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Modeling of steel frames with semi rigid Joints

A.**N.T.Ihaddoudène [1] , M. Saidani[2,a] and M.Chemrouk[3]**

[1, 3] Built Environment Research Laboratory, Faculty of Civil Engineering, U.S.T.H.B., Algiers, Algeria

[2] Faculty of Engineering and Computing, Department of Built Environment, Coventry University, Coventry, England, UK.

Emails: nacer_i@yahoo.fr, m.saidani@coventry.ac.uk, epau_chemrouk@yahoo.fr

Abstract

Beam to column connections are usually considered either as perfectly rigid or as nominally pinned. However, as is now well established, this does not represent the actual behaviour of the connection, and hence the impact it has on the behaviour of the structure itself, as connections are often semi-rigid. The present paper presents a mechanical model of the connections, where the rigidity of the joint is represented by means of rotational and translational springs introducing the concept of non deformable element of nodes, describing relative displacements and rotations between the nodes and the elements of the structure. Examples are provided to illustrate the simplicity and efficiency of the method and comparisons to recently published results have proved the generalization, the simplicity and the efficiency of the proposed model.

Keywords: Rigid, Semi-rigid, Connection, Mechanical model, Frames, Buckling

Notations:

k_1, k_2 : Elastic constants of the springs in rotation at nodes "i" and "j", respectively

C_1^v, C_2^v: Elastic constants of vertical springs at nodes "i" and "j", respectively.

C_1^u, C_2^u : Elastic constants of axial springs at nodes "i" and "j", respectively.
Ψ: Area of bending moment diagram for a simply supported beam.
nl, ml: Distance to left support and right support respectively, from gravity centre Ψ
ω: Flexural rigidity per unit length, $\frac{EI}{l}$
Δ_i: Relative vertical displacement between nodes "i" and "j"
V_i, M_i, V_j, M_j: Reactions at nodes "i" and "j", in local reference.
$\overline{F_e}$: Vector force in local reference.

Introduction

Conventional analysis of steel frames assumes either perfectly rigid or pinned connections. The rigidity of most connections behaves, in fact, somewhat between these two extremes and incorporating the effect of connection flexibility of the frame becomes necessary.

To assess the real behaviour of the frame, it is therefore necessary to incorporate the effect of connection flexibility of the frame (Bjorhovde et. al. [1]; Gerstle [2]; Kishi and Chen [3]; Saidani [4]; Ihaddoudène and Chemrouk [5]).

In the past few decades, extensive research works on static loading tests for different types have been carried out and conducted on the commonly used connections from which large collections of test data have been reported (Jaspart [6]; Azizinamini et al [7]; Nethercot and Zandonini [8]).

Some researchers such as Kishi and Chen [3], have collected available experimental results and constructed steel connection data banks that provided the user with not only the test data, but also some predictive equations, however, not every structural engineer has access to the data base of experimental results.

For the beam-moment connections, it is clearly shown (Kishi and Chen [3]; Jaspart [6]; Azizinamini et al [7]; Nethercot and Zandonini [8]; Atamaz Sibai [9]; Aribert et al [10]) that the moment-rotation relationship is non-linear for all types of connections and varies depending on connection flexibility.

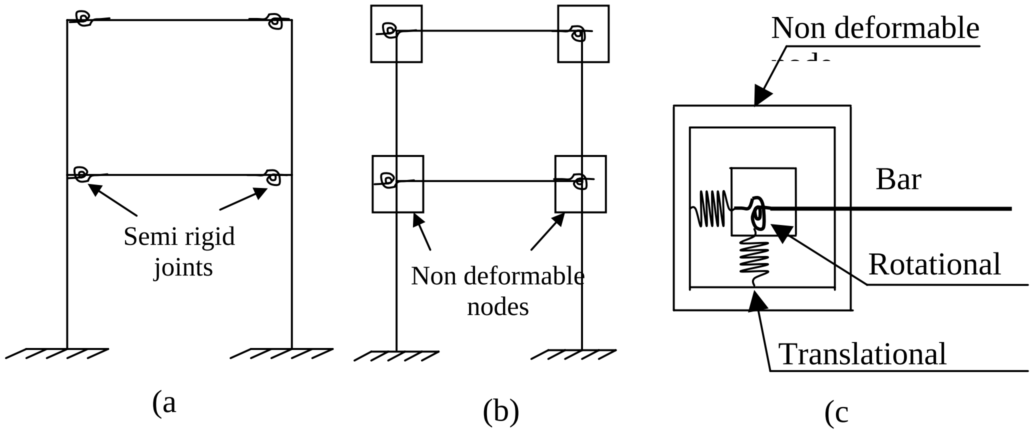
Several mathematical models have been proposed to fit the moment-rotation curves from experimental data. These models vary widely in their complexity to describe $(M-\theta)$ curves of connections. It is presented in its exponential form (Ihaddoudène and Chemrouk [5]) by the equation:

$$\Theta = kM^{\alpha} \tag{1}$$

Because of the high number of the parameters influencing the behaviour of connections, accurate modeling of such behaviour becomes complex. Globally, initial rigidity and the ultimate moment of the connection are the two most significant characteristics to define the behaviour of a joint (Bjorhovde et. al. [1]; Ihaddoudène and Chemrouk [5].

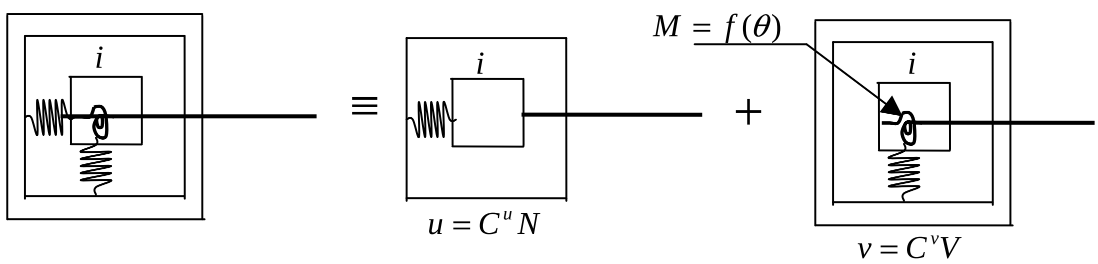
Mechanical model

To incorporate the effect of semi rigid joints on steel frames (Fig.1a), the concept of a non-deformable element of node (Fig.1b) describing relative displacements and rotations between the nodes and the elements of the structure is introduced. The adopted model (Ihaddoudène and Chemrouk [5]) is based on the analogy of three springs with two translational and one rotational (Fig.1c) and the effect of each spring is considered separately (Fig.2).



(a): Semi rigid joints; (b): Non deformable node; (c): Bar element and non deformable node.

Figure 1: Mechanical model adopted



(a): Bar element and non deformable node;(b): Axial spring; (c):Translational and rotational springs

Figure 2 :Analogy of three springs

Equilibrium equations and rotational deformations

2.1.1. Bar element subjected to axial force:

$$N_{ij} = -N_{ji} = N \tag{2.a}$$

$$\Delta_{j,1} = \frac{Nl}{EA} + C_1^u N + C_2^u N \tag{2.b}$$

2.1.2. Bar element subjected to transversal forces: When considering the actions of the vertical and rotational springs, the equilibrium equations are expressed as:

$$V_i + V_j - R = 0 \tag{3.a}$$

$$M_i + M_j + RZ - V_j l = 0 \tag{3.b}$$

The rotational deformations are for the node "i" (see Figure.3)

$$\Theta_i = \frac{1}{l}(\Delta_i - C_1^v V_i + C_2^v V_j) + \frac{m\Psi}{\omega l} + \frac{M_i}{3\omega} + k_1 M_i - \frac{M_j}{6\omega} \tag{3.c}$$

and for the node "j",

$$\Theta_j = \frac{1}{l}(\Delta_i - C_1^v V_i + C_2^v V_j) - \frac{n\Psi}{\omega l} + \frac{M_j}{3\omega} + k_2 M_j - \frac{M_i}{6\omega} \tag{3.d}$$

In a simplified form, these equations can be written as:

$$2(A_1 + B)M_i - (1 - 2B)M_j = 6\omega\theta_i - 6\omega\frac{\Delta}{l} - 6m\frac{\Psi}{l} \tag{3.e}$$

$$-(1 - 2B)M_i + 2(A_2 + B)M_j = 6\omega\theta_j - 6\omega\frac{\Delta}{l} + 6n\frac{\Psi}{l} \tag{3.f}$$

Where $A_1 = (1 + 3k_1\omega)$, $A_2 = (1 + 3k_2\omega)$ and $B = \dfrac{3\omega(C_1^v + C_2^v)}{l^2}$

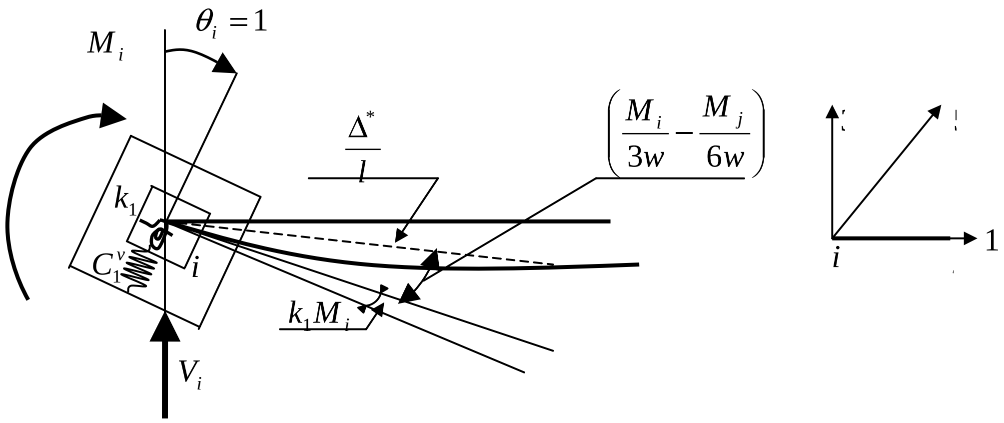


Figure 3: Unit rotation of node "i"

where $\Delta^* = (\Delta_i - C_1^v V_i + C_2^v V_j)$

Rigidity of the bar element

2.2.1. Rigidity of the bar element for the displacement $\Delta_{j,1}$ in direction 1:

By imposing the unit displacement to the node "j", the axial effort in the bar element is expressed as:

$$\Delta_{j,1} = N\left(\frac{l}{EA} + C_1^u + C_2^u\right) = 1 \tag{4.a}$$

$$N = \frac{EA}{l + EA(C_1^u + C_2^u)} = a_{11}\frac{EA}{l} \tag{4.b}$$

where: $a_{11} = \dfrac{1}{1 + \dfrac{EA}{l}(C_1^u + C_2^u)}$

2.2.2. Rigidity of the bar element for the displacement of the node "i" in the transversal direction:

By imposing to node "i", a unit displacement $\Delta_{i,3} = 1$ in the direction 3(see Figure.4)

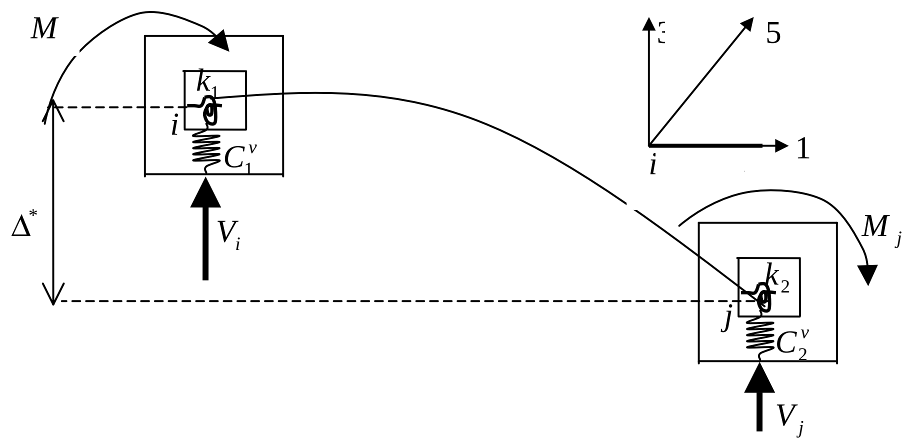


Figure 4: Unit displacement of node "i"

Replacing the limit conditions $\Delta_{i,3} = \Delta_i = 1$ and $\Theta_{i,5} = \Theta_{j,5} = \Psi = R = 0$ in the equations (3.e) and (3.f) leads to:

$$M_i = -\frac{(1 + 2A_2)}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2}\frac{6\omega}{l} \tag{5.a}$$

$$M_j = -\frac{(1 + 2A_1)}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2}\frac{6\omega}{l} \tag{5.b}$$

$$V_i = \frac{1 + A_1 + A_2}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2}\frac{12\omega}{l^2} \tag{5.c}$$

2.2.3. Rigidity of the bar element to the rotation around the axe 5

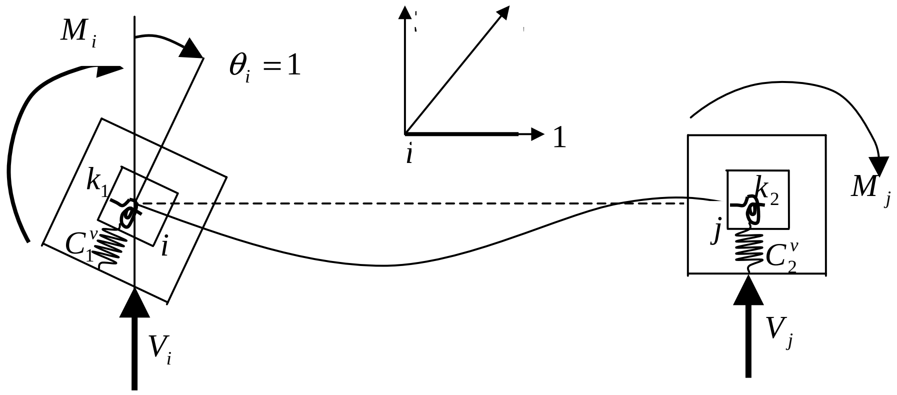


Figure 5: Unit rotation of node $\theta_i = 1$

Imposing to the node "i" of the structure the unit rotation $\theta_i = \theta_{i,5} = 1$ around the axe 5, replacing the limit conditions $\theta_i = \theta_{i,5} = 1$, $\Delta = \theta_j = \Psi = R = 0$ in the equations (3.e) and (3.f), one obtain:

$$M_i = \frac{3(A_2 + B)}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2} 4\omega \quad (6.a)$$

$$M_j = \frac{3(1 - 2B)}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2} 2\omega \quad (6.b)$$

$$V_i = -\frac{1 + 2A_2}{4(A_1 + B)(A_2 + B) - (1 - 2B)^2} \frac{6\omega}{l} \quad (6.c)$$

2.2.4. Reactions of the bar element for fixed ends:

If the nodes "i" and "j" are fixed in their initial positions, the internal efforts at each end of the bar element can be determined from the equations (3.e) and (3.f) by imposing the limit conditions: $\Delta = \theta_i = \theta_j = 0$.

$$M_i = -\frac{6\Psi\left[2m(A_2 + B) - n(1 - 2B)\right]}{l\left[4(A_1 + B)(A_2 + B) - (1 - 2B)^2\right]} \quad (7.a)$$

$$M_j = \frac{6\Psi\left[2n(A_1 + B) - m(1 - 2B)\right]}{l\left[4(A_1 + B)(A_2 + B) - (1 - 2B)^2\right]} \quad (7.b)$$

Stiffness matrix

To establish the modified stiffness matrix considering the effect of connection flexibility, the direct method is used, i.e. the rigidity k_{ij} of an element is the reaction in the direction "j" due to a unit displacement in the direction "i".

The nodes of the beam are represented by non deformable frames at each ends. Considering in the mechanical model adopted (Ihaddoudène and Chemrouk [5]), the beam with different rotational flexibilities k_1 and k_2 at ends i and j respectively. In order to establish the different elements of the stiffness matrix $\overline{K_e}$ in local reference, equilibrium equations and rotational deformations are considered for each element k_{ij}.

The stiffness matrix, $\overline{K_e}$, in local coordinates is given by:

$$\overline{K_e} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \quad (8.a)$$

Terms k_{2j} of the stiffness matrix (8.a), for instance in bending, are obtained by letting $\theta_i = 1$, $\Delta_i = 0$; $\theta_j = 0$; $\Psi = R = 0$; $B = 0$ in the Eq. (3.e) and (3.f).

For linear behaviour $\theta = kM^{\alpha}$ where $\alpha = 1$

Which gives:

$$k_{21} = -\frac{18\,\omega(1+2k_2\omega)}{l\left[4(1+3k_1\omega)(1+3k_2\omega)-1\right]} \quad (8.b)$$

$$k_{22} = \frac{12\omega(1+3k_2\omega)}{4(1+3k_1\omega)(1+3k_2\omega)-1} \quad (8.c)$$

$$k_{23} = -k_{21} \quad (8.d)$$

$$k_{24} = \frac{6\omega}{4(1+3k_1\omega)(1+3k_2\omega)-1} \quad (8.e)$$

The same procedure is followed in deriving all the terms of the local stiffness matrix $\overline{K_e}$.

Nodal load vector

When the beam with different flexibilities k_1 and k_2 at both fixed ends, is subjected to an external load "q", the nodal load vector is established in local reference as:

$$\overline{F_e} = \left[\overline{X_i},\ \ \overline{Y_i},\ \ \overline{M_i},\ \ \overline{X_j},\ \ \overline{Y_j},\ \ \overline{M_j}\right]^T = \left[0,\ \ -V_i,\ \ -M_i,\ \ 0,\ \ -V_j,\ \ -M_j\right]^T \quad (9)$$

In global reference, the stiffness matrix and the vector force are obtained respectively as:

$$\mathrm{K_e} = \mathrm{T_e^T}.\overline{\mathrm{K_e}}.\mathrm{T_e} \quad (10.a)$$

$$F_e = T_e^T.\overline{F_e} = \left[V_i\sin\beta,\ \ -V_i\cos\beta,\ \ -M_i,\ \ V_j\sin\beta,\ \ -V_j\cos\beta,\ \ -M_j\right]^T \quad (10.b)$$

In which $\mathrm{T_e}$ is the transformation stiffness matrix given by:

$$T_e = \left[\begin{array}{ccc|ccc} \cos\beta & \sin\beta & 0 & 0 & 0 & 0 \\ -\sin\beta & \cos\beta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \cos\beta & \sin\beta & 0 \\ 0 & 0 & 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

The angle β defines the orientation of the element with respect to the global reference system.
Finally, the internal forces are calculated by using the well known equation:

$$[K_e]\{U_e\} = \{F_e\} \tag{11}$$

Buckling consideration

The system of equilibrium equation is now expressed in the deformed state of the bar element, which connections are semi rigid. Here, the effects of semi rigid joints on the buckling load of steel frames are considered.
In order to express the terms k_{2j}, for instance, of the stiffness matrix, let us considering a column with semi rigid joints (Fig.6), the equilibrium equations are expressed as:

$$H_i = H_j = H \tag{12.a}$$

$$M_x = Ny + Hx - M_j \tag{12.b}$$

$$M_i = Hl - M_j \tag{12.c}$$

The governing differential equations is given as follows:

$$EIy''(x) = -Ny - Hx + M_j \tag{13.a}$$

$$y''(x) + \alpha^2 y = -\frac{Hx + M_j}{EI} \tag{13.b}$$

Where $\alpha^2 = \frac{N}{EI}$

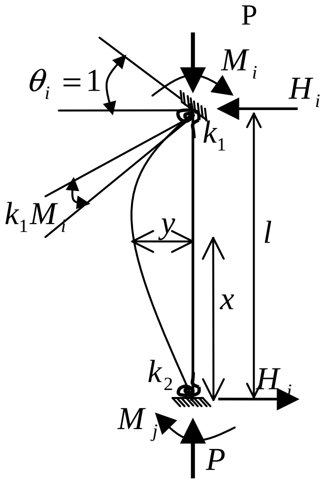


Figure 6: Bar element with semi rigid joints

The general solution of Eq.(12) is:

$$y(x) = A\cos\alpha x + B\sin\alpha x + y_p \tag{13.c}$$

Where y_p, A and B are respectively the particular solution and integration constants to be determined using the boundary conditions at the ends supports:

$y(0)=0$ leads to $B=-\dfrac{M_j}{\alpha^2 EI}$ (13.d)

$y(l)=0$ leads to $A=\dfrac{1}{\alpha^2 EI\sin\alpha l}\left[M_j(\cos\alpha l-1)+Hl\right]$ (13.e)

Hence,

$$y(x)=\frac{\sin\alpha x}{\alpha^2 EI\sin\alpha l}\left[M_j(\cos\alpha l-1)+Hl\right]-\frac{M_j\cos\alpha x}{\alpha^2 EI}-\frac{Hx+M_j}{\alpha^2 EI} \tag{14}$$

The derivative:

$$y'(x)=\frac{\alpha\cos\alpha x}{\alpha^2 EI\sin\alpha l}\left[M_j(\cos\alpha l-1)+Hl\right]+\frac{\alpha M_j\sin\alpha x}{\alpha^2 EI}-\frac{H}{\alpha^2 EI} \tag{15}$$

The reactions H, M_i and M_j are determined employing the boundary conditions

$$y'(0)=k_2 M_j \tag{16.a}$$

$$y'(l)=-1+k_1 M_i \tag{16.b}$$

We obtain :

$$H=\frac{w}{l}\xi_1(v) \tag{17.a}$$

$$M_j=w\phi_1(v) \tag{17.b}$$

$$M_i=w\phi_2(v) \tag{17.c}$$

Where $v=\alpha l$

From which the terms of k_{2j} are derived (see Table 1)

Table 1: k_{2j} expressions in several situations

Both axial force and semi rigid connections are accounted	Only axial force is accounted
$k_{21}=\dfrac{EI}{l^2}\dfrac{v^2\left[1-\cos v+k_2 vw\sin v\right]}{D}$	$k_{21}=\dfrac{EI}{l^2}\dfrac{v^2\left[1-\cos v\right]}{(2-2\cos v-v\sin v)}$
$k_{22}=\dfrac{EI}{l}\dfrac{v\left[\sin v-v\cos v+k_2 v^2 w\sin v\right]}{D}$	$k_{22}=\dfrac{EI}{l}\dfrac{v\left[\sin v-v\cos v\right]}{(2-2\cos v-v\sin v)}$
$k_{23}=-\dfrac{EI}{l^2}\dfrac{v^2\left[1-\cos v+k_2 vw\sin v\right]}{D}$	$k_{23}=-\dfrac{EI}{l^2}\dfrac{v^2\left[1-\cos v\right]}{(2-2\cos v-v\sin v)}$
$k_{24}=\dfrac{EI}{l}\dfrac{v\left[v-\sin v\right]}{D}$	$k_{24}=\dfrac{EI}{l}\dfrac{v\left[v-\sin v\right]}{(2-2\cos v-v\sin v)}$

Where $D=(2-2\cos v-v\sin v)+\xi(v,k_1,k_2)$

The same procedure is employed to derive all the components of the matrix. The buckling load for the frame is obtained when the determinant is equal to zero, i.e:

$$\det[K] = 0 \tag{18}$$

Example

The frame below [11] which has a single span $L = 20m$, a story height $h_1 = 20m$ and $h_2 = 15m$ is subjected to concentrated and distributed load $P = 12kN$, $q = 2.4kN/ml$ respectively. Different flexibilities have been considered; as for the columns AB and DC, they are k_1, k_2 and k_6, k_5 respectively, for the beam they are k_3, k_4 as shown in Figure.7.

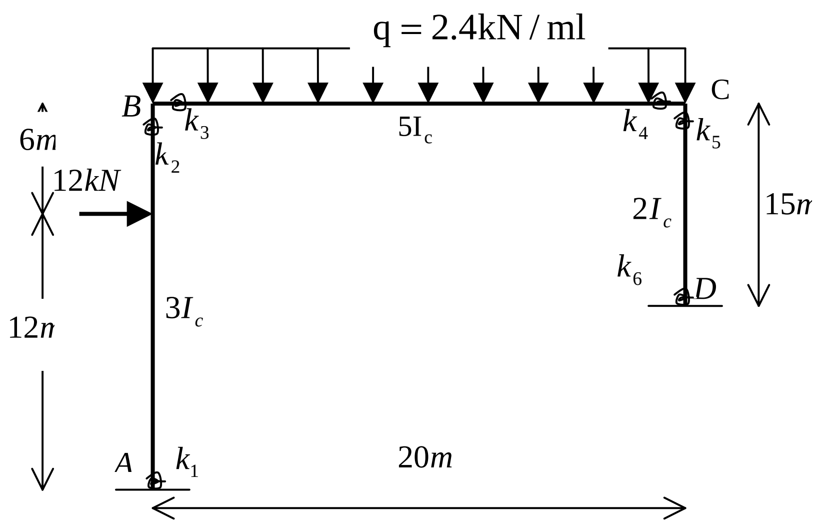


Figure 7: Example of the reference [11]

In bending, the rotational spring is the essential component and hence the equations of rotational deformations (7.a) and (7.b) are reduced and expressed as:

$$M_i = -\frac{6\Psi[2m(1+3k_2\omega)-n]}{l[4(1+3k_1\omega)(1+3k_2\omega)-1]}$$

$$M_j = \frac{6\Psi[2n(1+3k_1\omega)-m]}{l[4(1+3k_1\omega)(1+3k_2\omega)-1]}$$

$$V_i = \frac{M_i + M_j - RZ}{l}$$

Table 2 below summarises the calculation data for each element:

Table 2: Example data [11]

Elements	1	2	3 (No charge)
Ψ [m²]	$432m^2$	$1600m^2$	-
m	0.444	0.5	-
n	0.555	0.5	
$1+3k_i w$	2.5	1.666	3.333
$1+3k_j w$	2.0	2.5	1.25
M_i [kN.m]	-9.262	-61.28	-
M_j [kN.m]	17.684	35.744	-

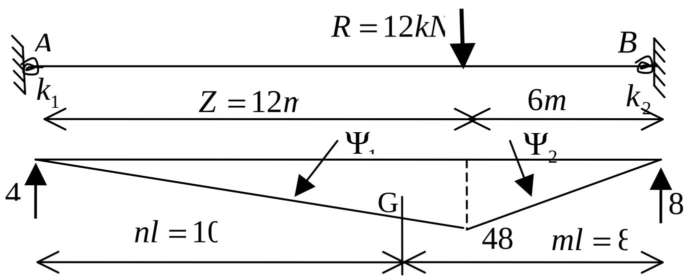


Figure 7: a Area of bending moment diagram for a simply supported beam.

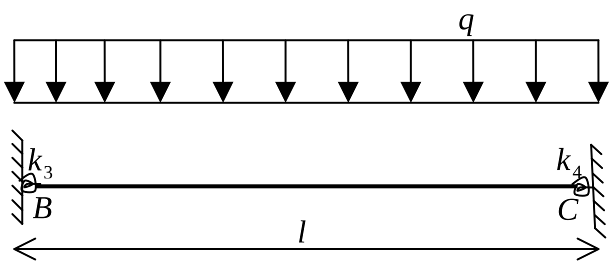


Figure 7 : b. Beam element

The load vector force is then:

$$F = \begin{bmatrix} M_1 \\ M_2 \\ H \end{bmatrix} = \begin{bmatrix} -17.684 + 61.28 \\ -35.744 \\ 8.468 \end{bmatrix} = \begin{bmatrix} 43.592 \\ -35.744 \\ 8.468 \end{bmatrix}$$

The matrix stiffness of the structure is obtained by assembling the elements and the internal forces are calculated by using the equation with six degree of freedom:

$$[K_e]\{U_e\} = \{F_e\}$$

The final bending moment diagram is:

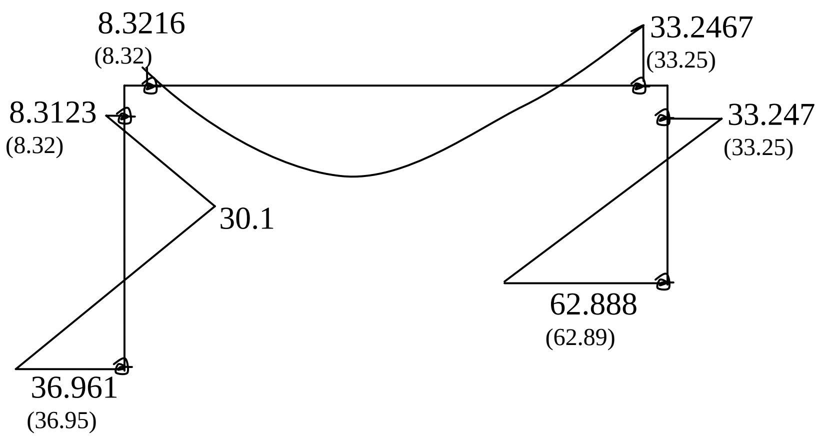


Figure 8: Bending moment diagram (kN.m), present study and [11]

The proposed method is in excellent agreement with the numerical results obtained by the author of reference [11].

Conclusions

The influence of the flexibility of connections in steel frames is investigated and a simple method of analysis and design is provided through a mechanical model for the joints.

The significant role of semi rigid joints on the buckling load on the frame is also included in the analysis of steel frames for linear buckling effect.

Illustrative examples of simple frame are examined and comparisons between the results obtained are similar, suggesting the proposed model is adequate and may be a useful tool in the analysis of steel frames with semi-rigid joints.

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