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Author post-print (accepted) deposited in CURVE May 2012

Original citation & hyperlink:

Batanović, V. , Petrović, D. and Petrović, R. (2009) Fuzzy logic based algorithms for maximum covering location problems. Information Sciences, volume 179 (1-2): 120–129.

<http://dx.doi.org/10.1016/j.ins.2008.08.019>

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Fuzzy logic based algorithms for maximum covering location problems

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Abstract:

This paper concerns a class of maximum covering location problems in networks in uncertain environments. It is assumed that: (a) relative weights of demand nodes are either deterministic or imprecise, described by linguistic expressions, (b) potential facility site locations are limited to network nodes. The concept of coverage is extended to include a degree of node coverage which means that the borders between the subset of covered demand nodes and the subset of uncovered demand nodes are inexact. The acceptable service distance/traveling times from a facility site to demand nodes are modelled by fuzzy sets. Three new algorithms for choosing the best facility locations are developed which assume that (1) demands at all nodes are equally important, (2) relative weights of demand at nodes are deterministic and (3) weights of demand at nodes are

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imprecise and described by linguistic terms, respectively. The algorithms are based on searching among potential facility nodes by applying comparison operations on discrete fuzzy sets. It is shown how to extend the proposed algorithms from one-site to multi-site covering problems. Illustrative examples of selecting locations for logistics centres in a distribution company are given.

Keywords: Uncertainty, Fuzzy sets, Network covering, Location, Facility site.

1. Introduction

In this paper, a class of network based facility location problems in uncertain environments is treated. The problems under consideration belong to the class of maximum covering problems defined as follows: given a connected network with demand at nodes, locate one or more facility sites at nodes in such a way as to maximise the coverage of demand nodes. The covered nodes are within the acceptable service distance or within the acceptable travelling service time from a facility site.

In the conventional covering problems, it is commonly assumed that there is a critical distance or critical time within which the demand node is fully covered, while beyond this distance/time it is not covered at all. In this paper, we extended the concept of node coverage by supposing that the borders between the subset of covered nodes and the subset of uncovered nodes are inexact. Following this idea the notion of fuzzy coverage is introduced with an appropriate measure of coverage decreasing from full coverage to non-coverage.

Still another source of uncertainty in covering problems treated in this paper is imprecise weights of demand nodes. It is assumed here that the weights of demand nodes are described by vague linguistic terms that can be appropriately modelled using fuzzy values.

Three covering problem variants which include the notion of fuzzy coverage are defined and considered:

Problem 1: Demands at all nodes are equally important and the facility location objective is to determine one or more facility nodes which maximize a measure of belief that the largest number of demand nodes is covered.

Problem 2: Amounts of demands at nodes are different either by size or by importance. A deterministic relative weight is associated to each demand node, which is proportional to the

amount of demand at the node. The objective is to determine one or more facility nodes which maximise a measure of belief that the largest amount of demand at demand nodes is covered.

Problem 3: Amounts of demands at nodes are specified as imprecise weight values. They are described by vague linguistic terms defined by fuzzy sets. The objective is to determine one or more facility nodes which maximise a measure of belief that the largest coverage of demand at demand nodes labelled by their weights is obtained.

In all the three problems one site solution is considered first and then the attention is extended to the corresponding multi-site problems.

Fuzzified versions of the network covering models presented in this paper are particularly suitable for real world logistic tasks where both imprecisions in travelling times between network nodes and vaguely described weights or importance of demand nodes are very common.

The paper is organized as follows. In Section 2 the relevant literature is given and discussed. Section 3 provides assumptions and notation used in the three covering models. Section 4 gives three new algorithms for solving the corresponding one-site fuzzy covering problems. In Section 5, an extension to multi-site problems is discussed. In Section 6 illustrative examples are given. The procedure for discrete fuzzy numbers comparison, based on the probabilistic approach, and used in the three algorithms, is given in Appendix.

2. Literature review

Network covering problems have a long and wealthy history. The problems have been defined in a variety of ways which have given rise to a large number of generalizations, extensions and modifications in problem solving. Particularly, the past three decades have shown a fast growth of the output of published materials.

In the following, we provide a brief review of theoretical and applied contributions to the field. In view of the rather large amount of literature on the subject, the reference list is, needless to say, incomplete and holds only the items closely related to this paper. The reviewed models and algorithms are classified into the following categories: general, approximate solutions, stochastic/fuzzy, and anti-covering models.

General models. Generally, it is claimed in the literature that facility location models have been applied in the public and private sectors for years, giving emphasis on discrete location problems. The very challenging and stimulating papers on network covering problems are provided by Church and ReVelle [6], Schilling et al, [26], Owen and Daskin [18] and Drezner and Wesolowsky [12]. These papers have received a strong theoretical interest and a wide attention of practitioners considering the questions on how to select locations for manufacturing units, commodity distribution, logistic centres for maintenance, and places for emergency service facilities, such as fire stations and ambulances. ReVelle provided a perspective on location science in [25]. In [12], the selection of the best location was viewed as the problem of allocating customers to facilities with changing costs. Drezner and Hamacher [11] gave a state-of-the-art review of available location models. A visionary paper by Daskin [8] offered an answer on the question what one should know about location modelling, and presented a taxonomy of location problems.

Approximate solutions. It is already in the earliest node covering bibliographies, dating from the mid-seventies, where one may find that maximum covering problems are computationally complex and not easy to solve. In the case of general network considerations, they are usually NP-hard combinatorial optimization problems. The algorithms which solve the covering problems optimally (for example, Downs and Camm's branch and bound algorithm [9]) take a lot of time and, therefore, they cannot be applied to large scale problems if the solution is to be used in real time. A number of authors suggested approximate solutions for the minimum set covering problem where the objective is to find a minimum size facility set covering all nodes, or for the

maximum covering problem where the objective is to select a subset of facilities in such a way as to maximise the sum of weights associated with demand sites. For example, Affif et al, [1] proposed an efficient heuristics, Lorena and Lopes [16] developed a classical genetic algorithm and Resende [24] applied a greedy randomized adaptive search procedure. Klose and Drexl [14] reviewed some contributions to the state-of-the-art probabilistic, multiobjective network location models. Recently, Sugihara [28] proposed a greedy based algorithm for a modified problem defined as to find a facility set of a predetermined size which maximizes the sum of the weights of vertices covered in a given capacitated graph. A genetic-like algorithm, GASUB, for determining a given number of facility locations from a set of potential locations so as to optimize a predetermined fitness function is given in [20].

Stochastic / fuzzy models. Initially, the complexity of network covering problems has limited much of the literature to deterministic cases. Later on, stochastic formulations attempted to capture uncertainty by considering explicitly the probability distribution of uncertain input parameters such as demands at nodes or distance values in networks ([29] and [17]). Darzentas [7] developed a fuzzy model for the facility location problem based on fuzzy set partitioning, where parameters associated to nodes are not crisp, but described by fuzzy sets. Bhattacharya and Tiwary [5] considered imprecise weights in the Weber location problem with the objective to minimise the sum of transportation cost. In addition, it was supposed that the costs per unit distance were not precisely stated. A location problem with future uncertainty in the data was treated in [10]. The authors assumed some possible scenarios about the future values of the input parameters with the objective to find the location that will best accommodate the possible scenarios. Later on, the concept of a gradual coverage is introduced in [4] by introducing two distances. A demand point is fully covered if it is within the lower distance, not covered at all if it is beyond the larger distance, and, if its distance is between the two distances, a level of coverage is determined using a 'decay function'. A stochastic set-covering location model where the probability of each customer being covered is not less than a specified critical value was

proposed by Hwang [13]. Perez et al, [21] claimed that in real applications the facility locations can be full of linguistic vagueness that can be appropriately modelled using networks with the fuzzy values which describe nodes, weights or importance of nodes, lengths of paths, etc.

Anti-covering models. The curiosity of the network covering theory and applications is that in addition to the conventional maximum covering problems, the attention of some authors have been focussed on inverse tasks – the minimum node/weights covering applicable when locating an obnoxious facility (see, for example, [2] and [3]). Let us notice that all the three algorithms presented in this paper, designed to solve the maximum covering problem can be easily adapted to solve the anti-covering problems, too.

3. Assumptions and notation

Consider a discrete network location model where users with demand are located at network nodes. The problem is to determine one or more network nodes to place facilities which will cover demand nodes in an optimal way. Suppose that a connected network $\mathfrak{G} = (N, L)$ consists of a collection N of nodes together with a set L of unordered pairs of nodes from N , called links. Each link has an associated length (or time) parameter. It is also assumed that there exist the paths between each pair of nodes. The distances among nodes are computed as shortest paths in graph [23]. Conventionally, the basic parameter in the treated location problem is the given acceptable service distance or traveling service time. Each demand node can only be considered covered if there is a facility located at some node within the given distance/time. Otherwise, demand node is uncovered.

In the formulation of the fuzzy location problems defined in this paper, the following notation is used:

$I \subseteq N$, set of potential locations of service facilities; if facilities may be located at any node of the network then $I = N$,

I , total number of facility nodes,

J , total number of demand nodes,

$J \subseteq N$, set of demand nodes; the amount of demand is specified for each demand node $j \in J$,

either: (1) by a deterministic parameter $w_j, j = 1, \dots, J$; or (2) by a weight of demand node that is described by a vague linguistic term defined by triangular fuzzy numbers on a subjectively defined scale (1 ÷ 5) as follows:

a low demand node, denoted by $\mathbf{A}=(x; 1,1,5)$,

a moderate demand node, denoted by $\mathbf{B}=(x; 1,3,5)$ and

a high demand node, denoted by $\mathbf{C}=(x; 1,5,5)$,

where $(x; a, m, b)$ denotes the membership function of the triangular fuzzy number \mathbf{X} defined by the linear relationship:

$$\mathbf{X}(x; a, m, b) = \begin{cases} (x - a)/(m - a) & \text{if } x \in [a, m] \\ (b - x)/(b - m) & \text{if } x \in [m, b] \\ 0 & \text{otherwise} \end{cases}$$

d_{ij} , shortest distance between nodes i and j ; distances $d_{ij}, i = 1, \dots, I, j = 1, \dots, J$ are arranged in $I \times J$ matrix D ,

\mathbf{R} , covered demand node is a fuzzy set with stepwise decreasing membership function $\mu(r)$, defined by K points by the pairs $(r_1, \mu(r_1)), \dots, (r_k, \mu(r_k)), \dots, (r_K, \mu(r_K))$, see Fig 1.

A degree of closeness between facility node $i, i = 1, \dots, I$ and demand node $j, j = 1, \dots, J$ is determined as follows:

if $d_{ij} \leq r_1$, demand node j is covered by facility node i with degree $\mu(r_1)=1$,

if $r_k < d_{ij} \leq r_{k+1}$, demand node j is covered by facility node i with degree $\mu(r_{k+1})$,

$k = 1, \dots, K-1$,

if $d_{ij} > r_K$, demand node j is uncovered by facility i ,

[Insert Fig. 1 here]

$m_i(r_k)$, number of demand nodes within r_k distance from facility node i , $i = 1, \dots, I$, $k = 1, \dots, K$,
 $m_{A,i}(r_k)$, $m_{B,i}(r_k)$, $m_{C,i}(r_k)$, number of *low*, *medium* and *high demand nodes*, respectively, within
 r_k distance from facility node i , $i = 1, \dots, I$, $k = 1, \dots, K$,

$J_i(r_k)$, subset of demand nodes within r_k distance from facility node i , $i = 1, \dots, I$, $k = 1, \dots, K$,

$p_i(r_k)$, percentage of demand nodes within r_k distance from facility node i , $i = 1, \dots, I$,
 $k = 1, \dots, K$,

\mathbf{p}_i , a discrete and finite fuzzy set

$$\mathbf{p}_i = \sum_{k=1}^K \mu(r_k) / p_i(r_k), \quad i = 1, \dots, I,$$

$s_i(r_k)$, percentage of the amount of covered demand nodes within r_k distance from facility node
 i ,

$$i = 1, \dots, I, k = 1, \dots, K,$$

\mathbf{s}_i , a discrete and finite fuzzy set

$$\mathbf{s}_i = \sum_{k=1}^K \mu(r_k) / s_i(r_k), \quad i = 1, \dots, I,$$

$t_i(r_k)$, measure of coverage of demand within r_k distance from facility node i , $i = 1, \dots, I$, $k =$
 $1, \dots, K$, when the weights at demand nodes are linguistic terms

\mathbf{t}_i a discrete and finite fuzzy set

$$\mathbf{t}_i = \sum_{k=1}^K \mu(r_k) / t_i(r_k), \quad i = 1, \dots, I,$$

$\mathbf{Q}_i(r_k)$ a triangular fuzzy set which represents the overall fuzzy coverage of demand from
facility node i within r_k distance,

P_{i^*} , measure of belief that the facility node $i^* \in I$ is better than any other facility node in the
sense that i^* covers the largest percentage of demand nodes,

S_{i^*} , measure of belief that the facility node $i^* \in I$ is better than any other facility node in the sense that i^* covers the largest percentage of the amount of demands at all demand nodes,

T_{i^*} , measure of belief that the facility node $i^* \in I$ is better than any other facility node with respect to the total coverage of demand at demand nodes labeled by their weights.

A question arises on how to determine a membership function for the fuzzy set *covered demand node*. Various experimental methods can be used to determine membership function $\mu(r)$. One of the conceptually very simple methods that can be easily applied in practice is the Horizontal method of membership estimation [19]. In this method, determination of membership functions' parameters relies on some experimental findings collected from a group of experts who are asked to answer questions such as: can r_k be an acceptable service distance between a demand and a facility node. The membership degree of the selected value r_k is taken as a ratio of the number of positive replies to the total number of responses.

4. Algorithms

In what follows, we shall explain three algorithms for choosing the best facility location in a network with respect to demand coverage, in the presence of uncertainty. *Algorithm 1*, *Algorithm 2* and *Algorithm 3* solve *Problem 1*, *2* and *3*, respectively. The essential part of the algorithms is the comparison of discrete and finite fuzzy sets. Different approaches to the comparison of discrete fuzzy sets have been suggested in the literature (e.g. ,[15]). In [27] it was postulated that the results of fuzzy number comparisons need to be given in the form of fuzzy values instead of real values. One efficient comparison method which, in addition, generates a measure of belief that one fuzzy set is greater than the other one, and determines a fuzzy sets ranking is applied in these three algorithms [22] (see Appendix).

Algorithm 1

Step 1.

Calculate the elements d_{ij} , $i = 1, \dots, I$, $j = 1, \dots, J$, and create matrix D .

This can be done by applying one among many customary algorithms for determining the shortest routes in a graph, for example, well known Dantzig algorithm or Polack's matrix algorithm.

Step 2.

For each $i = 1, \dots, I$ and each r_k , $k = 1, \dots, K$, determine $m_i(r_k)$ and calculate $p_i(r_k) = m_i(r_k) / I$.

Step 3.

For each $i = 1, \dots, I$, create a discrete and finite fuzzy set \mathbf{p}_i shown in Fig 2:

$$\mathbf{p}_i = \sum_{k=1}^K \mu(r_k) / p_i(r_k),$$

[Insert Fig. 2 here]

Step 4.

Using the comparison method given in Appendix, compare discrete fuzzy sets $\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_I$ and find \mathbf{p}_{i^*} for which fuzzy relations $\mathbf{p}_{i^*} \geq \mathbf{p}_1, \dots, \mathbf{p}_{i^*} \geq \mathbf{p}_I$ are true. The $i^* \in I$ selected is the best facility node with respect to the number of demand nodes covered.

Step 5.

Determine P_{i^*} , i.e., the corresponding measure of belief that i^* is better then or at least equal to any other facility node.

Algorithm 2

Step 1.

Calculate the elements d_{ij} , $i = 1, \dots, I$, $j = 1, \dots, J$, and create matrix D .

Step 2.

For each $i = 1, \dots, I$, and each r_k , $k = 1, \dots, K$ determine subset $J_i(r_k)$ and calculate

$$s_i(r_k) = \left\{ \sum_{j \in J_i(r_k)} w_j \right\} / \left\{ \sum_{j \in J} w_j \right\}$$

Step 3.

For each $i = 1, \dots, I$, create a discrete and finite fuzzy set s_i shown in Fig 3:

$$s_i = \sum_{k=1}^K \mu(r_k) / s_i(r_k),$$

[Insert Fig. 3 here]

Step 4.

Using the comparison method given in Appendix, compare discrete fuzzy sets s_1, s_2, \dots, s_I and find s_{i^*} for which fuzzy relations $s_{i^*} \geq s_1, \dots, s_{i^*} \geq s_I$ are true. The $i^* \in I$ selected is the best facility node with respect to the percentage of the amount of covered demands at demand nodes.

Step 5.

Determine S_{i^*} , i.e., the corresponding measure of belief that i^* is better then or at least equal to any other facility node.

Algorithm 3

Step 1.

Calculate the elements d_{ij} , $i = 1, \dots, I$, $j = 1, \dots, J$, and create matrix D .

Step 2.

For each $i = 1, \dots, I$ and each $r_k, k = 1, \dots, K$ determine $m_{A,i}(r_k), m_{B,i}(r_k), m_{C,i}(r_k)$ using fuzzy arithmetic operations \cdot and \oplus , (see, [19]), create a triangular fuzzy sets $\mathbf{Q}_i(r_k)$:

$$\mathbf{Q}_i(r_k) = m_{A,i}(r_k) \cdot \mathbf{A} \oplus m_{B,i}(r_k) \cdot \mathbf{B} \oplus m_{C,i}(r_k) \cdot \mathbf{C}$$

Step 3.

For each $i = 1, \dots, I$ and each $r_k, k = 1, \dots, K$ defuzzify $\mathbf{Q}_i(r_k)$ by applying Centre-of-gravity method [19]. It determines a representative scalar $t_i(r_k)$ of a fuzzy set $\mathbf{Q}_i(r_k)$ as an element in the fuzzy set domain at which a line perpendicular to the axis passes through the centre of the area formed by the corresponding membership function:

$$t_i(r_k) = [\text{defuz } \mathbf{Q}_i(r_k)]$$

Step 4.

For each $i = 1, \dots, I$, create a discrete and finite fuzzy set \mathbf{t}_i shown in Fig 4:

$$\mathbf{t}_i = \sum_{k=1}^K \mu(r_k) / t_i(r_k),$$

[Insert Fig. 4 here]

Step 5.

Using the comparison method given in Appendix compare discrete fuzzy sets $\mathbf{t}_1, \mathbf{t}_2, \dots, \mathbf{t}_I$ and find \mathbf{t}_{i^*} for which fuzzy relations $\mathbf{t}_{i^*} \geq \mathbf{t}_1, \dots, \mathbf{t}_{i^*} \geq \mathbf{t}_I$ are true. The i^* selected is the best facility node with respect to the total coverage of demand at demand nodes labeled by their weights.

Step 6.

Determine T_{i^*} , i.e., the corresponding measure of belief that i^* is better then or at least equal to any other facility node is T_{i^*} .

5. Extension to the problems of locating two, three and more facility sites

All node covering models discussed to this point assume that only one facility site has to be located. The presented methodology can be extended to the problems to locate two, three, and up to L -facility sites. In such cases, there is only one additional task which is necessary to accomplish.

First, consider the problem to locate two-facility sites. At the beginning it is necessary to calculate the shortest distances $d_{(i', i''), j}$ between each pair (i', i'') , $i', i'' \in I$ of potential locations of service facilities and demand node $j \in J$. It is simply done by calculating:

$$d_{(i', i''), j} = \min (d_{i', j}, d_{i'', j}), i', i'' \in I, j \in J.$$

The distances $d_{(i', i''), j}$ can be arranged in a distance matrix D^2 with the number of rows equal to

$$\binom{I}{2} = \frac{I!}{2!(I-2)!}, \text{ each representing a potential facility pair, and the number of columns equal } J.$$

After that, choosing the best facility location pair is performed following one among the three algorithms given in Section 3.

In a very similar way L best facility location sites can be chosen by generating matrix D^L and then applying algorithms presented in Section 3. The number of possible location

configurations and consequently the number of rows in D^L is $\binom{I}{L} = \frac{I!}{L!(I-L)!}$. Thus, choosing

the best among the possible location configurations would be computationally prohibitive for large values of I and L .

6. Illustrative example

Consider a distribution company with $N = 15$ retailers and $I = 3$ potential locations for depots, spatially distributed on a geographical area and modelled by an undirected network, Fig. 5.

The node 1, 10, and 12 are potential locations for depots. The parameters associated to the links are travel times between nodes in some time units. Matrix D of the shortest distances between the potential locations for facility service and demand nodes is given in Table 1. Fuzzy set *covered demand nodes* \mathbf{R} is given by four pairs: $\mathbf{R} = (20, 1), (24, .8), (28, .5), (30, .3)$.

[Insert Fig. 5 here]

[Insert Table 1 here]

Three cases are considered. In each case, the demands at demand nodes are specified in a different way.

Case 1: All demand nodes have equal weights.

The problem is to determine the best location for a depot which maximises the measure of belief that the largest number of retailers is covered.

In Case 1 the results are obtained by applying *Algorithm 1* and are given in Table 2.

[Insert Table 2 here]

Location 12 is better than Location 1 or Location 10. The measure of belief that location 12 is better or at least equal as location 1 is 0.64 and the measure of belief that location 12 is better or at least equal as location 10 is 0.50.

Case 2: Demand at retailers are described by the linguistic terms as follows:

low demand retailers are: 4, 6, 7

medium demand retailers are: 1, 2, 5, 9, 10, 12, 13, 14, 15

high demand retailers are: 3, 8, 11

The problem is to determine the best location for a depot which maximises the measure of belief that the largest coverage of demand at retailers, labeled by their weights, is obtained.

In Case 2, *Algorithm 3* is applied and the results are given in Table 3.

[Insert Table 3 here]

The measure of belief that location 12 is better or at least equal as location 1 is 0.64 and the measure of belief that location 12 is better or at least equal as location 10 is 0.53.

Case 3: Potential depot location configurations are pairs of nodes (1, 10), (1, 12), (10, 12).

The problem is to determine the pair of locations for depots which maximises the measure of belief that the largest coverage of demand at retailers, labeled by their weights, is obtained.

In order to choose the best pair of locations in Case 3, D^2 matrix is calculated first. It is given in Table 4. Then *Algorithm 3* is applied and the results are given in Table 5.

[Insert Table 4 here]

[Insert Table 5 here]

The measure of belief that the pair of nodes (10, 12) is better or at least equal as the pair of nodes (1, 10) is 1, and the measure of belief that the pair (10, 12) is better or equal as the pair (1, 12) is 0.78.

7. Discussion and conclusion

Locating facilities in a network in such a way as to maximise demand coverage at demand nodes in the presence of uncertainty has been recognized as a complex problem, both conceptually and computationally. A large number of models concerning facility location problems developed so far are limited to the simplified deterministic cases where the demand at facility locations and the distances between them are precisely known.

In this paper, an attempt is made to formulate a class of network covering problems which imitate, as close as possible, the reality of a wide class of facility location tasks. Following this, an important point is identified: in real practice the description of facility location is commonly given full of linguistic vagueness.

The first step to cope with the vague nature of the coverage problems is a realistic assumption that the weight or the importance of the demand at a node has to be given by a vague linguistic term, and not simply, by some deterministic value. In this paper three primary terms *low demand*, *medium demand* and *high demand*, modelled by triangular fuzzy sets, are used to describe the weights of demand nodes. Further on, it is easy to extend the vocabulary list and include more linguistic terms. For example, two more terms such as *very low demand* and *very high demand* can be easily generated by applying appropriate intensification operators, i.e. by using fuzzy hedges, and still preserve triangular fuzzy sets as their descriptors.

The next step in developing a covering model which is important for a wide range of real situations is to introduce the concept of a gradual coverage of demand nodes. A realistic assumption is made that each demand node is covered if it is within a vaguely expressed acceptable service distance/time. It is shown that the use of fuzzy sets with a step-wise decreasing membership function is an appropriate way to describe the practical meaning of node coverage. If a demand node is within a given optimistic distance/time from a facility site the demand node is fully covered, and if it is beyond a given pessimistic distance/time, it is not covered at all. For a distance/time between the optimistic and pessimistic values a gradual coverage decreases from 1 to 0, following the step-wise membership function.

Three new algorithms with the embedded concept of fuzzy node coverage, which assume either equal, precisely known or imprecisely specified relative weights of demand nodes, are the main contributions of the paper. The algorithms are based on the searching among nodes – candidates for sitting facilities and on establishing a fuzzy preference structure on a number of discrete fuzzy sets which describe coverage efficiency. The selection of the best facility sites is supported by calculating an associated measure of belief that one facility site is the best with respect to the node coverage.

For one-site problems, the algorithms involve one-dimensional searching among potential facility nodes. Searching time depends linearly on the number of potential locations of service facilities, and, in line with that, the algorithms are efficient for medium size network location problems where the number of demand nodes and/or the number of potential facility sites is of the order of hundreds. For multi-site problems the use of presented algorithms in practice is restricted to smaller networks since the searching time rises in a combinatorial way, following the formula for the number of combinations of a given number of elements divided in classes. The algorithms are suited for smaller networks where the number of demand nodes and/or the number of potential facility sites are of the order of tens.

The models presented and the algorithms developed increase our capacity for solving real world covering problems in uncertain environments.

Appendix

Comparison of discrete fuzzy sets

This Appendix describes an efficient procedure for determining a fuzzy preference structure on a number of discrete fuzzy sets. The measure of belief $b(\mathbf{M} < \mathbf{N})$ that a fuzzy relation $\mathbf{M} < \mathbf{N}$ between two discrete fuzzy sets \mathbf{M} and \mathbf{N} is true is defined as the probability that a crisp value $m \in S(\mathbf{M})$ is less than a crisp value $n \in S(\mathbf{N})$ [22]:

$$b(\mathbf{M} < \mathbf{N}) = \text{Prob}(m < n) \quad \text{A.1}$$

where $S(\mathbf{M})$ and $S(\mathbf{N})$ represent the supports of the fuzzy sets \mathbf{M} and \mathbf{N} , respectively. The support of a fuzzy set is defined as a crisp set that contains all the elements of the fuzzy set that have nonzero membership degrees. Let $S(\mathbf{M}) = \{m_1, m_2, \dots, m_{U_m}\}$ and $S(\mathbf{N}) = \{n_1, n_2, \dots, n_{U_n}\}$.

Let us introduce the following:

$P_M(m_l)$ - the probability that \mathbf{M} takes a given crisp value $m_l \in S(\mathbf{M})$

$\Phi_M(m_l)$ - the probability that \mathbf{M} takes a crisp value which is smaller or equal to $m_l \in S(\mathbf{M})$

Then,

$$P_M(m_l) = \mu_M(m_l) \left[\sum_{j=1}^{U_m} \mu_M(m_j) \right]^{-1} \quad \text{A.2}$$

$$\Phi_M(m_l) = \sum_{j=1}^l P_M(m_j) \quad \text{A.3}$$

Consequently, $1 - \Phi_M(m_l)$ is the probability that \mathbf{M} takes a crisp value which is greater than $m_l \in S(\mathbf{M})$.

Analogous expressions hold for a discrete fuzzy number \mathbf{N} . Then, under a realistic assumption that random values defined on $S(\mathbf{M})$ and $S(\mathbf{N})$ are independent (A.1) becomes:

$$b(\mathbf{M} < \mathbf{N}) = \sum_{l=1}^{U_m} \text{Prob}[m = m_l \wedge n > m_l] = \sum_{l=1}^{U_m} P_M(m_l)[1 - \Phi_N(m_l)] \quad \text{A.4}$$

The following relationship holds:

$$b(\mathbf{M} < \mathbf{N}) = 1 - b(\mathbf{M} \geq \mathbf{N}) \quad \text{A.5}$$

In the case when I discrete fuzzy sets $\mathbf{N}_1, \dots, \mathbf{N}_i, \dots, \mathbf{N}_I$ are given, the measure of belief $b(\mathbf{M} < \mathbf{N}_1, \dots, \mathbf{M} < \mathbf{N}_i, \dots, \mathbf{M} < \mathbf{N}_I)$ that fuzzy relations $\mathbf{M} < \mathbf{N}_1, \dots, \mathbf{M} < \mathbf{N}_i, \dots, \mathbf{M} < \mathbf{N}_I$ are true is:

$$b(\mathbf{M} < \mathbf{N}_1, \dots, \mathbf{M} < \mathbf{N}_i, \dots, \mathbf{M} < \mathbf{N}_I) = \min\{b(\mathbf{M} < \mathbf{N}_1), \dots, b(\mathbf{M} < \mathbf{N}_i), \dots, b(\mathbf{M} < \mathbf{N}_I)\} \quad \text{A.6}$$

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Figure Captions

Fig. 1 Fuzzy set *covered demand node* with a stepwise decreasing membership function

Fig. 2. Discrete fuzzy set \mathbf{p}_i

Fig. 3. Discrete fuzzy set \mathbf{s}_i

Fig. 4. Discrete fuzzy set \mathbf{t}_i

Fig. 5. Undirected network with 15 nodes and 19 links

Tables

Table 1. Matrix D of the shortest distances between depots and retailers

Table 2. Case 1: Number and percentage of retailers covered

Table 3. Case 2: Number of **A**, **B**, **C** types of retailers covered

Table 4. Matrix D^2 of the shortest distances between the pairs of depots and retailers

Table 5. Case 3: Number of **A**, **B**, **C** types of retailers covered by each pair of depot location cites

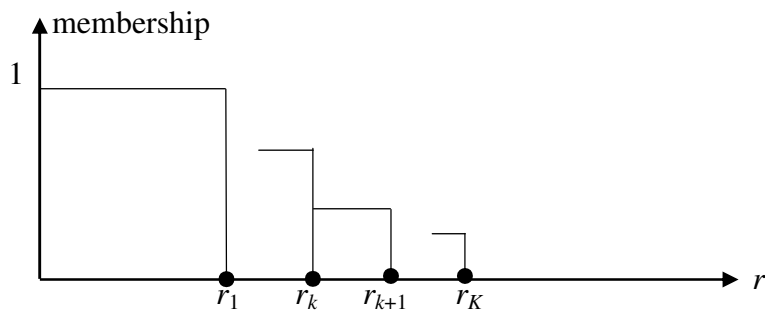


Fig. 1 Fuzzy set "Covered demand node"

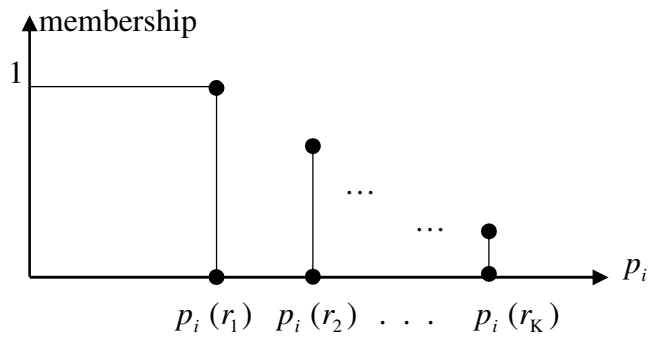


Fig. 2. Discrete fuzzy set \mathbf{p}_i

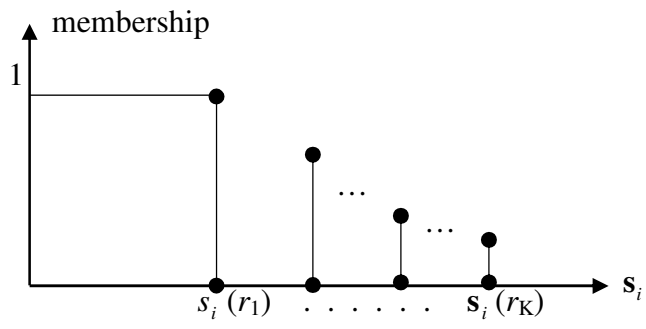


Fig. 3. Discrete fuzzy set s_i

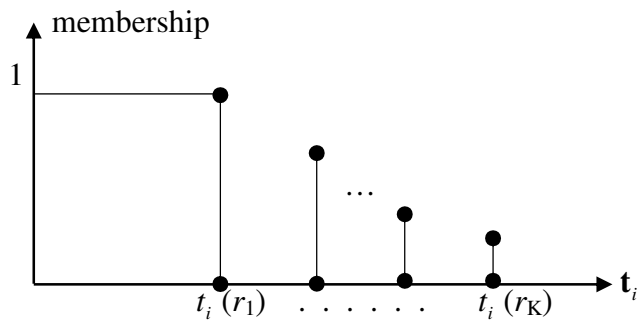


Fig. 4. Discrete fuzzy set t_i

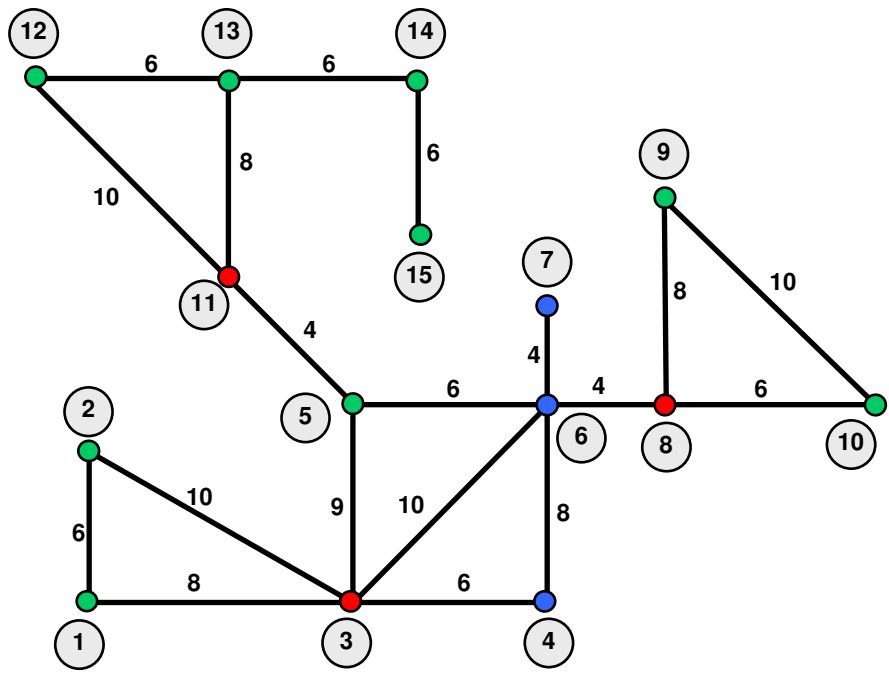


Fig. 5. Undirected network with 15 nodes and 19 links

Table 1. Matrix D of the shortest distances between facility nodes and demand nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	—	6	8	14	17	18	22	22	30	28	21	31	29	35	41
10	28	30	20	20	16	10	14	6	10	—	20	30	28	34	40
12	30	32	22	28	14	20	24	24	32	30	10	—	6	12	18

Table 2. Case 1: Number and percentage of nodes covered

	Distances $\mu(r_k) / r_k$							
	1 / 20		0.8 / 24		0.5 / 28		0.3 / 30	
Facility node	No. of nodes covered	% of nodes	No. of nodes covered	% of nodes	No. of nodes covered	% of nodes	No. of nodes covered	% of nodes
1	6	40	9	60	11	73	12	80
10	9	60	9	60	11	73	12	80
12	7	47	10	67	11	73	13	87

Table 3. Case 2: Number of A, B, C types of nodes covered

Facility node <i>i</i>	Distances $\mu(r_k) / r_k$							
	1 / 20		0.8 / 24		0.5 / 28		0.3 / 30	
	No. of nodes covered A B C	t_i	No. of nodes covered A B C	t_i	No. of nodes covered A B C	t_i	No. of nodes covered A B C	t_i
1	2 3 2	21	3 3 3	22	3 5 3	33	3 6 3	36
10	3 3 3	27	3 3 3	27	3 5 3	33	3 6 3	36
12	1 5 1	21	2 5 3	31	3 5 3	33	3 7 3	39

Table 4. Matrix D^2 of the shortest distances between the pairs of facility nodes and demand nodes

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	B	B	C	A	B	A	A	C	B	B	C	B	B	B	B
1, 10	—	6	8	14	16	10	14	6	10	—	20	30	28	34	40
1, 12	—	6	8	14	14	18	22	22	30	28	28	—	6	12	18
10, 12	28	30	20	20	14	10	14	6	10	—	10	—	6	12	18

Table 5. Case 3: Number of A, B, C types of nodes covered by each pair of facility location cites

Pairs of facility nodes	Distances $\mu(r_k) / r_k$															
	1 / 20			0.8 / 24			0.5 / 28			0.3 / 30						
	Nodes covered			Nodes covered			Nodes covered			Nodes covered						
	A	B	C	$t_{i',i''}$	A	B	C	$t_{i',i''}$	A	B	C	$t_{i',i''}$	A	B	C	$t_{i',i''}$
1, 10	3	4	3	30	3	5	3	32	3	6	3	36	3	7	3	39
1, 12	2	7	2	33	3	7	3	39	3	8	3	42	3	8	3	42
10, 12	3	7	3	39	3	7	3	39	3	8	3	42	3	9	3	45