Laminar-turbulent boundary-layer transition over a rough rotating disk

F. Zoueshtiagh, R. Ali, A. J. Colley, P. J. Thomas, and P. W. Carpenter

Original citation:
Zouestiagh, F; Ali, R; Colley, A.J; Thomas, P.J and Carpenter, P.W. (2003) Laminar-turbulent boundary-layer transition over a rough rotating disk. Physics of Fluids 15, 2441 (2003); doi: 10.1063/1.1586916

http://dx.doi.org/10.1063/1.1586916

AIP Publishing

Laminar-turbulent boundary-layer transition over a rough rotating disk

F. Zoueshtiagh, R. Ali, A. J. Colley, P. J. Thomas, and P. W. Carpenter

Citation: Physics of Fluids **15**, 2441 (2003); doi: 10.1063/1.1586916
View online: http://dx.doi.org/10.1063/1.1586916
View Table of Contents: http://aip.scitation.org/toc/phf/15/8
Published by the American Institute of Physics

Laminar-turbulent boundary-layer transition over a rough rotating disk

F. Zoueshtiagh[a)]
Laboratoire de Physique et Mécanique des Milieux Hétérogènes (PMMH) UMR 7636 CNRS-ESPCI, Ecole Supérieure de Physique et Chimie Industrielles (ESPCI) 10, rue Vauquelin, 75231 Paris Cedex 05, France

R. Ali,[b)] A. J. Colley, P. J. Thomas,[c)] and P. W. Carpenter[d)]
Fluid Dynamics Research Centre, School of Engineering, University of Warwick, Coventry CV4 7AL, United Kingdom

(Received 17 December 2002; accepted 6 May 2003; published 2 July 2003)

Boundary-layer transition over a disk spinning under water is investigated. Transitional Reynolds numbers, Re_c, and associated boundary-layer velocity profiles are determined from flow-visualizations and hot-film measurements, respectively. The value of Re_c and the velocity profiles are studied as a function of the disk's surface roughness. It is found that transition over rough disks occurs in a similar fashion to that over smooth disks, i.e., abruptly and axisymmetrically at well-defined radii. Wall roughness has little effect on Re_c until a threshold relative roughness is reached. Above the threshold Re_c decreases sharply. The decrease is consistent with the drop one expects for our flow for the absolute instability discovered by Lingwood [J. Fluid Mech. **299**, 17 (1995); **314**, 373 (1996); **331**, 405 (1997)]. This indicates that the Lingwood absolute instability may continue to play a major role in the transition process even for large relative roughness. [DOI: 10.1063/1.1586916]

The influence of surface roughness on boundary-layer flow has been extensively studied for various geometries, such as flat plates and pipes. See Floryan[1] for a review of the effect of surface roughness on the stability of flow over flat plates and in channels. Here, we investigate the effects of roughness on laminar-turbulent transition in a rotating-disk flow.

The conceptually simple rotating-disk flow, with its undisturbed flow field given by the similarity solution derived by Kármán,[2] has long been the classic paradigm for experimental, theoretical and computational studies of three-dimensional boundary-layer instability and transition. The dominant instability is the Type I cross-flow vortex originally described by Gregory *et al.*[3] Extensive reviews are given by Reed and Saric,[4] and more recently by Cooper and Carpenter.[5] With regard to the effect of surface roughness on transition over rotating disks, to our knowledge the only relevant studies are those by Granville[6] and more recently Spalart.[7] The former analyzes the velocity profiles on the basis of three-dimensional boundary-layer theory. Similarity-law correlations for various types of irregular roughness at high shearing stresses are calculated and show excellent agreement with the experimentally obtained velocity profiles of Theodorsen and Regier.[8] Spalart[7] conducted direct numerical simulations of the flow with the aim to study cross-flow vortices. He briefly reports on the effect of "random stationary disturbances" that he suggests is analogous to natural roughness.

Our experimental arrangement (Fig. 1) is similar to that of Colley *et al.*[9] It consists of a disk of diameter 388 mm, which spins with a rotational speed, Ω_D, about a vertical axis of rotation in filtered water. The disk is mounted within a tank of diameter 1 m and height of 650 mm. For the purposes of flow visualization, a circular lid, of diameter 555 mm, is placed over the disk. This prevented disturbances forming on the water surface. The gap between the lid and the disk surface is 20 mm.

Disks of different roughness levels were manufactured by permanently attaching appropriately sized quartz granules to the surface of a circular smooth glass disk. Each disk can be mounted on the rotating support structure. The granules were sieved using a sieve shaker (Endecott, Octagon 2000) to obtain three different grades of roughness. The three sample grain diameter sizes chosen were $d_G = 170 \pm 10$, 335 ± 20, and 1325 ± 75 μm. The experimental data for the sand-coated, rough disks are compared with data for the flow over smooth and spray-painted glass disks. The surface finish of these two disks was determined with a Rank–Taylor–Hobson Talysurf facility in a similar manner to Colley *et al.*[9] The average surface roughness of the smooth glass disk is 0.006 ± 0.003 μm and that of the spray-painted disk is 7.15 μm with highest elevation of $+35.27$ μm and lowest recesses of -27.38 μm. Note that natural roughness has a random distribution in terms of the distribution, density, and roughness height of the grains. However, experimentally no meaningful work can be conducted without some control over at least one or more of these variables. In light of this, our method effectively allows control over the roughness height. The distribution and density of the grains remains random.

Concentrated Kalliroscope fluid was added to the water inside the tank. This facilitated flow visualization in conjunction with a horizontal light sheet illuminating the fluid in a

[a)]Electronic mail: farzam@pmmh.espci.fr
[b)]Electronic mail: R.Ali@warwick.ac.uk
[c)]Electronic mail: pjt@eng.warwick.ac.uk
[d)]Electronic mail: pwc@eng.warwick.ac.uk

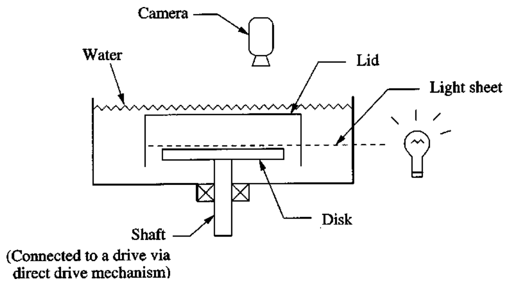

FIG. 1. Sketch illustrating experimental setup.

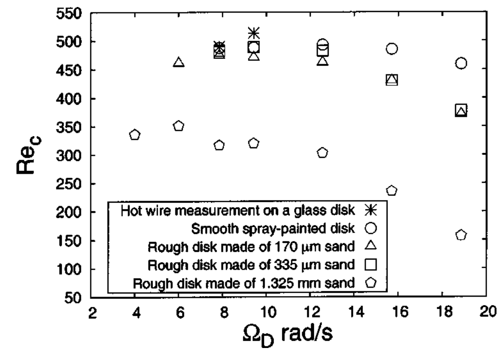

FIG. 2. Variation of transitional Reynolds number, Re_c, as a function of rotational velocity, Ω_D, for smooth and rough disks.

plane adjacent and parallel to the disk surface (Fig. 1). The fluid motion was captured by a video camera mounted above the tank.

For the hot-film measurements, the lid was replaced with an annular shroud. This surrounded the disk along its periphery. This configuration was chosen following an earlier study[10] which revealed that it permitted much cleaner signals to be obtained with the hot-film probe with no perceptible change in the time-averaged flow field. The water depth above the disk's surface was set at 295 mm. Measurements were made in filtered tap water with the filtering system capable of removing particles larger than 0.4 μm in diameter. The water temperature was 18.1 °C with temperature fluctuations less than ± 0.5 °C. A TSI IFA 300 constant-temperature hot-film anemometry system was used. All experiments were carried out with a single-sensor probe previously calibrated in the water tank. The radial and vertical location of the hot film probe was determined with a computer-controlled traverse system to an accuracy of less than 5 μm with a repeatability of ± 0.5 μm. Data acquisition was triggered by the optoelectronic detection of a reference timing mark on the shaft connected to motor driving the disk. For the measurements of the radial velocity component the sensor of the probe was aligned parallel to the disk surface at a right angle to the disk radius. Correspondingly, the sensor was oriented radially for measuring the azimuthal velocity component. Each component of the velocity was evaluated from an average of 60 measurements.

The flow variables are nondimensionalized in the usual manner. The laminar boundary-layer displacement thickness, $\sqrt{\nu/\Omega_D}$ (where ν is the kinematic viscosity of the fluid), and the local linear velocity of the disk, $r\Omega_D$ (where r is the radial coordinate), are used as reference quantities; so that the dimensionless wall-normal and radial coordinates are written as

$$\zeta = z\sqrt{\Omega_D/\nu}, \tag{1a}$$

$$\mathrm{Re} = r\sqrt{\Omega_D/\nu}, \tag{1b}$$

where the notation Re reflects the practice of regarding the nondimensional radial coordinate as a Reynolds number. The radial and azimuthal time-averaged velocity components are made nondimensional as follows:

$$F(\zeta) = v_r/(\Omega_D r), \tag{2a}$$

$$G(\zeta) = v_\phi/(\Omega_D r). \tag{2b}$$

The theoretical values for $F(\zeta)$ and $G(\zeta)$ for the laminar boundary layer can be found from the von Kármán similarity solution in tabulated form in Owen and Rogers.[11]

Lingwood[12] observed that previous experimental investigations of transition over rotating disks showed transition as occurring abruptly at a well-defined circular boundary. Moreover all the transitional Reynolds numbers reported were within a few percent of 513. She suggested that this was evidence of the role of absolute instability in the transition process. She then went on to demonstrate, both theoretically and experimentally, that the rotating-disk flow is absolutely unstable[12,13] with a critical Reynolds number close to the experimentally observed transition point. Our experiments show that, similar to smooth disks, transition over rough rotating disks still occurs abruptly and axisymmetrically at well-defined radii. Video recordings of the flow visualization show a visible circular boundary separating the flow in the central part of the disk from the flow further outwards. Unfortunately the quality of the flow-visualization recordings does not permit them to be reproduced here. Nor was the flow visualization good enough to discern fine features such as the cross-flow vortices inboard of the transition point. Nevertheless, the circular boundary was qualitatively similar to those typically seen over smooth disks in the previous experimental studies cited by Lingwood.[12] Laminar-turbulent transition occurs in the vicinity of this boundary. This was verified by comparing the Reynolds number obtained from the flow visualization and hot-film measurements for spray-painted and uncoated glass disks. The hot-film measurements over the uncoated glass disk gave transitional Reynolds number of 497 ± 7 and 513 ± 7.5 for disk speeds of 7.85 and 9.42 rad/s, respectively. In comparison, the video recordings revealed that the visible boundary over the spray-painted disk was located at Reynolds numbers of around 488 for both rotational speeds. The agreement between the figures obtained from the two different evaluation techniques shows that the transitional Reynolds number can be determined with sufficient accuracy from a visual inspection of the video recordings. The estimated maximum error associated with measuring the location of the boundary from the recordings is approximately 4% for low rotation rates around 4 rad/s and up to 20% for the highest rotation rates around 18.85 rad/s.

Figure 2 shows the video-determined values of the transitional Reynolds number Re_c as a function of the rotation rate Ω_D. The transition-point data for the three sand-coated disks are compared to those for the smooth, spray-painted disk. In addition two available data points obtained from hot-film measurements for flow over the plain glass disk are also included. As expected, the transitional Reynolds number for the smooth disk remains fairly constant with rotational speed up to 16 rad/s. The last data point at 18.85 rad/s is considerably lower. This is indicative of increased background disturbance at this highest rotational speed, so the data for 18.85 rad/s should probably be disregarded.

For rough disks one can define a relative roughness height as the ratio of mean grain diameter and boundary-layer thickness given by $d_G/\sqrt{\nu/\Omega_D}$. The boundary-layer thickness decreases with rising rotational speed and, hence, the relative roughness height increases correspondingly. For rough disks the transitional Reynolds number must be expected to begin to decrease once a certain critical value of the relative roughness is exceeded. Figure 2 reveals that first significant differences between data for a smooth disk and a disk covered with the smallest particles of diameters $d_G = 170\ \mu$m appear above approximately $\Omega_D = 12.5$ rad/s. Using $\nu = 1$ mm^2/s one then finds a value of 0.6 as the estimate for the critical relative roughness height. The value probably represents an overestimate because differences between smooth and rough disks must already exist at $\Omega_D < 12.5$ rad/s where they are, however, below the measurement accuracy.

Schlichting and Gersten[14] and Floryan[1] cite experiments by Feindt,[15] who investigated the influence of distributed roughness on critical transitional Reynolds numbers. Feindt performed measurements in a convergent and a divergent channel of circular cross section with a cylinder covered with sand placed axially in them. Feindt's data[1,14,15] show that for a flow with free-stream velocity U the critical transitional Reynolds number decreases steeply when the value $Ud_G/\nu = 120$ is exceeded. This value can be compared to the present experiment. Figure 2 shows that for $\Omega_D = 12.5$ rad/s the transitional Reynolds number for flow over the disk was of the order of $\mathrm{Re}_c = 475$. Using Eq. (1b) one sees that this corresponds to transition at a radial location $r_c = 134.4$ mm where the azimuthal component of the flow velocity is $v_\phi = \Omega_D r_c = 1680$ mm/s. Using this value of v_ϕ, $\nu = 1$ mm^2/s, and $d_G = 170\ \mu$m one gets $v_\phi d_G/\nu = 286$. Similar to the relative roughness height this is likely to represent an overestimate. The data do not enable one to give a reasonable estimate for the associated error. While the value of 286 is larger than Feindt's[1,14,15] value of 120 for channel flow both values are, however, of comparable magnitude. In that sense our measurements are consistent with Feindt's conclusions which are based on results obtained in a fundamentally different experimental flow configuration.

Figures 3 and 4 show the measured velocity profiles, $F(\zeta)$ and $G(\zeta)$, for the smooth, and 170 and 335 μm sand-roughened disks. The measurements were carried out for different Reynolds numbers at a rotation speed of, $\Omega_D = 7.85$ rad/s. The solid line in each figure represents the von Kármán similarity solution. The velocity-profile data for the smooth disks collected here with the annular shroud in place are almost identical to those obtained by Colley *et al.*[9] on a setup similar to the one depicted in Fig. 1 with a lid over the disk. Hence, this quite substantial change of the experimental

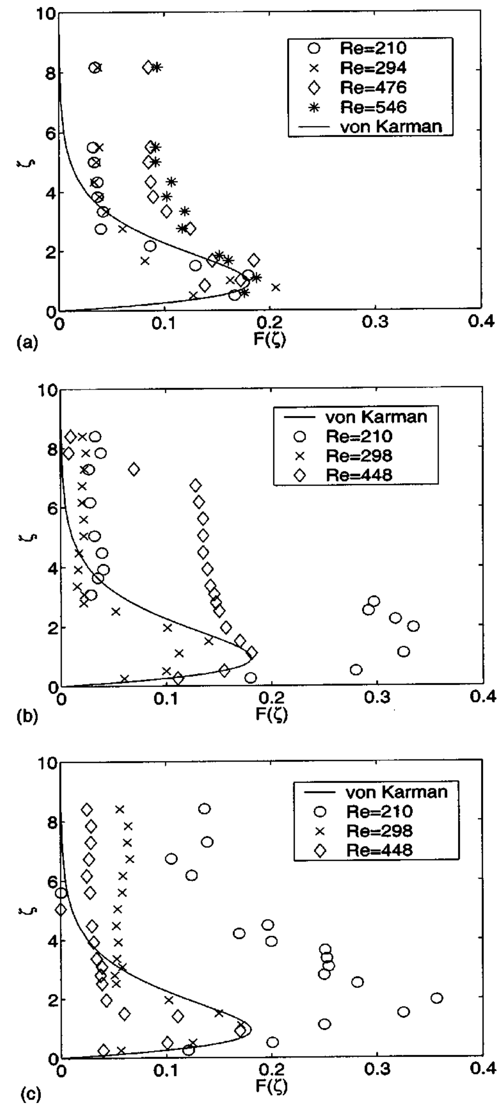

FIG. 3. Hot-wire measurements of the mean velocity profiles in the radial direction for (a) a smooth disk (b) a 170 μm sand roughened disk, (c) a 335 μm sand roughened disk. The solid line shows the profiles derived by the von Kármán similarity solution calculated by Rogers and Lance (Ref. 16).

boundary conditions did not lead to measurable effects on the mean flow field above the disk.

The velocity profiles plotted in Figs. 3 and 4 clearly show that there is some residual rotational motion in the flow outside the boundary layer. We suspect that this residual motion is a feature of all rotating-disk experiments in water. It may be associated with the finite dimensions of the water-filled chambers or drums containing the rotating disk. There have been several previous experiments on disks spinning in water. Mean velocity profiles are, however, not reported in any of them apart from our own.[9] This is no doubt because of the difficulties of calibrating the hot-film probes. We went to considerable trouble and expense to do this (see Ref. 9).

Figure 4 reveals that there is a slight undulatory variation of the azimuthal velocity $G(\zeta)$ for $2<\zeta<4$ and lower values of Re. This variation is also indicative of residual rotational motion. It is consistent with the theory for Kármán flows with finite azimuthal velocity as $\zeta\rightarrow\infty$ (see, e.g., Lingwood[17]). Such flows are characterized by the type of undulatory variation observed here experimentally.

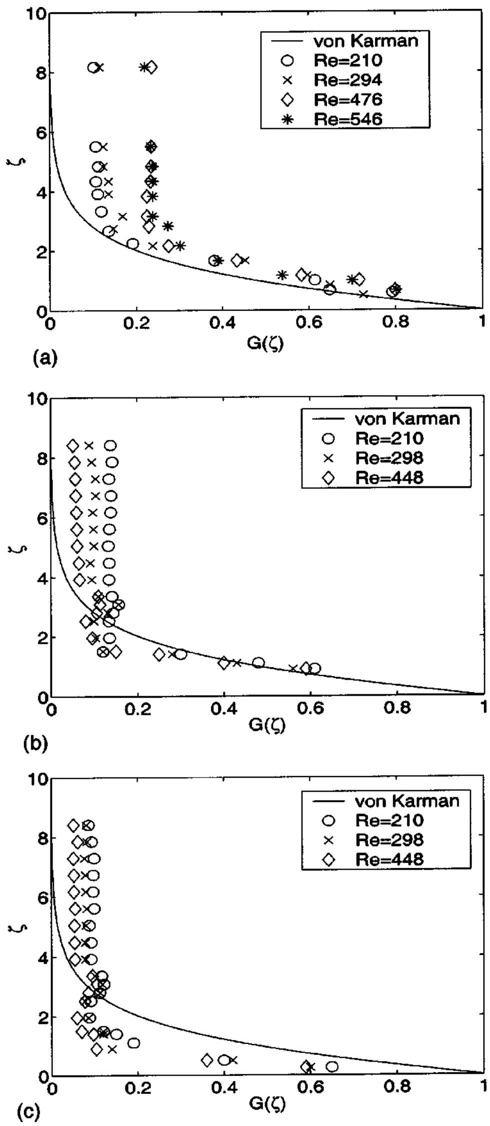

FIG. 4. Hot-wire measurements of the mean velocity profiles in the azimuthal direction for (a) a smooth disk, (b) a 170 μm sand roughened disk, (c) a 335 μm sand roughened disk. The solid line shows the profiles derived by the von Kármán similarity solution calculated by Rogers and Lance (Ref. 16).

It is possible to use Lingwood's[17] theoretical results to make rough estimates of the effect of the residual rotational motion on the critical Reynolds number for absolute instability and therefore, by implication, on the transitional Reynolds number. For the lower values of Re, at least, the ratio of the residual rotational flow speed outside the boundary layer to that of the disk is around 0.1. This corresponds to a Rossby number of $\mathrm{Ro} \approx -0.9$ in Lingwood's theory, as opposed to $\mathrm{Ro} = -1$ for the usual Kármán flow with no residual rotational motion in the bulk of the fluid. For this relatively small difference the critical Reynolds number for absolute instability drops from 507.3 to approximately 471.1, based on linear interpolation between the data for $\mathrm{Ro} = -1$ and $\mathrm{Ro} = -0.8$ of Table 3 in Ref. 17. Thus, we could expect to see a slightly lower transitional Reynolds number when there is residual rotational motion in the flow above the boundary layer. This seems to be reflected in the value of the transitional Reynolds numbers plotted in Fig. 2 for $\Omega_D = 7.85$ rad/s, the value at which the velocity profiles in Figs. 3 and 4 were measured. In this context, it may be worth remarking on the experimental results of Jarre *et al.*[18] Their apparatus was similar to our earlier configuration[9] and so presumably had similar velocity profiles—although the profiles were not measured. The results of Jarre *et al.*[18] indicate that the transitional Reynolds number in their experiments was between 488 and 509.

The main general conclusions of interest are that the wall roughness has little effect on transitional Reynolds numbers of rotating-disk flow until a threshold relative roughness is reached. The magnitude of the threshold relative roughness is similar to the value reported in the literature[1,14] for the experiments of Feindt[15] for channel flow. For relative roughness values above the threshold the transitional Reynolds number decreases sharply, but transition still occurs at a sharp, fixed circular boundary. This suggests that absolute instability continues to play a major role in the transition process even for large relative roughness.

[1] J. M. Floryan, "Stability of wall-bounded shear layers in the presence of stimulated distributed surface roughness," J. Fluid Mech. **335**, 29 (1997).
[2] Th. von Kármán, "Über laminar und turbulente Reibung," Z. Angew. Math. Mech. **1**, 233 (1921).
[3] N. Gregory, J. T. Stuart, and W. S. Walker, "On the stability of three-dimensional boundary-layers with application to the flow due to rotating disk," Philos. Trans. R. Soc. London, Ser. A **248**, 155 (1955).
[4] H. L. Reed and W. S. Saric, "Stability of three-dimensional boundary layers," Annu. Rev. Fluid Mech. **21**, 235 (1989).
[5] A. J. Cooper and P. W. Carpenter, "The stability of the rotating-disk boundary-layer flow over a compliant wall. 1. Type I and Type II instabilities," J. Fluid Mech. **350**, 231 (1997).
[6] P. S. Granville, "The torque and turbulent boundary layer of rotating disks with smooth and rough surfaces, and on drag reducing polymer solutions," J. Ship Res. **17**, 181 (1973).
[7] P. R. Spalart, "On the cross-flow instability near a rotating disk," Proceedings R. A. S. Boundary-layer Transition and Control, Royal Aero. Society, 1991, pp. 22.1–22.13.
[8] Th. Theodorsen and A. Regier, "Experiments on drag of revolving disks, cylinders and streamline rods at high speeds," National Advisory Committee for Aeronautics Technical Report No. 793, 1944.
[9] A. J. Colley, P. J. Thomas, P. W. Carpenter, and A. J. Cooper, "An experimental study of boundary-layer transition over a rotating, compliant disk," Phys. Fluids **11**, 3340 (1999).
[10] A. J. Colley, P. W. Carpenter, and P. J. Thomas, "Experiments on boundary-layer transition over a rotating compliant disc: New experimental method," Abstracts of the Fourth EUROMECH Fluid Mechanics Conference, Eindhoven, Netherlands, 19–23 November 2000, p. 158.
[11] J. M. Owen and R. H. Rogers, *Rotor-Stator Systems*, Flow and Heat Transfer in Rotating-disk Systems Vol. 1 (Research Studies, Taunton, UK, 1989).
[12] R. J. Lingwood, "Absolute instability of the boundary layer on a rotating disk," J. Fluid Mech. **299**, 17 (1995).
[13] R. J. Lingwood, "An experimental study of absolute instability of the rotating-disk boundary-layer flow," J. Fluid Mech. **314**, 373 (1996).
[14] H. Schlichting and K. Gersten, *Boundary-Layer Theory*, 8th revised and enlarged edition (Springer-Verlag, Berlin, 2000).
[15] E. G. Feindt, "Untersuchungen über die Abhängigkeit des Umschlages laminar-turbulent von der Oberflächenrauhigkeit und der Druckverteilung," Diss. Braunschweig, 1956; Jb. 1956 Schiffbautechn. Gesellschaft **50**, 180 (1957).
[16] M. H. Rogers and G. N. Lance, "The rotationally symmeteric flow of a viscous fluid in the presence of an infinite rotating disk," J. Fluid Mech. **7**, 617 (1960).
[17] R. J. Lingwood, "Absolute instability of the Ekman layer and related rotating flows," J. Fluid Mech. **331**, 405 (1997).
[18] S. Jarre, P. Le Gal, and M. P. Chauve, "Experimental analysis of the instability of the boundary layer over a rotating disk," Europhys. Lett. **14**, 649 (1991).