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Frequency-Locked Loop Based Estimation of Single-Phase Grid Voltage Parameters

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***Abstract*—Estimation of Amplitude, instantaneous phase and frequency of single-phase grid voltage signal have been studied in this paper. The proposed approach uses a novel circular limit cycle oscillator (CLO) coupled with frequency-locked loop (FLL). Due to the nonlinear structure of the CLO, the proposed frequency adaptive CLO technique is robust against various perturbations faced in the practical settings *e.g.* discontinuous jump of phase, frequency and amplitude. Global stability analysis of the CLO and local stability analysis of the frequency adaptive CLO has been performed. Experimental results demonstrate the effectiveness of the proposed technique over a very recent technique proposed in the literature.**

I. Introduction

AMPLITUDE, phase and frequency are the fundamental parameters of single-phase grid signal. These fundamental parameters play significant role in various control, estimation, measurement and monitoring application in the context of smart grid [1], [2]. As a result, the estimation of these parameters is very important in the presence of various uncertainties (*e.g.* disturbance, non-smooth jumps) faced by the real-life electric power systems.

There are several widely accepted class of techniques available in the literature [3]–[16]. Frequency based techniques like discrete Fourier transform (DFT) or recursive DFT [5] can be used for parameter estimation. However, DFT suffers from spectral leakage while the recursive DFT may be subject to accumulation error. Least squares (LS) and its variants like weighted or recursive LS [8] are also found in the literature. LS can not handle singular matrix while for weighted LS exact knowledge of the weights are required. Kalman filter (KF) is another technique that can be used for the grid voltage parameter estimation [6]. However, prior knowledge of the covariance matrix is required. Moreover, it is computationally burdensome for low-cost real-time computing hardware.

Another popular class of technique is the well known phase-locked loop (PLL). PLL and its various variants [4], [7], [17] are widely used in the literature and successfully applied in various practical applications. PLLs are computationally simple, easy to tune and can provide accurate estimate of the time varying parameters. Classical PLL algorithms suffer in unbalanced condition and the fast dynamic response generally comes at a cost of accuracy. Enhanced PLL (EPLL) [4] overcome these limitations but at a higher computational cost than the classical PLL.

Recently two nonlinear techniques namely second order generalized integrator frequency-locked loop (SOGI-FLL) [7] and adaptive notch filter (ANF) [9] became very popular. These methods have good performance however they suffer during voltage swells and sags. Although these techniques are nonlinear in nature but they are initial condition dependent. The main fundamental building block of these methods rely on the linear harmonic oscillator. Linear oscillators are not structurally stable as such small perturbation may change the equilibrium. Moreover, the oscillation amplitude is initial conditions dependent. For global stability analysis of various variants of FLL based technique, please consult [13], [14]. To overcome the limitations of linear oscillator based methods, nonlinear oscillator as the main building block can be useful. This has been explored in [10]. However, [10] can't estimate the amplitude in the presence of discontinuous amplitude jumps.

In this article, we propose the application of a novel nonlinear circular limit cycle oscillator (CLO) to estimate the fundamental parameters of the single-phase grid voltage signal. The special feature of this oscillator is that it has an almost globally asymptotically stable closed orbit in the phase-plane. This makes the proposed technique global *i.e.* its convergence doesn't depend on the initial conditions. This is a considerable advantage over the existing literature. However, the circular limit cycle oscillator is not frequency adaptive. To overcome this limitation, we couple the CLO with the FLL technique proposed in [7]. The coupled system works on a wide range of initial conditions for the FLL part and for any initial conditions for the CLO part.

The rest of the article is organized as follows: Section II gives the details of the proposed technique while the experimental results are given in Section III. Finally, Section IV concludes this article.

II. Proposed Technique

A. Basics of Circular Limit Cycle Oscillator

In the phase-plane analysis of second-order nonlinear systems, some time an isolated periodic trajectory can be observed. This isolated periodic trajectory is known as limit cycle. The existence of a stable limit cycle implies sustained robust oscillation. This is a considerable advantage over linear harmonic oscillator which is not structurally stable. Circular-LCO (CLO) are a special type of LCO where the shape of

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the limit cycle is circular and independent of the oscillator parameters. In this work, we propose the following CLO model adapted from [18],

$$\dot{x} = y\omega_n \tag{1a}$$
$$\dot{y} = -x\omega_n - y\left(x^2 + y^2 - 1\right) \tag{1b}$$

where x and y are the state variables and ω_n is the angular frequency of the sustained oscillation. The solutions of the CLO are $x(t) = -\cos(\omega_n t), y(t) = \sin(\omega_n t)$. As the solution of $y(t)$ is similar to the single-phase grid voltage signal, CLO (1) can be considered as a proxy of the grid voltage signal. Then using proper feedback mechanism, any change in the grid voltage parameters, can be easily tracked using model (1).

A particular feature of CLO (1) is that it has an unstable equilibrium which is the origin and an almost globally asymptotically stable limit cycle which is the circle of radius 1 denoted by $x^2 + y^2 = 1$. It means, any trajectory that originates anywhere in the phase-plane, will converge to the circle of radius 1 except the one at origin. To verify this claim, the instability of the origin and the almost global asymptotic stability (A-GAS) of the limit cycle need to be proved.

Instability of the origin: Through linearization of eq. (1) at the origin, it can be found that the eigenvalues are given by $\frac{1}{2}\left(1 \pm \sqrt{1 - 4\omega_n^2}\right)$. This clearly demonstrates the instability as the real part is always positive.

A-GAS of the limit cycle: This property can be demonstrated by converting eq. (1) into polar coordinates. In polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$ and $\theta = \arctan(y/x)$. Then the dynamics of CLO in polar coordinates are given by:

$$\begin{aligned} r\dot{r} &= x\dot{x} + y\dot{y} \\ \dot{r} &= -r\sin^2(\theta)\left(r^2 - 1\right) \end{aligned} \tag{2}$$

$$\begin{aligned} \dot{\theta} &= (\dot{y}x - \dot{x}y)/r^2 \\ \dot{\theta} &= -\sin(\theta)\cos(\theta)\left(r^2 - 1\right) - \omega_n \end{aligned} \tag{3}$$

From eq. (2) and (3), we get that if $r = 1$, then $\dot{r} = 0$ and $\dot{\theta} = -\omega_n$, which implies a clockwise circular limit cycle of radius 1 with angular frequency ω_n like the grid signal angular frequency. The stability of the limit cycle can be proved by considering $V = 0.5\left(x^2 + y^2 - 1\right)^2$ as a Lyapunov function [19]. Then the proof of A-GAS property is straightforward and avoided here for the purpose of brevity.

B. Frequency Adaptive CLO for parameter estimation

Eq. (1) has A-GAS property which makes it a very suitable model for single-phase grid voltage parameter estimation. However, the angular frequency ω_n is constant. As such, the CLO will face difficulty in the presence of varying grid frequency. To overcome this issue, a frequency adaptation property needs to be introduced. For this purpose, frequency-locked loop (FLL) [7] can be very useful. CLO coupled with FLL and feedback mechanism to introduce the actual grid voltage is given by:

$$\dot{x} = y\omega \tag{4a}$$
$$\dot{y} = -\alpha(y - \chi)\omega - x\omega - y\left(x^2 + y^2 - 1\right) \tag{4b}$$
$$\dot{z} = \beta(y - \chi)x\omega \tag{4c}$$

where α, β are the positive gains, $\omega = \omega_n + 2\pi z$ is the angular frequency with ω_n denoting the nominal grid frequency, $y - \chi$ denotes the estimation error of the grid signal by the CLO and the grid voltage signal χ is given by:

$$\begin{aligned} \chi &= A_g \sin(\theta_g) \\ \dot{\theta}_g &= -\omega_g \end{aligned} \tag{5}$$

where A_g,θ_g and ω_g are the amplitude, instantaneous phase and frequency of the actual grid signal. Original CLO (1) oscillates at ω_n, however, the frequency adaptive CLO oscillates with the actual grid frequency ω_n, thanks to the feedback mechanism and the coupling of FLL. It is to be noted here that CLO (4) can't handle DC bias in its current form. To make the CLO DC bias robust, the DC bias can be considered as an additional state variable in (4) as used in [20]. This solves the problem of DC bias. Details are avoided here for space limitation.

In the steady state, $-\alpha(y - \chi)\omega \to 0$. Then the dynamics of eq. (4a) and (4b) are similar to the original CLO (1). As such the solutions are also similar i.e. $x(t) = -A_g\cos(\theta_g)$ and $y(t) = A_g\sin(\theta_g)$. Then the following formula gives the frequency, phase and amplitude of the actual grid signal χ:

$$\omega_g = \omega_n + 2\pi z \tag{6a}$$
$$\theta_g = \arctan\{y/(-x)\} \tag{6b}$$
$$A_g = x^2 + y^2 \tag{6c}$$

C. Local analysis of frequency adaptive CLO

To analyze the local stability of the frequency adaptive CLO, in this Section, we would resort to the polar coordinates as in Section II-B. Prior to that, to couple the dynamics of the grid signal (5) into the frequency adaptive CLO (4), let us introduce the instantaneous phase error as: $e_\theta = \theta - \theta_g$. Then the dynamics of (4) and e_θ in polar coordinates are given as:

$$\dot{r} = r\sin^2(e_\theta + \theta_g)(A^2 - 1) - \alpha e_{x_2-\chi}\omega\sin(e_\theta + \theta_g) \tag{7a}$$
$$\dot{e}_\theta = \omega_g - \omega + \frac{\sin\{2(e_\theta + \theta_g)\}}{2(A^2 - 1)^{-1}} - \frac{\omega\cos(e_\theta + \theta_g)}{r(\alpha e_{x_2-\chi})^{-1}} \tag{7b}$$
$$\dot{z} = \beta e_{x_2-\chi}\omega r\cos(e_\theta + \theta_g) \tag{7c}$$

where $e_{x_2-\chi} = r\sin(e_\theta + \theta_g) - A_g\sin\theta_g$. The desired equilibrium of eq. (7) is given by:

$$x^\star = \{r = A = A_g, e_\theta = 0, z = (\omega_g - \omega_n)/2\pi\}$$

Without losing any generality, for the sake of computational simplicity, we assume that $A_g = 1$. Then the system matrix of eq. (7) linearized at the equilibrium is given by:

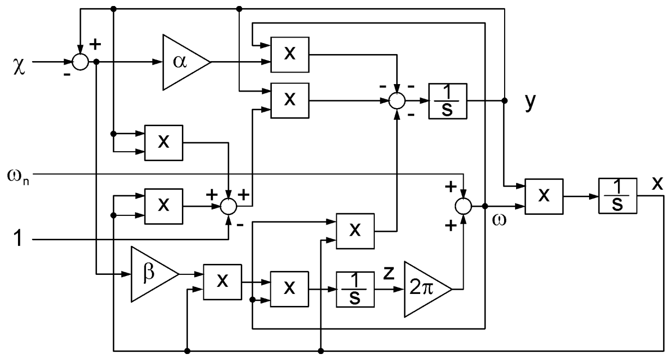

Figure 1. Block diagram of the frequency adaptive CLO.

$$J(x^{\star}) = \begin{bmatrix} (2+\alpha\omega_g)\{\cos^2(\theta_g - 1)\} & -\alpha\omega_g \sin(2\theta_g)/2 & 0 \\ -(2+\alpha\omega_g)\sin(2\theta_g)/2 & -\alpha\omega_g\cos^2(\theta_g) & -2\pi \\ \beta\omega_g \sin(2\theta_g)/2 & \beta\omega_g \cos^2(\theta_g) & 0 \end{bmatrix} \tag{8}$$

The eigenvalues of matrix $J(x^{\star})$ can be calculated from the following equation:

$$\lambda^3 + (2\sin^2(\theta_g) + \alpha\omega_g)\lambda^2 + 2\pi\beta\omega_g(1-\sin^2(\theta_g))\lambda + 4\pi\beta\omega_g\sin^2(\theta_g)(1-\sin^2(\theta_g)) = 0 \tag{9}$$

From eq. (9), it can be seen that for any $\alpha, \beta > 0$, $\mathrm{Re}(\lambda_J) \leq 0$ as $-1 \leq \sin(\theta_g) \leq 1$, which implies that the polynomial (9) is Hurwitz. This implies the local stability of the equilibrium point except the case when $\sin(\theta_g) = 0$. When $\sin(\theta_g) = 0$ only marginal stability can be guaranteed as in this case $\lambda_{1,2} = 0$ while $\lambda_3 < 0$. The block diagram of the frequency adaptive CLO is given in Fig. 1.

D. Tuning of the parameters α and β

In the stability analysis part, we have shown that for any $\alpha, \beta > 0$, the local stability is guaranteed. However, this doesn't give any idea on the selection of α and β. To select α and β, let us assume that $\theta_g = 0$. Then the polynomial (9) reduces to:

$$\lambda^2 + \alpha\omega_g\lambda + 2\pi\beta\omega_g = 0 \tag{10}$$

By comparing the polynomial (10) to the denominator polynomial of a second-order transfer function *i.e.*

$$\lambda^2 + 2\zeta\omega_0\lambda + \omega_0^2 = 0 \tag{11}$$

we found that $\omega_o = \sqrt{2\pi\beta\omega_g}$ and $\zeta = \frac{\alpha\omega_g}{2\omega_o}$. Then for a damping ratio of $\zeta = \frac{1}{\sqrt{2}}$, the following formula can be a good starting point for selecting α and β:

$$\alpha = \sqrt{\frac{4\pi\beta}{\omega_g}} = \sqrt{\frac{2\beta}{f_g}} \tag{12}$$

where experience showed that $\beta \leq f_g$.

Table I
SUMMARY OF THE RESULTS.

Test condition ⟶	+5Hz. freq.		$+40^{\circ}$ phase	
Characteristics↓	CLO	EPLL	CLO	EPLL
Settling time ± 0.1Hz. (cycles)	≈ 1	≈ 2.8	≈ 2	≈ 3.35
Max. freq. error (Hz.)	5	5	5.3	4.3
Max. phase error	9°	9.5°	40°	40°
Max. amp. error (p.u.)	0.07	0.04	0.12	0.05
Test condition ⟶	-0.2p.u. amp.		$+0.1$p.u. DC bias	
Characteristics↓	CLO	EPLL	CLO	EPLL
Settling time ± 0.1Hz. (cycles)	≈ 0.5	≈ 2.5	≈ 1	≈ 1
Max. freq. error (Hz.)	0.85	0.8	0.68	0.41
Max. phase error	6.6°	6.6°	3.5°	4.7°
Max. amp. error (p.u.)	0.2	0.2	0.091	0.07

III. EXPERIMENTAL STUDY

DC bias robust enhanced phase-locked loop (EPLL) [20] has been selected as the comparison technique. Proposed frequency adaptive CLO and EPLL are implemented experimentally using dSPACE 1104 board with a sampling frequency of 8KHz and discretized using the third-order Adams-Bashforth method. The parameters of the proposed technique are selected as $\alpha = \sqrt{2}$ and $\beta = 20$. EPLL parameters are selected as: $\mu_0 = 85$; $\mu_1 = 200$, $\mu_3 = 400$ and $\mu_2 = 20000$. For the DC bias test, we modified the CLO following the ideas given in [20] and used $\mu_0 = 85$ as the parameter for the DC bias estimation part like EPLL. In other cases, DC bias part was turned-off in both techniques for the sake of fare comparison. Following [16], step changes in a) frequency. b) ref. magnitude, c) phase, and d) DC offset are considered as test scenarios. Details are given in Table I.

Experimental results for case a) and d) are shown in Fig. 2. Case b) and c) are avoided here due to space limitation. Summary of the results for four cases are given in Table I. Except the DC bias case, CLO significantly outperformed EPLL in terms of settling time. Moreover, in terms of peak phase or frequency error, CLO performed better than EPLL most of the times. However, in terms of peak amplitude error, CLO always lagged behind EPLL. This is partly related to the gain tuning. Peak amplitude error is mostly determined by the rapidity of the FLL part *i.e.* gain γ. Lower value of γ increases the convergence time while decreases the peak amplitude error. As such this is a trade-off for the practising engineers.

IV. CONCLUSION

This article demonstrated a nonlinear technique for phase, frequency and amplitude estimation of the single-phase grid voltage signal. The nonlinear technique used a novel circular limit cycle oscillator in conjunction with the well known frequency-locked loop. By transforming the closed-loop system into polar coordinates, global and local stability analysis have been performed for the CLO and CLO-FLL respectively. Comparative analysis with EPLL has been performed using four challenging test cases involving non-smooth amplitude, phase, frequency and DC bias jump. The proposed technique outperformed the comparison technique. The proposed approach didn't consider harmonics which could be a possible

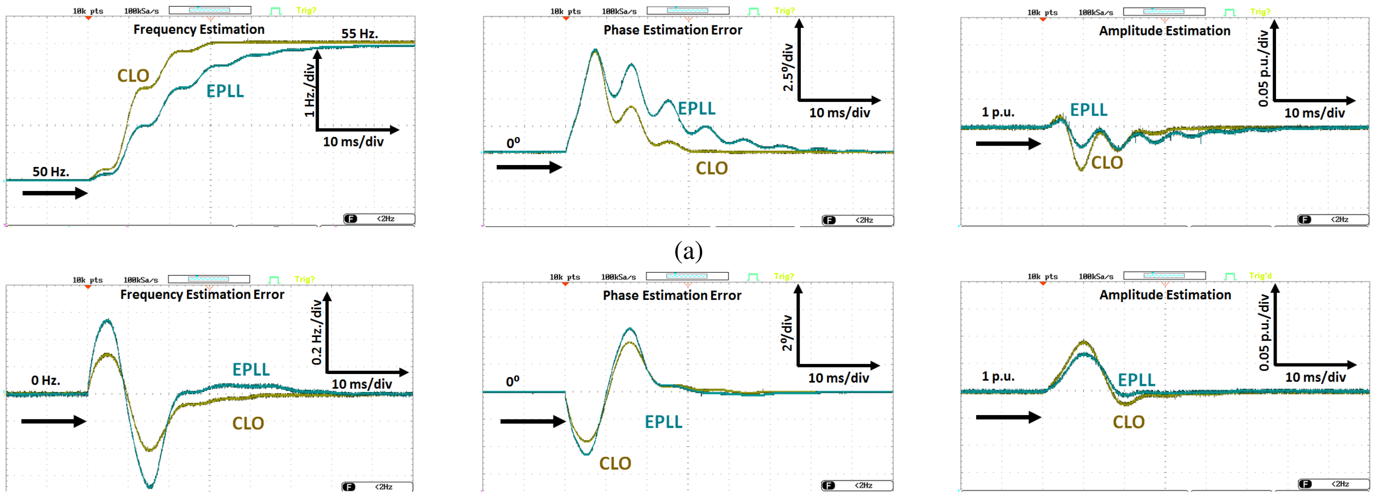

Figure 2. Experimental Results. (A): change in frequency from 50Hz. to 55Hz. (B): change of DC magnitude from 0 to 0.1p.u. Arrow indicates the time when change happens.

future research topic. The application of the proposed technique to various interesting areas *e.g.* grid synchronization is planned to be done in the future. In this work, we haven't considered rigorous mathematical analysis of the FLL part. This is also going to be considered in a future work.

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