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Interface crack between dissimilar one-dimensional hexagonal quasicrystals with piezoelectric effect

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Abstract

In this paper, an interface crack between dissimilar one-dimensional (1D) hexagonal quasicrystals with piezoelectric effect under anti-plane shear and in-plane electric loadings has been studied. By using integral transform techniques the mixed boundary value problem for the interface crack was reduced to the solution of singular integral equations, which can be further reduced to solving Riemann-Hilbert problems with an exact solution. An analytic full-field solution for phonon and phason stresses, electric fields, electric displacement in the cracked bi-materials is given, and of particular interest, the analytical expression of the phonon and phason stresses, and electric displacements along the interface has been obtained. The crack sliding displacements (CSDs) of the interface crack are provided, and it is found that the phonon and phason stress distributions inside the dissimilar quasicrystal material are independent on the material properties under the anti-plane shear and in-plane electric loadings. The results of the stress intensity factors energy release rate indicate that the crack propagation can either be enhanced or retarded depending on the magnitude and direction of the electric loadings.

Keywords: Interface crack; One-dimensional (1D) quasicrystal materials; Singular integral equations; Riemann-Hilbert problem; Crack sliding displacement.

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1 Introduction

Quasicrystals (QCs) were discovered as a new solid structure and material by Shechtman et al. [1] in 1984, and there has been a fascination with investigating their physical, chemical, structural and mechanical properties. Much interest is being motivated by the ever-expanding technological exploitation within, for example, the aerospace, automobile and nuclear fuel industries of their most desirable properties. QCs possess unique atomic structures with perfect long-range orientational order and long-range quasiperiodic translation order, which are different from the conventional crystalline materials and non-crystalline materials [2]. To characterize the structural properties of QCs, two kinds of displacement fields are needed. One is a phonon displacement field, u_i, which describes the usual physical or parallel space, and the other is a phason displacement field, w_j, which describes the complementary or perpendicular space. It is noted that the phason displacement field is diffusive due to the elementary excitation associated with the phason mode and describes the local rearrangements of unit-cells. A one-dimensional (1D) quasicrystal is defined as a three-dimensional solid whose atomic structures are periodic in the ($x_1 - x_2$) - plane and quasiperiodic in the normal direction of the plane, thus there only exists a phason displacement w_3 along the x_3- axis for describing the quasilattices, and 1D QCs show a property of transverse isotropy [3].

Experiments have shown that Qcs are quite brittle and defects such as dislocations and cracks in QCs have been observed. When QCs are subjected external loadings in service, the propagation of defects may lead to damage and/or failure of these materials. Therefore, Fracture analysis of quasicrystal materials (QCs) is meaningful both in theoretical studies and practical applications. Fan [4] systematically presented the mathematical theory of quasicrystalline elasticity, and solved many boundary value problems in QCs. Dislocation problems in one-dimensional hexagonal quasicrystals have been investigated by Li and Fan [5] and Fan et al. [6]. General solutions for one-dimensional hexagonal quasicrystals have been derived by Chen et al. [7] and Wang [8] for the static and dynamic problems, respectively. Li and Li [9] presented three-dimensional general solutions for static problems in thermo-elasticity of 1D hexagonal QCs. Li et al. [10] investigated a Griffith crack in a pentagonal QC and obtained analytical solutions for the stress field and stress intensity factors near the crack tips. Conservation laws in elasticity of QCs have been established to cognize the influence of phason displacements on the mechanical behaviors of QCs, and collinear periodic cracks of antiplane sliding mode in 1D hexagonal quasicrystal have been investigated [11, 12]. Wang and Pan [13] derived analytical solutions for some defect problems in one-dimensional hexagonal and two-dimensional octagonal quasicrystals, and exact expressions for all the field variables are found, also observed is the shielding or anti-shielding effect on the crack-tip due to the neighboring dislocation. Li and Fan [14] applied complex variable method to obtain the exact solutions for two semi-infinite collinear cracks in a strip of 1D hexagonal QC. By using the Stroh formalism, Guo and Lu [15] investigated the problem of four cracks originating from an elliptical hole in 1D hexagonal QC under anti-plane shear loadings. The two-dimensional problem of an elliptic hole or a crack in three-dimensional quasicrystals subject to far field loadings was analyzed based on complex potential method [16]. Guo et al. [17] investigated the anti-plane fracture problem for a finite crack in a 1D hexagonal quasicrystal strip by using Fourier transforms and the technique of dual integral equations, and the expressions for stress, displacements and field intensity factors of the phonon and phason fields near the crack tip were obtained exactly. Potential theory method was used to solve planar crack problems in an infinite space of 1D hexagonal QCs, and the mode I problems of three common planar cracks (a penny-shaped crack, an external circular crack and a half-infinite crack) have been solved in a systematic manner [18]. Path-independent integral in fracture mechanics of quasicrystals has been derived to evaluate the fracture parameters in QCs with the atomic arrangement quasiperiodic in one-, two- or three-direction, and the relation between stress intensity factors and energy release rate was discussed [19]. Crack path prediction and crack deflection in 1D quasicrystals were investigated theoretically within a quasi-static framework [20].

Piezoelectricity is an important physical property of quasicrystals, and it has been studied from a theoretical viewpoint [21]. Due to the piezoelectric effect, quasicrystal materials are expected to be exploited as sensors and actuators in smart structures. Using complex variable function method and conformal mapping technique, Yang and Li [22] investigated the anti-plane shear problem of two symmetric cracks originating from an elliptical hole in 1D hexagonal piezoelectric QC. General solutions of plane problem in 1D quasicrystal piezoelectric materials have been obtained based on the fundamental equations of piezoelectricity of QCs, and the application in fracture mechanics of QCs was discussed [23]. Fan et al. [24] obtained the fundamental solutions of three-dimensional cracks in one-dimensional hexagonal piezoelectric quasicrystals. A non-uniformly generally loaded anti-plane crack in a half-space of a one-dimensional piezoelectric quasicrystal was studied by using an extension of the classical continuous dislocation layer method [25]. By using the integral transform technique, the problem of two collinear mode-III cracks in 1D hexagonal piezoelectric quasicrystal strip has been reduced to solving a standard singular integral equation, and exact closed-form solutions have been obtained for the cracks either parallel or perpendicular to the strip boundaries [26, 27]. Li et al. [28] performed the fracture analysis of a transversely isotropic piezoelectric quasicrystal cylinder under axial shear, and found that the quasicrystal fracture is governed by the phonon or phason field, depending on the phonon-phason loading ratio. The problem of a moving crack in 1D hexagonal piezoelectric quasicrystals was studied under the action of anti-plane shear and in-plane electric field, and the result shows that the coupled elastic fields inside piezoelectric QCs depend on the speed of crack propagation [29].

In reality, the interface crack between quasicrystal materials or quasicrystal and non-quasicrystal materials has important engineering application background, for example, smart structures or components made of dissimilar quasicrystal materials, and in this case, crack may appear on the interface across which material properties change abruptly. Interfacial cracks of antiplane sliding mode between usual elastic materials and quasicrystals were investigated by Shi et al. [30] and the role of the phason displacement play in crack extension was studied. The extended displacement discontinuity boundary integral-differential equation method has been adopted by Zhao et al. [31] and Dang et al. [32] to analyze a three-dimensional interface crack in one-dimensional hexagonal thermo-electro-elastic quasicrystal bi-material.

To the authors' knowledge, the interface crack problem between dissimilar 1D piezoeledtric quasicrystals under anti-plane shear and in-plane electric loadings has not been solved. In this paper, by using the integral transforms and the technique of singular integral equations, the mixed boundary value problem of the interface crack between dissimilar QCs has been reduced to corresponding Riemann-Hilbert problems, and exact solutions of the full-fields in the cracked bi-materials were obtained. The field intensity factors of the phonon and phason stresses, the crack sliding displacement (CSD) and the energy release rate were studied in detail.

2 Problem statement

We consider an interface crack of length $2a$ between dissimilar one-dimensional (1D) piezoelectric QCs with point group 6*mm*, which has a quasiperiodic poling axis, denoted as z-axis, and an isotropic periodic plane, denoted as xoy-plnae, as shown in Fig. 1. For convenience, a set of Cartesian coordinate system (x, y) is attached to the crack. Assume that a uniform shear stresses, T_0, H_0, and uniform electric displacement D_0 are applied on the far-field boundaries.

According to the quasicrystal elasticity theory, the generalized Hooke's law follows

$$
\begin{aligned}
\sigma_{xx} &= C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} - e_{31}E_z \\
\sigma_{yy} &= C_{12}\varepsilon_{xx} + C_{11}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} - e_{31}E_z \\
\sigma_{zz} &= C_{13}\varepsilon_{xx} + C_{13}\varepsilon_{yy} + C_{33}\varepsilon_{zz} + R_2 w_{zz} - e_{33}E_z \\
\sigma_{xy} &= 2C_{66}\varepsilon_{xy} \\
\sigma_{xz} &= \sigma_{zx} = 2C_{44}\varepsilon_{zx} + R_3 w_{zx} - e_{15}E_x \\
\sigma_{yz} &= \sigma_{zy} = 2C_{44}\varepsilon_{zy} + R_3 w_{zy} - e_{15}E_y \\
H_{zx} &= 2R_3\varepsilon_{zx} + K_2 w_{zx} - d_{15}E_x \\
H_{zy} &= 2R_3\varepsilon_{zy} + K_2 w_{zy} - d_{15}E_y \\
H_{zz} &= R_1\varepsilon_{xx} + R_1\varepsilon_{yy} + R_2\varepsilon_{zz} + K_1 w_{zz} - d_{33}E_z \\
D_x &= 2e_{15}\varepsilon_{zx} + d_{15}w_{zx} + \lambda_{11}E_x \\
D_y &= 2e_{15}\varepsilon_{zy} + d_{15}w_{zy} + \lambda_{11}E_y \\
D_z &= e_{31}\varepsilon_{xx} + e_{31}\varepsilon_{yy} + e_{33}\varepsilon_{zz} + d_{33}w_{zz} + \lambda_{33}E_z
\end{aligned} \tag{1}
$$

where σ_{ij} and H_{ij} are the stresses in the phonon and the phason fields, respectively; ε_{ij} and w_{ij} are the phonon and phason strains, respectively; D_i and E_i are the electric displacements and electric field, respectively; C_{ij} and K_i are elastic constants in phonon and phason fields, respectively; R_i is the phonon-phason coupling elastic constant, e_{ij} and d_{ij} are the piezoelectric constants, and λ_{jj} the dielectric permittivities, and $C_{66} = (C_{11} - C_{12})/2$. The subscripts i, j stand for the coordinate x or y or z.

Under anti-plane mechanical loading and in-plane electric loading with reference to the *xoy*-plane, deformation involved is independent on the spatial variable *z*. Consequently, there are only non-vanishing out-of-plane displacements of phonon and phason fields and in-plane electric fields, i.e.,

$$
\begin{aligned}
&u_x = u_y = 0,\ \ u_z = u_z(x, y), \\
&w_z = w_z(x, y), \\
&E_x = E_x(x, y),\ \ E_y = E_y(x, y),\ \ E_z = 0
\end{aligned} \tag{2}
$$

The gradient equations are as follows:

$$
\begin{aligned}
&\varepsilon_{xx} = u_{x,x},\ \ \varepsilon_{yy} = u_{y,y},\ \ \varepsilon_{zz} = u_{z,z} \\
&\varepsilon_{yz} = \frac{1}{2}(u_{y,z} + u_{z,y}),\ \ \varepsilon_{zx} = \frac{1}{2}(u_{x,z} + u_{z,x}),\ \ \varepsilon_{xy} = \frac{1}{2}(u_{x,y} + u_{y,x})
\end{aligned} \tag{3-1}
$$

$$
w_{zx} = w_{z,x},\ \ w_{zy} = w_{z,y},\ \ w_{zz} = w_{z,z} \tag{3-2}
$$

$$
E_x = -\phi_{,x},\ \ E_y = -\phi_{,y},\ \ E_z = -\phi_{,z} \tag{3-3}
$$

where the subscript comma denotes a partial differentiation with respect to the coordinate.

The constitutive equations under the anti-plane deformation are as follows

$$
\begin{aligned}
\sigma_{zx} &= 2C_{44}\varepsilon_{zx} + R_3 w_{zx} - e_{15}E_x = C_{44}u_{z,x} + R_3 w_{z,x} + e_{15}\phi_{,x} \\
\sigma_{zy} &= 2C_{44}\varepsilon_{zy} + R_3 w_{zy} - e_{15}E_y = C_{44}u_{z,y} + R_3 w_{z,y} + e_{15}\phi_{,y} \\
H_{zx} &= 2R_3\varepsilon_{zx} + K_2 w_{zx} - d_{15}E_x = R_3 u_{z,x} + K_2 w_{z,x} + d_{15}\phi_{,x} \\
H_{zy} &= 2R_3\varepsilon_{zy} + K_2 w_{zy} - d_{15}E_y = R_3 u_{z,y} + K_2 w_{z,y} + d_{15}\phi_{,y} \\
D_x &= 2e_{15}\varepsilon_{zx} + d_{15}w_{zx} + \lambda_{11}E_x = e_{15}u_{z,x} + d_{15}w_{z,x} - \lambda_{11}\phi_{,x} \\
D_y &= 2e_{15}\varepsilon_{zy} + d_{15}w_{zy} + \lambda_{11}E_y = e_{15}u_{z,y} + d_{15}w_{z,y} - \lambda_{11}\phi_{,y}
\end{aligned} \tag{4}
$$

and the equilibrium equations for the anti-plane deformation problem are

$$
\begin{aligned}
&\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = 0 \\
&\frac{\partial H_{zx}}{\partial x} + \frac{\partial H_{zy}}{\partial y} = 0 \\
&\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0
\end{aligned} \tag{5}
$$

By considering the gradient equations and the constitutive equations and neglecting body forces and free charge, the governing equations for the anti-plane problem of the 1D quasicrystal material with piezoelectric effect can be obtained as

$$
\begin{aligned}
&C_{44}\nabla^2 u_z + R_3\nabla^2 w_z + e_{15}\nabla^2\phi = 0 \\
&R_3\nabla^2 u_z + K_2\nabla^2 w_z + d_{15}\nabla^2\phi = 0 \\
&e_{15}\nabla^2 u_z + d_{15}\nabla^2 w_z - \lambda_{11}\nabla^2\phi = 0
\end{aligned} \tag{6}
$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian operator in the variables x and y, u_z is the out-of-plane displacement, ϕ is the electric potential,

w_z is the displacement of the phason field, C_{44} and K_2 are the elastic constants for the phonon field and phason field, respectively; e_{15} and d_{15} are the piezoelectric constants, R_3 the phonon-phason coupling elastic constant, and λ_{11} the dielectric permittivities.

For the upper half-space occupied by quasicrystal material-I, the governing equations are

$$\begin{aligned} C_{44}^I \nabla^2 u_z^I + R_3^I \nabla^2 w_z^I + e_{15}^I \nabla^2 \phi^I &= 0 \\ R_3^I \nabla^2 u_z^I + K_2^I \nabla^2 w_z^I + d_{15}^I \nabla^2 \phi^I &= 0 \\ e_{15}^I \nabla^2 u_z^I + d_{15}^I \nabla^2 w_z^I - \lambda_{11}^I \nabla^2 \phi^I &= 0 \end{aligned} \tag{7}$$

and for the lower half-space occupied by quasicrystal material-II, the governing equations read

$$\begin{aligned} C_{44}^{II} \nabla^2 u_z^{II} + R_3^{II} \nabla^2 w_z^{II} + e_{15}^{II} \nabla^2 \phi^{II} &= 0 \\ R_3^{II} \nabla^2 u_z^{II} + K_2^{II} \nabla^2 w_z^{II} + d_{15}^{II} \nabla^2 \phi^{II} &= 0 \\ e_{15}^{II} \nabla^2 u_z^{II} + d_{15}^{II} \nabla^2 w_z^{II} - \lambda_{11}^{II} \nabla^2 \phi^{II} &= 0 \end{aligned} \tag{8}$$

where the superscripts "*I*" and "*II*" denote the properties and quantities in the upper and lower half-spaces, respectively.

Comparing the basic equations for the 1D hexagonal piezoelectric QC materials to those for transversely isotropic magnetoelectroelastic (MEE) materials under anti-plane deformation [33-38], one can find the analogy relations as listed in Table 1. It is clear to see that the governing equations of both the 1D hexagonal piezoelectric QCs and magnetoelectroelastic materials are of the same form, which can be reduced to Laplacian equations when the determinant of the material constants is generally non-zero. In column 3 in Table 1, u_z is the out-of-plane displacement, ϕ is the electric potential, φ is the magnetic potential, C_{44} the elastic constant, e_{15} the piezoelectric constant, h_{15} the piezomagnetic constant, λ_{11} the dielectric permittivities, d_{11} the electromagnetic constant, and μ_{11} is magnetic permeabilities. With the analogy relation mentioned in Table 1, one can get the solutions for 1D piezoelectric QCs by using the same method for MEE. It is noted that similar analogy relations between 1D hexagonal thermo-electro-elastic QCs and magnetoelectrothermoelastic materials have been used to analyze a three-dimensional arbitrary shaped interface crack in a 1D hexagonal thermo-electro-elastic quasicrystal bi-material [31].

When the bi-materials made of dissimilar 1D hexagonal piezoelectric quasicrystals are subjected to constant electromechancial loadings at infinity, the regular conditions are:

$$\sigma_{zy}(x,\infty) = T_0 \qquad (|x| < \infty) \tag{9-1}$$

$$H_{zy}(x,\infty) = H_0 \qquad (|x| < \infty) \tag{9-2}$$

$$D_y(x,\infty) = D_0 \qquad (|x| < \infty) \tag{9-3}$$

The continuity conditions along the bonded interface between the dissimilar quasicrystal materials at the plane $y = 0$ are:

$$\sigma^I_{zy}(x,0^+) = \sigma^{II}_{zy}(x,0^-) \qquad (|x| < \infty) \tag{10-1}$$

$$H^I_{zy}(x,0^+) = H^{II}_{zy}(x,0^-) \qquad (|x| < \infty) \tag{10-2}$$

$$D^I_y(x,0^+) = D^{II}_y(x,0^-) \qquad (|x| < \infty) \tag{10-3}$$

The crack surfaces are assumed free of traction in the phonon and phason fields, and the interface crack problem may be solved under the following mixed boundary conditions:

$$\sigma_{zy}^{I}(x,0^{+})=\sigma_{zy}^{II}(x,0^{-})=0 \qquad (|x|<a) \tag{11-1}$$

$$u_{z}^{I}(x,0^{+})=u_{z}^{II}(x,0^{-}) \qquad (|x|\geq a) \tag{11-2}$$

$$H_{zy}^{I}(x,0^{+})=H_{zy}^{II}(x,0^{-})=0 \qquad (|x|<a) \tag{12-1}$$

$$w_{z}^{I}(x,0^{+})=w_{z}^{II}(x,0^{-}) \qquad (|x|\geq a) \tag{12-2}$$

For the electrical boundary conditions, the crack surfaces may be assumed to be either imperpeable, permeable or partially permeable, similar to the case in the classical piezoelectric materials [39]. In this study, we consider both the electrically impermeable and permeable cracks, and comparisons of the results will be made.

For the electrically impermeable crack case, the boundary conditions on the crack faces are:

$$D_{y}^{I}(x,0^{+})=D_{y}^{II}(x,0^{-})=0 \qquad (|x|<a) \tag{13-1}$$

$$\phi^{I}(x,0^{+})=\phi^{II}(x,0^{-}) \qquad (|x|\geq a) \tag{13-2}$$

For the electrically permeable crack case, the boundary conditions along the crack face plane are:

$$\phi^{I}(x,0^{+})=\phi^{II}(x,0^{-}) \qquad (|x|<\infty) \tag{14}$$

3 Method of solution

Due to the symmetry property of the problem, it is sufficient to consider a half of the bi-materials, i.e., $x\geq 0$. By applying the technique of integral transform to Eqs. (7, 8), the displacements in the phonon and phason fields, and the electric potential in the dissimilar one-dimensional hexagonal quasicrystal materials can be expressed as:

$$u_{z}^{I}(x,y)=\int_{0}^{\infty}A^{I}(\xi)\exp(-\xi y)\cos(\xi x)d\xi+a_{0}^{I}y, \qquad (y\geq 0) \tag{15}$$

$$w_z^I(x,y)=\int_0^\infty B^I(\xi)\exp(-\xi y)\cos(\xi x)d\xi+b_0^I y\,,\qquad (y\ge 0) \tag{16}$$

$$\phi^I(x,y)=\int_0^\infty C^I(\xi)\exp(-\xi y)\cos(\xi x)d\xi+d_0^I y\,,\qquad (y\ge 0) \tag{17}$$

$$u_z^{II}(x,y)=\int_0^\infty A^{II}(\xi)\exp(\xi y)\cos(\xi x)d\xi+a_0^{II} y\,,\qquad (y\le 0) \tag{18}$$

$$w_z^{II}(x,y)=\int_0^\infty B^{II}(\xi)\exp(\xi y)\cos(\xi x)d\xi+b_0^{II} y\,,\qquad (y\le 0) \tag{19}$$

$$\phi^{II}(x,y)=\int_0^\infty C^{II}(\xi)\exp(\xi y)\cos(\xi x)d\xi+d_0^{II} y\,,\qquad (y\le 0) \tag{20}$$

where $A^I(\xi)$, $A^{II}(\xi)$, $B^I(\xi)$, $B^{II}(\xi)$, $C^I(\xi)$, $C^{II}(\xi)$ are unknown functions to be solved, $a_0^I, a_0^{II}, b_0^I, b_0^{II}, d_0^I, d_0^{II}$ are constants related to the far-field loading conditions and are defined in Appendix A.

A simple calculation leads to the components of stresses in the phonon and phason fields, and the electric displacements in the upper half-space ($y\ge 0$) as follows:

$$\sigma_{zy}^I(x,y)=T_0-\int_0^\infty \xi\left[C_{44}^I A^I(\xi)+R_3^I B^I(\xi)+e_{15}^I C^I(\xi)\right]\exp(-\xi y)\cos(\xi x)d\xi \tag{21}$$

$$\sigma_{zx}^I(x,y)=-\int_0^\infty \xi\left[C_{44}^I A^I(\xi)+R_3^I B^I(\xi)+e_{15}^I C^I(\xi)\right]\exp(-\xi y)\sin(\xi x)d\xi \tag{22}$$

$$H_{zy}^I(x,y)=H_0-\int_0^\infty \xi\left[R_3^I A^I(\xi)+K_2^I B^I(\xi)+d_{15}^I C^I(\xi)\right]\exp(-\xi y)\cos(\xi x)d\xi \tag{23}$$

$$H_{zx}^I(x,y)=-\int_0^\infty \xi\left[R_3^I A^I(\xi)+K_2^I B^I(\xi)+d_{15}^I C^I(\xi)\right]\exp(-\xi y)\sin(\xi x)d\xi \tag{24}$$

$$D_y^I(x,y)=D_0-\int_0^\infty \xi\left[e_{15}^I A^I(\xi)+d_{15}^I B^I(\xi)-\lambda_{11}^I C^I(\xi)\right]\exp(-\xi y)\cos(\xi x)d\xi \tag{25}$$

$$D_x^I(x,y)=-\int_0^\infty \xi\left[e_{15}^I A^I(\xi)+d_{15}^I B^I(\xi)-\lambda_{11}^I C^I(\xi)\right]\exp(-\xi y)\sin(\xi x)d\xi \tag{26}$$

The stresses in the phonon and phason fields and the electric displacement in the lower half-space ($y\le 0$) can be expressed as:

$$\sigma_{zy}^{II}(x,y)=T_0+\int_0^\infty \xi\left[C_{44}^{II} A^{II}(\xi)+R_3^{II} B^{II}(\xi)+e_{15}^{II} C^{II}(\xi)\right]\exp(\xi y)\cos(\xi x)d\xi \tag{27}$$

$$\sigma_{zx}^{II}(x,y)=-\int_0^\infty \xi\left[C_{44}^{II} A^{II}(\xi)+R_3^{II} B^{II}(\xi)+e_{15}^{II} C^{II}(\xi)\right]\exp(\xi y)\sin(\xi x)d\xi \tag{28}$$

$$H_{zy}^{II}(x,y)=H_0+\int_0^\infty \xi\left[R_3^{II} A^{II}(\xi)+K_2^{II} B^{II}(\xi)+d_{15}^{II} C^{II}(\xi)\right]\exp(\xi y)\cos(\xi x)d\xi \tag{29}$$

$$H_{zx}^{II}(x,y) = -\int_0^\infty \xi\left[R_3^{II}A^{II}(\xi) + K_2^{II}B^{II}(\xi) + d_{15}^{II}C^{II}(\xi)\right]\exp(\xi y)\sin(\xi x)d\xi \tag{30}$$

$$D_y^{II}(x,y) = D_0 + \int_0^\infty \xi\left[e_{15}^{II}A^{II}(\xi) + d_{15}^{II}B^{II}(\xi) - \lambda_{11}^{II}C^{II}(\xi)\right]\exp(\xi y)\cos(\xi x)d\xi \tag{31}$$

$$D_x^{II}(x,y) = -\int_0^\infty \xi\left[e_{15}^{II}A^{II}(\xi) + d_{15}^{II}B^{II}(\xi) - \lambda_{11}^{II}C^{II}(\xi)\right]\exp(\xi y)\sin(\xi x)d\xi \tag{32}$$

3.1 Impermeable crack case

The satisfaction of the continuity conditions in Eqs. (10) leads to the following relations:

$$\begin{Bmatrix} A^{II}(\xi) \\ B^{II}(\xi) \\ C^{II}(\xi) \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{Bmatrix} A^{I}(\xi) \\ B^{I}(\xi) \\ C^{I}(\xi) \end{Bmatrix}, \tag{33}$$

where

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} = -\begin{bmatrix} C_{44}^{II} & R_3^{II} & e_{15}^{II} \\ R_3^{II} & K_2^{II} & d_{15}^{II} \\ e_{15}^{II} & d_{15}^{II} & -\lambda_{11}^{II} \end{bmatrix}^{-1} \begin{bmatrix} C_{44}^{I} & R_3^{I} & e_{15}^{I} \\ R_3^{I} & K_2^{I} & d_{15}^{I} \\ e_{15}^{I} & d_{15}^{I} & -\lambda_{11}^{I} \end{bmatrix}, \tag{34}$$

Introduce the following three jump functions as

$$\begin{aligned} u(x) &= u_z^I(x,0^+) - u_z^{II}(x,0^-) \\ w(x) &= w_z^I(x,0^+) - w_z^{II}(x,0^-) \\ \varphi(x) &= \phi^I(x,0^+) - \phi^{II}(x,0^-) \end{aligned} \tag{35}$$

By using the Fourier inverse transform, the unknown functions $A^I(\xi)$, $B^I(\xi)$, $C^I(\xi)$ can be expressed by the integrals of the jump functions $u(x)$, $w(x)$, $\varphi(x)$ as follows:

$$\begin{Bmatrix} A^{I}(\xi) \\ B^{I}(\xi) \\ C^{I}(\xi) \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} \begin{Bmatrix} \frac{2}{\pi}\int_0^a u(x)\cos(x\xi)dx \\ \frac{2}{\pi}\int_0^a w(x)\cos(x\xi)dx \\ \frac{2}{\pi}\int_0^a \varphi(x)\cos(x\xi)dx \end{Bmatrix}, \tag{36}$$

where

$$\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix} = \begin{bmatrix} 1-\alpha_{11} & -\alpha_{12} & -\alpha_{13} \\ -\alpha_{21} & 1-\alpha_{22} & -\alpha_{23} \\ -\alpha_{31} & -\alpha_{32} & 1-\alpha_{33} \end{bmatrix}^{-1}, \tag{37}$$

Following the procedure in [40, 41], one can find that the satisfaction of the mixed boundary conditions (11-13) leads to the following integral equations about the unknown functions $A^I(\xi)$, $B^I(\xi)$, $C^I(\xi)$:

$$\begin{aligned} &\int_0^\infty \xi A^I(\xi)\cos(x\xi)d\xi = a_0^I \\ &\int_0^\infty \xi B^I(\xi)\cos(x\xi)d\xi = b_0^I \qquad (0 \le x < a) \\ &\int_0^\infty \xi C^I(\xi)\cos(x\xi)d\xi = d_0^I \end{aligned} \tag{38}$$

where the constants a_0^I, b_0^I, d_0^I are defined in the Appendix A.

The substitution of Eqs. (36) into the Eqs. (38) leads to the following singular integral equations:

$$\begin{aligned} &\int_{-a}^{a} \frac{u(t)}{t-x}dt = f_A x + t_A \\ &\int_{-a}^{a} \frac{w(t)}{t-x}dt = f_B x + t_B \\ &\int_{-a}^{a} \frac{\varphi(t)}{t-x}dt = f_C x + t_C \end{aligned} \tag{39}$$

where t_A, t_B, t_C are constants to be determined from the far-field regular conditions, and f_A, f_B, f_C are constants defined as

$$\begin{Bmatrix} f_A \\ f_B \\ f_C \end{Bmatrix} = -\pi \begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix}^{-1} \begin{Bmatrix} a_0^I \\ b_0^I \\ d_0^I \end{Bmatrix} \tag{40}$$

To solve the singular integral equations (39), following Muskhelishvili [42], we introduce the following functions of $z = x + iy$ as

$$
\begin{aligned}
U(z) &= \frac{1}{2\pi i}\int_{-a}^{a}\frac{u(t)}{t-z}dt \\
W(z) &= \frac{1}{2\pi i}\int_{-a}^{a}\frac{w(t)}{t-z}dt \\
\Phi(z) &= \frac{1}{2\pi i}\int_{-a}^{a}\frac{\varphi(t)}{t-z}dt
\end{aligned}
\tag{41}
$$

which are analytic in the whole complex plane with a cut along the segment $-a \le x \le a$ of the real axis. The boundary values of the continuous extension on this segment to the left and right are determined by the Sokhotskii-Plemelj formulas [42]

$$
\begin{aligned}
&U^{+}(x)+U^{-}(x) = \frac{1}{\pi i}\int_{-a}^{a}\frac{u(t)}{t-x}dt \\
&U^{+}(x)-U^{-}(x) = u(x)
\end{aligned}
\tag{42-1}
$$

$$
\begin{aligned}
&W^{+}(x)+W^{-}(x) = \frac{1}{\pi i}\int_{-a}^{a}\frac{w(t)}{t-x}dt \\
&W^{+}(x)-W^{-}(x) = w(x)
\end{aligned}
\tag{42-2}
$$

$$
\begin{aligned}
&\Phi^{+}(x)+\Phi^{-}(x) = \frac{1}{\pi i}\int_{-a}^{a}\frac{\varphi(t)}{t-x}dt \\
&\Phi^{+}(x)-\Phi^{-}(x) = \varphi(x)
\end{aligned}
\tag{42-3}
$$

where the signs "+" and "-" denote the limiting values of the functions $U(z)$, $W(z)$, $\Phi(z)$ at $y=0$ from the positive and negative y-axis, respectively.

The following Riemann-Hilbert problem can be obtained by substituting Eqs. (42) into Eqs. (39):

$$
U^{+}(x) = -U^{-}(x) + \frac{1}{\pi i}\left[f_A x + t_A\right] = -U^{-}(x) + F_A(x) \tag{43-1}
$$

$$
W^{+}(x) = -W^{-}(x) + \frac{1}{\pi i}\left[f_B x + t_B\right] = -W^{-}(x) + F_B(x) \tag{43-2}
$$

$$
\Phi^{+}(x) = -\Phi^{-}(x) + \frac{1}{\pi i}\left[f_C x + t_C\right] = -\Phi^{-}(x) + F_C(x) \tag{43-3}
$$

The solution of the Riemann-Hilbert problem can be obtained as [42]:

$$U(z)=\frac{X(z)}{2\pi i}\int_{-a}^{a}\frac{F_A(t)}{X^+(t)(t-z)}dt \tag{44-1}$$

$$W(z)=\frac{X(z)}{2\pi i}\int_{-a}^{a}\frac{F_B(t)}{X^+(t)(t-z)}dt \tag{44-2}$$

$$\Phi(z)=\frac{X(z)}{2\pi i}\int_{-a}^{a}\frac{F_C(t)}{X^+(t)(t-z)}dt \tag{44-3}$$

where $X(z)$ is the particular solution of the homogeneous, Riemann-Hilbert problem which is bounded near the ends $x=\pm a$, $X^+(t)$ is the value of $X(z)$ on the left boundary of the discontinuity, and

$$X(z)=\sqrt{(z+a)(z-a)} \tag{45}$$

Considering that the differences of the functions $u,\ w,\ \varphi$ vanish at infinity, one should require the equality conditions $U(\infty)=0,\ W(\infty)=0,\ \Phi(\infty)=0$ to hold. This leads to the condition

$$\int_{-a}^{a}\frac{1}{X^+(t)}(f_A t+t_A)dt=0 \tag{46-1}$$

$$\int_{-a}^{a}\frac{1}{X^+(t)}(f_B t+t_B)dt=0 \tag{46-2}$$

$$\int_{-a}^{a}\frac{1}{X^+(t)}(f_C t+t_C)dt=0 \tag{46-3}$$

and by recalling the fact that $X^+(t)=i\sqrt{a^2-t^2}$ is an even function of t, one can obtain from (43, 46) that

$$t_A=t_B=t_C=0 \tag{47}$$

$$F_A(x)=\frac{f_A x}{i\pi};\ \ F_B(x)=\frac{f_B x}{i\pi};\ \ F_A(x)=\frac{f_C x}{i\pi} \tag{48}$$

The general solution of the Riemann-Hilbert problems (43) can be obtained as

$$U(z) = \frac{f_A}{2}\left[z - X(z)\right] \tag{49-1}$$

$$W(z) = \frac{f_B}{2}\left[z - X(z)\right] \tag{49-2}$$

$$\Phi(z) = \frac{f_C}{2}\left[z - X(z)\right] \tag{49-3}$$

where the constants f_A, f_B, f_C are defined in Eqs. (40).

It is noted that the exact nature of the singularity and the correct form of the asymptotic distribution of the unknown functions at and near the singular points is determined by using the complex function theory and investigating the properties of the corresponding Riemann-Hilbert problems. The above method and procedure for solving the anti-plane interface crack problem could be extended to solving in-plane interface crack problems and even three-dimensional interface crack problems.

By substituting Eqs. (49) into Eqs. (42), the displacement jump function $u(x)$, i.e., the crack sliding displacement (CSD) of the interface crack under anti-plane shear deformation can be obtained as

$$u(x) = -\frac{f_A}{\pi}\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{50}$$

Similar procedure can be applied to get the solutions of $w(x)$ and $\varphi(x)$ as

$$w(x) = -\frac{f_B}{\pi}\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{51}$$

$$\varphi(x) = -\frac{f_C}{\pi}\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{52}$$

The crack sliding displacements (CSDs) for the phonon and phason fields and the crack electric potential jump can be obtained as:

$$u_z^I(x,0^+) = a_0^I\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{53-1}$$

$$u_z^{II}(x,0^-) = (\alpha_{11}a_0^I + \alpha_{12}b_0^I + \alpha_{13}d_0^I)\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{53-2}$$

$$w_z^I(x,0^+) = b_0^I\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{54-1}$$

$$w_z^{II}(x,0^-) = (\alpha_{21}a_0^I + \alpha_{22}b_0^I + \alpha_{23}d_0^I)\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{54-2}$$

$$\phi^I(x,0^+) = d_0^I\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{55-1}$$

$$\phi^{II}(x,0^-) = (\alpha_{31}a_0^I + \alpha_{32}b_0^I + \alpha_{33}d_0^I)\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{55-2}$$

It is noted that during the derivation of the Eqs. (53-55), the following identity has been used [35]

$$\int_0^\infty \cos(x\xi)\cos(t\xi)d\xi = \frac{\pi}{2}\delta(x-t), \tag{56}$$

where $\delta(x-t)$ is the Dirac delta function, which takes the value 1 when $x = t$, and otherwise takes the value 0.

The analytical solutions of the unknown functions $A^I(\xi)$, $B^I(\xi)$, $C^I(\xi)$ are derived as

$$\begin{Bmatrix} A^I(\xi) \\ B^I(\xi) \\ C^I(\xi) \end{Bmatrix} = -\frac{aJ_1(\xi a)}{\pi\xi}\begin{bmatrix} \beta_{11} & \beta_{12} & \beta_{13} \\ \beta_{21} & \beta_{22} & \beta_{23} \\ \beta_{31} & \beta_{32} & \beta_{33} \end{bmatrix}\begin{Bmatrix} f_A \\ f_B \\ f_C \end{Bmatrix}, \tag{57}$$

where $J_1(\;)$ is the Bessel function of the first kind, and the following integral identity has been used in the derivation [43]:

$$\int_0^a \sqrt{a^2 - x^2}\cos(x\xi)dx = \frac{\pi a}{2\xi}J_1(a\xi), \tag{58}$$

By substituting Eqs. (57) into Eqs. (21-26) and using some integral identities listed in the Appendix A, the full-field solutions can be derived. The phonon and phason stresses and electric displacements in the upper part of the cracked bi-materials are obtained as

$$\sigma_{zy}^{I}(x,y)=T_0\frac{r}{\sqrt{r_1 r_2}}\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{59-1}$$

$$\sigma_{zx}^{I}(x,y)=T_0\frac{r}{\sqrt{r_1 r_2}}\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{59-2}$$

$$\sigma_{zy}^{I}(x,y)+i\sigma_{zx}^{I}(x,y)=T_0\frac{z}{\sqrt{z^2-a^2}},\quad (y\geq 0) \tag{59}$$

$$H_{zy}^{I}(x,y)=H_0\frac{r}{\sqrt{r_1 r_2}}\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{60-1}$$

$$H_{zx}^{I}(x,y)=H_0\frac{r}{\sqrt{r_1 r_2}}\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{60-2}$$

$$H_{zy}^{I}(x,y)+iH_{zx}^{I}(x,y)=H_0\frac{z}{\sqrt{z^2-a^2}},\quad (y\geq 0) \tag{60}$$

$$D_{y}^{I}(x,y)=D_0\frac{r}{\sqrt{r_1 r_2}}\cos\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{61-1}$$

$$D_{x}^{I}(x,y)=D_0\frac{r}{\sqrt{r_1 r_2}}\sin\left(\theta-\frac{\theta_1+\theta_2}{2}\right),\quad (y\geq 0) \tag{61-2}$$

$$D_{y}^{I}(x,y)+iD_{x}^{I}(x,y)=D_0\frac{z}{\sqrt{z^2-a^2}},\quad (y\geq 0) \tag{61}$$

where $z=x+iy$, and the polar coordinates $r,\ r_1,\ r_2,\ \theta,\ \theta_1,\ \theta_2$ are defined respectively as

$$\begin{aligned} &r=\sqrt{x^2+y^2},\ r_1=\sqrt{(x-a)^2+y^2},\ r_2=\sqrt{(x+a)^2+y^2},\\ &\theta=\cos^{-1}\left(\frac{x}{r}\right),\ \theta_1=\cos^{-1}\left(\frac{x-a}{r_1}\right),\ \theta_2=\cos^{-1}\left(\frac{x+a}{r_2}\right), \end{aligned}\quad (y\geq 0) \tag{62}$$

The expression of the phonon and phason stresses and electric displacements in the lower half-space is in the same form as those in the upper half-space, while the corresponding angles $\theta,\ \theta_1,\ \theta_2$ are defined as

$$\theta=-\cos^{-1}\left(\frac{x}{r}\right),\ \theta_1=-\cos^{-1}\left(\frac{x-a}{r_1}\right),\ \theta_2=-\cos^{-1}\left(\frac{x+a}{r_2}\right),\quad (y\leq 0) \tag{63}$$

Near the right crack tip, the asymptotic solution of the singular phonon and phason stresses, and electric displacements in the bi-materials can be expressed as

$$\sigma_{zy}(r_1,\theta_1)=T_0\frac{\sqrt{a}}{\sqrt{2r_1}}\cos\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{64-1}$$

$$\sigma_{zx}(r_1,\theta_1)=-T_0\frac{\sqrt{a}}{\sqrt{2r_1}}\sin\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{64-2}$$

$$H_{zy}(r_1,\theta_1)=H_0\frac{\sqrt{a}}{\sqrt{2r_1}}\cos\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{65-1}$$

$$H_{zx}(r_1,\theta_1)=-H_0\frac{\sqrt{a}}{\sqrt{2r_1}}\sin\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{65-2}$$

$$D_y(r_1,\theta_1)=D_0\frac{\sqrt{a}}{\sqrt{2r_1}}\cos\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{66-1}$$

$$D_x(r_1,\theta_1)=-D_0\frac{\sqrt{a}}{\sqrt{2r_1}}\sin\left(\frac{\theta_1}{2}\right),\qquad(-\pi\le\theta_1\le\pi)\tag{66-2}$$

It is observed that the field components of phonon stresses, phason stresses and electric displacements are governed by a $1/\sqrt{r_1}$ crack-tip singular behavior, as exhibited near the crack tip in an isotropic elastic homogeneous medium. The distributions of the phonon and phason stresses and electric displacements in the dissimilar one-dimensional hexagonal piezoelectric quasicrystals are independent on the material properties of the bi-materials under the state of anti-plane deformation and in-plane electric loading.

3.2 Permeable crack case

In the case that a permeable interface crack is located on the interface between dissimilar 1D hexagonal piezoelectric quasicrystals, the satisfaction of the boundary conditions in Eqs. (10) and (14) lead to the following relation

$$\begin{Bmatrix} A^{II}(\xi) \\ B^{II}(\xi) \\ C^{II}(\xi) \end{Bmatrix} = [\boldsymbol{\alpha}] \begin{Bmatrix} A^{I}(\xi) \\ B^{I}(\xi) \end{Bmatrix} = \begin{bmatrix} \alpha_{11}^{p} & \alpha_{12}^{p} \\ \alpha_{21}^{p} & \alpha_{22}^{p} \\ \alpha_{31}^{p} & \alpha_{32}^{p} \end{bmatrix} \begin{Bmatrix} A^{I}(\xi) \\ B^{I}(\xi) \end{Bmatrix} \tag{67}$$
$$C^{I}(\xi) = C^{II}(\xi)$$

where the constant matrix $[\boldsymbol{\alpha}]$ is defined as

$$[\boldsymbol{\alpha}] = \begin{bmatrix} \alpha_{11}^{p} & \alpha_{12}^{p} \\ \alpha_{21}^{p} & \alpha_{22}^{p} \\ \alpha_{31}^{p} & \alpha_{32}^{p} \end{bmatrix} = -\begin{bmatrix} C_{44}^{II} & R_{3}^{II} & e_{15}^{I} + e_{15}^{II} \\ R_{3}^{II} & K_{2}^{II} & d_{15}^{I} + d_{15}^{II} \\ e_{15}^{II} & d_{15}^{II} & -(\lambda_{11}^{I} + \lambda_{11}^{II}) \end{bmatrix}^{-1} \begin{bmatrix} C_{44}^{I} & R_{3}^{I} \\ R_{3}^{I} & K_{2}^{I} \\ e_{15}^{I} & d_{15}^{I} \end{bmatrix} \tag{68}$$

Following the procedure in the case of an impermeable crack, we introduce the following two jump functions as

$$\begin{aligned} u(x) &= u_{z}^{I}(x,0^{+}) - u_{z}^{II}(x,0^{-}) \\ w(x) &= w_{z}^{I}(x,0^{+}) - w_{z}^{II}(x,0^{-}) \end{aligned} \tag{69}$$

By using the Fourier inverse transform, the unknown functions $A^{I}(\xi)$ and $B^{I}(\xi)$ can be expressed by the integrals of the jump functions $u(x)$, $w(x)$ as follows:

$$\begin{Bmatrix} A^{I}(\xi) \\ B^{I}(\xi) \end{Bmatrix} = \begin{bmatrix} 1-\alpha_{11}^{p} & -\alpha_{12}^{p} \\ -\alpha_{21}^{p} & 1-\alpha_{22}^{p} \end{bmatrix}^{-1} \begin{Bmatrix} \dfrac{2}{\pi}\displaystyle\int_{0}^{a} u(x)\cos(x\xi)dx \\ \dfrac{2}{\pi}\displaystyle\int_{0}^{a} w(x)\cos(x\xi)dx \end{Bmatrix}, \tag{70}$$

where α_{ij}^{p} $(i, j = 1,2)$ are defined in Eq. (68).

The satisfaction of the mixed boundary conditions (11, 12) leads to the following integral equations about the unknown functions $A^{I}(\xi)$ and $B^{I}(\xi)$:

$$\begin{aligned} \int_{0}^{\infty} \xi A^{I}(\xi)\cos(x\xi)d\xi &= R_{A} \\ \int_{0}^{\infty} \xi B^{I}(\xi)\cos(x\xi)d\xi &= R_{B} \end{aligned}, \qquad (0 \le x < a) \tag{71}$$

where

$$\begin{Bmatrix} R_{A} \\ R_{B} \end{Bmatrix} = \begin{bmatrix} C_{44}^{I} + e_{15}^{I}\alpha_{31}^{p} & R_{3}^{I} + e_{15}^{I}\alpha_{32}^{p} \\ R_{3}^{I} + d_{15}^{I}\alpha_{31}^{p} & K_{2}^{I} + d_{15}^{I}\alpha_{32}^{p} \end{bmatrix}^{-1} \begin{Bmatrix} T_{0} \\ H_{0} \end{Bmatrix} \tag{72}$$

The substitution of Eqs. (70) into the Eqs. (71) leads to the following singular integral equations of Cauchy type:

$$\begin{Bmatrix} \int_{-a}^{a} \frac{u(t)}{t-x} dt \\ \int_{-a}^{a} \frac{w(t)}{t-x} dt \end{Bmatrix} = -\pi \begin{bmatrix} 1-\alpha_{11}^{p} & -\alpha_{12}^{p} \\ -\alpha_{21}^{p} & 1-\alpha_{22}^{p} \end{bmatrix} \begin{Bmatrix} R_A x \\ R_B x \end{Bmatrix} \tag{73}$$

Following the procedure in the section for the impermeable interface crack, the solutions of these singular integral equations can be obtained analytically as

$$\begin{Bmatrix} u(x) \\ w(x) \end{Bmatrix} = \begin{bmatrix} 1-\alpha_{11}^{p} & -\alpha_{12}^{p} \\ -\alpha_{21}^{p} & 1-\alpha_{22}^{p} \end{bmatrix} \begin{Bmatrix} R_A \sqrt{a^2 - x^2} \\ R_B \sqrt{a^2 - x^2} \end{Bmatrix} \tag{74}$$

The crack sliding displacements (CSDs) for the phonon and phason fields can be obtained as:

$$u_z^I(x,0^+) = R_A \sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{75-1}$$

$$u_z^{II}(x,0^-) = (\alpha_{11}^p R_A + \alpha_{12}^p R_B)\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{75-2}$$

$$w_z^I(x,0^+) = R_B \sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{76-1}$$

$$w_z^{II}(x,0^-) = (\alpha_{21}^p R_A + \alpha_{22}^p R_B)\sqrt{a^2 - x^2} \quad , \qquad (|x| < a) \tag{76-2}$$

Again by using the integral identity in Eq. (58), the phonon and phason stresses and electric displacements in the upper part of the cracked bi-materials under the permeable crack assumption can be obtained as

$$\sigma_{zy}^I(x,y) + i\sigma_{zx}^I(x,y) = T_0 \frac{z}{\sqrt{z^2 - a^2}} \tag{77}$$

$$H_{zy}^I(x,y) + iH_{zx}^I(x,y) = H_0 \frac{z}{\sqrt{z^2 - a^2}} \tag{78}$$

$$D_y^I(x,y) + iD_x^I(x,y) = (D_0 - R_0) + R_0 \frac{z}{\sqrt{z^2 - a^2}} \tag{79}$$

where, the constant R_0 is defined as

$$R_0 = (e_{15}^I - \lambda_{11}^I \alpha_{31}^p) R_A + (d_{15}^I - \lambda_{11}^I \alpha_{32}^p) R_B \tag{80}$$

It is clear that the phonon and phason stresses obtained for the permeable crack are the same as those for the impermeable crack, while the distribution of the electric displacement around the permeable crack is different from that for the impermeable crack. Also need to mention that the crack sliding displacements (CSDs) for the phonon and phason fields are different for the permeable and impermeable cracks, as shown in Eqs. (75, 76) for the permeable crack case and in Eqs. (53, 54) for the impermeable crack case, respectively.

4 Field intensity factors and energy release rate

After the full-field solution of the phonon and phason stresses and electric displacements have been obtained, the field intensity factors for the phonon and phason stresses can be determined as

$$K_{III} = \lim_{x \to a^+} \sqrt{2\pi (x-a)} \sigma_{zy}(x,0) = T_0 \sqrt{\pi a} \tag{81}$$

$$K_H = \lim_{x \to a^+} \sqrt{2\pi (x-a)} H_{zy}(x,0) = H_0 \sqrt{\pi a} \tag{82}$$

and the electric displacement intensity factor can be defined as

$$K_D = \lim_{x \to a^+} \sqrt{2\pi (x-a)} D_y(x,0) = D_0 \sqrt{\pi a} \tag{83-1}$$

for the electrically impermeable crack case, and

$$K_D = \lim_{x \to a^+} \sqrt{2\pi (x-a)} D_y(x,0) = R_0 \sqrt{\pi a} \tag{83-2}$$

for the electrically permeable crack case.

It is noted that the intensity factors defined in Eqs. (81, 82) hold for both the impermeable and permeable cracks, while the electric displacement intensity factor for the electrically impermeable crack is different from that for the electrically permeable crack, as shown in Eqs. (83).

The intensity factors for phonon stresses, phason stresses and electric displacements in the dissimilar one-dimensional hexagonal piezoelectric quasicrystals are decoupled for the impermeable crack. These results are in agreement with those for crack problems in a homogeneous piezoelectric medium. For the permeable crack case, the electric displacement intensity factor does not depend on the electric loading but only on the phonon and phason stresses applied.

It is noted that the field intensity factors defined above only demonstrate the information of the corresponding field quantities on the crack face plane, and therefore the decoupled field intensity factors could not show the coupling effects of the phonon, phason and electric fields. The field intensity factors are not used as the fracture criterion for the anti-plane crack problem in this work.

From the viewpoint of energy balance, the energy release rate for the interface crack between dissimilar 1D hexagonal piezoelectric quasicrystals is analyzed s follows. Assume that under applied loadings the crack tip advances along the crack plane from $x = a$ to $x = a + \delta$ ($\delta << a$), then the energy release rate at the crack tip $x = a$ per unit length during this process is

$$G = \lim_{\delta \to 0^+} \frac{1}{2\delta} \int_0^{\delta} \left\{ \sigma_{zy}(r,0) u(\delta - r) + H_{zy}(r,0) w(\delta - r) + D_y(r,0) \varphi(\delta - r) \right\} dr \qquad (84)$$

for the electrically impermeable crack.

The substitution of Eqs. (50-52) and (59-61) leads to the expression of the energy release rate for the interface crack between dissimilar 1D hexagonal piezoelectric quasicrystals as

$$G=\frac{1}{4}\{\mathbf{K}\}^T\left[[\mathbf{I}]+\left[\mathbf{M}^{\mathbf{II}}\right]^{-1}\left[\mathbf{M}^{\mathbf{I}}\right]\right]\left[\mathbf{M}^{\mathbf{I}}\right]^{-1}\{\mathbf{K}\} \tag{85}$$

$$\{\mathbf{K}\}=\{K_{III}\quad K_H\quad K_D\}^T=\begin{Bmatrix}K_{III}\\ K_H\\ K_D\end{Bmatrix}=\begin{Bmatrix}T_0\\ H_0\\ D_0\end{Bmatrix}\sqrt{\pi a} \tag{86}$$

$$[\mathbf{I}]=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\ \left[\mathbf{M}^{\mathbf{I}}\right]=\begin{bmatrix}C_{44}^I&R_3^I&e_{15}^I\\R_3^I&K_2^I&d_{15}^I\\e_{15}^I&d_{15}^I&-\lambda_{11}^I\end{bmatrix},\ \left[\mathbf{M}^{\mathbf{II}}\right]=\begin{bmatrix}C_{44}^{II}&R_3^{II}&e_{15}^{II}\\R_3^{II}&K_2^{II}&d_{15}^{II}\\e_{15}^{II}&d_{15}^{II}&-\lambda_{11}^{II}\end{bmatrix} \tag{87}$$

where K_D is defined in Eq. (83-1) for the electric displacement intensity factor for an electrically impermeable interface crack.

When the upper and lower parts of the bi-materials are the same, i.e., $\mathbf{M}^{\mathbf{II}}=\mathbf{M}^{\mathbf{I}}$, we can have the energy release rate for a crack in a homogeneous 1D hexagonal piezoelectric quasicrystal as

$$G=\frac{1}{2}\{\mathbf{K}\}^T[\mathbf{M}]^{-1}\{\mathbf{K}\} \tag{88}$$

where $\mathbf{M}=\mathbf{M}^{\mathbf{I}}=\mathbf{M}^{\mathbf{II}}$ for the homogeneous material case.

It is shown that the energy release rate of the interface crack is dependent on the applied loadings, the geometric size of the crack and the material properties, as shown in Eqs. (85-87). Therefore, the energy release rate can be used as the fracture criterion of the quasicrystal materials. If there is no phason field, the energy release rate obtained above can be reduced to that for crack problem in piezoelectric materials.

For the electrically permeable crack case, the energy release rate at the crack tip $x=a$ per unit length is defined as

$$G=\lim_{\delta\to 0^+}\frac{1}{2\delta}\int_0^\delta\{\sigma_{zy}(r,0)u(\delta-r)+H_{zy}(r,0)w(\delta-r)\}dr \tag{89}$$

and the integral calculation leads to the following expression as

$$G=\frac{1}{4}\{\mathbf{K}_2\}^T[[\mathbf{I}_2]-[\boldsymbol{\alpha}_2]][\mathbf{M}_2]^{-1}\{\mathbf{K}_2\} \tag{90}$$

$$\{\mathbf{K}_2\} = \{K_{III} \quad K_H\}^T = \begin{Bmatrix} K_{III} \\ K_H \end{Bmatrix} \tag{91}$$

$$[\mathbf{I}_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ [\boldsymbol{\alpha}_2] = \begin{bmatrix} \alpha_{11}^p & \alpha_{12}^p \\ \alpha_{21}^p & \alpha_{22}^p \end{bmatrix}, \ [\mathbf{M}_2] = \begin{bmatrix} C_{44}^I + e_{15}^I \alpha_{31}^p & R_3^I + e_{15}^I \alpha_{32}^p \\ R_3^I + d_{15}^I \alpha_{31}^p & K_2^I + d_{15}^I \alpha_{32}^p \end{bmatrix} \tag{92}$$

It is noted that during the derivation of the energy release rate, the following integral identity has been used

$$\int_0^\delta \sqrt{\frac{\delta - r}{r}} dr = \frac{\pi}{2} \delta \tag{93}$$

For the limiting case when the upper part and the lower part of the bi-materials are the same quasicrystal materials, there is the fact that

$$\begin{bmatrix} \alpha_{11}^p & \alpha_{12}^p \\ \alpha_{21}^p & \alpha_{22}^p \\ \alpha_{31}^p & \alpha_{32}^p \end{bmatrix} = -\begin{bmatrix} C_{44} & R_3 & 2e_{15} \\ R_3 & K_2 & 2d_{15} \\ e_{15} & d_{15} & -2\lambda_{11} \end{bmatrix}^{-1} \begin{bmatrix} C_{44} & R_3 \\ R_3 & K_2 \\ e_{15} & d_{15} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \tag{94}$$

where the superscripts "*I*" and "*II*" have been deleted, and the energy release rate for an electrically permeable crack in a homogeneous 1D hexagonal quasicrystal material can be obtained as

$$G = \frac{1}{2} \{K_{III} \quad K_H\} \begin{bmatrix} C_{44} & R_3 \\ R_3 & K_2 \end{bmatrix}^{-1} \begin{Bmatrix} K_{III} \\ K_H \end{Bmatrix} \tag{95}$$

It can be observed that the energy release rate for the electrically permeable interface crack between dissimilar one-dimensional quasicrystal materials is independent on the electric displacement loading applied, and this is different from the result for the electrically impermeable crack case, in which the electric displacement has a contribution to the energy release rate.

5 Numerical results and discussions

In this section, the analytic solution obtained in the previous section are used to evaluate the distribution of the stresses of phonon and phason fields, and electric displacements near the interface crack, and of particular interest are the CSDs and phonon and phason stresses along the bonded interface between the dissimilar 1D hexagonal piezoelectric quasicrystal materials.

The material properties of the dissimilar 1D hexagonal piezoelectric quasicrystals used in the following numerical calculation are taken as [27, 44]:

$$\begin{aligned} &C_{44}^{I}=3.55\times10^{10}(\mathrm{N/m^2}),\ e_{15}^{I}=17\ (\mathrm{C/m^2}),\ K_2^{I}=0.15\times10^{9}(\mathrm{N/m^2}),\\ &R_3^{I}=1.765\times10^{9}(\mathrm{N/m^2}),\ d_{15}^{I}=17\ (\mathrm{C/m^2}),\ \lambda_{11}^{I}=15.1\times10^{-9}(\mathrm{C^2/Nm^2}) \end{aligned} \tag{96}$$

for the upper part of the bi-materials, and

$$\begin{aligned} &C_{44}^{II}=5.0\times10^{10}(\mathrm{N/m^2}),\ e_{15}^{II}=-0.318\ (\mathrm{C/m^2}),\ K_2^{II}=0.3\times10^{9}(\mathrm{N/m^2}),\\ &R_3^{II}=1.2\times10^{9}(\mathrm{N/m^2}),\ d_{15}^{II}=-0.16\ (\mathrm{C/m^2}),\ \lambda_{11}^{II}=8.25\times10^{-12}(\mathrm{C^2/Nm^2}) \end{aligned} \tag{97}$$

for the lower part of the bi-materials.

Without loss of generality, the applied uniform phonon stress at far-field is taken as $T_0 = 5.0\ \mathrm{MPa}$. The crack sliding displacements (CSDs) for the interface crack between dissimilar 1D quasicrystal piezoelectric materials are plotted in Fig. 2 for different electric displacement loadings when the phason stress vanishes, i.e., $H_0 = 0$. It is observed that the CSDs in the upper half-space are different from those in the lower half-space due to the dissimilar material properties. The magnitude of CSDs increases as the electric displacement change from positive values to negative values. When there is no electric displacement loading applied to the dissimilar quasicrystal piezoelectric materials, the CSDs for the interface crack under different phason stresses at infinity are plotted in Fig. 3. It is shown that the effect of the phason stresses in the considered range have little effect on the crack sliding displacements.

When the phason stresses change from -1000Pa to +1000Pa, the magnitude of the CSD decreases accordingly, as shown in the enlarged picture. It is noted that at the time being, it is unclear how to assign and measure the phason loading. If the phason loading is assumed free at infinity, then the field intensity factor for phason stresses is zero, i.e., $K_H = 0$, as shown from Eq. (82), and the phason stresses at the crack tip do not show the singular behavior.

Fig. 4 displays the distributions of the electric displacements on the crack face plane ahead of the right crack-tip. As shown in Eqs. (61, 79), the electric displacements are singular near the crack tip, but the distribution of electric displacement for an impermeable crack is different from that for a permeable crack. It can be readily observed that at the far-field along the interface plane, the normalized electric displacement is one, which means that the undisturbed electric displacement is equal to the uniform electric displacement applied at infinity.

Fig. 5 shows the normalized energy release rate G/G_0 of an impermeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials under different electric displacement loadings at infinity. The value of G_0 is defined as the energy release rate when there is only phonon stress applied at infinity. The energy release rate decreases first as the phason stress increases from negative to positive values, and after it reaches a minimum, it increases as the phason stress increases. For different electric displacement loadings, the energy release rates are different. When the phason stress is of a particular value, the minimum of the energy release rate can be obtained. This result may be useful for the fracture design of the quasicrystal materials.

The normalized energy release rates G/G_0 of a permeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials are displayed in Fig. 6. Obviously, for this electrically permeable crack, the energy release rate is not dependent on the electric displacement loading. Comparing the energy release rates for the electrically impermeable and permeable cracks, it shows that the energy release rates for the permeable crack are positive, while the ERRs for the impermeable crack could be negative depending on the electro-mechanical loadings applied.

6 Conclusions

An interface crack between dissimilar quasicrystal materials with piezoelectric effect under anti-plane deformation and in-plane electric loading is investigated using integral transform method and singular integral equations technique. The mixed boundary value problems of the interface crack were reduced to solving Riemann-Hilbert problems with exact solutions. The full-field solutions for the phonon and phason stresses, and electric displacements in the dissimilar 1D hexagonal quasicrystal materials have been obtained analytically, and the crack sliding displacements (CSDs), field intensity factors and energy release rate are obtained. The electric displacements near the crack tips are different for the electrically impermeable and permeable cracks. When the phason stress is of a particular value, the minimum of the energy release rate can be obtained. The analytical results obtained for the interface crack problem may be useful for the structure design of the new quasicrystal composite materials.

Acknowledgement

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Appendix A

By considering the regular conditions at infinity, the constants $a_0^I, a_0^{II}, b_0^I, b_0^{II}, d_0^I, d_0^{II}$ in Eqs. (15-20) are defined as:

$$\begin{Bmatrix} a_0^I \\ b_0^I \\ d_0^I \end{Bmatrix} = \begin{bmatrix} C_{44}^I & R_3^I & e_{15}^I \\ R_3^I & K_2^I & d_{15}^I \\ e_{15}^I & d_{15}^I & -\lambda_{11}^I \end{bmatrix}^{-1} \begin{Bmatrix} T_0 \\ H_0 \\ D_0 \end{Bmatrix} \tag{A.1}$$

$$\begin{Bmatrix} a_0^{II} \\ b_0^{II} \\ d_0^{II} \end{Bmatrix} = \begin{bmatrix} C_{44}^{II} & R_3^{II} & e_{15}^{II} \\ R_3^{II} & K_2^{II} & d_{15}^{II} \\ e_{15}^{II} & d_{15}^{II} & -\lambda_{11}^{II} \end{bmatrix}^{-1} \begin{Bmatrix} T_0 \\ H_0 \\ D_0 \end{Bmatrix} \tag{A.2}$$

The following integral identities [43, 45] have been used in the derivation of the full-field solutions:

$$\int_0^\infty J_1(\xi c)\exp(iz\xi)d\xi = \frac{1}{c}\left[1 - \frac{z}{\sqrt{z^2 - c^2}}\right] \tag{A.3}$$

$$\int_0^\infty \xi^{-1} J_1(\xi c)\exp(iz\xi)d\xi = \frac{i}{c}\left[z - \sqrt{z^2 - c^2}\right] \tag{A.4}$$

where $z = x + iy$.

$$\int_0^\infty J_1(\xi c)\exp(-\xi y)\cos(x\xi)d\xi = \frac{1}{c}\left[1 - \frac{r}{\sqrt{r_1 r_2}}\cos\left(\theta - \frac{\theta_1 + \theta_2}{2}\right)\right] \tag{A.5}$$

$$\int_0^\infty J_1(\xi c)\exp(-\xi y)\sin(x\xi)d\xi = -\frac{1}{c}\frac{r}{\sqrt{r_1 r_2}}\sin\left(\theta - \frac{\theta_1 + \theta_2}{2}\right) \tag{A.6}$$

$$\int_0^\infty \frac{1}{\xi} J_1(\xi c)\exp(-\xi y)\cos(x\xi)d\xi = \frac{1}{c}\sqrt{r_1 r_2}\sin\left(\frac{\theta_1 + \theta_2}{2}\right) - \frac{r}{c}\sin(\theta) \tag{A.7}$$

$$\int_0^\infty \frac{1}{\xi} J_1(\xi c)\exp(-\xi y)\sin(x\xi)d\xi = -\frac{1}{c}\sqrt{r_1 r_2}\cos\left(\frac{\theta_1+\theta_2}{2}\right)+\frac{r}{c}\cos(\theta) \qquad \text{(A.8)}$$

where the polar coordinates $r,\ r_1,\ r_2,\ \theta,\ \theta_1,\ \theta_2$ are defined in Eqs. (62, 63).

The following identities for the integrals of some generalized functions are used in the derivation of the integral equations (40):

$$\int_0^\infty \sin(\xi x)\cos(\xi t)d\xi = \frac{x}{x^2-t^2} = \frac{1}{2}\left[\frac{1}{x+t}+\frac{1}{x-t}\right] \qquad \text{(A.9)}$$

The identity in Eq. (93) is actually a special form of the Beta function [46]

$$\int_0^1 \left(\frac{t}{1-t}\right)^q dt = \frac{q\pi}{\sin(q\pi)}, \qquad (|\mathrm{Re}(q)|<1) \qquad \text{(A.10)}$$

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Captions for Figures and Table

Figure 1. An interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

Figure 2. CSD for the interface crack under different electric loadings when $H_0 = 0$.

Figure 3. CSD for the interface crack under different phason stresses when $D_0 = 0$.

Figure 4. Normalized electric displacement $D_y(x,0)/D_0$ ahead of the interface crack.

Figure 5. Normalized energy release rate G/G_0 of an impermeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

Figure 6. Normalized energy release rate G/G_0 of a permeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

Table 1. The analogy relations between 1D hexagonal quasicrystal piezoelectric QCs and magnetoelectroelastic (MEE) materials.

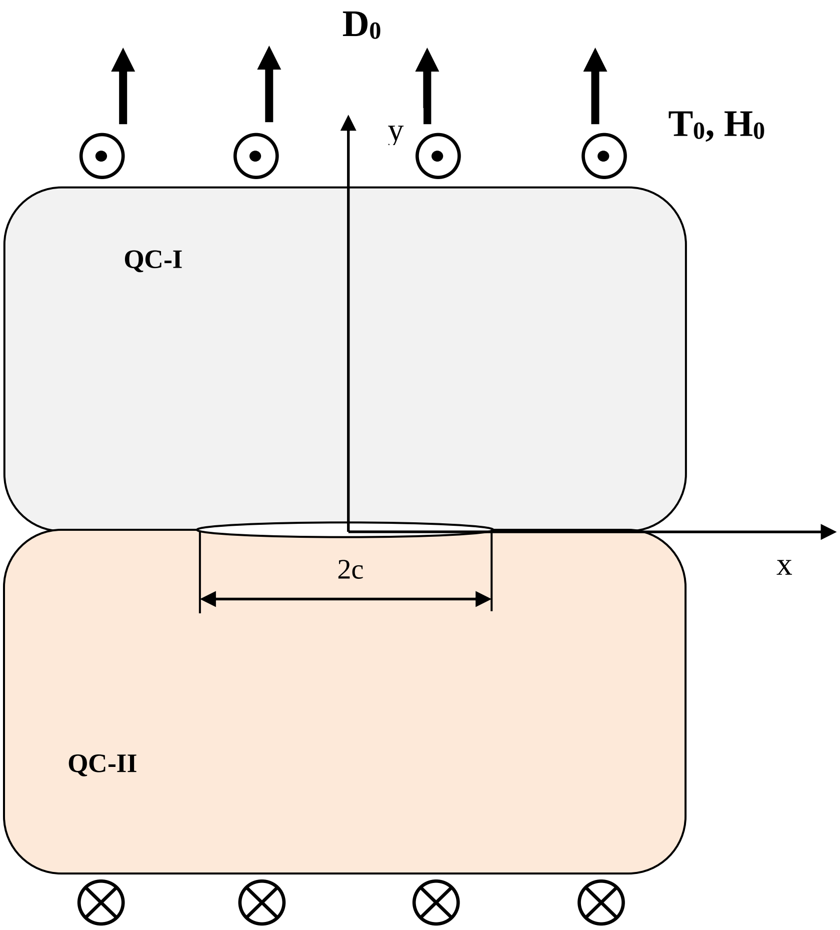

Fig. 1. An interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

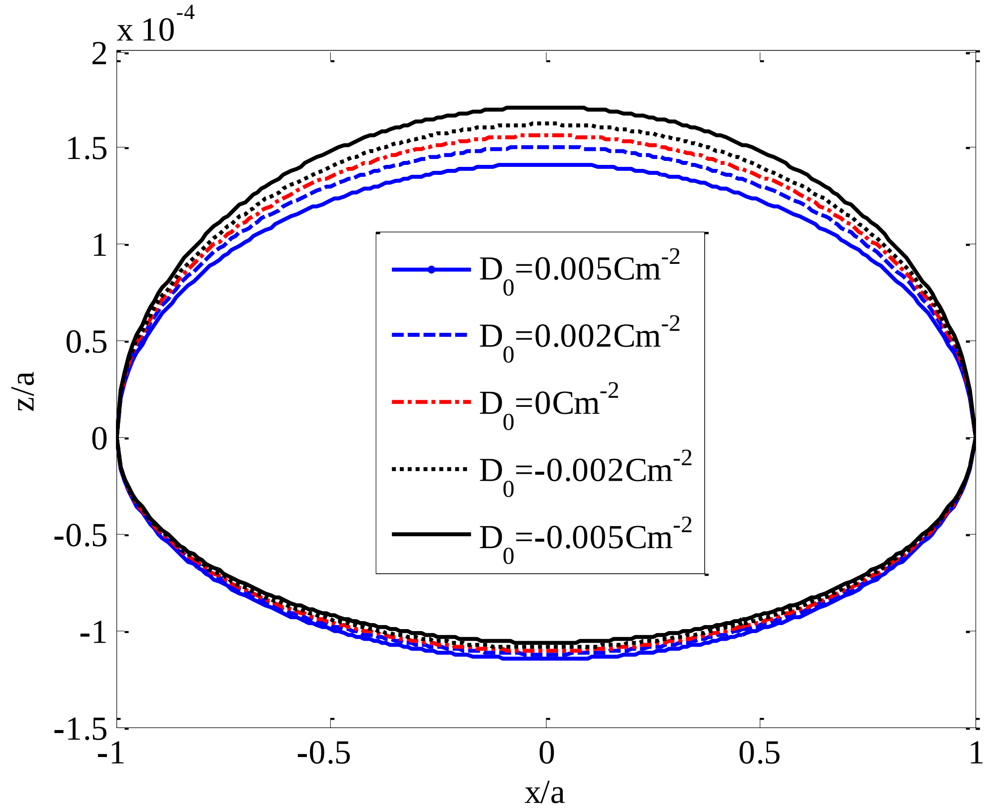

Fig. 2. CSD for the interface crack under different electric loadings when $H_0 = 0$.

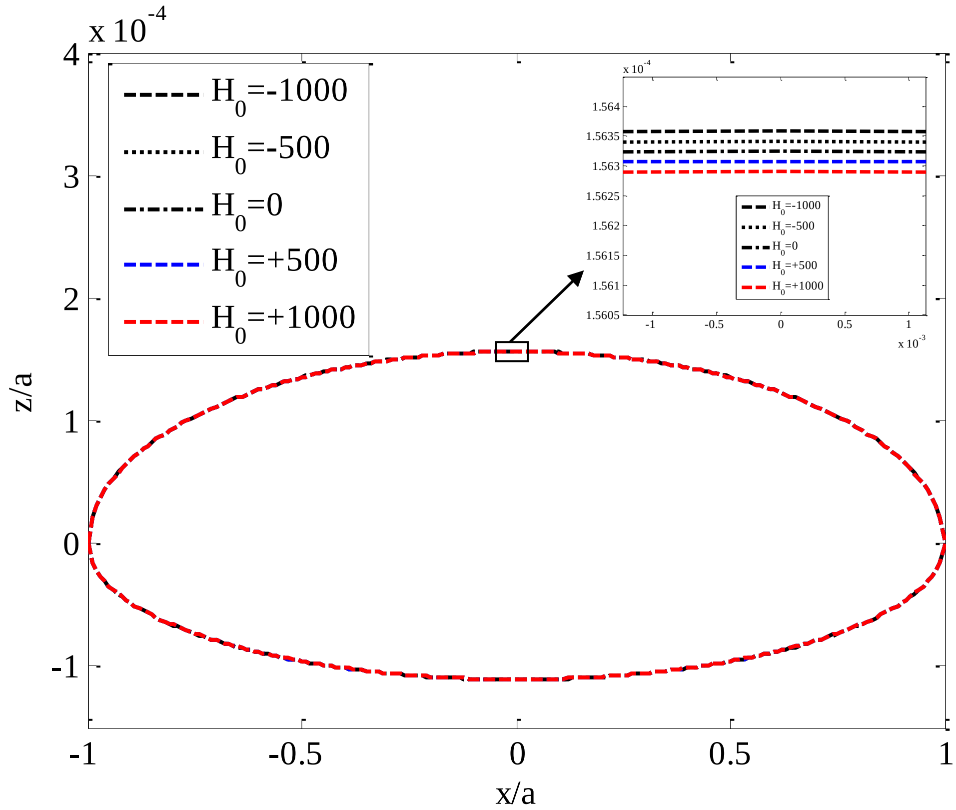

Fig. 3. CSD for the interface crack under different phason stresses when $D_0 = 0$.

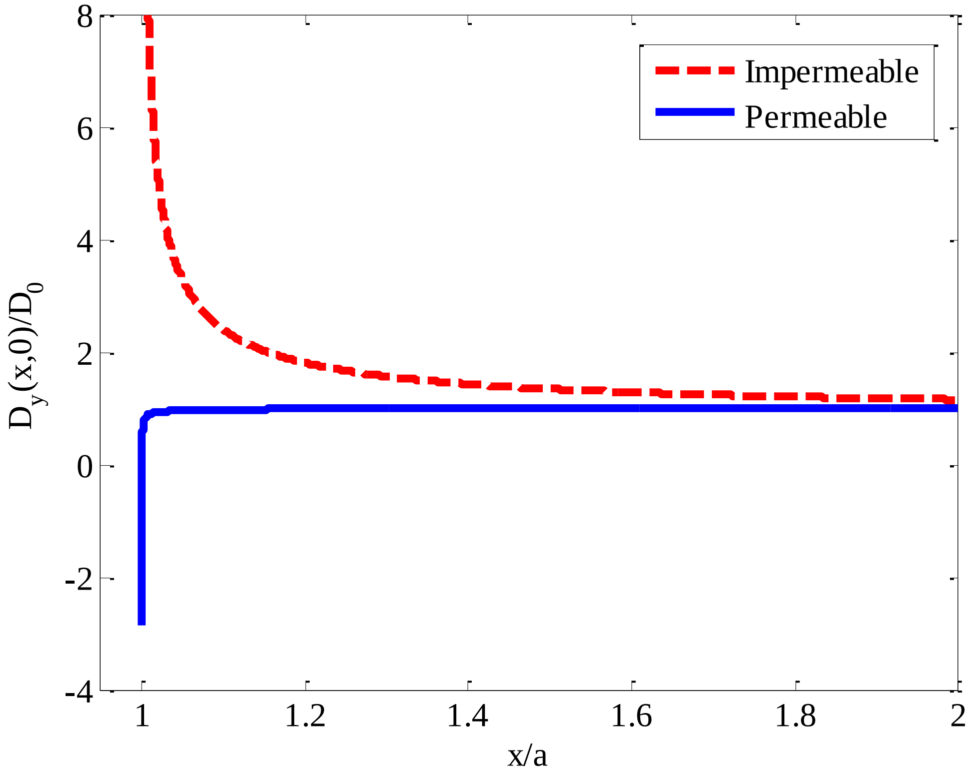

Fig. 4. Normalized electric displacement $D_y(x,0)/D_0$ ahead of the interface crack.

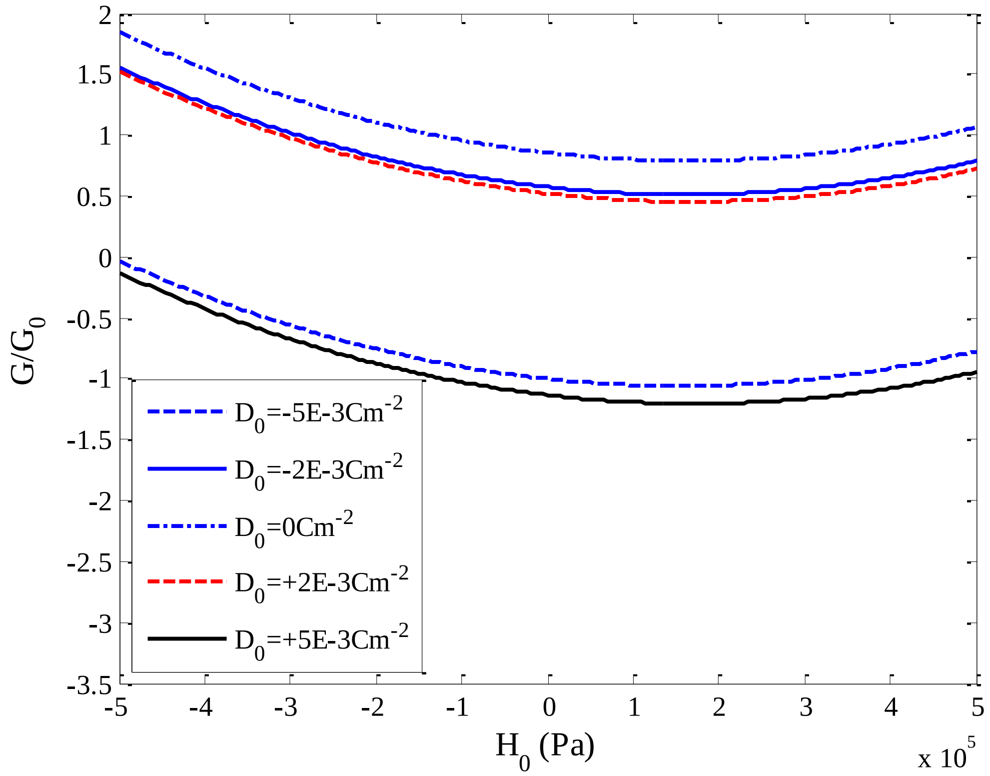

Fig. 5. Normalized energy release rate G/G_0 of an impermeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

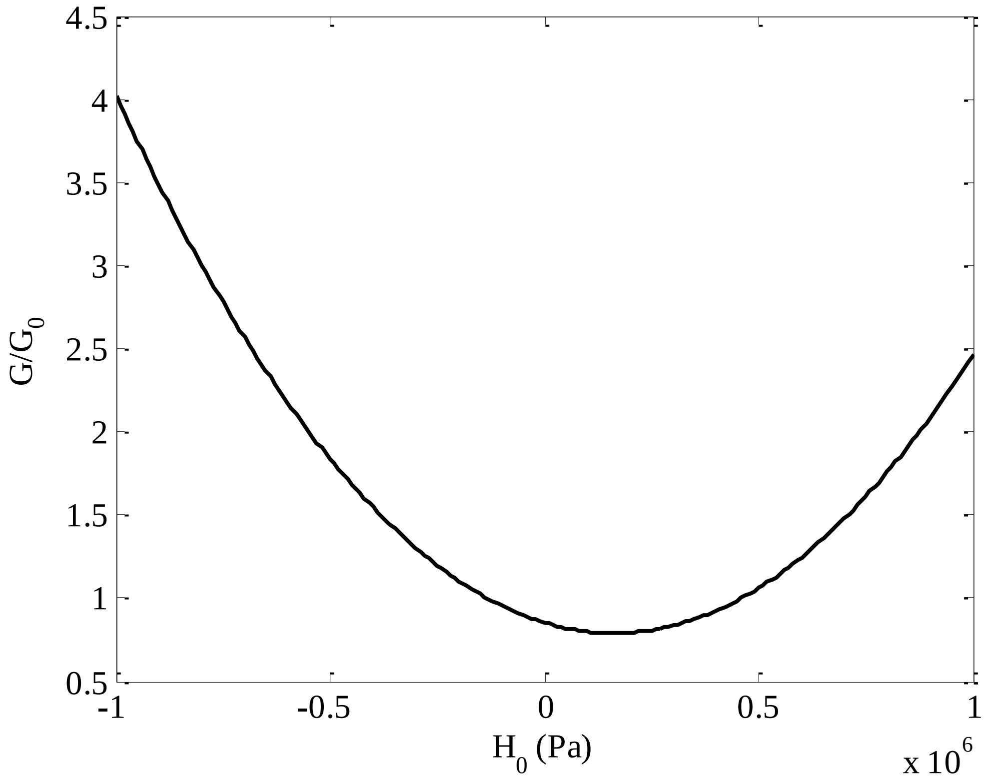

Fig. 6. Normalized energy release rate G/G_0 of a permeable interface crack between dissimilar 1D hexagonal quasicrystal piezoelectric materials.

Table 1. The analogy relations between 1D hexagonal quasicrystal piezoelectric QCs and magnetoelectroelastic (MEE) materials.

Material	1D QC	MEE
Extended displacements	u_z, w_z, ϕ	u_z, φ, ϕ
Extended stresses	σ_{zx}, σ_{zy} H_{zx}, H_{zy} D_x, D_y	σ_{zx}, σ_{zy} B_x, B_y D_x, D_y
Coefficients	C_{44}, e_{15}, λ_{11} R_3 d_{15} K_2	C_{44}, e_{15}, λ_{11} h_{15} d_{11} μ_{11}