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Properties of input-output Hammerstein-bilinear structure with application to an industrial air handling unit

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Abstract. When developing mathematical models, especially for control, the practical interest lies in relatively simple extensions of linear structures that offer improved modelling capabilities. In this paper a discrete-time input-output Hammerstein-bilinear structure is introduced and its properties are discussed in detail. It consists of a cascade connection of a static nonlinearity followed by a dynamic bilinear system. By combining advantages of constituent subsystems the Hammerstein-bilinear structure allows for both an input dependant dynamic behaviour (particular property of bilinear systems) and an increased flexibility of the steady-state characteristic (particular property of Hammerstein models) to be obtained simultaneously. Modelling capabilities of such structure are evaluated on an air-handling unit that is a part of an industrial heating, ventilation and air-conditioning system.

1. Introduction

Most, if not all, physical systems are inherently nonlinear, hence they can be satisfactorily approximated by standard linear modelling techniques only over a rather narrow range of operation. A need for more flexible structures that allow models to be adequate over wider operating ranges stems, therefore, rather naturally and has become a prominent and an important topic of research in the control community, see for instance [1], [2] and [3]. When developing mathematical models, especially for control, the practical interest lies in relatively simple extensions of linear structures that offer improved modelling capabilities and whose properties are well understood. Among others, the frequently considered nonlinear models include: Hammerstein systems (HS) [4], Wiener systems (WS) [5], bilinear systems (BS) [6] and Lure models [7]. A HS comprises of a static nonlinearity followed by a dynamic linear subsystem, whilst a WS model is a dual of a HS with the order of these elements reversed. A Lure model also consists of the same two elements but with a static nonlinearity located in the feedback. A BS is a structure where the output is a function of a product between the output and the input (or state, in general). Each of these models can be used to mimic certain qualitative behaviour of the process of interest. A detailed comparison and discussion of their specific features can be found in [8]. Although relatively simple, by providing enhanced modelling capabilities and permitting design of efficient dedicated control schemes, these structures have been demonstrated to be of a great pragmatic utility in many industrial control problems, see [9], [10], [11] and [12]. In this paper, a discrete-time input-output Hammerstein-bilinear system (HBS) structure is considered

that consists of a cascade connection of a static nonlinearity followed by a dynamic BS. Such a model structure combines advantages of its constituent subsystems allowing for both an input dependant dynamic behaviour (particular property of BS) and an increased flexibility of the steady-state characteristic (particular property of HS) to be obtained simultaneously. Although the more general class of nonlinear autoregressive with exogenous input (NARX) models, to which the HBS belongs, has received interest in the literature, see for instance [13], [7], [3], the HBS on its own has been given limited attention. This paper aims at improving this situation by focusing directly on the HBS structure and its properties. Also, an exemplary application to industrial problem is presented.
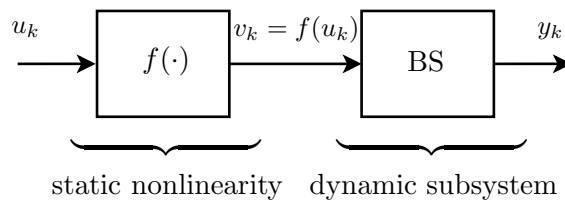
Figure 1. Diagrammatic representation of a Hammerstein-bilinear structure.

2. Definition of Hammerstein-bilinear structure

HBS structure is a member of a general family of NARX models, more specifically, it belongs to a sub-class of models retaining affinity with respect to the output signals, i.e. so-called output affine models [14]. It is constructed as a cascade connection of a static (memoryless) nonlinearity followed by a dynamic time-invariant BS, see Figure 1, and is given by

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{i=1}^{n_b} b_i v_{k-i} + \sum_{j=1}^{n_a} \sum_{i=1}^{n_b} \eta_{ij} v_{k-i} y_{k-j}, \qquad v_k = f(u_k) \qquad (1)$$

with the bilinearity defined as a product between system output y_k and the intermediate input variable v_k, and $f(\cdot)$ being a general scalar static nonlinear function. To facilitate subsequent identification it is advantageous to restrict $f(\cdot)$ to functions that are smooth, linear in the parameters and have continuous derivative. Such functions that are commonly used are polynomials, see [4], [15] and [12], where HS models are considered. (Because the focus is placed on properties of the HBS structure, additional signals modelling disturbances are not considered.) The input u_k, before entering the dynamic BS and yielding the overall system output y_k, undergoes a nonlinear transformation through $f(\cdot)$ to form the intermediate input signal v_k. Not all bilinear coefficients must necessarily be present in (1), hence a particular structure can be obtained by setting selected η_{ij} to zero. Driven by control pragmatism, considerations are restricted here to structures whose dynamic part is not purely bilinear, i.e. not all b parameters are simultaneously null. The HBS can be interpreted as a generalisation of both of its constituent subsystems, i.e. the HS and BS structures. In particular, a BS is obtained from (1) by setting $u_k = v_k$, i.e. by selecting $f(x) = x$, which gives

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{i=1}^{n_b} b_i u_{k-i} + \sum_{j=1}^{n_a} \sum_{i=1}^{n_b} \eta_{ij} u_{k-i} y_{k-j}. \qquad (2)$$

Similarly, a HS is obtained by setting $\eta_{ij} = 0 \ \forall \ i, j$ in (1), which leads to

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{i=1}^{n_b} b_i v_{k-i}, \qquad v_k = f(u_k). \qquad (3)$$

Also, a linear structure can be obtained by imposing both restrictions simultaneously, i.e. $\eta_{ij} = 0 \; \forall i, j$ and $u_k = v_k$.

The HS structure is of particular interest when modelling processes with nonlinear actuators such as flow controlling valves. Also, more generally, it is useful in situation where the dynamic behaviour can be approximated sufficiently well via a linear model, whilst nonlinearity is manifested in the overall steady-state characteristic, see [4] and [15]. A BS model structure is useful when dealing with applications where there is heat exchange and/or transfer of heat is involved, see [16] and [17]. As well, there are examples of BS arising naturally that can be found when modelling various chemical processes, where, quite commonly the exogenous inputs are flow rates.

Because the HBS structure comprises HS and BS as its special cases it is applicable in situations where both the steady-state characteristic and the dynamics are nonlinear. The HBS can, alternatively, be expressed as the following NARX difference equation

$$y_k = \sum_{j=1}^{n_a} a_j y_{k-j} + \sum_{i=1}^{n_b} b_i f(u_{k-i}) + \sum_{j=1}^{n_a} \sum_{i=1}^{n_b} \eta_{ij} f(u_{k-i}) y_{k-j}. \tag{4}$$

By postulating that the function $f(\cdot)$ is linear in the parameters, denoted by α, the difference equation (4), becomes bilinear in both i.e. input-output and its coefficients, i.e. a, b, η and α. This property significantly facilitates the parameter estimation problem. Analogously to the classification of input-output BS, HBS structures can be divided into three main categories: i) subdiagonal $\eta_{ij} = 0 \; \forall j > i$, ii) superdiagonal $\eta_{ij} = 0 \; \forall j < i$ and iii) diagonal $\eta_{ij} = 0 \; \forall j \neq i$. Note that from a physical point of view the class of superdiagonal HBS appears to be a somewhat peculiar because it suggests that future values of input are correlated with past values of system output. The HBS representation (4) can be re-expressed such that the resulting structure exhibits input (or, alternatively, output) dependency of the parameters. The corresponding system with input dependent parameters a_j^k is given by

$$y_k = \sum_{j=1}^{n_a} a_j^k y_{k-j} + \sum_{i=1}^{n_b} b_i f(u_{k-i}), \qquad\qquad a_j^k = a_j + \sum_{i=1}^{n_b} \eta_{ij} f(u_{k-i}). \tag{5}$$

In terms of a quasi-transfer function representation, the HBS can be expressed as

$$y_k = G(q^{-1}) f(u_k), \tag{6}$$

where the input dependent quasi-transfer function of the constituent BS is given by

$$G(q^{-1}) = \frac{y_k}{v_k} = \frac{\sum_{i=1}^{n_b} q^{-i} b_i}{1 - \sum_{j=1}^{n_a} q^{-j} a_j^k} = \frac{\sum_{i=1}^{n_b} q^{-i} b_i}{1 - \sum_{j=1}^{n_a} q^{-j} \left[a_j + \sum_{i=1}^{n_b} \eta_{ij} f(u_{k-i}) \right]}. \tag{7}$$

From a structural point of view, an advantage resulting from the interpretation of the constituent BS model as a linear time-varying system is that standard well understood notions from classical linear system theory such as system time-constants, damping/natural frequency and steady-state gain are to large extent retained. As a consequence, many existing methods developed for linear systems, including both control and identification, can be readily extended and transferred to the BS case.

3. Steady-state properties

Considering (6)-(7) the steady-state characteristic of the HBS can be expressed as a combination of the steady-states of the constituent static nonlinearity and the BS. Therefore, before discussing

steady-state properties of the HBS structure, the steady-state behaviour of the component subsystems is analysed first in detail.

The steady-state characteristic of the overall HBS is given by

$$y_{ss} = g_{BS}(v_{ss})v_{ss} \tag{8}$$

where the subscript ss denotes the steady-state value. The steady-state gain g_{BS} of the component BS is given by

$$g_{BS}(v_{ss}) = \frac{y_{ss}}{v_{ss}} = \frac{\sum_{i=1}^{n_b} b_i}{1 - \sum_{j=1}^{n_a}\left[a_j + \sum_{i=1}^{n_b} \eta_{ij} f\left(u_{ss}\right)\right]} = \frac{\bar{b}}{1 - \bar{a} - \bar{\eta}v_{ss}}, \tag{9}$$

where $\bar{a} = \sum_{j=1}^{n_a} a_j$, $\bar{b} = \sum_{i=1}^{n_b} b_i$ and $\bar{\eta} = \sum_{i=1}^{n_a} \sum_{j=1}^{n_b} \eta_{ij}$. It is observed that the steady-state characteristic of the HBS is, firstly, nonlinear, and, secondly, dependent on both the input and the form of $f(\cdot)$. Also, note that $\bar{a} + \bar{\eta}v_{ss} \neq 1$ for y_{ss} to remain bounded.

3.1. Steady-state behaviour of Hammerstein systems

In the case of HS, see (3), the steady-state gain is equal to the static nonlinearity $f(\cdot)$ scaled by a constant gain g_L of the linear dynamic subsystem, i.e.

$$y_{ss} = g_L f(u_{ss}), \qquad\qquad g_L = \frac{y_{ss}}{v_{ss}} = \frac{\sum_{i=1}^{n_b} b_i}{1 - \sum_{j=1}^{n_a} a_j} = \frac{\bar{b}}{1 - \bar{a}} \tag{10}$$

with the assumption that $\bar{a} \neq 1$. In contrast to the BS model, the HS structure can exhibit IM, if the static input nonlinearity is non-strictly monotonic. However, similarly to the BS structure, it is also inherently incapable of exhibiting OM. The steady-state asymptotes depend on a particular form of the static input nonlinearity $f(\cdot)$, hence cannot be characterised for a generic case.

3.2. Steady-state behaviour of bilinear subsystem

Depending on a value of $\bar{\eta}$ the steady-state of a BS, see (2), can exhibit either saturation or 'burst' characteristics. Whilst a constant steady-state gain is obtained if $\bar{\eta} = 0$, in which case the BS is so-called static-linear, see [14]. Note that a particular case for which the condition $\bar{\eta} = 0$ is satisfied occurs when $\eta_{ij} = 0$ $\forall i, j$, i.e. when the BS is reduced to a linear system. An illustrative plot showing representative steady-state characteristics of an arbitrary BS for a range of different values of $\bar{\eta}$ is given in Figure 2(a). By limiting consideration only to the quadrant where both $u_{ss} \geq 0$ and $y_{ss} \geq 0$, the three darker curves above the black dashed line correspond to the cases where $\bar{\eta} > 0$, i.e. a so-called positive bilinearity (burst - typical of exothermic chemical processes). Whilst the remaining three brighter curves below the black dashed line correspond to $\bar{\eta} < 0$, i.e. a so-called negative bilinearity (typical of many industrial systems with saturation nonlinearities) [11].

A crucial property of a model is the extent to which it can mimic the steady-state characteristics of a process. In this connection, the existence of a steady-state multiplicity is of interest. Following [2], the input (output) multiplicity is defined as the property where a single steady-state output (input) corresponds to multiple steady-state inputs (outputs). The input multiplicity (IM) property appears to be more frequently encountered phenomena than the output multiplicity (OM) and is related to the notion of an 'optimal' or 'sweet' point in the process steady-state curve [8]. However, the OM has also been encountered in many industrial problems, see [7] and the references therein. For the IM to exist the steady-state characteristic must not be strictly monotonic or, equivalently, must not be invertible.

(a) An arbitrary BS.

(b) An arbitrary HBS.
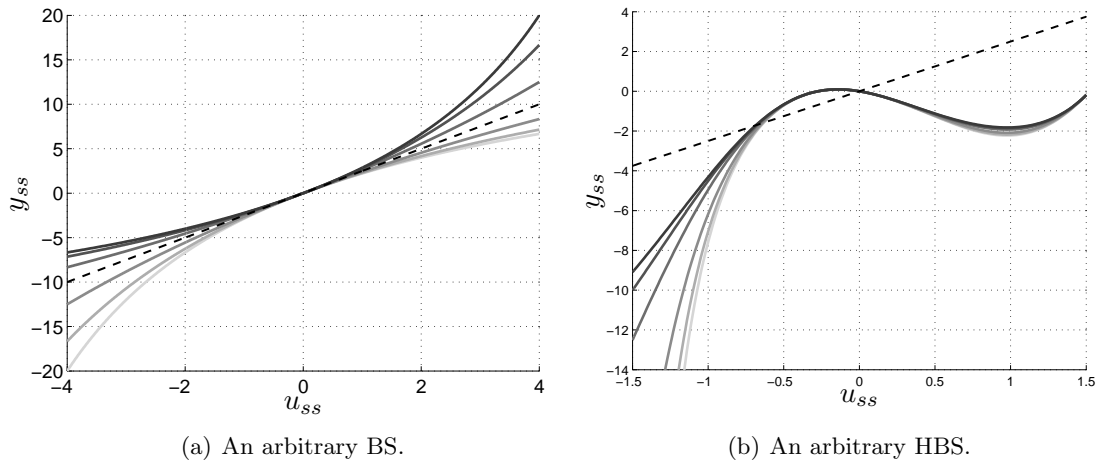
Figure 2. Illustrative plots showing representative steady-state characteristics for a range of different values of $\bar{\eta}$ parameter. The dashed black line corresponds to a linear system case.

Considering the constituent BS, it is observed that neither IM nor OM are possible. This fact follows from a property that the corresponding steady-state curve is strictly monotonic, which, for instance, can be useful when designing nonlinear compensators, see [18] and [11]. However, for completeness, it is worth noting that generally, in fact, the IM is feasible in a somewhat degenerate case when y_{ss} is constant. Recalling the previously postulated assumption that $\bar{a} + \bar{\eta}u_{ss} \neq 1$, this condition is satisfied if $\bar{b} = 0$ or $u_{ss} = 0$ or $\bar{a} = 1$. Similarly, a degenerate OM can also be exhibited in general when u_{ss} is constant. The steady-state input as a function of a steady-state output is given by

$$u_{ss} = \frac{(1 - \bar{a})y_{ss}}{\bar{b} + \bar{\eta}y_{ss}}, \tag{11}$$

where it is assumed that $\bar{b} + \bar{\eta}y_{ss} \neq 0$. Considering (11), the degenerate OM is present if $\bar{a} = 1$ or $y_{ss} = 0$ or $\bar{b} = 0$. Driven by utility for control, these degenerate forms of IM and OM are, however, deliberately excluded in the remainder of this paper, hence BS structures are treated as not being able to exhibit any form of multiplicity. Also, it can be verified that the horizontal asymptote of the steady-state characteristic, obtained for $u_{ss} \to \pm\infty$, is given by $-\bar{b}/\eta$, whilst the vertical asymptote, obtained for $y_{ss} \to \pm\infty$, is given by $(1 - \bar{a})/\eta$.

3.3. Steady state behaviour of Hammerstein-bilinear structure
Since composed of HS and BS models, the HBS inherits properties of the constituent structures allowing for an extended range of potential steady-state characteristics to be obtained, when compared to those feasible for BS. In particular, this includes an important capability of exhibiting IM. This property is illustrated in Figure 2(b) for an arbitrarily parametrised HBS with $f(\cdot)$ selected as a third order polynomial and a range of different values of $\bar{\eta}$. Similarly as in the case of HS, the asymptotes of the steady-state characteristic of the overall system cannot be characterised for a generic case. Furthermore, the HBS structure can behave as a static-linear system in two cases. Firstly, if the function $f(\cdot)$ is linear, hence the HBS is reduced effectively to a BS submodel. In this case the condition for static-linearity is satisfied when $\bar{\eta} = 0$. Secondly, if the static nonlinear function $f(\cdot)$ is selected as an inverse of the steady-state characteristic corresponding to the component BS, cf. (11), i.e. $v_{ss} = \frac{(1-\bar{a})u_{ss}}{b+\bar{\eta}u_{ss}}$ assuming $\bar{b} + \bar{\eta}u_{ss} \neq 0$. Note that because the steady-state characteristic of BS is strictly monotonic its inverse exists.

4. Dynamic properties

Similarly as when discussing behaviour in steady-state, dynamic properties of the HBS structure depend on its constituent components and hence dynamic properties of HS and BS structures are analysed first.

4.1. Dynamics of Hammerstein subsystem

One of the important attributes characterising the class of HS structures is that the eigenvalues of the corresponding linearised HS model are constant thus independent of an operating point. Therefore, the qualitative behaviour of HS models do not change with the reference and is determined exclusively by the linear subsystem, see [2], [1]. A direct consequence is that HS structures are not desirable when modelling processes with operating point dependant dynamics, such as those with a time-varying time-constant. Also, it is interesting to note that if the static nonlinearity is swapped with the dynamic linear subsystem leading to a WS, the same observation is no longer valid, see [15]. Consequently, HS are useful in cases where the system dynamics is reasonably linear and nonlinearities can be attributed to the steady-state behaviour.

4.2. Dynamics of bilinear subsystem

Consider the quasi-transfer function representation of a BS, which is obtained from (7) by imposing $v_k = u_k$, i.e.

$$G(q^{-1}) = \frac{\sum_{i=1}^{n_b} q^{-i} b_i}{1 - \sum_{j=1}^{n_a} q^{-j} \left[a_j + \sum_{i=1}^{n_b} \eta_{ij} u_{k-i} \right]}. \tag{12}$$

It is observed from (12) that both static and dynamic behaviour is input dependant. Consequently, the (time-varying equivalent) poles of the BS structure are determined by the instantaneous roots of the denominator in (12). This can be illustrated by a simple, yet representative, example of a first order diagonal BS model expressed in a quasi-transfer function form as $G(q^{-1}) = \frac{q^{-1} b_1}{1 - q^{-1}(a_1 + \eta_{11} u_{k-1})}$ with the time-varying discrete pole located at $a_1 + \eta_{11} u_{k-1}$. Considering, for simplicity, the case of $a_1 + \eta_{11} u_{k-1} \geq 0$ and utilising a direct relationship between the continuous-time and discrete-time plane, the corresponding time-constant of a continuous-time counterpart is given by $-T_s / \ln (a_1 + \eta_{11} u_{k-1})$, where T_s denotes the sampling time. It can be deduced that with the expression $a_1 + \eta_{11} u_{k-1}$, hence the input, increasing in magnitude, the corresponding time-constant increases, i.e. the system dynamics becomes faster, and vice-versa.

Figure 3(a) shows this phenomenon for the introduced exemplary first order diagonal BS. Considering top right panel it is observed that, first, because $\eta_{11} < 0$ the BS exhibits a saturation steady-state nonlinearity and, second, that the dynamics become faster with an increasing magnitude of the input. This is verified by the upper right and the lower left panels, where the changing location of a discrete pole over time is plotted. It is seen that the pole moves towards the origin with the increasing magnitude of the input. The lower right panel shows the corresponding location of the time-constant of the continuous-time counterpart. It is observed that the equivalent time-constant decreases, proportionally to an inverse of the logarithm, confirming that the system becomes progressively faster. Moreover, it can be verified that if $a_1 + \eta_{11} u_{k-1} < 0$ the analysed BS will exhibit dynamics with damped oscillations.

Since the dynamics is affected by the input, the stability of BS structure is input dependent. This property is clearly observed in the case of the introduced exemplary first order diagonal BS model, where it is required that the time-varying pole stays within the unit circle, i.e. $|a_1 + \eta_{11} u_{k-1}| < 1$. This condition implies that the magnitude of the input must be restrained to $\left(\frac{1-a_1}{\eta_{11}}, \frac{-1-a_1}{\eta_{11}} \right)$. Also note that $|a_1| < 1$ for stability of the autoregressive linear part. With reference to [19] and [20], the corresponding sufficient conditions for a general BS structure to

(a) First order diagonal BS.   (b) Exemplary HBS.
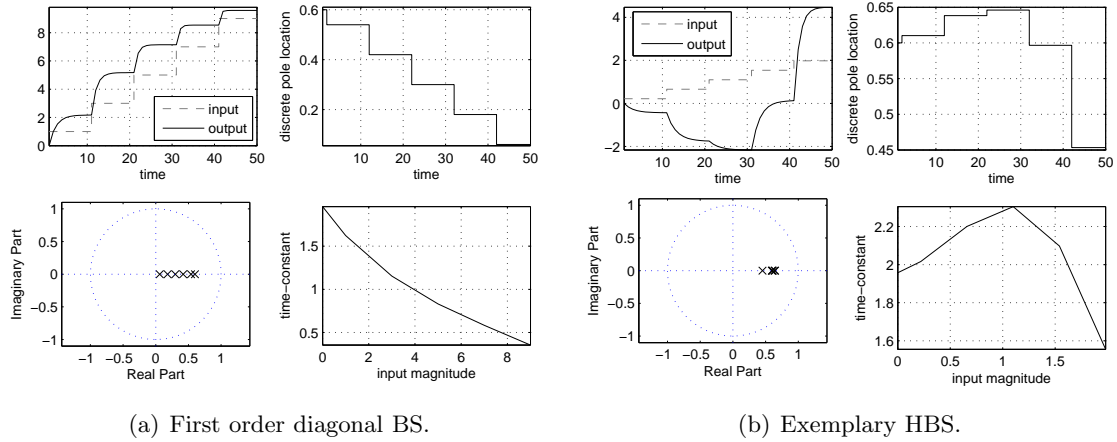
Figure 3. Illustrative plots showing dependence of system dynamics on input.

remain bounded are given by

$$|p_i| < 1, \qquad\qquad M_u < \frac{\Pi_{i=1}^{n_a}(1 - |p_i|)}{\sum_{i=1}^{n_a} \sum_{j=1}^{n_b} |\eta_{ij}|}, \qquad (13)$$

where M_u denotes a finite magnitude of the input and p_i correspond to the n_a poles of a linear part of the BS structure, i.e. roots of the polynomial $1 - \sum_{j=1}^{n_a} q^{-j} a_j$.

4.3. Dynamics of Hammerstein-bilinear structure

Dynamic properties of the HBS are defined by the two component substructures. Note that this observation is in stark contrast to a HS structure, where the static nonlinearity has no impact on the overall qualitative system behaviour. In the case of the HBS structure such dependency is present because the dynamic part is input dependent, hence, is implicitly influenced by the nonlinear function that transforms it. Consequently and differently to the BS case, denominator coefficients of the HBS structure, cf. (7), do not directly depend on the instantaneous values of the input u_k but on the instantaneous values of the intermediate input v_k. This property increases the potential range of feasible dynamic behaviour that can be obtained, in particular, a range of possible dependency of system poles on the input is wider than that of the BS structure.

This observation is illustrated in Figure 3(b), where it is shown that the dependency of the discrete pole location on the input magnitude may well be of a non-monotonic nature - a phenomenon infeasible for BS models. The HBS structure comprises the arbitrarily parametrised exemplary first order diagonal BS defined previously followed by an arbitrary third order polynomial, i.e. $f(x) = \sum_{i=1}^{3} \alpha_i x^i$. It is observed that the dynamics of the system, becomes slower initially only to get faster once the input magnitude exceeds approximately 1.1. Consequently, the relationship between the time-constant of the corresponding continuous-time counterpart and the input magnitude is also distinctively dissimilar, which is observed in the lower right panel. Such a non-monotonic behaviour of the apparent time-constant was reported in [4] when modelling a distillation column with input and output being percentage change in a reflux flow and in the top composition, respectively.

The discussed reference dependent dynamics of the HBS structure, cf. equation (4), can be demonstrated explicitly by linearisation around a working point, denoted (u_k^*, y_k^*), and by

examining subsequently the poles of the resulting linearised counterpart, i.e.

$$\Delta y_k = \sum_{j=1}^{n_a} \frac{\partial y_k}{\partial y_{k-j}}\Big|_{(u_k^*, y_k^*)} \Delta y_{k-j} + \sum_{i=1}^{n_b} \frac{\partial y_k}{\partial u_{k-i}}\Big|_{(u_k^*, y_k^*)} \Delta u_{k-i}, \tag{14}$$

where the deviation variables are denoted with Δ, e.g. $\Delta y_k = y_k - y_k^*$. The partial derivatives in (14) are given by

$$\frac{\partial y_k}{\partial y_{k-j}} = a_j + \sum_{i=1}^{n_b} f(u_{k-i})\eta_{ij}, \qquad \frac{\partial y_k}{\partial u_{k-i}} = \frac{\partial f(u_{k-i})}{\partial u_{k-i}}\left(b_i + \sum_{j=1}^{n_a} y_{k-j}\eta_{ij}\right). \tag{15}$$

The transfer function of the deviations from the working point is expressed as

$$\frac{\Delta y_k}{\Delta u_k} = \frac{\sum_{i=1}^{n_b} q^{-i}\left[\frac{\partial f(u_{k-i})}{\partial u_{k-i}}\Big|_{(u_k^*)}\left(b_i + \sum_{j=1}^{n_a} y_{k-j}^*\eta_{ij}\right)\right]}{1 - \sum_{j=1}^{n_a} q^{-j}\left[a_j + \sum_{i=1}^{n_b} f(u_{k-i}^*)\eta_{ij}\right]}. \tag{16}$$

Considering (16), it is seen that the local dynamics of the HBS is governed by the roots of the denominator, whose coefficients are functions of both the working point and $f(\cdot)$. Note that a similar observation is valid with respect to the numerator coefficients, too. This shows, therefore, that, first, the dynamics of the HBS structure is reference dependent and, second, that it also depends on the selection of $f(\cdot)$.

Conditions for stability of the HBS structure can be inferred from the analogy to the BS, see (13), where the crucial difference is that M_u is substituted with M_v, denoting a finite magnitude of the intermediate input signal v_k. Only when $f(\cdot)$ is strictly monotonic, by using the relation $u_k = f^{-1}(v_k)$, this condition can be expressed explicitly in terms of the magnitude M_u of the actual input that excites the HBS structure.

Table 1. Overview of properties ($\sqrt{}$ - presence, $\times$ - absence) of selected model structures.

Structure / Property	IDS	IM	OM	IDD
linear	$\times$	$\times$	$\times$	$\times$
Hammerstein	$\times$	$\sqrt{}$	$\times$	$\times$
Wiener	$\times$	$\sqrt{}$	$\times$	$\sqrt{}$
bilinear	$\sqrt{}$	$\times$	$\times$	$\sqrt{}$
Hammerstein-bilinear	$\sqrt{}$	$\sqrt{}$	$\times$	$\sqrt{}$
Lure	$\sqrt{}$	$\times$	$\sqrt{}$	$\sqrt{}$

5. Discussion of properties of Hammerstein-bilinear structure with relation to other nonlinear models

Crucial properties of all discussed structures are juxtaposed in Table 1 that is partially based on that given in the survey paper [14]. Linear model structures are also included for completeness. The following abbreviations are used: IDS - input dependent stability, IM - input multiplicity, OM - output multiplicity, IDD - input dependent dynamics. Generally, HS structure can be treated as a first step towards an increase of modelling flexibility of ordinary linear structures. It allows a nonlinear steady-state gain with IM to be obtained but the transient behaviour remains unchanged over the entire operating range. The next step could be to consider the WS structure which additionally offers the IDD. However, it is worth emphasising, citing [8], that

'this behaviour is near the limit of the Wiener model's qualitative capability' and achievable at a cost of complex static elements that are undesirable in practice. In contrast, BS are inherently flexible enough to exhibit strong IDD, which comes at a price of IDS, thus a need of restricting magnitude of the input, cf. (13). Also, contrary to both HS and WS, BS structures lack possibility of IM and can exhibit only a single family of steady-states curves, i.e. hyperbolic, as shown in Figure 2. It must be added that although structurally WS possess capability of IM, to facilitate identification and control, it is, in fact, usually assumed that the static nonlinearity is invertible. This means that the potential for IM is inevitably lost, see [21], [22], [2], [15] for such cases. Moreover, WS models seem to be less popular than for instance HS structures, which is likely due to increased difficulty in their identification [1]. HBS structures add to the repertoire of BS the potential of IDS and widen variety of feasible steady-states and dynamics, at the same time retaining capability of a strong IDD. In applications where the process exhibits OM other nonlinear structures, such as Lure models, are required. Note that Lure models, similarly as HBS, are capable of exhibiting IDS and IDD, but in contrast to HBS are unable of IM. In [14] nonlinear structures with static nonlinearities in the forward path, such as HS and WS, are classified as being 'mildly' nonlinear, whilst BS, due to IDS, are placed in the 'intermediately' nonlinear class. Lure models, because of OM, are categorised as 'strongly' nonlinear. Following this reasoning, the HBS structures, although remain in an 'intermediately' nonlinear class, can be considered as a one step further towards a strongly nonlinear class of models.
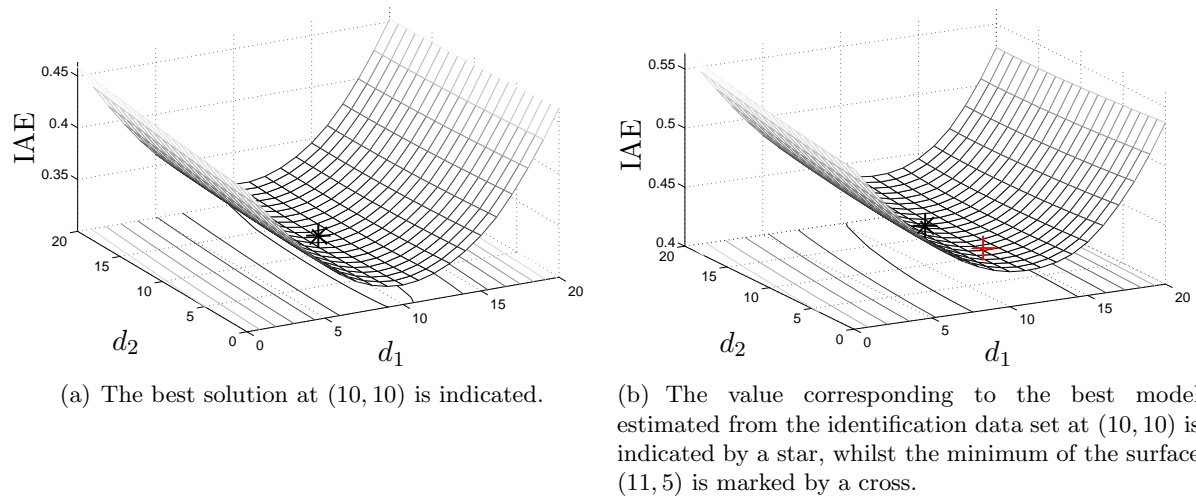
6. Industrial case study

In this section the HBS structure is used for modelling an air handling unit, which is a part of an industrial heating, ventilation and air-conditioning system used to maintain specific tight and stable environmental conditions during the production of blood glucose test strips. The main goal of modelling is subsequent control synthesis and tuning to optimise the overall energy consumption.

The system is represented schematically in Figure 4 and consists of a standard air-to-water heat exchanger, where an inflow air is cooled down by cold water. In nominal conditions the inflow air, cold water mass-flow and the inflow cold water temperature are all approximately constant. Therefore, for modelling purposes only two inputs, i.e. inflow air temperature and valve stem position, and a single output of interest, i.e. outflow air temperature, are considered. The main motivation for considering the HBS structure for this particular application follows from first principles modelling conducted previously in [23] and [24]. These investigations have uncovered that, firstly, it is advantageous to account explicitly for the nonlinear valve characteristic and approximate it by a HS-type nonlinearity. Secondly, that the dynamics of the air handling unit is operating point dependant and can be modelled by a BS. Thirdly, that the bilinear product term has physically meaningful significance and stems directly from energy
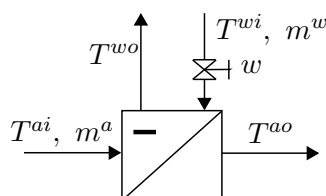
Figure 4. Schematic representation of the air-handling unit, where: w - valve stem position [%], T^{ai} - inflow (on coil) air temperature [K], T^{ao} - outflow (discharge) air temperature [K], T^{wi} - inflow cold water temperature [K], T^{wo} - outflow cold water temperature [K], m^w - cold water mass-flow rate [kg/s], m^a - inflow air mass-flow [kg/s].

(a) The best solution at $(10, 10)$ is indicated.

(b) The value corresponding to the best model estimated from the identification data set at $(10, 10)$ is indicated by a star, whilst the minimum of the surface $(11, 5)$ is marked by a cross.

Figure 5. (a) - identification and (b) - validation results in terms of the IAE criterion plotted in function of delays corresponding to the two inputs.

balance equations. Consequently, the following HBS model structure is proposed to model the air-handling unit

$$T_k^{ao} = a_1 T_{k-1}^{ao} + b_1 v_{k-1-d_1} + \eta_{11} T_{k-1}^{ao} v_{k-1-d_1} + b_1^* T_{k-1-d_2}^{ai}, \qquad v_k = \sum_{l=1}^{5} \alpha_i w_k^l, \qquad (17)$$

where d_1 and d_2 denote unknown transport delays. Although properties of single-input single-output HBS structures were discussed in this paper exclusively, their extension to two-input single-output case defined by (17) should be straightforward, especially because only w_k is transformed through the static polynomial nonlinearity of 5th order.

Since both equations in (17) are separately linear in their parameters, this property is exploited by making use of a so-called bilinear parametrisation method, see [25]. This approach has been applied for the purpose of HS identification in [26] and identification of errors-in-variables problems in [27], where it has been shown to possess numerically appealing properties. Its consistency and convergence properties when applied to HS have been examined in [28].

Two sets of data sampled at 5s, referred to as the identification and validation data set, are used giving $7,800$ and $9,000$ data points, respectively. Valve position is scaled to 10 to ease plotting, also all data is normalised by subtracting bases that are assumed to be the corresponding first values of signals. The identification algorithm is stopped when a maximal number of iterations, selected as 100, is reached or when $\|\hat{\theta}_i - \hat{\theta}_{i-1}\|_2 < 10^{-6}$, where $\hat{\theta}_i$ is the estimate of the parameter vector at i-th iteration. To quantify the identification results two performance criteria are used, i.e. the normalised integral absolute error (IAE) and the coefficient of determination (R_T^2), defined, respectively, as follows IAE $= \frac{1}{N} \sum_{k=1}^{N} |y_k - \hat{y}_k|$ and $R_T^2 = 100 \left(1 - \|y - \hat{y}\|_2^2 / \|y - \bar{y}\|_2^2 \right)$, where $\hat{y}_k$ is the identified model output, y and $\hat{y}$ are vectors built from the actual output and the model output, respectively, and $\bar{y}$ denotes the mean value of y. The notation $\|\cdot\|_2$ stands for the square norm of a vector and N is number of data samples.

Because not only the parameters but also the transport delays are unknown, the identification has to be repeated for all combinations of d_1 and d_2 in a predefined range. Results obtained in terms of the IAE criterion with both delays restricted to $[0, \ 20]$ samples are shown in Figure 5(a), whilst the corresponding validation results are presented in Figure 5(b). The smallest value of IAE=0.316, marked in Figure 5(a) by a star, is obtained at $(d_1, d_2) = (10, 10)$. For completeness,
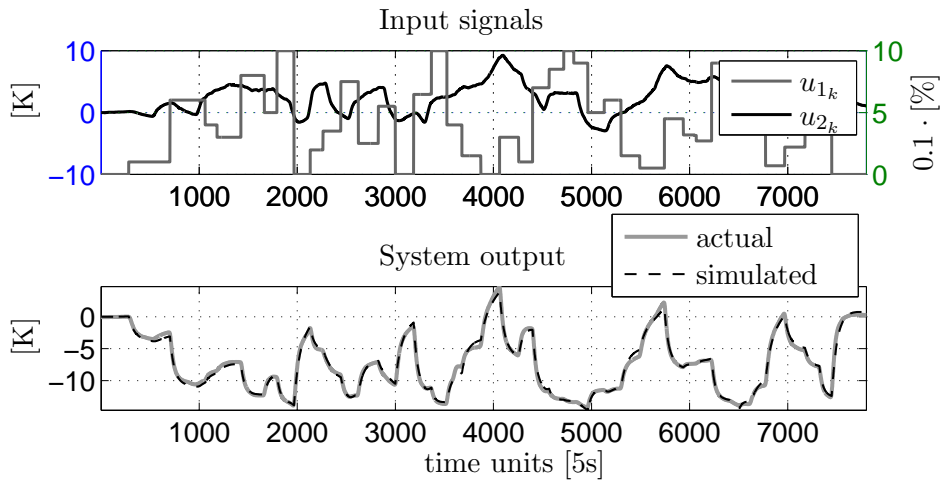
(a) Identification dataset, $R_T^2 = 99.2\%$.
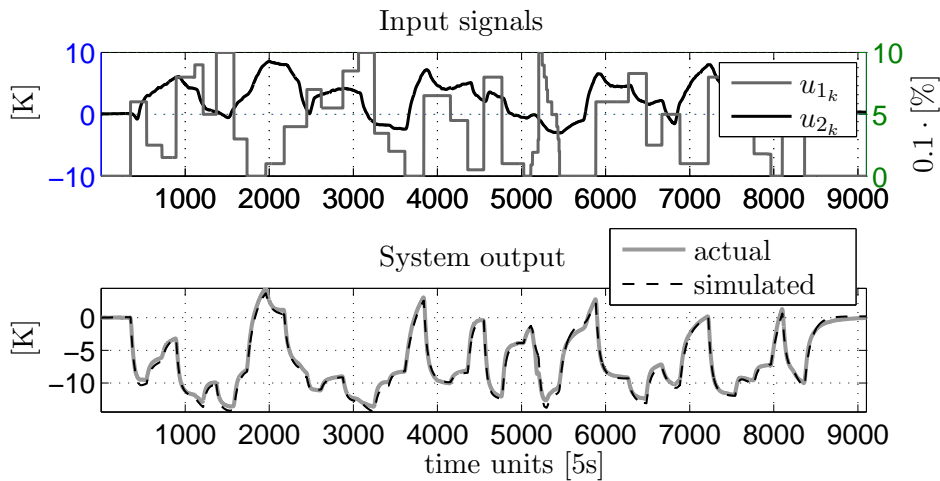
(b) Validation dataset, $R_T^2 = 98.6\%$

Figure 6. Results showing performance of the identified HBS model.

the value corresponding to $(10, 10)$ is also marked in Figure 5(b), where the actual minimum at $(11, 5)$ is denoted by a cross. Considering the coordinates of the minima in both figures it is noted that whilst the first coordinate is similar, the second differs quite significantly. However, by analysing shape of surfaces, it is observed that in both cases they resemble three dimensional parabolas with a gradient visibly greater in the direction of d_1 than d_2. This means that the selection of d_1 is considerably more important than d_2. Consequently, the apparent discrepancy in the second coordinate between the two data sets is of minor significance. This observation is supported also by a relatively small difference of the validation IAE values corresponding to the points $(10, 10)$ and $(11, 6)$ which are 0.431 and 0.428, respectively. Therefore, it is $d_1 = d_2 = 10$ that is selected. Visual results in terms of the simulated output together with the input signals used are given in Figure 6(a) and 6(b) for the identification and validation data set, respectively. It can be observed that in both cases the model output matches the actual system output very well. The corresponding values of the R_T^2 criteria are 99.2% and 98.6%, for the identification and validation data set, respectively, indicating that on average only approximately 1% of output variation remains unexplained by the identified model. The estimated parameters of the model

(17) together with corresponding standard deviations are: $\hat{a}_1 = 9.846 \cdot 10^{-1} \pm 8.215 \cdot 10^{-5}$, $\hat{b}_1 = -4.977 \cdot 10^{-3} \pm 2.536 \cdot 10^{-5}$, $\hat{b}_1^* = -1.697 \cdot 10^{-4} \pm 1.715 \cdot 10^{-6}$, $\hat{\eta}_{11} = 8.813 \cdot 10^{-3} \pm 8.304 \cdot 10^{-5}$, $\hat{\alpha}_1 = 6.233 \cdot 10^{-1} \pm 2.629 \cdot 10^{-2}$, $\hat{\alpha}_2 = 9.918 \cdot 10^{-2} \pm 2.296 \cdot 10^{-3}$, $\hat{\alpha}_3 = -3.752 \cdot 10^{-3} \pm 6.810 \cdot 10^{-5}$, $\hat{\alpha}_4 = 4.886 \cdot 10^{-5} \pm 8.208 \cdot 10^{-7}$, $\hat{\alpha}_5 = -2.104 \cdot 10^{-7} \pm 3.449 \cdot 10^{-9}$. It is observed that the standard deviations are all at least of an order smaller than the values of the corresponding estimated parameters. These results indicate high confidence in the estimates and hence, in turn, also in the identified HBS model.

7. Summary
A class of Hammerstein-bilinear systems (HBS) has been introduced and its properties of main practical interest for modelling have been analysed and discussed in detail. An industrial case study has demonstrated application of HBS structures to an exemplary real-world problem.

Acknowledgments

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