfy the above properties. The customer arrival processes of the two queues are independent batch Markovian arrival processes (BMAPs) [9].

2) Polling models with across-queue state-dependent service discipline

For this type of polling models, there is at least one queue whose service discipline depends on the states of other queues. Some examples are listed in the following.

In [10], the polling model consists of one server and two infinite-buffer queues. The first queue is served according to the exhaustive service discipline until it is empty, at this time, if the second queue is not empty, the server switches to the second queue. During the service time of the second queue, (a) if the second queue is not empty while the number of customers in the first queue exceeds a certain threshold, the server switches to the first queue immediately; (b) if the second queue is empty while the number of customers in the first queue does not exceed a certain threshold, the server still switches to the first queue. The customer arrival processes of the queues are independent Poisson processes. The joint queue length distribution was determined.

In [11], [12], the polling models consist of one server and three infinite-buffer queues. The first queue is served according to the exhaustive service discipline until it is empty, then the server switches to the second queue. During the service time of the second queue, if there exist some customers arriving at the first queue, the service in progress (if any) is interrupted and the server switches to the first queue immediately; otherwise, the second queue is served until it is empty, and then the server switches to the third queue. During the service time of the third queue, if there exist some customers arriving at the first queue, the service in progress (if any) is interrupted and the server switches to the first queue immediately; otherwise if the number of customers in the second queue exceeds a certain threshold, the service in progress (if any) is interrupted and the server switches to the second queue immediately, otherwise the third queue is served until it is empty, and then the server switches to the second queue. In the second and third queue, the interrupted service will be started from beginning again in the next cycle. For each queue, customers arrive independently according to the Poisson process. In [11], the exact heavy-traffic limits of the polling system were derived first, and then based on these results, an approximation of the tail asymptotics of the stable queues was provided, which describes the heavy-traffic behaviors more distinctly. In [12], the stationary joint queue length distributions were derived, and then the behaviors in the light-traffic and heavy-traffic scenarios were presented, finally, interpolation approximations of the mean sojourn times were provided.

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In [13], Cao and Xie proposed a cyclic polling model with BMAP and across-queue state-dependent service discipline, and analyzed its stability. In this polling model, there are one server and two infinite-buffer queues. The customers arrive at the two queues according to two independent BMAPs; the server is entitled to serve the two queues in a cyclic manner. The customers in the first queue have the higher service priority, and they are served according to the gated service; the customers in the second queue have the lower service priority, and they are served according to the across-queue state-dependent time-limited service discipline. As the length of the first queue increases, the mean predetermined service time of the second queue either decreases or remains the same.

For more polling models, see [14]–[21] and references therein. According to the existing literature, in the current researches on polling models, the customer arrival processes are mostly assumed to be the Poisson processes. For the Poisson arrival process, the customers arrive independently, and the inter arrival times are independent and identically distributed exponential random variables. Hence, the Poisson arrival process can't effectively describe the arrival characteristics of the video streaming that have correlated frames. Fortunately, the BMAP can capture the batch, correlated and bursty nature of the video streaming [22]–[25]. Moreover, it includes the Poisson process, the PH-renewal process, the Markov-modulated Poisson process, the Markovian arrival process as special cases. In addition, the weighted round-robin (WRR) policies are commonly used to transmit data with different priorities [26], [27]. In WRR, the used weights are set statically according to the prior traffic information. In the dynamic weighted round-robin (DWRR), the weights are set dynamically according to the time-varying characteristics of traffic. It was shown in [28], [29] that DWRR can achieve better performance than WRR, without the prior traffic information. Based on this result and the dependencies among the frames generated by H.264 and H.265, it is inferred that the time division transmission policy with across-buffer state-dependent property can effectively improve the transmission quality of the compressed video. The transmission policy attached to one buffer can be dynamically adjusted based on the time-varying characteristics of frames in other buffers with higher priorities. Therefore, the polling models with BMAP and across-queue state-dependent service discipline are worth studying. And they are effective analysis tools to guide the design and optimization of time division transmission policy, for transmitting the video compressed by H.264 and H.265. However, there have been very few researches in this regard. To the best of our knowledge, in 2017, a polling model of this type was proposed firstly [13], and up to now its performance measures have not been analyzed. The method proposed in [8] is not suitable for the model in [13], since the service disciplines need to be independent of the history of the model, whereas in [13], the service time of the second queue depends on the length of the first queue. In [30], Vishnevsky et al. indicated that a polling model can be analyzed by the decomposition of the polling model into a set of vacation queueing models. But, this method is also not suitable for the model in [13], due to the across-queue state-dependent service discipline attached to the second queue. Indeed, the BMAP and across-queue state-dependent service discipline lead to the large state space and several coupled state transition processes, which complicate the performance analysis.

In this paper, we will analyze the performance of the cyclic polling model, as presented in [13]. The buffer sizes of the two queues are finite in our analysis. The motivation of this paper includes two aspects. First, the performance analysis of the cyclic polling model in this paper can be used as a basis of analyzing the cyclic polling model in [13] with infinite-buffer queues, by increasing the buffer sizes. Second, since the polling models with BMAP and across-queue state-dependent discipline can be used to guide the design and optimization of time division transmission policies for transmitting the video compressed by H.264 and H.265, we will explore the method to analyze this type of polling model by concentrating on the two-queue model, whenever possible, suggest extensions to the multi-queue model in the future.

B. OUR MAIN CONTRIBUTIONS

By constructing a supplementary embedded Markov chain and applying the matrix-analytic method based on the semi-regenerative process [31], some important performance measures of the polling model presented are analyzed.

- The expressions of three joint queue length stationary distributions are obtained, including: the joint queue length stationary distribution, at queue 1 polling epochs when the server arrives at the first queue; the joint queue length stationary distribution, at queue 2 polling epochs when the server arrives at the second queue; the joint queue length stationary distribution at arbitrary time.
- The expressions of customer blocking probabilities in different queues are derived.
- The expressions of customer mean waiting times in different queues are obtained.

In addition, the analysis method applied in this paper can provide inspiration for analyzing the polling models with BMAP and across-queue state-dependent service discipline.

C. NOTATIONS

Throughout this paper, unless otherwise stated, notations are used as follows. $\mathrm{N} = \{0, 1, 2, \cdots\}$; $\mathrm{N}^+ = \{1, 2, 3, \cdots\}$; $\mathbf{e}$ denotes a column vector of appropriate size consisting of 1's; $\mathbf{e}_k$ $(k \in \mathrm{N}^+)$ denotes a k-dimensional column vector consisting of 1's; $\mathbf{0}$ denotes a vector or matrix of appropriate size consisting of 0's; $\mathbf{0}_k$ $(k \in \mathrm{N}^+)$ denotes a $k \times k$ matrix consisting of 0's; $\mathbf{0}_{k_1 \times k_2}$ $(k_1, k_2 \in \mathrm{N}^+)$ denotes a $k_1 \times k_2$ matrix consisting of 0's; $\mathbf{I}$ denotes an identity matrix of appropriate size; $\mathbf{I}_k$ $(k \in \mathrm{N}^+)$ denotes a $k \times k$ identity matrix. For any $n_1, n_2 \in \mathrm{N}$, if $n_1 = n_2$, then $\delta_{n_1,n_2} = 1$; otherwise, $\delta_{n_1,n_2} = 0$. Given two sets A and B, $A \setminus B =$

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$\{x|x \in A \text{ and } x \notin B\}$. Given a matrix $\mathbf{A}$ whose elements (which may be blocks) are indexed by $(i,j) \in \Omega_1 \times \Omega_2$, where the set Ω_1 consists of the row indices which are all either scalars or row vectors with the same dimension, and the set Ω_2 consists of the column indices which are all either scalars or row vectors with the same dimension, $\mathbf{A}$ can be denoted by $\mathbf{A} = \left(\mathbf{A}_{i,j} : i \in \Omega_1, j \in \Omega_2\right)$, where $\mathbf{A}_{i,j}$ represents the (i,j)-th element, and no matter the indices are scalars or vectors, the elements in each row (column) are arranged in the lexicographical order among the corresponding row (column) indices. Given a row vector $\mathbf{B}$ whose elements (which may be blocks) are indexed by $i \in \Omega$, where the set Ω consists of the indices which are all either scalars or row vectors with the same dimension, $\mathbf{B}$ can be denoted by $\mathbf{B} = (\mathbf{B}_i : i \in \Omega)$, where $\mathbf{B}_i$ represents the i-th element, and no matter the indices are scalars or vectors, the elements are arranged in the lexicographical order among the corresponding indices. $\mathbf{B}^T$ represents the transposition of the row vector $\mathbf{B}$; whether or not blocks, $\mathbf{B}_i$, $i \in \Omega$, are the transposed atomic elements.

The rest of this paper is organized as follows. The system model is presented in Section II. In Section III, firstly a supplementary embedded Markov chain is constructed; and then the joint queue length stationary distributions at queue 1 polling epochs and at queue 2 polling epochs are analyzed. In Section IV, the joint queue length stationary distribution at arbitrary time is analyzed. The blocking probabilities and waiting times of customers in different queues are analyzed in Section V. In Section VI, a numerical example is carried out to illustrate the calculations of performance measures which have been analyzed, and some numerical experiments are carried out to show the effectiveness of the proposed polling model. Finally, the conclusion is given in Section VII.

II. MODEL DESCRIPTION

The cyclic polling model considered in this paper consists of a single server and two finite-buffer queues. The customers arrive at the two queues according to two independent BMAPs. Upon arrival, if there is not enough space in the buffer, a part of the current batch will be rejected. The customers in the first queue have the higher service priority than the customers in the second queue. The server attends the two queues in a cyclic manner. The first queue is served according to the gated service discipline. The second queue is served according to an across-queue state-dependent time-limited service discipline with the preemptive repeat-different property. Namely, the predetermined time of the server's visit to the second queue is time-limited, and its probability distribution function depends on the length of the first queue at the instant when the server started to depart from the first queue last time. As the length of the first queue increases, the mean predetermined limited time either decreases or remains the same. Because of the preemptive repeat-different property, in the second queue, the service in progress (if any) is interrupted when the predetermined limited time expires. The interrupted service will be started

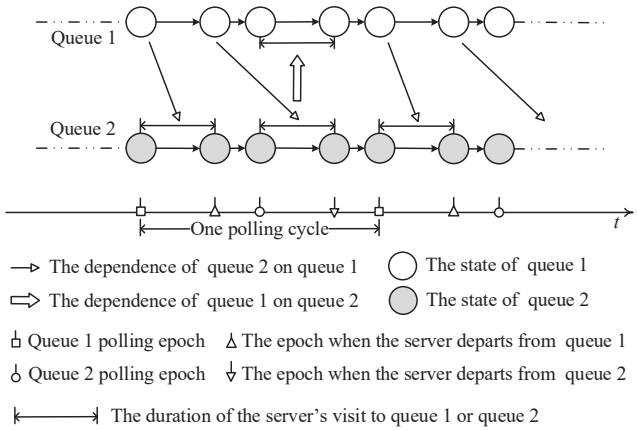

FIGURE 3. The dependency diagram of two queues.

from beginning again in the next cycle, and its service time is newly sampled from the same service time distribution of the customers in the second queue. In addition, for the two service disciplines, the service orders are first in first out (FIFO); and the switchover times of the server transferring from a queue to the other one are considered. Fig. 3 shows the dependency of two queues. The following assumptions are made.

(1) The length of a queue counts the number of customers whose services are not finished in the queue.
(2) The length of a queue is always less than the buffer size of the queue, either when the service of a customer in the queue just terminates or when a customer just departs from the queue after being served.
(3) When the server arrives at each queue, it immediately begins to serve the customers (if any), and the service progress is not broken until the current service period ends according to the used service discipline.

The ι-th queue is called the queue ι and the customer in the queue ι is called the ι-customer, where unless otherwise stated, $\iota \in \{1,2\}$ throughout this paper. The epoch just when the server arrives at any queue is called the polling epoch; the epoch just when the server arrives at queue ι is called the queue ι polling epoch. Without loss of generality, suppose that each cycle begins at queue 1 polling epochs. If queue 1 is not empty at the queue 1 polling epoch, the server starts to serve the present 1-customers until the services of these 1-customers are completed, and then the server departs from queue 1; otherwise the server immediately departs from queue 1. After departing from queue 1, the server transfers to queue 2. If queue 2 is not empty at the queue 2 polling epoch, the server starts to serve the 2-customers until the predetermined limited time for queue 2 expires or queue 2 is empty (whichever occurs first), and then the server departs from queue 2; otherwise the server immediately departs from queue 2. After departing from queue 2, the server transfers to queue 1. And the next cycle will begin.

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The buffer size of queue ι is denoted by Q_ι $\left(Q_\iota \in \mathrm{N}^+\right)$. The predetermined limited time for queue 2 is denoted by the random variable H_j, which obeys the exponential distribution with the parameter γ_j $(0 < \gamma_j < \infty,\ j \in \{0, 1, \cdots, Q_1 - 1\})$, where j denotes the length of queue 1 when the server last departed from queue 1. For $j_1, j_2 \in \{0, 1, \cdots, Q_1 - 1\}$, if $j_1 > j_2$, then $\gamma_{j_1} \geq \gamma_{j_2}$. The switchover time of the server transferring from queue ι to the other queue is denoted by the random variable R_ι, which obeys the general distribution with the distribution function $R_\iota(t)$ and the mean r_ι, where $t \in [0, +\infty)$, $R_\iota(0) = 0$ and $r_\iota \in (0, +\infty)$. For the same queue, the service times of the customers are independent and identically distributed. The service time of the ι-customer is denoted by the random variable B_ι, which obeys the general distribution with the distribution function $B_\iota(t)$ and the mean b_ι, where $t \in [0, +\infty)$, $B_\iota(0) = 0$ and $b_\iota \in (0, +\infty)$. According to Theorem 9.14 in [32], any probability distribution on $[0, +\infty)$ can be approximated by a probability distribution of phase type (PH-distribution). Moreover, in analyzing the queueing model with BMAP by the matrix-analytic method, some numerical integrals can be avoided by applying the PH-distribution. So, suppose that $R_\iota(t)$ has the phase type representation $(\boldsymbol{\alpha}_\iota, \mathbf{R}_\iota)$ of order m_{R_ι}, where $m_{R_\iota} \in \mathrm{N}^+$ and $\boldsymbol{\alpha}_\iota \mathbf{e} = 1$, and that $B_\iota(t)$ has the phase type representation $(\boldsymbol{\beta}_\iota, \mathbf{B}_\iota)$ of order m_{B_ι}, where $m_{B_\iota} \in \mathrm{N}^+$ and $\boldsymbol{\beta}_\iota \mathbf{e} = 1$. From Theorem 2.2.1 in [33], for a PH-distribution, if the given representation is reducible, its irreducible representation can be obtained by deleting the superfluous states of the Markov chain corresponding to the given representation. So, suppose that the representations of the PH-distributions involved in this paper are all irreducible.

The BMAP corresponding to the ι-customers is denoted by the BMAP-ι, which is defined by a two-dimensional continuous-time Markov chain $X^{(\iota)}(t)$ on the state space $S^{(\iota)}$,

$$X^{(\iota)}(t) = \{N_\iota(t), V_\iota(t); t \geq 0\},$$
$$S^{(\iota)} = \{(i_\iota, v_\iota) : i_\iota \in \mathrm{N}, v_\iota \in M_\iota\},$$

where $M_\iota = \{1, 2, \cdots, m_\iota\}$, $m_\iota \in \mathrm{N}^+$. $N_\iota(t)$ represents a counting variable denoting the number of arrivals in $(0, t]$; $V_\iota(t)$ represents a phase variable denoting the phase of the BMAP-ι at time t.

$\mathbf{P}^{(\iota)}(n, t)$, $n \in \mathrm{N}$, $t \geq 0$, is defined as a $m_\iota \times m_\iota$ matrix,

$$\mathbf{P}^{(\iota)}(n, t) = \left(\mathbf{P}^{(\iota)}_{v_\iota, v'_\iota}(n, t) : v_\iota, v'_\iota \in M_\iota\right),$$

where $\mathbf{P}^{(\iota)}_{v_\iota, v'_\iota}(n, t)$ represents the following conditional probability,

$$\mathbf{P}^{(\iota)}_{v_\iota, v'_\iota}(n, t) = P\left\{N_\iota(t) = n, V_\iota(t) = v'_\iota | N(0) = 0, V(0) = v_\iota\right\}.$$

$\mathbf{P}^{(\iota)}(n, t)$ satisfies the following Chapman-Kolmogorov equations,

$$\mathbf{P}'^{(\iota)}(n, t) = \sum_{j=0}^{n} \mathbf{P}^{(\iota)}(j, t)\, \mathbf{D}^{(\iota)}_{n-j}, \quad n \in \mathrm{N},\ t \geq 0, \tag{1}$$

$$\mathbf{P}^{(\iota)}(0, 0) = \mathbf{I}_{m_\iota}. \tag{2}$$

$\mathbf{D}^{(\iota)}_j$ $(j \in \mathrm{N})$ is a $m_\iota \times m_\iota$ matrix. For $v_\iota, v'_\iota \in M_\iota$ and $v_\iota \neq v'_\iota$, $\left(\mathbf{D}^{(\iota)}_0\right)_{v_\iota, v'_\iota}$ is nonnegative and characterizes the transition intensity of $X^{(\iota)}(t)$ from the state (i_ι, v_ι) to the state (i_ι, v'_ι), $i_\iota \in \mathrm{N}$; for $v_\iota \in M_\iota$, $\left(\mathbf{D}^{(\iota)}_0\right)_{v_\iota, v_\iota}$ is negative, and its opposite characterizes the transition intensity of $X^{(\iota)}(t)$ from the state (i_ι, v_ι) to any other state in $S^{(\iota)}$. For $j \in \mathrm{N}^+$ and $v_\iota, v'_\iota \in M_\iota$, $\left(\mathbf{D}^{(\iota)}_j\right)_{v_\iota, v'_\iota}$ is nonnegative and characterizes the transition intensity of $X^{(\iota)}(t)$ from the state (i_ι, v_ι) to the state $(i_\iota + j, v'_\iota)$, $i_\iota \in \mathrm{N}$.

The matrix generating function of $\mathbf{D}^{(\iota)}_j$ $(j = 0, 1, 2, \cdots)$ is defined as

$$\mathbf{D}^{(\iota)}(z) = \sum_{j=0}^{\infty} \mathbf{D}^{(\iota)}_j z^j, \quad |z| \leq 1. \tag{3}$$

$\mathbf{D}^{(\iota)}(1)\mathbf{e} = \mathbf{0}$, and $\mathbf{D}^{(\iota)}(1)$ is briefly denoted by $\mathbf{D}^{(\iota)}$ throughout this paper. Assume that $\mathbf{D}^{(\iota)} \neq \mathbf{D}^{(\iota)}_0$, thus based on Theorem 1.3.17 of [34], the matrix $\mathbf{D}^{(\iota)}_0$ is stable. $\mathbf{D}^{(\iota)}$ can be viewed as the infinitesimal generator of the irreducible continuous-time Markov chain $v^{(\iota)}_t$ $(t \geq 0)$, which is the underlying Markov chain of the BMAP-ι and has the state space M_ι. The stationary distribution of $v^{(\iota)}_t$ is denoted by $\boldsymbol{\theta}_\iota$, such that $\boldsymbol{\theta}_\iota \mathbf{D}^{(\iota)} = \mathbf{0}$ and $\boldsymbol{\theta}_\iota \mathbf{e} = 1$. The average arrival rate of BMAP-ι is defined as $\lambda_\iota = \boldsymbol{\theta}_\iota \sum_{j=1}^{\infty} j \mathbf{D}^{(\iota)}_j \mathbf{e}$.

The matrix generating function of $\mathbf{P}^{(\iota)}(n, t)$, $n = 0, 1, 2, \cdots$, is defined as

$$\mathbf{P}^{(\iota)}(z, t) = \sum_{n=0}^{\infty} \mathbf{P}^{(\iota)}(n, t)\, z^n, \quad |z| \leq 1. \tag{4}$$

From (1), (2), (3) and (4), the first derivative of $\mathbf{P}^{(\iota)}(z, t)$ with respect to t satisfies

$$\mathbf{P}'^{(\iota)}(z, t) = \mathbf{P}^{(\iota)}(z, t)\, \mathbf{D}(z),$$
$$\mathbf{P}^{(\iota)}(z, 0) = \mathbf{I}_{m_\iota}.$$

Moreover, there is the following relation,

$$\mathbf{P}^{(\iota)}(z, t) = e^{\mathbf{D}^{(\iota)}(z)t}, \quad |z| \leq 1.$$

Assume that R_ι, B_ι, H_j, BMAP-ι $(\iota = 1, 2;\ j = 0, 1, \cdots, Q_1 - 1)$ are mutually independent. The joint arrival process of the BMAP-1 and the BMAP-2 can be defined by the four-dimensional continuous-time Markov chain $Y(t) =$

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$\{N_1(t), N_2(t), V_1(t), V_2(t); t \geq 0\}$. For $Y(t)$, a $\bar{m} \times \bar{m}$ matrix $\mathbf{P}(n_1, n_2, t), t \geq 0, n_1, n_2 \in \mathrm{N}$, is introduced as follows.

$$\mathbf{P}(n_1, n_2, t) = \Big(\mathbf{P}_{(v_1,v_2),(v_1',v_2')}(n_1, n_2, t) : (v_1, v_2), (v_1', v_2') \in M_1 \times M_2\Big),$$

where $\bar{m} = m_1 m_2$, and the $\big((v_1, v_2), (v_1', v_2')\big)$-th element represents the following conditional probability,

$$\begin{aligned}&\mathbf{P}_{(v_1,v_2),(v_1',v_2')}(n_1, n_2, t)\\ &= P\{N_1(t) = n_1, N_2(t) = n_2, V_1(t) = v_1', V_2(t) = v_2'\\ &\quad |N_1(0) = 0, N_2(0) = 0, V_1(0) = v_1, V_2(0) = v_2\}.\end{aligned}$$

$\mathbf{P}(n_1, n_2, t)$ satisfies the following relation,

$$\mathbf{P}(n_1, n_2, t) = \mathbf{P}^{(1)}(n_1, t) \otimes \mathbf{P}^{(2)}(n_2, t), \tag{5}$$

where the symbol $\otimes$ denotes the Kronecker product operation. Based on (4) and (5), it can be shown that $\mathbf{P}(n_1, n_2, t)$ satisfies the following Chapman-Kolmogorov equations,

$$\begin{aligned}\mathbf{P}'(n_1, n_2, t) &= \sum_{j_1=0}^{n_1} \mathbf{P}(j_1, n_2, t)\left(\mathbf{D}^{(1)}_{n_1-j_1} \otimes \mathbf{I}_{m_2}\right)\\ &\quad + \sum_{j_2=0}^{n_2} \mathbf{P}(n_1, j_2, t)\left(\mathbf{I}_{m_1} \otimes \mathbf{D}^{(2)}_{n_2-j_2}\right), \quad (6)\end{aligned}$$

$$\mathbf{P}(0, 0, 0) = \mathbf{I}_{\bar{m}}. \tag{7}$$

The matrix generating function of $\mathbf{P}(n_1, n_2, t)$, $n_1, n_2 = 0, 1, 2, \cdots$, is defined as

$$\begin{aligned}&\mathbf{P}(z_1, z_2, t)\\ &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} \mathbf{P}(n_1, n_2, t) z_1^{n_1} z_2^{n_2}, \quad |z_1| \leq 1, |z_2| \leq 1.\end{aligned}$$

Based on (4) and (5), there is the following relation,

$$\mathbf{P}(z_1, z_2, t) = e^{\left(\mathbf{D}^{(1)}(z_1) \oplus \mathbf{D}^{(2)}(z_2)\right)t}, \quad |z_1| \leq 1, |z_2| \leq 1,$$

where the symbol $\oplus$ denotes the Kronecker sum operation. As the definition of [35], $\mathbf{A} \oplus \mathbf{B} = \mathbf{A} \otimes \mathbf{I}_b + \mathbf{I}_a \otimes \mathbf{B}$, where a and b denote the orders of the matrices $\mathbf{A}$ and $\mathbf{B}$, respectively.

III. TWO JOINT QUEUE LENGTH STATIONARY DISTRIBUTIONS AT THE POLLING EPOCHS

In this section, we will analyze the joint queue length stationary distributions at queue 1 polling epochs and at queue 2 polling epochs. In order to prevent some details of the state transitions of the cyclic polling model being ignored, the supplementary embedded Markov chain at service completion and switchover termination epochs needs to be constructed firstly. This supplementary embedded Markov chain is the basis not only for analyzing the two joint queue length stationary distributions at the polling epochs, but also for analyzing the joint queue length stationary distribution at arbitrary time.

A. THE SUPPLEMENTARY EMBEDDED MARKOV CHAIN AT SERVICE COMPLETION AND SWITCHOVER TERMINATION EPOCHS

Define an event that either a service completion or a switchover termination just occurs, and let T_n denote the instant when this event occurs at the n-th time, where $n \in \mathrm{N}^+$ and $T_n \in [0, +\infty)$. Notice that at the instant just when a switchover of the server terminates, the server just arrives at either queue 1 or queue 2. Without loss of generality, it is assumed that T_1 is the instant just when the switchover of the server from queue 2 to queue 1 terminates, or the instant just when the server arrives at queue 1.

Consider the state of the cyclic polling model at T_n, i.e.

$$\xi(T_n) = \big(\Phi(T_n), L_1(T_n), L_2(T_n), V_1(T_n), V_2(T_n)\big).$$

The value assignment rules for $\Phi(T_n)$ are listed in Table 2. $V_\iota(T_n)$ denotes the phase of the BMAP-ι at time T_n, where $V_\iota(T_n) \in M_\iota$. $L_\iota(T_n)$ denotes the length of queue ι at time T_n, where $L_\iota(T_n)$ takes its values as the following way.

(a) If $\Phi(T_n) = (1, i, k)$, then $L_1(T_n) \in \mathbb{L}_1^{(1,i-k)}$ and $L_2(T_n) \in \mathbb{L}_2^{(1)}$, where

$$\begin{aligned}\mathbb{L}_1^{(1,i-k)} &= \{i-k, i-k+1, \cdots, Q_1 - 1\},\\ \mathbb{L}_2^{(1)} &= \{0, 1, \cdots, Q_2\};\end{aligned}$$

(b) if $\Phi(T_n) = (s_1, j)$, then $L_1(T_n) \in \mathbb{L}_1^{(s_1,j)}$ and $L_2(T_n) \in \mathbb{L}_2^{(s_1)}$, where

$$\begin{aligned}\mathbb{L}_1^{(s_1,j)} &= \{j, j+1, \cdots, Q_1\},\\ \mathbb{L}_2^{(s_1)} &= \{0, 1, \cdots, Q_2\};\end{aligned}$$

(c) if $\Phi(T_n) = (2, j)$, then $L_1(T_n) \in \mathbb{L}_1^{(2,j)}$ and $L_2(T_n) \in \mathbb{L}_2^{(2)}$, where

$$\begin{aligned}\mathbb{L}_1^{(2,j)} &= \{j, j+1, \cdots, Q_1\},\\ \mathbb{L}_2^{(2)} &= \{0, 1, \cdots, Q_2 - 1\};\end{aligned}$$

(d) if $\Phi(T_n) = s_2$, then $L_1(T_n) \in \mathbb{L}_1^{(s_2)}$ and $L_2(T_n) \in \mathbb{L}_2^{(s_2)}$, where

$$\mathbb{L}_1^{(s_2)} = \{0, 1, \cdots, Q_1\}, \quad \mathbb{L}_2^{(s_2)} = \{0, 1, \cdots, Q_2\}.$$

$\xi(T_n)$ is briefly denoted by

$$\xi_n = (\Phi_n, L_{1,n}, L_{2,n}, V_{1,n}, V_{2,n}).$$

The discrete-time stochastic process $\{\xi_n; n \in \mathrm{N}^+\}$ is constructed, and it is a homogeneous Markov chain on the state

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TABLE 2. The value assignment rules for $\Phi(T_n)$

$\Phi(T_n)$	The value assignment rule
s_2	If a switchover of the server from queue 2 to queue 1 just terminates at time T_n.
(s_1, j)	If a switchover of the server from queue 1 to queue 2 just terminates at time T_n, and the length of queue 1 was j when the server started to depart from queue 1 last time, where $j \in \{0, 1, \cdots, Q_1 - 1\}$.
$(2, j)$	If the service of a 2-customer just completes at time T_n, and the current predetermined limited service time of queue 2 is H_j, where $j \in \{0, 1, \cdots, Q_1 - 1\}$.
$(1, i, k)$	If the service of the k-th 1-customer just completes at time T_n, given that the length of queue 1 was i when the current service period started, where $i \in \{1, 2, \cdots, Q_1\}$, $k \in \{1, 2, \cdots, i\}$.

space $S_\xi = S_\xi^{(1)} \cup S_\xi^{(s_1)} \cup S_\xi^{(2)} \cup S_\xi^{(s_2)}$, where

$$S_\xi^{(1)} = \bigcup_{i=1}^{Q_1} \bigcup_{k=1}^{i} \{ (1, i, k) \} \times \mathbb{L}_1^{(1,i-k)} \times \mathbb{L}_2^{(1)} \times M_1 \times M_2,$$

$$S_\xi^{(s_1)} = \bigcup_{j=0}^{Q_1-1} \{ (s_1, j) \} \times \mathbb{L}_1^{(s_1,j)} \times \mathbb{L}_2^{(s_1)} \times M_1 \times M_2,$$

$$S_\xi^{(2)} = \bigcup_{j=0}^{Q_1-1} \{ (2, j) \} \times \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)} \times M_1 \times M_2,$$

$$S_\xi^{(s_2)} = \{s_2\} \times \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)} \times M_1 \times M_2.$$

A matrix $\mathbf{P}\{\phi', l_1', l_2' | \phi, l_1, l_2\}$ is introduced,

$$\mathbf{P}\{\phi', l_1', l_2' | \phi, l_1, l_2\} = \Big(\mathbf{P}_{(v_1,v_2),(v_1',v_2')} \{\phi', l_1', l_2' | \phi, l_1, l_2\} : (v_1, v_2), (v_1', v_2') \in M_1 \times M_2 \Big),$$

where the $\big((v_1, v_2), (v_1', v_2') \big)$-th element represents the following conditional probability,

$$\mathbf{P}_{(v_1,v_2),(v_1',v_2')} \{\phi', l_1', l_2' | \phi, l_1, l_2\} = P\{\xi_{n+1} = (\phi', l_1', l_2', v_1', v_2') \,|\, \xi_n = (\phi, l_1, l_2, v_1, v_2)\}.$$

Let the states in S_ξ be listed in the order, i.e. first the states in $S_\xi^{(1)}$, second the states in $S_\xi^{(s_1)}$, third the states in $S_\xi^{(2)}$ and fourth the states in $S_\xi^{(s_2)}$ are listed in the lexicographical order. Based on this sequence, the one-step transition probability matrix $\mathbf{M}$ of the Markov chain $\{\xi_n; n \in \mathrm{N}^+\}$ is constructed as the following,

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_3 & \mathbf{M}_4 \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_5 & \mathbf{M}_6 \\ \mathbf{M}_7 & \mathbf{M}_8 & \mathbf{0} & \mathbf{0} \end{pmatrix},$$

where the matrices $\mathbf{M}_i$ $(i = 1, 2, \cdots, 8)$ describe the transition probabilities among the states in S_ξ, see Fig. 4. It can be seen that, some blocks in $\mathbf{M}$ are zero matrices. That is because the swithover times of the server between the two queues are not ignored. The structure of $\mathbf{M}$ will provide conveniences for the following analysis, see Section III-B and Section III-C. The details about the matrices $\mathbf{M}_i$ $(i = 1, 2, \cdots, 8)$ will be given respectively in the following.

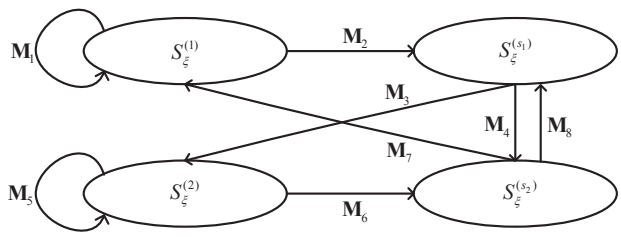

FIGURE 4. The schematic diagram of the one-step transitions among the states in S_ξ.

For making some expressions concise, two notations are introduced firstly. Let

$$\boldsymbol{\delta}(n_1, n_2) = \begin{pmatrix} \delta_{n_1,n_2} & 1 - \delta_{n_1,n_2} \end{pmatrix},$$

$$\langle \mathbf{\Gamma}(n_1, n_2) \rangle = \begin{pmatrix} \mathbf{\Gamma}(\overline{n_1}, \overline{n_2}) & \mathbf{\Gamma}(n_1, \overline{n_2}) \\ \mathbf{\Gamma}(\overline{n_1}, n_2) & \mathbf{\Gamma}(n_1, n_2) \end{pmatrix},$$

where $\mathbf{\Gamma}$ is a universal symbol, and it will be replaced by the required symbols in different cases. The meanings and calculation formulas of $\mathbf{\Gamma}(\overline{n_1}, \overline{n_2})$, $\mathbf{\Gamma}(n_1, \overline{n_2})$, $\mathbf{\Gamma}(\overline{n_1}, n_2)$ and $\mathbf{\Gamma}(n_1, n_2)$ are presented in Appendix A.

(1) The matrix $\mathbf{M}_1$ describes the one-step transitions among the states in $S_\xi^{(1)}$. Within $S_\xi^{(1)}$, the state with $\phi = (1, i, i)$ can not transfer to any other states, where $i \in \{1, 2, \cdots, Q_1\}$; the state with $\phi = (1, i, k)$ can transfer to the state with $\phi = (1, i, k+1)$, where $i \in \{2, 3, \cdots, Q_1\}$, $k \in \{1, 2, \cdots, i-1\}$. So, $\mathbf{M}_1$ has the following structure.

$$\mathbf{M}_1 = diag \begin{pmatrix} \mathbf{M}_{1,1} & \mathbf{M}_{1,2} & \cdots & \mathbf{M}_{1,Q_1} \end{pmatrix},$$

where $\mathbf{M}_{1,1} = \mathbf{0}_{\varsigma_1}$, $\varsigma_n = \bar{m}(Q_1 - n + 1)(Q_2 + 1)$, $n \in \{0, 1, \cdots, Q_1\}$; and

$$\mathbf{M}_{1,i} = \begin{pmatrix} \mathbf{0} & (\mathbf{M}_{1,i})_{1,2} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & (\mathbf{M}_{1,i})_{i-1,i} \\ \mathbf{0}_{\varsigma_1 \times \varsigma_i} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix},$$

$i \in \{2, 3, \cdots, Q_1\}$. $(\mathbf{M}_{1,i})_{k,k+1}$, $i \in \{2, 3, \cdots, Q_1\}$, $k \in \{1, 2, \cdots, i-1\}$, consists of matrix blocks which

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are indexed by $\left(\left(l_1,l_2\right),\left(l_1',l_2'\right)\right)$, $(l_1,l_2)\in\mathbb{L}_1^{(1,i-k)}\times\mathbb{L}_2^{(1)}$, $(l_1',l_2')\in\mathbb{L}_1^{(1,i-k-1)}\times\mathbb{L}_2^{(1)}$, where

$$\left(\left(\mathbf{M}_{1,i}\right)_{k,k+1}\right)_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{\left(1,i,k+1\right),l_1',l_2'\middle|\left(1,i,k\right),l_1,l_2\right\}.$$

Given i and k, consider each $(l_1,l_2)\in\mathbb{L}_1^{(1,i-k)}\times\mathbb{L}_2^{(1)}$,

(a) for $(l_1',l_2')\in\{l_1-1,l_1,\cdots,Q_1-1\}\times\{l_2,l_2+1,\cdots,Q_2\}$,

$$\left(\left(\mathbf{M}_{1,i}\right)_{k,k+1}\right)_{(l_1,l_2),(l_1',l_2')} = \boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{B}_1\left(l_1'-l_1+1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T \times\left(Q_1,l_1'+1\right);$$

(b) for the other $(l_1',l_2')\in\mathbb{L}_1^{(1,i-k-1)}\times\mathbb{L}_2^{(1)}$,

$$\left(\left(\mathbf{M}_{1,i}\right)_{k,k+1}\right)_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

(2) The matrix $\mathbf{M}_2$ describes the one-step transitions of the states from $S_\xi^{(1)}$ to $S_\xi^{(s_1)}$. Consider the states in $S_\xi^{(1)}$, only the state with $\phi=(1,i,i)$ can enter into $S_\xi^{(s_1)}$, where $i\in\{1,2,\cdots,Q_1\}$. So, $\mathbf{M}_2$ has the following structure.

$$\mathbf{M}_2=\begin{pmatrix}\mathbf{M}_{2,1,0} & \mathbf{M}_{2,1,1} & \cdots & \mathbf{M}_{2,1,Q_1-1}\\ \mathbf{M}_{2,2,0} & \mathbf{M}_{2,2,1} & \cdots & \mathbf{M}_{2,2,Q_1-1}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{M}_{2,Q_1,0} & \mathbf{M}_{2,Q_1,1} & \cdots & \mathbf{M}_{2,Q_1,Q_1-1}\end{pmatrix},$$

where

$$\mathbf{M}_{2,1,j}=\mathbf{M}_{2,j},\qquad j\in\{0,1,\cdots,Q_1-1\},$$

$$\mathbf{M}_{2,i,j}=\begin{pmatrix}\mathbf{0}_{S_{Q_1-1}^{i-1}\times\varsigma_j}\\ \mathbf{M}_{2,j}\end{pmatrix},\qquad i\in\{2,3,\cdots,Q_1\},\quad j\in\{0,1,\cdots,Q_1-1\},$$

and $S_n^k=\bar{m}\left(Q_2+1\right)[n+(n-1)+(n-2)+\cdots+(n-k+1)]$. $\mathbf{M}_{2,j}$, $j\in\{0,1,\cdots,Q_1-1\}$, consists of matrix blocks which are indexed by $\left(\left(l_1,l_2\right),\left(l_1',l_2'\right)\right)$, $(l_1,l_2)\in\mathbb{L}_1^{(1,0)}\times\mathbb{L}_2^{(1)}$, $(l_1',l_2')\in\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)}$, where

$$\left(\mathbf{M}_{2,j}\right)_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{\left(s_1,j\right),l_1',l_2'\middle|\left(1,i,i\right),l_1,l_2\right\}.$$

Given j, consider each $(l_1,l_2)\in\left(\mathbb{L}_1^{(1,0)}\setminus\{j\}\right)\times\mathbb{L}_2^{(1)}$, for any $(l_1',l_2')\in\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)}$,

$$\left(\mathbf{M}_{2,j}\right)_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};$$

consider each $(l_1,l_2)\in\{j\}\times\mathbb{L}_2^{(1)}$,

(a) for $(l_1',l_2')\in\mathbb{L}_1^{(s_1,j)}\times\{l_2,l_2+1,\cdots,Q_2\}$,

$$\left(\mathbf{M}_{2,j}\right)_{(l_1,l_2),(l_1',l_2')} = \boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{R}_1\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right),$$

(b) for the other $(l_1',l_2')\in\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)}$,

$$\left(\mathbf{M}_{2,j}\right)_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}}.$$

(3) The matrix $\mathbf{M}_3$ describes the one-step transitions of the states from $S_\xi^{(s_1)}$ to $S_\xi^{(2)}$. In this case, the state with $\phi=(s_1,j)$ in $S_\xi^{(s_1)}$ may only transfer to the state with $\phi=(2,j)$ in $S_\xi^{(2)}$, where $j\in\{0,1,\cdots,Q_1-1\}$. So, $\mathbf{M}_3$ has the following structure.

$$\mathbf{M}_3=diag\begin{pmatrix}\mathbf{M}_{3,0} & \mathbf{M}_{3,1} & \cdots & \mathbf{M}_{3,Q_1-1}\end{pmatrix},$$

where $\mathbf{M}_{3,j}$, $j\in\{0,1,\cdots,Q_1-1\}$, consists of matrix blocks which are indexed by $\left(\left(l_1,l_2\right),\left(l_1',l_2'\right)\right)$, $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)}$, $(l_1',l_2')\in\mathbb{L}_1^{(2,j)}\times\mathbb{L}_2^{(2)}$, where

$$\left(\mathbf{M}_{3,j}\right)_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{\left(2,j\right),l_1',l_2'\middle|\left(s_1,j\right),l_1,l_2\right\}.$$

Given j, consider each $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\{0\}$, for any $(l_1',l_2')\in\mathbb{L}_1^{(2,j)}\times\mathbb{L}_2^{(2)}$,

$$\left(\mathbf{M}_{3,j}\right)_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};$$

consider each $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\left(\mathbb{L}_2^{(s_1)}\setminus\{0\}\right)$,

(a) for $(l_1',l_2')\in\{l_1,l_1+1,\cdots,Q_1\}\times\{l_2-1,l_2,\cdots,Q_2-1\}$,

$$\left(\mathbf{M}_{3,j}\right)_{(l_1,l_2),(l_1',l_2')} = \boldsymbol{\delta}\left(Q_2,l_2'+1\right)\times\left\langle\overline{\mathbf{H}_j}\circ\mathbf{B}_2\left(l_1'-l_1,l_2'-l_2+1\right)\right\rangle\times\boldsymbol{\delta}^T\left(Q_1,l_1'\right),$$

(b) for the other $(l_1',l_2')\in\mathbb{L}_1^{(2,j)}\times\mathbb{L}_2^{(2)}$,

$$\left(\mathbf{M}_{3,j}\right)_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}}.$$

(4) The matrix $\mathbf{M}_4$ describes the one-step transitions of the states from $S_\xi^{(s_1)}$ to $S_\xi^{(s_2)}$. Any state in $S_\xi^{(s_1)}$ may enter into $S_\xi^{(s_2)}$. So, $\mathbf{M}_4$ has the following structure.

$$\mathbf{M}_4=\begin{pmatrix}\mathbf{M}_{4,0} & \mathbf{M}_{4,1} & \mathbf{M}_{4,2} & \cdots & \mathbf{M}_{4,Q_1-1}\end{pmatrix}^T,$$

where $\mathbf{M}_{4,j}$, $j\in\{0,1,\cdots,Q_1-1\}$, consists of matrix blocks which are indexed by $\left(\left(l_1,l_2\right),\left(l_1',l_2'\right)\right)$, $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)}$, $(l_1',l_2')\in\mathbb{L}_1^{(s_2)}\times\mathbb{L}_2^{(s_2)}$, where

$$\left(\mathbf{M}_{4,j}\right)_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{s_2,l_1',l_2'\middle|\left(s_1,j\right),l_1,l_2\right\}.$$

Given j, consider each $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\{0\}$,

(a) for $(l_1',l_2')\in\{l_1,l_1+1,\cdots,Q_1\}\times\mathbb{L}_2^{(s_2)}$

$$\left(\mathbf{M}_{4,j}\right)_{(l_1,l_2),(l_1',l_2')} = \boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{R}_2\left(l_1'-l_1,l_2'\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right),$$

(b) for the other $(l_1',l_2')\in\mathbb{L}_1^{(s_2)}\times\mathbb{L}_2^{(s_2)}$,

$$\left(\mathbf{M}_{4,j}\right)_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};$$

consider each $(l_1,l_2)\in\mathbb{L}_1^{(s_1,j)}\times\left(\mathbb{L}_2^{(s_1)}\setminus\{0\}\right)$,

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(a) for $(l_1', l_2') \in \{l_1, l_1+1, \cdots, Q_1\} \times \{l_2, l_2+1, \cdots, Q_2\}$,

$$\begin{aligned}&(\mathbf{M}_{4,j})_{(l_1,l_2),(l_1',l_2')}\\&= \sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2} \boldsymbol{\delta}(Q_2, l_2+i_2)\left\langle \overline{\mathbf{B}_2} \circ \mathbf{H}_j(i_1,i_2)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1+i_1)\,\boldsymbol{\delta}(Q_2, l_2')\\&\quad\times \left\langle \mathbf{R}_2(l_1'-l_1-i_1, l_2'-l_2-i_2)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1'),\end{aligned}$$

(b) for the other $(l_1', l_2') \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$

$$(\mathbf{M}_{4,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

(5) The matrix $\mathbf{M}_5$ describes the one-step transitions among the states in $S_\xi^{(2)}$. Within $S_\xi^{(2)}$, the state with $\phi = (2, j)$ may only transfer to the state with the same $\phi = (2, j)$, where $j \in \{0, 1, \cdots, Q_1 - 1\}$. So, $\mathbf{M}_5$ has the following structure.

$$\mathbf{M}_5 = diag\left(\mathbf{M}_{5,0} \quad \mathbf{M}_{5,1} \quad \cdots \quad \mathbf{M}_{5,Q_1-1}\right),$$

where $\mathbf{M}_{5,j}$, $j \in \{0, 1, \cdots, Q_1 - 1\}$, consists of matrix blocks which are indexed by $\left((l_1, l_2), (l_1', l_2')\right)$, $(l_1, l_2), (l_1', l_2') \in \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)}$, where

$$(\mathbf{M}_{5,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{(2, j), l_1', l_2' | (2, j), l_1, l_2\right\}.$$

Given j, consider each $(l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \{0\}$, for any $(l_1', l_2') \in \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)}$,

$$(\mathbf{M}_{5,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}};$$

consider each $(l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \left(\mathbb{L}_2^{(2)} \setminus \{0\}\right)$,

(a) for $(l_1', l_2') \in \{l_1, l_1+1, \cdots, Q_1\} \times \{l_2-1, l_2, \cdots, Q_2-1\}$,

$$\begin{aligned}&(\mathbf{M}_{5,j})_{(l_1,l_2),(l_1',l_2')}\\&= \boldsymbol{\delta}(Q_2, l_2'+1)\\&\quad\times \left\langle \overline{\mathbf{H}_j} \circ \mathbf{B}_2(l_1'-l_1, l_2'-l_2+1)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1'),\end{aligned}$$

(b) for the other $(l_1', l_2') \in \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)}$,

$$(\mathbf{M}_{5,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

(6) The matrix $\mathbf{M}_6$ describes the one-step transitions of the states from $S_\xi^{(2)}$ to $S_\xi^{(s_2)}$. Any state in $S_\xi^{(2)}$ may enter into $S_\xi^{(s_2)}$. So, $\mathbf{M}_6$ has the following structure.

$$\mathbf{M}_6 = \left(\mathbf{M}_{6,0} \quad \mathbf{M}_{6,1} \quad \mathbf{M}_{6,2} \quad \cdots \quad \mathbf{M}_{6,Q_1-1}\right)^T,$$

where $\mathbf{M}_{6,j}$, $j \in \{0, 1, \cdots, Q_1 - 1\}$, consists of matrix blocks which are indexed by $\left((l_1, l_2), (l_1', l_2')\right)$, $(l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)}$, $(l_1', l_2') \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$, where

$$(\mathbf{M}_{6,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{s_2, l_1', l_2' | (2, j), l_1, l_2\right\}.$$

Given j, consider each $(l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \{0\}$,

(a) for $(l_1', l_2') \in \{l_1, l_1+1, \cdots, Q_1\} \times \mathbb{L}_2^{(s_2)}$,

$$\begin{aligned}&(\mathbf{M}_{6,j})_{(l_1,l_2),(l_1',l_2')}\\&= \boldsymbol{\delta}(Q_2, l_2')\left\langle \mathbf{R}_2(l_1'-l_1, l_2')\right\rangle \boldsymbol{\delta}^T(Q_1, l_1'),\end{aligned}$$

(b) for the other $(l_1', l_2') \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$,

$$(\mathbf{M}_{6,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}};$$

consider each $(l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \left(\mathbb{L}_2^{(2)} \setminus \{0\}\right)$,

(a) for $(l_1', l_2') \in \{l_1, l_1+1, \cdots, Q_1\} \times \{l_2, l_2+1, \cdots, Q_2\}$,

$$\begin{aligned}&(\mathbf{M}_{6,j})_{(l_1,l_2),(l_1',l_2')}\\&= \sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2} \boldsymbol{\delta}(Q_2, l_2+i_2)\left\langle \overline{\mathbf{B}_2} \circ \mathbf{H}_j(i_1,i_2)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1+i_1)\,\boldsymbol{\delta}(Q_2, l_2')\\&\quad\times \left\langle \mathbf{R}_2(l_1'-l_1-i_1, l_2'-l_2-i_2)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1'),\end{aligned}$$

(b) for the other $(l_1', l_2') \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$,

$$(\mathbf{M}_{6,j})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

(7) The matrix $\mathbf{M}_7$ describes the one-step transitions of the states from $S_\xi^{(s_2)}$ to $S_\xi^{(1)}$. Only the state with $\phi = (1, i, 1)$ in $S_\xi^{(1)}$ can be reached, where $i \in \{1, 2, \cdots, Q_1\}$. So, $\mathbf{M}_7$ has the following structure.

$$\mathbf{M}_7 = \left(\mathbf{M}_{7,1,1} \quad \mathbf{M}_{7,1,2} \quad \mathbf{M}_{7,1,3} \quad \cdots \quad \mathbf{M}_{7,1,Q_1}\right),$$

where

$$\mathbf{M}_{7,1,1} = \mathbf{M}_{7,1},$$
$$\mathbf{M}_{7,1,i} = \left(\mathbf{M}_{7,i} \quad \mathbf{0}_{\varsigma_0 \times S_{Q_1}^{i-1}}\right), \quad i \in \{2, 3, \cdots, Q_1\}.$$

$\mathbf{M}_{7,i}$, $i \in \{1, 2, \cdots, Q_1\}$, consists of matrix blocks which are indexed by $\left((l_1, l_2), (l_1', l_2')\right)$, $(l_1, l_2) \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$, $(l_1', l_2') \in \mathbb{L}_1^{(1,i-1)} \times \mathbb{L}_2^{(1)}$, where

$$(\mathbf{M}_{7,i})_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\left\{(1, i, 1), l_1', l_2' | s_2, l_1, l_2\right\}.$$

Given i, $i \in \{1, 2, \cdots, Q_1\}$, consider $(l_1, l_2) \in \left(\mathbb{L}_1^{(s_2)} \setminus \{i\}\right) \times \mathbb{L}_2^{(s_2)}$, for any $(l_1', l_2') \in \mathbb{L}_1^{(1,i-1)} \times \mathbb{L}_2^{(1)}$,

$$(\mathbf{M}_{7,i})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}};$$

consider $(l_1, l_2) \in \{i\} \times \mathbb{L}_2^{(s_2)}$,

(a) for $(l_1', l_2') \in \mathbb{L}_1^{(1,i-1)} \times \{l_2, l_2+1, \cdots, Q_2\}$,

$$\begin{aligned}&(\mathbf{M}_{7,i})_{(l_1,l_2),(l_1',l_2')}\\&= \boldsymbol{\delta}(Q_2, l_2')\left\langle \mathbf{B}_1(l_1'-l_1+1, l_2'-l_2)\right\rangle\\&\quad\times \boldsymbol{\delta}^T(Q_1, l_1'+1),\end{aligned}$$

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(b) for the other $(l_1', l_2') \in \mathbb{L}_1^{(1,i-1)} \times \mathbb{L}_2^{(1)}$,

$$(\mathbf{M}_{7,i})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

(8) The matrix $\mathbf{M}_8$ describes the one-step transitions of the states from $S_\xi^{(s_2)}$ to $S_\xi^{(s_1)}$. Only the state with $\phi = (s_2, 0)$ in $S_\xi^{(s_1)}$ can be reached. So, $\mathbf{M}_8$ has the following structure.

$$\mathbf{M}_8 = \begin{pmatrix} \mathbf{M}_{8,0} & \mathbf{0}_{\varsigma_0 \times S_{Q_1}^{Q_1-1}} \end{pmatrix},$$

where $\mathbf{M}_{8,0}$ consists of matrix blocks which are indexed by $\big((l_1,l_2),(l_1',l_2')\big)$, $(l_1,l_2) \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}$, $(l_1',l_2') \in \mathbb{L}_1^{(s_1,0)} \times \mathbb{L}_2^{(s_1)}$, where

$$(\mathbf{M}_{8,0})_{(l_1,l_2),(l_1',l_2')} = \mathbf{P}\big\{(s_1,0), l_1', l_2' | s_2, l_1, l_2\big\}.$$

Consider $(l_1,l_2) \in \left(\mathbb{L}_1^{(s_2)} \setminus \{0\}\right) \times \mathbb{L}_2^{(s_2)}$, for any $(l_1',l_2') \in \mathbb{L}_1^{(s_1,0)} \times \mathbb{L}_2^{(s_1)}$,

$$(\mathbf{M}_{8,0})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}};$$

consider $(l_1,l_2) \in \{0\} \times \mathbb{L}_2^{(s_2)}$,

(a) for $(l_1',l_2') \in \mathbb{L}_1^{(s_1,0)} \times \{l_2, l_2+1, \cdots, Q_2\}$,

$$\begin{aligned}&(\mathbf{M}_{8,0})_{(l_1,l_2),(l_1',l_2')}\\ &= \boldsymbol{\delta}(Q_2, l_2') \langle \mathbf{R}_1 (l_1', l_2' - l_2) \rangle \boldsymbol{\delta}^T (Q_1, l_1'),\end{aligned}$$

(b) for the other $(l_1',l_2') \in \mathbb{L}_1^{(s_1,0)} \times \mathbb{L}_2^{(s_1)}$,

$$(\mathbf{M}_{8,0})_{(l_1,l_2),(l_1',l_2')} = \mathbf{0}_{\bar{m}}.$$

The states of the Markov chain $\{\xi_n; n \in \mathrm{N}^+\}$ satisfy the properties.

Property 1: For the zero state $(s_2, 0, 0, v_1, v_2)$, $v_1 \in M_1$, $v_2 \in M_2$, it can be reached from any state in S_ξ. The reason is that the BMAP-1 and the BMAP-2 are independent; $\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$ are irreducible; $\mathbf{D}_0^{(1)}$ and $\mathbf{D}_0^{(2)}$ are stable.

Property 2: There is a state subspace denoted by $\hat{S}_\xi$, which consists of the states $(s_2, 0, 0, v_1, v_2)$, $v_1 \in M_1$, $v_2 \in M_2$, and the states which can be reached from any of the states $(s_2, 0, 0, v_1, v_2)$, $v_1 \in M_1$, $v_2 \in M_2$. So, $\hat{S}_\xi$ is irreducible and aperiodic. From this and Property 1, it follows that the Markov chain $\{\xi_n; n \in \mathrm{N}^+\}$ has the stationary distribution in its state space S_ξ.

B. THE JOINT QUEUE LENGTH STATIONARY DISTRIBUTION AT QUEUE 1 POLLING EPOCHS

Consider the event that a switchover of the server from queue 2 to queue 1 just terminates. Let T_n' $(n \in \mathrm{N}^+)$ be the instant when this event occurs at the n-th time. The state of the cyclic polling model at T_n' is denoted by $\xi'(T_n') = \big(L_1(T_n'), L_2(T_n'), V_1(T_n'), V_2(T_n')\big)$, where $L_\iota(T_n')$ and $V_\iota(T_n')$ represent the same meanings as the ones corresponding to $\xi(T_n)$. $\xi'(T_n')$ is briefly denoted by $\xi_n' = \big(L_{1,n}', L_{2,n}', V_{1,n}', V_{2,n}'\big)$. The discrete-time stochastic process $\{\xi_n'; n \in \mathrm{N}^+\}$ is constructed, and it is a homogeneous Markov chain on the state space $S_\xi' = \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)} \times M_1 \times M_2$. Based on the one-step transition probability matrix $\mathbf{M}$ of the Markov chain $\{\xi_n; n \in \mathrm{N}^+\}$, the one-step transition probability matrix $\mathbf{W}$ of the Markov chain $\{\xi_n'; n \in \mathrm{N}^+\}$ can be given by

$$\begin{aligned}\mathbf{W} = &\left[\mathbf{M}_{7,1} + \sum_{i=2}^{Q_1} \mathbf{M}_{7,i} \prod_{k=2}^{i} (\mathbf{M}_{1,i})_{k-1,k}\right]\\ &\times \sum_{j=0}^{Q_1-1} \mathbf{M}_{2,j} \left[\mathbf{M}_{4,j} + \mathbf{M}_{3,j} \left(\sum_{k=0}^{\infty} \mathbf{M}_{5,j}^k\right) \mathbf{M}_{6,j}\right]\\ &+ \mathbf{M}_{8,0} \left[\mathbf{M}_{4,0} + \mathbf{M}_{3,0} \left(\sum_{k=0}^{\infty} \mathbf{M}_{5,0}^k\right) \mathbf{M}_{6,0}\right]. \quad (8)\end{aligned}$$

Based on Property 1 and Property 2, it can be proved by contradiction that the Markov chain $\{\xi_n'; n \in \mathrm{N}^+\}$ has the stationary distribution in S_ξ'. Let the probability vector $\boldsymbol{\omega}$ denote the stationary distribution of $\{\xi_n'; n \in \mathrm{N}^+\}$, i.e.

$$\boldsymbol{\omega} = \left(\boldsymbol{\omega}_{(l_1,l_2)} : (l_1,l_2) \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}\right),$$

where

$$\begin{aligned}\boldsymbol{\omega}_{(l_1,l_2)} &= \left(\left(\boldsymbol{\omega}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1,v_2) \in M_1 \times M_2\right),\\ \left(\boldsymbol{\omega}_{(l_1,l_2)}\right)_{(v_1,v_2)} &= \lim_{n\to\infty} P\{\xi_n' = (l_1,l_2,v_1,v_2)\}.\end{aligned}$$

$\boldsymbol{\omega}$ is also the joint queue length stationary distribution at queue 1 polling epochs, where $\left(\boldsymbol{\omega}_{(l_1,l_2)}\right)_{(v_1,v_2)}$ represents the stationary probability that, at queue 1 polling epochs, the length of queue 1 is l_1, the length of queue 2 is l_2, the phase of the BMAP-1 is v_1, and the phase of the BMAP-2 is v_2. $\boldsymbol{\omega}$ satisfies the following relations,

$$\begin{cases}\boldsymbol{\omega}\mathbf{W} = \boldsymbol{\omega},\\ \boldsymbol{\omega}\mathbf{e} = 1.\end{cases} \quad (9)$$

Based on the GTH algorithm [36], $\boldsymbol{\omega}$ can be obtained by solving the system of linear equations (9).

C. THE JOINT QUEUE LENGTH STATIONARY DISTRIBUTION AT QUEUE 2 POLLING EPOCHS

Consider the event that a switchover of the server from queue 1 to queue 2 just terminates. Let T_n'' $(n \in \mathrm{N}^+)$ be the instant when this event occurs at the n-th time. The state of the cyclic polling model at T_n'' is denoted by

$$\xi''(T_n'') = \big(\Phi(T_n''), L_1(T_n''), L_2(T_n''), V_1(T_n''), V_2(T_n'')\big),$$

where $\Phi(T_n'')$, $L_\iota(T_n'')$ and $V_\iota(T_n'')$ represent the same meanings as the ones corresponding to $\xi(T_n)$. $\xi''(T_n'')$ is briefly denoted by $\xi_n'' = \big(\Phi_n'', L_{1,n}'', L_{2,n}'', V_{1,n}'', V_{2,n}''\big)$. The discrete-time stochastic process $\{\xi_n''; n \in \mathrm{N}^+\}$ is constructed, and it is a homogeneous Markov chain on the state space $S_\xi^{(s_1)}$.

Based on Property 1 and Property 2, it can be proved by contradiction that the Markov chain $\{\xi_n''; n \in \mathrm{N}^+\}$ has the stationary distribution in $S_\xi^{(s_1)}$. Let the probability vector $\boldsymbol{\theta}$ denote the stationary distribution of $\{\xi_n''; n \in \mathrm{N}^+\}$.

$$\boldsymbol{\theta} = \begin{pmatrix}\boldsymbol{\theta}^{(s_1,0)} & \boldsymbol{\theta}^{(s_1,1)} & \cdots & \boldsymbol{\theta}^{(s_1,Q_1-1)}\end{pmatrix},$$

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where for each $j \in \{0, 1, \cdots, Q_1 - 1\}$,

$$\boldsymbol{\theta}^{(s_1,j)} = \left(\boldsymbol{\theta}^{(s_1,j)}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1^{(s_1,j)} \times \mathbb{L}_2^{(s_1)}\right),$$
$$\boldsymbol{\theta}^{(s_1,j)}_{(l_1,l_2)} = \left(\left(\boldsymbol{\theta}^{(s_1,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\theta}^{(s_1,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \lim_{n\to\infty} P\left\{\xi''_n = \left((s_1, j), l_1, l_2, v_1, v_2\right)\right\}.$$

There is the following relationship between $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$.

$$\boldsymbol{\theta}^{(s_1,0)} = \boldsymbol{\omega}\left\{\left[\mathbf{M}_{7,1} + \sum_{i=2}^{Q_1} \mathbf{M}_{7,i} \prod_{k=2}^{i} (\mathbf{M}_{1,i})_{k-1,k}\right] \mathbf{M}_{2,0} + \mathbf{M}_{8,0}\right\}, \tag{10}$$

$$\boldsymbol{\theta}^{(s_1,j)} = \boldsymbol{\omega}\left[\mathbf{M}_{7,1} + \sum_{i=2}^{Q_1} \mathbf{M}_{7,i} \prod_{k=2}^{i} (\mathbf{M}_{1,i})_{k-1,k}\right] \times \mathbf{M}_{2,j}, \quad j \in \{1, 2, \cdots, Q_1 - 1\}. \tag{11}$$

Let the probability vector $\boldsymbol{\eta}$ denote the joint queue length stationary distribution at queue 2 polling epochs.

$$\boldsymbol{\eta} = \left(\boldsymbol{\eta}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1 \times \mathbb{L}_2\right),$$

where $\mathbb{L}_1 = \{0, 1, \cdots, Q_1\}$, $\mathbb{L}_2 = \{0, 1, \cdots, Q_2\}$,

$$\boldsymbol{\eta}_{(l_1,l_2)} = \left(\left(\boldsymbol{\eta}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\eta}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \sum_{j=0}^{l_1} \left(\boldsymbol{\theta}^{(s_1,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)}. \tag{12}$$

$\left(\boldsymbol{\eta}_{(l_1,l_2)}\right)_{(v_1,v_2)}$ represents the stationary probability that, at queue 2 polling epochs, the length of queue 1 is l_1, the length of queue 2 is l_2, the phase of the BMAP-1 is v_1, and the phase of the BMAP-2 is v_2.

IV. THE JOINT QUEUE LENGTH STATIONARY DISTRIBUTION AT ARBITRARY TIME

From Property 2, the Markov chain $\{\xi_n; n \in \mathrm{N}^+\}$ has the stationary distribution in the state space S_ξ. Let the stationary distribution be denoted by the probability vector $\boldsymbol{\pi}$, which satisfies the following relations,

$$\boldsymbol{\pi}\mathbf{M} = \boldsymbol{\pi}, \quad \boldsymbol{\pi}\mathbf{e} = 1. \tag{13}$$

$\boldsymbol{\pi}$ can be divided into four parts, i.e.

$$\boldsymbol{\pi} = \left(\boldsymbol{\pi}^{(1)} \quad \boldsymbol{\pi}^{(s_1)} \quad \boldsymbol{\pi}^{(2)} \quad \boldsymbol{\pi}^{s_2}\right).$$

(1)

$$\boldsymbol{\pi}^{(1)} = \left(\boldsymbol{\pi}^{(1,1,1)} \quad \boldsymbol{\pi}^{(1,2,1)} \quad \boldsymbol{\pi}^{(1,2,2)} \quad \cdots \quad \boldsymbol{\pi}^{(1,Q_1,1)} \quad \boldsymbol{\pi}^{(1,Q_1,2)} \quad \cdots \quad \boldsymbol{\pi}^{(1,Q_1,Q_1)}\right),$$

where for $i \in \{1, 2, \cdots, Q_1\}$, $k \in \{1, 2, \cdots, i\}$,

$$\boldsymbol{\pi}^{(1,i,k)} = \left(\boldsymbol{\pi}^{(1,i,k)}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1^{(1,i-k)} \times \mathbb{L}_2^{(1)}\right),$$
$$\boldsymbol{\pi}^{(1,i,k)}_{(l_1,l_2)} = \left(\left(\boldsymbol{\pi}^{(1,i,k)}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\pi}^{(1,i,k)}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \lim_{n\to\infty} P\left\{\xi_n = \left((1, i, k), l_1, l_2, v_1, v_2\right)\right\};$$

(2)

$$\boldsymbol{\pi}^{(s_1)} = \left(\boldsymbol{\pi}^{(s_1,0)} \quad \boldsymbol{\pi}^{(s_1,1)} \quad \cdots \quad \boldsymbol{\pi}^{(s_1,Q_1-1)}\right),$$

where for $j \in \{0, 1, \cdots, Q_1 - 1\}$,

$$\boldsymbol{\pi}^{(s_1,j)} = \left(\boldsymbol{\pi}^{(s_1,j)}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1^{(s_1,j)} \times \mathbb{L}_2^{(s_1)}\right),$$
$$\boldsymbol{\pi}^{(s_1,j)}_{(l_1,l_2)} = \left(\left(\boldsymbol{\pi}^{(s_1,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\pi}^{(s_1,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \lim_{n\to\infty} P\left\{\xi_n = \left((s_1, j), l_1, l_2, v_1, v_2\right)\right\};$$

(3)

$$\boldsymbol{\pi}^{(2)} = \left(\boldsymbol{\pi}^{(2,0)} \quad \boldsymbol{\pi}^{(2,1)} \quad \cdots \quad \boldsymbol{\pi}^{(2,Q_1-1)}\right),$$

where for $j \in \{0, 1, \cdots, Q_1 - 1\}$,

$$\boldsymbol{\pi}^{(2,j)} = \left(\boldsymbol{\pi}^{(2,j)}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1^{(2,j)} \times \mathbb{L}_2^{(2)}\right),$$
$$\boldsymbol{\pi}^{(2,j)}_{(l_1,l_2)} = \left(\left(\boldsymbol{\pi}^{(2,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\pi}^{(2,j)}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \lim_{n\to\infty} P\left\{\xi_n = \left((2, j), l_1, l_2, v_1, v_2\right)\right\};$$

(4)

$$\boldsymbol{\pi}^{s_2} = \left(\boldsymbol{\pi}^{s_2}_{(l_1,l_2)} : (l_1, l_2) \in \mathbb{L}_1^{(s_2)} \times \mathbb{L}_2^{(s_2)}\right),$$

where

$$\boldsymbol{\pi}^{s_2}_{(l_1,l_2)} = \left(\left(\boldsymbol{\pi}^{s_2}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2\right),$$
$$\left(\boldsymbol{\pi}^{s_2}_{(l_1,l_2)}\right)_{(v_1,v_2)} = \lim_{n\to\infty} P\{\xi_n = (s_2, l_1, l_2, v_1, v_2)\}.$$

Based on the relations in (13) and the structures of $\mathbf{M}$ and $\boldsymbol{\pi}$, there are the following relations.

$$\boldsymbol{\pi}^{(1,i,1)} = \boldsymbol{\pi}^{s_2}\mathbf{M}_{7,i}, \quad i \in \{1, 2, \cdots, Q_1\}, \tag{14}$$

$$\boldsymbol{\pi}^{(1,i,k)} = \boldsymbol{\pi}^{(1,i,k-1)}(\mathbf{M}_{1,i})_{k-1,k}, \quad i \in \{2, 3, \cdots, Q_1\}, \quad k \in \{2, 3, \cdots, i\}; \tag{15}$$

$$\boldsymbol{\pi}^{(s_1,0)} = \boldsymbol{\pi}^{s_2}\mathbf{M}_{8,0} + \sum_{i=1}^{Q_1} \boldsymbol{\pi}^{(1,i,i)}\mathbf{M}_{2,0}, \tag{16}$$

$$\boldsymbol{\pi}^{(s_1,j)} = \sum_{i=1}^{Q_1} \boldsymbol{\pi}^{(1,i,i)}\mathbf{M}_{2,j}, \; j \in \{1, 2, \cdots, Q_1 - 1\}; \tag{17}$$

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$$\boldsymbol{\pi}^{(2,j)} = \boldsymbol{\pi}^{(s_1,j)}\mathbf{M}_{3,j}(\mathbf{I}-\mathbf{M}_{5,j})^{-1},$$
$$j \in \{0,1,\cdots,Q_1-1\}. \quad (18)$$

From (14), (15), (16), (17) and (18), it can be seen that if $\boldsymbol{\pi}^{s_2}$ is given, the other parts of $\boldsymbol{\pi}$ can be calculated directly. There is a constant c $(0<c<\infty)$, such that $\boldsymbol{\pi}^{s_2} = c\boldsymbol{\omega}$. So, the stationary distribution $\boldsymbol{\pi}$ can be obtained as the following way. First, set $\boldsymbol{\pi}^{s_2} = \boldsymbol{\omega}$; then from the relations (14), (15), (16), (17) and (18), the vector $\boldsymbol{\pi}$ is calculated; finally, the stationary distribution is obtained by normalizing $\boldsymbol{\pi}$.

Let
$$\tilde{S}_\xi = \tilde{S}_\xi^{(1)} \bigcup \tilde{S}_\xi^{(s_1)} \bigcup \tilde{S}_\xi^{(2)} \bigcup \tilde{S}_\xi^{(s_2)},$$
where
$$\tilde{S}_\xi^{(1)} = \bigcup_{i=1}^{Q_1}\bigcup_{k=1}^{i}\{(1,i,k)\}\times\mathbb{L}_1^{(1,i-k)}\times\mathbb{L}_2^{(1)},$$
$$\tilde{S}_\xi^{(s_1)} = \bigcup_{j=0}^{Q_1-1}\{(s_1,j)\}\times\mathbb{L}_1^{(s_1,j)}\times\mathbb{L}_2^{(s_1)},$$
$$\tilde{S}_\xi^{(2)} = \bigcup_{j=0}^{Q_1-1}\{(2,j)\}\times\mathbb{L}_1^{(2,j)}\times\mathbb{L}_2^{(2)},$$
$$\tilde{S}_\xi^{(s_2)} = \{s_2\}\times\mathbb{L}_1^{(s_2)}\times\mathbb{L}_2^{(s_2)}.$$

From $\{\xi_n; n\in\mathrm{N}^+\}$, a Markov renewal process $\{\xi_n, T_n; n\in\mathrm{N}^+\}$ can be constructed. The mean time between two successive renewals of the Markov renewal process $\{\xi_n, T_n; n\in\mathrm{N}^+\}$ is denoted by τ, and it can be calculated by
$$\begin{aligned}\tau =& \sum_{i=1}^{Q_1}\sum_{k=1}^{i}\sum_{l_1=i-k}^{Q_1-1}\sum_{l_2=0}^{Q_2}\boldsymbol{\pi}_{(l_1,l_2)}^{(1,i,k)}\mathbf{m}_{(l_1,l_2)}^{(1,i,k)}\\ &+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{Q_1}\sum_{l_2=0}^{Q_2}\boldsymbol{\pi}_{(l_1,l_2)}^{(s_1,j)}\mathbf{m}_{(l_1,l_2)}^{(s_1,j)}\\ &+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{Q_1}\sum_{l_2=0}^{Q_2-1}\boldsymbol{\pi}_{(l_1,l_2)}^{(2,j)}\mathbf{m}_{(l_1,l_2)}^{(2,j)}\\ &+\sum_{l_1=0}^{Q_1}\sum_{l_2=0}^{Q_2}\boldsymbol{\pi}_{(l_1,l_2)}^{s_2}\mathbf{m}_{(l_1,l_2)}^{s_2}. \end{aligned} \quad (19)$$

(1) Given $\left((1,i,k),l_1,l_2\right)\in\tilde{S}_\xi^{(1)}$, $\mathbf{m}_{(l_1,l_2)}^{(1,i,k)}$ is a $\bar{m}$-dimensional column vector,
$$\mathbf{m}_{(l_1,l_2)}^{(1,i,k)} = \left(\left(\mathbf{m}_{(l_1,l_2)}^{(1,i,k)}\right)_{(v_1,v_2)} : (v_1,v_2)\in M_1\times M_2\right)^T,$$
where the (v_1,v_2)-th element denotes the mean time from a renewal with the state $\left((1,i,k),l_1,l_2,v_1,v_2\right)$ to the next renewal. For any $\left((1,i,k),l_1,\ l_2\right)\in\tilde{S}_\xi^{(1)}$,
$$\mathbf{m}_{(l_1,l_2)}^{(1,i,k)} = \begin{cases} b_1\mathbf{e}_{\bar{m}}, & \text{for } k<i,\\ r_1\mathbf{e}_{\bar{m}}, & \text{for } k=i.\end{cases}$$

(2) Given $\left((s_1,j),l_1,l_2\right)\in\tilde{S}_\xi^{(s_1)}$, $\mathbf{m}_{(l_1,l_2)}^{(s_1,j)}$ is a $\bar{m}$-dimensional column vector,
$$\mathbf{m}_{(l_1,l_2)}^{(s_1,j)} = \left(\left(\mathbf{m}_{(l_1,l_2)}^{(s_1,j)}\right)_{(v_1,v_2)} : (v_1,v_2)\in M_1\times M_2\right)^T,$$
where the (v_1,v_2)-th element denotes the mean time from a renewal with the state $\left((s_1,j),l_1,l_2,v_1,v_2\right)$ to the next renewal. Consider each $\left((s_1,j),\ l_1,l_2\right)\in\tilde{S}_\xi^{(s_1)}$,

(a) for $l_2=0$,
$$\mathbf{m}_{(l_1,l_2)}^{(s_1,j)} = r_2\mathbf{e}_{\bar{m}};$$

(b) for $l_2\neq 0$,
$$\begin{aligned}\mathbf{m}_{(l_1,l_2)}^{(s_1,j)} &= \int_{t=0}^{\infty}\Big\{1-\int_{x=0}^{t}\left[1-H_j(x)\right]dB_2(x)\\ &\quad-\int_{x=0}^{t}\left[1-B_2(x)\right]dH_j(x)\\ &\quad\times R_2(t-x)\Big\}dt\mathbf{e}_{\bar{m}}\\ &= \Big\{\boldsymbol{\beta}_2\left(\gamma_j\mathbf{I}_{m_{B_2}}-\mathbf{B}_2\right)^{-1}\mathbf{e}_{m_{B_2}}-\boldsymbol{\alpha}_2\mathbf{R}_2^{-1}\mathbf{e}_{m_{R_2}}\\ &\quad\times\left[\gamma_j\boldsymbol{\beta}_2\left(\gamma_j\mathbf{I}_{m_{B_2}}-\mathbf{B}_2\right)^{-1}\mathbf{e}_{m_{B_2}}\right]\Big\}\mathbf{e}_{\bar{m}}.\end{aligned}$$

(3) Given $\left((2,j),l_1,l_2\right)\in\tilde{S}_\xi^{(2)}$, $\mathbf{m}_{(l_1,l_2)}^{(2,j)}$ is a $\bar{m}$-dimensional column vector,
$$\mathbf{m}_{(l_1,l_2)}^{(2,j)} = \left(\left(\mathbf{m}_{(l_1,l_2)}^{(2,j)}\right)_{(v_1,v_2)} : (v_1,v_2)\in M_1\times M_2\right)^T,$$
where the (v_1,v_2)-th element denotes the mean time from a renewal with the state $\left((2,j),l_1,l_2,v_1,v_2\right)$ to the next renewal. Consider each $\left((2,j),l_1,\ l_2\right)\in\tilde{S}_\xi^{(2)}$,

(a) for $l_2=0$,
$$\mathbf{m}_{(l_1,l_2)}^{(2,j)} = r_2\mathbf{e}_{\bar{m}};$$

(b) for $l_2\neq 0$,
$$\begin{aligned}\mathbf{m}_{(l_1,l_2)}^{(2,j)} &= \int_{t=0}^{\infty}\Big\{1-\int_{x=0}^{t}\left[1-H_j(x)\right]dB_2(x)\\ &\quad-\int_{x=0}^{t}\left[1-B_2(x)\right]dH_j(x)\\ &\quad\times R_2(t-x)\Big\}dt\mathbf{e}_{\bar{m}}\\ &= \Big\{\boldsymbol{\beta}_2\left(\gamma_j\mathbf{I}_{m_{B_2}}-\mathbf{B}_2\right)^{-1}\mathbf{e}_{m_{B_2}}-\boldsymbol{\alpha}_2\mathbf{R}_2^{-1}\mathbf{e}_{m_{R_2}}\\ &\quad\times\left[\gamma_j\boldsymbol{\beta}_2\left(\gamma_j\mathbf{I}_{m_{B_2}}-\mathbf{B}_2\right)^{-1}\mathbf{e}_{m_{B_2}}\right]\Big\}\mathbf{e}_{\bar{m}}.\end{aligned}$$

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(4) Given $(s_2, l_1, l_2) \in \tilde{S}_{\xi}^{(s_2)}$, $\mathbf{m}_{(l_1,l_2)}^{s_2}$ is a $\bar{m}$-dimensional column vector,

$$\mathbf{m}_{(l_1,l_2)}^{s_2} = \left(\left(\mathbf{m}_{(l_1,l_2)}^{s_2} \right)_{(v_1,v_2)} : (v_1, v_2) \in M_1 \times M_2 \right)^T,$$

where the (v_1, v_2)-th element denotes the mean time from a renewal with the state $(s_2, l_1, l_2, v_1, v_2)$ to the next renewal. Consider each $(s_2, l_1, l_2) \in \tilde{S}_{\xi}^{(s_2)}$,

(a) for $l_1 = 0$,
$$\mathbf{m}_{(l_1,l_2)}^{s_2} = r_1 \mathbf{e}_{\bar{m}};$$

(b) for $l_1 \neq 0$,
$$\mathbf{m}_{(l_1,l_2)}^{s_2} = b_1 \mathbf{e}_{\bar{m}}.$$

Let $\chi(t) = \left(L_1(t), L_2(t), V_1(t), V_2(t)\right)$ denote the state of the cyclic polling model at arbitrary time t $(t \geq 0)$, where $L_\iota(t)$ and $V_\iota(t)$ represent the same meanings as the ones corresponding to $\xi(T_n)$ in Section III-A, $L_\iota(t) \in \mathbb{L}_\iota$, $V_\iota(t) \in M_\iota$. Consider the stochastic process $\{\chi(t); t \geq 0\}$ on the state space $S = \mathbb{L}_1 \times \mathbb{L}_2 \times M_1 \times M_2$, and assume that for any sample path $t \to \chi(t)$, it is right continuous and has left-hand limit. According to the definition 10.6.1 of [31], $\{\chi(t); t \geq 0\}$ is a semi-regenerative process with the corresponding Markov renewal process $\{\xi_n, T_n; n \in \mathrm{N}^+\}$.

For $(\phi, l_1, l_2) \in \tilde{S}_{\xi}$, $(l_1', l_2') \in \mathbb{L}_1 \times \mathbb{L}_2$, define the matrix $\mathbf{K}\left((\phi, l_1, l_2), (l_1', l_2'), t\right)$,

$$\begin{aligned}
&\mathbf{K}\left((\phi, l_1, l_2), (l_1', l_2'), t\right) \\
&= \Big(\mathbf{K}_{(v_1,v_2),(v_1',v_2')}\left((\phi, l_1, l_2), (l_1', l_2'), t\right) \\
&\qquad : (v_1, v_2), (v_1', v_2') \in M_1 \times M_2 \Big),
\end{aligned}$$

where the $\left((v_1, v_2), (v_1', v_2')\right)$-th element denotes the following conditional probability,

$$\begin{aligned}
&\mathbf{K}_{(v_1,v_2),(v_1',v_2')}\left((\phi, l_1, l_2), (l_1', l_2'), t\right) \\
&= P\Big\{ L_1(t) = l_1',\ L_2(t) = l_2',\ V_1(t) = v_1',\ V_2(t) = v_2', \\
&\qquad t < T \mid \Phi(0) = \phi,\ L_1(0) = l_1,\ L_2(0) = l_2, \\
&\qquad V_1(0) = v_1,\ V_2(0) = v_2 \Big\},
\end{aligned}$$

T is a random variable denoting the time from the renewal with the state $(\phi, l_1, l_2, v_1, v_2)$ to the next renewal of the Markov renewal process $\{\xi_n, T_n; n \in \mathrm{N}^+\}$. The expression for each $\mathbf{K}\left((\phi, l_1, l_2), (l_1', l_2'), t\right)$ is given in Appendix B. And

$$\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{\phi} = \int_{t=0}^{\infty} \mathbf{K}\left((\phi, l_1, l_2), (l_1', l_2'), t\right) dt < \infty,$$

where the calculation formula of each $\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{\phi}$ is also given in Appendix B.

Based on Property 1 and Property 2, $\{\xi_n, T_n; n \in \mathrm{N}^+\}$ can enter into the state subspace $\hat{S}_{\xi}$ eventually and is an irreducible aperiodic recurrent process in $\hat{S}_{\xi}$. According to Theorem 10.6.12 of [31], $\{\chi(t); t \geq 0\}$ has the stationary distribution denoted by $\boldsymbol{p}$, as $t \to \infty$.

$$\boldsymbol{p} = \left(\boldsymbol{p}_{(l_1',l_2')} : (l_1', l_2') \in \mathbb{L}_1 \times \mathbb{L}_2 \right),$$

where

$$\boldsymbol{p}_{(l_1',l_2')} = \left(\left(\boldsymbol{p}_{(l_1',l_2')} \right)_{(v_1',v_2')} : (v_1', v_2') \in M_1 \times M_2 \right),$$
$$\left(\boldsymbol{p}_{(l_1',l_2')} \right)_{(v_1',v_2')} = \lim_{t \to \infty} P\left\{ \chi(t) = (l_1', l_2', v_1', v_2') \right\}.$$

$\boldsymbol{p}$ is also the joint queue length stationary distribution at arbitrary time, where $\left(\boldsymbol{p}_{(l_1',l_2')} \right)_{(v_1',v_2')}$ represents the stationary probability that, at arbitrary time, the length of queue 1 is l_1', the length of queue 2 is l_2', the phase of the BMAP-1 is v_1', and the phase of the BMAP-2 is v_2'. $\boldsymbol{p}$ can be calculated as follows. Given $(l_1', l_2') \in \mathbb{L}_1 \times \mathbb{L}_2$,

$$\boldsymbol{p}_{(l_1',l_2')} = \frac{1}{\tau} \sum_{(\phi,l_1,l_2) \in \tilde{S}_{\xi}} \boldsymbol{\pi}_{(l_1,l_2)}^{\phi} \mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{\phi}.$$

(1) For $l_1' = 0, l_2' = 0$,

$$\begin{aligned}
\boldsymbol{p}_{(0,0)} = &\frac{1}{\tau} \Bigg\{ \sum_{i=1}^{Q_1} \boldsymbol{\pi}_{(0,0)}^{(1,i,i)} \mathbf{K}_{(0,0),(0,0)}^{(1,i,i)} \\
&+ \boldsymbol{\pi}_{(0,0)}^{(s_1,0)} \mathbf{K}_{(0,0),(0,0)}^{(s_1,0)} + \boldsymbol{\pi}_{(0,0)}^{(2,0)} \mathbf{K}_{(0,0),(0,0)}^{(2,0)} \\
&+ \boldsymbol{\pi}_{(0,0)}^{s_2} \mathbf{K}_{(0,0),(0,0)}^{s_2} \Bigg\};
\end{aligned} \tag{20}$$

(2) For $l_1' = 0, l_2' \in \{1, 2, \cdots, Q_2\}$,

$$\begin{aligned}
\boldsymbol{p}_{(0,l_2')} = \frac{1}{\tau} \Bigg\{ &\sum_{i=1}^{Q_1} \sum_{l_2=0}^{l_2'} \boldsymbol{\pi}_{(0,l_2)}^{(1,i,i)} \mathbf{K}_{(0,l_2),(0,l_2')}^{(1,i,i)} \\
&+ \sum_{l_2=0}^{l_2'} \boldsymbol{\pi}_{(0,l_2)}^{(s_1,0)} \mathbf{K}_{(0,l_2),(0,l_2')}^{(s_1,0)} \\
&+ \sum_{l_2=0}^{\Delta_{l_2'}} \boldsymbol{\pi}_{(0,l_2)}^{(2,0)} \mathbf{K}_{(0,l_2),(0,l_2')}^{(2,0)} \\
&+ \sum_{l_2=0}^{l_2'} \boldsymbol{\pi}_{(0,l_2)}^{s_2} \mathbf{K}_{(0,l_2),(0,l_2')}^{s_2} \Bigg\},
\end{aligned} \tag{21}$$

where $\Delta_{l_2'} = \delta_{Q_{2,l_2'}} (Q_2 - 1) + \left(1 - \delta_{Q_{2,l_2'}}\right) l_2'$;

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(3) For $l_1' \in \{1, 2, \cdots, Q_1\}$, $l_2' = 0$,

$$\begin{aligned}
&\boldsymbol{p}_{(l_1',0)} \\
&= \frac{1}{\tau}\Bigg\{\sum_{l_1=1}^{\Delta_{l_1'}}\sum_{i=2}^{Q_1}\sum_{\eta=1}^{\min\{l_1,i-1\}}\boldsymbol{\pi}_{(l_1,0)}^{(1,i,i-\eta)}\mathbf{K}_{(l_1,0),(l_1',0)}^{(1,i,i-\eta)} \\
&\quad+\sum_{i=1}^{Q_1}\sum_{l_1=0}^{\Delta_{l_1'}}\boldsymbol{\pi}_{(l_1,0)}^{(1,i,i)}\mathbf{K}_{(l_1,0),(l_1',0)}^{(1,i,i)} \\
&\quad+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{l_1'}\boldsymbol{\pi}_{(l_1,0)}^{(s_1,j)}\mathbf{K}_{(l_1,0),(l_1',0)}^{(s_1,j)} \\
&\quad+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{l_1'}\boldsymbol{\pi}_{(l_1,0)}^{(2,j)}\mathbf{K}_{(l_1,0),(l_1',0)}^{(2,j)} \\
&\quad+\sum_{l_1=0}^{l_1'}\boldsymbol{\pi}_{(l_1,0)}^{s_2}\mathbf{K}_{(l_1,0),(l_1',0)}^{s_2}\Bigg\},
\end{aligned} \tag{22}$$

where $\Delta_{l_1'} = \delta_{Q_{1,l_1'}}(Q_1 - 1) + \left(1 - \delta_{Q_{1,l_1'}}\right) l_1'$;

(4) For $l_1' \in \{1, 2, \cdots, Q_1\}$, $l_2' \in \{1, 2, \cdots, Q_2\}$,

$$\begin{aligned}
&\boldsymbol{p}_{(l_1',l_2')} \\
&= \frac{1}{\tau}\Bigg\{\sum_{l_1=1}^{\Delta_{l_1'}}\sum_{i=2}^{Q_1}\sum_{\eta=1}^{\min\{l_1,i-1\}}\sum_{l_2=0}^{l_2'}\boldsymbol{\pi}_{(l_1,l_2)}^{(1,i,i-\eta)}\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{(1,i,i-\eta)} \\
&\quad+\sum_{i=1}^{Q_1}\sum_{l_1=0}^{\Delta_{l_1'}}\sum_{l_2=0}^{l_2'}\boldsymbol{\pi}_{(l_1,l_2)}^{(1,i,i)}\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{(1,i,i)} \\
&\quad+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{l_1'}\sum_{l_2=0}^{l_2'}\boldsymbol{\pi}_{(l_1,l_2)}^{(s_1,j)}\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{(s_1,j)} \\
&\quad+\sum_{j=0}^{Q_1-1}\sum_{l_1=j}^{l_1'}\sum_{l_2=0}^{\Delta_{l_2'}}\boldsymbol{\pi}_{(l_1,l_2)}^{(2,j)}\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{(2,j)} \\
&\quad+\sum_{l_1=0}^{l_1'}\sum_{l_2=0}^{l_2'}\boldsymbol{\pi}_{(l_1,l_2)}^{s_2}\mathbf{K}_{(l_1,l_2),(l_1',l_2')}^{s_2}\Bigg\}.
\end{aligned} \tag{23}$$

Remark 1: The cyclic polling model presented in this paper is consistent with the one presented by [13], except that the buffer sizes are finite. In [13], the stability condition of the cyclic polling model with infinite-buffer queues was analyzed. Under this stability condition, the joint queue length stationary distributions of the cyclic polling model with infinite-buffer queues can be obtained asymptotically, by increasing the buffer sizes of the corresponding cyclic polling model with finite-buffer queues.

V. CUSTOMER BLOCKING PROBABILITIES AND WAITING TIMES

Let $k \in \mathrm{N}^+$, $n_1 \in \{0, 1, \cdots, Q_1\}$, $n_2 \in \{0, 1, \cdots, Q_2\}$. Consider the event that, at arbitrary time the lengths of queue 1 and queue 2 are n_1 and n_2 respectively, and at the same time there is a batch arrival of size k in queue 1. The probability that the event occurs can be expressed as $\boldsymbol{p}_{(n_1,n_2)}\frac{k\left(\mathbf{D}_k^{(1)}\otimes\mathbf{I}_{m^{(2)}}\right)}{\lambda_1}\mathbf{e}_{\bar{m}}$. When the size of the batch arriving at queue 1 is k, if the length of queue 1 is n_1 $(k > Q_1 - n_1)$, the probability that any customer in this batch is blocked is $\frac{k-(Q_1-n_1)}{k}$. So, the probability that any customer arriving at queue 1 is blocked can be expressed as the following.

$$\begin{aligned}
&P_{1,BL} \\
&= \sum_{n_1=0}^{Q_1}\sum_{n_2=0}^{Q_2}\boldsymbol{p}_{(n_1,n_2)} \\
&\quad\times\sum_{k=Q_1-n_1+1}^{\infty}\frac{k\left(\mathbf{D}_k^{(1)}\otimes\mathbf{I}_{m^{(2)}}\right)}{\lambda_1}\times\frac{k-(Q_1-n_1)}{k}\mathbf{e}_{\bar{m}} \\
&= \sum_{n_1=0}^{Q_1}\sum_{n_2=0}^{Q_2}\boldsymbol{p}_{(n_1,n_2)} \\
&\quad\times\sum_{k=Q_1-n_1+1}^{\infty}\lambda_1^{-1}\left(\mathbf{D}_k^{(1)}\otimes\mathbf{I}_{m^{(2)}}\right)\left[k-(Q_1-n_1)\right]\mathbf{e}_{\bar{m}}.
\end{aligned} \tag{24}$$

Consider the event that, at arbitrary time the lengths of queue 1 and queue 2 are n_1 and n_2 respectively, and at the same time there is a batch arrival of size k in queue 2. The probability that the event occurs can be expressed as $\boldsymbol{p}_{(n_1,n_2)}\frac{k\left(\mathbf{I}_{m^{(1)}}\otimes\mathbf{D}_k^{(2)}\right)}{\lambda_2}\mathbf{e}_{\bar{m}}$. When the size of the batch arriving at queue 2 is k, if the length of queue 2 is n_2 $(k > Q_2 - n_2)$, the probability that any customer in this batch is blocked is $\frac{k-(Q_2-n_2)}{k}$. So, the probability that any customer arriving at queue 2 is blocked can be expressed as the following.

$$\begin{aligned}
&P_{2,BL} \\
&= \sum_{n_1=0}^{Q_1}\sum_{n_2=0}^{Q_2}\boldsymbol{p}_{(n_1,n_2)} \\
&\quad\times\sum_{k=Q_2-n_2+1}^{\infty}\frac{k\left(\mathbf{I}_{m^{(1)}}\otimes\mathbf{D}_k^{(2)}\right)}{\lambda_2}\times\frac{k-(Q_2-n_2)}{k}\mathbf{e}_{\bar{m}} \\
&= \sum_{n_1=0}^{Q_1}\sum_{n_2=0}^{Q_2}\boldsymbol{p}_{(n_1,n_2)} \\
&\quad\times\sum_{k=Q_2-n_2+1}^{\infty}\lambda_2^{-1}\left(\mathbf{I}_{m^{(1)}}\otimes\mathbf{D}_k^{(1)}\right)\left[k-(Q_2-n_2)\right]\mathbf{e}_{\bar{m}}.
\end{aligned} \tag{25}$$

Let $\bar{W}_1$ and $\bar{W}_2$ represent the mean waiting times of customers in queue 1 and queue 2. It is notice that the customer waiting time is defined as the time, from the instant when the customer arrives at the queue (queue 1 or queue 2) until the instant when the service of the customer is finished.

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According to Little's law, $\bar{W}_1$ and $\bar{W}_2$ can be obtained as follows.

$$\bar{W}_1 = \bar{L}_1/[\lambda_1 (1 - P_{1,BL})], \tag{26}$$

$$\bar{W}_2 = \bar{L}_2/[\lambda_2 (1 - P_{2,BL})], \tag{27}$$

where $\bar{L}_1$ and $\bar{L}_2$ are respectively the mean lengths of queue 1 and queue 2 at arbitrary time, i.e.

$$\bar{L}_1 = \sum_{n_1=0}^{Q_1} \sum_{n_2=0}^{Q_2} n_1 \boldsymbol{p}_{(n_1,n_2)} \mathbf{e}_{\bar{m}}, \tag{28}$$

$$\bar{L}_2 = \sum_{n_1=0}^{Q_1} \sum_{n_2=0}^{Q_2} n_2 \boldsymbol{p}_{(n_1,n_2)} \mathbf{e}_{\bar{m}}. \tag{29}$$

VI. NUMERICAL EXPERIMENTS

In this section, some numerical experiments will be carried out to illustrate the calculations of three important performance measures and to show the effectiveness of the proposed polling model, respectively.

A. CALCULATIONS OF THREE IMPORTANT PERFORMANCE MEASURES

A numerical example is given to illustrate the calculations of the joint queue length distributions, the customer blocking probabilities and the customer mean waiting times. The numerical example has the following parameters. The BMAP-1 is set as,

$$\mathbf{D}_0^{(1)} = \begin{pmatrix} -0.4650 & 0.1953 \\ 0.5089 & -2.8710 \end{pmatrix},$$

$$\mathbf{D}_1^{(1)} = \begin{pmatrix} 0.1927 & 0 \\ 0.2544 & 1.4328 \end{pmatrix}, \mathbf{D}_2^{(1)} = \begin{pmatrix} 0.0770 & 0 \\ 0.1018 & 0.5731 \end{pmatrix}.$$

The BMAP-2 is set as,

$$\mathbf{D}_0^{(2)} = \begin{pmatrix} -2.1527 & 0.9044 \\ 2.3559 & -13.2916 \end{pmatrix},$$

$$\mathbf{D}_1^{(2)} = \begin{pmatrix} 0.8917 & 0 \\ 1.1780 & 6.6332 \end{pmatrix}, \mathbf{D}_2^{(2)} = \begin{pmatrix} 0.3566 & 0 \\ 0.4712 & 2.6533 \end{pmatrix}.$$

The phase type representation of $B_1(t)$ is $(\boldsymbol{\beta}_1, \mathbf{B}_1)$, where

$$\boldsymbol{\beta}_1 = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}, \ \mathbf{B}_1 = \begin{pmatrix} -24.9900 & 24.9900 \\ 0 & -24.9900 \end{pmatrix}.$$

The phase type representation of $B_2(t)$ is $(\boldsymbol{\beta}_2, \mathbf{B}_2)$, where

$$\boldsymbol{\beta}_2 = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}, \ \mathbf{B}_2 = \begin{pmatrix} -37.4850 & 37.4850 \\ 0 & -37.4850 \end{pmatrix}.$$

The phase type representation of $R_1(t)$ is $(\boldsymbol{\alpha}_1, \mathbf{R}_1)$, where $\boldsymbol{\alpha}_1 = 1, \mathbf{R}_1 = -10$. The phase type representation of $R_2(t)$ is $(\boldsymbol{\alpha}_2, \mathbf{R}_2)$, where $\boldsymbol{\alpha}_2 = 1$, $\mathbf{R}_2 = -10$. Let $Q_1 = 3$ and $Q_2 = 3$. $H_j(t)$, $j = 0, 1, 2$, are the exponential distributions with the parameters $\gamma_0 = 0.5$, $\gamma_1 = 1$ and $\gamma_2 = 2$, respectively.

(1) The matrices $\mathbf{M}_i$ $(i = 1, 2, \cdots, 8)$ are calculated firstly, according to the corresponding formulas given in Appendix A and Section III-A; then, based on the relation (8), the one-step transition probability matrix $\mathbf{W}$ is obtained; finally, the joint queue length stationary distribution at queue 1 polling epochs, namely $\boldsymbol{\omega}$, is computed by solving the system of linear equations given in (9).

$$\boldsymbol{\omega} = \left(\boldsymbol{\omega}_{(l_1,l_2)} : (l_1, l_2) \in \{0,1,2,3\} \times \{0,1,2,3\}\right),$$

where $\boldsymbol{\omega}_{(l_1,l_2)} = \left(\left(\boldsymbol{\omega}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in \{1,2\} \times \{1,2\}\right)$. $\boldsymbol{\omega}$ is shown in Table 3, in which the (v_1, v_2)-element of the (l_1, l_2)-th block represents the probability $\left(\boldsymbol{\omega}_{(l_1,l_2)}\right)_{(v_1,v_2)}$. For example, $\left(\boldsymbol{\omega}_{(0,0)}\right)_{(1,1)} = 0.54659$, $\left(\boldsymbol{\omega}_{(0,0)}\right)_{(1,2)} = 0.06751$, $\left(\boldsymbol{\omega}_{(0,0)}\right)_{(2,1)} = 0.08188$, $\left(\boldsymbol{\omega}_{(0,0)}\right)_{(2,2)} = 0.01003$.

(2) Given $\boldsymbol{\omega}$, the joint queue length stationary distribution at queue 2 polling epochs, namely $\boldsymbol{\eta}$, can be calculated based on (10), (11) and (12).

$$\boldsymbol{\eta} = \left(\boldsymbol{\eta}_{(l_1,l_2)} : (l_1, l_2) \in \{0,1,2,3\} \times \{0,1,2,3\}\right),$$

where $\boldsymbol{\eta}_{(l_1,l_2)} = \left(\left(\boldsymbol{\eta}_{(l_1,l_2)}\right)_{(v_1,v_2)} : (v_1, v_2) \in \{1,2\} \times \{1,2\}\right)$. $\boldsymbol{\eta}$ is shown in Table 4, in which the (v_1, v_2)-element of the (l_1, l_2)-th block represents the probability $\left(\boldsymbol{\eta}_{(l_1,l_2)}\right)_{(v_1,v_2)}$.

(3) Given $\boldsymbol{\omega}$, the stationary distribution $\boldsymbol{\pi}$ of the supplementary embedded Markov chain is firstly calculated as the following steps, (a) set $\boldsymbol{\pi}^{s_2} = \boldsymbol{\omega}$; (b) from the relations (14), (15), (16), (17) and (18), the vector $\boldsymbol{\pi}$ is calculated; (c) the stationary distribution $\boldsymbol{\pi}$ is obtained by normalization. Secondly, the mean duration τ is calculated by the formula (19); and the matrices $\mathbf{K}^{\phi}_{(l_1,l_2),(l_1',l_2')}$, $(\phi, l_1, l_2) \in \tilde{S}_\xi$, $(l_1', l_2') \in \mathbb{L}_1 \times \mathbb{L}_2$, are obtained according to the corresponding formulas given in Appendices A and B. Finally, the joint queue

TABLE 3. The joint queue length stationary distribution at queue 1 polling epochs

l_1 \ l_2	0	1	2	3
0	0.54659	0.05233	0.02856	0.01445
	0.06751	0.02145	0.01522	0.01545
	0.08188	0.00721	0.00388	0.00183
	0.01003	0.00303	0.00211	0.00202
1	0.02862	0.00386	0.00225	0.00146
	0.00363	0.00142	0.00108	0.00136
	0.02081	0.00246	0.00139	0.00080
	0.00262	0.00095	0.00070	0.00080
2	0.01330	0.00187	0.00110	0.00074
	0.00169	0.00068	0.00052	0.00068
	0.01199	0.00154	0.00089	0.00055
	0.00152	0.00058	0.00044	0.00053
3	0.00269	0.00047	0.00029	0.00024
	0.00035	0.00016	0.00013	0.00019
	0.00560	0.00091	0.00055	0.00041
	0.00072	0.00031	0.00025	0.00035

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TABLE 4. The joint queue length stationary distribution at queue 2 polling epochs

l_1 \ l_2	0	1	2	3
0	0.47573 0.04837	0.09209 0.02717	0.05574 0.02248	0.04290 0.03546
	0.07962 0.00800	0.01576 0.00456	0.00961 0.00380	0.00758 0.00615
1	0.01047 0.00095	0.00310 0.00070	0.00205 0.00065	0.00214 0.00143
	0.00984 0.00089	0.00284 0.00066	0.00187 0.00060	0.00189 0.00129
2	0.00452 0.00041	0.00136 0.00030	0.00091 0.00028	0.00097 0.00063
	0.00484 0.00043	0.00147 0.00033	0.00098 0.00031	0.00103 0.00068
3	0.00048 0.00004	0.00019 0.00004	0.00014 0.00004	0.00019 0.00010
	0.00121 0.00010	0.00046 0.00009	0.00033 0.00009	0.00042 0.00024

TABLE 5. The joint queue length stationary distribution at arbitrary time

l'_1 \ l'_2	0	1	2	3
0	0.42353 0.04801	0.09230 0.03087	0.05473 0.02660	0.03368 0.03040
	0.06690 0.00747	0.01462 0.00479	0.00874 0.00415	0.00552 0.00487
1	0.02583 0.00291	0.00680 0.00207	0.00423 0.00185	0.00316 0.00253
	0.01948 0.00217	0.00498 0.00151	0.00309 0.00135	0.00225 0.00182
2	0.01128 0.00129	0.00303 0.00092	0.00189 0.00083	0.00143 0.00113
	0.01104 0.00124	0.00293 0.00088	0.00183 0.00079	0.00138 0.00109
3	0.00218 0.00025	0.00066 0.00019	0.00042 0.00017	0.00036 0.00026
	0.00461 0.00053	0.00135 0.00040	0.00086 0.00036	0.00069 0.00052

length stationary distribution at arbitrary time, namely $\boldsymbol{p}$, is computed by the formulas (20), (21), (22) and (23).

$$\boldsymbol{p} = \left(\boldsymbol{p}_{(l'_1,l'_2)} : (l'_1, l'_2) \in \{0,1,2,3\} \times \{0,1,2,3\}\right),$$

where $\boldsymbol{p}_{(l'_1,l'_2)} = \left(\left(\boldsymbol{p}_{(l'_1,l'_2)}\right)_{(v'_1,v'_2)} : (v'_1, v'_2) \in \{1,2\} \times \{1,2\}\right)$. $\boldsymbol{p}$ is shown in Table 5, in which the (v'_1, v'_2)-element of the (l'_1, l'_2)-th block represents the probability $\left(\boldsymbol{p}_{(l'_1,l'_2)}\right)_{(v'_1,v'_2)}$.

(4) Based on the joint queue length stationary distribution $\boldsymbol{p}$ shown in Table 5, (a) according to the expression given in (24), the customer blocking probability $P_{1,BL}$ of queue 1 can be calculated, i.e. $P_{1,BL} = 0.0544$; according to the expression given in (25), the customer blocking probability $P_{2,BL}$ of queue 2 can be calculated, i.e. $P_{2,BL} = 0.2095$. And then according to the expressions (26) and (27), the mean waiting times of customers in queue 1 and queue 2 can be calculated, i.e. $\bar{W}_1 = 0.2680$ and $\bar{W}_2 = 0.2159$.

B. EFFECTIVENESS OF THE PROPOSED POLLING MODEL

In this subsection, numerical experiments are carried out to show the effectiveness of the proposed polling model. The main features of the proposed polling model lie in the across-queue state-dependent service discipline attached to queue 2. Namely, the duration of the server's visit to queue 2 is predetermined and time-limited. This duration has the probability distribution function $H_j(t)$, which depends on the length j of queue 1 at the instant when the server started to depart from queue 1 last time, where $j \in \{0, 1, 2, \cdots, Q_1\}$. Therefore, we will show the effectiveness of the proposed polling model by comparing two classes of service disciplines. In the first class, the predetermined duration of the server's visit to queue 2 obeys the fixed probability distribution. In the second class, the predetermined duration of the server's visit to queue 2 depends on the state of queue 1.

The numerical experiments are carried out with the following parameters. The BMAP-1 is set as,

$$\mathbf{D}_0^{(1)} = \begin{pmatrix} -0.4650 & 0.1953 \\ 0.5089 & -3.8710 \end{pmatrix},$$

$$\mathbf{D}_3^{(1)} = \begin{pmatrix} 0.1927 & 0 \\ 0.2544 & 0.4328 \end{pmatrix}, \mathbf{D}_5^{(1)} = \begin{pmatrix} 0.0770 & 0 \\ 0.1018 & 2.5731 \end{pmatrix},$$

where the average arrival rate $\lambda'_1 = 3.6291$. The BMAP-2 is set as,

$$\mathbf{D}_0^{(2)} = \begin{pmatrix} -0.4305 & 0.1809 \\ 0.4712 & -3.6583 \end{pmatrix},$$

$$\mathbf{D}_5^{(2)} = \begin{pmatrix} 0.1783 & 0 \\ 0.2356 & 0.3266 \end{pmatrix}, \mathbf{D}_7^{(2)} = \begin{pmatrix} 0.0713 & 0 \\ 0.0942 & 2.5307 \end{pmatrix},$$

where the average arrival rate $\lambda_2 = 5.0374$. The phase type representation of $B_1(t)$ is $(\boldsymbol{\beta}_1, \mathbf{B}_1)$, where

$$\boldsymbol{\beta}_1 = \begin{pmatrix} 0.5 & 0.5 \end{pmatrix}, \ \mathbf{B}_1 = \begin{pmatrix} -9.9960 & 9.9960 \\ 0 & -9.9960 \end{pmatrix}.$$

The phase type representation of $B_2(t)$ is $(\boldsymbol{\beta}_2, \mathbf{B}_2)$, where $\boldsymbol{\beta}_2 = \boldsymbol{\beta}_1$ and $\mathbf{B}_2 = \mathbf{B}_1$. The phase type representation of $R_1(t)$ is $(\boldsymbol{\alpha}_1, \mathbf{R}_1)$, where $\boldsymbol{\alpha}_1 = 1$, $\mathbf{R}_1 = -10$. The phase type representation of $R_2(t)$ is $(\boldsymbol{\alpha}_2, \mathbf{R}_2)$, where $\boldsymbol{\alpha}_2 = 1$, $\mathbf{R}_2 = -10$. Let $Q_1 = Q_2 = 10$. For $H_j(t)$ $(j = 0, 1, 2, \cdots, 10)$, we compare the following cases in Fig. 5.

- $H_j(t)$ is exponentially distributed with constant parameter, regardless of j ($j \in \{0, 1, 2, \cdots, 10\}$). The following cases, in which each one has a constant parameter, are considered.

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(a) **CV 0.11.**[1] For $j \in \{0, 1, 2, \cdots, 10\}$, $\gamma_j = 9.0909$, i.e. $h_j = 0.11$.
(b) **CV 0.13.** For $j \in \{0, 1, 2, \cdots, 10\}$, $\gamma_j = 7.6923$, i.e. $h_j = 0.13$.
(c) **CV 0.15.** For $j \in \{0, 1, 2, \cdots, 10\}$, $\gamma_j = 6.6670$, i.e. $h_j = 0.15$.
(d) **CV 0.17.** For $j \in \{0, 1, 2, \cdots, 10\}$, $\gamma_j = 5.8824$, i.e. $h_j = 0.17$.
(e) **CV 0.19.** For $j \in \{0, 1, 2, \cdots, 10\}$, $\gamma_j = 5.2632$, i.e. $h_j = 0.19$.

- $H_j(t)$ is exponentially distributed with the parameter varying with j ($j = 0, 1, 2, \cdots, 10$). The following case is considered.

(f) **AQSD value.**[2] For $j = 0, 1, 2, 3, 4$, $\gamma_j = 5$, 5.5556, 6.2500, 7.1429, 8.3333 respectively, i.e. $h_j = 0.2$, 0.18, 0.16, 0.14, 0.12 respectively; for $j = 5, 6, 7, 8, 9, 10$, $\gamma_j = 10$, i.e. $h_j = 0.1$.

For each of the above cases related to $H_j(t)$ ($j = 0, 1, 2, \cdots, 10$), let the parameters of BMAP-1 vary as $(0.1k + 1)\mathbf{D}_0^{(1)}$, $(0.1k + 1)\mathbf{D}_3^{(1)}$ and $(0.1k + 1)\mathbf{D}_5^{(1)}$, where $k = 1, 2, \cdots, 8$ (meanwhile, the average arrival rate $\lambda_1 = (0.1k+1)\lambda_1'$ increases from 3.9920 to 6.5324). The variations of customer 1 mean waiting time and customer 2 mean waiting time are displayed in Fig. 6 and Fig. 7 respectively.

It is shown in Fig. 6 and Fig. 7 that, given the case "CV 0.15", if the parameter γ_j (or h_j) is adjusted as the cases "CV 0.13" and "CV 0.11" respectively, customer 1 mean waiting time is reduced, while customer 2 mean waiting time is increased; if γ_j (or h_j) is adjusted as the cases "CV 0.17" and "CV 0.19" respectively, customer 1 mean waiting time is increased, while customer 2 mean waiting time is reduced; if γ_j (or h_j) is adjusted as the case "AQSD value", customer 1 mean waiting time and customer 2 mean waiting time can be reduced simultaneously. In addition, according to the characteristics of the proposed polling model, for each queue, if the customer mean waiting time decreases, the mean queue length and the customer blocking probability also decrease.

From the above observations, compared with the fixed service discipline, the across-queue state-dependent service discipline can reduce the customer mean waiting times, the mean queue lengths and the customer blocking probabilities of different queues simultaneously. The reason for these behaviors is that, the across-queue state-dependent service discipline can accommodate the fluctuation of the customer arrival process appropriately. This also show the effectiveness of the proposed polling model in optimizing the transmission of compressed video.

VII. CONCLUSION

According to the characteristics of H.264 and H.265 standards, a polling model with BMAP and across-queue state-dependent service discipline is proposed and analyzed in this paper. In this polling model, there are one server and

[1] "CV" stands for "constant value".
[2] "AQSD" stands for "across-queue state-dependent".

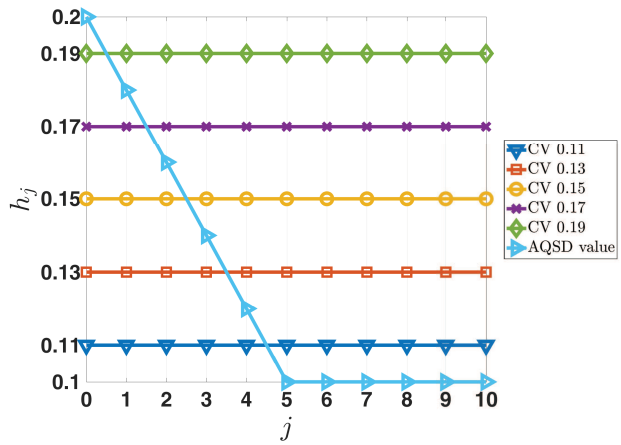

FIGURE 5. For different cases related to $H_j(t)$ ($j = 0, 1, 2, \cdots, 10$), the variation of the parameter $h_j = 1/\gamma_j$ as the length j of queue 1 varies.

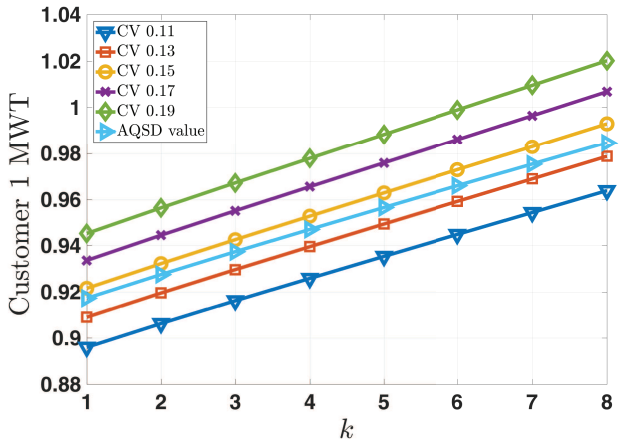

FIGURE 6. For different cases related to $H_j(t)$ ($j = 0, 1, 2, \cdots, 10$), the variation of customer 1 mean waiting time (MWT) as λ_1 varies, where $\lambda_1 = (0.1k + 1)\lambda_1'$ ($k = 1, 2, \cdots, 8$).

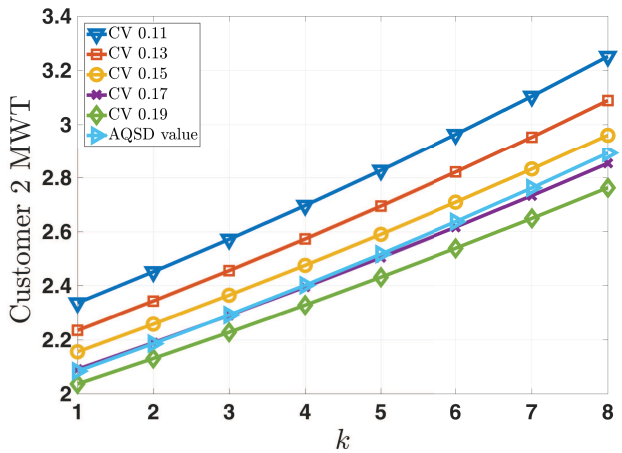

FIGURE 7. For different cases related to $H_j(t)$ ($j = 0, 1, 2, \cdots, 10$), the variation of customer 2 mean waiting time (MWT) as λ_1 varies, where $\lambda_1 = (0.1k + 1)\lambda_1'$ ($k = 1, 2, \cdots, 8$).

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two finite-buffer queues; the customers arrive at the two queues as two independent BMAPs; the server is entitled to serve the two queues in a cyclic manner; the first queue is served according to the gated service discipline; the second queue is served according to the across-queue state-dependent time-limited service discipline. Since the service discipline attached to the second queue is an across-queue state-dependent one, the existing methods for analyzing polling models with BMAP are not suitable for the model of this paper. Fortunately, the classical matrix-analytic method can be applied. By constructing a supplementary embedded Markov chain, the joint queue length stationary distributions at queue 1 polling epochs and at queue 2 polling epochs are obtained firstly; and then based on the semi-regenerative process and the stationary distribution of the supplementary embedded Markov chain, the joint queue length stationary distribution at arbitrary time is obtained; finally, based on the above results, the blocking probabilities of customers in different queues are given, and according to Little's law, the mean waiting times of customers in different queues can be calculated.

It has been shown from the numerical experiments that, compared with the fixed service discipline, the across-queue state-dependent service discipline can reduce the customer mean waiting times, the mean queue lengths and the customer blocking probabilities of different queues simultaneously. This indicates that the polling model proposed in this paper is suitable to design and optimize appropriate time division transmission policies for the video compressed by H.264 and H.265. These policies can be used in network devices and data centers to transmit the collected or stored video data to the destination through wireless networks.

.

APPENDIX A FUNDAMENTAL CONDITIONAL PROBABILITIES

For the calculations of some variables involved in Sections III and IV, some fundamental conditional probabilities and their calculation formulas will be given in this section. Given two general functions $F(t)$ and $G(t)$ on $[0,\infty)$, suppose that $F(t)$ has the phase type representation $(\boldsymbol{\alpha},\mathbf{T})$ of order m_F, where $m_F \in \mathrm{N}^+$ and $\boldsymbol{\alpha}\mathbf{e}=1$; and $G(t)$ has the phase type representation $(\boldsymbol{\beta},\mathbf{S})$ of order m_G, where $m_G \in \mathrm{N}^+$ and $\boldsymbol{\beta}\mathbf{e}=1$. Let

$$\mathbf{P}(\overline{n_1},n_2,t)=\sum_{i_1=n_1}^{\infty}\mathbf{P}(i_1,n_2,t), \tag{30}$$

$$\mathbf{P}(n_1,\overline{n_2},t)=\sum_{i_2=n_2}^{\infty}\mathbf{P}(n_1,i_2,t), \tag{31}$$

$$\mathbf{P}(\overline{n_1},\overline{n_2},t)=\sum_{i_1=n_1}^{\infty}\sum_{i_2=n_2}^{\infty}\mathbf{P}(i_1,i_2,t). \tag{32}$$

A. $\mathbf{F}(N_1,N_2)$, $\mathbf{F}(\overline{N_1},N_2)$, $\mathbf{F}(N_1,\overline{N_2})$, $\mathbf{F}(\overline{N_1},\overline{N_2})$

Let

$$\mathbf{F}(n_1,n_2)=\int_{t=0}^{\infty}\mathbf{P}(n_1,n_2,t)dF(t), \tag{33}$$

$$\mathbf{F}(\overline{n_1},n_2)=\int_{t=0}^{\infty}\mathbf{P}(\overline{n_1},n_2,t)dF(t), \tag{34}$$

$$\mathbf{F}(n_1,\overline{n_2})=\int_{t=0}^{\infty}\mathbf{P}(n_1,\overline{n_2},t)dF(t), \tag{35}$$

$$\mathbf{F}(\overline{n_1},\overline{n_2})=\int_{t=0}^{\infty}\mathbf{P}(\overline{n_1},\overline{n_2},t)dF(t), \tag{36}$$

where $n_1,n_2\in\mathrm{N}$.

(1) The $\big((v_1,v_2),(v_1',v_2')\big)$-th element of $\mathbf{F}(n_1,n_2)$, i.e. $\mathbf{F}_{(v_1,v_2),(v_1',v_2')}(n_1,n_2)$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1,v_2) at the initial time, after the time interval $\mathcal{T}$ which obeys the distribution $F(t)$, the joint number of the arrivals from the two BMAPs is (n_1,n_2), and the joint phase of the two BMAPs is (v_1',v_2'), where $(v_1,v_2),(v_1',v_2')\in M_1\times M_2$.

(2) The $\big((v_1,v_2),(v_1',v_2')\big)$-th element of $\mathbf{F}(\overline{n_1},n_2)$, i.e. $\mathbf{F}_{(v_1,v_2),(v_1',v_2')}(\overline{n_1},n_2)$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1,v_2) at the initial time, after the time interval $\mathcal{T}$ which obeys the distribution $F(t)$, the number of the arrivals from the BMAP-1 is equal to or greater than n_1, the number of the arrivals from the BMAP-2 is n_2, and the joint phase of the two BMAPs is (v_1',v_2'), where $(v_1,v_2),(v_1',v_2')\in M_1\times M_2$. From the relations (30), (33) and (34), $\mathbf{F}(\overline{n_1},n_2)$ can be expressed in terms of $\mathbf{F}(i_1,n_2)$ as the following,

$$\mathbf{F}(\overline{n_1},n_2)=\sum_{i_1=0}^{\infty}\mathbf{F}(i_1,n_2)-\sum_{i_1=0}^{n_1-1}\mathbf{F}(i_1,n_2). \tag{37}$$

(3) The $\big((v_1,v_2),(v_1',v_2')\big)$-th element of $\mathbf{F}(n_1,\overline{n_2})$, i.e. $\mathbf{F}_{(v_1,v_2),(v_1',v_2')}(n_1,\overline{n_2})$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1,v_2) at the initial time, after the time interval $\mathcal{T}$ which obeys the distribution $F(t)$, the number of the arrivals from the BMAP-1 is n_1, the number of the arrivals from the BMAP-2 is equal to or greater than n_2, and the joint phase of the two BMAPs is (v_1',v_2'), where $(v_1,v_2),(v_1',v_2')\in M_1\times M_2$. From the relations (31), (33) and (35), $\mathbf{F}(n_1,\overline{n_2})$ can be expressed in terms of $\mathbf{F}(n_1,i_2)$ as the following,

$$\mathbf{F}(n_1,\overline{n_2})=\sum_{i_2=0}^{\infty}\mathbf{F}(n_1,i_2)-\sum_{i_2=0}^{n_2-1}\mathbf{F}(n_1,i_2). \tag{38}$$

(4) The $\big((v_1,v_2),(v_1',v_2')\big)$-th element of $\mathbf{F}(\overline{n_1},\overline{n_2})$, i.e. $\mathbf{F}_{(v_1,v_2),(v_1',v_2')}(\overline{n_1},\overline{n_2})$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1,v_2) at the initial time, after the time interval $\mathcal{T}$ which obeys the distribution $F(t)$, the number of the arrivals from the BMAP-1 is equal to or greater

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than n_1, the number of the arrivals from the BMAP-2 is equal to or greater than n_2, and the joint phase of the two BMAPs is (v_1', v_2'), where $(v_1, v_2), (v_1', v_2') \in M_1 \times M_2$. From the relations (32), (33) and (36), $\mathbf{F}(\overline{n_1}, \overline{n_2})$ can be expressed in terms of $\mathbf{F}(i_1, i_2)$ as the following,

$$\begin{aligned}&\mathbf{F}(\overline{n_1}, \overline{n_2})\\ &= \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\mathbf{F}(i_1, i_2) - \sum_{i_1=0}^{n_1-1}\sum_{i_2=0}^{\infty}\mathbf{F}(i_1, i_2)\\ &\quad - \sum_{i_1=0}^{\infty}\sum_{i_2=0}^{n_2-1}\mathbf{F}(i_1, i_2) + \sum_{i_1=0}^{n_1-1}\sum_{i_2=0}^{n_2-1}\mathbf{F}(i_1, i_2). \quad (39)\end{aligned}$$

From (37), (38) and (39), it can be seen that the calculations of $\mathbf{F}(\overline{n_1}, n_2)$, $\mathbf{F}(n_1, \overline{n_2})$ and $\mathbf{F}(\overline{n_1}, \overline{n_2})$ can be reduced to the calculations of the fundamental terms $\mathbf{F}(n_1, n_2)$, $\sum_{n_1=0}^{\infty}\mathbf{F}(n_1, n_2)$, $\sum_{n_2=0}^{\infty}\mathbf{F}(n_1, n_2)$ and $\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\mathbf{F}(n_1, n_2)$.

B. $\bar{\mathbf{F}}(N_1, N_2)$, $\bar{\mathbf{F}}(\overline{N_1}, N_2)$, $\bar{\mathbf{F}}(N_1, \overline{N_2})$, $\bar{\mathbf{F}}(\overline{N_1}, \overline{N_2})$

Let

$$\bar{\mathbf{F}}(n_1, n_2) = \int_{t=0}^{\infty}\mathbf{P}(n_1, n_2, t)\,[1 - F(t)]\,dt, \quad (40)$$

$$\bar{\mathbf{F}}(\overline{n_1}, n_2) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, n_2, t)\,[1 - F(t)]\,dt, \quad (41)$$

$$\bar{\mathbf{F}}(n_1, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(n_1, \overline{n_2}, t)\,[1 - F(t)]\,dt, \quad (42)$$

$$\bar{\mathbf{F}}(\overline{n_1}, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, \overline{n_2}, t)\,[1 - F(t)]\,dt, \quad (43)$$

where $n_1, n_2 \in \mathrm{N}$.

The $((v_1, v_2), (v_1', v_2'))$-th element of $\bar{\mathbf{F}}(n_1, n_2)$, i.e. $\bar{\mathbf{F}}_{(v_1,v_2),(v_1',v_2')}(n_1, n_2)$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1, v_2) at the initial time, before the time interval $\mathcal{T}$ expires, where $\mathcal{T}$ obeys the distribution $F(t)$, the joint number of the arrivals from the two BMAPs is (n_1, n_2), and the joint phase of the two BMAPs is (v_1', v_2'), where $(v_1, v_2), (v_1', v_2') \in M_1 \times M_2$. The meanings of the elements of $\bar{\mathbf{F}}(\overline{n_1}, n_2)$, $\bar{\mathbf{F}}(n_1, \overline{n_2})$ and $\bar{\mathbf{F}}(\overline{n_1}, \overline{n_2})$ are explicit, by referring to Appendix A-A and the above description of $\bar{\mathbf{F}}(n_1, n_2)$.

Similarly to (37), (38) and (39), $\bar{\mathbf{F}}(\overline{n_1}, n_2)$, $\bar{\mathbf{F}}(n_1, \overline{n_2})$ and $\bar{\mathbf{F}}(\overline{n_1}, \overline{n_2})$ can be expressed in terms of $\bar{\mathbf{F}}(i_1, n_2)$, $\bar{\mathbf{F}}(n_1, i_2)$ and $\bar{\mathbf{F}}(i_1, i_2)$, respectively; moreover their calculations can be reduced to the calculations of the fundamental terms $\bar{\mathbf{F}}(n_1, n_2)$, $\sum_{n_1=0}^{\infty}\bar{\mathbf{F}}(n_1, n_2)$, $\sum_{n_2=0}^{\infty}\bar{\mathbf{F}}(n_1, n_2)$ and $\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\bar{\mathbf{F}}(n_1, n_2)$.

C. $\bar{\mathbf{G}} \circ \mathbf{F}(N_1, N_2)$, $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{N_1}, N_2)$, $\bar{\mathbf{G}} \circ \mathbf{F}(N_1, \overline{N_2})$, $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{N_1}, \overline{N_2})$

Let

$$\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2) = \int_{t=0}^{\infty}\mathbf{P}(n_1, n_2, t)\,[1 - G(t)]\,dF(t), \quad (44)$$

$$\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, n_2) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, n_2, t)\,[1 - G(t)]\,dF(t), \quad (45)$$

$$\bar{\mathbf{G}} \circ \mathbf{F}(n_1, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(n_1, \overline{n_2}, t)\,[1 - G(t)]\,dF(t), \quad (46)$$

$$\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, \overline{n_2}, t)\,[1 - G(t)]\,dF(t), \quad (47)$$

where $n_1, n_2 \in \mathrm{N}$.

The $((v_1, v_2), (v_1', v_2'))$-th element of $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$, i.e. $\bar{\mathbf{G}} \circ \mathbf{F}_{(v_1,v_2),(v_1',v_2')}(n_1, n_2)$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1, v_2) at the initial time, after the time interval $\mathcal{T}_1$, which obeys the distribution $F(t)$ and is less than the time interval $\mathcal{T}_2$, where $\mathcal{T}_2$ obeys the distribution $G(t)$, the joint number of the arrivals from the two BMAPs is (n_1, n_2), and the joint phase of the two BMAPs is (v_1', v_2'), where $(v_1, v_2), (v_1', v_2') \in M_1 \times M_2$. The meanings of the elements of $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, n_2)$, $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, \overline{n_2})$ and $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, \overline{n_2})$ are explicit, by referring to Appendix A-A and the above description of $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$.

Similarly to (37), (38) and (39), $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, n_2)$, $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, \overline{n_2})$ and $\bar{\mathbf{G}} \circ \mathbf{F}(\overline{n_1}, \overline{n_2})$ can be expressed in terms of $\bar{\mathbf{G}} \circ \mathbf{F}(i_1, n_2)$, $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, i_2)$ and $\bar{\mathbf{G}} \circ \mathbf{F}(i_1, i_2)$, respectively; moreover their calculations can be reduced to the calculations of the fundamental terms $\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$, $\sum_{n_1=0}^{\infty}\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$, $\sum_{n_2=0}^{\infty}\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$ and $\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\bar{\mathbf{G}} \circ \mathbf{F}(n_1, n_2)$.

D. $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(N_1, N_2)$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{N_1}, N_2)$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(N_1, \overline{N_2})$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{N_1}, \overline{N_2})$

Let

$$\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2) = \int_{t=0}^{\infty}\mathbf{P}(n_1, n_2, t)\,[1 - G(t)]\,[1 - F(t)]\,dt, \quad (48)$$

$$\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, n_2) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, n_2, t)\,[1 - G(t)]\,[1 - F(t)]\,dt, \quad (49)$$

$$\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(n_1, \overline{n_2}, t)\,[1 - G(t)]\,[1 - F(t)]\,dt, \quad (50)$$

$$\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, \overline{n_2}) = \int_{t=0}^{\infty}\mathbf{P}(\overline{n_1}, \overline{n_2}, t)\,[1 - G(t)]\,[1 - F(t)]\,dt, \quad (51)$$

where $n_1, n_2 \in \mathrm{N}$.

The $((v_1, v_2), (v_1', v_2'))$-th element of $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$, i.e. $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}_{(v_1,v_2),(v_1',v_2')}(n_1, n_2)$, represents the conditional probability that, given that the joint phase of the two BMAPs is (v_1, v_2) at the initial time, after the time interval $\mathcal{T}$, which is less than not only the time interval $\mathcal{T}_1$ but also the time interval $\mathcal{T}_2$, where $\mathcal{T}_1$ obeys the distribution $F(t)$ and $\mathcal{T}_2$ obeys the distribution $G(t)$, the joint number of the arrivals from the two BMAPs is (n_1, n_2), and the joint phase of the two BMAPs is (v_1', v_2'), where $(v_1, v_2), (v_1', v_2') \in M_1 \times M_2$. The meanings of the elements of $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, n_2)$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, \overline{n_2})$ and $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, \overline{n_2})$ are explicit, by referring to Appendix A-A and the above description of $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$.

Similarly to (37), (38) and (39), $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, n_2)$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, \overline{n_2})$ and $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(\overline{n_1}, \overline{n_2})$ can be expressed in terms

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of $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(i_1, n_2)$, $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, i_2)$ and $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(i_1, i_2)$, respectively; moreover their calculations can be reduced to the calculations of the fundamental terms $\bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$, $\sum_{n_1=0}^{\infty} \bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$, $\sum_{n_2=0}^{\infty} \bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$ and $\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \bar{\mathbf{G}} \circ \bar{\mathbf{F}}(n_1, n_2)$.

E. CALCULATION FORMULAS OF THE FUNDAMENTAL TERMS

The calculations of fundamental conditional probabilities given in Appendices A-A, A-B, A-C and A-D can be reduced to the calculations of some fundamental terms. Before giving the formulas for calculating these fundamental terms, Theorem 1 is given firstly.

Theorem 1: Given a $m \times m$ matrix $\mathbf{\Omega}$ and let

$$\mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2) = \int_{t=0}^{\infty} \mathbf{P}(n_1, n_2, t) \otimes e^{\mathbf{\Omega} t} dt,$$

where $m \in \mathrm{N}^+$ and $n_1, n_2 \in \mathrm{N}$, if $\mathbf{\Omega}$ is stable, then

(1) $\mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2)$, $n_1, n_2 \in \mathrm{N}$, can be calculated by the following recursion formulas,

$$\begin{aligned}
&\mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2) = \mathbf{U}_{n_1,n_2}, \quad n_1, n_2 \in \mathrm{N}, \\
&\mathbf{U}_{0,0} = -\left(\mathbf{D}_0^{(1)} \oplus \mathbf{D}_0^{(2)} \oplus \mathbf{\Omega}\right)^{-1}, \\
&\mathbf{U}_{n_1,n_2} = \left[\sum_{j_1=0}^{n_1-1} \mathbf{U}_{j_1,n_2} \left(\mathbf{D}_{n_1-j_1}^{(1)} \otimes \mathbf{I}_{m_2} \otimes \mathbf{I}_m\right)\right. \\
&\qquad \left. + \sum_{j_2=0}^{n_2-1} \mathbf{U}_{n_1,j_2} \left(\mathbf{I}_{m_1} \otimes \mathbf{D}_{n_2-j_2}^{(2)} \otimes \mathbf{I}_m\right)\right] \\
&\qquad \times \mathbf{U}_{0,0}, \qquad n_1 + n_2 \neq 0;
\end{aligned}$$

(2) $\sum_{n_1=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2)$, $n_2 \in \mathrm{N}$, can be calculated by the following recursion formulas,

$$\begin{aligned}
&\sum_{n_1=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2) = \mathbf{U}_{n_2}^{(1)}, \quad n_2 \in \mathrm{N}, \\
&\mathbf{U}_0^{(1)} = -\left(\mathbf{D}^{(1)} \oplus \mathbf{D}_0^{(2)} \oplus \mathbf{\Omega}\right)^{-1}, \\
&\mathbf{U}_{n_2}^{(1)} = \sum_{j_2=0}^{n_2-1} \mathbf{U}_{j_2}^{(1)} \left(\mathbf{I}_{m_1} \otimes \mathbf{D}_{n_2-j_2}^{(2)} \otimes \mathbf{I}_m\right) \\
&\qquad \times \mathbf{U}_0^{(1)}, \quad n_2 \in \mathrm{N}^+;
\end{aligned}$$

(3) $\sum_{n_2=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2)$, $n_1 \in \mathrm{N}$, can be calculated by the following recursion formulas,

$$\begin{aligned}
&\sum_{n_2=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2) = \mathbf{U}_{n_1}^{(2)}, \quad n_1 \in \mathrm{N}, \\
&\mathbf{U}_0^{(2)} = -\left(\mathbf{D}_0^{(1)} \oplus \mathbf{D}^{(2)} \oplus \mathbf{\Omega}\right)^{-1}, \\
&\mathbf{U}_{n_1}^{(2)} = \sum_{j_1=0}^{n_1-1} \left[\mathbf{U}_{j_1}^{(2)} \left(\mathbf{D}_{n_1-j_1}^{(1)} \otimes \mathbf{I}_{m_2} \otimes \mathbf{I}_m\right)\right] \\
&\qquad \times \mathbf{U}_0^{(2)}, \quad n_1 \in \mathrm{N}^+;
\end{aligned}$$

(4) $\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2)$ can be calculated by the following formula,

$$\sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2) = -\left(\mathbf{D}^{(1)} \oplus \mathbf{D}^{(2)} \oplus \mathbf{\Omega}\right)^{-1}.$$

Proof: Since the matrix $\mathbf{\Omega}$ is stable, $\mathbf{\Omega}$ is invertible and its eigenvalues are all negative. Let μ_i $(i = 1, 2, \cdots, m)$ denote the eigenvalues of $\mathbf{\Omega}$, where μ_i may be equal to μ_j for $i \neq j$. By using the Jordan canonical form method [37], $e^{\mathbf{\Omega} t}$ can be expressed in terms $e^{\mu_i t}$ $(i = 1, 2, \cdots, m)$; moreover, it follows that $\lim_{t \to \infty} e^{\mathbf{\Omega} t} = \mathbf{0}$. The matrices $\mathbf{D}_0^{(1)}$ and $\mathbf{D}_0^{(2)}$ are stable; the matrices $\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$ are semi-stable [38]. So, based on the eigenvalue property of Kronecker sum [39] , the matrices $\mathbf{D}_0^{(1)} \oplus \mathbf{D}_0^{(2)} \oplus \mathbf{\Omega}$, $\mathbf{D}^{(1)} \oplus \mathbf{D}_0^{(2)} \oplus \mathbf{\Omega}$, $\mathbf{D}_0^{(1)} \oplus \mathbf{D}^{(2)} \oplus \mathbf{\Omega}$ and $\mathbf{D}^{(1)} \oplus \mathbf{D}^{(2)} \oplus \mathbf{\Omega}$ are all invertible.

Let $\mathbf{U}_{n_1,n_2} = \mathbf{\Psi}(\mathbf{\Omega}, n_1, n_2)$, $n_1, n_2 \in \mathrm{N}$. By the partial integration, $\mathbf{U}_{n_1,n_2}$ can be expressed as follows.

$$\begin{aligned}
\mathbf{U}_{n_1,n_2} &= \mathbf{P}(n_1, n_2, t) \otimes \left(e^{\mathbf{\Omega} t} \mathbf{\Omega}^{-1}\right)\Big|_{t=0}^{\infty} \\
&\quad - \int_{t=0}^{\infty} \mathbf{P}'(n_1, n_2, t) \otimes \left(e^{\mathbf{\Omega} t} \mathbf{\Omega}^{-1}\right) dt \\
&= -\delta_{n_1+n_2,0} \left(\mathbf{I}_{\bar{m}} \otimes \mathbf{\Omega}^{-1}\right) \\
&\quad - \int_{t=0}^{\infty} \mathbf{P}'(n_1, n_2, t) \otimes \left(e^{\mathbf{\Omega} t} \mathbf{\Omega}^{-1}\right) dt.
\end{aligned}$$

(a) For $n_1 + n_2 = 0$,

$$\mathbf{U}_{0,0} = -\mathbf{I}_{\bar{m}} \otimes \mathbf{\Omega}^{-1} - \int_{t=0}^{\infty} \mathbf{P}'(0, 0, t) \otimes \left(e^{\mathbf{\Omega} t} \mathbf{\Omega}^{-1}\right) dt. \tag{52}$$

Substitute the relation (6) into (52), there is the following relation.

$$\mathbf{U}_{0,0} = -\mathbf{I}_{\bar{m}} \otimes \mathbf{\Omega}^{-1} - \mathbf{U}_{0,0} \left[\left(\mathbf{D}_0^{(1)} \oplus \mathbf{D}_0^{(2)}\right) \otimes \mathbf{\Omega}^{-1}\right]. \tag{53}$$

Post-multiply the relation (53) by $\mathbf{I}_{\bar{m}} \otimes \mathbf{\Omega}$, then the calculation formula of $\mathbf{U}_{0,0}$ can be obtained.

$$\mathbf{U}_{0,0} = -\left(\mathbf{D}_0^{(1)} \oplus \mathbf{D}_0^{(2)} \oplus \mathbf{\Omega}\right)^{-1}.$$

(b) For $n_1 + n_2 \neq 0$,

$$\mathbf{U}_{n_1,n_2} = -\int_{t=0}^{\infty} \mathbf{P}'(n_1, n_2, t) \otimes \left(e^{\mathbf{\Omega} t} \mathbf{\Omega}^{-1}\right) dt. \tag{54}$$

Substitute the relation (6) into (54), there is the following relation.

$$\begin{aligned}
\mathbf{U}_{n_1,n_2} &= -\int_{t=0}^{\infty} \sum_{j_1=0}^{n_1} \left[\mathbf{P}(j_1, n_2, t) \otimes e^{\mathbf{\Omega} t}\right] \qquad (55) \\
&\quad \times \left(\mathbf{D}_{n_1-j_1}^{(1)} \otimes \mathbf{I}_{m_2} \otimes \mathbf{\Omega}^{-1}\right) dt \\
&\quad - \int_{t=0}^{\infty} \sum_{j_2=0}^{n_2} \left[\mathbf{P}(n_1, j_2, t) \otimes e^{\mathbf{\Omega} t}\right] \qquad (56) \\
&\quad \times \left(\mathbf{I}_{m_1} \otimes \mathbf{D}_{n_2-j_2}^{(2)} \otimes \mathbf{\Omega}^{-1}\right) dt
\end{aligned}$$

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$$
\begin{aligned}
&= -\sum_{j_1=0}^{n_1}\left[\mathbf{U}_{j_1,n_2}\left(\mathbf{D}_{n_1-j_1}^{(1)}\otimes\mathbf{I}_{m_2}\otimes\boldsymbol{\Omega}^{-1}\right)\right]\\
&\quad-\sum_{j_2=0}^{n_2}\left[\mathbf{U}_{n_1,j_2}\left(\mathbf{I}_{m_1}\otimes\mathbf{D}_{n_2-j_2}^{(2)}\otimes\boldsymbol{\Omega}^{-1}\right)\right]\\
&\quad-\mathbf{U}_{n_1,n_2}\left(\mathbf{D}_0^{(1)}\otimes\mathbf{I}_{m_2}\otimes\boldsymbol{\Omega}^{-1}\right.\\
&\quad\left.+\mathbf{I}_{m_1}\otimes\mathbf{D}_0^{(2)}\otimes\boldsymbol{\Omega}^{-1}\right). \qquad (57)
\end{aligned}
$$

Post-multiply the relation (57) by $\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}$ leads to

$$
\begin{aligned}
&\mathbf{U}_{n_1,n_2}\left(\mathbf{I}\otimes\boldsymbol{\Omega}\right)\\
&= -\sum_{j_1=0}^{n_1}\left[\mathbf{U}_{j_1,n_2}\left(\mathbf{D}_{n_1-j_1}^{(1)}\otimes\mathbf{I}_{m_2}\otimes\mathbf{I}_{m}\right)\right]\\
&\quad-\sum_{j_2=0}^{n_2}\left[\mathbf{U}_{n_1,j_2}\left(\mathbf{I}_{m_1}\otimes\mathbf{D}_{n_2-j_2}^{(2)}\otimes\mathbf{I}_{m}\right)\right]\\
&\quad-\mathbf{U}_{n_1,n_2}\left[\left(\mathbf{D}_0^{(1)}\oplus\mathbf{D}_0^{(2)}\right)\otimes\mathbf{I}_m\right]. \qquad (58)
\end{aligned}
$$

From the relation (58), the calculation formula of $\mathbf{U}_{n_1,n_2}$ can be obtained.

$$
\begin{aligned}
\mathbf{U}_{n_1,n_2} = &\left[\sum_{j_1=0}^{n_1}\mathbf{U}_{j_1,n_2}\left(\mathbf{D}_{n_1-j_1}^{(1)}\otimes\mathbf{I}_{m_2}\otimes\mathbf{I}_{m}\right)\right.\\
&\left.+\sum_{j_2=0}^{n_2}\mathbf{U}_{n_1,j_2}\left(\mathbf{I}_{m_1}\otimes\mathbf{D}_{n_2-j_2}^{(2)}\otimes\mathbf{I}_{m}\right)\right]\\
&\times\mathbf{U}_{0,0}.
\end{aligned}
$$

The result (1) of Theorem 1 has been proved. Based on the result (1), the results (2), (3) and (4) can be proved, and the processes of these proofs are omitted. ■

The following formulas are introduced.

$$
\begin{aligned}
&\boldsymbol{\Phi}(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2,\boldsymbol{\Omega}_3,n_1,n_2)\\
&=\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_1\right)\boldsymbol{\Psi}\left(\boldsymbol{\Omega}_2,n_1,n_2\right)\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_3\right),\\
&\boldsymbol{\Phi}_1(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2,\boldsymbol{\Omega}_3,n_2)\\
&=\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_1\right)\sum_{n_1=0}^{\infty}\boldsymbol{\Psi}\left(\boldsymbol{\Omega}_2,n_1,n_2\right)\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_3\right),\\
&\boldsymbol{\Phi}_2(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2,\boldsymbol{\Omega}_3,n_1)\\
&=\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_1\right)\sum_{n_2=0}^{\infty}\boldsymbol{\Psi}\left(\boldsymbol{\Omega}_2,n_1,n_2\right)\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_3\right),\\
&\boldsymbol{\Phi}_{12}(\boldsymbol{\Omega}_1,\boldsymbol{\Omega}_2,\boldsymbol{\Omega}_3)\\
&=\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_1\right)\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\boldsymbol{\Psi}\left(\boldsymbol{\Omega}_2,n_1,n_2\right)\left(\mathbf{I}_{\bar{m}}\otimes\boldsymbol{\Omega}_3\right),
\end{aligned}
$$

where $\boldsymbol{\Omega}_1$ is a row vector of appropriate size, $\boldsymbol{\Omega}_2$ is a stable matrix of appropriate size, and $\boldsymbol{\Omega}_3$ is a column vector of appropriate size.

According to the product property of Kronecker product, the formulas for calculating the fundamental terms, which are presented in Appendices A-A, A-B, A-C and A-D, are listed in Table 6.

APPENDIX B THE EXPRESSION OF $\mathbf{K}\left(\left(\phi,L_1,L_2\right),\left(L_1',L_2'\right),T\right)$ AND THE CALCULATION FORMULA OF $\mathbf{K}^{\phi}_{(L_1,L_2),(L_1',L_2')}$

For convenience, let $\mathbb{L}_{1,l_1}=\{l_1,l_1+1,\cdots,Q_1\}$ and $\mathbb{L}_{2,l_2}=\{l_2,l_2+1,\cdots,Q_2\}$, where $l_1\in\{0,1,\cdots,Q_1\}$ and $l_2\in\{0,1,\cdots,Q_2\}$; let

$$
\left\langle\mathbf{P}\left(n_1,n_2,t\right)\right\rangle=\begin{pmatrix}\mathbf{P}\left(\overline{n_1},\overline{n_2},t\right) & \mathbf{P}\left(n_1,\overline{n_2},t\right)\\ \mathbf{P}\left(\overline{n_1},n_2,t\right) & \mathbf{P}\left(n_1,n_2,t\right)\end{pmatrix},
$$

where $t\geq 0$ and $n_1,n_2\in\mathbf{N}$.

(1) Given $\left(\left(1,i,k\right),l_1,l_2\right)\in\tilde{S}_{\xi}^{(1)}$,

(a) consider $\left(l_1',l_2'\right)\in\left(\mathbb{L}_1\times\mathbb{L}_2\right)\setminus\left(\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}\right)$,

$$
\mathbf{K}\Big(\left(\left(1,i,k\right),l_1,l_2\right),\left(l_1',l_2'\right),t\Big)=\mathbf{0}_{\bar{m}},
$$

$$
\mathbf{K}^{(1,i,k)}_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};
$$

(b) consider $\left(l_1',l_2'\right)\in\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}$, for $i=k$,

$$
\begin{aligned}
&\mathbf{K}\Big(\left(\left(1,i,i\right),l_1,l_2\right),\left(l_1',l_2'\right),t\Big)\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{P}\left(l_1'-l_1,l_2'-l_2,t\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right)\\
&\quad\times\left[1-R_1\left(t\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{K}^{(1,i,i)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\overline{\mathbf{R}_1}\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right);
\end{aligned}
$$

for $i\neq k$,

$$
\begin{aligned}
&\mathbf{K}\Big(\left(\left(1,i,k\right),l_1,l_2\right),\left(l_1',l_2'\right),t\Big)\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{P}\left(l_1'-l_1,l_2'-l_2,t\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right)\\
&\quad\times\left[1-B_1\left(t\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{K}^{(1,i,k)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\overline{\mathbf{B}_1}\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right).
\end{aligned}
$$

(2) Given $\left(\left(s_1,j\right),l_1,l_2\right)\in\tilde{S}_{\xi}^{(s_1)}$,

(a) consider $\left(l_1',l_2'\right)\in\left(\mathbb{L}_1\times\mathbb{L}_2\right)\setminus\left(\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}\right)$,

$$
\mathbf{K}\Big(\left(\left(s_1,j\right),l_1,l_2\right),\left(l_1',l_2'\right),t\Big)=\mathbf{0}_{\bar{m}},
$$

$$
\mathbf{K}^{(s_1,j)}_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};
$$

(b) consider $\left(l_1',l_2'\right)\in\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}$, for $l_2=0$,

$$
\begin{aligned}
&\mathbf{K}\Big(\left(\left(s_1,j\right),l_1,l_2\right),\left(l_1',l_2'\right),t\Big)\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{P}\left(l_1'-l_1,l_2'-l_2,t\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right)\\
&\quad\times\left[1-R_2\left(t\right)\right],
\end{aligned}
$$

$$
\begin{aligned}
&\mathbf{K}^{(s_1,j)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\overline{\mathbf{R}_2}\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right);
\end{aligned}
$$

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TABLE 6. The calculation formulas of the fundamental terms

Fundamental term	Formula	Parameters[1]
$\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1,n_2)$	$\mathbf{\Omega}_1=\boldsymbol{\alpha}$
$\sum_{n_1=0}^{\infty}\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_1(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_2)$	$\mathbf{\Omega}_2=\mathbf{T}$
$\sum_{n_2=0}^{\infty}\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_2(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1)$	$\mathbf{\Omega}_3=-\mathbf{T}\mathbf{e}_{m_F}$
$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_{12}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3)$	
$\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1,n_2)$	$\mathbf{\Omega}_1=\boldsymbol{\alpha}$
$\sum_{n_1=0}^{\infty}\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_1(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_2)$	$\mathbf{\Omega}_2=\mathbf{T}$
$\sum_{n_2=0}^{\infty}\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_2(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1)$	$\mathbf{\Omega}_3=\mathbf{e}_{m_F}$
$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_{12}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3)$	
$\bar{\mathbf{G}}\circ\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1,n_2)$	$\mathbf{\Omega}_1=\boldsymbol{\beta}\otimes\boldsymbol{\alpha}$
$\sum_{n_1=0}^{\infty}\bar{\mathbf{G}}\circ\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_1(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_2)$	$\mathbf{\Omega}_2=\mathbf{S}\oplus\mathbf{T}$
$\sum_{n_2=0}^{\infty}\bar{\mathbf{G}}\circ\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_2(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1)$	$\mathbf{\Omega}_3=-\mathbf{e}_{m_G}\otimes(\mathbf{T}\mathbf{e}_{m_F})$
$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\bar{\mathbf{G}}\circ\mathbf{F}(n_1,n_2)$	$\mathbf{\Phi}_{12}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3)$	
$\bar{\mathbf{G}}\circ\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1,n_2)$	$\mathbf{\Omega}_1=\boldsymbol{\beta}\otimes\boldsymbol{\alpha}$
$\sum_{n_1=0}^{\infty}\bar{\mathbf{G}}\circ\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_1(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_2)$	$\mathbf{\Omega}_2=\mathbf{S}\oplus\mathbf{T}$
$\sum_{n_2=0}^{\infty}\bar{\mathbf{G}}\circ\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_2(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3,n_1)$	$\mathbf{\Omega}_3=\mathbf{e}_{m_F m_G}$
$\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\bar{\mathbf{G}}\circ\bar{\mathbf{F}}(n_1,n_2)$	$\mathbf{\Phi}_{12}(\mathbf{\Omega}_1,\mathbf{\Omega}_2,\mathbf{\Omega}_3)$	

[1] From Theorem 1.3.17 of [34], the matrices $\mathbf{T}$ and $\mathbf{S}$ are all stable; and based on the eigenvalue property of Kronecker sum [39], the matrix $\mathbf{S}\oplus\mathbf{T}$ is also stable.

for $l_2\neq 0$,

$$\begin{aligned}
&\mathbf{K}\Big(\big((s_1,j),l_1,l_2\big),(l_1',l_2'),t\Big)\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\mathbf{P}(l_1'-l_1,l_2'-l_2,t)\rangle\boldsymbol{\delta}^T(Q_1,l_1')\\
&\quad\times[1-B_2(t)][1-H_j(t)]\\
&\quad+\sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2}\int_{x=0}^{t}\boldsymbol{\delta}(Q_2,l_2+i_2)\langle\mathbf{P}(i_1,i_2,x)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1+i_1)\\
&\quad\times[1-B_2(x)]dH_j(x)\boldsymbol{\delta}(Q_2,l_2')\\
&\quad\times\langle\mathbf{P}(l_1'-i_1-l_1,l_2'-i_2-l_2,t-x)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1')[1-R_2(t-x)],
\end{aligned}$$

$$\begin{aligned}
&\mathbf{K}^{(s_1,j)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\overline{\mathbf{B}_2}\circ\overline{\mathbf{H}_j}(l_1'-l_1,l_2'-l_2)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1')\\
&\quad+\sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2}\boldsymbol{\delta}(Q_2,l_2+i_2)\langle\overline{\mathbf{B}_2}\circ\mathbf{H}_j(i_1,i_2)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1+i_1)\boldsymbol{\delta}(Q_2,l_2')\\
&\quad\times\langle\overline{\mathbf{R}_2}(l_1'-i_1-l_1,l_2'-i_2-l_2)\rangle\boldsymbol{\delta}^T(Q_1,l_1').
\end{aligned}$$

(3) Given $\big((2,j),l_1,l_2\big)\in\tilde{S}_{\xi}^{(2)}$,

(a) consider $(l_1',l_2')\in(\mathbb{L}_1\times\mathbb{L}_2)\setminus(\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2})$,

$$\mathbf{K}\Big(\big((2,j),l_1,l_2\big),(l_1',l_2'),t\Big)=\mathbf{0}_{\bar{m}},$$

$$\mathbf{K}^{(2,j)}_{(l_1,l_2),(l_1',l_2')}=\mathbf{0}_{\bar{m}};$$

(b) consider $(l_1',l_2')\in\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}$,

for $l_2=0$,

$$\begin{aligned}
&\mathbf{K}\Big(\big((2,j),l_1,l_2\big),(l_1',l_2'),t\Big)\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\mathbf{P}(l_1'-l_1,l_2'-l_2,t)\rangle\boldsymbol{\delta}^T(Q_1,l_1')\\
&\quad\times[1-R_2(t)],
\end{aligned}$$

$$\begin{aligned}
&\mathbf{K}^{(2,j)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\overline{\mathbf{R}_2}(l_1'-l_1,l_2'-l_2)\rangle\boldsymbol{\delta}^T(Q_1,l_1');
\end{aligned}$$

for $l_2\neq 0$,

$$\begin{aligned}
&\mathbf{K}\Big(\big((2,j),l_1,l_2\big),(l_1',l_2'),t\Big)\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\mathbf{P}(l_1'-l_1,l_2'-l_2,t)\rangle\boldsymbol{\delta}^T(Q_1,l_1')\\
&\quad\times[1-B_2(t)][1-H_j(t)]\\
&\quad+\sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2}\int_{x=0}^{t}\boldsymbol{\delta}(Q_2,l_2+i_2)\\
&\quad\times\langle\mathbf{P}(i_1,i_2,x)\rangle\boldsymbol{\delta}^T(Q_1,l_1+i_1)\\
&\quad\times[1-B_2(x)]dH_j(x)\boldsymbol{\delta}(Q_2,l_2')\\
&\quad\times\langle\mathbf{P}(l_1'-l_1-i_1,l_2'-l_2-i_2,t-x)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1')[1-R_2(t-x)],
\end{aligned}$$

$$\begin{aligned}
&\mathbf{K}^{(2,j)}_{(l_1,l_2),(l_1',l_2')}\\
&=\boldsymbol{\delta}(Q_2,l_2')\langle\overline{\mathbf{B}_2}\circ\overline{\mathbf{H}_j}(l_1'-l_1,l_2'-l_2)\rangle\\
&\quad\times\boldsymbol{\delta}^T(Q_1,l_1')
\end{aligned}$$

$$+\sum_{i_1=0}^{l_1'-l_1}\sum_{i_2=0}^{l_2'-l_2}\boldsymbol{\delta}\left(Q_2,l_2+i_2\right)\left\langle\overline{\mathbf{B}_2}\circ\mathbf{H}_j\left(i_1,i_2\right)\right\rangle$$
$$\times\boldsymbol{\delta}^T\left(Q_1,l_1+i_1\right)\boldsymbol{\delta}\left(Q_2,l_2'\right)$$
$$\times\left\langle\overline{\mathbf{R}_2}\left(l_1'-i_1-l_1,l_2'-i_2-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right).$$

(4) Given $(s_2,l_1,l_2)\in\tilde{S}_{\xi}^{(s_2)}$,

(a) consider $(l_1',l_2')\in(\mathbb{L}_1\times\mathbb{L}_2)\setminus(\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2})$,

$$\mathbf{K}\big(\left(s_2,l_1,l_2\right),\left(l_1',l_2'\right),t\big)=\mathbf{0}_{\bar{m}},$$

$$\mathbf{K}^{s_2}_{(l_1,l_2),\left(l_1',l_2'\right)}=\mathbf{0}_{\bar{m}};$$

(b) consider $(l_1',l_2')\in\mathbb{L}_{1,l_1}\times\mathbb{L}_{2,l_2}$, for $l_1=0$,

$$\mathbf{K}\big(\left(s_2,l_1,l_2\right),\left(l_1',l_2'\right),t\big)$$
$$=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{P}\left(l_1'-l_1,l_2'-l_2,t\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right)$$
$$\times\left[1-R_1\left(t\right)\right],$$

$$\mathbf{K}^{s_2}_{(l_1,l_2),\left(l_1',l_2'\right)}$$
$$=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\overline{\mathbf{R}_1}\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right);$$

for $l_1\neq 0$,

$$\mathbf{K}\big(\left(s_2,l_1,l_2\right),\left(l_1',l_2'\right),t\big)$$
$$=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\mathbf{P}\left(l_1'-l_1,l_2'-l_2,t\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right)$$
$$\times\left[1-B_1\left(t\right)\right],$$

$$\mathbf{K}^{s_2}_{(l_1,l_2),\left(l_1',l_2'\right)}$$
$$=\boldsymbol{\delta}\left(Q_2,l_2'\right)\left\langle\overline{\mathbf{B}_1}\left(l_1'-l_1,l_2'-l_2\right)\right\rangle\boldsymbol{\delta}^T\left(Q_1,l_1'\right).$$

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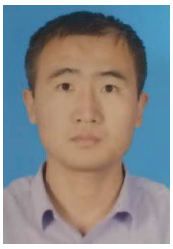

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