

# System identification and control of the broken river

Foo, M., Ooi, S. K. & Weyer, E.

Author post-print (accepted) deposited by Coventry University's Repository

**Original citation & hyperlink:**

Foo, M, Ooi, SK & Weyer, E 2014, 'System identification and control of the broken river' IEEE Transactions on Control Systems Technology, vol. 22, no. 2, pp. 618-634.

<https://dx.doi.org/10.1109/TCST.2013.2253103>

DOI 10.1109/TCST.2013.2253103

ISSN 1063-6536

Publisher: Institute of Electrical and Electronics Engineers (IEEE)

**© 2013 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.**

Copyright © and Moral Rights are retained by the author(s) and/ or other copyright owners. A copy can be downloaded for personal non-commercial research or study, without prior permission or charge. This item cannot be reproduced or quoted extensively from without first obtaining permission in writing from the copyright holder(s). The content must not be changed in any way or sold commercially in any format or medium without the formal permission of the copyright holders.

This document is the author's post-print version, incorporating any revisions agreed during the peer-review process. Some differences between the published version and this version may remain and you are advised to consult the published version if you wish to cite from it.

# System Identification and Control of the Broken River

Mathias Foo, Su Ki Ooi, and Erik Weyer, *Member, IEEE*

**Abstract**—In this paper control system designs are proposed for the Broken River in Victoria, Australia. The aim of the control system is to improve water resource management and operation for the benefit of irrigators and the environment. Both centralised and decentralised control schemes are considered. The decentralised scheme consists of a number of PI and I controllers, while the centralised scheme is a model predictive controller (MPC). The controllers are designed based on simple models obtained using system identification methods. In a realistic simulation scenario, the control systems compared very favorably with current manual operation offering increased operational flexibility with a significant potential for substantial water savings, improved level of service to irrigators and improved environmental benefits.

**Index Terms**—Control systems, MPC, System Identification, Modelling, River systems, Environmental systems.

## I. INTRODUCTION

Water is a scarce resource in many parts of the world, and sensible use and good management of the available water resources are becoming increasingly important. In Australia, after a decade of droughts in the southern states, there is a push to explore new farming practices and strategies for managing the water resources in order to prepare for a drier future. The water has to be shared among many users, and recent legislative changes have given water for environmental purposes higher priority compared to water for irrigation [1]. Water resource management at the basin scale is a complex problem and calls for an interdisciplinary approach including agricultural science, engineering, ecology, hydrology, economics, social sciences, etc. The research described in this paper is part of the Farms, Rivers, and Markets (FRM) project, which was initiated by Uniwater, a joint research initiative by The University of Melbourne and Monash University in response to the above challenges [2].

As the name suggests, the project consists of three key components. The aim of the Farms project is to explore new farming practices and how various sources of water can be used in flexible combination to make farming operations more resilient. The Rivers project is concerned with developing a better understanding of the hydrology and the development of systems for managing the water which are capable of handling

the needs of irrigators and the environment in a cooperative way. The Market project aims at developing new water products and services with appropriate legislation better suited to future requirements from irrigators and the environment.

Modelling and control have important parts to play in water management since well designed control and monitoring systems will allow for a more efficient distribution of water without creating undesirable environmental or ecological conditions along the river. Potential benefits include accurate and timely delivery of water to irrigators and the environment, water can be ordered by irrigators on a shorter notice leading to more flexible farming, improved environmental outcomes, and a larger amount of water can be commanded for targeted use, e.g. for irrigation or flooding of wetlands or traded out of the catchment.

The application of modelling and control to improve the operations of irrigation channels in Australia has been a large success (see e.g. [3] - [7]), and it is hoped that similar outcomes also can be achieved for the operations of rivers. However, there are major differences between rivers and irrigation channels which mean that solutions in one area cannot be blindly copied to the other. In a river, there are much larger distances between the points where the flows can be regulated, and hence the time delay in the models of the river reaches are much larger. Moreover, compared to an irrigation channel, which often can be viewed as a sequence of storages connected by regulation gates, there are much less possibilities for in-stream storage of water in a river. The control objectives and the operational requirements are also different. For an irrigation channel there are few or no ecological or environmental constraints (algae growth being one exception [8]), while they can be very important for a river. For example, the flow in an irrigation channel can be reduced to zero for a long period of time, but this is clearly not possible for a river. For an irrigation channel one would typically aim at minimising releases subject to satisfying demand. This may partly be an objective for a river, but minimising releases may mean that a downstream river is starved of water. For a river, due to the fewer points where the flow can be regulated, there are much larger time delays between the points of supply and the points of demand, and this makes it much more difficult to satisfy demand for water on a short notice. On the other hand, fewer regulation points means that error propagation phenomena as observed in an irrigation channel (see e.g. [3] or [9]) where setpoint errors and control actions amplify as the effect of disturbances propagate through a network of channels, is less of a concern for a river.

In this paper we consider modelling and control of a river

M. Foo is with Asia Pacific Center for Theoretical Physics (APCTP), Hogil Kim Memorial Building #501 POSTECH, San 31, Hyoja-dong, Nam-gu, Pohang, Gyeongbuk, 790-784, Korea e-mail: mfoo@apctp.org

S. K. Ooi (Corresponding Author) is with NICTA VRL, Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, VIC 3010, Australia e-mail: suki@unimelb.edu.au

E. Weyer is with the Department of Electrical and Electronic Engineering, The University of Melbourne, Parkville, VIC 3010, Australia e-mail: ewey@unimelb.edu.au

for the purpose of improving the service to the irrigators and the environment. The focus is on the Broken River in Victoria, Australia which has been used as a case study for the FRM project, and it is a well studied river and catchment from many different perspectives (hydrology, agriculture, ecology, etc.). There are a number of works on control of rivers with the focus on optimisation of the operation of hydro-electric power plants, minimising the energy cost of pumping or ensuring that the river is navigable etc. (see e.g. [10] - [17]). Different from the above works, this work focuses on improved management of water resources to the benefit of irrigators and the environment.

Rivers are traditionally modelled using the Saint Venant equations, but they are difficult to use for control design since they are nonlinear partial differential equations. In this paper we use system identification techniques applied to measured data from the Broken River to derive simple models for river reaches useful for control design. Moreover, we also validate the Saint Venant equations models against measured data, and these models are used in the simulation study.

Two different types of control system are considered. The first one is a decentralised scheme consisting of a number of PI and I controllers. This is a relatively simple scheme, and it can be implemented without a major infrastructure upgrade. It may therefore more easily gain acceptance from the management and the operators while providing substantially improved performance compared to the current manual operations. The second control scheme is a centralised MPC controller. Due to a number of constraints on the operations (e.g. minimum environmental flows, maximum allowed daily flow changes, minimum and maximum water levels), MPC is ideally suited for this problem due to its ability to handle constraints (see e.g. [18]), and as shown in this paper it does perform better than the decentralised schemes.

One of the aims of the FRM project has been to demonstrate the benefit of automatic control systems for the operations of the Broken River. However, as is often the case when a control system is proposed for a new application, all the infrastructure required in order to implement the control system is not available, and an investment case needs to be made for the improved infrastructure, in this case an improved Supervisory Control and Data Acquisition (SCADA) systems and more importantly, changes to the civil engineering infrastructure (gates and weirs) which are very costly. The performance of the control system is therefore assessed through a year long simulation scenario based on recent historical data suitably adjusted for known and predicted future trends, e.g. buy back of water rights from the irrigators (known) and changed demand patterns due to new farming practices better suited for reduced availability of water due to an anticipated drier future.

The paper is organised as follows. In the next section a description of the Broken River is given. The control objectives are described in detail in Section III, before the system identification models used for control design are introduced in Section IV. Section V covers control design, while the realistic year long simulation scenario is presented in Section VI. The performance of the control systems and the benefits compared to current manual operations are discussed in Section VII.

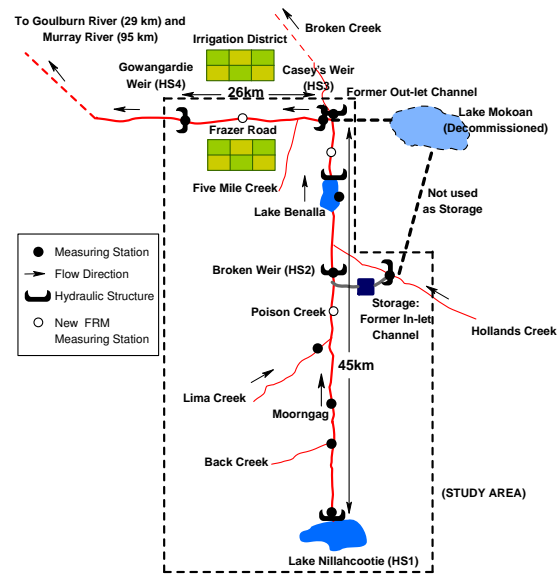


Fig. 1. Approximate top view of the Broken River (not to scale).

Concluding remarks are given in Section VIII.

## II. BROKEN RIVER

Figure 1 shows a map of the Broken River. The whole Broken basin covers  $7724\text{km}^2$  of catchment area, and it is part of the Murray Darling Basin. The river originates from Lake Nillahcootie which is a dammed lake which can store  $40 \times 10^6\text{m}^3$  (40GL) of water. It flows north for about 45km before turning west at Casey's Weir and flows into the Goulburn River after another 55km. The Goulburn River flows into the Murray River, Australia's longest river. The two main tributaries are Lima Creek and Hollands Creek which flow into the upper part of the Broken River. Broken Creek flows out from the river just upstream of Casey's Weir. The study area we consider is from Lake Nillahcootie (HS1) to Gowangardie Weir (HS4) and covers  $2500\text{km}^2$  of the catchment area.

Within the study area there are four weirs, Broken Weir, Benalla Weir, Casey's Weir and Gowangardie Weir. The last three are free overfall weirs (see photo in Figure 2) where the flow cannot be regulated. The places where the flow in the river can currently be regulated are at the out-let of Lake Nillahcootie and at Broken Weir. The flow into Broken Creek just upstream of Casey's Weir can also be regulated. Until a few years ago Lake Mokoan which was an artificially constructed lake, also contributed to the flow in the lower Broken. Lake Mokoan is now decommissioned, and the area is being returned to a wetland. However, the former in-let channel to Lake Mokoan will be used as an off-stream storage for water, and it is currently under construction. The capacity of the storage is  $3 \times 10^5\text{m}^3$  (300ML), and both the flow into the storage upstream of Broken Weir and the flow out of the storage through Hollands Creek into the Broken River will be regulated. Note that the decommissioning of Lake Mokoan means that it is not possible to compare a simulated performance of a control system against historical records



Fig. 2. Picture of Casey's Weir which is a free overfall weir with a fish ladder (seen in the foreground).

since the water previously supplied from Lake Mokoan must now be supplied from Lake Nillahcootie.

The water levels are measured at Lake Nillahcootie, Back Creek, Moorngag, Lima Creek, Broken Weir, Hollands Creek, in Lake Benalla, Casey's Weir, Broken Creek and Gowan-gardie Weir (indicated by the black circles shown in Figure 1). After the completion of the off-stream storage in the former in-let channel to Lake Mokoan, the levels and the flows in and out of the storage will also be measured. Note that all flow "measurements" are water level measurements which have been converted to flows using rating curves.

The environment is protected through minimum flow requirements ranging from natural flow to  $0.2894 \text{ m}^3/\text{s}$  (25ML/day) on a fortnightly average at a number of locations along the river [19]. Irrigators pump water directly from the river, and most of the demand for water is downstream of Casey's Weir. Hence there is a large distance between the point of supply (Lake Nillahcootie) and where the demand is. The system is a demand driven system<sup>1</sup>, and under current practice irrigators must order water four days in advance regardless of whether they are located close to or far from Lake Nillahcootie. All the water orders from irrigators and the environment are added up and another  $0.2315 \text{ m}^3/\text{s}$  (20ML/day) to  $0.3472 \text{ m}^3/\text{s}$  (30ML/day) is added as a safety margin before the flow is manually released from Lake Nillahcootie taking into account the approximate travel time to the place where the water is needed.

There is obviously a potential for improved operational efficiencies, e.g. in terms of accurate and timely delivery of water, by applying feedback control. However the performance of the control system will necessarily be limited by the long time delay between the point where the flow can be regulated and the point of demand. One possibility in order to overcome this difficulty is to install regulation gates at Casey's Weir which is closer to where the bulk of the demand is. In order for new regulation gates to be effective, it is important that there is storage upstream of the gates such that additional water is available if the flow is increased above natural flow or there is room to store backed up water if the flow is reduced below natural flow. Fortunately, the weir pool upstream of Casey's Weir is quite large and can act as an in-stream storage.

In this work we will consider control systems with and

<sup>1</sup>Roughly stated: in a demand driven system the irrigators decide when they want water while in a supply driven system water is delivered on a roster determined by the water authority.

without regulation at Casey's Weir. Control systems without regulations at Casey's Weir can be implemented without costly upgrades to the civil engineering infrastructure, but the improved performance with regulation at Casey's Weir may justify the investment in regulation gates and upgrades to the weir.

### III. CONTROL OBJECTIVES

The purpose of the control system considered here is to improve water management in dry periods. It is not intended for flood prevention. The control objectives are discussed in more details below.

#### *Demand from the irrigators.*

The main objective is the accurate and timely delivery of ordered water. Irrigators order water four days in advance, but a reduction in ordering time will allow for more flexible farming practices and increased productivity since decision about watering can be delayed until more information (e.g. weather forecast or soil moisture content) is available. If the use of an automatic control system allows shorter ordering times, it would be an important improvement.

#### *Environmental objectives.*

Generally, one would like the flow in the river to be as close to natural flow as possible. However, in a semi regulated river such as the Broken River with a dam at the upstream end, it is difficult to characterise what natural flow is in a way useful for control. For example, a river may flood a certain area or cease to flow once every 20 years on average, but incorporating this randomness among the control objectives may not be sensible. Moreover, since the demand for irrigation water is high in the drier periods of the year when the natural flow is low, it is bound to be difficult to satisfy demand with "natural" flow. Parts of the ecological research under the FRM project have therefore been concerned with what aspects of natural flow are important for good environmental outcomes. Research has been undertaken in investigating how different species react to changed flow conditions, and to develop appropriate guidelines, (see [20]). This is still work in progress, but the following objectives have emerged.<sup>2</sup>

- Minimum average fortnightly environmental flows at several locations are stipulated in the current entitlement, [19].
- Avoid large day to day variations in the flows.
- From early spring to mid summer it is important to maintain slackwater<sup>3</sup> pockets along the river which is important for spawning and survival of larval and juvenile fish. Hence during this period the maximum flow in the river should be limited.

The above objectives are soft limitations in the sense that short periods of violation are allowed. For example, under natural occurring flood events the flow will violate limits on day to day variations.

<sup>2</sup>One future possibility is the creation of an "environmental manager" position. The environmental manager will order water on behalf of the environment, e.g. for flooding of a wetland, in the same way as irrigators order water for their crops.

<sup>3</sup>Here, slackwater is understood as small, shallow areas of still water which exhibit little or no discernible current.

### *Downstream users and the Goulburn River.*

Efficient control of the Broken River may reduce the amount of water ending up in the Goulburn River, and this may not be desirable since additional water may then have to be supplied to the Goulburn River from elsewhere. It will therefore be assumed that a certain amount of water has to be delivered to the Goulburn River.

### *Water releases and water levels.*

Subject to satisfying demand from the irrigators, the downstream users and the environment, the releases from Lake Nillahcootie should be kept small, increasing the possibility that there will be enough water for all users in the future. Furthermore, the water levels in weir pools and lakes should be in certain ranges. The water levels in the weir pools are important since they affect the ability to supply water and avoid flooding as weir pools act as storages of water. Lake Benalla is used for recreational purposes and for that reason the water levels in the lake should be within certain limits. The above water level limitations are also treated as soft limitations.

### *A. Summary of the control objectives*

Based on the discussions above the following control objectives are used in this paper.

- a) Satisfy the demand for water from the irrigators and the environment.
- b) Maintain the volume of the off-stream storage at 50% of full capacity.
- c) Maintain the flows over Broken Weir and Casey's Weir above  $0.2546 \text{ m}^3/\text{s}$  (22ML/day) to satisfy environmental minimum flow requirements.
- d) Maintain the water levels at Broken Weir, Lake Benalla and Casey's Weir within  $\pm 15\text{cm}$  of the setpoints of 175.15mAHD, 169.87mAHD and 163.07mAHD respectively (mAHD is meter Australian Height Datum, which is relative to mean sea level) which correspond to water depths of 2.15m, 2.25m, and 2.00m respectively.
- e) Maintain the flow over Gowangardie Weir at a desired setpoint in order to satisfy downstream demands (including water to the Goulburn River) and the environmental minimum flow requirements.
- f) Release as little water from Lake Nillahcootie as possible.
- g) Reduce the water ordering time for irrigators.
- h) Keep the flow from early spring to mid summer under  $1.3889 \text{ m}^3/\text{s}$  (120ML/day) in order to create slackwater pockets.
- i) Limit the average daily flow to be between 0.76 and 1.50 of the previous day's flow in all reaches [21].

The setpoint for the off-stream storage is selected somewhat arbitrary. It will also act as a rain rejection storage, and as it is currently under construction, the most suitable setpoint is not known yet.

Future market mechanisms for ordering and offering irrigation water may necessitate a supervisory system above the control system which decides the flows and volumes that can be offered to the irrigators. For example, assume the flow in a reach is  $0.2546 \text{ m}^3/\text{s}$  (22ML/day) and that this is also the

minimum environmental flow. The maximum flow the next day is then  $0.3819 \text{ m}^3/\text{s}$  (33ML/day) since the flow should be less than 1.5 times the previous day's flow, and only a flow of  $0.1273 \text{ m}^3/\text{s}$  (11ML/day) can be offered to irrigators on a one day notice. Hence, an order of a flow of  $0.1736 \text{ m}^3/\text{s}$  (15ML/day) can only be accepted on two or more days notice. The design of a supervisory system that decides what type of flows and volumes can be offered is beyond the scope of this paper.

### *B. Evaluation of the control system*

Future changes such as changed demand patterns, refined environmental guidelines and trading of water are not likely to significantly change the control objectives. These changes will affect setpoints, external input signals and upper and lower limits on constraints, but the overall objectives will stay more or less the same. How well the control system performs and the relative benefit compared to manual operation may be affected by these changes, and hence anticipated future trends should be taken into account when the control system is evaluated.

Current practice is that once an irrigator has placed an order for water and it has been approved, then the irrigator can pump water from the river regardless of whether sufficient water has been released. For the evaluation of the control system this practice will be followed, and a failure to supply sufficient water will manifest itself in water levels in weir pools going below minimum levels or that changes in the daily flows are too large.

The efficiency of the control system can be measured in terms of excess water which is the amount of water leaving the study area at Gowangardie Weir which is not required for environmental purposes, downstream users or the Goulburn River. Note that excess water is not "wasted" water since this water may come to good use in the Goulburn River or the Murray River.

## IV. MODELS

In this section, we introduce the models of the Broken River used for simulation and control design. For simulations the full Saint Venant equations are used while delay and integrator delay models are used for control design. By calibrating and testing the models against real data it is shown that both the Saint Venant equations and the delay and integrator delay models obtained using system identification techniques represent the dynamics of the reaches of the Broken River very well. In particular it is shown that the simple delay and integrator delay models capture the dynamics relevant for control (see also [22]). As some of the infrastructure is not yet in place, (e.g. the off-stream storage) obviously no operational data exists from these locations<sup>4</sup>. Hence in order to derive a complete model of the Broken River to be used for control design, we have also simulated the Saint Venant equations (although the storage is not yet completed, the physical dimensions are known), and used the simulated

<sup>4</sup>Even though the river is well instrumented, the old measurements are of varying quality, and many of the new sensors installed as part of the FRM projects were damaged in floods.

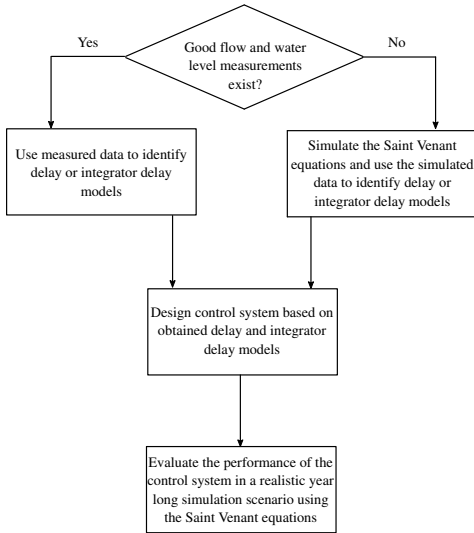


Fig. 3. Modelling, design and evaluation methodology.

data to obtain delay and integrator delay models using system identification techniques.

The modelling methodology employed in this paper is summarised in Figure 3. The approach of identifying low order dynamic models using data generated by a physically based simulation model (as in the right branch of Figure 3) is also known as meta-modelling or dynamic emulation modelling ([23], [24]), and our approach of using data from the Saint-Venant equations to identify delay and integrator delay models can be viewed as a data based dynamic emulation modelling approach to obtain a nominal emulation model ([24]).

In Section IV-B below we derive models from measurement data following the left path of Figure 3, while in Section IV-C we derive models for the remaining reaches using the approach in the right path of Figure 3.

#### A. Models for simulation

Traditionally, the Saint Venant equations, which are two nonlinear partial differential equations have been used to describe the dynamics of a river. Under the assumption that the flow is one-dimensional and the velocity is uniform, the Saint Venant equations are given by (see e.g. [25])

$$\begin{aligned} \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} &= q_L \\ \frac{\partial Q}{\partial t} + \left( \frac{gA}{T} - \frac{Q^2}{A^2} \right) \frac{\partial A}{\partial x} + \frac{2Q}{A} \frac{\partial Q}{\partial x} + gA(S_f - S_0) &= V_x q_L \end{aligned} \quad (1)$$

where  $Q$  is the flow,  $A$  is the wetted cross sectional area,  $T$  is the top width,  $g = 9.81\text{m/s}^2$  is the gravity constant,  $S_0$  is the bottom slope and  $S_f$  is the friction slope. The friction slope is given by  $S_f = n_f^2 Q^2 P^{4/3} A^{-10/3}$ , where  $P$  is the wetted perimeter and  $n_f$  is the Manning friction coefficient which represents the effect of flow resistance and river roughness.  $q_L$  is the lateral flow per unit length which is positive if it is an in-flow and negative if it is an out-flow.  $V_x$  is the velocity of the lateral flow in the direction of the spatial

variable  $x$ . The Saint Venant equations are solved numerically in space and time using the Preissmann scheme, which is a finite difference method (see e.g. [25]). These equations are solved together with the boundary conditions (e.g. the upstream and the downstream flows) and the initial conditions (e.g. the steady state solutions of Eqn. (1)).

The Saint Venant equations require knowledge of the physical characteristic of the river. For the Broken River, these characteristics are obtained from [27] and various other sources such as Google Maps. Even though the river meanders and the geometry changes, it has been shown in [22] and [28] that for the purposes in this paper the river can be well approximated using straight line approximations between hydraulic structures and calibrating the friction coefficients.

#### B. Models for control

The Saint Venant equations are difficult to use for control design since they are nonlinear partial differential equations. A common approach is to linearise the Saint Venant equations (see e.g. [29] - [31]) or to obtain linear models directly by using system identification techniques (see e.g. [5], [6]). In this work we use the latter approach. Two model structures are considered, the delay model and the integrator delay model. We use time index  $t$  for the continuous time models and  $n$  for the discrete time models.

1) *Time delay model:* We consider the reach between Casey's and Gowangardie Weirs. Figure 4 shows the measured flows and water levels from April to June 2001. The sampling interval is 15 minutes, and in total there are 8640 data samples. The flows are computed from the water levels using a rating curve. From Figure 4, we observe that the flow measurements

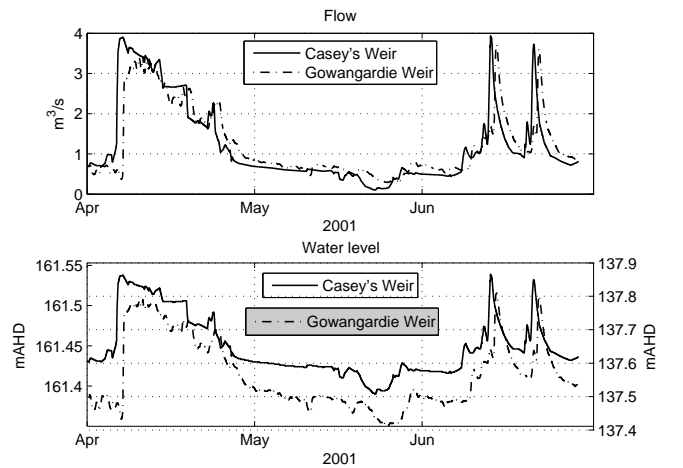


Fig. 4. Measurements at Casey's Weir and Gowangardie Weir from April to June 2001. Top: Flows. Bottom: Water levels at Casey's Weir (left-axis) and Gowangardie Weir (right-axis).

show a lag between Casey's and Gowangardie Weirs and hence the time delay model,

$$Q_G(t) = Q_C(t - \tau_{CG}) \quad (2)$$

is used where  $\tau_{CG}$  is the time delay and  $Q_C$  and  $Q_G$  denote the flows at Casey's and Gowangardie Weirs respectively. The

use of the time delay model is in agreement with the findings in [32].

Both Casey's and Gowangardie Weirs resemble sharp crested weirs where the flow can be approximated by ([33]),

$$Q_C(t) \approx c_C h_C^{3/2}(t) = c_C [y_C(t) - p_C]^{3/2} \quad (3)$$

$$Q_G(t) \approx c_G h_G^{3/2}(t) = c_G [y_G(t) - p_G]^{3/2} \quad (4)$$

where  $c_C$  and  $c_G$  are constants.  $h_C = y_C - p_C$  and  $h_G = y_G - p_G$  are the head over Casey's and Gowangardie Weirs respectively.  $p_C = 163.02\text{mAHD}$  and  $p_G = 138.54\text{mAHD}$  are the heights of Casey's and Gowangardie Weirs ([26]), and  $y_C$  and  $y_G$  are the water levels. Using Eqns. (3) and (4), Eqn. (2) can be rewritten as

$$y_G(t) = \theta_{CG,1} y_C(t - \tau_{CG}) + \theta_{CG,2} \quad (5)$$

where  $\theta_{CG,1} = (c_C/c_G)^{2/3}$  and  $\theta_{CG,2} = p_G - (c_C/c_G)^{2/3} p_C$  are unknown constants, which can be estimated from the observed data together with the time delay.

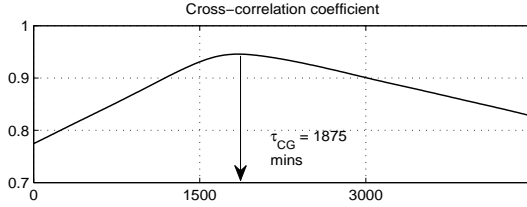


Fig. 5. Cross-correlation coefficient between flows at Casey's Weir and Gowangardie Weir.

The time delay,  $\tau_{CG}$  is estimated from the cross-correlation between the upstream and downstream flows. The cross-correlation coefficient is shown in Figure 5 and computed from the sampled data as follows

$$R_{Q_G Q_C}(\delta) = \frac{\sum_{n=\delta+1}^N [Q_G(n) - \bar{Q}_G][Q_C(n - \delta) - \bar{Q}_C]}{\sigma_{Q_G} \sigma_{Q_C}} \quad (6)$$

$$\hat{\delta}_{CG} = \underset{\delta}{\operatorname{argmax}} R_{Q_G Q_C}(\delta); \quad \delta = 0, 1, \dots \quad (7)$$

$$\hat{\tau}_{CG} = \hat{\delta}_{CG} \cdot T_s \quad (8)$$

Here  $\hat{\tau}_{CG}$  is the estimate of the delay in minutes and  $\hat{\delta}_{CG}$  is the estimate in samples.  $N = 8640$ ,  $\bar{Q}_i = \frac{1}{N} \sum_{n=1}^N Q_i(n)$ ,  $i = C, G$  are the average flows,  $\sigma_{Q_i} = \sqrt{\frac{1}{N} \sum_{n=1}^N (Q_i(n) - \bar{Q}_i)^2}$ ,  $i = C, G$  and  $T_s$  is the sampling period. The associated discrete time predictor for Eqn. (5) is given by

$$\begin{aligned} \hat{y}_G(n, \theta_{CG}, \hat{\delta}_{CG}) &= \theta_{CG,1} y_C(n - \hat{\delta}_{CG}) + \theta_{CG,2} \\ &= \varphi^T(n) \theta_{CG} \end{aligned} \quad (9)$$

where  $\varphi(n) = [y_C(n - \hat{\delta}_{CG}), 1]^T$ . The parameter vector  $\theta_{CG} = [\theta_{CG,1}, \theta_{CG,2}]^T$  is estimated using least squares, i.e.

$$\hat{\theta}_{CG, \delta_{CG}} = \left[ \sum_{n=\delta_{CG}+1}^N \varphi(n) \varphi^T(n) \right]^{-1} \left[ \sum_{n=\delta_{CG}+1}^N \varphi(n) y_G(n) \right] \quad (10)$$

The estimated values are  $\hat{\theta}_{CG,1} = 2.71$ ,  $\hat{\theta}_{CG,2} = -298.60$  and  $\hat{\tau}_{CG} = 1875$  minutes.

The time delay model is validated on a data set not used for estimation. In addition, we include the simulated water levels using the Saint Venant equations. In the Saint Venant equations the friction coefficient and the gate coefficients equivalent to  $c_C$  and  $c_G$  in Eqns. (3) and (4) are calibrated on data sets not used for validation ([28]). Figure 6 shows the measured water levels, predicted water levels using the time delay model and the simulated water levels from the calibrated Saint Venant equations. For quantitative comparison, the mean

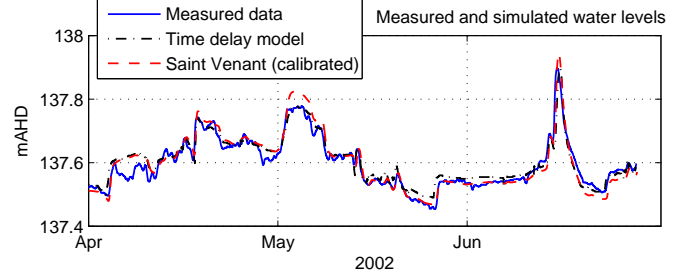


Fig. 6. Measured and simulated water levels at Gowangardie Weir from April to June 2002.

square errors, (MSE) between the predicted and the simulated water levels and the measured water levels are calculated. The MSE is given by

$$\frac{1}{N_v - \delta_{CG}} \sum_{n=\delta_{CG}+1}^{N_v} [y_G(n) - \hat{y}_G(n, \theta_{CG}, \delta_{CG})]^2 \quad (11)$$

where  $y_G$  is the measured water level and  $\hat{y}_G$  is the water level predicted by the time delay model or simulated using the calibrated Saint Venant equations.  $N_v = 8764$  is the number of data points in the data set in Figure 6. The MSEs are given in Table I. From Figure 6, it can be seen that both the time

TABLE I  
VALUES OF MSE.

| Time Delay Model ( $10^{-3} \text{m}^2$ ) | Saint Venant Equations ( $10^{-3} \text{m}^2$ ) |
|---|---|
| 0.73                                      | 0.52  |

delay model and the Saint Venant equations are accurate when compared to the real measured water levels. They pick up the trends in the data very well, and the MSE values in Table I are small.

2) *Effect of varying flow conditions*: It is known (see e.g. [34], [35]) that the flow conditions affect the time delay, and the variations in the delay will affect the robustness margins of a control system. Data sets with different flows were found, and the time delays were estimated using Eqns. (6) - (8). As expected, the estimated time delays decrease with higher flows as can be seen from Table II and Figure 7. The measurements during the high flows in 1995 and 1996 may not be accurate, but they do illustrate the point that the time delay decreases. Note the good agreement in Figure 7 between the time delays estimated using real data and those found using simulated data from the Saint Venant equations. In order to ensure robustness

of the control system, the time delays in the models used for control design in Section V are estimated based on the lowest flows and hence have the largest delays.

TABLE II  
FLOWS AT CASEY'S WEIR AND TIME DELAYS.

| Year | Flow Range                    | Mean flow             | Time delay   |
|------|-------------------------------|-----------------------|--------------|
| 1995 | 18.5 - 129.6m <sup>3</sup> /s | 54.6m <sup>3</sup> /s | 705 minutes  |
| 1996 | 19.1 - 47.0m <sup>3</sup> /s  | 26.4m <sup>3</sup> /s | 870 minutes  |
| 2003 | 0.6 - 5.8m <sup>3</sup> /s    | 3.3m <sup>3</sup> /s  | 1560 minutes |
| 2004 | 0.7 - 3.9m <sup>3</sup> /s    | 2.2m <sup>3</sup> /s  | 1695 minutes |

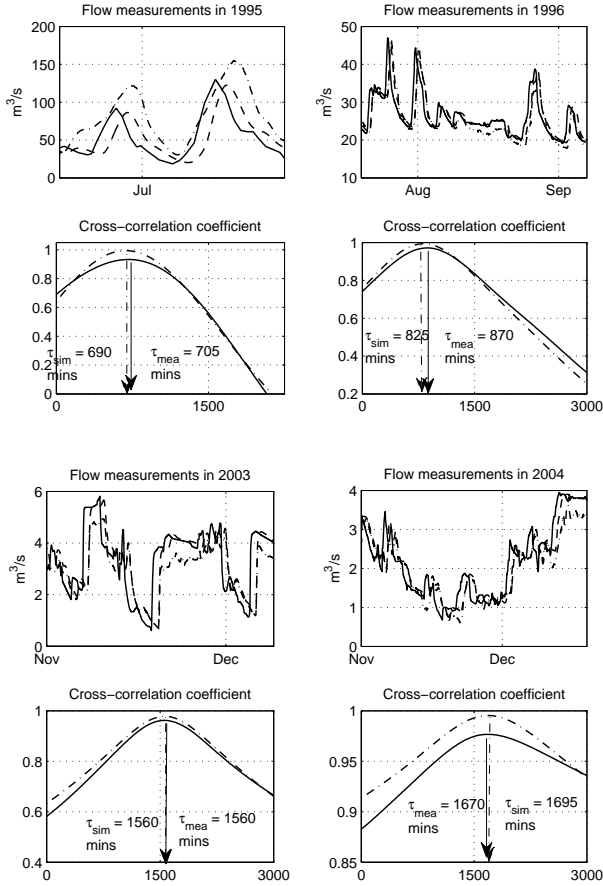


Fig. 7. Flows and the estimated time delays. Flow plots: Solid line: Casey's Weir, Dash-Dotted line: Gowangardie Weir (measured), Dashed line: Gowangardie Weir (simulated). Cross-correlation coefficient plots: Solid line: Cross-correlation coefficient using measured data. Dash-dotted line: Cross-correlation coefficient using measured data at Casey's Weir and simulated data at Gowangardie Weir.

3) *Integrator delay model*: In cases where the downstream flow can be regulated independently of the upstream flow, the time delay model is obviously not a good model. In addition, the time delay model also assumes that there is little storage. This may not be a valid assumption if the river flows through a large weir pool or a lake. In such cases, an integrator delay model is more appropriate. We consider the reach from Lake Benalla to Casey's Weir. For this reach there is an out-flow just upstream of Casey's Weir to Broken Creek. The measured data are shown in Figure 8. The sampling interval is 15 minutes

with a total of 6336 data samples. For Lake Benalla only water level measurements are available.

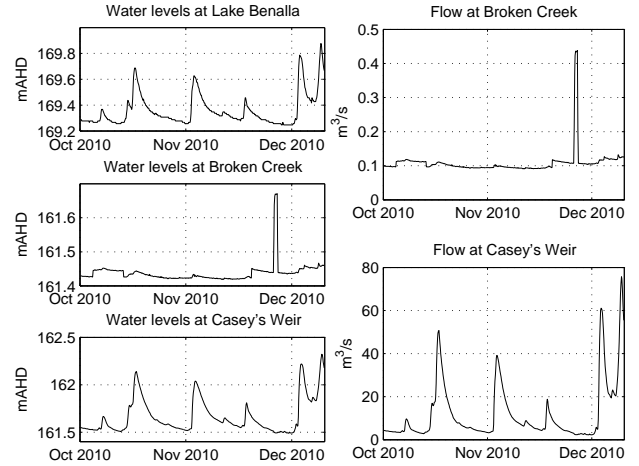


Fig. 8. Flow and water level for the reach from Lake Benalla to Casey's Weir.

Using a simplified mass balance equations, we obtain the integrator delay model

$$\dot{V}_C(t) = Q_{LB}(t - \tau_{LB}) - Q_C(t) - Q_{BC}(t) \quad (12)$$

where  $V_C$  is the volume,  $Q_{LB}$ ,  $Q_C$  and  $Q_{BC}$  are the flows at Lake Benalla, Casey's Weir and Broken Creek respectively and  $\tau_{LB}$  is the time delay. Both Casey's Weir and the weir at Lake Benalla resemble sharp crested weirs and thus the flows can be approximated respectively by Eqn. (3) and

$$Q_{LB}(t) \approx c_{LB}[y_{LB}(t) - p_{LB}]^{3/2} \quad (13)$$

where  $p_{LB} = 169.78\text{mAHD}$  ([26]) is the height of the weir, and  $y_{LB}$  is the water level.

Substituting Eqns. (13) and (3) into (12), assuming the volume is proportional to the water level at Casey's Weir<sup>5</sup> and using an Euler approximation for the derivative, we arrive at

$$y_C(n+1) = y_C(n) + \left(\frac{T_s}{A_s}\right) c_{LB}[y_{LB}(n - \delta_{LB}) - p_{LB}]^{3/2} - \left(\frac{T_s}{A_s}\right) c_C[y_C(n) - p_C]^{3/2} - \left(\frac{T_s}{A_s}\right) Q_{BC}(n) \quad (14)$$

where  $T_s$  is the sampling interval,  $\delta_{LB} = \tau_{LB}/T_s$ , and  $A_s$  is the nominal surface area of the weir pool. The associated Output Error (OE) type predictor for Eqn. (14) is

$$\begin{aligned} \hat{y}_C(n+1, \theta_{LBC}, \delta_{LB}) &= \hat{y}_C(n, \theta_{LBC}, \delta_{LB}) + \theta_{LBC,1}[y_{LB}(n - \delta_{LB}) - p_{LB}]^{3/2} \\ &+ \theta_{LBC,2}[\hat{y}_C(n, \theta_{LBC}, \delta_{LB}) - p_C]^{3/2} + \theta_{LBC,3}Q_{BC}(n) \end{aligned} \quad (15)$$

<sup>5</sup>The weir pool is very wide compared to the expected changes in the water level so no large error is introduced by using the local approximation that the volume is proportional to the water level.



where  $\theta_{LBC} = [\theta_{LBC,1}, \theta_{LBC,2}, \theta_{LBC,3}]^T = \left[ \left( \frac{T_s}{A_s} \right) c_{LB}, - \left( \frac{T_s}{A_s} \right) c_C, - \left( \frac{T_s}{A_s} \right) \right]^T$ . An OE-type model is used since it gives a good description of a system in the low frequency range (see e.g. [36]), which is of most interest for control design. Notice that this predictor only makes use of the initial value of the water level at Casey's Weir and makes predictions using the previously predicted water levels.

As before, the time delay,  $\tau_{LB} = \delta_{LB} \cdot T_s$  is estimated from the cross-correlation between the measurements at Lake Benalla and Casey's Weir, and it is shown in Figure 9. The

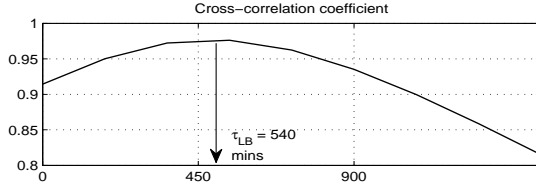


Fig. 9. Cross-correlations coefficient and estimated time delay for the reach from Lake Benalla to Casey's Weir.

parameter,  $\theta_{LBC}$  is estimated using the data set shown in Figure 8 using a prediction error method, i.e.

$$\hat{\theta}_{LBC} = \underset{\theta_{LBC}}{\operatorname{argmin}} \frac{1}{N - \hat{\delta}_{LB}} \sum_{n=\hat{\delta}_{LB}+1}^N [y_C(n) - \hat{y}_C(n, \theta_{LBC}, \hat{\delta}_{LB})]^2 \quad (16)$$

where  $N = 6336$  is the number of data points. The estimated parameters are  $\hat{\theta}_{LBC,1} = 2.200$ ,  $\hat{\theta}_{LBC,2} = -1.394$ ,  $\hat{\theta}_{LBC,3} = -0.226$  and  $\hat{\tau}_{LBC} = 540$  minutes. The value of  $\hat{\theta}_{LBC,1}$  is positive which is in agreement with an in-flow and the values of  $\hat{\theta}_{LBC,2}$  and  $\hat{\theta}_{LBC,3}$  are negative which is in agreement with out-flows.

The integrator delay model is validated on a data set not used for estimation, and the simulated water levels from the integrator delay model and the calibrated Saint Venant equations are shown in Figure 10 together with the measured water levels. The values of the MSE are given in Table III.

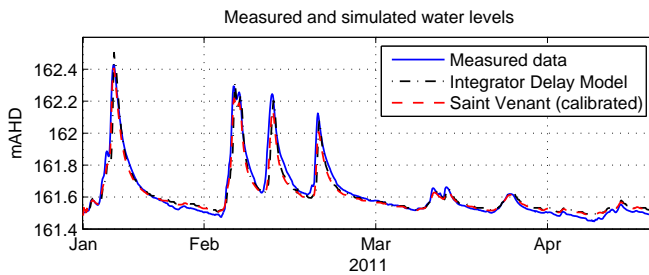


Fig. 10. Measured and simulated water levels at Casey's Weir from January to April 2011.

From Figure 10 it is observed that both the integrator delay model and the Saint Venant equations pick up the trends in the water level very well. The predictive capabilities of the integrator delay model are very good, bearing in mind that

the predictor only makes use of the initial value of the water level to make predictions. The integrator delay model has a larger MSE than the Saint Venant equations, but both MSE values are small.

TABLE III  
VALUES OF MSE.

| Integrator Delay Model           | Saint Venant Equations           |
|----------------------------------|----------------------------------|
| $2.82 \times 10^{-3} \text{m}^2$ | $0.83 \times 10^{-3} \text{m}^2$ |

4) *Summary*: In this section we have shown that the delay and integrator delay models represent the dynamics of reaches in the Broken River well. Moreover, we have shown that this is also the case for the calibrated Saint Venant equations. In the next subsection we give the complete set of models used for control design. The parameter values of the remaining models are found using system identification techniques as described above using simulated data from the Saint Venant equations. As we would like models estimated under low flows, we have also re-estimated the parameters of the model from Lake Benalla to Casey's Weir using data from the Saint Venant equations simulated under low flow.

### C. Models of Broken River for control design

Here we briefly describe the models used for design of the controllers in Section V. The same model structures as in the previous sections have been used, and terms representing irrigation offtakes and flows in creeks have been included. The continuous time models are used for the design of the decentralised controllers, while the discrete time models are used for the design of the centralised MPC controller.

1) *Reach Lake Nillahcootie to Broken Weir (Reach LNB)*: Reach LNB can be modelled using an integrator delay model

$$\dot{y}_B(t) = \tilde{c}_{LNB} [Q_{LN}(t - \tau_{LNB}) - Q_B(t) - Q_{Sin}(t) - Q_{offLNB}(t) + Q_{LC}(t - \tau_{LCB})] \quad (17)$$

where  $y_B$  is the water level at Broken Weir,  $Q_{LN}$ ,  $Q_B$  and  $Q_{Sin}$  are the flows at Lake Nillahcootie, Broken Weir and the in-let to the storage respectively.  $Q_{offLNB}$  represents the offtakes of water to irrigators and  $Q_{LC}$  is the in-flow from Lima Creek.  $\tau_{LNB}$  and  $\tau_{LCB}$  are the time delays from respectively Lake Nillahcootie and Lima Creek to Broken Weir.

In order to obtain a discrete time model for MPC an Euler approximation for the derivative is used, and we obtain

$$y_B(n) = y_B(n-1) + c_{LNB} [Q_{LN}(n - \delta_{LNB} - 1) - Q_B(n-1) - Q_{Sin}(n-1) - Q_{offLNB}(n-1) + Q_{LC}(n - \delta_{LCB} - 1)] \quad (18)$$

where  $c_{LNB} = \tilde{c}_{LNB} T_s$ ,  $\delta_{LNB} = \tau_{LNB}/T_s$  and  $\delta_{LCB} = \tau_{LCB}/T_s$ .

2) *Reach Lake Nillahcootie to off-stream storage (Reach LNS)*: The volume in the storage is modelled as

$$\dot{V}_S(t) = Q_{Sin}(t) - Q_{Sout}(t) \quad (19)$$

where  $V_S$  is the volume, and  $Q_{Sin}$  and  $Q_{Sout}$  are the in- and out-flows of the storage. In the decentralised control schemes, the volume in the storage is controlled by the flow releases from Lake Nillahcootie. Thus, the approximation  $Q_{Sin}(t) \approx Q_{LN}(t - \tau_{LNS}) - Q_B(t) + Q_{LC}(t - \tau_{LCS})$  is used. This is a reasonable approximation since the in-flow is controlled by a fast acting upstream level controller.  $\tau_{LNS}$  and  $\tau_{LCS}$  are the time delays. Hence, Eqn. (19) can be written as

$$\dot{V}_S(t) = Q_{LN}(t - \tau_{LNS}) - Q_{Sout}(t) - Q_B(t) + Q_{LC}(t - \tau_{LCS}) \quad (20)$$

For centralised control, the discrete time version of Eqn. (19) is used directly, i.e.

$$V_S(n) = V_S(n-1) + T_s [Q_{Sin}(n-1) - Q_{Sout}(n-1)] \quad (21)$$

3) *Reach off-stream storage to Lake Benalla (Reach SLB)*: This model is only used in the design of the centralised MPC to model the water level at Lake Benalla,  $y_{LB}$  which is subject to constraints. The model used is a discrete time integrator delay model

$$\begin{aligned} y_{LB}(n) = & y_{LB}(n-1) + c_{SLB} [Q_{Sout}(n - \delta_{SLB} - 1) \\ & - c_{LB}(y_{LB}(n-1) - p_{LB})^{3/2} + Q_B(n - \delta_{BLB} - 1) \\ & - Q_{offSLB}(n-1) + Q_{HC}(n - \delta_{HLB} - 1)] \quad (22) \end{aligned}$$

where  $Q_{LB}$  is the flow out of Lake Benalla,  $Q_{offSLB}$  represents the offtakes to irrigators and  $Q_{HC}$  is the in-flow from Hollands Creek.  $\delta_{SLB}$ ,  $\delta_{BLB}$  and  $\delta_{HLB}$  are the discrete time delays from the off-stream storage, Broken Weir and Hollands Creek to Lake Benalla respectively.  $c_{LB} = 10.15\text{m}^{3/2}/\text{s}$  is the weir constant.  $c_{SLB} = \tilde{c}_{SLB}T_s$  (see Table IV). Note that the flow at Lake Benalla cannot be regulated.

4) *Reach off-stream storage to Casey's Weir (Reach SC)*: This model is only used for decentralised control design. The reach is modelled using the integrator delay model

$$\begin{aligned} \dot{y}_C(t) = & \tilde{c}_{SC} [Q_{Sout}(t - \tau_{SC}) - Q_C(t) + Q_B(t - \tau_{BC}) \\ & - Q_{offSC}(t) - Q_{BC}(t) + Q_{HC}(t - \tau_{HCC})] \quad (23) \end{aligned}$$

where  $\tilde{c}_{SC}$  is a proportionality constant relating water level and volume,  $Q_C$  is the flow at Casey's Weir,  $Q_{offSC}$  represents the offtakes and  $Q_{BC}$  is the out-flow into Broken Creek.  $\tau_{SC}$ ,  $\tau_{BC}$  and  $\tau_{HCC}$  are the time delays. Due to the presence of a weir at the out-let of Lake Benalla, a first order plus integrator model, may seem more appropriate for this reach. However, operational and simulated data indicate that an integrator delay model is sufficient.

5) *Reach Lake Benalla to Casey's Weir (Reach LBC)*: For use in MPC, the water level at Casey's Weir is modelled using the discrete time integrator delay model

$$\begin{aligned} y_C(n) = & y_C(n-1) \\ & + c_{LBC} [c_{LB}(y_{LB}(n - \delta_{LBC} - 1) - p_{LB})^{3/2} - Q_C(n-1) \\ & - Q_{offLBC}(n-1) - Q_{BC}(n-1)] \quad (24) \end{aligned}$$

where  $Q_{offLBC}$  represents the offtakes, and  $\delta_{LBC}$  is the time delay from Lake Benalla to Casey's Weir.  $c_{LBC} = \tilde{c}_{LBC}T_s$  (see Table IV).

6) *Reach Casey's Weir to Gowangardie Weir (Reach CG)*: Reach CG is modelled as described in Section IV-B1, but with terms representing offtakes added.

7) *Reach off-stream Storage to Gowangardie Weir (Reach SG)*: When there is no regulation at Casey's Weir, the flow at Gowangardie Weir is controlled from the off-stream storage. Even though Lake Benalla and the weir pool at Casey's Weir are located within this stretch of the river, simulated and real data indicate that the time delay model

$$\begin{aligned} Q_G(t) = & Q_{Sout}(t - \tau_{SG}) + Q_B(t - \tau_{BG}) - Q_{offSG}(t) \\ & + Q_{HC}(t - \tau_{HCG}) \quad (25) \end{aligned}$$

is a good approximation where  $\tau_{SG}$ ,  $\tau_{BG}$  and  $\tau_{HCG}$  are the time delays.  $Q_{offSG}$  represents the offtakes of water to irrigators.

The parameters in the models are estimated using the simulated data from the Saint Venant equations using a prediction error method (Eqn. (16)) and are given together with the time delays in Table IV. Only the parameters and time delays of the continuous time models are given. The corresponding parameters and time delays of the discrete time models are found respectively by multiplication and division with the sampling period. The flows used in the simulations were in the range 0.3472-0.6944 m<sup>3</sup>/s (30-60ML/day). These flows are at the lower end of what is expected, and hence the time delays are at the largest, which means that they are the most unfavourable ones from a control perspective.

TABLE IV  
ESTIMATED PARAMETERS AND TIME DELAYS.

| Reach | Estimates                   | Time delays (minutes)  |
|-------|-----------------------------|--|
| LNB   | $\tilde{c}_{LNB} = 0.00060$ | $\tau_{LNB} = 2250, \tau_{LCB} = 1620$                       |
| LNS   | -                           | $\tau_{LNS} = 2250$  |
| SC    | $\tilde{c}_{SC} = 0.00027$  | $\tau_{SC} = 1350, \tau_{BC} = 1800,$<br>$\tau_{HCC} = 1850$ |
| SLB   | $\tilde{c}_{SLB} = 0.00020$ | $\tau_{SLB} = 425, \tau_{BLB} = 875,$<br>$\tau_{HLB} = 925$  |
| LBC   | $\tilde{c}_{LBC} = 0.00040$ | $\tau_{LBC} = 975$   |
| CG    | -                           | $\tau_{CG} = 1875$   |
| SG    | -                           | $\tau_{SG} = 3225, \tau_{BG} = 3675,$<br>$\tau_{HCG} = 3725$ |

## V. CONTROL

### A. Control strategies

Currently, the flows in the river are regulated by manually releasing water from Lake Nillahcootie in order to fulfil the water demand along the river. The purpose of this project is to demonstrate the benefit of control, so a simple decentralised control scheme which does not require any major infrastructure upgrade is first considered. Improved performance is expected if the flow can be regulated close to where most of the demand is, and hence, a decentralised configuration where the flow over Casey's Weir can be regulated is also considered. This configuration requires an upgrade in infrastructure, but it is still simple and there is potentially a large improvement in performance. Finally, centralised MPC is considered. This controller is more complex than the decentralised schemes, but MPC is ideal for the problem at hand due to its ability to handle constraints.

## B. Manual operation

Under current manual operation, the flow out of Lake Nillahcootie is adjusted daily by adding up future water orders which are known 4 days in advance. An extra 0.2315 m<sup>3</sup>/s (20ML/day) is added to account for uncertainties and transmission losses. There is no control at any other weir other than Lake Nillahcootie in manual operation.

## C. Decentralised control configurations

PI and I controllers are considered in the decentralised configurations. Due to the long time delays, feedforward of the known future orders of water is necessary. For demand driven systems, the most appropriate control configuration is distant downstream control, where the water level or flow at the downstream end of a reach is controlled by a gate at the upstream end. The distant downstream control configuration with feedforward is shown in Figure 11. The feedforward term is calculated as follows: All orders for water downstream of a gate including water orders in the downstream reaches are converted into an equivalent flow time series. The time series are shifted forward with the nominal time delay between the gate and the offtake point and then summed together to arrive at the feedforward flow. This feedforward action will compensate for the offtakes in a reach and for the outflows at the downstream end of the reach. The feedback controller

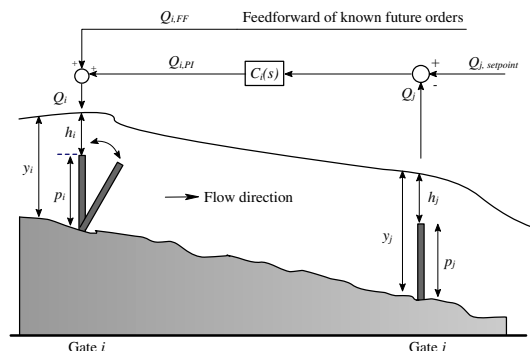


Fig. 11. Distant downstream control configuration with feedforward.

$C_i(s)$  will adjust for any discrepancies in the flow (or the water level) e.g. due to model mismatch or errors in the rating curve.  $C_i(s)$  is usually a PI controller. The feedforward and the feedback are added together to produce the required flow  $Q_i$  and the upstream gate  $i$  is positioned to release the flow. In the decentralised control schemes the flows in the creeks are treated as unknown disturbances.

1) *Decentralised control without regulation at Casey's Weir:* The reaches are controlled as shown in Figure 12. The release from Lake Nillahcootie is used to control the volume in the off-stream storage. The gate at the in-let to the storage is in an upstream level control configuration such that it controls the immediate upstream water level which is the water level at Broken Weir. Broken Weir is in flow mode where the measurement of the water level  $y_B$  is used to compute the

required gate position  $p_B$  in order to maintain a desired flow over the weir. The off-stream storage rather than Broken Weir is used to control the water level at Gowangardie Weir's since there is limited storage in the weir pool upstream of Broken Weir. Putting everything together, the control configuration shown in Figure 12 is obtained.

PI<sub>1</sub>, PI<sub>2</sub> and I are PI and I-only<sup>6</sup> controllers.  $e_{V_{Sin}}$ ,  $e_{y_B}$  and  $e_{Q_G}$  are the difference between the volume in the off-stream storage, the water level at Broken Weir and the flow over Gowangardie Weir and their respective setpoints.  $Q_{LN}$ ,  $Q_{Sin}$  and  $Q_{Sout}$  are the flows at Lake Nillahcootie, and in and out of the off-stream storage respectively.

2) *Decentralised control with regulation at Casey's Weir:* The difference from the configuration in Figure 12 is that the water level at Casey's Weir is controlled from the off-stream storage and the flow at Gowangardie Weir is controlled from Casey's Weir that is assumed upgraded to an adjustable weir.

3) *Decentralised control design:* The PI and I controllers are tuned using classical frequency response methods (see e.g. [37]) based on the models in Section IV. For further details see [38]. Due to the variation in the time delay with the flow, (see Section IV-B2), the controllers are tuned rather conservatively to ensure robustness. One may argue that since the controllers are designed for the largest time delays, it is not necessary to tune them conservatively. However, as it turns out, there is only marginal difference in the performance, since the feedforward terms account for most of the control effort. The controller parameters are given in Table V together with the phase and gain margins as well as the extra time delays that can be tolerated before the closed loop becomes unstable. These additional delays are well within the range of time delays observed.

TABLE V  
CONTROLLER PARAMETERS. PM: PHASE MARGIN, GM: GAIN MARGIN,  $\bar{\tau}$ : EXTRA TOLERABLE DELAY.

| Reach | $K_p$ | $T_i$ | PM (°) | GM (dB) | $\bar{\tau}$ (min) |
|-------|-------|-------|--------|---------|--------------------|
| Sin   | 1000  | 200   | 51.5   | 23.3    | 125                |
| LNS   | 0.3   | 50000 | 57.6   | 10.3    | 4800               |
| SC    | 100   | 20000 | 47.2   | 26.9    | 2784               |
| SG    | -     | 5000  | 53.0   | 7.7     | 4629               |
| CG    | -     | 4000  | 63.1   | 10.5    | 4408               |

## D. Model Predictive Control (MPC)

MPC is a control scheme where at each time step, the controller generates a sequence of control inputs (flow releases) based on the predicted behavior of the river over a finite horizon by solving an optimisation problem. The criterion reflects the control objectives, and constraints can be incorporated.

1) *State space model:* The models in Section IV-C are rewritten in state space form. Eqns (22) and (24) are nonlinear models due to the weir equation used at Lake Benalla (i.e.  $Q_{LB}(n) = c_{LB}(y_{LB}(n) - p_{LB})^{3/2}$ ). A linear model is obtained by linearising the weir equation around the water

<sup>6</sup>As the model for this reach is a pure time delay, there is not much improvement using a PI controller compared to the simpler I controller.

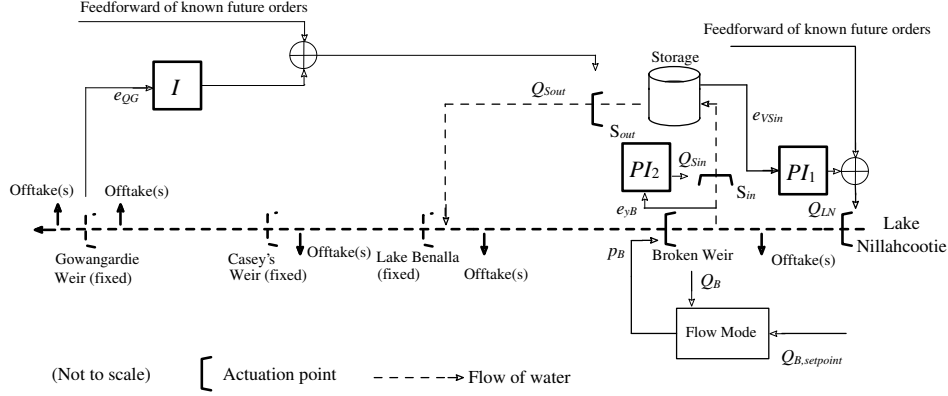


Fig. 12. Decentralised control configuration without regulation at Casey's Weir.

level setpoint  $y_{LB,sp}$ , that is  $Q_{LB}(n) \approx \check{c}_{LB,1}[y_{LB}(n) - y_{LB,sp}(n)] + \check{c}_{LB,2}$ , with  $\check{c}_{LB,2} = c_{LB}[y_{LB,sp}(n) - p_{LB}]^{3/2}$  and  $\check{c}_{LB,1} = \frac{3}{2}c_{LB}\sqrt{y_{LB,sp}(n) - p_{LB}}$ .

We assume that the flow at Casey's Weir can be regulated. The control variables are  $Q_{LN}$ ,  $Q_{Sin}$ ,  $Q_{Sout}$ ,  $Q_B$  and  $Q_C$ , and the controlled variables are  $y_B$ ,  $V_S$ ,  $y_C$ ,  $y_{LB}$  and  $Q_G$ . The state variables are the deviation of the controlled variables from their setpoints, e.g.  $x_{e,C}(n) = y_C(n) - y_{C,sp}(n)$  for the water level deviation at Casey's Weir, where  $y_{C,sp}(n)$  is the setpoint.

Let  $u_j(n) = Q_j(n)$ , where  $j = LN, Sin, Sout, B$  or  $C$ . Due to the time delays, we need a number of states to remember the past flows. Hence, we introduce the states,  $x_{j,i}(n) = u_j(n - i)$ .

In view of the large time delays, it would be impractical to choose a small sampling interval,  $T_s$  as this will introduce a large number of states and many input variables to optimise over. We chose,  $T_s = 360$  minutes. The discrete time delays are  $\delta_{LNB} = 6$ ,  $\delta_{SLB} = 1$ ,  $\delta_{BLB} = 2$ ,  $\delta_{LBC} = 3$  and  $\delta_{CG} = 5$ .

Equipped with these variables we obtain the following state space model

$$\begin{aligned}
x_{LN,1}(n) &= u_{LN}(n-1) \\
x_{LN,i+1}(n) &= x_{LN,i}(n-1), \quad i = 1, \dots, 5 \\
x_{e,B}(n) &= x_{e,B}(n-1) + c_{LNB}x_{LN,6}(n-1) \\
&\quad - c_{LNB}u_B(n-1) - c_{LNB}u_{Sin}(n-1) \\
&\quad + \tilde{Q}_{LC}(n-1) + d_B(n-1) + v_B(n-1) \\
x_{B,1}(n) &= u_B(n-1) \\
x_{B,2}(n) &= x_{B,1}(n-1) \\
x_{e,S}(n) &= x_{e,S}(n-1) + T_s u_{Sin}(n-1) \\
&\quad - T_s u_{Sout}(n-1) + v_S(n-1) \\
x_{Sout,1}(n) &= u_{Sout}(n-1) \\
x_{e,LB,1}(n) &= (1 - c_{SLB}\check{c}_{LB,1})x_{e,LB,1}(n-1) \\
&\quad + c_{SLB}x_{Sout,1}(n-1) + c_{SLB}x_{B,2}(n-1) \\
&\quad + \tilde{Q}_{HC}(n-1) + d_{LB}(n-1) + v_{LB}(n-1) \\
&\quad - c_{SLB}\check{c}_{LB,2} \\
x_{e,LB,i+1}(n) &= x_{e,LB,i}(n-1), \quad i = 1, \dots, 3
\end{aligned}$$

$$\begin{aligned}
x_{e,C}(n) &= x_{e,C}(n-1) + c_{LBC}\check{c}_{LB,1}x_{e,LB,4}(n-1) \\
&\quad - c_{LBC}u_C(n-1) - \tilde{Q}_{BC}(n-1) \\
&\quad + d_C(n-1) + v_C(n-1) + c_{LBC}\check{c}_{LB,2} \\
x_{C,1}(n) &= u_C(n-1) \\
x_{C,i+1}(n) &= x_{C,i}(n-1) \quad i = 1, \dots, 4 \\
x_{e,G}(n) &= x_{C,4}(n-1) + d_{CG}(n-1) - Q_{G,sp}(n-1)
\end{aligned} \tag{26}$$

where  $d_B$ ,  $d_{LB}$ ,  $d_C$  and  $d_G$  represent offtakes to irrigators,  $\tilde{Q}_{LC}$ ,  $\tilde{Q}_{HC}$  and  $\tilde{Q}_{BC}$  denote the effects of the flows in the creeks and  $v_B$ ,  $v_S$ ,  $v_C$  and  $v_{LB}$  denote setpoint changes.  $Q_{G,sp}$  is the flow setpoint at Gowangardie Weir.

In order to ensure zero steady state error in the presence of disturbances, the integral of the setpoint errors are augmented to the plant, i.e.

$$x_{int,l}(n) = x_{int,l}(n-1) + T_s x_{e,l}(n-1) \tag{27}$$

where  $l = B, S, C, LB$  and  $G$ . Eqns. (26) and (27) can be written as

$$x(n+1) = Ax(n) + Bu(n) + d(n) \tag{28}$$

where  $d(n)$  incorporates offtakes, setpoint changes, flows in creeks and terms due to the linearisation. Note that we have access to all states, and there is no need to design an observer.

2) *Optimisation criterion and constraints:* Based on the control objectives in Section III-A, the criterion to be minimised at time  $n$  is given by<sup>7</sup>

$$\begin{aligned}
J(n, u^T(n), \dots, u^T(n + N_P - 1)) \\
&= \sum_{k=1}^{N_P} x^T(n+k)Qx(n+k) + \sum_{k=0}^{N_P-1} (u^T(n+k)Ru(n+k) \\
&\quad + s_L^T(n+k)Q_{s,L}s_L(n+k) + s_H^T(n+k)Q_{s,H}s_H(n+k))
\end{aligned} \tag{29}$$

<sup>7</sup>We also investigated the performance of the controller with a terminal cost penalty where the first term in equation (29) was replaced by  $\sum_{k=1}^{N_P-1} x^T(n+k)Qx(n+k) + x^T(n+N_P)Px(n+N_P)$ . The matrix  $P$  was found as the solution to a Discrete Algebraic Riccati Equation (see e.g. [39]). There were only minor differences in performance. With the terminal cost penalty the amount of excess water was slightly reduced at the expense of a few more days with violations of the constraints.

Here  $N_P$  is the prediction horizon. The matrices  $Q$  and  $R$  are chosen below based on the control objectives while the matrices  $Q_{s,L}$  and  $Q_{s,H}$  are used to handle the soft constraints.  $s_L$  and  $s_H$  are slack variables which are zero when the constraints are satisfied and non-zero when the constraints are violated. Non-zero slack variables are penalised with large weights and hence the optimiser tries to maintain the slack variables at zero or close to zero.

One of the control objectives is to keep the releases from Lake Nillahcootie small, thus  $u_{LN}$  is assigned a weight  $r_{LN}$ . In order to ensure that most of the water to be used downstream is supplied from the off-stream storage rather than from the weir pool at Broken Weir,  $u_B$  is assigned a weight  $r_B$ . The remaining control actions are assigned a weight of 1. The integral of the setpoint errors were penalised with the weights  $q_{int,B}$ ,  $q_{int,S}$ ,  $q_{int,C}$ ,  $q_{int,LB}$  and  $q_{int,G}$ . The setpoint errors themselves were not penalized since they were taken care of by the constraints. Using these weights we can derive the  $Q$  and  $R$  matrices.

The formulation (29) is a standard MPC formulation. One may argue that a quadratic penalty on the flows is not natural since the value of the criterion is much more sensitive to flow changes at high flows than at low flows. In our case it does however, work quite well since the penalties are mainly used to limit releases from Lake Nillahcootie (subject to constraints on downstream flows and water levels) and to ensure that water is supplied from the offstream storage rather than Broken Weir.

The control objectives of ensuring minimum flows and water levels within given limits can be viewed as constraints. The minimum flow over Gowangardie Weir is  $0.2894 \text{ m}^3/\text{s}$  (25ML/day) while over Casey's and Broken Weirs it is  $0.2546 \text{ m}^3/\text{s}$  (22ML/day). Thus, the constraints  $0.2894 \text{ m}^3/\text{s} \leq Q_G(n)$  and  $0.2546 \text{ m}^3/\text{s} \leq Q_{C,B}(n)$  are used. These flow constraints are soft constraints. The slack variables  $s_{L,QG}$ ,  $s_{L,QC}$  and  $s_{L,QB}$  are used to represent how far the flows go below  $0.2894 \text{ m}^3/\text{s}$  and  $0.2546 \text{ m}^3/\text{s}$ , and they are penalised in the criterion with the weights  $q_{L,QG}$ ,  $q_{L,QC}$  and  $q_{L,QB}$ .

In order to create favourable slackwater conditions for ecology, all the flows are constrained to be less than  $1.3889 \text{ m}^3/\text{s}$  (120ML/day), i.e.  $Q_i(n) \leq 1.3889 \text{ m}^3/\text{s}$ , with  $i = LN, Sout, Sin, B, C$  and  $G$ . These are also soft constraints with associated slack variables  $s_{H,QLN}$ ,  $s_{H,QSout}$ ,  $s_{H,QSsin}$ ,  $s_{H,QB}$ ,  $s_{H,QC}$  and  $s_{H,QG}$ , and they are penalised in the criterion with the weights  $q_{H,QLN}$ ,  $q_{H,QSout}$ ,  $q_{H,QSsin}$ ,  $q_{H,QB}$ ,  $q_{H,QC}$  and  $q_{H,QG}$ .

The water levels  $y_B$ ,  $y_{LB}$  and  $y_C$  should be within 15cm from their setpoints. Thus, we get  $175.00\text{mAHD} \leq y_B(n) \leq 175.30\text{mAHD}$ ,  $169.72\text{mAHD} \leq y_{LB}(n) \leq 170.02\text{mAHD}$  and  $162.92\text{mAHD} \leq y_C(n) \leq 163.22\text{mAHD}$ . However, a tighter bound of 5cm was also used for robustness against model mismatch. Thus, we get a second set of constraints:  $175.10\text{mAHD} \leq y_B(n) \leq 175.20\text{mAHD}$ ,  $169.82\text{mAHD} \leq y_{LB}(n) \leq 169.92\text{mAHD}$  and  $163.02\text{mAHD} \leq y_C(n) \leq 163.12\text{mAHD}$ . These are also soft constraints with corresponding slack variables  $s_{L,yB5}$ ,  $s_{L,yLB5}$  and  $s_{L,yC5}$  for violation of the lower limits, and  $s_{H,yB5}$ ,  $s_{H,yLB5}$  and  $s_{H,yC5}$  for violation of the upper limits. These slack variables are penalised with weights  $q_{L,yB5}$ ,  $q_{L,yLB5}$ ,  $q_{L,yC5}$ ,  $q_{H,yB5}$ ,  $q_{H,yLB5}$  and  $q_{H,yC5}$ . In order

to ensure that the water levels do not exceed the  $\pm 15\text{cm}$  bound, additional slack variables,  $s_{L,yB15}$ ,  $s_{L,yLB15}$ ,  $s_{L,yC15}$ ,  $s_{H,yB15}$ ,  $s_{H,yLB15}$  and  $s_{H,yC15}$  are introduced and these slack variables are heavily penalised with weights  $q_{L,yB15}$ ,  $q_{L,yLB15}$ ,  $q_{L,yC15}$ ,  $q_{H,yB15}$ ,  $q_{H,yLB15}$  and  $q_{H,yC15}$ . For the storage, the hard constraint  $0 \text{ m}^3 \leq V_S(n) \leq 0.3 \times 10^6 \text{ m}^3$  was applied. The weights used in the matrices,  $Q$ ,  $R$ ,  $Q_{s,L}$  and  $Q_{s,H}$  are summarised in Table VI.

In order to ensure that the flows were between 0.76 and 1.50 of the previous day's flows, the following hard constraints were used

$$\begin{aligned} Q_j(n) &\geq 0.76 \left[ \sum_{l=1}^r Q_j(n-r-l+1) \right] - \sum_{l=1}^{r-1} Q_j(n-l) \\ Q_j(n) &\leq 1.5 \left[ \sum_{l=1}^r Q_j(n-r-l+1) \right] - \sum_{l=1}^{r-1} Q_j(n-l) \end{aligned} \quad (30)$$

where  $r = 1440\text{mins}/T_s = 4$  is the number of samples per day.  $j = LN, Sin, Sout, B$  and  $C$ .

Known future orders from irrigators are easily handled by including them directly in the prediction model [18]. The flow setpoint changes at Gowangardie Weir and the water levels setpoint changes at Casey's Weir, Lake Benalla and Broken Weir are also known in advance, and this information is also included in the prediction model. The in-flows from the creeks are assumed constant over the prediction horizon and equal to the last available measurement. This is different from decentralised control where they are treated as unknown disturbances. The prediction horizon,  $N_P$  is chosen to be 4 days (5760 minutes)<sup>8</sup>. The control problem is a quadratic programming problem, which was formulated using YALMIP [40] in MATLAB<sup>®</sup> and solved using the commercial package CPLEX 12.2 [41].

## VI. SIMULATION SCENARIO

Here we describe the simulation scenario which is used to assess the performance of the control systems. Even though the controllers are designed based upon system identification models, the river is simulated using the Saint Venant equations from Section IV, (see also [28], [42], [43]). As the study area we consider ends at Gowangardie Weir, all demand for water downstream of Gowangardie Weir including environmental water and water to be delivered to the Goulburn River are aggregated into a desired flow over Gowangardie Weir. As for in-flows from creeks, only the two main tributaries, Lima and Hollands Creeks, are considered. It is assumed that Lake Nillahcootie is always able to supply the flow required.

### A. External Inputs

The external inputs used in the simulation scenario are:

- i) *Demand for irrigation water.* The orders for irrigation water are mainly based on the historical water orders from

<sup>8</sup>We considered prediction horizons up to 10 days, but no significant improvement was observed. In order to keep the computational load small, we chose the prediction horizon of 4 days.

TABLE VI  
WEIGHTS USED IN THE MPC OBJECTIVE.

| Integral of setpoint errors |                      | Control actions |       | Soft constraints                                   |                      |
|-----------------------------|----------------------|-----------------|-------|--|----------------------|
| Parameter                   | Value                | Parameter       | Value | Parameter  | Value                |
| $q_{int,B}$                 | $30 \times 10^{-7}$  | $r_{LN}$        | 5     | $q_{L,yB5}, q_{H,yB5}, q_{L,yB15}, q_{H,yB15}$     | 20, 10, 10000, 10000 |
| $q_{int,S}$                 | $2 \times 10^{-7}$   | $r_{Sin}$       | 1     | $q_{L,yLB5}, q_{H,yLB5}, q_{L,yLB15}, q_{H,yLB15}$ | 20, 10, 10000, 10000 |
| $q_{int,C}$                 | $150 \times 10^{-7}$ | $r_{Sout}$      | 1     | $q_{L,yC5}, q_{H,yC5}, q_{L,yC15}, q_{H,yC15}$     | 60, 30, 10000, 10000 |
| $q_{int,LB}$                | $1 \times 10^{-7}$   | $r_B$           | 80    | $q_{L,QC}, q_{H,QC}$                               | 20, 10               |
| $q_{int,G}$                 | $90 \times 10^{-7}$  | $r_C$           | 1     | $q_{L,QB}, q_{H,QB}$                               | 20, 10               |
|                             |                      |                 |       | $q_{L,QG}, q_{H,QG}$                               | 20, 10               |
|                             |                      |                 |       | $q_{H,QLN}, q_{H,QSout}, q_{H,QSin}$               | 10, 10, 10           |

July 2006 to June 2007. This year has been chosen since it was a dry year. Recently, water has been bought back from the irrigators along Broken Creek and downstream of Gowangardie Weir, and the orders at these locations have been replaced by the orders from July 2007 to June 2008 since the demand was less in that year. This also creates a bimodal demand pattern with peaks in spring and autumn which is expected to be more representative for future farming practices. Spatially nearby offtakes points have been lumped together, so in total there are 18 offtakes points. As an example, the total flow demand for irrigation water between Casey's Weir and Gowangardie Weir in January 2007 is shown in Figure 13. Orders that according to the historical records were cancelled are also cancelled in the simulation. That is, for the cancelled orders there is a mismatch between the orders used in MPC and the feedforward calculations and those used in the simulations of the systems. Cancelled order totalled about  $0.25 \times 10^6 \text{ m}^3$  for the whole year.

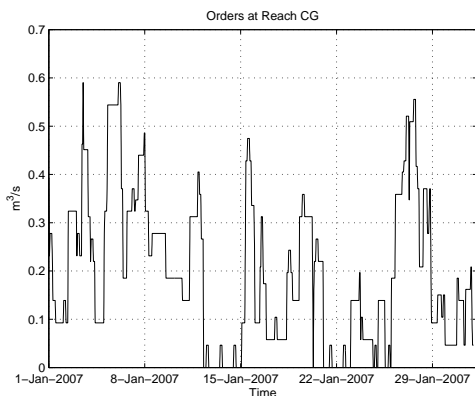


Fig. 13. Orders for water between Casey's and Gowangardie Weirs in January 2007.

ii) *Minimum environmental flows.* The minimum environmental flows are taken from [19]. These are the same as given in the control objectives in Section III-A. These flows only come into effect when the other demands for water are below the minimum environmental flows with the exception of Gowangardie Weir where the flow is added to the demand from the irrigators since there are also minimum environmental flow requirements downstream of Gowangardie Weir. For the environment, a natural varying flow is better than a constant flow.

One possibility is therefore to introduce time varying environmental flows. However, as irrigation demands and in-flows from creeks create variations anyway, we have used constant minimum environmental flows in this study.

iii) *In-flows from creeks.* The historical in-flows from Lima and Hollands Creeks from 2006-2007 scaled down with 65% to account for a possible drier future were used.

iv) *Evaporation and interaction between surface water and ground water.* Some reaches of the Broken River gain water from the ground water while others loose water. For most reaches the ground water discharge is minor, and the dynamics of the interactions are quite slow [44], so surface water/ground water interactions have not been included in the simulation scenario.

In [45] it is estimated that on average  $2.32 \times 10^6 \text{ m}^3$  evaporate yearly from the Broken River. The daily evaporation losses have been modelled as half a period of a sinusoid starting at 9am, reaching its maximum at 3pm and finishing at 9pm, and the amplitude has been scaled linearly with the maximum daily temperature from July 2006 to June 2007. The evaporation losses have been distributed among the reaches and the storage relative to their surface area and lumped together in five spatial evaporation points, one for each reach and the storage.

## VII. EVALUATION OF THE CONTROL SYSTEMS

As the purpose of the study is to demonstrate the benefit of control compared to current practice, manual operation was considered as the baseline case. In the simulation  $30.92 \times 10^6 \text{ m}^3$  of water was released from Lake Nillahcootie as shown in Table VII.  $8.122 \times 10^6 \text{ m}^3$  of the released water was excess water which is water leaving the study area at Gowangardie Weir which is neither required by the irrigators nor the environment. This water eventually ends up in the Goulburn River, and it is not "wasted" water. As an additional  $0.2315 \text{ m}^3/\text{s}$  (20ML/day) is released under manual operations there were only a few days where the environmental minimum flows were violated as can be seen from Table VII. However, the limits on the daily flow variations were often exceeded, e.g. they were violated on 52 days at Gowangardie Weir. Apart from ten days at Broken Weir and five days at Lake Benalla all water levels were within 15cm from setpoints. In the period from early spring to summer the flow exceeded  $1.3889 \text{ m}^3/\text{s}$  (120ML/day) on 18 days in some parts of the river, see Table VIII.

TABLE VII

TOTAL WATER RELEASED, EXCESS WATER, NUMBER OF DAYS WITH FLOW VIOLATIONS WHEN THE EMPHASIS WAS ON REDUCING THE RELEASES FROM LAKE NILLAHCOOTIE.

| Location  | Manual  | Decentralised |         | MPC      |       |
|---|---------|---------------|---------|----------|-------|
|   |         | no Casey      | Casey   | no Casey | Casey |
| Total water released ( $\times 10^6$ m <sup>3</sup> ) | 30.92   | 25.84         | 25.76   | 26.94    | 26.41 |
| Excess water ( $\times 10^6$ m <sup>3</sup> )         | 8.122   | 2.315         | 2.235   | 3.415    | 2.884 |
| $Q_G < 80\%Q_{min}$                                   | 1 day   | 10 days       | 0 day   | 1 day    | 0 day |
| $Q_G < 0.76Q_{G,prev}$ or $> 1.5Q_{G,prev}$           | 52 days | 63 days       | 25 days | 0 day    | 0 day |
| $Q_C < 0.76Q_{C,prev}$ or $> 1.5Q_{C,prev}$           | 12 days | 26 days       | 34 days | 0 day    | 0 day |
| $Q_{LB} < 0.76Q_{LB,prev}$ or $> 1.5Q_{LB,prev}$      | 6 days  | 14 days       | 9 days  | 0 day    | 0 day |
| $Q_B < 0.76Q_{B,prev}$ or $> 1.5Q_{B,prev}$           | 17 days | 0 day         | 0 day   | 0 day    | 0 day |

TABLE VIII

NUMBER OF DAYS THE WATER LEVELS WERE MORE THAN 15 CM BELOW/ABOVE SETPOINTS AND THE FLOWS WERE GREATER THAN 1.3889 M<sup>3</sup>/S (120ML/DAY).

| Location                 | Broken Weir |      | Lake Benalla |      | Casey's Weir |      |
|--------------------------|-------------|------|--------------|------|--------------|------|
|                          | Level       | Flow | Level        | Flow | Level        | Flow |
| Manual                   | 0/10        | 6    | 0/5          | 12   | 0/0          | 0    |
| Decentralised (no Casey) | 0/0         | 0    | 0/15         | 5    | 0/0          | 0    |
| Decentralised            | 0/0         | 0    | 0/11         | 3    | 0/0          | 0    |
| Decentralised (2.5 days) | 0/0         | 0    | 0/1          | 9    | 8/1          | 0    |
| MPC (no Casey)           | 0/0         | 0    | 0/0          | 0    | 0/0          | 0    |
| MPC                      | 0/0         | 0    | 0/0          | 0    | 0/0          | 0    |
| MPC (2.5 days)           | 0/0         | 0    | 0/0          | 3    | 0/0          | 0    |
| MPC (1 days)             | 0/0         | 0    | 0/0          | 6    | 0/0          | 0    |

#### A. Decentralised control

Next it was investigated what a decentralised control system could achieve using only the existing civil engineering infrastructure and the rain rejection storage, (i.e. there was no regulation at Casey's Weir). The ordering times were four days as before, and the emphasis was on minimising the releases from Lake Nillahcootie. The flow setpoint at Gowangardie Weir was set as the sum of the orders placed downstream of Gowangardie Weir plus 0.3472 m<sup>3</sup>/s which corresponds to the minimum flow requirement of 0.2894 m<sup>3</sup>/s with an extra 20% added. Due to the long time delay at the downstream end of the river, an additional 20% is added to the feedforward flow computed as described in Section V-C with the aim to improve the performance. In this case only 25.84  $\times 10^6$  m<sup>3</sup> was released, and the excess water was reduced to 2.315  $\times 10^6$  m<sup>3</sup> as can be seen from Table VII. Not surprisingly, this reduction came at the expense of more violations of the minimum environmental flows. At Gowangardie Weir the flow was below minimum environmental flow on 23 days, but the flow was less than 80% of the minimum environmental flow for only 10 of these days, and there were no violations of the minimum environmental flows at the other locations. The limits on the daily flow variations were violated a bit more often than under manual operation (e.g. 63 vs 52 days at Gowangardie Weir). In the spring/summer period the flow exceeded 1.3889 m<sup>3</sup>/s in some parts of the river on five days, see Table VIII.

As expected, when the flow at Casey's Weir can be regulated, the performance of the decentralised control scheme improved. The setpoint at Gowangardie Weir was computed as described above but there was no flow added to the feedforward term, i.e. the feedforward flow was now computed

as described in Section V-C. In this case 25.76  $\times 10^6$  m<sup>3</sup> of water was released which is slightly lower than without regulation at Casey's Weir. There were far fewer violations of the constraints. There were no violation of the minimum environmental flow constraints at Gowangardie Weir. This is due to that the flow can now be regulated at Casey's Weir, and faster responses at Gowangardie Weir can be achieved. Moreover, there were only 25 days for which the daily flow variations exceeded the limits at Gowangardie Weir and only 3 days in the spring/summer period where the flow exceeded 1.3889 m<sup>3</sup>/s in some parts of the river.

Due to the large offtakes, the feedforward terms in the control dominate over the feedback terms. The feedforward terms are also the main reason for the violations of the constraints on the daily flow variation. These breaches can be reduced by imposing saturation limits, but such limits will also cause more frequent large deviations in the water levels, particularly at Casey's Weir.

Next the emphasis was changed from releasing less water to reducing the ordering times for the irrigators. The aim was now to deliver the same amount of water to the Goulburn River as under manual operation. As the amount of excess water was reduced with 5.957  $\times 10^6$  m<sup>3</sup> using decentralised control, the flow setpoint at Gowangardie Weir was increased with 0.1889 m<sup>3</sup>/s (16.32ML/day) which over a year adds up to 5.957  $\times 10^6$  m<sup>3</sup>. The ordering time was reduced to 2.5 days. In this case, as expected, a similar volume of water as with manual operation was released (see Table IX), but only 2.165  $\times 10^6$  m<sup>3</sup> was regarded as excess water since it was a deliberate operational decision that 5.957  $\times 10^6$  m<sup>3</sup> should be delivered to the Goulburn River and not just a byproduct of the operational procedures as it was under manual operation. The number of days the constraints on the flow variations were violated has been reduced as can be seen from Table IX. The other results were fairly similar to the case where the emphasis was on minimising the releases. The main difference in the negative direction was that the water level at Casey's Weir was more than 15cm below setpoints on eight days compared to none in the other cases.

#### B. MPC

The scenarios above were repeated with MPC. In the first scenario, using only the existing infrastructure and 4 days ordering times, 26.94  $\times 10^6$  m<sup>3</sup> was released from Lake Nillahcootie and the amount of excess water was 3.415  $\times 10^6$  m<sup>3</sup>. The flow at Gowangardie Weir was below minimum

TABLE IX

TOTAL WATER RELEASED, EXCESS WATER, NUMBER OF DAYS WITH FLOW VIOLATIONS FOR DIFFERENT ORDERING TIMES AND CONTROL STRATEGIES.

| Ordering Time   | 4 days  |               |        | 2.5 days      |        | 1 day  |
|---|---------|---------------|--------|---------------|--------|--------|
| Control Strategy                                      | Manual  | Decentralised | MPC    | Decentralised | MPC    | MPC    |
| Total water released ( $\times 10^6$ m <sup>3</sup> ) | 30.920  | 25.760        | 26.409 | 31.647        | 31.081 | 31.165 |
| Excess water ( $\times 10^6$ m <sup>3</sup> )         | 8.122   | 2.235         | 2.884  | 2.165         | 2.318  | 2.403  |
| $Q_G < 80\%Q_{min}$                                   | 1 day   | 0 days        | 0 day  | 0 day         | 0 day  | 0 day  |
| $Q_G < 0.76Q_{G,prev}$ or $> 1.5Q_{G,prev}$           | 52 days | 25 days       | 0 day  | 6 days        | 0 day  | 0 day  |
| $Q_C < 0.76Q_{C,prev}$ or $> 1.5Q_{C,prev}$           | 12 days | 34 days       | 0 day  | 15 days       | 0 day  | 0 day  |
| $Q_{LB} < 0.76Q_{LB,prev}$ or $> 1.5Q_{LB,prev}$      | 6 days  | 9 days        | 0 day  | 4 days        | 0 day  | 0 day  |
| $Q_B < 0.76Q_{B,prev}$ or $> 1.5Q_{B,prev}$           | 17 days | 0 day         | 0 day  | 0 day         | 0 day  | 0 day  |

environmental flow for 36 days, but the flow was less than 80% of the minimum environmental flow for only 1 day. There were no violations of minimum environmental flows at the other locations. Moreover, there were no violations of the limits on the daily flow variations, all water levels were within 15cm of setpoints and the flow in spring/summer did not exceed 1.3889 m<sup>3</sup>/s in any reach demonstrating MPC's abilities to handle constraints.

When the flow at Casey's Weir could be regulated, an improvement in performance was observed. The release from Lake Nillahcootie was  $26.409 \times 10^6$  m<sup>3</sup>, and the excess water was  $2.884 \times 10^6$  m<sup>3</sup>. The flow at Gowangardie Weir was now below minimum environmental flow on 18 days, but the flow was never less than 80% of the minimum environmental flow. All other results were similar to when the flow over Casey's Weir could not be regulated.

In the second scenario, the ordering times were reduced to 2.5 and 1 day while delivering the same amount of water to the Goulburn River as under manual operation. In order to supply the additional water, the flow setpoint at Gowangardie Weir was increased with 0.1661 m<sup>3</sup>/s (14.35ML/day). For the ordering times of 2.5 and 1 days, the excess water was  $2.318 \times 10^6$  m<sup>3</sup> and  $2.403 \times 10^6$  m<sup>3</sup> respectively. The flow at Gowangardie Weir was below the minimum environmental flow on 4 and 6 days respectively for 2.5 and 1 day ordering which are significantly fewer than before. This is expected since the flow setpoint is increased. All water levels were within 15cm of setpoints on all days, see Table VIII. The flow was above 1.3889 m<sup>3</sup>/s for at most 6 days at Lake Benalla, but the maximum flow on these days was only 1.4815 m<sup>3</sup>/s.

### C. Discussion

From the above results one can see the benefit of control. With control one can either reduce the releases or the ordering times for the irrigators. Most importantly, control allows operators to command a larger volume of water and make operative decisions on how it should be used. The results also demonstrate the excellent abilities of MPC to deal with constraints. If we use water levels being more than 15cm below setpoint as a proxy for not being able to supply water, then we had 100% satisfaction in demand from irrigators, apart from under decentralised control with 2.5 days ordering time in which case the water level at Casey's Weir dropped below 15cm on 8 days.

The ordering times that can be achieved are very dependent on the flows in the river and the orders from the irrigators, and

as mentioned in Section III-A, the control system should be supplemented with a supervisory system which at the time of ordering declines or reschedules water orders if they cannot be delivered without breaching the environmental flow limits.

The simulation study showed that control systems allow for a more accurate and timely delivery of water to irrigators while ensuring that the environmental and ecological water needs are satisfied. However, in order to implement the control system, improved infrastructure is required. A valid question is then whether investment of capital funds in a control system is worthwhile compared to the current manual operating system. As part of the FRM project, an investment case was carried out, and the results showed that the control system would likely generate a large benefit to the Broken River catchment community, (see [46]). However, the study was based only on the amount of water "saved", since the benefit of shorter ordering times and improved environmental outcomes, which are likely to be the most significant benefits, are difficult to quantify in monetary terms.

## VIII. CONCLUSION

Management of natural water resources is becoming increasingly important, and in this paper we have considered modelling and control of the Broken River with the aim of improving the environmental outcomes and the service to irrigators. By experimental validation it was found that simple delay and integrator delay models which were identified from observed data, captured the important dynamics for control. These models are much easier to use for control design than the Saint Venant equations. Both decentralised and centralised controllers were designed using the delay and integrator delay models. Through a realistic year long simulation scenario it was found that a control system offers significant advantages compared to the current manual operation. In particular, it was found that a control system allows the operators to command a larger portion of the water resource with the benefit that releases and/or ordering times for the irrigators can be reduced. As expected, additional actuation points improved the performance, and MPC performed better than decentralised control, particularly when it came to satisfying constraints on the water levels and flows.

## ACKNOWLEDGMENT

This work was supported by The Farms Rivers and Markets Project, an initiative of Uniwater and funded by the National Water Commission, the Victorian Water Trust, The Dookie



Farms 2000 Trust (Tallis Trust) and the University of Melbourne and supported by the Departments of Sustainability and Environment and Primary Industry, the Goulburn Broken Catchment Management Authority and Goulburn-Murray Water. The first author also gratefully acknowledges the financial support from National ICT Australia (NICTA). NICTA is funded by the Australian Government as represented by the Department of Broadband, Communications and the Digital Economy and the Australian Research Council through the ICT Centre of Excellence program.

## REFERENCES

- [1] Commonwealth Environmental Water, "Commonwealth Environmental Water," Available via <http://www.environment.gov.au/ewater/index.html>. Cited 25 Dec 2012., 2012.
- [2] FRM. <http://www.frm.unimelb.edu.au/default.htm>. Cited 25 Dec 2012., Farms, Rivers and Market Project, 2012.
- [3] M. Cantoni, E. Weyer, Y. Li, S. K. Ooi, I. Mareels, and M. Ryan, "Control of large-scale irrigation networks," *Proceedings of IEEE Special Issue on the Technology of Networked Control Systems*, vol. 95, pp. 75–91, 2007.
- [4] I. Mareels, E. Weyer, S. K. Ooi, M. Cantoni, Y. Li, and G. Nair, "Systems engineering for irrigation systems: Successes and challenges," *Annual Reviews in Control*, vol. 29, pp. 191–204, August 2005.
- [5] S. K. Ooi and E. Weyer, "Control design for an irrigation channel from physical data," *Control Engineering Practice*, vol. 16, pp. 1132–1150, 2008.
- [6] E. Weyer, "System identification of an open water channel," *Control Engineering Practise*, vol. 9, pp. 1289–1299, 2001.
- [7] E. Weyer, "Control of irrigation channels," *IEEE Transactions on Control Systems Technology*, vol. 16, pp. 664–675, 2008.
- [8] O. Fovet, G. Belaud, X. Litrico, S. Charpentier, C. Bertrand, P. Dollet, and C. Hugodot, "A model for fixed algae management in open channels using flushing flows," *River Research and Applications*, 2011.
- [9] Y. Li, "Robust control of open water channels," Ph.D. Thesis, Department of Electrical and Electronic Engineering, The University of Melbourne, 2006.
- [10] H. Linke, "A model-predictive controller for optimal hydro-power utilization of river reservoirs," *Proceedings of IEEE Multiconference on Systems and Control, Yokohama, Japan*, pp. 1868–1873, 2010.
- [11] A. Sahin and M. Morari, "Decentralized model predictive control for a cascade of river power plants," *Intelligent Systems, Control and Automation: Science and Engineering*, vol. 42, pp. 463–486, 2010.
- [12] P.-J. van Overloop, R. Negenborn, B. De Schutter, and N. van de Giesen, "Predictive control for national water flow optimization in The Netherlands," in *Intelligent Infrastructures*, R. R. Negenborn, Z. Lukszo, and H. Hellendoorn, Eds. Dordrecht, The Netherlands: Springer, 2010, pp. 439–461.
- [13] A. Castelletti, F. Pianosi, and R. Soncini-Sessa, "Receding horizon control for water resources management," *Applied Mathematics and Computation*, vol. 204, no. 2, pp. 621–631, 2008.
- [14] X. Litrico, "Robust IMC Flow Control of SIMO Dam-River Open-Channel Systems," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 432–437, 2002.
- [15] B. Sohlberg and M. Sernfält, "Grey box modelling for river control," *Journal of Hydroinformatics*, vol. 4, pp. 265–280, 2002.
- [16] M. Papageorgiou and A. Messmer, "Flow control of a long river stretch," *Automatica*, vol. 25, 1989.
- [17] B. Foss, J.E. Haug, J. Alne and S. Aam, "User experience with on-line predictive river flow regulation," *IEEE Transactions on Power Systems*, vol. 4, no. 3, pp. 1089–1094, 1989.
- [18] J. Maciejowski, *Predictive control with constraints*. Prentice Hall, 2002.
- [19] Bulk Entitlement, "Bulk entitlements and environment entitlements, Victorian Water Register, the Department of Sustainability and Environment," Available via <http://www.waterregister.vic.gov.au/public/reports/bulkentitlements.aspx>. Cited 25 Dec 2012, 2012.
- [20] B. Farquharson, T. Ramilan, M. Stewardson, C. Beverly, G. Vietz, B. George, D. Dassanayake, and M. Sammonds, "Water sharing for the environment and agriculture in the Broken catchment," *55th Annual AARES National Conference, Melbourne, Australia*, pp. 1–20, 2011.
- [21] M. Stewardson, "Private communication with Michael Stewardson (FRM Rivers subproject) of Department of Infrastructure Engineering, The University of Melbourne." 2010.
- [22] M. Foo, "Modelling and control design of river systems," Ph.D. Thesis, Department of Electrical and Electronic Engineering, The University of Melbourne, 2012.
- [23] P.C. Young and M. Ratto, "Statistical Emulation of Large Linear Dynamic Models," *Techometrics*, vol. 53, no. 1, pp. 29–43, 2011.
- [24] A. Castelletti, S. Galelli, M. Ratto, R. Soncini-Sessa and P.C. Young, "A general framework for Dynamic Emulation Modelling in environmental problems," *Environmental modelling and Software*, vol. 34, pp. 5–18, 2012.
- [25] M. H. Chaudhry, *Open-Channel Flow*. Prentice Hall, 1993.
- [26] GBCMA, "<http://www.gbcma.vic.gov.au/>, Goulburn-Broken Catchment Management Authority, Australia." Tech. Rep., 2009.
- [27] Goulburn Murray Water, "North East Region Monitoring Site Summary Report." Tech. Rep., 2004.
- [28] M. Foo, N. Bedjaoui, and E. Weyer, "Segmentation of a river using the Saint Venant equations," *Proceedings of IEEE Multiconference on Systems and Control, Yokohama, Japan*, pp. 848–853, 2010.
- [29] M. Papageorgiou and A. Messmer, "Continuous-time and discrete-time design of water flow and water level regulators," *Automatica*, vol. 21, no. 6, pp. 649–661, 1985.
- [30] X. Litrico and D. Georges, "Robust continuous-time and discrete-time flow control of a dam-river system. (i) Modelling," *Applied Mathematical Modelling*, vol. 23, pp. 795–805, 1999.
- [31] X. Litrico, V. Fromion, J-P. Baume, C. Arranja and M. Rijo, "Experimental validation of a methodology to control irrigation canals based on Saint-Venant equations," *Control Engineering Practice*, vol. 13, no. 11, pp. 1425–1437, 2005.
- [32] M. Maxwell and S. Warnick, "Modelling and identification of the Sevier River system," *Proceedings of the American Control Conference*, 2006.
- [33] M. Bos, *Discharge measurement structures*. International Institute for Land Reclamation and Improvement/ILRI, Wageningen, The Netherlands, 1978.
- [34] X. Litrico and J.-B. Pomet, "Nonlinear modelling and control of a long river stretch," *Proceedings of the 2003 European Control Conference, Cambridge, UK*, 2003.
- [35] M. Thomassin, T. Bastogne, and A. Richard, "Identification of a managed river reach by a Bayesian approach," *IEEE Transactions on Control System Technology*, vol. 17, no. 2, pp. 353–365, 2009.
- [36] L. Ljung, *System Identification: Theory For The User*, 2nd Ed. Prentice Hall, 1999.
- [37] K. Ogata, *Modern Control Engineering*, 5th Ed. Prentice Hall, 2009.
- [38] S. K. Ooi, M. Foo, and E. Weyer, "Control of the Broken River," *Proceedings of the IFAC 18th World Congress, Milan, Italy*, pp. 627–632, 2011.
- [39] D. Chmielewski and V. Manousiouthakis, "On constrained infinite-time linear quadratic optimal control," *Systems and Control Letters*, vol. 29, pp. 121–129, 1996.
- [40] J. Löfberg, "Yalmip : A toolbox for modeling and optimization in MATLAB®," *Proceedings of CACSD Conference, Taipei, Taiwan*, 2004. [Online]. Available: <http://users.isy.liu.se/johanl/yalmip>
- [41] IBM ILOG CPLEX, "<http://www-01.ibm.com/software/integration/optimization/cplex-optimizer>, IBM ILOG CPLEX Optimizer," Tech. Rep., 2010.
- [42] M. Foo, S. K. Ooi, and E. Weyer, "Modelling of river for control design," *Proceedings of IEEE Multiconference on Systems and Control, Yokohama, Japan*, pp. 1862–1867, 2010.
- [43] M. Foo, S. Ooi, and E. Weyer, *Modelling of Rivers for Control Design* Chapter 21 in System Identification, Environmental Modelling and Control System Design. Eds: Wang, L., Garnier, H. and Jakeman, T. . Springer, pp. 423–447, 2012.
- [44] J.F. Costelloe, G. Sites, J. Moreau, J. and A. Western, "Using hydraulic and chemical data to determine groundwater contribution to the Broken River, Victoria", *34th IAHR World Congress, Brisbane, Australia*, pp. 1611-1618, 2011.
- [45] Sinclair Knight Merz, "Broken River and Broken Creek loss reduction concept study stage 1 report," Tech. Rep., 2005.
- [46] J. Langford, Ed., *Farms, Rivers and Markets Project 2012, Farms, Rivers and Markets: Overview Report, Victoria*. The University of Melbourne, 2012.