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## The Performance of Observer-based Residuals for Detecting Intermittent Faults: the Limitations

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### Abstract

In this paper a broad nonlinear system is considered. Attention is focused upon both performance of a high-gain observer-based residual and the investigation of residual effectiveness for detecting faults in actuators/components. Residual performances for different fault positions and various system complexities are compared. Both qualitative and quantitative evidence for selected fault positions indicated the performance and the effectiveness of the residuals decrease by ascending the system complexity. The poor performance of residuals in the more complex system may cause **No Fault Found (NFF)**. The methods may be extended to the more general class of nonlinear systems and different observers. Efficiency of the proposed approach is demonstrated through the intermittent failure case in a vehicle suspension system.

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**Keywords:** Nonlinear control systems; Fault detection; Observer-based residual, Mass-Spring-Damper system; Intermittent faults; No Fault Found; Vehicle suspension system.

### 1. Introduction

Faults are generally categorized according to whether they have developed slowly during the operation of a system usually characteristic of gradual component wear (incipient fault); arisen suddenly like a step change as a result of a sudden breakage (abrupt faults); or accrued in discrete intervals attributed to component degradation or unknown system interactions (intermittent faults). Intermittent faults can manifest in any system, mechanical or electronic, in an unpredictable manner, and if left unattended over time they may evolve into serious and persistent faults. The assumed unpredictability of an intermittent fault means that it cannot be easily predicted, detected nor is it necessarily repeatable during maintenance testing, thus faults of this nature raise many concerns in the realm of through-life engineering of products [1]. However, an intermittent fault, which is missed during standardised maintenance testing, by its very definition will reoccur at some time in the future. This therefore poses an ever increasing challenge in the maintenance of electronic, mechanical and hydraulic equipment. A substantial portion of malfunctions attributed to intermittent faults as tested healthy and may be categorized as No Fault Found (NFF) [2]. When the fault is not intermittent and the fault symptoms are

consistent (hard fault), it is not difficult to isolate and repair. However, a fault that persists for a very short duration and manifests itself intermittently and only during a particular set of operational stresses can be extremely difficult to identify and isolate [3]. In general, intermittent faults typically tend to worsen with time, until eventually becoming substantial enough to be detected with conventional test equipment [4]. Hence, developing the capability for early detection and isolation of the intermittent fault can help to avoid major system breakdowns [5]. Faults can occur in actuators, process components or the sensors. Sensor faults are of particular importance, as they could affect the system performance, or result in a catastrophic mechanical failure. Model-based fault detection schemes can be powerful tools in determining sensor and actuator faults. The concept is to compare the behaviour of an actual process to that of a nominal fault-free model of the process driven by the same input signals. Model-based approaches are more powerful than data-driven signal-processing-based approaches [6] because they rely much more upon physical knowledge of the process and its interactions whereas signal processing techniques rely on large quantities of data to be recorded that may not be practical.

A model-based fault detection scheme consists of two main stages: residual generation and residual evaluation. The objective of designing residuals is to define a signal that can be

compared to the appropriate measurements and estimations and then evaluated for possible presence of faults [7]. Early research on fault diagnosis, based on software and hardware, have been given. The robust observer-based method of generating residuals based on software is well-known. Such residuals have been designed based on adaptive observers [8], sliding-mode observers [8], bilinear observers [9], quasi-linear observers [10], neural-network-based adaptive observers [11], nonlinear high-gain observers [12], nonlinear canonical form observers [13] and nonlinear observer based on the existence of linearizing transformations [14]. Robust observer based-residuals, based on polynomial models, have been found effective, specially for hydraulic systems [15, 16] and the residuals generated by high-gain observers [18], have wide applicability. However, a wide study of the effectiveness of the residuals for higher dimensional nonlinear systems, with few output measurement has not been attempted. Limited evidence from a study of multi-tanks hydraulic benchmark system [19], food chain system and pipeline system [20] show that residual performances degrades significantly when system complexity increase. This issue and its relation to NFF with more details, will be addressed for the vehicle suspension systems in this paper.

The main object of the paper is to examine the effectiveness of a well established high-gain observer-based residual to detect the intermittent faults. In addition it is shown that the poor performance of residuals for more complex system may cause NFF events.

This paper is constructed as follows: Section 2 gives a system description, derives several models and maps and considers equilibrium points and control. Design of the observer-based residual for the considered system is addressed in Section 3 while a numerical example is provided in Section 4 to investigate the limits to fault detection as system complexity increases. Conclusions are presented in Section 5.

### Nomenclature

|          |                           |
|----------|---------------------------|
| $C_n$    | restoring force of damper |
| $e$      | error                     |
| $f_i$    | intermittent fault        |
| $f_s$    | sensor fault              |
| $F_n$    | restoring force of spring |
| $g$      | gravity                   |
| $g_s$    | nonlinearity              |
| $c$      | damper constant           |
| $k$      | spring constant           |
| $L$      | length of spring          |
| $m$      | mass                      |
| $n$      | number of masses          |
| $N$      | dimension of the system   |
| $n_c$    | choice of output          |
| $p_n$    | fault position            |
| $r_n$    | displacement              |
| $r_s$    | residual                  |
| $u_n$    | applied control           |
| $y$      | output                    |
| $\eta_y$ | additive offset           |

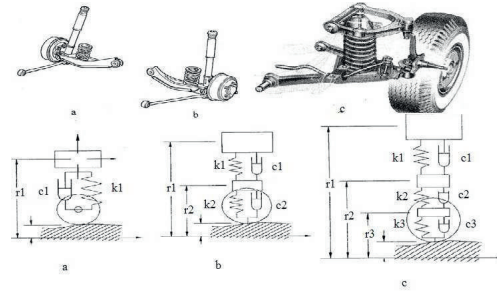


Fig. 1: The model vehicle suspension system.

## 2. System Description

Consider the class of nonlinear systems defined by the state-space form:

$$\begin{aligned}\dot{x}(t) &= h_x(x, u, g_s, f_i) \\ y(t) &= h_y(x, f_s)\end{aligned}\quad (1)$$

If the nonlinear function  $h_x(x, u, g_s, f_i)$  is differentiable with respect to the state  $x(t)$ , then this class of the system may be expressed in terms of a linear forced part, and nonlinear state dependent controlled part [21] and [22]:

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t) + Sg_s(x, u, t) + K_i f_i(t) \\ y(t) &= Cx(t) + K_{ss} f_s(t)\end{aligned}\quad (2)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$  and  $y(t) \in \mathbb{R}^p$  represent the state, input and output vectors respectively.  $A \in \mathbb{R}^{n \times n}$ ,  $B \in \mathbb{R}^{n \times m}$ ,  $C \in \mathbb{R}^{p \times n}$ ,  $S \in \mathbb{R}^{n \times s}$ ,  $K_i \in \mathbb{R}^{n \times r}$  and  $K_{ss} \in \mathbb{R}^{p \times i}$  are known matrices,  $f_i$  and  $f_s$  present the intermittent and sensor faults respectively. The function  $g_s(x, u, t) \in \mathbb{R}^s$  represents the known nonlinearity function.

To illustrate the application of the results obtained in sections 2 – 4, consider the dynamic characteristics of a model vehicle suspension system treating it as a Mass-Spring-Damper (M-S-D) system shown in Figure 1 and 2 where  $n$  masses, springs and dampers are connected together in series [23] and [24].

More thorough analysis of a full suspension system are quite complex involving all four tire/suspension systems acting independently. The quarter-car suspension model can be considered in the three levels of complexity shown in Figure 1. The one-degree of freedom model shown in Figure 1a considers displacement  $r_1$  of the sprung mass  $m_1$  of the vehicle and the primary suspension stiffness  $k_1$  and damping  $c_1$  only. Here the unsprung mass (mass of the wheels and other components such as lower control arms) and the mass of the tires are not considered. The two degree of freedom model shown in Figure 1b accounts for the dynamics of the unsprung mass as well and introduces a second equation of motion and degree of freedom for the displacement  $r_2$  of the unsprung mass  $m_2$ , springs and dampers with  $k_2$  and  $c_2$ . In this model, the tires are massless. A three-degree of freedom model is shown in Figure 1c where the dynamics of the tires are added to the analysis by treating them as a mass spring damper as well (see Figure 2), [25], and [26].

A mass-spring-damper system is usually modeled by a set of differential equations. The system comprises of a finite number

of masses, springs and dampers on a line. In fact it is assumed that  $n$  masses, springs and dampers are connected together serially. The model that will be developed here could be extended so that the user is able to select any number of springs, dampers and masses to connect together to build the final system.

The dynamic of the  $n$ -th mass is given by

$$\ddot{r}_n = (m_n)^{-1} \left[ -c_n(\dot{r}_n - \dot{r}_{n-1}) - k_n(r_n - r_{n-1} - L_n) + c_{n+1}(\dot{r}_{n+1} - \dot{r}_n) + k_{n+1}(r_{n+1} - r_n - L_{n+1}) + g + u_n + g_s + f_i \right] \quad (3)$$

where  $m_n$  represents the mass of  $n^{th}$  mass,  $r_n$  represents the displacement from a reference position of the  $n^{th}$  mass,  $c_n$  represents the restoring force of  $n^{th}$  damper,  $k_n$  represents the stiffness of  $n^{th}$  spring,  $L_n$  represents the length of  $n^{th}$  spring,  $g$  represents the gravity,  $u_n$  is the control applied on the  $n^{th}$  mass,  $g_s$  is the nonlinearity and  $f_i$  is the possible fault in the system. Hence the dynamic of the  $n^{th}$  mass with  $n$  degrees of freedom may be rewritten in the following form

$$m_n \ddot{r}_n = F_{n+1} - C_{n+1} + F_n - C_n + g + u_n + g_s + f_i \quad (4)$$

where  $F_n$  represents the restoring force of the  $n^{th}$  spring and  $C_n$  represents the restoring force of the  $n^{th}$  damper.

For relatively small displacements, restoring forces in (4), can be considered as linear function of displacements

$$\begin{aligned} F_n &= k_n(r_n - r_{n-1} - L_n) \\ C_n &= c_n(\dot{r}_n - \dot{r}_{n-1}). \end{aligned} \quad (5)$$

Also a situation in which the spring and damper restoring forces depend nonlinearly on displacement, hardening spring, where, beyond a certain displacement, large force increments are obtained for small displacement increments, case (5), can be rewritten as:

$$\begin{aligned} F_n &= k_{n1}(x_n - x_{n-1} - L_n) + k_{n2}(x_n - x_{n-1} - L_n)^3 \\ C_n &= c_{n1}(\dot{x}_n - \dot{x}_{n-1}) + c_{n2}(\dot{x}_n - \dot{x}_{n-1})^3 \end{aligned} \quad (6)$$

see [27].

To obtain the state-space equation of the M-S-D system define

$$X_n = r_n - L_n \quad (7)$$

from Figure (2b), where  $X_n$  is the amount of the stretch of the corresponding spring. Then the displacement  $r$  may be represented as

$$r = WX + L \quad (8)$$

where  $r = [r_1 \dots r_n]^T$ ,  $X = [X_1 \dots X_n]^T$ ,  $L = [L_1 \dots L_n]^T$  are extended vectors and

$$W = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{pmatrix}. \quad (9)$$

Hence, the the system equation in  $W$ -form is presented as

$$W\ddot{x} = M^{-1}(-CW\dot{x}) - M^{-1}(K(WX + L)) + M^{-1}g_s + M^{-1}g + M^{-1}\bar{U} + M^{-1}\bar{f}_i \quad (10)$$

with  $\bar{g}_s = [g_{s1} \dots g_{sn}]^T$ ,  $\bar{U} = [u_1 \dots u_n]^T$ ,  $\bar{f}_i = [f_{i1} \dots f_{in}]^T$  and

$$M = \begin{pmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & m_n \end{pmatrix}.$$

Now define  $w = W(X - \bar{X})$ , where  $\bar{X}$  is the equilibrium point. Then the system (10) in  $w$ -form is

$$\ddot{w} = M^{-1}(-C\dot{w}) + M^{-1}(-K(w + W\bar{X})) + M^{-1}\bar{g}_s + M^{-1}g + M^{-1}\bar{U} + M^{-1}\bar{f}_i. \quad (11)$$

Note that at equilibrium ( $\dot{w} = 0$ ,  $w = W\bar{X}$ ,  $\dot{w} = 0$  and  $\ddot{w} = 0$ ), (11) will find the following form

$$M^{-1}KW\bar{X} = M^{-1}g. \quad (12)$$

Thus the system equation (11) can be rewritten as

$$\ddot{w} = -M^{-1}C\dot{w} - M^{-1}Kw + M^{-1}\bar{g}_s + M^{-1}\bar{U} + M^{-1}\bar{f}_i \quad (13)$$

or equivalently, in terms of its state space representation

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & I_n \\ -M^{-1}K & -M^{-1}C \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}\bar{g}_s \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}\bar{f}_i \end{pmatrix} + \begin{pmatrix} 0 \\ M^{-1}\bar{U} \end{pmatrix} \quad (14)$$

where  $x_1 = w$  and  $x_2 = \dot{w}$ . The system output is  $y = Cx + \eta_y$ , where  $C \in \mathbb{R}^{n \times n}$  and  $\eta_y$  is an additive offset (output error/sensor fault) on each output.

When the system is consist of two masses, springs and dampers, then the system equation (14) without fault terms is presented as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{1}{m_1}[-k_1x_1 - c_1x_2 + k_2x_3 + c_2x_4 + u_1 + g_{s1}(x_1, x_2, x_3, x_4)] \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{1}{m_2}[k_1x_1 + c_1x_2 - 2k_2x_3 - 2c_2x_4 - u_1 + u_2 - g_{s1}(x_1, x_2, x_3, x_4) + g_{s2}(x_1, x_2, x_3, x_4)]. \end{aligned} \quad (15)$$

### 3. Fault Detection Filter

Not all the states  $x(t)$  can be directly measured (as is commonly the case), therefore we can design an observer,  $\hat{y}(t)$  to estimate them, while measuring only the output  $y(t)$ . The observer is basically a model of the plant; it has the same input and follows a similar differential equation. An extra term compares the actual measured output  $y(t)$  to the estimated output of the observer  $\hat{y}(t)$ ; minimising this error will cause the estimated states  $\hat{x}(t)$  to tend towards the values of the actual real-system states  $x(t)$ . It is conventional to write the combined equations for the system plus observer using the original state  $x(t)$  plus the error state [19],

$$e(t) = x(t) - \hat{x}(t). \quad (16)$$

In general the fault detection system consists of two parts, 1) residual generation, 2) residual evaluation [28].

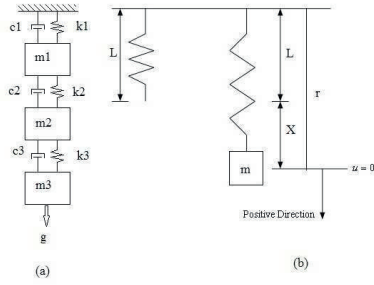


Fig. 2: The mass-spring-damper system.

### 3.1. Residual Generation

While a suitable observer is chosen for every case, if the error system stability is satisfied, then the following scalar observer-based residual can be generated for each output to detect the intermittent faults

$$r_s(t) = (y(t) - \hat{y}(t)) = \zeta C e(t) + \zeta K_{ss} f_s(t) \quad (17)$$

where  $\zeta \in \mathbb{R}^{n \times p}$ , is a suitable weighting matrix to be designed. The problem can be stated as finding  $\zeta$ , such that the following aims are achieved [29]:

- The effect of unknown input and disturbance signals on the residual signal are as small as possible while the effect of fault signal is as large as possible.
- The effect of parametric uncertainties on residual signal are as small as possible.
- The fault detection system is robust stable in the presence of exogenous signals and uncertainties.

The object is to show that the residuals are differing from zero when faults have occurred; however, the residual tends to zero in "no fault" situation.

### 3.2. Residual Evaluation

A common choice of evaluation signal is the 2-norm:

$$r_{s_{eval}} = \|r_s\|_2 \triangleq \sqrt{\int_0^{\infty} |r_s(\tau)|^2 d\tau} \quad (18)$$

Since the evaluation function (18) can not be realised exactly, because the value of  $\|r_s\|_2$  is not known until  $t = \infty$ , and it is reasonable to assume that faults could be detected, if they occur over finite time interval. Therefore equation (18) could be modified to

$$r_{s_{eval}} = \|r_s(t)\|_2 \triangleq \sqrt{\int_0^t |r_s(\tau)|^2 d\tau} \quad (19)$$

where  $\tau$  is the time window and it is finite [27].

## 4. Simulation Outcomes

In this section the object is to investigate several M-S-D systems with the aim of showing a system dependent phenomenon

which limits the effectiveness of the observer-based residuals.

### 4.1. Intermittent fault

Collapsing suspension due to coil spring failure seems to be a growing problem - caused by a combination of recent harsh winter conditions and weight-saving designs. A plastic coating is applied to coil springs when they are made to reduce the risk of corrosion. Over time, contact between coils as the spring is repeatedly compressed in service can cause damage to this coating. Most often coil spring failure seems to be caused by corrosion, accelerated by salt applied to the roads in winter. In other hand Electrolytic action between the salt solution, formed by road salting, and the iron in the spring generates free hydrogen atoms which enter the steel and can cause microscopic cracking. Cracks propagate and combine, ultimately leading to failure of the spring, (www.theaa.com). Cracks and corrosion both can be classified as Intermittent faults, which will start with small failure in a short time but will get stronger and longer until it ends up with complete spring failure eventually.

Assume that at each failure, the length of spring will change unexpectedly. In other word a fault in position  $i$  is a change in the length of the  $i$ -th spring, so that  $L = (L_0 + f_i L_0)$  in position  $i$ , and  $L = L_0$  in all other sections of the system, where  $L_0$  is the initial length of the spring.

Also, it is defined that  $f_i(t)$  is a time varying of the form  $f_i(t) = dd_i y_{n_c}(t)$ , where  $dd_i$ , the maximum fault amplitudes, are constant and  $y_{n_c}$  is the designers's choice of output.

Hence the intermittent fault,  $f_i(t)$ , could be generated as combination of impulses at different amplitudes which will occurred in discrete intervals. We could model the fault as follows

$$f_i(t) = \begin{cases} 0 & \text{for } 0 \leq t < 55s \\ f_{i1} & \text{for } 55s \leq t < 60s \\ 0 & \text{for } 60s \leq t < 120s \\ f_{i2} & \text{for } 120s \leq t < 145s \\ 0 & \text{for } 145s \leq t < 190s \\ f_{i3} & \text{for } 190s \leq t < 260s \\ 0 & \text{for } 260s \leq t < 270s \\ f_{i4} & \text{for } 270s \leq t < 400s \end{cases} \quad (20)$$

where  $dd_1 = 0.0025$ ,  $dd_2 = 0.01$ ,  $dd_3 = 0.15$  and  $dd_4 = 0.25$  are constants,  $n_c = 1, \dots, n$  is the choice of output and  $t$  indicates the time.

### 4.2. Simulation Conditions

For numerical example a general  $N = 2n$  dimension M-S-D system is considered. This system has a maximum of  $n$  inputs and  $n$  outputs. There is no disturbances, and the effect of a single fault  $f_i(t)$  depends on the choice of  $K_i$  in (2). Here  $\eta_y(t)$  is the term defining the sensor fault  $f_s(t)$  of the form  $\eta_0 \sin(t)$  and the intermittent fault is of the form (20).

Note that the length of spring,  $L$ , is  $1m$ , the mass  $m$  is  $1Kg$  and  $g = 9.8 \frac{N}{m}$  is the gravity.

### 4.3. Residual effectiveness investigation

For each system by keeping as many factors as possible the same, such as input  $u$ , residual speed of response, residual

design parameters and observer design parameters, try to simulate the residual performance as system complexity or  $n$  increases.

To investigate the residual effectiveness for increasing  $n$  the following steps are performed:

1. An intermittent fault of the form (20) is applied, in the one of the springs,  $p_i$ , ( $1 \leq i \leq n$ ).
2. An additive offset (output error),  $\eta_y$ , is made on each output  $y_i$ , so that  $y_i(t) = x_i(t) + \eta_y$  where  $\eta_y = \eta_0 \sin(t)$ .
3. The residual of the form (17) with corresponding observer is designed.
4. The number  $\eta_0$ , bounding the output error, is varied until the effect of the fault  $f_i(t)$  on the residual, denoted by  $R_{f_{max}}$  in Table 1, is approximately equal to the effect of the noise,  $\eta_y$ , on the residual denoted by  $R_{\eta_y}$  in Table 1. This condition is denoted by  $\frac{\|R_{f_{max}}\|}{\|R_{\eta_y}\|} = 1$ . It indicates a limit on the error. Increasing  $\eta_0$  further means that, if  $\eta_y$  is present throughout the time frame, its effect on the residual (17) would mask the effect due to the fault  $f_i(t)$ . This condition may cause No Fault Found events, [19].
5. Finally the steps are repeated for different number of masses.

#### 4.4. Results of investigation

The numerical results for the system mentioned above are summarised in Table 1 and Figures 3 – 7.

For M-s-D system, Figures 3 – 7 show, some results in graphical form when implementing the residual with a fault  $f_i(t)$  is presented. For each case a nearly limited condition for  $\eta_0$  is chosen. Table 1 is driven from the data and compares some important numbers for each  $n$ . for some cases the residual  $R_{f_{max}}$  is so small and that it can not be distinguished from modeling error and control effects without filtering action.

Table 1:  $R_{f_{max}}$ ,  $R_{\eta_y}$  and  $\eta_0$  for fault  $f_i(t)$ , where  $n_c = 1$  and  $l = 1m$ .

| $n$ | $P_n$ | $R_{f_{max}}$          | $R_{\eta_y} = \frac{\eta_0}{R_{f_{max}}}$ | $\eta_0$ |
|-----|-------|------------------------|---|----------|
| 2   | 1     | $2.67 \times 10^{-4}$  | $0.149 \times 10^4$                       | 0.04     |
| 2   | 2     | $2.10 \times 10^{-5}$  | $5.86 \times 10^5$                        | 1.2      |
| 3   | 1     | $3.07 \times 10^{-4}$  | $0.048 \times 10^4$                       | 0.15     |
| 3   | 2     | $5.82 \times 10^{-5}$  | $0.154 \times 10^5$                       | 0.9      |
| 3   | 3     | $2.96 \times 10^{-5}$  | $0.405 \times 10^5$                       | 1.2      |
| 4   | 1     | $3.64 \times 10^{-4}$  | $0.68 \times 10^4$                        | 0.25     |
| 4   | 2     | $1.062 \times 10^{-4}$ | $0.68 \times 10^4$                        | 0.73     |
| 4   | 3     | $7.14 \times 10^{-5}$  | $0.14 \times 10^5$                        | 1        |
| 4   | 4     | $3.66 \times 10^{-5}$  | $0.34 \times 10^5$                        | 1.25     |
| 5   | 1     | $4.60 \times 10^{-4}$  | $0.65 \times 10^4$                        | 0.3      |
| 5   | 2     | $1.56 \times 10^{-4}$  | $0.38 \times 10^4$                        | 0.6      |
| 5   | 3     | $1.13 \times 10^{-4}$  | $0.70 \times 10^4$                        | 0.8      |
| 5   | 4     | $7.62 \times 10^{-5}$  | $0.14 \times 10^5$                        | 1.1      |
| 5   | 5     | $3.93 \times 10^{-5}$  | $0.33 \times 10^5$                        | 1.3      |
| 6   | 1     | $7.05 \times 10^{-4}$  | $0.049 \times 10^4$                       | 0.35     |
| 6   | 2     | $2.48 \times 10^{-4}$  | $0.24 \times 10^4$                        | 0.59     |
| 6   | 3     | $1.85 \times 10^{-4}$  | $0.43 \times 10^4$                        | 0.8      |
| 6   | 4     | $1.43 \times 10^{-4}$  | $0.67 \times 10^4$                        | 0.97     |
| 6   | 5     | $9.95 \times 10^{-5}$  | $0.11 \times 10^5$                        | 1.1      |
| 6   | 6     | $5.19 \times 10^{-5}$  | $0.25 \times 10^5$                        | 1.3      |

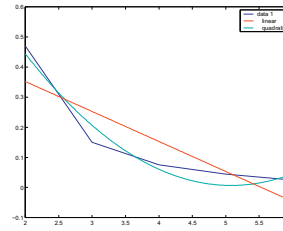


Fig. 3:  $\eta_0$  against  $n = 6$ ,  $n_c = n$  and  $P_n = 2$ .

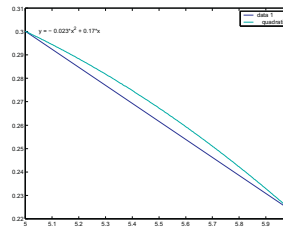


Fig. 4:  $\eta_0$  against  $n = 6$ ,  $n_c = n$  and  $P_n = 5$ .

Note that a straight line can be fitted to the data as a limiting line for fault detection ( $\log(\eta_0)$ ). A quadratic curve is more accurate but is poor when extrapolations are made for more complex systems, [19]. Figures 2 and 3 display  $\eta_0$ , as the data pair ( $\log(\eta_0), n$ ) is fitted for each system. These graphs clearly show that the effectiveness of the residuals decreases as  $n$  increases.

In Figure 2, consider that  $\eta_y$  is determined as a sensor fault, then if the magnitude of  $\log \eta_0$  is greater than 0.3 or (equivalently  $\eta_0 \geq 0.0003$ ), a fault in the second position is only detectable for a system with 2 masses or less. But in Figure 3, a fault in position 5 is detectable for a system with 5 masses or less. For the system with more masses, residual cannot detect the fault  $f_i(t)$ , and is masked by the effect of the sensor fault  $f_s(t)$ .

Figures 5 – 7 show that the intermittent faults detection may be delayed due to the effect of the sensor faults causing No Fault Found (NFF). Figures 5 and 7 show that the complete masking due to the effect of the sensor fault has been occurred and made the intermittent fault detection impossible. Figure 6 also show that a sensor fault has masked part of the intermittent fault.

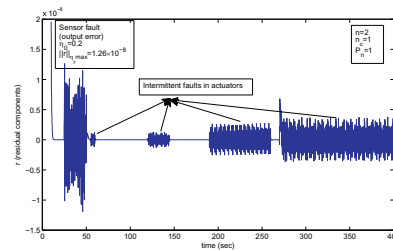


Fig. 5: Sensor faults masked the intermittent fault,  $n = 2$ ,  $n_c = 1$  and  $P_n = 1$  for  $\eta_0 = 0.1$ .



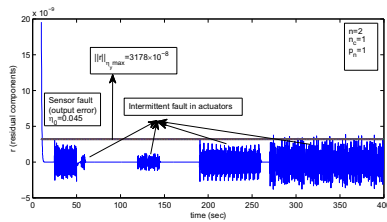


Fig. 6: Sensor faults masked part of the intermittent fault,  $n = 2$ ,  $n_c = 1$  and  $P_n = 1$  for  $\eta_0 = 0.045$ .

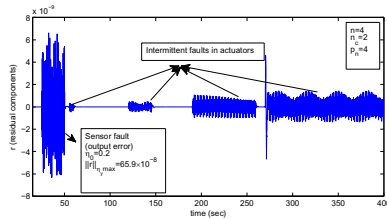


Fig. 7: Sensor faults masked the intermittent fault,  $n = 4$ ,  $n_c = 2$  and  $P_n = 4$  for  $\eta_0 = 0.2$ .

## 5. Conclusions

The development of state-space models and several transformations for use in satisfying the objectives concerning fault detection for a nonlinear M-S-D systems have been discussed. In particular, an observer-based residual has been proposed for the state space model.

An extensive investigation has been made into the effectiveness and performance of the residuals based on an observer design. For each system and a fixed controller, a specific form of sensor fault and a specific input structure, residual performance has been investigated for detecting the intermittent faults of different positions.

The effectiveness of a nonlinear observer-based residuals has been shown to be limited by the system complexity. The evidence has been shown that both residual effectiveness and quality of residual performance decreases as  $n$  increases. Residuals effectiveness can change with fault position, when  $n$  is fixed. However, the residual effectiveness is not only dependent on these two factors.

The simulations also show that the sensor fault may be able to mask the effect of the intermittent faults in the actuator/components, resulting a very late detection of the intermittent faults and NFF.

Future investigation is needed to compare the performance of different observers to detect the intermittent faults as one of the main root causes of NFF in the presence of the sensor faults and unknown inputs/disturbances.

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