Financial Contagion in a Core–Periphery Interbank Network

Sui, P., Tanna, S. & Zhou

Financial Contagion in a Core–Periphery Interbank Network

Peng Sui[*]   Sailesh Tanna[†]   Dandan Zhou[‡]

Abstract

This paper studies financial contagion in a core-periphery interbank network where core banks are large in balance sheet size while periphery banks are smaller and link only with the core banks. Core banks are all bilaterally linked and intermediate liquidity for periphery banks. We establish analytic conditions under which financial contagion propagates in the core-periphery network and examine the extent to which heterogeneity associated with size and number of banks affects these conditions. We show that the failure of core banks does not necessarily imply contagious failure of periphery banks; the core-periphery network structure exhibits a 'robust-yet-fragile' tendency with increased size of core banks; and the resilience of the network to contagion depends on the number of core banks, the number of periphery banks, and the level of interbank liquidity intermediated between the core banks. We also find that, under certain conditions, the core-periphery network is more resilient than the complete network with increased size of core banks.

[*] The Center for Economic Research, Shandong University, No. 27 Shanda Nanlu, Jinan, 250100, P. R. China. Email: peng.sui@sdu.edu.cn.

[†] Corresponding author. Research Centre for Financial and Corporate Integrity and School of Economics, Finance and Accounting, Faculty of Business and Law, Coventry University, Priory Street, Coventry, CV1 5FB, UK. Email: s.tanna@coventry.ac.uk

[‡] The Center for Economic Research, Shandong University, No. 27 Shanda Nanlu, Jinan, 250100, P. R. China. Email: zhoudandan@sdu.edu.cn.

1. Introduction

Banking connections can be ideally represented as a network in which nodes represent banks, links reflect interbank lending and borrowing, and the structure characterizing their interdependence represents the configuration of the banking system. Network connections enable banks to diversify risk but they also provide channels through which shocks can spread by financial contagion (Glasserman and Young, 2016). The global financial crisis demonstrated that the failure of one bank can cause contagious failure of other banks owing to their interconnected links, potentially generating systemic risk in the banking system (Yellen, 2013).[1]

The theoretical literature on financial networks shows that the magnitude of financial contagion depends, among other things, on the structure of the interbank network (e.g. Allen and Gale, 2000, Freixas et al. 2000, Acemoglu et al., 2015; Castiglionesi and Eboli, 2018). As Glasserman and Young (2016) assert, different characteristics of interbank networks and their interconnectedness can have different implications for financial contagion.

This paper contributes to the literature on financial networks by presenting a theoretical model to assess financial contagion in a stylized core-periphery interbank network that is found in many financial systems across the world. We establish conditions under which financial contagion propagates in the core-periphery network, and examine the extent to which heterogeneity associated with size and number of banks affects these conditions. We define the core-periphery network as follows: core banks are large in balance sheet size and are all bilaterally linked with each other;

---

[1] The prominent example is in the aftermath of Lehman Brother's bankruptcy where we witnessed a collapse of trading activity in the interbank market because of heightened counterparty risk associated with direct interbank exposures; see Gabrieli and Georg (2014) for an explanation of interbank market freeze from a network perspective.

periphery banks are relatively small and link only with the core banks; core banks intermediate liquidity among themselves and for their directly linked periphery banks.

The study of above defined core-periphery network is motivated by an overwhelming number of empirical studies which reveal three stylized features of interbank markets. First, the interbank markets are tiered. Boss et al. (2004) showed that the Austrian interbank market can best be represented by a tiered community structure where, within every community, many second tier banks only link with a first tier core bank, whereas core banks link with each other. Similar results are found in the Italian interbank market, e-MID (De Masi et al., 2006; Iori et al., 2008), the US Fedwire system (Soramäki et al., 2007), the US Federal Funds market (Bech and Atalay, 2010), and the UK interbank system (Langfield et al., 2014). Other studies also confirm the core-periphery structure for the German interbank system (Craig and von Peter, 2014), the Dutch interbank market (in't Veld and van Lelyveld, 2014), and the Italian interbank market (Fricke and Lux, 2015). Second, the aforementioned studies all find that banks belonging to the first tier are larger in balance sheet size than banks in the second tier; in other words, the periphery banks are smaller relative to the core banks. The third stylized feature is that core banks not only share liquidity risk but also intermediate interbank deposits with their directly linked periphery banks which do not have the facility to extend credits among themselves. By using the core-periphery algorithm, Craig and von Peter (2014), Fricke and Lux (2015), and in't Veld and van Lelyveld (2014) confirm that interbank markets exhibit core-periphery structures that are clustered around a tight set of large core banks which intermediate liquidities for their many but relatively smaller periphery banks.

In deriving results and analyzing contagion in a core-periphery network, we first extend the classic Allen and Gale (2000) model to an economy with many sectors, each

of which contains a large bank and many small banks. A large bank exchanges interbank deposits with large banks in other sectors to share liquidity risk, and each large bank intermediates liquidity for all the small banks in the same sector. In this set-up, we examine financial contagion by assuming the failure of a core bank and the failure of a periphery bank, respectively. Differently from Allen and Gale (2000), our analysis shows that contagious failure of one bank does not necessarily imply contagious failure of all banks, because the loss due to default depends not only on the amount of interbank claims other banks have at the defaulting bank, but also on the liquidation value of that bank.[2] An important implication of this finding is that the failure of core banks does not necessarily lead to contagious failure of periphery banks. This result also contrasts with studies on targeted shocks under a fat-tailed financial network (e.g., Gai et al., 2011), which assert that the financial network is very vulnerable to the failure of core banks simply because of their high connectivity.

Our analysis then demonstrates the importance of having large core banks in a network. With increasing size of the core banks, the core-periphery network exhibits a robust-yet-fragile tendency, adding to the current research on financial networks which suggests the robust-yet-fragile property exists in terms of average connectivity (e.g., Gai and Kapadia, 2010). The resilience of the network to contagion also increases with greater number of core banks, which is intuitive since increasing the number of core banks decreases the relative weight of the interbank linkage between the core banks. A similar effect holds when increasing the number of periphery banks while holding the level of cross-sector liquidity risk-sharing constant.

---

[2] Allen and Gale (2000) study financial contagion in symmetric financial networks, in which once one of the neighbours of a defaulting bank becomes bankrupt the rest of the banks in the connected network must fail. This is because in symmetric networks the loss due to default decreases monotonically with the number of contagious failures.

We next analyze the effect of interbank liquidity intermediation on financial contagion. We show that, conditional on a small number of core banks, if adding a periphery bank increases the core banks' intermediation activity within the sector, then the network is more resilient to contagion; whereas if adding a periphery bank increases interbank intermediation among sectors then contagion is more likely to propagate. The effect reverses when there are many core banks. The implication of this finding is that if there are few core banks, the network is more resilient to contagion if the intermediation is mostly done within the sector; whereas if there are many core banks the network resilience increases with the amount of cross-sector intermediation. This result is similar to the work of Acemoglu et al. (2015) and Morris (2000) on modeling contagion in weighted networks.

Finally, following the approach of previous studies (e.g. Allen and Gale, 2000; Acemoglu et al., 2015; Castiglionesi and Eboli, 2018), we compare the resiliency to contagion of the core-periphery network and the complete network where every bank is connected to all other banks. We find that, under certain conditions, the core-periphery network is more resilient with increasing size of core banks.

The rest of the paper is structured as follows. Section 2 discusses related literature. Section 3 presents the model, after which section 4 shows that Pareto-optimal allocation can be decentralized in the core-periphery network. Section 5 investigates financial contagion assuming the failure of one core bank and one periphery bank, respectively. Section 6 analyzes the network heterogeneities. Section 7 compares the resiliency of the core-periphery and complete networks. Section 8 concludes. In the working paper version of this paper (Sui, Tanna and Zhou, 2019), we also include an Appendix - available from the authors upon request – which contains proofs of some of the propositions outlined in this paper.

2. Related Literature

The paper is part of a growing literature that examines how an initial shock to a bank, which leads to interbank defaults through direct linkages, propagates within a given interbank network.[3] As Hüser (2015) and Glasserman and Young (2016) reveal in their comprehensive surveys on the interbank networks literature, the majority of these studies rely on assessing financial contagion using (average) connectivity as a channel for the transmission of liquidity shocks.[4] In their seminal work, Allen and Gale (2000) argue that networks with higher connectivity yield lower risk of contagion. However, Freixas et al. (2000) and Brusco and Castiglionesi (2007) find that the extent of contagion is greater the larger the number of interbank connections. Eboli (2013) shows that the complete network is fragile to large shocks but more resilient to relatively small shocks. Acemoglu et al. (2015) show that less connectivity is better able to prevent contagion in the presence of large shocks, whereas a complete network is more resilient to small shocks.

While the aforementioned papers assess financial contagion through connectivity, other studies use numerical simulation to study contagion in interbank networks typically with star or core-periphery structure. For example, Nier et al. (2007) explore the consequences of a targeted shock in a star network with one large core bank connecting many small periphery banks. Their analysis focuses on the relationship

---

[3] While the core-periphery network can be regarded as determined endogenously within our framework (via decentralized allocation), we do not study a network formation game in a strategic sense, which views the network as emerging from the trade-off between efficiency and stability. The existence of this network as an equilibrium structure based on strategic interaction is discussed in Farboodi (2014), Babus and Hu (2017) and Castiglionesi and Navarro (2016).

[4] Some studies analyse financial contagion using random or fat-tailed networks. For instance, Gai and Kapadia (2010) reveal features of an interbank network exhibiting a robust-yet-fragile property, whereby high connectivity increases risk-sharing and reduces the probability of contagious failure, but when contagion occurs, more links allow the possibility of widespread default cascades. Gai et al. (2011), who study a fat-tailed network, find that failure of a bank with a large number of links makes financial contagion almost certain for a very wide range of parameter values under a targeted shock.

between number of periphery banks and contagion. Assuming a shock hits the large bank, defaults increase initially with the number of periphery banks but then decrease with more periphery banks in the network, revealing a non-monotonic relationship between connectivity and contagion.[5] Krause and Giansante (2012) study the size effect of the core banks in a tiered (core-periphery) network. They show that losses from a periphery bank are unlikely to spread as the larger size of the core banks allows them to absorb losses more easily and thus contagious failures will be limited - a result similar to the random attack in a scale-free network with well-connected banks having large balance sheet size. Elliott et al. (2014) consider financial contagion in a core-periphery network formed by the cross-holding of liabilities. Assuming the failure of a periphery bank, they simulate how the weight of interbank linkage between core banks affects financial contagion. They find a non-monotonicity in the integration: widespread contagions occur if core banks have middle level of cross-holdings, whereas the contagions are less severe if this level is either low or high.

A recent paper by Castiglionesi and Eboli (2018) presents analytic results on comparing the resilience of the complete interbank network and the star-shaped network in which there is one large core bank connecting with many small periphery banks. Differently from our paper, they apply flow network theory to analyse liquidity flows in interbank networks, and conclude that the star network is more resilient to systemic risk than the complete network. We add to their analysis by comparing the resilience between the complete network and the core-periphery network which represents a collection of star networks with all core banks bilaterally linked.

---

[5] Freixas et al. (2000) also examine contagious failure in a star network with one core bank and homogenous banking size. They show that the effect of a shock on the center bank is more severe than on a periphery bank since the center bank is well connected.

At a more general level, Glasserman and Young (2016) point out that one of the key lessons of the current literature is that we cannot draw any conclusions regarding the contagion effect of interbank connections without accounting for the influence of other factors, such as heterogeneity in size and weights of links. As they argue, the interaction between these factors and the network topology have not yet been fully understood at a theoretical level. Our paper, in this sense, aims to fill a gap by considering three levels of heterogeneity, namely: the number of (periphery and core) banks, the size of the core banks, and the weight of interbank linkage. By bringing together different types of heterogeneity in a single core-periphery interbank network, we show that some established results from previous studies may not necessarily hold.[6] Also, the analysis of how financial contagion spreads across the network in the presence of interbank intermediation has, to the best of our knowledge, not been investigated before.[7]

3. The Model

The economy lasts for three dates, $t = 0, 1, 2$, and consists of n sectors, where n is an even number. Each sector is denoted by i, $i \in \{1, 2, 3, \dots, n\}$, and contains the same number of regions m where $m \geq 3$.[8] Each region is denoted by ij, where $j \in \{1, 2, 3, \dots, m\}$ and $ij \in \{1, 2, 3, \dots, n\} \times \{1, 2, 3, \dots, m\}$. There is a continuum of *ex*

---

[6] For example, as we shall see, the effect of the number of periphery banks depends also on the liquidity characteristics of the additional bank, and the size effect of the core banks is affected by the weight of link between core banks.

[7] In Elliott et al. (2014), the total amount of cross-holdings for each core bank is exogenously given. Therefore, the greater integration between core banks means less link weight between the core bank and periphery banks, the core banks hence become more resistant to peripheral failures. Our analysis is different from theirs because, when taking the role of interbank intermediaries for core banks into account, the total cross-holdings for core banks depends on several factors hence not exogenous.

[8] The notion of a region or sector can be interpreted as different categories of banks focusing on lending to different industries, or it can be a geographical metaphor that banks lend to different regions in spatial terms. The regional structure can have many interpretations as long as different regions receive different liquidity shocks. An interbank network thus plays an important role in redistributing liquidities.

ante identical consumers in each region. The population in region im is k times larger than other regions in each sector. We call im large regions and $ij, \forall j \neq m$, small regions.

Each consumer, endowed with one unit of homogeneous consumption good at date 0, is uncertain about her liquidity preference in the future date. That is, with probability ω_{ij} a consumer in region ij is an early consumer who values only date 1 consumption C_1 with utility $u(C_1)$, and with probability $1 - \omega_{ij}$ she is a late consumer and values only date 2 consumption C_2 with utility $u(C_2)$; where $u(\cdot)$ is a neoclassical utility function, i.e., $u'(\cdot) > 0, \ u''(\cdot) < 0$.

The probability can take one of the two values which can be either high or low, $\omega_{ij} \in \{\omega_L, \omega_H\}, \ 0 < \omega_L < \omega_H < 1$. There are two equally likely states of nature, $S = \{S_1, S_2\}$. In each region, if the probability of being an early consumer is ω_L in one state, it is ω_H in the other state. We normalize the measure of the set of consumers in a small region to be equal to one. Since liquidity shocks are independent, the law of large numbers holds in each region. We can denote ω_H and ω_L equivalently as different amounts of early consumers in small regions, and, ω_H^m and ω_L^m as different amounts of early consumers in large regions, where $\omega_H^m = k\omega_H$ and $\omega_L^m = k\omega_L$. Denote by λ $(k\lambda)$ the average demand for liquidity in each small (large) region. Since the realization of state S_1 and S_2 is equally probable in each region, we have

$$\lambda = \frac{\omega_H + \omega_L}{2} = \frac{\omega_H^m + \omega_L^m}{2k}.$$

Let Δ be the excess liquidity demand with respect to average liquidity in small region, we can write

$$\omega_H = \lambda + \Delta \ \text{ and } \ \omega_L = \lambda - \Delta \tag{1}$$

for small regions, and

$$\omega_H^m = k(\lambda + \Delta) \ \text{ and } \ \omega_L^m = k(\lambda - \Delta) \tag{2}$$

for large regions.

The value of probability varies within and across regions. However, aggregate liquidity demand is the same in two states.[9] In each sector, there are u small regions negatively correlated with v small regions and with the large region, where $u + v = m - 1$ and $k + v \neq u$.[10] Each large region positively correlates with $n/2 - 1$ large regions in other sectors, and negatively correlates with the rest of the $n/2$ large regions. To explain the correlations in more detail, suppose the economy consists of two sectors with four regions in each sector, and, the population in large regions is twice the size in small regions; that is $n = 2$, $m = 4$ and $k = 2$. Also suppose $u = 1$ and $v = 2$. Then, two large regions 14 and 24 are negatively correlated: so if region 14 experiences high proportion of early consumer ω_H, then region 24 will experience ω_L, and *vice versa*. In each sector, there is one small region negatively correlated with the large region, and there are two small regions positively correlated with the large region. Without loss of generality, suppose in state 1 large region 14 experiences ω_H. There is one small region in sector 1 experiencing ω_L ; and there are two small regions experiencing ω_H . In sector 2, large region 24 experiences ω_L ; one small region experiences ω_H ; and two small regions experience ω_L. Using (1) and (2), the total amount of early consumers is $2(3 + 2)\lambda = 10\lambda$ in state 1. In state 2, the total number of early consumers is also

[9] When the liquidity shock occurs, a region either experiences a high proportion or a low proportion of early consumers. This assumption of symmetric liquidity shock is also used by Allen and Gale (2000) and Castiglionesi and Eboli (2018). In Castiglionesi and Eboli (2018), the liquidity shock has an expected value equal to zero. In our model, unlike Allen and Gale (2000), the expected amount of early consumer is equal to λ for small regions and $k\lambda$ for large regions.

[10] The correlations between regions create incentives for liquidity risk-sharing. As we shall see later, for $k + v \neq u$, complete risk-sharing can be reached only through sectoral connection. If $k + v = u$, complete liquidity risk sharing can be done within each sector. In this case a star network in each sector is an equilibrium. Under the assumption of costly connection, this n-disconnected star network Pareto dominates the core-periphery network. Since the main focus of the paper is on contagion effects under the core-periphery network, we assume that complete liquidity risk-sharing cannot be achieved within sectors, i.e., $k + v \neq u$. See footnote 13 for further discussion of the issue of network formation.

10λ and the liquidity shock for each region is just the opposite to state 1. In any state the aggregate liquidity demand is the same or, equivalently, there is no excess liquidity demand in the economy. Table 1 shows the details of the realization of consumers' liquidity shocks for the general case.

	11	...	1u	1u+1	...	1m-1	1m
S_1	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_H^m = k(\lambda + \Delta)$
S_2	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_L^m = k(\lambda - \Delta)$
	21	...	2u	2u+1	...	2m-1	2 m
S_1	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_L^m = k(\lambda - \Delta)$
S_2	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_H^m = k(\lambda + \Delta)$
	31	...	3u	3u+1	...	3m-1	3 m
S_1	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_H^m = k(\lambda + \Delta)$
S_2	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_L^m = k(\lambda - \Delta)$
	⋮		⋮	⋮		⋮	⋮
	n1	...	nu	nu+1	...	nm-1	nm
S_1	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_L^m = k(\lambda - \Delta)$
S_2	$\omega_L = \lambda - \Delta$		$\omega_L = \lambda - \Delta$	$\omega_H = \lambda + \Delta$		$\omega_H = \lambda + \Delta$	$\omega_H^m = k(\lambda + \Delta)$

Table 1: Regional Liquidity Shocks at Date 1.

There are two types of asset in the economy: short asset and long asset. One unit of consumption good invested in short asset produces a gross return of 1 unit of consumption good after one period. The long asset has higher return, $R > 1$, but requires two periods to mature. The long asset can be liquidated prematurely at date 1 yielding r unit of consumption good for each unit invested at date 0, where $1 > r > 0$.

4. Decentralized Allocation in the Core-Periphery Network

We now show that a Pareto-optimal allocation can be decentralized in the core-periphery network. Since consumers are *ex ante* identical and there is no aggregate uncertainty, Pareto-optimal allocation is state independent. A central planner only needs to solve

$$\max \quad \lambda u(C_1) + (1 - \lambda)u(C_2),$$

subject to feasibility conditions for different regions at different dates. The feasibility constraint at date 0 is

$$x + y = 1$$

for small region (multiplied by k for large region), where x and y denote the amount of endowments invested in the long and short asset, respectively. At date 1 the planner needs to satisfy

$$\lambda C_1 = y.$$

The average fraction of liquidity demand is λ ($k\lambda$) for each small (large) region, and each early consumer is promised C_1. The demand for early consumption is equal to the return from short asset.[11] At date 2, there is on average $1 - \lambda$ ($k(1 - \lambda)$) fraction of liquidity demand in each small (large) region. We have

$$(1 - \lambda)C_2 = Rx.$$

The return from the long asset has to meet the deposit demand from late consumers. The solution to the maximization problem is

$$u'(C_1^*) = Ru'(C_2^*), \tag{3}$$

which also satisfies the first-order condition, where C_1^* and C_2^* are the optimal consumption value at date 1 and 2, respectively. The optimal portfolios for small and large regions are then

---

[11] Since $R > 1$, all the feasibility conditions must be equal. It is never optimal to carry over any short asset from date 1 to date 2.

$$(y, x) = (\lambda C_1^*, (1 - \lambda)C_2^*/R) \qquad (4)$$

and

$$(ky, kx) = (k\lambda C_1^*, k(1 - \lambda)C_2^*/R) \qquad (5)$$

respectively. It is easy to verify that condition (3) implies $C_2^* > C_1^*$, meaning that the optimal solution is also incentive efficient.

To illustrate, consider large regions only. Let hm denote the odd numbered large region, where $h \in \{1, 3, 5, \dots, n - 1\}$ and lm be the even numbered large region, where $l \in \{2, 4, 6, \dots, n\}$. Suppose the economy experiences state S_1, where the amount of early consumers in each region hm and lm are ω_H^m and ω_L^m, respectively. According to asset allocation (5), each large region has $k\lambda C_1^*$ units of short asset. Equation (2) implies region hm has excess demand of $k\Delta C_1^*$ units of liquidity which is equal to the excess liquidity supply in region lm. Since the number of regions in h is the same as the regions in l, the central planner can reallocate $k\Delta C_1^*$ from each lm to each hm. At date 2, the transfer reverses. Each lm has excess demand of $k\Delta C_2^*$ consumption goods which is equal to the excess supply in each hm. Same reallocation process applies also to small regions.

Next consider the banking solution. Regional banking industry is competitive. We assume that there is a representative bank in each region which maximizes depositors' expected utility, subject to a zero-profit constraint. At date 0, consumers deposit their consumption goods at their regional bank; in exchange, each consumer receives a deposit contract which promises an amount of consumption, $C_{1,ij}^S$ or $C_{2,ij}^S$, depending on when and in which state she chooses to withdraw. Bank ij then allocates the deposits in short asset, long asset and the interbank on behalf of the depositors, where the interbank deposit contract is the same as consumer's deposit contract.

The fluctuation in liquidity demand among regions motivates banks to share liquidity risk by exchanging interbank deposit at date 0, so that banks with liquidity surplus can provide liquidity for banks with liquidity shortage. We next show that banks can offer each of their depositors a state independent deposit contract which gives Pareto-optimal consumption, i.e., $\left(C_{1,ij}^{S}, C_{2,ij}^{S} \right) = (C_1^*, C_2^*), \forall ij, \forall S$, if they form the core-periphery network.

For simplicity, we call a bank in a large region the large bank and a bank in a small region the small bank. Let the large bank exchange Δ amount of deposits with each of the small banks in the same sector, and exchange $2|k + v - u|\Delta/n$ with each of the other large banks $jm, \forall j \neq i.$[12] The date 0 budget constraint for each small bank is

$$x + y + \Delta = 1 + \Delta,$$

where the left-hand side (LHS) is the total assets consisting of long asset, short asset and the claims of Δ interbank deposits in the large bank from the same sector, and the right-hand side (RHS) is the liabilities comprising consumers' deposit and the large bank's interbank deposit claims. The budget constraint for each large bank is

$$k(x + y) + \left[(m - 1) + (n - 1)\frac{2|k + v - u|}{n} \right]\Delta = k + \left[(m - 1) + (n - 1)\frac{2|k + v - u|}{n} \right]\Delta.$$

The term in the square bracket on the LHS is large bank's total lending consisting of Δ to each small bank, with the sum of $m - 1$ small banks in sector i, and $2|k + v - u|\Delta/n$ to each large bank, with $n - 1$ large banks in total. The square bracket on the RHS is the total interbank borrowing from all small banks in the same sector and large banks. Figure 1 illustrates the archetypal interbank network with a core-periphery structure.

---

[12] Castiglionesi and Eboli (2018) characterize the minimum interbank deposit that achieves the full coverage of liquidity risk in different networks. According to their analysis, we can assert that the equilibrium interbank deposit each bank holds in the core-periphery network (and the complete network - see Section 7) is also minimum to achieve efficient allocation. Interested readers may refer to the proofs of Proposition 1-3 in Castiglionesi and Eboli (2018).
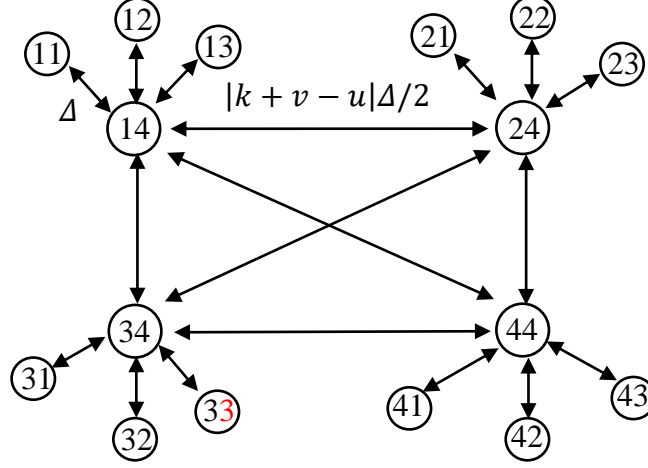
Figure 1: The figure illustrates a core-periphery financial network with $m = 4$ and $n = 4$, where core banks $\{14, 24, 34, 44\}$ are large and periphery banks $ij, j \neq 4$, are small. The interbank deposit exchange is $|k + v - u|\Delta/2$ between each two core banks, and Δ between each periphery bank and core bank.

Given the cross holdings of interbank deposits, we then have[13]

Proposition 1 *Pareto-optimal allocation can be achieved in the core-periphery network, in which each large bank exchanges Δ deposits with each small bank in the same sector and exchanges $2|k + v - u|\Delta/n$ deposits with each of the other large banks. Large banks are core banks that intermediate liquidity for small banks. Small banks are periphery banks each of which links only with a core bank.*

The proof of Proposition 1 is in Appendix A.1 (Sui et al., 2019). To grasp the intuition, it is easy to see that all banks' budget constraints at date 0 can be reduced to date 0 feasibility constraint. We only need to prove that under the core-periphery

---

[13] One may argue about the multiplicity of equilibrium structures. One possible refinement is to introduce information asymmetry on liquidity correlation at date 0. In this case, the core-periphery network can be a strict bilateral equilibrium (see Goyal and Vega-Redondo, 2007; and also Jackson, 2008 for discussion on the stability of different equilibrium concepts). It is because any one bank, or two banks by coordinating their actions, would be worse off by deviating from the core-periphery network. For example, any periphery bank would be worse off dropping the link with the core bank because there is no risk sharing opportunity; and, any two periphery banks have no incentive to drop the link with their core banks and, instead, exchange Δ with each other, because there is a positive probability that they are positively correlated. Same reason holds for the core banks. Since our main focus is on the contagion process in the core-periphery network (and the analysis of network formation games is beyond the scope of this paper), the current model suffices to yield the core-periphery equilibrium network for the analysis.

network, banks' budget constraints at date 1 and 2 are also the same as the feasibility conditions for the central planner.

First consider the periphery banks. For each periphery bank having ω_H amount of early consumers, it liquidates the interbank deposits from the core bank, and for each periphery bank experiencing ω_L liquidity demand, the core bank liquidates Δ deposits from it. Suppose the economy is in state S_1 and $k + v > u$. There is an excess demand for liquidity in sector h, so each core bank in region hm then liquidates the interbank deposits from core bank in region lm in order to satisfy the deposit withdrawal for its own early consumers as well as for the periphery banks. Each bank in lm is willing to meet the deposit demand because each sector l has excess supply of liquidity which is the same as the excess demand in each sector h. At date 2, the situation reverses and the same argument applies. Suppose $k + v < u$, the interbank deposit withdrawal among core banks reverses at date 1. Since the number of banks in hm and in lm are the same, there is no aggregate excess liquidity demand in any of the cases. The symmetric states imply that deposit contracts are state independent. Each bank can therefore offer (C_1^*, C_2^*) to its depositors.

5. Financial Contagion

The interbank deposit exchange can cause financial contagion if there is an excess liquidity demand at date 1. This section studies how financial contagion propagates in the core-periphery network by considering a liquidity shock on a core and a periphery bank respectively.

Table 2 depicts the realization of liquidity shocks with excess liquidity demand. Assume there are two additional states, S_3 and S_4. In state S_3, all banks face average demand for liquidity except for the core bank in nm which faces liquidity demand of

16

$k\lambda + \varepsilon$. In state S_4, periphery bank 11 faces liquidity demand $\lambda + \varepsilon$ whereas other banks experience average amount of early consumers. Both S_3 and S_4 occur with infinitely small probability so that date 0 asset allocations do not change.[14]

	11	12	...	1m	...	n1	n2	...	nm
S_3	λ	λ	...	$k\lambda$	...	λ	λ	...	$k\lambda + \varepsilon$
S_4	$\lambda + \varepsilon$	λ	...	$k\lambda$	...	λ	λ	...	$k\lambda$

Table 2: Regional Liquidity Shocks with Excess Aggregate Liquidity Demand.

Note that banks follow a pecking order in liquidating different assets. Intuitively, liquidating short assets is least costly because the return is one, whereas by liquidating one unit of interbank deposit, bank give up C_2^* units of date 2 consumption and obtain C_1^* units of date 1 consumption which incur a cost of $C_2^* / C_1^* > 1$ given condition (3). By liquidating one unit of long asset banks receive r unit of date 1 consumption and give up R units of future consumption. We assume r is small such that

$$\frac{R}{r} > \frac{C_2^*}{C_1^*} > 1. \tag{6}$$

Condition (6) implies that, in the presence of excess liquidity demand, all banks prefer liquidating their interbank deposits to long asset. This mutual withdrawal cancels out the effect of interbank liquidity risk-sharing, and, as a result, the only way to satisfy excess liquidity demand is by liquidating long asset.[15] A bank defaults if it cannot meet liquidity demand at date 1, and the value of deposit is no longer C_1^*. Instead, all

---

[14] The assumption of infinitely small probability is important in deriving the equilibrium network that is Pareto optimal. With positive probability of excess aggregate liquidity, first-best allocation cannot be achieved in decentralized economy because markets for liquidity risk-sharing are incomplete. As a result, feasible asset allocations for banks are difficult to characterize; see Allen and Gale (2000) for further explanation of the difficulties in imposing positive probability.

[15] The justification for this argument is that banks suffer from coordination failure because in the presence of excess liquidity demand, liquidating long asset is a public good and every bank tries to free ride from this action. See Leitner (2005) for the model in which banks could coordinate by incurring private bail-out.

depositors receive the liquidation value of the deposit from the failed bank, denoted by $\tilde{C}_{ij}$, where $\tilde{C}_{ij} \leq C_1^*$. If the liquidation value is too low, the neighbouring banks will suffer from contagious failure. We next assess the conditions under which contagion occurs by assuming the failure of one core bank and one periphery bank, respectively.

5.1.Shock on a Core Bank

Consider state S_3. Core bank in region nm (hereafter bank nm) has excess demand of εC_1^* deposit withdrawal. Condition (6) implies that it has to meet the excess demand by liquidating long asset. Bank nm can liquidate some long assets without defaulting, if it keeps no less than $[k(1 - \lambda) - \varepsilon]C_1^* /R$ units of long asset to satisfy date 2 deposit withdrawal.[16] The available long asset that can be liquidated without causing bankruptcy is the capital buffer for bank nm, which is

$$CB_{nm} = r\left[kx - \frac{[k(1 - \lambda) - \varepsilon]C_1^*}{R}\right]. \tag{7}$$

Bank nm will not fail if and only if $\varepsilon C_1^* \leq CB_{nm}$. Otherwise, it will default and all its depositors withdraw at date 1; it has to liquidate all the long assets and the liquidation value of the deposits is $\tilde{C}_{nm}$. Let $\bar{C}_{nm}$ denote the maximum liquidation value of bank nm, which is the value of the deposits given that all the bank nm's neighbouring banks can meet bank nm's deposit withdrawal at value C_1^* . In what follows, we use the maximum liquidation value to investigate the necessary conditions for financial contagion.[17] Once bank nm fails, contagion first propagates to all the periphery banks

---

[16] For $C_2 = C_1^*$, although late consumers are worse off, they are indifferent between withdrawing at date 1 and 2. For $C_1 > C_2$, the incentive constraint is violated and the late consumers are better off withdrawing at date 1.

[17] The values of deposit are interdependent in the financial network. We follow Allen and Gale (2000) and Babus (2016) in studying only the maximum liquidation values. It would be much easier and more explanatory than finding the exact value of liquidation for each bank which depends on the entire network structure. Eisenberg and Noe (2001) propose a solution concept to characterize the existence and the uniqueness of a vector of payments that clears a network. Castiglionesi and Eboli (2018), on the other

in sector n and other core banks. If other core banks' capital buffers cannot satisfy the losses due to default of bank nm, they will fail and contagion continues to propagate to the rest of the periphery banks. Proposition 2 presents the contagion thresholds for periphery banks in sector n, other core banks and the rest of the periphery banks.

Proposition 2 *Suppose bank nm fails in state S_3. The necessary condition for periphery bank $nj, \forall j \neq m$, not to default is*

$$\Delta \cdot (C_1^* - \bar{C}_{nm}) \leq CB_p; \tag{8}$$

the necessary condition for core bank $im, \forall i \neq n$, to survive from the contagion is

$$\frac{2|k + v - u|}{n} \Delta \cdot (C_1^* - \bar{C}_{nm}) \leq CB_C. \tag{9}$$

If core bank $im, \forall i \neq n$, fails, periphery banks $ij, \forall i \neq n, j \neq m$, are safe only if

$$\Delta \cdot (C_1^* - \bar{C}_m) \leq CB_P. \tag{10}$$

The proof of Proposition 2 and the derivations of $\bar{C}_{nm}$ and $\bar{C}_m$ are in Appendix A.2 (Sui et al., 2019). The intuitions for the threshold conditions are the following. Consider condition (8). Upon failure of bank nm, periphery banks in n can only retrieve $\bar{C}_{nm}$ for each unit of the interbank deposit, where $C_1^* \geq \bar{C}_{nm}$. The interbank deposit exchanges are Δ for each periphery bank and core bank. The LHS thus measures the total *Losses Due to Default* (*LDD*) of core bank nm for each periphery bank in sector n. The RHS is the periphery bank's capital buffer, where

$$CB_P = r \left[x - \frac{(1 - \lambda)C_1^*}{R} \right],$$

which is the available long asset it can liquidate without defaulting. Condition (8) shows that a periphery banks in sector n will not default only if the capital buffer is greater than or equal to the *LDD* of bank nm.

---

hand, use flow network theory to characterize the maximum flow of interbank payments and endogenize the liquidation value of the failed bank.

Condition (9) is the threshold condition for other core bank $im, \forall i \neq n$. Although periphery banks in sector n and other core banks receive the same liquidation value $\bar{C}_{nm}$, their amounts of deposits at nm are different: the amount exchanged between each pair of core banks is $2|k + v - u|\Delta/n$. The LHS of condition (9) thus measures the *LDD* for core bank $im, \forall i \neq n$ and the RHS is their capital buffer denoted by CB_C. Since core banks are k times larger than periphery banks, it is easy to show $CB_C = kCB_P$.

Once threshold condition (9) is violated, all the core banks fail. Contagion continues to propagate to the rest of the periphery banks $ij, \forall i \neq n, j \neq m$. The contagion threshold is now governed by (10), where the LHS is the *LDD* of core bank $im, \forall i \neq n$. Note that now the liquidation value of the deposit is $\bar{C}_m$, which is the maximum liquidation value of core bank $im, \forall i \neq n$. We can verify that $\bar{C}_m < \bar{C}_{nm}$. Since all the core banks are effectively bankrupt, the maximum liquidation value of bank nm, $\bar{C}_{nm}$, no longer holds and decreases to a lower value $\tilde{C}_{nm}$. The maximum liquidation value of core bank $im, \forall i \neq n$ is then conditional on a lower liquidation value of bank nm, hence $\bar{C}_m < \bar{C}_{nm}$. This also means that, conditional on the failure of all core banks, violation of threshold condition (8) ensures that condition (10) is violated; in other words, contagious failure of periphery banks in sector n implies contagious failure of all the periphery banks.

5.2. Shock on a Periphery Bank

Next consider state S_4. Periphery bank 11 is hit by an exogenous liquidity shock. Let CB_{11} denote bank 11's capital buffer. It is safe if and only if $\varepsilon C_1^* \leq CB_{11}$, where

$$CB_{11} = r\left[x - \frac{(1 - \lambda - \varepsilon)C_1^*}{R}\right].$$

Suppose bank 11 fails, then financial contagion first propagates to core bank $1m$. If bank $1m$'s capital buffer cannot meet the *LDD* of periphery bank 11, it then defaults and contagion continues to propagate to the rest of the periphery banks in sector 1 and other core banks. If other core banks suffer from contagious failure then the shock keeps on propagating to the rest of the periphery banks. Proposition 3 characterizes all the contagion thresholds due to initial failure of a periphery bank.

Proposition 3 *Suppose bank 11 fails in state S_4. Core bank $1m$ is safe if and only if*

$$\Delta \cdot \left(C_1^* - \overline{C}_{11} \right) \leq CB_C. \tag{11}$$

If core bank $1m$ fails, periphery bank $1j, \forall j \neq 1, m$, will not default only if

$$\Delta \cdot \left(C_1^* - \overline{C}_{1m} \right) \leq CB_P; \tag{12}$$

the necessary condition for core bank $im, \forall i \neq 1$, to survive from the contagion is

$$\frac{2|k + v - u|}{n} \Delta \cdot \left(C_1^* - \overline{C}_{1m} \right) \leq CB_C. \tag{13}$$

If core bank $im, \forall i \neq 1$, fails, periphery banks $ij, \forall i \neq 1, j \neq m$, are safe only if

$$\Delta \cdot \left(C_1^* - \overline{C}_{m'} \right) \leq CB_P. \tag{14}$$

See Appendix A.3 (Sui et al., 2019) for detailed proof of Proposition 3. $\overline{C}_{11}$, $\overline{C}_{1m}$ and $\overline{C}_{m'}$ represent the maximum liquidation value of periphery bank 11, core bank $1m$, and the rest of the core bank $im, \forall i \neq 1$, respectively.[18] The derivation of these results is similar to Proposition 2. Note that the liquidation value of core bank $1m$, $\overline{C}_{1m}$, is greater than $\overline{C}_{m'}$ for the same reason as we discussed above. Thus, conditional on contagious failure of core banks $im, \forall i \neq 1$, when threshold condition (12) fails to hold, it leads to default of all the periphery banks, i.e., threshold condition (14) is also violated.

5.3. Discussion

---

[18] Note that $\overline{C}_{1m} \neq \overline{C}_{nm}$ and $\overline{C}_{m'} \neq \overline{C}_m$. See Appendix A.2 and A.3 (Sui et al., 2019) for the derivations.

In the core-periphery network, the heterogeneities in the amount of interbank deposit exchanges and bank sizes imply that different banks have different contagion thresholds. Proposition 2 and 3 show that contagion threshold depends on three factors: the liquidation value of interbank deposit, the amount of interbank deposits a bank has at the defaulting bank, and banks' capital buffers. The more interbank deposits a bank has at defaulting bank the less likely the threshold condition holds; the greater the liquidation value of the defaulting bank the more likely that the capital buffer exceeds the *LDD*, and the greater the capital buffer, e.g. CB_C, the more likely it can satisfy the threshold condition.

This result is different from Allen and Gale (2000) in which the symmetric structure and identical bank size imply that the liquidation value decreases monotonically with the number of contagious failures. In other words, contagious failure of one bank leads to contagious failure of all banks in the interconnected network. By contrast, Proposition 2 and 3 show that, in the core-periphery network, contagious failure of one bank does not necessarily imply contagious failure of all banks. Formally, propositions 2 and 3 can be summed up as follows:

Corollary *For $|k + v - u| \leq nk/2$, contagious failure of more than one core bank implies the failure of all banks in the network; whereas, for $|k + v - u| > nk/2$, contagious failure of core banks does not necessarily imply the failure of all banks.*

To elaborate on the corollary, compare condition (8) with (9) in Proposition 2. Although periphery banks $nj, \forall j \neq m$ and core banks $im, \forall i \neq n$ receive the same liquidation value of interbank deposit from core bank nm, their capital buffer and amount of interbank deposit exchange with bank nm are different. Since $CB_C = kCB_P$, we can rewrite condition (9) as

$$\frac{2|k + v - u|}{nk} \Delta \cdot (C_1^* - \bar{C}_{nm}) \leq CB_p,$$

where $2|k + v - u|\Delta/nk$ can be interpreted as the amount of interbank deposit exchange between core banks per unit of capital buffer. Thus, if the per unit amount of interbank deposit exchange between core banks is less than the amount of deposit exchange between periphery bank and core bank (Δ), which effectively implies that $|k + v - u| < nk/2$, then contagious failure of other core banks $im, \forall \ i \neq n$, must imply the failure of periphery banks in sector n, but the reverse relation does not necessarily hold; that is, contagious failure of periphery banks in sector n does not necessarily imply contagious failure of other core banks. On the other hand, if $|k + v - u| > nk/2$, then contagious failure of periphery banks in sector n must imply the failure of other core banks $im, \forall \ i \neq n$, whereas contagious failure of other core bank does not necessarily imply the failure of periphery banks in sector n. If the per unit amount of interbank deposit exchange is the same between core banks, and between periphery and core bank, i.e. $|k + v - u| = nk/2$, then threshold conditions (8) and (9) are the same.

Suppose condition (9) does not hold and all the core banks fail. Since the liquidation value $\bar{C}_m$ is less than $\bar{C}_{nm}$, for $|k + v - u| \leq nk/2$ condition (10) cannot hold and the rest of the periphery banks $ij, \forall i \neq n, j \neq m$ also fail; whereas for $|k + v - u| > nk/2$, it is not necessarily the case because although periphery banks $ij, \forall i \neq n, j \neq m$, receive lower liquidation value, the amount of interbank claims it has at the core bank is less than the claims each core bank has at bank nm. However, as we have shown above, violation of condition (8) ensures the failure all the rest of the periphery banks. It is easy to show that the same argument applies to the threshold conditions (12), (13) and (14) in Proposition 3.

The corollary also stands in contrast to the studies on targeted shocks under a fat-tailed network structure, which stress the importance of core banks in the process of contagion. Albert et al. (2000) show that fat-tailed networks are vulnerable to a targeted attack on well-connected nodes. Their result is replicated by Gai et al. (2011) who explore the consequences of a targeted shock on financial contagion. They argue that, for concentrated or tiered financial networks, the failure of core banks almost surely implies the failure of periphery banks for a wide range of parameter values, because the core banks connect to a large part of the network which leads to severe contagion. We show that financial contagion is not determined only by the number of links. The contagion thresholds also depend crucially on the level of capital buffer in the core banks and the amount of interbank deposits on each link. These two factors are implicitly determined by the size of the core banks k, the number of core banks n, the number of periphery banks m, and the liquidity characteristics of periphery banks, v and u, which determines the amount of interbank deposits each core bank needs to intermediate between sectors.[19] In the following section, we study how these factors affect contagion thresholds, and hence the resilience of the network.

6. Analysis of Network Heterogeneity

We analyze the effects of network heterogeneity by varying, in turn, the size of core banks, the number of core banks, and the number of periphery banks. We study the

---

[19] To show how the liquidity characteristics of periphery banks affects contagion thresholds, consider the core-periphery network shown in Figure 1 as an example, where $m = n = 4$. Suppose $k = 2$, $v = 0$ and $u = 3$, that is, in each sector, the core bank is twice the size of three periphery banks which are all negatively correlated with the core bank. The per unit interbank deposit exchange between core banks is then $\Delta/4$ which is less than the per unit interbank deposit exchange between periphery bank and core bank (Δ). The first part of the corollary holds. Now suppose $k = 2$, $v = 3$ and $u = 0$, the three periphery banks are all positively correlated with the core bank in each sector . Then per unit interbank deposit exchange between core banks becomes 1.25Δ, which is greater than Δ. In this case, contagious failure of core banks does not necessarily imply the failure of periphery banks. Section 6.3 discusses more on the liquidity characteristics of periphery banks.

changes in these parameters on the contagion thresholds such that that the changes do not affect the achievement of efficient allocation in Section 4. That is, when we vary the size of core banks, k, we change the size of the population in all large regions. It is easy to see that Pareto optimal allocation can still be achieved in the core-periphery network. In varying the number of core banks, we assume many efficient disconnected core-periphery networks with the same liquidity characteristics of individual banks. For example, two disconnected efficient core-periphery networks, as shown in Figure 2, can merge into a larger core-periphery network - as in Figure 1 - such that it achieves efficient allocation where each core bank now connects with greater number of core banks. When considering the number of periphery banks, we change the number of small regions in all sectors so that the decentralized allocation in the core-periphery network is Pareto optimal.
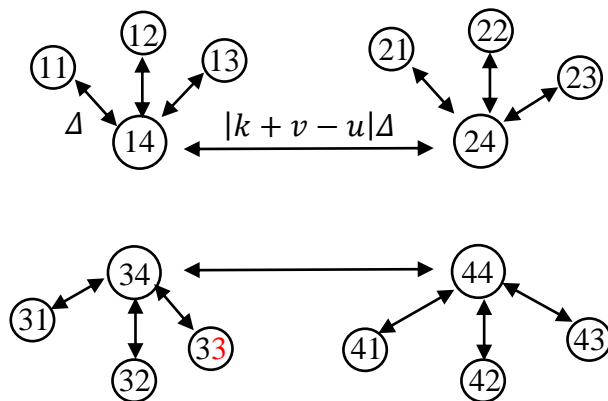

Figure 2: The figure illustrates two disconnected interbank networks, each of which achieves efficient allocation by exchanging $|k + v - u|\Delta$ interbank deposits between core banks, and Δ between each periphery bank and core bank.

6.1. Size of Core Banks

Consider how the size of core bank relative to periphery bank (k) affects the contagion thresholds in proposition 2 and 3. In state S_3, bank nm will not fail if and only if $\varepsilon C_1^* \leq CB_{nm}$. Substituting out CB_{nm} using (7) and (5) we have

$$(R - r)\varepsilon C_1^* \leq rk(1 - \lambda)(C_2^* - C_1^*), \tag{15}$$

where the RHS is positive given the first-order condition (3). Condition (15) is more likely to hold as k increases. This result is intuitive: large size of core banks means there are more late consumers sharing a given liquidity shock, hence core banks have more capital buffer to satisfy the excess liquidity demand.

Suppose bank nm fails. The contagion threshold for periphery banks in sector n is governed by condition (8). Substituting out $\bar{C}_{nm}$ in (8) using (A.1) and taking the derivative of the LHS with respect to k, we have

$$\Delta \cdot \frac{(1 - \lambda)(C_1^* - \frac{r}{R}C_2^*)\left[(m - 1) + \frac{2|v - u|}{n}(n - 1)\right]}{\left\{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta\right\}^2}. \tag{16}$$

The pecking order condition (6) implies that $C_1^* > rC_2^*/R$ in the numerator, so (16) is positive. The periphery banks in sector n are more likely to fail as the size of the core bank nm increases. In other words, the maximum liquidation value of nm, $\bar{C}_{nm}$, decreases with k.[20] The intuition is the following. The greater size of core bank means it is less likely to fail. However, once it fails, there are more late consumers withdrawing at date 1. This means that the core bank has to liquidate more long assets at low liquidation value r. The greater amount of premature liquidation therefore decreases the liquidation value of interbank deposit *per capita*.

Next consider condition (9). Although $\bar{C}_{nm}$ decreases with the size of core bank nm, the capital buffer for other core banks also increases with k. The contagion threshold depends on the amount of interbank deposits exchanged between core banks. Suppose $k + v < u$, then as k increases, liquidity risk-sharing weighs more within sectors than among sectors. It is more likely that $|k + v - u| < nk/2$, which means

---

[20] It is easy to verify that this condition holds also for other maximum liquidation values of core bank, i.e., $\bar{C}_{1m}$, $\bar{C}_m$ and $\bar{C}_{m'}$.

contagious failure of periphery banks in sector n does not necessarily imply contagious failure of other core banks. However, since condition (8) is less likely to hold for large k, the failure of periphery banks in sector n decreases the liquidation value of nm to a lower value $\tilde{C}_{nm}$. In this case, the size effect on contagion threshold (9) is ambiguous. Suppose $|k + v - u| > nk/2$ and $k + v > u$. As k increases, interbank deposit exchange increases between core banks, which implies that condition (9) is less likely to hold relative to condition (8); the failure of core bank nm implies contagious failure of all the other core banks.

As we have shown in Section 5, subject to the failure of all the core banks, violation of condition (8) means that condition (10) does not hold. Since (8) is less likely to hold when the size of the core banks is large, it then implies that, once all the core banks fail, the rest of the periphery banks must fail.

Now consider state S_4. According to proposition 3, contagion threshold for bank $1m$ is subject to condition (11). Since $CB_C = kCB_P$, we can rewrite (11) as

$$R(C_1^* - \bar{C}_{11}) \cdot \Delta \le rk(1 - \lambda)(C_2^* - C_1^*). \tag{17}$$

For large size of the core bank $1m$, condition (17) is more likely to hold for the same reason as discussed in condition (15).

Suppose bank $1m$ defaults, it must be true that condition (11) is violated and the contagion threshold for the periphery banks in sector 1 is subject to condition (12). If failure of bank $1m$ leads to contagious failure of the rest of periphery banks in sector 1, then the following condition must hold:

$$(C_1^* - \bar{C}_{11}) \le k(C_1^* - \bar{C}_{1m}), \tag{18}$$

where the LHS can be interpreted as the *LDD* per unit of interbank deposit for core bank $1m$, and the RHS, which is from condition (12), is the per unit *LDD* for each periphery bank in sector 1 while taking account of the capital buffer difference $CB_C =$

kCB_P. Since the maximum liquidation value $\bar{C}_{1m}$ decreases with the size of the core bank for the same reason above, the failure of the large core bank implies contagious failure of all periphery banks in the same sector.

Similar to the derivation of (18), if the failure of core bank $1m$ implies contagious failure of the rest of the core banks, then it must be that

$$(C_1^* - \bar{C}_{11}) \leq \frac{2|k + v - u|}{n}(C_1^* - \bar{C}_{1m}). \tag{19}$$

Since $\bar{C}_{1m}$ decreases with k, then, as k increases, condition (19) is more likely to hold for $k + v > u$. For $k + v < u$, as k increases, the amount of interbank deposit held between core banks decreases, condition (19) does not necessarily hold. However, as k increases further, interbank deposit exchange increases again between core banks, and condition (19) is likely to hold. There is a possible non-monotonic effect in the contagion process between core banks with respect to k: for $k + v < u$, increasing the size of core banks decreases the interbank deposit cross holding between core banks. This implies that contagion could be contained within sector 1 and it may lead to greater resilience for the rest of the sectors in the network. However, when the size of core banks continues to increase, liquidity risk-sharing weighs more between sectors and it makes the network less resilient to contagion. For the same reason above, contagious failure of all core banks implies violation of condition (14), hence all the rest of periphery banks fail.

The next proposition summarizes the analysis above:

Proposition 4 *Increasing the size of core banks increases their resilience to the initial liquidity shock and to the failure of periphery banks. For core banks that are large enough, the failure of a core bank leads to contagious failure of all periphery banks in the same sector; while contagious failure of one core bank leads to failure of all banks.*

The proposition implies that the size effect of the core banks exhibits a robust-yet-fragile property. That is, bigger size of the core banks makes the core-periphery network more resilient to liquidity shocks. However, once a large core bank defaults, the network is more likely to incur widespread contagion.

6.2. Number of Core Banks

We next examine how the increase in the number of core banks, n, affects the contagion thresholds. It is easy to see that increasing n increases the maximum liquidation value of core banks, $\bar{C}_{nm}$ and $\bar{C}_{1m}$ (see A.1 and A.4) as well as decreases the amount of interbank deposit exchanges between core banks, which implies that conditions (9) and (13) are more likely to hold. Therefore, financial contagion is less likely to propagate from one sector to other sectors. This is simply due to diversification effect of liquidity risk-sharing. The increase of $\bar{C}_{nm}$ and $\bar{C}_{1m}$ also has positive effect on the resilience of the periphery banks in sector *n* and 1, as conditions (8) and (12) are more likely to hold. Therefore, the large number of core banks makes the network more resilient not only to cross-sectoral contagion but also to contagion from core banks to periphery banks within each sector. It is more likely that $|k + v - u| < nk/2$ for large *n* and the first part of the corollary in section 5 holds, which again suggests the robust-yet-fragile property of the network with respect to contagious failure of core banks.

6.3. Number of Periphery Banks

Consider the effect of increasing the number of periphery banks, m. Since, as we shall see below, the effect of varying one periphery bank at a time depends on the liquidity characteristic of the bank, in order to isolate the contagion effect of m, we first vary a pair of periphery banks which are negatively correlated. Condition (8), after substituting out $\bar{C}_{nm}$ using (A.1), becomes

$$\Delta \cdot \left\{ \frac{k(1-\lambda)\left(C_1^* - \frac{r}{R}C_2^*\right)}{k + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta} \right\} \leq CB_P, \qquad (20)$$

where m appears only in the denominator of the LHS. The *LDD* for periphery bank in

sector n therefore decreases with the number of periphery banks. Since other core

banks receive the same liquidation value of core bank nm as periphery banks in

sector n, condition (9) is also more likely to hold.

We next examine whether the contagious failure of the core bank would imply the

failure of periphery banking as m increases. We have shown that the failure of core

bank $1m$ implies contagious failure of the periphery banks in sector 1 if condition (18)

holds. Substituting out $\bar{C}_{11}$ and $\bar{C}_{1m}$ from (18) using (A.3) and (A.4), we have

$$\frac{(1-\lambda)(C_1^* - \frac{r}{R}C_2^*)}{k(1+\Delta)} \leq \frac{k(1-\lambda)\left(C_1^* - \frac{r}{R}C_2^*\right) + \Delta(C_1^* - \tilde{C}_{11})}{k + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta}, \qquad (21)$$

where again m appears only in the denominator of the RHS. That is, when the number

of the pair increases, condition (21) is less likely to hold. Or, equivalently, as the pair

of periphery banks increases, the maximum liquidation value of core bank $1m$ is more

likely to be greater than the maximum liquidation value of periphery bank 11. By the

same argument, for $|k+v-u| > nk/2$, conditions (10) and (14) are also more likely

to hold. Thus periphery banks are more resilient to the failure of core banks, which

reflects the second part of the corollary. The result is intuitive: more periphery banks

imply that there are more late consumers among the periphery banks sharing the risk of

contagion from the core bank; this in turn increases the liquidation value of the core

bank.

Next, consider adding one periphery bank at a time. The effect depends on the

number of the core banks and the characteristic of the additional bank. To illustrate,

taking the derivative of (20) with respect to m, for $k + v > u$, it would appear

that $\bar{C}_{nm}$ increases with v, and the effect is opposite when increasing u. However, this is not necessarily the case. Consider two extreme cases: first, there are only two core banks $(n = 2)$; second, the number of the core banks is large enough such that other core banks will not suffer from contagious failure of one of the core banks, that is, conditions (9) and (13) hold for any liquidation value.

Suppose $k + v > u$, and $n = 2$. When increasing v, core banks have to intermediate for the "v" periphery banks by exchanging more interbank deposits across sectors. Substituting out $\bar{C}_{nm}$ from condition (9) and taking the derivative of LHS with respect to Δ, we have

$$\frac{k(1 - \lambda)\left(C_1^* - \frac{r}{R}C_2^*\right)(k + v - u)}{[k + (k + 2v)\cdot\Delta]^2}.$$

The *LDD* increases with the amount of interbank deposit exchange between core banks, and condition (9) is less likely to hold. Since the liquidation value of core bank nm is more likely to decrease to $\tilde{C}_{nm}$, condition (8) is less likely to hold. On the other hand, when increasing u, liquidity risk-sharing and intermediation are mostly done within sectors which decreases the level interbank deposit exchange among core banks. So both condition (8) and (9) are more likely to hold. However, the effect is non-monotonic when u is large enough such that $k + v < u$. The same argument applies to condition (12) and (13).

Now suppose $k + v > u$ and there are many core banks. The effect is the opposite. Since n is large, increasing the level of cross-sector intermediation, i.e. the "v" periphery banks, increases $\bar{C}_{nm}$, whereas increasing u decreases $\bar{C}_{nm}$, because liquidity risk-sharing and intermediation concentrates more within sectors rather than spreading out evenly among the sectors. As u continue to increase, conditions (8) and

(12) are then more likely to hold again. Thus, the non-monotonic effect is the opposite compared to the case when n is small.

We summarise the analysis in sections (6.2) and (6.3) in the following proposition:

Proposition 5 *Ceteris paribus, the resilience of the network increases with the number of core banks and the number of periphery banks, respectively. For small number of core banks, the network is more (less) resilient if an additional periphery bank decreases (increases) the level interbank intermediation between core banks; for number of core banks that is large enough, the opposite is true.*

This proposition, in a nutshell, highlights the impact of network heterogeneity associated with increasing the number of core banks and the number of periphery banks in a way that is comparable to the results of Acemoglu et al. (2015) and Morris (2000). They consider "δ-connected" financial network and "γ-cohesive" network, respectively, which are measures of the connectiveness among different "clusters" in the network. They show that in order to achieve resilience, the weight of links has to be unevenly distributed such that contagion is contained in one part and does not propagate to another part of the network. In our analysis, as *n* increases, the amount of interbank deposit exchange between core banks decreases, which is equivalent to having a firewall of weakly connected core banks. As a result, contagious failure does not easily propagate from one core bank to other core banks and to the periphery banks in other sectors. Similarly, when considering the number of periphery banks, if the addition of a periphery bank increases core banks' intermediation activity within the sector, it also weakens the weight of links between core banks and makes the sector more resilient to cross-sector contagion. Moreover, when considering the resilience of the network as a whole, Proposition 5 suggests that if core banks are already susceptible to contagious failure from the default of one core bank, the network is more resilient when the

32

intermediations are mostly done within sectors; whereas, if n is large enough such that diversification leads to strong resilience to cross-sector contagion, the network is more resilient when interbank intermediation is mostly among the core banks.

7. Core-Periphery versus Complete Network

This section compares the resilience of the core-periphery network with a complete network in which all large banks and small banks have *nm*-1 links, implying that they are all bilaterally linked.[21] We first show that Pareto-optimal allocation can be decentralized in the complete network. We then derive contagion thresholds by considering an initial default of one large bank and one small bank, respectively. Without loss of generality, suppose the economy is in state S_1 in which large banks hm face high proportion of early consumers ω_H. The total number of banks which face high proportion of early withdrawal is $\frac{n}{2} + \frac{n}{2}(v + u)$, where the first component of this expression $(n/2)$ is the total number of large banks experiencing a high proportion of early withdrawal; and the second component is the total number of small banks facing high proportion of early withdrawal, of which $nv/2$ is the number of small banks from the sectors in which large banks are positively correlated with bank hm, and $nu/2$ is the number of small banks from the sectors in which large banks are negatively correlated with bank hm.[22] Since $u + v = m - 1$, the total number of banks which experience high proportion of early withdrawal is $nm/2$; which is the same as the number of banks facing a low proportion of early consumers.

Let each small bank exchange $2\Delta/nm$ amount of deposits with each bank; and let each large bank exchange $2\Delta/nm$ amount of deposits with each small banks and exchange $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ with each large banks.[23] The date 0 budget constraint for each small bank is

$$x + y + \frac{2(nm-1)}{nm}\Delta = 1 + \frac{2(nm-1)}{nm}\Delta,$$

where the LHS is the total assets consisting of long asset, short asset and total claims of interbank deposits in $nm - 1$ banks, and RHS is the liabilities comprising consumers' deposit and the total interbank deposit claims from $nm - 1$ banks. The budget constraint for each large bank is

$$k(x + y) + \left[\frac{2(m-1)}{m} + \frac{2(n-1)}{n}\left(k - \frac{m-1}{m}\right)\right]\Delta = k + \left[\frac{2(m-1)}{m} + \frac{2(n-1)}{n}\left(k - \frac{m-1}{m}\right)\right]\Delta.$$

The term in the square brackets on the LHS is large banks' total interbank lending which consists of $2\Delta/nm$ to each small bank with the sum of $n(m - 1)$ small banks, and $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ to each large bank with $n - 1$ large banks in total. The term in the square brackets on the RHS is the total interbank borrowing from all the small banks and large banks. Given the cross holdings of interbank deposits at date 0, we have

Proposition 6 *Pareto-optimal allocation can be achieved in the complete network, in which each small bank exchanges $2\Delta/nm$ interbank deposits with each of the other banks, and each large bank exchanges $2\Delta/nm$ with each small bank and exchanges $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ with each of the other large banks.*

The detailed proof of Proposition 6 is in Appendix A.4 (Sui et al., 2019). The intuition is the following: All banks' budget constraints at date 0 can be reduced to date

---

[23] For each bank, the effective interbank deposit exchange is with negatively correlated banks. For each large bank the total effective interbank deposit exchange is with $(nm - n)/2$ small banks which means there are still $\left(k - \frac{m-1}{m}\right)\Delta$ liquidity shock needs to be shared with $n - 1$ large banks. Hence, each large bank exchanges $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ with each of the other large banks.

0 feasibility constraint. For any small bank that faces ω_H at date 1, it withdraws $2\Delta/nm$ from all banks that faces ω_L or ω_L^m. The total liquidity received is Δ. For any small bank that faces ω_L, it has to meet the deposit withdrawal of $2\Delta/nm$ from each bank facing ω_H, the total interbank deposit withdrawal is also Δ. Given equation (1) there is no excess liquidity demand for any small banks. For any large bank that faces ω_H^m, it withdraws $2\Delta/nm$ from each of the $(nm - n)/2$ small banks that face ω_L and withdraws $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ from each of the $n/2$ large banks that faces ω_L^m; the total liquidity received is $k\Delta$. For any large bank that experiences ω_L^m, it has to meet the deposit withdrawal of $2\Delta/nm$ from each small banks that faces ω_L and $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ from each large banks that faces ω_H^m. The total interbank deposit withdrawal is also $k\Delta$. Given (2), there is no excess liquidity demand for any large banks. The same argument holds at date 2. Each bank can thus offer (C_1^*, C_2^*) to its depositors.

Next consider states S_3 and S_4. Since banks' sizes are heterogenous, contagion thresholds for large banks and small banks are different. Let $\bar{C}_{nm}^c$ and $\bar{C}_{11}^c$ denote the maximum liquidation values of bank nm and 11 under the complete network. Proposition 7 presents the contagion thresholds in state S_3 and S_4:

Proposition 7 *Suppose large bank nm fails in state S_3. The necessary condition for small banks not to default is*

$$\frac{2}{nm}\Delta \cdot (C_1^* - \bar{C}_{nm}^c) \leq CB_P, \tag{22}$$

and the necessary condition for all the large banks $im, \forall i \neq n$, to be safe is

$$\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta \cdot (C_1^* - \bar{C}_{nm}^c) \leq CB_C. \tag{23}$$

Suppose bank 11 fails in state S_4. The necessary condition for other small banks not to default is

$$\frac{2}{nm}\Delta \cdot \left(C_1^* - \bar{C}_{11}^c\right) \le CB_P, \tag{24}$$

and the necessary condition for all the large banks to survive is

$$\frac{2}{nm}\Delta \cdot \left(C_1^* - \bar{C}_{11}^c\right) \le CB_C. \tag{25}$$

The proof of Proposition 7 is in Appendix A.5 (Sui et al., 2019). The intuition is similar to the ones in Proposition 3 and 4. $\left(C_1^* - \bar{C}_{nm}^c\right)$ and $\left(C_1^* - \bar{C}_{11}^c\right)$ measure the *LDD* per unit of interbank deposit. The LHS of the four conditions thus measure the total amount of *LDD* during the defaults of bank nm and 11. Compare threshold condition (22) with (23). Since $CB_C = kCB_P$ and $k > 1$, the *LDD* per unit of capital buffer for large banks is greater than for small banks,[24] i.e., condition (22) is easier to hold than condition (23). Thus contagious failure of all large banks does not necessarily imply contagious failure of all small banks. In state S_4 the *LDD* per unit of capital buffer is smaller for large banks than for small banks; violation of threshold condition (24) does not necessarily imply that condition (25) does not hold, whereas the opposite is true.

We now compare the resilience between the core-periphery and complete network in which bank nm fails. Taking the derivative of $\bar{C}_{nm}^c$ (A.6) with respect to k, we have

$$-\frac{\frac{2(m-1)}{nm}\left(C_1^* - \frac{r}{R}C_2^*\right)\Delta}{\left\{k + \left[\frac{2(m-1)}{nm} + \frac{2(n-1)}{n}k\right]\Delta\right\}^2}. \tag{26}$$

Since $C_1^* > rC_2^*/R$, (26) is negative. The maximum liquidation values of bank nm, $\bar{C}_{nm}^c$, decrease with k.[25] Thus, as k increases condition (22) and (23) are

---

[24] If $k = 1$, interbank deposit exchanges are the same between small banks and between large banks, i.e. $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta = \frac{2}{nm}\Delta$. For $k > 1$, we have $\left(k - \frac{m-1}{m}\right) > \frac{k}{m}$, that is deposit exchange per unit capital buffer between two large banks is greater than the deposit exchange between a large bank and a small bank.

[25] The intuition is the same as condition (16): once the large bank fails, there are more late consumers withdrawing from large bank hence more long assets being liquidated at low scrap value r, which decreases the liquidation value of interbank deposit *per capita*.

unlikely to hold. Suppose k is large enough such that threshold (22) does not hold. This in turn implies that condition (23) does not hold, so all the banks in the complete network fail.

Comparing condition (8) with (22), the interbank deposit exchange between periphery bank and core bank is $nm/2$ times larger than the deposit exchange between small bank and large bank. Also, since condition (16) shows that $\bar{C}_{nm}$ also decreases with k. Thus, as k increases, *LDD* in (8) is greater than in (22). The failure of all the banks in the complete network implies the failure of periphery banks in sector n.

Next consider contagion thresholds for other core banks. The analysis in Section 6.1 shows that condition (9) depends on the amount of interbank deposit exchange between core banks per unit of capital buffer. If $|k + v - u| > nk/2$ and $k + v > u$, then as k increases condition (9) is less likely to hold relative to condition (8). Thus, the failure of core bank nm implies contagious failure of all banks. However, if $k + v < u$, as k increases, interbank deposit exchange between core banks per unit of capital buffer deceases; liquidity risk-sharing weighs more within sectors than among sectors. We then have $|k + v - u| < nk/2$. In this case, contagious failure of periphery banks in sector n does not necessarily imply contagious failure of other core banks. Proposition 8 summarizes the analysis above

Proposition 8 *Consider state S_3 in which bank nm fails. For k that is large enough such that threshold condition (22) does not hold, the core-periphery network is more resilient than the complete network if $k + v < u$.*

Next consider the failure of bank 11 in state S_4. Under the core-periphery network, threshold condition (11) shows that *LDD* per unit capital buffer decreases with k, which means (11) is more likely to hold. This is also true for the threshold condition (25) for large banks in the complete network. The threshold condition for

small bank (24), however, is independent of the size of large banks. This implies that, for k that is large enough such that contagion thresholds for large banks in both the core-periphery bank and the complete network are satisfied, it is still possible that all small banks in the complete network suffer from contagious failure. We then have

Proposition 9 *Consider state S_4 in which bank 11 fails. For k that is large enough such that threshold condition (11) holds, the core-periphery network is more resilient than the complete network.*

Propositions (8) and (9) accord with the findings of Castiglionesi and Eboli (2018). They show that contagion thresholds in the star network are strictly greater than the thresholds in the complete network, in all cases. That is because the ratio of consumer deposits to interbank deposits in star is less than the ratio of the banks in the complete network; intuitively, the larger the ratio the smaller the flow of losses that propagates within the network. Their argument also applies in the core-periphery network which can be seen as a hybrid structure with both star and complete structures. Large banks form a complete network between themselves, and each of them forms a star network with small banks in their sector. If $k + v < u$, there are more small banks negatively correlated with the large bank and other small banks in each sector. As k increases, liquidity risk-sharing can be done mostly within sectors; interbank deposit exchange between core banks hence decreases, which implies that the ratio of consumer deposits to interbank deposits of all the core banks decreases. In this case the core-periphery network is more resilient than the complete network. If $k + v > u$, this ratio for core banks becomes greater in order to achieve efficient allocation; the core-periphery network then becomes relatively less resilient.

8. Conclusion

This paper has examined financial contagion in a stylized core-periphery interbank network where large core banks intermediate liquidity between themselves and for their directly connected smaller periphery banks. We contribute to the theoretical literature on interbank networks by examining the effects of heterogeneity associated with the number of banks, the size of core banks, and the weight of interbank linkages within a single core-periphery structure. Our analysis involves deriving conditions under which direct linkages in the core-periphery network exhibits financial contagion from liquidity shocks to the core and periphery banks in turn. Furthermore, we study the contagion effects of liquidity intermediation both across sectors where core banks operate, and within sectors where core banks intermediate liquidity for the periphery banks. Within this network structure, we scrutinize the conditions under which heterogeneity associated with size and number of banks dampens or fuels contagion. Finally, we compare the resilience of the core-periphery network and the complete network to contagion.

Our overall findings are that the network resilience depends on the size of core banks, the number of core banks and the number of periphery banks. We first show the failure of core banks does not necessarily imply contagious failure of periphery banks. We then show that the resilience of the network increases with the number of core banks, as well as with the number of periphery banks while holding the amount of cross-sector liquidity risk-sharing constant. If an additional periphery bank increases the core banks' intermediation activity within the sector, it makes the network more resilient to financial contagion; whereas if the additional periphery bank increases intermediation among sectors then the financial contagion is more likely to propagate to the rest of the network. The effect is the opposite when there are more core banks in the network.

39

Finally, we show that, under certain conditions, the core-periphery network is more resilient than the complete network with increased size of core banks. However, the relative resiliency of the two networks depends on the amount of interbank deposits held by the core banks.

Our analysis has policy implications for the propagation of shocks among banks that are deemed to be "too big to fail" and "too interconnected to fail". Following the global financial crisis, these banks are perceived to pose substantial risk and there have been concerns about reducing their size and connectivity to ensure stability and soundness of the financial system (Volcker, 2012). This paper shows that large size of the core banks and the dense interconnectedness among them could in fact act as a buffer to prevent contagion not only within but across sectors, thereby increasing the resilience of the whole financial network. The stability of the network depends not only on the number of interbank connections but also on the weight of the interbank linkages. We show that "too interconnected to fail" is a concern only in the case where small number of core banks intermediate large amounts of interbank deposits among them.

The present paper has studied financial contagion in a given core-periphery network. We believe that an analytic study of the trade-off between the risk of contagion and the benefit of liquidity risk-sharing within a core-periphery network is an important area for further research, along with a detailed analysis of the resilience to contagion of the core-periphery network vis-à-vis the complete network based on considerations other than the increased size of core banks that we conducted in this study.

References

Acemoglu, D., Ozdaglar, A. and Tahbaz-Salehi, A. (2015). Systemic risk and stability in financial networks. *The American Economic Review*, 105(2), pp.564–608.

Albert, R., Jeong, H. and Barabási, A.-L. (2000). Error and attack tolerance of complex networks. *Nature*, 406(6794), pp.378–382.

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108(1), pp.1-33.

Babus, A. (2016). The formation of financial networks. *The RAND Journal of Economics*, 47(2), pp.239–272.

Babus, A., and Hu, T. W. (2017). Endogenous intermediation in over-the-counter markets. *Journal of Financial Economics*, 125(1), pp.200-215.

Bech, M.L. and Atalay, E. (2010). The topology of the federal funds market. *Physica A: Statistical Mechanics and its Applications*, 389(22), pp.5223–5246.

Boss, M., Elsinger, H., Summer, M., and Thurner, S. (2004). Network topology of the interbank market. *Quantitative Finance*, 4(6), pp.677-684.

Brusco, S. and Castiglionesi, F. (2007). Liquidity coinsurance, moral hazard, and financial contagion. *The Journal of Finance*, 62(5), pp.2275-2302.

Castiglionesi, F., and Eboli, M. (2018). Liquidity flows in interbank networks. *Review of Finance*, *22*(4), 1291-1334.

Castiglionesi, F. and N. Navarro (2016). (In)Efficient interbank networks, Manuscript, Tilburg University.

Craig, B. and von Peter, G. (2014). Interbank tiering and money center banks. *Journal of Financial Intermediation*, 23(3), pp.322–347.

De Masi, G., Iori, G. and Caldarelli, G. (2006). Fitness model for the Italian interbank money market. *Physical Review E*, 74(6), pp.66112.

Eboli, M. (2013). A Flow Network Analysis of Direct Balance-sheet Contagion in Financial Networks., Kiel Institute for the World Economy, *Working Paper* No. 1862.

Eisenberg, L. and Noe, T.H. (2001). Systemic risk in financial systems. *Management Science*, 47(2), pp.236–249.

Elliott, M., Golub, B. and Jackson, M.O. (2012). Financial networks and contagion. *American Economic Review*, 104 (10), pp.3115-53.

Farboodi, M (2014). Intermediation and voluntary exposure to counter-party risk, *Working Paper*, University of Chicago.

Freixas, X., Parigi, B.M. and Rochet, J.-C. (2000). Systemic risk, interbank relations, and liquidity provision by the central bank. *Journal of Money, Credit and Banking*, pp.611–638.

Fricke, D. and Lux, T. (2015). Core–periphery structure in the overnight money market: evidence from the e-mid trading platform. *Computational Economics*, 45(3), pp.359–395.

Gabrieli, S. and Georg, C. P. (2014). A network view on interbank market freezes. *Bundesbank Discussion Paper*, No. 44.

Gai, P., Haldane, A., and Kapadia, S. (2011). Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5), pp.453-470.

Gai, P. and Kapadia, S. (2010). Contagion in financial networks. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Science,* 466(2120), pp.2401-2423.

Glasserman, P. and Young, P. (2016). Contagion in financial networks", *Journal of Economic Literature*, 54(3), pp.779-831.

Goyal, S. and Vega-Redondo, F. (2007). Structural holes in social networks, *Journal of Economic Theory*, 137(1), pp. 460-492.

Hüser, A. C. (2015). Too interconnected to fail: A survey of the interbank networks literature, *Journal of Network Theory in Finance*, 1(3), pp. 1-50.

in't Veld, D., and van Lelyveld, I. (2014). Finding the core: Network structure in interbank markets. *Journal of Banking and Finance*, 49, pp.27–40.

Iori, G., Masi, G. D., Precup, O. V., Gabbi, G., and Caldarelli, G. (2008). A network analysis of the italian overnight money market. *Journal of Economic Dynamics and Control*, 32(1), pp.259 -278.

Jackson, M.O. (2008). Social and economic networks. *Princeton University Press*.

Langfield, S., Liu, Z., and Ota, T. (2014). Mapping the UK interbank system. *Journal of Banking and Finance*, 45, pp.288-303.

Krause, A., and Giansante, S. (2012). Interbank lending and the spread of bank failures: A network model of systemic risk. *Journal of Economic Behavior and Organization*, 83(3), pp.583-608.

Leitner, Y. (2005). Financial networks: Contagion, commitment, and private sector bailouts. *The Journal of Finance*, 60, pp.2925–2953.

Morris, S. (2000). Contagion. *The Review of Economic Studies*, 67(1), pp.57–78.

Nier, E., Yang, J., Yorulmazer, T., and Alentorn, A. (2007). Network models and financial stability. *Journal of Economic Dynamics and Control*, 31(6), pp.2033–2060.

Soramäki, K., Bech, M. L., Arnold, J., Glass, R. J., and Beyeler, W. E. (2007). The topology of interbank payment flows. *Physica A: Statistical Mechanics and its Applications*, 379(1):317-333.

Sui, P., Tanna, S., and Zhou, D. (2019). Financial Contagion in a Core-Periphery Interbank Network, *Working Paper*, Coventry University. Available from: https://pureportal.coventry.ac.uk/en/persons/sailesh-tanna/publications/

Volcker, P., 2012. Unfinished business in financial reform. *International Finance* 15, pp.125–135.

Yellen, J.L. (2013). Challenges confronting monetary policy. In *Speech at the 2013 National Association for Business Economics Policy Conference, Washington, DC, March*.

Appendix

A.1. Proof of Proposition 1

Without loss of generality, suppose the economy is in state S_1. In each sector h, there are v periphery banks each having ω_H amount of deposit withdrawal and u periphery banks each facing ω_L amount of early consumer. Each core bank hm has ω_H^m early consumers. In each sector l, there are v number of periphery banks facing ω_L and u number of periphery banks facing ω_H amount of early consumer. Each core bank lm has ω_L^m early consumers. All deposits are valued at C_1.

First consider periphery banks that experience ω_H. Let each of them withdraw Δ deposits from the core bank. The budget constraint is then

$$\omega_H C_1 = y + \Delta C_1.$$

Each periphery bank has y units of return from short asset. Since $\omega_H = \lambda + \Delta$, the budget constraint can be reduced to the planner's feasibility condition at date 1. Next consider periphery banks facing ω_L amount of early consumer, let core bank liquidate Δ interbank deposits from each of them. The total liquidity demand is then $(\omega_L + \Delta)$. Its budget constraint is then

$$(\omega_L + \Delta)C_1 = y.$$

Since $\omega_L = \lambda - \Delta$, the constraint is again the same as the planner's feasibility condition.

Now consider core banks in the case where $k + v > u$. For bank hm, the liquidity demand is ω_H^m from the early consumers, Δ from each periphery bank having ω_H, and $2(k + v - u)\Delta/n$ from other core bank in $hm, \forall h \neq h$. Each bank hm has ky units of return from short asset, liquidates Δ interbank deposits from each periphery bank experiencing ω_L, and withdraws $2(k + v - u)\Delta/n$ deposits from each core bank $im, \forall i \neq i$. The budget constraint for core bank hm is

$$\left\{\omega_H^m + \left[v + \frac{2(k+v-u)}{n}\left(\frac{n}{2}-1\right)\right]\Delta\right\} \cdot C_1 = ky + \left\{u + \frac{2(k+v-u)}{n}(n-1)\right\}\Delta \cdot C_1.$$

Since $\omega_H^m = k(\Delta + \lambda)$, the equation also can be simplified to the central planner's feasibility condition at date 1. Bank lm each faces ω_L^m early consumers, Δ amount of deposit withdrawal from periphery bank having ω_H, and $2(k+v-u)\Delta/n$ deposit demand from each bank hm. The supply of the liquidity is the return from short asset, ky, and the claims of Δ deposits from each periphery bank having ω_L early consumers. The budget constraint is

$$\left\{\omega_L^m + \left[u + \frac{2(k+v-u)}{n}\cdot\frac{n}{2}\right]\Delta\right\} \cdot C_1 = ky + v\Delta \cdot C_1,$$

Since $\omega_L^m = k(\lambda - \Delta)$, the budget constraint is the same as the planner's feasibility condition at date 1.

We next consider the case where $k+v<u$. The budget constraint for core bank lm becomes

$$\left\{\omega_L^m + \left[u + \frac{2(u-k-v)}{n}\cdot\left(\frac{n}{2}-1\right)\right]\Delta\right\} \cdot C_1 = ky + \left\{v + \frac{2(u-k-v)}{n}\cdot(n-1)\right\}\Delta \cdot C_1.$$

Since there are more periphery banks facing ω_H early consumers, bank lm liquidates all the interbank deposits from other core bank $im, \forall i \neq i$. Other bank $lm, \forall l \neq l$ also liquidates its deposit at lm. Core bank hm faces $2(u-k-v)\Delta/n$ deposit withdrawal from each bank lm. The budget constraint for bank hm is

$$\left\{\omega_H^m + \left[v + \frac{2(u-k-v)}{n}\cdot\frac{n}{2}\right]\Delta\right\} \cdot C_1 = ky + u\Delta \cdot C_1,$$

The constraints for lm and hm can be reduced to the planner's feasibility condition. All banks' budget constraints at date 1 are thus the same as the planner's feasibility constraint.

At date 2, there are v periphery banks facing $(1-\omega_H)$ amount of deposit withdrawal from late consumers and u periphery banks facing $(1-\omega_L)$ amount of late consumer, in each sector h. Each core bank hm has $(1-\omega_H^m)$ early consumers.

In each sector l, there are v number of periphery banks having liquidity demand $(1 - \omega_L)$ and u number of periphery banks facing $(1 - \omega_H)$. Each core bank lm has $(1 - \omega_L^m)$ late consumers. All date 2 deposits are valued at C_2.

Each periphery bank facing $(1 - \omega_L)$ late consumers liquidates Δ interbank deposits from the core bank, and each periphery bank facing $(1 - \omega_H)$ late consumers meets the deposit withdrawal of Δ by the core bank. The budget constraints for the two types of periphery bank are

$$(1 - \omega_L)C_2 = Rx + \Delta C_2$$

and

$$[(1 - \omega_H) + \Delta]C_2 = Rx,$$

respectively. Each periphery bank has Rx units of consumption goods from the return of the long asset. Since $\omega_L = \lambda - \Delta$ and $\omega_H = \lambda + \Delta$, the budget constraints for periphery banks are hence the same as the planner's feasibility condition at date 2.

Consider the case where $k + v > u$. For each core bank hm, the total liquidity demand consists of $(k - \omega_H^m)$ from the late consumers deposit withdrawal, Δ deposit withdrawal from each periphery bank having $(1 - \omega_L)$, and $2(k + v - u)\Delta/n$ deposit withdrawal from each core bank in lm. Each bank hm has kRx return of the long asset, and liquidates Δ interbank deposits from each periphery bank experiencing $(1 - \omega_H)$. The budget constraint for bank hm is

$$\left\{(k - \omega_H^m) + \left[u + \frac{2(k + v - u)}{n} \cdot \frac{n}{2}\right]\Delta\right\} \cdot C_2 = kRx + v\Delta \cdot C_2$$

Core bank lm each faces $(1 - \omega_L^m)$ late consumers and Δ amount of deposit withdrawal from periphery bank having $(1 - \omega_L)$. The total supply of the liquidity consists of the return from the long asset kRx, claims of Δ deposits from each periphery bank having $(1 - \omega_H)$ late consumers, and $2(k + v - u)\Delta/n$ interbank deposit withdrawal from each bank hm. The budget constraint is

$$[(k - \omega_L^m) + v\Delta] \cdot C_2 = kRx + \left[u + \frac{2(k + v - u)}{n} \cdot \frac{n}{2}\right]\Delta \cdot C_2,$$

Since $\omega_H^m = k(\Delta + \lambda)$ and $\omega_L^m = k(\lambda - \Delta)$, the budget constraints for core bank hm and lm are both reduced to planner's feasibility condition at date 2.

Finally consider the case where $k + v < u$. There are more late consumers in sector h than sector l. Core bank hm liquidates all the interbank deposits from core bank lm. The budget constraint for bank hm becomes

$$[(1 - \omega_H^m) + u\Delta] \cdot C_2 = kRx + \left\{v + \frac{2(u - k - v)}{n} \cdot \frac{n}{2}\right\}\Delta \cdot C_2.$$

Core bank lm faces $2(u - k - v)\Delta/n$ deposit withdrawal from each core bank hm. The budget constraint for core bank hm is

$$\left\{(1 - \omega_L^m) + \left[v + \frac{2(u - k - v)}{n} \cdot \frac{n}{2}\right]\Delta\right\} \cdot C_2 = ky + u\Delta \cdot C_2.$$

The simplification of the above two budget constraints is again the planner's feasibility condition at date 2. The same argument applies to state S_2. The deposit contract in competitive banking system with core-periphery financial network hence replicates the central planner's optimal allocation.

A.2. Proof of Proposition 2

Suppose bank nm fails in state S_3, then all the depositors in bank nm withdraw at date 1. The demand for deposits consists of k from its regional consumers, Δ from each periphery bank in sector n, and $2|k + v - u|\Delta/n$ from each core bank $im, \forall i \neq n$. The total value of deposits is

$$\left\{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta\right\}\tilde{C}_{nm}.$$

The liquidity supply of bank nm consists of ky units of short asset, kx units of long asset valued at r, Δ interbank deposits from periphery bank $nj, \forall j \neq m$, valued at $\tilde{C}_{nj}$, and $2|k + v - u|\Delta/n$ from core bank $im, \forall\ i \neq n$, valued at $\tilde{C}_{im}$. The sum is

$$k(y + rx) + \Delta \cdot \sum_{nj,j \neq m} \tilde{C}_{nj} + \frac{2|k + v - u|}{n} \Delta \cdot \sum_{im,i \neq n} \tilde{C}_{im}.$$

The liquidation value of bank nm's deposits is determined by the clearing condition

$$\tilde{C}_{nm} = \frac{k(y + rx) + \Delta \cdot \sum_{nj,j \neq m} \tilde{C}_{nj} + \frac{2|k + v - u|}{n} \Delta \cdot \sum_{im,i \neq n} \tilde{C}_{im}}{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta}.$$

If all the other banks are safe, $\tilde{C}_{nm}$ becomes

$$\bar{C}_{nm} = \frac{k(y + rx) + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta C_1^*}{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta}, \tag{A.1}$$

where $\bar{C}_{nm}$ is the maximum liquidation value of bank nm.

The necessary condition for periphery bank $nj, \forall j \neq m$, not to default is

$$(\lambda + \Delta)C_1^* \leq y + \Delta \bar{C}_{nm} + CB_P$$

where the LHS is the deposit withdrawal from the early consumers and bank nm, and the RHS is the total liquidity available without causing default which comprises short asset y, claims of deposit Δ from bank nm valued at $\bar{C}_{nm}$, and the capital buffer CB_P. Using (4), the inequality can be simplified as

$$\Delta \cdot (C_1^* - \bar{C}_{nm}) \leq CB_P.$$

Next consider the contagion threshold for core bank $im, \forall i \neq n$. The necessary condition for core bank $im, \forall i \neq n$, to survive from the contagion is

$$\left(k\lambda + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right]\Delta\right) \cdot C_1^*$$

$$\leq ky + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 2)\right]\Delta \cdot C_1^* + \frac{2|k + v - u|}{n}\Delta \cdot \bar{C}_{nm}$$

$$+ CB_C,$$

where the LHS is the deposit withdrawal from the early consumers $k\lambda$, Δ is interbank deposit withdrawal from each periphery bank in the same sector and $2|k + v - u|\Delta/n$ withdrawal from each core bank. The RHS is the total liquidity available without

causing bankruptcy which comprises short asset ky, claims of deposit Δ from each periphery bank valued at C_1^*, $2|k + v - u|\Delta/n$ claims each from $(n - 2)$ core banks, also valued at C_1^*, $2|k + v - u|\Delta/n$ from core bank nm which is valued at $\bar{C}_{nm}$, and the capital buffer denoted by CB_C. Using (5), the above inequality is reduced to

$$\frac{2|k + v - u|}{n}\Delta \cdot (C_1^* - \bar{C}_{nm}) \leq CB_C.$$

Suppose the above condition does not hold, then all the core banks fail and contagion continues to propagate to the rest of the periphery banks in the network. Consider the maximum liquidation value for the core bank $im, \forall i \neq n$, denoted by $\bar{C}_m$. The total value of deposits is

$$\left\{ k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1) \right] \cdot \Delta \right\} \bar{C}_m.$$

The liquidity supply of bank nm consists of ky units of short asset, kx units of long asset valued at r, Δ interbank deposits from periphery bank $nj, \forall j \neq m$, valued at C_1^*, and $2|k + v - u|(n - 2)\Delta/n$ from core bank $im, \forall \ i \neq i, n$, valued also at $\bar{C}_m$. Since all the core banks are effectively bankrupt, the maximum liquidation value of bank nm no longer holds and the liquidation value decreases to a lower value $\tilde{C}_{nm}$. The sum is

$$k(y + rx) + (m - 1)\Delta C_1^* + \frac{2|k + v - u|}{n}(n - 2) \cdot \Delta \bar{C}_m + \frac{2|k + v - u|}{n}\Delta \ \tilde{C}_{nm}.$$

We then have

$$\bar{C}_m = \frac{k(y + rx) + (m - 1)\Delta C_1^* + \frac{2|k + v - u|}{n}(n - 2) \cdot \Delta \bar{C}_m + \frac{2|k + v - u|}{n}\Delta \ \tilde{C}_{nm}}{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1) \right] \cdot \Delta}. \quad \text{(A.2)}$$

Periphery banks $ij, \forall i \neq n, j \neq m$, are safe only if

$$(\lambda + \Delta)C_1^* \leq y + \Delta\bar{C}_m + CB_P$$

which can be reduced to

$$\Delta \cdot (C_1^* - \bar{C}_m) \le CB_P$$

Comparing (A.2) with (A.1) it is easy to see that $\bar{C}_m < \bar{C}_{nm}$.

A.3. Proof of Proposition 3

Suppose bank 11 defaults in state S_4, The demand for deposits consists of 1 from its regional consumers, Δ from core bank $1m$. The total value of deposits is

$$(1 + \Delta)\tilde{C}_{11},$$

The liquidity supply of bank 11 consists of y units of short asset, x units of long asset valued at r, and Δ interbank deposits from core bank $1m$, valued at $\tilde{C}_{1m}$. The sum is

$$y + rx + \Delta\tilde{C}_{1m}.$$

The maximum liquidation value of bank 11 is then determined by

$$\bar{C}_{11} = \frac{y + rx + \Delta C_1^*}{1 + \Delta}, \tag{A.3}$$

Core bank $1m$ is safe if and only if

$$\left(k\lambda + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right]\Delta\right) \cdot C_1^*$$

$$\le ky + \left[(m-2) + \frac{2|k+v-u|}{n}(n-1)\right]\Delta \cdot C_1^* + \Delta\bar{C}_{11} + CB_C.$$

Rearranging the inequality and applying (5), the inequality becomes

$$\Delta \cdot (C_1^* - \bar{C}_{11}) \le CB_C.$$

Suppose this condition fails to hold, then contagion will propagate to the rest of periphery banks in sector 1 and other core banks. Consider the maximum liquidation value of bank $1m$, denoted by $\bar{C}_{1m}$. Bank $1m$'s total liquidity demand is

$$\left\{k + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta\right\} \cdot \bar{C}_{1m}.$$

The liquidity supply of bank nm consists of ky units of short asset, kx units of long asset valued at r, Δ interbank deposits from each periphery bank $nj, \forall j \ne 1, m$, valued at C_1^*, and $2|k+v-u|\Delta/n$ from each core bank $im, \forall\ i \ne i, n$, valued also

at C_1^* . Since core bank $1m$ defaults, the maximum liquidation value of bank 11 decreases to a lower value $\tilde{C}_{11}$. The total claims from bank 11 is then $\Delta \tilde{C}_{11}$. The sum of liquidity supply is then

$$k(y + rx) + \left[(m-2) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta C_1^* + \Delta \tilde{C}_{11}$$

We then have

$$\bar{C}_{1m} = \frac{k(y+rx) + \left[(m-2) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta C_1^* + \Delta \tilde{C}_{11}}{k + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta}. \qquad \text{(A.4)}$$

First consider periphery bank $1j, \forall j \neq 1, m$. It is safe only if

$$(\lambda + \Delta)C_1^* \leq y + \Delta\bar{C}_{1m} + CB_P$$

which can be simplified as

$$\Delta \cdot \left(C_1^* - \bar{C}_{1m}\right) \leq CB_P$$

Other core banks $im, \forall i \neq 1$, will not fail only if

$$\left(k\lambda + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right]\Delta\right) \cdot C_1^*$$

$$\leq ky + \left[(m-1) + \frac{2|k+v-u|}{n}(n-2)\right]\Delta \cdot C_1^* + \frac{2|k+v-u|}{n}\Delta \cdot \bar{C}_{1m} + CB_C$$

which can be reduced to

$$\frac{2|k+v-u|}{n}\Delta \cdot (C_1^* - \bar{C}_{1m}) \leq CB_C.$$

Suppose this condition does not hold, then the maximum liquidation value for the core bank $im, \forall i \neq 1$, is denoted by $\bar{C}_{m'}$. The total liquidity demand is

$$\left\{k + \left[(m-1) + \frac{2|k+v-u|}{n}(n-1)\right] \cdot \Delta\right\} \bar{C}_{m'}.$$

The liquidity supply of bank nm consists of ky units of short asset, kx units of long asset valued at r, Δ interbank deposits from periphery bank $nj, \forall j \neq m$, valued at C_1^*, and $2|k+v-u|\Delta/n$ from core bank $im, \forall\ i \neq i, n$, valued also at $\bar{C}_{m'}$. Since all the core banks are effectively bankrupt, the maximum liquidation value of bank $1m$ no longer holds and the liquidation value decreases to a lower value $\tilde{C}_{1m}$. The sum is

$$k(y + rx) + (m - 1)\Delta C_1^* + \frac{2|k + v - u|}{n}(n - 2) \cdot \Delta \bar{C}_{m'} + \frac{2|k + v - u|}{n}\Delta \ \tilde{C}_{1m}.$$

The maximum liquidation value of the core bank $im, i \neq 1$, is determined by

$$\bar{C}_{m'}$$

$$= \frac{k(y + rx) + (m - 1)\Delta C_1 + \frac{2|k + v - u|}{n}(n - 2) \cdot \Delta \bar{C}_{m'} + \frac{2|k + v - u|}{n}\Delta \ \tilde{C}_{1m}}{k + \left[(m - 1) + \frac{2|k + v - u|}{n}(n - 1)\right] \cdot \Delta} \qquad (A.5)$$

Periphery bank $ij, \forall i \neq 1, j \neq m$, is safe only if

$$(\lambda + \Delta)C_1^* \leq y + \Delta \bar{C}_{m'} + CB_P,$$

which can be reduce to

$$\Delta \cdot (C_1^* - \bar{C}_{m'}) \leq CB_P.$$

Comparing (A.4) with (A.5), we have $\bar{C}_{m'} < \bar{C}_{1m}$.

A.4. Proof of Proposition 6: Decentralized Allocation in Complete Network

First consider small banks that experience ω_H. Let each of them withdraw all the interbank deposits from the other banks. The date 1 budget constraint is then

$$\left[\omega_H + \frac{nm - 2}{nm}\Delta\right] \cdot C_1 = 1 + \frac{2(nm - 1)}{nm}\Delta \cdot C_1,$$

where $(nm - 2)\Delta/nm$ is the total interbank deposit demand from other $(nm - 2)/nm$ banks that also faces ω_H. Each small bank has y units of return from short asset. Since $\omega_H = \lambda + \Delta$, the budget constraint can be reduced to the planner's feasibility condition at date 1. Next consider small banks facing ω_L. Let each of them withdraw the interbank deposits from other banks which also face ω_L and keep the interbank claims from banks facing ω_H. Its budget constraint is then

$$\left[\omega_L + \frac{2(nm - 1)}{nm}\Delta\right] \cdot C_1 = 1 + \frac{nm - 2}{nm}\Delta \cdot C_1.$$

Since $\omega_L = \lambda - \Delta$, the constraint is again the same as the planner's feasibility condition.

Now consider large banks that experience ω_H. Let each of them withdraw all the interbank deposits from the other banks. The budget constraint is then

$$\left\{\omega_H^m + \left[\frac{m-1}{m} + \frac{n-2}{n}(k - \frac{m-1}{m})\right]\Delta\right\} \cdot C_1 = ky + \left[\frac{2(m-1)}{m} + \frac{2(n-1)}{n}(k - \frac{m-1}{m})\right]\Delta \cdot C_1.$$

Now consider large banks that experience ω_L. Let each of them withdraw the interbank deposits from other banks which also face ω_L and keep the interbank claims from banks facing ω_H. Its budget constraint is then

$$\left\{\omega_L^m + \left[\frac{2(m-1)}{m} + \frac{2(n-1)}{n}(k - \frac{m-1}{m})\right]\Delta\right\} \cdot C_1 = ky + \left[\frac{m-1}{m} + \frac{n-2}{n}(k - \frac{m-1}{m})\right]\Delta \cdot C_1.$$

At date 2, small banks facing $(1 - \omega_L)$ late consumers liquidate $2\Delta/nm$ interbank deposits from each of the $nm/2$ banks facing $(1 - \omega_H)$ late consumers. The budget constraint is

$$(1 - \omega_L)C_2 = Rx + \Delta C_2$$

And the budget constraint for each small facing $(1 - \omega_H)$ is

$$[(1 - \omega_H) + \Delta]C_2 = Rx,$$

Since $\omega_L = \lambda - \Delta$ and $\omega_H = \lambda + \Delta$, the budget constraints for periphery banks are hence the same as the planner's feasibility condition at date 2. Each large bank facing $(1 - \omega_L)$ withdraw $2\Delta/nm$ from each small bank and $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ from each large bank who experience low proportion of late consumers. The budget constraint is

$$(k - \omega_L^m)C_2 = kRx + \left[\frac{m-1}{m} + (k - \frac{m-1}{m})\right]\Delta C_2$$

And the budget constraint for each large bank facing $(1 - \omega_H)$ is

$$\left\{(k - \omega_H^m) + \left[\frac{m-1}{m} + (k - \frac{m-1}{m})\right]\Delta\right\} \cdot C_2 = kRx$$

The simplification of the above two budget constraints is again the planner's feasibility condition at date 2. The deposit contract in competitive banking system with complete network replicates the central planner's optimal allocation.

A.5. Proof of Proposition 7

Suppose bank nm fails in state S_3, then all the depositors in bank nm withdraw at date 1. The demand for deposits consists of k from its regional consumers, $2\Delta/nm$ from each small bank, and $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ from each large bank. The total value of deposits is

$$\left\{k + \left[(nm - n)\frac{2}{nm} + (n-1)\frac{2}{n}\left(k - \frac{m-1}{m}\right)\right] \cdot \Delta\right\} \tilde{C}_{nm}^c$$

which can be simplified as

$$\left\{k + \left[\frac{2(m-1)}{nm} + \frac{2(n-1)}{n}k\right] \cdot \Delta\right\} \tilde{C}_{nm}^c$$

The liquidity supply of bank nm consists of ky units of short asset, kx units of long asset valued at r, $2\Delta/nm$ interbank deposits from each small bank $ij, \forall j \neq m$, valued at $\tilde{C}_{ij}^c$, and $\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta$ from each large bank $im, \forall\ i \neq n$, valued at $\tilde{C}_{im}^c$. The sum is

$$k(y + rx) + \frac{2}{nm}\Delta \cdot \sum_{ij, j \neq m} \tilde{C}_{ij}^c + \frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta\ \cdot \sum_{im, i \neq n} \tilde{C}_{im}^c.$$

If all the other banks are safe, $\tilde{C}_{nm}$ becomes

$$\bar{C}_{nm}^c = \frac{k(y + rx) + \left[\frac{2(m-1)}{nm} + \frac{2(n-1)}{n}k\right]\Delta \cdot C_1^*}{k + \left[\frac{2(m-1)}{nm} + \frac{2(n-1)}{n}k\right]\Delta}, \tag{A.6}$$

where $\bar{C}_{nm}^c$ is the maximum liquidation value of bank nm.

The necessary condition for all the small banks not to default is

$$\left(\lambda + \frac{2}{nm}\Delta\right) \cdot C_1^* \leq y + \frac{2}{nm}\Delta \cdot \bar{C}_{nm}^c + CB_P$$

Using (4), the inequality can be simplified as

$$\frac{2}{nm}\Delta \cdot (C_1^* - \bar{C}_{nm}^c) \leq CB_P.$$

Next consider the contagion threshold for all the large banks. The necessary condition for core bank $im, \forall i \neq n$, to survive from the contagion is

$$\left[k\lambda + \frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta\right] \cdot C_1^* \leq ky + \frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta \cdot \bar{C}_{nm}^c + CB_C$$

Using (5), the above inequality is reduced to

$$\frac{2}{n}\left(k - \frac{m-1}{m}\right)\Delta \cdot (C_1^* - \bar{C}_{nm}^c) \leq CB_C.$$

Suppose bank 11 defaults in state S_4, The demand for deposits consists of 1 from its consumer depositors and $2\Delta/nm$ from each of the $(nm - 1)$ banks. The total value of deposits is

$$\left(1 + \frac{2(nm-1)}{nm}\Delta\right) \cdot \tilde{C}_{11}^c,$$

The liquidity supply consists of y units of short asset, x units of long asset valued at r, and $2\Delta/nm$ interbank deposits from each bank. The maximum liquidation value of bank 11 is then determined by

$$\bar{C}_{11}^c = \frac{y + rx + \frac{2(nm-1)}{nm}\Delta \cdot C_1^*}{1 + \frac{2(nm-1)}{nm}\Delta},$$

The necessary condition for the other small banks not to default is

$$\left(\lambda + \frac{2}{nm}\Delta\right) \cdot C_1^* \leq y + \frac{2}{nm}\Delta \cdot \bar{C}_{11}^c + CB_P$$

Using (4), the inequality can be simplified as

$$\frac{2}{nm}\Delta \cdot \left(C_1^* - \bar{C}_{11}^c\right) \leq CB_P.$$

Next consider the contagion threshold for all the large banks. The necessary condition for large banks to survive from the contagion is

$$(k\lambda + \frac{2}{nm}\Delta) \cdot C_1^* \leq ky + \frac{2}{nm}\Delta \cdot \bar{C}_{11}^c + CB_C$$

Using (5), the above inequality is reduced to

$$\frac{2}{nm}\Delta \cdot \left(C_1^* - \bar{C}_{11}^c\right) \leq CB_C.$$