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Original citation:
Martineau, S; Burnham, K.J; Haas, O.C.L; Andrews, G. and Heely, A. (2004) Four-term bilinear PID controller applied to an industrial furnace *Control Engineering Practice* 12 (4), 457–464. DOI 10.1016/S0967-0661(03)00147-3

http://dx.doi.org/10.1016/S0967-0661(03)00147-3

Publisher:
Pergamon and Elsevier on behalf of the International Federation of Automatic Control (IFAC)

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Control Engineering Practice 12 (2004) 457–464

CONTROL ENGINEERING PRACTICE

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Four-term bilinear PID controller applied to an industrial furnace

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Received 16 January 2003; accepted 8 July 2003

Abstract

PID controllers are widely used in many industries and provide acceptable performances with no specific requirement for mathematical knowledge of the plant. However, these controllers, which are tuned for one operating point, are based on the assumption that local linearity holds for the plant to be controlled. When considering operation over a range, this assumption may become invalid, and it is at this juncture that the notion of bilinearisation is raised as providing a way forward. Application of a bilinear control strategy to a high-temperature industrial furnace is described and the results in terms of improved performance are presented.

Keywords: Bilinear; Control; Design; Gas; Industrial; PID; Temperature

1. Introduction

The three-term PID control systems are widely used in the process industries where they provide satisfactory and robust performance, when operated about the point of tuning, i.e. where local linearity holds. Via feedback of the system output the standard PID controller has the ability to eliminate steady-state offsets through integral action and it can also 'anticipate' the future through its derivative action. In continuous form, the PID algorithm may be expressed as

$$u(t) = K\left(e(t) + \frac{1}{T_i}\int_0^t e(\tau)\,\mathrm{d}\tau + T_d\frac{\mathrm{d}e(t)}{\mathrm{d}t}\right). \qquad (1)$$

As all practical systems exhibit nonlinear behaviour, the resulting system performance, when use is made of a standard PID controller (Aström & Hägglund, 1995) with fixed gains, is reduced; especially when the controller operates over a wide region about the point of tuning. One solution to alleviate this problem is to continually retune the PID parameters over the operating range (i.e. gain scheduling). Another is to detune the PID controller to enable a wider range of operation. In practice, because of constraints on time, availability of personnel and running costs of plant, the latter solution is commonly adopted, despite the fact that the plant operates sub-optimally.

Prompted by the fact that all practical systems exhibit nonlinear behaviour, coupled with the fact that bilinear structures are able to model nonlinear phenomena more accurately than linear structures, the Control Theory and Applications Centre (CTAC) of Coventry University has been involved, in collaboration with industry, in designing bilinear control strategies for nonlinear industrial plant. In particular, work has focused on improving the consistency and operating performance of high temperature gas-fired industrial furnaces. The bilinear PID (BPID) controller which is described in this paper, is considered to offer a realistic compromise between the standard PID controller and other rather more complex alternatives (Narendra, 1996), which have similarly been proposed by academic research groups over the last decade.

Research on bilinear systems dates back to the 1960s but intensity of activity in this area increased following the major breakthrough made by Mohler in the early 1970s with his work on modeling and control of nuclear reactor dynamics (Mohler & Shen, 1970). In Mohler (1973), a class of nonlinear bilinear models for biological and physical processes is surveyed and the main characteristics of bilinear systems are described. It is

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doi:10.1016/S0967-0661(03)00147-3

shown in Bruni, Dipillo, and Koch, (1974), that bilinear systems exhibit operating point dependent dynamical behaviour and this is representative of a wide range of practical systems. A good overview of bilinear system theory and applications may be found in Mohler and Kolodziej (1980). Work on bilinear systems within the CTAC began in the mid-1980s and a number of contributions have been made. The work of the CTAC has extended the linear self-tuning framework to the bilinear case (Burnham, 1991), thus facilitating the design of self-tuning controllers for bilinear systems. A bilinear form of generalised predictive control was developed in Goodhart (1991) and implemented for the first time in real-time on an industrial furnace. In Disdell (1995) the controller was extended and applied to a multi-zone multi-burner industrial furnace application. In Dunoyer (1996) the link between continuous and discrete time bilinear descriptions was investigated and an appropriate discretisation and bilinearisation scheme was proposed. In an attempt to fulfil the needs of improved control whilst at the same time providing a solution that would be welcomed by industry, a compromise between complexity and operational efficiency was found: the BPID controller, described in Minihan (2001).

This paper presents the trials which were carried out over a 5 month period at AvestaPolarit, Sheffield, UK, when the BPID controller was applied to a continuously operated gas-fired industrial furnace. Similar results have already been published for shorter periods of operation (Martineau, Burnham, Minihan, Gaura, & Haas, 2001a; Martineau et al., 2001b), however to confirm the promising results obtained, a prolonged period was considered necessary. The paper is based on that presented at the UKACC Control 2002, Postgraduate Symposium, held at Sheffield, UK (Martineau, 2002).

2. The bilinear approach

Within the classes of nonlinear systems, bilinear systems represent a sub-class which are defined to be linear in both state and control, with the nonlinearity (bilinearity) occurring as a product between state and control (Mohler, 1973). The state-space representation of a continuous single-input single-output (SISO) bilinear system is given by:

$$\dot{x}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t) + u(t)\boldsymbol{N}\boldsymbol{x}(t),$$
$$y(t) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t), \tag{2}$$

where $\boldsymbol{x} \in \Re^n$ is a vector of state variables and u, y, $\in \Re$ are the control input and process output variables, respectively. $\boldsymbol{A}$ is the $n \times n$ matrix of real constants, $\boldsymbol{b}$ is the $n \times 1$ vector of real constants, $\boldsymbol{N}$ is the $n \times n$ matrix of real constants, comprising the bilinear coefficients, and $\boldsymbol{c}$ is the $n \times 1$ output vector of real constants (Dunoyer, 1996). The matrices are defined as follows:

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & . & . & 0 & 0 \\ 0 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & 1 & 0 \\ 0 & 0 & . & . & 0 & 1 \\ -\alpha_0 & -\alpha_1 & . & . & -\alpha_{n-2} & -\alpha_{n-1} \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ 0 \\ 1 \end{bmatrix},$$

$$\boldsymbol{N} = \begin{bmatrix} 0 & 0 & . & . & 0 & 0 \\ 0 & 0 & . & . & 0 & 0 \\ . & . & . & . & . & . \\ . & . & . & . & 0 & 0 \\ 0 & 0 & . & . & 0 & 0 \\ \rho_0 & \rho_1 & . & . & \rho_{n-2} & \rho_{n-1} \end{bmatrix},$$

$$\boldsymbol{c}^{\mathrm{T}} = \begin{bmatrix} \beta_0 & \beta_m & 0 & 0 \end{bmatrix}. \tag{3}$$

Combining the terms in the state equation of Eq. (2) a SISO bilinear system can be expressed by:

$$\dot{\boldsymbol{x}}(t) = [\boldsymbol{A} + u(t)\boldsymbol{N}]\boldsymbol{x}(t) + \boldsymbol{b}u(t) \tag{4}$$

from which it is clear that the dynamic and steady-state response characteristics are input dependent. Indeed, for any bilinear system of the form Eq. (2), the steady-state output Y_{ss}, (the subscript ss denotes steady-state value) corresponding to a steady-state input U_{ss} is given by:

$$Y_{\mathrm{ss}} = \frac{\beta_0 U_{\mathrm{ss}}}{\alpha_0 - \rho_0 U_{\mathrm{ss}}}. \tag{5}$$

The steady-state input/output characteristics for the three different cases of the bilinear term are illustrated in Fig. 1 (Dunoyer, 1996). Clearly, if ρ_0 is zero, Eq. (5) represents a linear system—hence linear systems may be considered as a special subclass. A consequence of this close relationship between linear and bilinear systems is

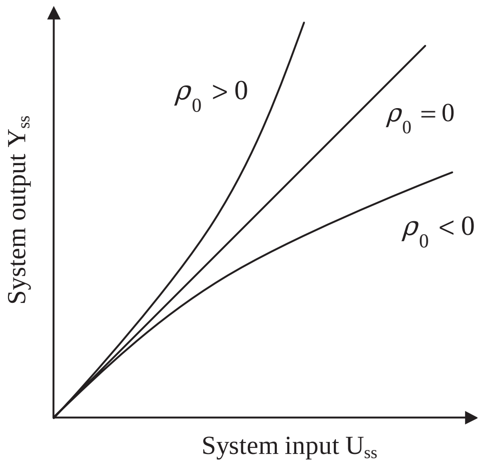

Fig. 1. Steady-state input/output characteristics of a bilinear system.

that many techniques developed for linear systems can be extended and applied to the bilinear case. Positive values of ρ_0 result in a gain which increases as U_{ss} increases, typical of exothermic chemical processes. Conversely, negative ρ_0 produces a gain, which decreases as U_{ss} increases, leading to eventual saturation, and is typical of many industrial systems. Should a system exhibit bilinear characteristics of the form illustrated in Fig. 1, then it is pertinent to consider adopting a bilinear systems modelling and control approach.

3. Design and implementation of the BPID controller

Another advantage of adopting a bilinear model, as opposed to a linear model, for a system exhibiting nonlinear characteristics of the form outlined above is that the order of the resulting bilinear model is significantly lower than a corresponding linear model. When considering industrial furnace applications it has been found (Goodhart, Burnham, & James, 1994; Dunoyer, Burnham, Heeley, & Marcroft, 1998b) that a bilinear model of the form:

$$\dot{y}(t) = -\alpha_0 y(t) + \beta_0 u(t) + \rho_0 u(t) y(t) \tag{6}$$

is sufficient. In this work this structure is used as the basis for modelling individual zones in a multi-zone industrial furnace.

As the control algorithm is required to be implemented in discrete time, the SISO bilinear system, Eq. (6) can be expressed in discrete time form as

$$y(t) = -ay(t-1) + bu(t-1) + \eta u(t-1)y(t-1), \tag{7}$$

where the parameters a, b and η are defined (Dunoyer, Balmer, Burnham, & James, 1998a) by

$$a = -\mathrm{e}^{-\alpha_0 Th}, \quad b = \frac{\beta_0}{\alpha_0}[1 - \mathrm{e}^{-\alpha_0 Th}],$$
$$\eta = \frac{\rho_0}{\alpha_0}[1 - \mathrm{e}^{-\alpha_0 Th}],$$
$$\text{with } h = -\frac{1}{\alpha_0 T}\ln\left[\frac{\alpha_0 \mathrm{e}^{-\alpha_0 Th} - \rho_0 u}{\alpha_0 - \rho_0}\right] \text{ or}$$
$$h \approx 1 + \frac{\rho_0 Tu}{2}. \tag{8}$$

The BPID control system essentially compromises a standard linear PID controller cascaded with a bilinear compensator. The concept of the approach may be visualised as illustrated schematically in Fig. 2. Essentially, by designing a bilinear compensator which results in linearising a given nonlinear plant, a tuned BPID control system is automatically achieved. With reference to Eq. (7), the objective is to find an expression for the value of $u(t)$ which results in a linearisation, i.e. the bilinear term is nullified. Making use of the backward shift operator and rearranging Eq. (7) into a 'quasi-linear' transfer function form (in which $u(t-1)$ is assumed to be constant between samples) it may be deduced that the bilinear compensator takes the form $1/(1 + (\eta/b)y(t-1))$, where η/b is the tuning parameter, denoted as K_b. Use of such a bilinear compensator leads to an overall linearised system.

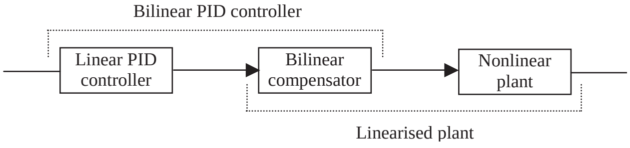

Fig. 2. Basic concept of the bilinear compensator.

Whilst the standard PID controller gives desired performance when the controller operates at the point of tuning, the inclusion of the bilinear compensator enables the resulting controller to maintain this level of control over a wider range of operation about the point of tuning. To ensure that the performance of the PID controller is maintained at the point of tuning, a numerator term is included in the compensator to give a null compensation at this point. The bilinear compensator is therefore given by

$$\frac{1 + K_b r_0}{1 + K_b y(t-1)}, \tag{9}$$

where r_0 is the set point at which the PID controller was tuned. The BPID results in a four-term controller (see Fig. 3), which comprises the existing three-term PID controller and an additional bilinear term (Minihan, Burnham, Haas, Hibberd, & Shields, 1999).

The existing PID controller terms may be tuned by making use of standard commercial packages or experienced plant engineers. The bilinear term may be obtained using methods similar to those developed for autotuning existing 3-term controllers. The method preferred here is based on a least-squares fit to measured plant data. Hence, with a minimum knowledge of the plant, use of the BPID can provide improved overall plant performance (Minihan, 2001). In practice, it is found that even an approximate value of the bilinear term gives rise to improvements.

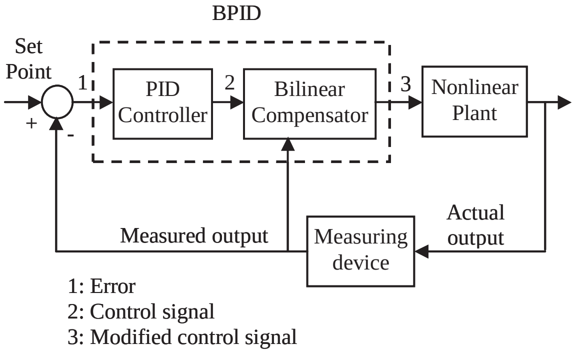

Fig. 3. Illustrating the configuration of the BPID controller implemented with the process.

Whereas linearisation holds at a point, bilinearisation holds over a predefined range; this is considered to be highly beneficial when dealing with practical industrial systems (Dunoyer, 1996).

4. The furnace process

4.1. Description of the annealing line

The process, under consideration, is a continuously operated gas-fired industrial furnace, based at AvestaPolarit Ltd, Sheffield, UK. Trials have been carried out on the individual zones of the furnace, which forms part of an annealing line, used to process stainless steel strip. The annealing line, illustrated in Fig. 4, is composed of three physically separated furnaces, each of which is subdivided into three zones.

The single-side fired furnaces have been designed to allow high rates of gas circulation, and each zone has its own temperature set point and control loop. To reach the required temperature, each of the zones 1–7 use three pairs of regenerative burners in which each pair switches over a ninety second firing cycle, i.e. one (burner) fires whilst the second (regenerator) exhausts waste combustion gases, and stores the wasted heat in a ceramic chamber.

Regenerative burner technology allows a gas saving of 30–50% (Disdell, 1995) by making use of the stored heat to pre-heat the air/gas mixture after the changeover. Although this technique is efficient, it does give rise to an undesirable oscillation in zone temperature. The burners, positioned within no. 2 furnace, as illustrated in Fig. 5, are fired sequentially, i.e. Pair 1, Pair 2, …, Pair 9 at a 10 s interval, giving rise to a 180 s firing cycle for a given pair.

The temperatures inside the furnace are measured by a thermocouple positioned at the middle of each zone and sampled every 125 ms. This measuring device is influenced by the firing of the central burners (e.g. Pair 6 in Zone 4) so that the measured temperature exhibits an oscillation having a 180 s period. Another oscillatory mode of 30 s is present as a consequence of the firing of the adjacent burner pairs (i.e. Pair 3 and Pair 9), as the switch over of the pair of burners appears every 30 s in each zone.

Zones 8 and 9 make use of recuperative burners, which are less efficient and require the use of more gas to maintain the set point temperature. Control of each furnace is carried out by an Eurotherm T640 control system, with each zone having its own control loop. The temperature is effected by regulating the speed of an air fan, and subsequently the gas flow, to obtain the desired air/gas ratio.

The photograph in Fig. 6 presents Furnace No. 1 of the annealing line based at AvestaPolarit.

4.2. Simulation results for one zone of the furnace

A Simulink model of the AvestaPolarit annealing line was developed to replicate the dynamic characteristics of the plant. Whereas the actual multi-zone furnace systems are highly complex processes, the model used here is restricted to replicate the variation of the temperature of a single zone of the furnace, which corresponds to one control loop. Two strategies were

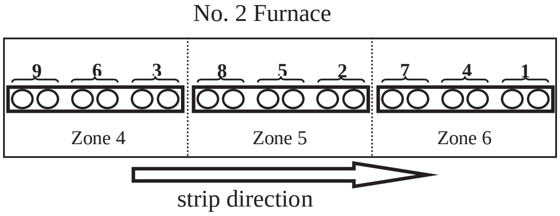

Fig. 5. Schematic of No. 2 Furnace illustrating arrangement of burners.

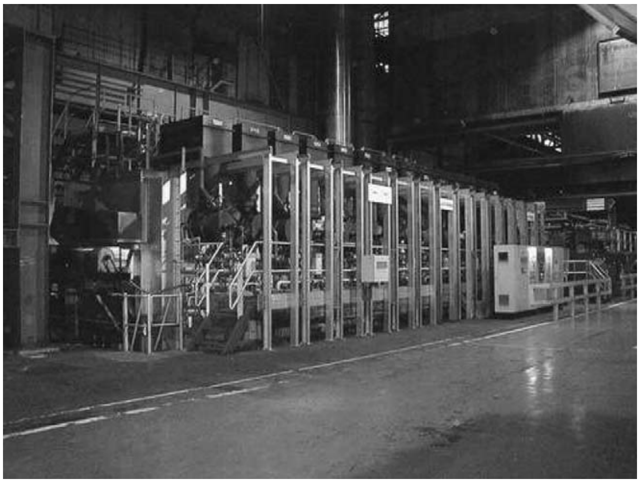

Fig. 6. Photograph of furnace No. 1 of the annealing line based at AvestaPolarit, Sheffield.

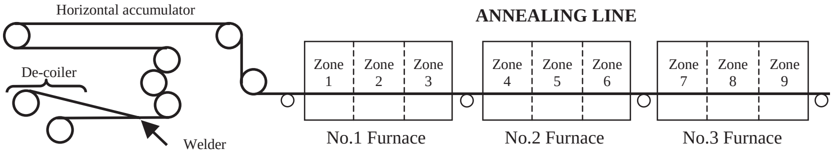

Fig. 4. Schematic of the AvestaPolarit annealing line.

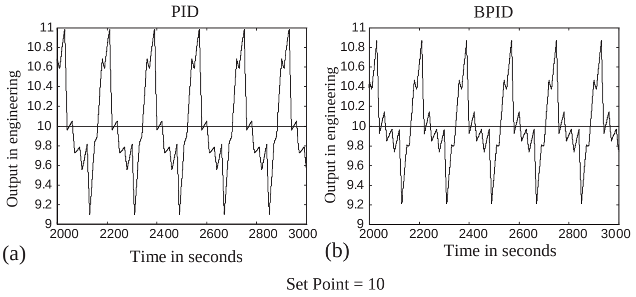

Fig. 7. Simulation results.

compared, the standard PID control strategy previously used within the T640 and the BPID control strategy where a bilinear compensator is combined with the PID controller, as illustrated in Fig. 3. The BPID controller used the existing PID terms with the additional bilinear term being tuned using methods outlined in Section 3.

Fig. 7 illustrates the temperature output of the furnace zone with both strategies. The output in engineering units corresponds to a ratio of the actual zone temperature, i.e. 10 units corresponds to 1000°C. It can be observed that the BPID controller reduces the variation in temperature and a more detailed observation indicates that the degree of asymmetry is also reduced. This indicates that the BPID controller tends to linearise the overall system around a given set point, hence leading to an increased consistency over a range. A key question prior to the trials was: will a similar degree of consistency be achieved when dealing with plant and process uncertainties in a production environment?

5. On-site trials

The control of furnace temperature using the current PID controllers (Eurotherm T640) is already considered to be good, but the variability of product, high throughput rates and high material costs mean that even a relatively small improvement in controller performance could give rise to potential substantial benefits. Consequently corrections to the incremental controller output when controlling temperature at a given operating point should lead to a corresponding improvement in overall consistency, resulting in reduced temperature swings and improved product quality. Additionally, a reduced temperature swing within the furnace could also have a knock-on effect leading to increased throughput and/or reduced temperature set point for a given product type; both of which should result in a further potential increase in overall plant efficiency.

Comparative real-time trials over short periods have already been reported and these have consistently indicated the potential of the bilinear controller. During these short periods, it was observed that the average temperature deviation from the set point and the mean level of the control signal, indicating fuel flow, were reduced, when use was made of BPID control strategy. An improvement on the standard deviation of the control signal was also observed during these short periods. To provide quantifiable evidence regarding the performance of the BPID controller, a study on the usage of gas was also carried out. The reported improvements had witnessed energy savings in the region of 3–4% (Martineau et al., 2001a, b; Martineau et al., 2002), as well as marginal improvements in set point tracking. However, differences in performance appeared when different steel products were encountered. Consequently it was considered vital that real-time trials over prolonged periods of normal plant operation involving a range of steel products would be the only 'sure way' of providing the necessary substantive evidence.

For the purpose of the trial, data was initially recorded under normal PID control. This was carried out in October and November 2001. During the period covered, the product mix was considered to be typical. For the purpose of providing a sound basis for statistical analysis/comparison, the trial included recorded measurements of fuel flow for 750 coils. Each measurement corresponded to a period where a given steel strip is processed through the furnace, typically between 20 and 30 min depending on the speed of the line, which also relates to mass flow consistency of product. The trial involving BPID control took place during December 2001 and January/February 2002, with over 1000 measurements of fuel flow being recorded. Then, with knowledge of line speed and details of product dimensions, this flow measurement data was converted into MJ/tonne and tonnes/h, noting that the product of these two quantities is power in units of MJ/h. Due to the commercial nature of the work, the average power consumption values have been normalised (Martineau, 2002).

The product range is made up from four product types, denoted T1, T2, T3 and T4, and three basic widths, small W1, medium W2 and large W3. Width is selected as an important parameter because this is observed to affect the circulation of gases within the furnace, hence affects the system behaviour; but more significantly it has an effect on the thermal load placed upon the furnace. As a consequence, the product mix consists of 12 different types, where each product type has a different emissivity, quality and characteristics (width and gauge). This would lead to different energy consumption for each group and potentially different savings depending on individual product characteristics.

It is instructive at this point to examine the categories of product firstly on a width-by-width basis and secondly on a basis of product types. In each case the

PID and BPID are compared in pairs. In the case of the three widths (see Fig. 8), it is clear that the BPID consistently uses less power. Perhaps more importantly, it is observed that the greater the width the greater the improvement, thus reinforcing the argument of consistency with the BPID expending less power for the higher throughput rates.

When considering the product types (see Fig. 9), the BPID produced the more favourable results with the exception of product type T4. A close examination of the data revealed that there was insufficient data to draw meaningful comparisons for product type T4 (for all three widths). Similarly, it was considered that there was insufficient data for product types W1-T2 and W3-T2. Consequently, of the 12 product types, only seven were considered to be representative of sufficiently large sample sizes and, therefore, suitable for the purpose of statistical analyses. The product types not selected represent only 3% of the total recorded data. The proportion of products for PID and BPID trials are presented in the pie charts of Figs. 10 and 11. The data not selected are referred to as ‘others’. From the pie charts, it can be observed that two products, W1-T1 and W2-T1, represent about 50% of the products encountered from the PID and BPID trials, whilst two other product types represent together less than 10%, i.e. W1-T3 and W3-T3.

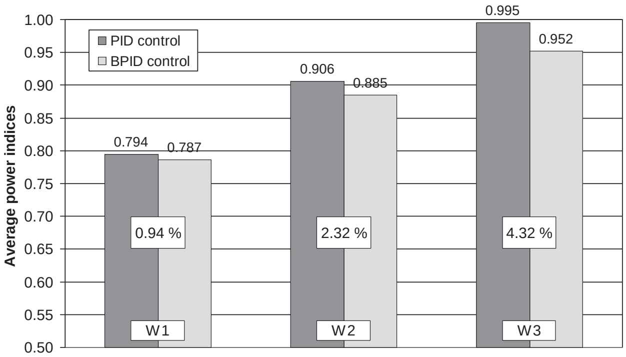

Fig. 8. Average power indices for different widths of steel. The improvement for each case is expressed as a percentage.

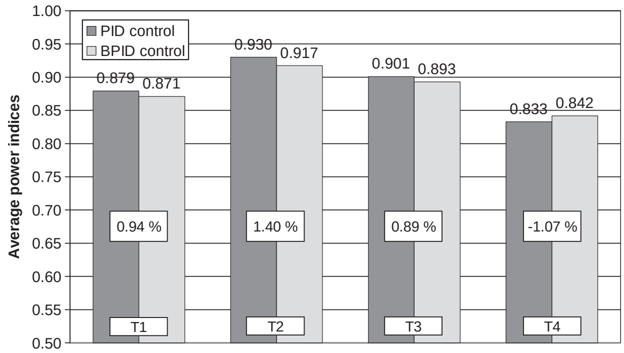

Fig. 9. Average power indices for different types of steel. The improvement for each case is expressed as a percentage.

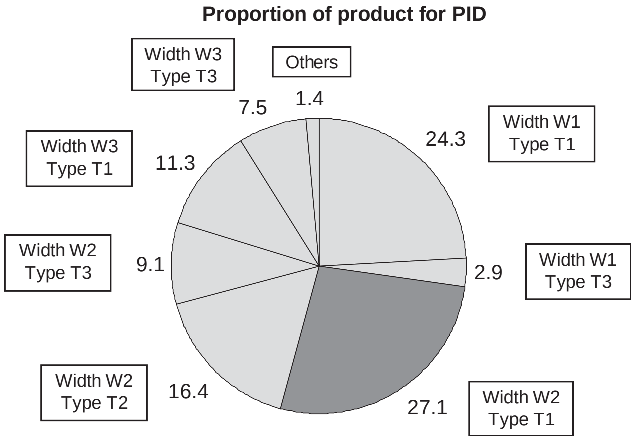

Fig. 10. Proportion of product for PID trials expressed as a percentage.

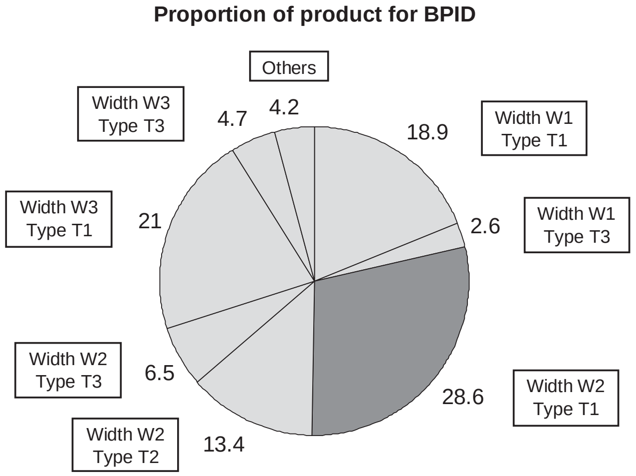

Fig. 11. Proportion of product for BPID trials expressed as a percentage.

Having segmented the data into the various product types to be included in the analysis, it is instructive to compare the average power consumption on a product-by-product basis for PID and BPID. A comparison of results in terms of average power consumption per product is presented in Fig. 12. It is interesting to note that for 5 out of the 7 product types considered the BPID gives rise to reduced power consumption. The two cases where an increase in power with the BPID control strategy is observed correspond to the product types W1-T3 and W2-T3. This particular product type has a high level of emissivity. It also corresponds to a small proportion of the product mix (about 10%), and consistency could be considered to outweigh energy considerations in this case.

The standard deviation of power consumption for the seven selected product types is provided in Fig. 13. It is noted that for 5 of the 7 selected product types,

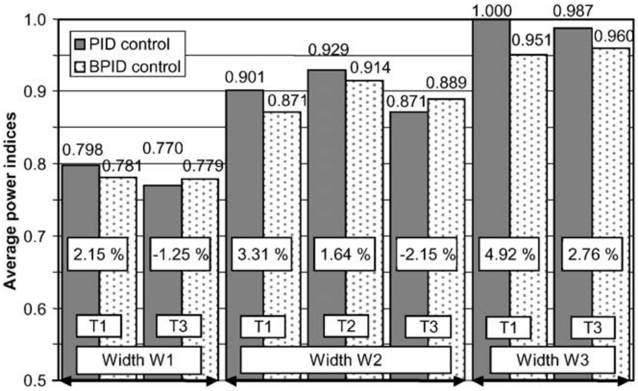

Fig. 12. Average power indices for PID and BPID control strategy. The improvement for each case is expressed as a percentage.

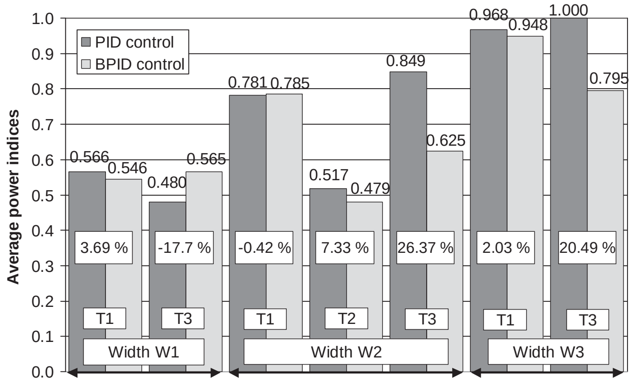

Fig. 13. Standard deviation indices for PID and BPID control strategy. The improvement for each case is expressed as a percentage.

the standard deviation is lower when using BPID. This reduced standard deviation reinforces the claim to improved consistency when use is made of the BPID controller. An examination of Figs. 12 and 13 reveals that the worse case (in terms of the deviation towards higher power consumption) for BPID is not as extreme as for PID even in cases where the difference in the average power consumption is small.

It is clear that the largest energy savings on one particular product was found to be almost 5%, with the second largest saving being over 3%. The latter product forms almost 30% of a typical production schedule. Other savings were of the order 1.6–2.8%. The variability of the data on a product-by-product basis was also analysed, and it was found that, for five out of the seven products, use of the BPID gives rise to a reduction in the standard deviation. This indicates that the extreme case for BPID in terms of energy expenditure will be lower than the PID for the same average value. This finding supports the claim for improved consistency of performance, i.e. the response of the controller to variations in load demand is more consistent over a range.

6. Conclusion

This paper has presented the benefits of combining a bilinear compensator and an existing PID control strategy. The resulting BPID controller has been applied to a continuously operated multi-zone furnace at AvestaPolarit Ltd, Sheffield, UK. Trials have confirmed quantifiable improvements in terms of reduced gas usage when applying the BPID controller compared to the standard PID controller. An average energy saving of some 3–4% has been found. Since the completion of the trials in February 2002, the BPID controller has been operating continuously on the furnace.

This successful application demonstrates the potential benefit of the BPID by reducing the gas consumption, hence costs. The BPID may be regarded as a natural extension to the standard PID controller. The BPID, which can be easily implemented and tuned, provides adaptivity, through an assumed nonlinear controller model structure, with robustness being provided by the existing three-term PID controller. In practice, the compensator may be cascaded with an existing PID as a retrofit device, or integrated within a standard PID to form a BPID scheme.

Acknowledgements

The authors wish to acknowledge the support from the Energy Technology Support Unit (ETSU) in pursing this work, as well as colleagues at AvestaPolarit Ltd, Sheffield, UK and Eurotherm Ltd, Worthing, UK.

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